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Costly External Finance, Reallocation, and Aggregate Productivity

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Abstract

Empirical studies document that resource reallocation across production units plays an important role in accounting for aggregate productivity growth in the U.S. manufacturing. Distortions in financial market could hinder the reallocation process and hence may adversely affect aggregate productivity growth. This paper studies the quantitative impact of costly external finance on aggregate productivity through resource reallocation across firms with idiosyncratic productivity shocks. A partial equilibrium model calibrated to the U.S. manufacturing data shows that costly external finance causes inefficient output reallocation from high productivity firms to low productivity firms and as a result leads to a 1 percent loss in aggregate TFP.

Key words: Costly external finance; Reallocation; Output weighted aggregate productivity

Introduction

This paper studies the quantitative impact of financial frictions on aggregate productivity in a setting with heterogenous firms. Recently there has been an increased interest in understanding the microeconomic dynamics of aggregate productivity growth. Corresponding to this literature is a surge of empirical work that exploits establishment-level data to explore the relationship between microeconomic productivity dynamics and aggregate productivity growth. Representative work includes Baily, Hulten and Campbell (1992), Bartelsman and Dhrymes (1998), Foster, Haltiwanger and Krizan (2001), and etc. A common theme of these studies is to decompose aggregate productivity growth into several parts to characterize the contributions of within plant productivity growth and reallocation, where the latter includes the contribution of reallocation among continuing establishments and the impact of entry and exit [1]. Despite that their findings vary with the specific data sets and decomposition methodologies used, a uniform finding in these studies is an important role of reallocation in accounting for aggregate productivity growth in the U.S. manufacturing. For instance, Foster, Haltiwanger and Krizan (2001) document that reallocation accounts for about half of overall multifactor productivity growth in U.S. manufacturing for the period 1977 to 1987.

Distortions in product, labor, credit market and policies can all slow aggregate productivity growth by hindering the reallocation process among heterogenous producers. However, works that explain and quantify these impacts remain little. Restuccia and Rogerson (2003) explores the quantitative impacts of policy distortions on aggregate productivity in a stationary equilibrium with heterogeneous plants. They show that policy distortions that create heterogeneity in the prices faced by individual producers lead to misallocation of resources across heterogeneous plants, and as a result can lead to sizable decreases in output and measured TFP. This paper is along the same line of Restuccia and Rogerson (2003), while the distortion we focus on is financial frictions.

Frictions in financial market constrain a firm's ability to finance profitable investment opportunities, and as a result, may lead to misallocation of resources among heterogenous producers and therefore hamper the growth of aggregate productivity. This paper formulates a simple partial equilibrium model to quantitatively assess this adverse effect. We abstract from modeling the microfoundations of financial frictions. Instead, financial market imperfections are summarized into a simple external finance cost function capturing the basic idea that external funds are more costly than internal funds if financial imperfections present (see Fazzari, Hubard and Peterson (1988)). Then the costly external finance function is incorporated into a standard capital accumulation problem of a firm with idiosyncratic productivity shocks.

Firm entry and exit are excluded from the baseline model. That is, we focus on the impact of external finance on reallocation among continuing establishments. This simplification is taken based on two considerations. First, the simplification is not a big deviation from the Compustat U.S. manufacturing data we use to calibrate the model, as the Compustat firms are relatively large and mature and do not exhibit a lot of entry and exit [2]. Like many studies on external finance (see Whited (1992, 2005), Gomes (2001), and etc.), we use Compustat data to calibrate the model since it provides detailed financial data such as firms' debt, equity issuance, interest expenses, and so on, which are crucial information for our model, while a richer data set for the U.S. manufacturing like LRD lacks such information. On average, the Compustat firms in our sample account for 82% of the total employment of U.S. manufacturing, so the data set provides a good representation of U.S. manufacturing. Second, reallocation among continuing establishments is itself an important contributor to productivity growth in U.S. manufacturing, as many empirical studies have documented. Baily, Hulten and Campbell (1992) find that reallocation of output shares to more productive plants within stayers accounts for nearly half of the TFP growth for the 1972-77 period and about one third of the rapid productivity growth in the 1980s. Foster, Haltiwanger and Krizan (2001) find that reallocation within continuing plants accounts for 26% of overall multifactor productivity growth in U.S. manufacturing for the 1977-87 period. Therefore, examining how external finance influences reallocation among continuing establishments alone is important for us to understand the impact of external finance on aggregate productivity dynamics. But we do recognize the significant role entry and exit may play as the other important component of reallocation, so in a later section we also give a discussion on how the results of the baseline model may change if considering firm entry and exit.

The model is simulated to compute the stationary properties of the industry, which are then compared with the properties of a stationary equilibrium with costless external finance. The results show that costly external finance leads to a reallocation of output shares from high-productivity firms to low productivity firms such that the output-weighted aggregate TFP is 1 percent less than it would be if external finance is costless. This is a significant loss considering that aggregate TFP growth for the U.S. manufacturing has averaged less than 1 percent a year in the 1970s and 1980s (according to the NBER Manufacturing Productivity Database, see Bartelsman and Gray (1996)). In a discussion we show that this quantitative result does not hinge on the partial equilibrium analysis adopted and considering firm entry and exit is unlikely to change its magnitude significantly. A comparative static analysis shows that the adverse impact of costly external finance on aggregate productivity increases with the return to scale, the persistence of productivity shocks, the variability of productivity across firms, and external finance costs. Since the Compustat firms in our data sample may exhibit less diversity in firm level productivities than a richer data set like LRD would suggest and may face lower external finance costs than an average manufacturing firm would face, our result may underestimate the quantitative impact of costly external finance on aggregate productivity growth for the U.S. manufacturing. A re-calibration is desirable when a richer data set incorporating finance and performance information becomes available [3]. However, a discussion suggests that a re-calibration is not expected to change this quantitative result dramatically.

This paper also gives interesting implications for the impact of financial market frictions on output growth, which has been an important research issue. A majority of this literature discusses this issue within the framework of neoclassical growth models that abstract from heterogeneity in production units. Not surprisingly, much of the literature has been concerned with understanding the role of aggregate accumulation and how aggregate accumulation is affected by financial market frictions. However, the empirical evidence shows that it is not only the level of factor accumulation that matters for aggregate output but how these factors are allocated across heterogenous production units. In our model, costly external finance decreases aggregate output through two channels. One is the traditional channel–capital accumulation. Costs associated with external finance increase the aggregate relative price of capital, and as a result decrease aggregate investment and lower aggregate capital accumulation. The other channel is through resource reallocation across heterogenous firms which results in a lower aggregate productivity. Our results show that with costly external finance, the reallocation leads to 0.3 percent loss in aggregate output, which is about a third of its impact on aggregate productivity. The small magnitude of this effect may suggest that for the U.S. economy, the traditional neoclassical model is not a bad framework for characterizing the long run consequences of financial frictions on aggregate output despite that it ignores the effect through resource reallocation across heterogenous production units.

The rest of the paper is organized as follows. To help formulate the model and understand the results, Section 1 reviews a popular measurement of aggregate productivity and a decomposition methodology of aggregate productivity growth widely adopted by the empirical studies. Section 2 describes the model. Section 3 details the calibration and simulation methods. Section 4 describes the results from the baseline model. Section 5 discusses the robustness of the main quantitative result to several variations of the analysis. And Section 6 concludes.

1 Measurement of Aggregate Productivity and Decomposition of Aggregate Productivity Growth

A lot of empirical studies use the sum of output (or employment) weighted firm/plant level TFP (or labor productivity) to measure the aggregate productivity of an industry. According to Baily, Hulten and Campbell (1992), the definition of aggregate productivity is as follows. Suppose the production function for plant i in period t is

$$Q_{it} = F(K_{it}, L_{it}, M_{it}),$$

where K, L and M are capital, labor and intermediate inputs, respectively. The plant level TFP is defined as

$$ln TFP_{it} = lnQ_{it} - \alpha_K lnK_{it} - \alpha_L lnL_{it} - \alpha_M lnM_{it},$$

where α_K, α_L and α_M are return to scale factors for capital, labor and intermediate inputs respectively. Then the level of productivity for the industry in year t is represented by the following index:

$$TFP_t = \sum_i \theta_{it} TFP_{it},$$

where θ_{it} is the output share of the ith plant in industry output.

The industry productivity growth is typically decomposed into several parts characterizing the relative contributions of the stayers, the entrants and the exits. According to Baily, Hulten and Campbell (1992), the change in industry productivity between $t-\tau$ and t can be decomposed into 3 parts.

$$\Delta TFP_t = \sum_{i\in C} \theta_{i,t-\tau} \Delta TFP_{it} + \sum_{i\in C} (\theta_{it} - \theta_{i,t-\tau}) TFP_{it} + \Big(\sum_{i\in N} \theta_{it} TFP_{it} - \sum_{i\in X} \theta_{i,t-\tau} TFP_{i,t-\tau} \Big). \quad (1)$$

The first term reflects the contribution of within plant productivity growth to aggregate productivity growth. The last two terms reflect the contribution of reallocation, where the second term reflects the contribution of reallocation of shares within continuing plants, and the last term reflects the contribution of net entry.

In this paper, we formulate a version of the growth model in which capital accumulation and production is carried out by heterogenous firms with idiosyncratic productivity shocks. We compare the steady state output-weighted aggregate productivity in two cases: external finance is costly and costless. In other words, we consider the change in aggregate productivity from $t-\tau$ to t, imagining that in period $t-\tau$ the industry is in the steady state with costless external finance, while in period t the industry is in the steady state with costly external finance. It is shown that in the decomposition equation (1), the first term is zero, since the two periods have exactly the same productivity distribution. The third term is also zero since firm entry and exit is excluded in the model. Therefore the change in aggregate productivity is completely characterized by the second term–reallocation of output shares across heterogenous firms due to costly external finance.

2 The Model

The analysis is of partial equilibrium type, in that it focuses on a single firm's dynamic capital accumulation problem. When assessing the aggregate implications of costly external finance, a large number of such firms are considered. A discussion in Section 5 shows that a more complex general equilibrium analysis would not change the main results.

The firm is infinitely lived. That is, we exclude firm entry and exit from the analysis. In a later section, we discuss how the results would change if considering firm entry and exit. In period t, the firm's operating cash flow is generated by a profit function given by [4]

$$\pi(k_t, z_t) = e^{z_t} k_t^{\alpha},\ \alpha < 1.$$

Here, k_t is the firm's capital stock at the beginning of period t. Capital depreciates at rate δ and must be decided one period in advance. The relative price of capital good is p. z_t is the firm's idiosyncratic total factor productivity (TFP) shock. It is assumed to follow a $AR(1)$ process given by

$$z_{t+1} = \rho z_t + \varepsilon_{t+1},$$

where ε follows a truncated normal distribution with zero mean, standard deviation of σ and finite support $[-10\sigma, 10\sigma]$. Note that the firm's TFP in period t is e^{z_t}, according to the definition in Section 1.

The firm can finance its investment in capital by internal funds or borrowing from the financial market. As in Gomes (2001) and Whited (2004), we assume that financial market imperfections exist and are summarized with a simple external finance cost function that takes the linear form given by

$$\lambda = \lambda_0 + \lambda_1 \times \text{amount of external funds}.$$

Equivalently, there is a fixed cost λ_0 and per unit cost λ_1 associated with external finance. This specification is intended to capture a variety of costs of going to financial market to raise capital, which would include the fixed and variable costs of public stock offerings, costs of monitoring the firm and the discounted present value of any premia associated with external debt and equity finance. Clearly the firm will only choose to use external finance when it exhausts internal funds and current investment opportunities justify the additional cost of external funds.

The firm's problem is to choose its capital stock to maximizes its expected discounted sum of future net cash flow, taking the price of capital good p as given. It has the following recursive formulation.

$$\begin{aligned} V(k,z) &= \max_{k' \geq 0} \quad \pi(k,z) - p\,i(k,k') - \lambda_0 I\{p\,i(k,k') > \pi(k,z)\} \\ &\quad -\lambda_1 \max\{p\,i(k,k') - \pi(k,z), 0\} + \beta E_{z'|z} V(k',z'), \end{aligned} \tag{2}$$

where $i(k,k') = k' - (1-\delta)k$, and $I\{\cdot\}$ is an indicator function. The right-hand side of (2) specifies the decisions the firm has to make. The first four terms reflect the current net cash flow: profits minus investment spending and financing costs. The last term is the expected continuation value.

Notice that in the model firms can only save through real assets (capital). We abstract from firm savings in cash holding or other financial assets. Allowing for these other forms of savings would give firms more means of transferring funds across periods, and as a result may alleviate firms' financing constraints due to costly external finance. However, this simplification should not be quantitatively significant for the question we aim to address because in the data a majority of investing funds is used for capital expenditure. In the data sample we use to calibrate the model, capital expenditure accounts for 86 percent of total investing funds, while funds used for cash holdings and short-term financial assets are only 7 percent. Also, in our calibration, we restrict investment to be capital expenditure only.

Applying standard arguments of dynamic programming, one can show that a unique solution to this problem exists and establish some useful properties of the value function.

Proposition 1 *For a given p, there is a unique function $V(k,z)$ that satisfies (1); $V(k,z)$ is continuous and increasing in both k and z, and concave in k.*

Associated with this solution there is a decision rule concerning capital accumulation, denoted by $k'(k,z)$. If external finance is costless ($\lambda_0 = \lambda_1 = 0$), $k'(k,z)$ would be a function of current productivity shock z only, i.e., it is independent of current capital stock. Costly external finance introduces dependence of k' on k. The following proposition characterizes the decision rule $k'(k,z)$.

Proposition 2 *For a given z, there exists $0 < k_1(z) < k_2(z)$ and $0 < k'_e(z) < k'_u(z)$, such that*
(i) For $k < k_1(z)$, the firm resorts to external finance and $k'(k,z) = k'_e(z)$;
(ii) For $k_1(z) \leq k \leq k_2(z)$, the firm's investment is constrained by its profits, i.e.,$k'(k,z) = (1-\delta)k + \frac{\pi(k,z)}{p}$;
(iii) For $k > k_2(z)$, the firm's investment achieves its unconstrained level, i.e., $k'(k,z) = k'_u(z)$.

PROOF: See Appendix A.2.

Proposition 2 states that for a given current productivity level, if the firm's current capital stock is relatively small, using external finance is profitable. But since the profit function exhibits decreasing return to scale, when the firm's capital stock passes some level ($k_1(z)$), current investment opportunities would not justify the additional cost of external finance and hence the firm's investment is constrained by its operating profit. If the firm's capital stock is big enough (greater than $k_2(z)$) such that it could generate enough cash flow to finance desired level of investment, the firm's investment is no longer financially constrained.

Figure 1 plots the policy function $k'(k, z)$ for a low level of current productivity z and a high level of z. The figure is based on the baseline parameterization to be described in next section. In both plots, the solid line corresponds to the case of costly external finance, and the dashed line corresponds to costless external finance. Note that with costly external finance, k' depends on k in the way described in Proposition 2. The figure also shows that with costly external finance $k'(k, z)$ may be discontinuous at $k_1(z)$ (For characterization of $k_1(z)$, see the proof of Proposition 2 in Appendix A.2). This is due to the nonlinearity introduced by a fixed external finance cost.

A comparison of the two plots shows that the constrained region with a high productivity is larger than the constrained region with a low productivity (both $k_1(z)$ and $k_2(z)$ are larger with a higher z), implying that high productivity firms are more seriously impacted by costly external finance. Another finding is that the unconstrained level of k' with costly external finance ($k'_u(z)$) is bigger than the efficient level corresponding to costless external finance (These two are equal only at the highest level of z), implying that with costly external finance firms have an incentive to over-accumulate capital when they are not financially constrained, a behavior similar to "precautionary saving" by households subject to borrowing constraints. This precautionary saving motive is stronger for lower productivity firms as the gap between the two unconstrained levels of k' is larger with a lower productivity. In summary, Figure 1 implies that higher productivity firms are more financially constrained and have less incentive to over accumulate capital when they are able to do so. As a result, the adverse effect of costly external finance on capital accumulation is more severe for higher productivity firms than for lower productivity firms. This property will help explain why the presence of costly external finance has an adverse effect on aggregate productivity, as will be clear in a later section.

Proposition 2 implies that small firms (with smaller capital stock) resorts to external finance more often. This seems to contradict the commonly held belief that small firms are more financially constrained and rely on internal funds more heavily. We compute external finance ratios by asset class for Compustat manufacturing firms during the 1989-2003 period, as reported in Table 1. A strong negative relationship is found between external finance ratios and the total assets of firms. That is, smaller firms have higher external finance ratios than larger firms [5]. Since Compustat firms are mainly large mature firms, it's not clear whether this relationship holds for all manufacturing firms. However, this finding suggests that the commonly held belief may not hold uniformly in the data.

(Insert Table 1 here)

3 Calibration and Simulation

To execute a quantitative analysis, we need to set values for parameters of the model, including the relative price of capital good, p, the discount factor, β, the depreciation rate of capital, δ, the return to scale, α, the parameters describing the productivity shock, ρ and σ, and parameters in the external finance cost function, λ_0 and λ_1. The data we use to estimate or calibrate the parameters is taken from the Compustat North American industry annual file. We only consider firms in the manufacturing sector (with SIC codes between 2000 and 3999) during the period of 1989 to 2003. This time period is chosen since there are substantial changes in the reporting and accounting methods since 1988. Observations with missing data are deleted from the sample. Similar to Whited (1992) and Gilchrist and Himmelberg (1995), we exclude observations with large changes in the book value of capital stock, considering that they may indicate expansions or contractions of firms at margins other than capital expenditure (See Appendix A.1.1 for details). Finally we end up with an unbalanced panel of firms from 1989 to 2003 with between 2210 and 3265 observations per year. Appendix A.1.2 gives a detailed description of the variables in this data sample.

First, we normalize p to 1. Following Cooper and Ejarque (2001), we set β to 0.95. The external finance cost function was estimated by Smith (1977) and Altinkilic and Hansen (2000), both using data on costs associated with new equity issuance. Their estimates for λ_1 are 0.028 and 0.0241 respectively. Since in the data external finance mainly takes the form of debt finance rather than equity finance [6], we re-estimate this parameter by a panel regression of interest expenses of debt on debt issuance [7]. It gives a similar result, $\lambda_1 = 0.028$. Since λ_0 is sensitive to units of measure, it is estimated together with α, δ, ρ and σ to match five moments of the data. The first moment is the mean annual investment rate defined as the ratio of total investment to total capital stock, which is 0.17 for the data sample. The second moment is the cross-sectional average investment rate, which is 0.22. The third moment is the cross-sectional standard deviation of investment rate, which is 0.19. The fourth moment is the autocorrelation of investment rate, which is 0.21. In constructing investment rates for each firm at each year, the book values of the gross capital stock are converted into its replacement values following the perpetual inventory method described in Salinger and Summers (1983). Appendix A.1.3 gives a detailed description of this procedure. The last moment is the fraction of total investment financed externally, i.e. the ratio of external finance used for investment to total investment. Compustat does not have enough information to directly calculate this moment. But it can be reasonably approximated by the ratio of total external finance to total uses of funds, which is 0.072, since in the data sample 86% of total uses of funds are for new capital purchase. These five moments are selected for their informativeness about the underlying structural parameters as well as their prominence in the literature.

To demonstrate that these five moments provide identification of the five parameters to be estimated, Table 2 presents how their values change with respect to small changes in each parameter. In the table, parameterization (1) is a benchmark parameterization, where the parameters are set to values commonly used in the literature. In particular, the annual depreciation rate δ is set to 10%; the return to scale parameter α is set to 0.975, a value close to the standard CRS assumption; parameters governing the productivity shocks ρ and σ are set to 0.95 and 0.01 respectively, values that are commonly used in the RBC literature;

the fixed external finance cost λ_0 is set to 1000, a positive but very small number relative to the average amount of external finance in equilibrium (which is about 1.13×10^{30} under parameterization (1)). Parameterization (2) considers a 10% change in δ relative to the benchmark with all other parameters unchanged, parameterization (3) and (4) consider a 1% change in α and ρ respectively, and parameterization (5) and (6) consider a 10% change in σ and λ_0 respectively. The results indicate that the five moments we choose are sensitive to changes in the parameters. In particular, the investment rate (I/K) is very sensitive to changes in δ, the cross sectional average and standard deviation of investment rates are sensitive to changes in all parameters, the autocorrelation of investment rates is sensitive to changes in α and ρ, and the external finance ratio is very sensitive to changes in α, ρ, σ and increases in λ_0. So we conclude that these moments provide identification of the parameters to be estimated.

(Insert Table 2 here.)

Here is a brief description of the estimation procedure. A more detailed description is given in Appendix A.3. For arbitrary values of the parameters to be estimated, the productivity shock is approximated by a 10-state Markov chain and the firm's problem is solved by value function iteration to obtain the decision rules $k'(k, z)$. Using the decision rules, an invariant distribution of firms over capital stock and productivity types, $\mu(k, z)$, is computed, which is independent of the initial distribution of (k, z). Then we draw 20,000 firms from the invariant firm distribution and carry out the simulation for 15 periods (Our data sample covers 15 years) to form an artificial panel data set. The five moments are computed for this artificial data set and compared with the corresponding data moments. This procedure is continued until the distance between the moments of the simulated data and the actual data moments is minimized. Considering the potential discontinuity introduced by the fixed external finance cost and the discretization of the state space, we use a simulated annealing algorithm as described in Goffe, Ferrier and Rogers (1994) to perform the minimization. Table 3 summarizes the estimated parameter values and matched moments.

(Insert Table 3 here.)

The high degree of nonlinearities in the solution makes it hard to match all moments exactly. Nevertheless the approximation appears reasonably close, as shown in Table 3. Note that the estimated value of α is 0.8993, which is pretty close to 1, suggesting that the technology does not substantially depart from constant return to scale. This is consistent to many of previous studies (See Burnside (1996) and Gomes (2001)). Cooper and Haltiwanger (2006) give a much lower α of about 0.6 using the LRD plant level data. Their estimate does not contradict ours since Compustat file is composed of bigger and more mature firms as compared to LRD. The estimated depreciation rate is 0.17, higher than those of most previous studies based on data before 1990s. Considering the rapid technological progress since 1990s, a higher depreciation rate of capital seems reasonable. The estimated degree of persistence and variability in productivity shocks is consistent with Gomes (2001). But the variability is much smaller than that of Cooper and Haltiwanger (2005). The fixed cost of external finance λ_0 is estimated to be about 608, which is about 0.2% of the average size of external finance in the stationary equilibrium.

4 Results

With the parameters determined, the question outlined in the Introduction can be addressed. This section summarizes the quantitative impacts of costly external finance on aggregate productivity, capital accumulation and output. A comparative static analysis is executed to see how these impacts are affected by the primitives of the model. Finally, we briefly discuss whether considering a general equilibrium analysis and adding firm entry and exit would change the results.

4.1 Impact of Costly External Finance on Aggregate Productivity

To evaluate the quantitative impact of costly external finance on aggregate productivity, we compute the output-weighted aggregate productivity and compare it with the productivity measure we would obtain if external finance is costless, i.e. if all parameter values are the same as in Table 3 except that $\lambda_0 = 0$ and $\lambda_1 = 0$. As described in Section 2, to compute the output-weighted aggregate productivity, a distribution of output shares across different productivity types is needed. The invariant measure of firms over capital stock and productivity, $\mu(k, z)$, enables us to do so. As reported in Table 4, the output-weighted aggregate productivity with costly external finance is 1.0395, while its costless counterpart is 1.0496. This implies a 1% loss in aggregate productivity due to costly external finance. According to the NBER manufacturing productivity database, the aggregate TFP growth for U.S. manufacturing is far less than 1 percent a year and sometimes negative in the 1970s and 1980s except the period 1982-87 (see Bartelsman and Gray (1996) for details). So our result suggests that the adverse impact of costly external finance on aggregate TFP growth is quantitatively significant. Let us examine this result from several aspects by comparing the two steady state distributions with and without costly external finance.

(Insert Table 4 here.)

First, as illustrated in Figure 2, the productivity distributions with costly or costless external finance are the same: firms with each of the 10 productivity types account for 10% of all firms. So in Table 4 the average productivity is 1 in both cases. Therefore the productivity change due to within firm productivity change is zero, i.e., the first item in the decomposition of aggregate productivity growth (equation (1)) is zero. So the 1% loss in output-weighted aggregate productivity due to costly external finance is completely through the second item–reallocation of output shares. This is shown clearly in Figure 3, which plots the distribution of output shares across productivity types for the two cases. Note that with costly external finance, the output shares of firms with high level productivities are smaller than their costless counterparts, while the output shares of firms with low productivities are larger than their costless counterparts. It follows that the presence of costly external finance leads to a shift of output shares from high productivity firms to low productivity firms and hence results in a lower aggregate productivity. The driving force underlying this result is the distortion in firms' investment behavior due to costly external finance. As discussed earlier, the adverse effect that costly external finance decreases capital accumulation is more severe for high productivity firms than for low productivity firms. Consequently, the output of high productivity firms is more seriously impacted by costly external finance than low productivity firms and as a result costly external finance leads to a reallocation of output shares from high productivity to low productivity firms.

(Insert Figure 2 and Figure 3 here)

Finally, Figure 4 plots the firm distribution over capital stock in the two cases. If external finance is costless, firms with the same productivity will have the same capital stock, and as a result the firm distribution is a uniform distribution over the 10 efficient levels of capital stock corresponding to the 10 productivity types. While with costly external finance, since firms are financially constrained in achieving their efficient size, the resulting firm distribution is skewed to the right, with a majority of firms having low capital stock while only a small fraction of firms having very high capital stock. This feature of the model is consistent with the data.

(Insert Figure 4 here)

4.2 Impact of Costly External Finance on Output through Reallocation

According to Table 4, costly external finance decreases aggregate output by 6.9 percent. This is achieved trough two channels. One is the traditional channel–capital accumulation. As shown in Table 4, costly external finance decreases aggregate capital accumulation by 7.3 percent. Notice that these results hinge on a partial equilibrium analysis. That is, we keep the price of capital unchanged, $p = 1$, when solving the costless problem. In a general equilibrium setting, the price of capital goods would increase to discourage investment as investment demand rises. As a result, the aggregate capital accumulation and aggregate output with costless external finance would not be as large as reported in Table 4. So the adverse effects of costly external finance on aggregate capital accumulation and output in a general equilibrium analysis would be smaller than suggested by Table 4. Here, our focus is on the impact of costly external finance on output through the second channel–resource misallocation which results in lower aggregate productivity.

To quantify this impact, we do another experiment. When solving the costless problem, we vary the price of capital good p, such that aggregate capital stock is the same as its counterpart with costly external finance. In this way, we keep the aggregate capital accumulation the same in both cases. Any change in aggregate output is completely through changes in aggregate productivity. The result is summarized in Table 5. Note that the impact on output-weighted aggregate productivity is not affected by the change of capital price, suggesting that the former result regarding the quantitative impact of costly external finance on aggregate productivity does not hinge on the partial equilibrium analysis adopted. This property will be explored further in a later discussion. Table 5 shows that a 1 percent decrease in aggregate productivity due to costly external finance leads to about 0.3 percent decrease in aggregate output, which seems a small effect on aggregate output.

(Insert Table 5 here)

A large literature that attempts to explore the relationship between financial market frictions and output growth adopts the framework of neoclassical growth models that abstracts from heterogeneity in production units. Therefore much of this literature has been concerned with understanding the role of aggregate accumulation and how aggregate accumulation is affected by financial frictions. The role of reallocation is completely neglected.

With heterogeneous firms, the model can characterize both roles of aggregate accumulation and reallocation, where the role of reallocation is characterized by the change in output-weighted aggregate productivity. The quantitative analysis above suggests that the impact of costly external finance on aggregate output through reallocation is not quite significant, despite that a thorough evaluation of the relative importance of aggregate accumulation and reallocation requires a general equilibrium analysis.

4.3 Comparative Statics

The previous results are based on the baseline calibration. In this section, we execute a comparative static analysis to see how the effects of external finance vary with key parameters of the model. We consider the effects of changes in the return to scale, in the persistence and variability of the productivity shocks, and in the external finance costs. For each new parameterization, the firm's problem is re-solved and the model is simulated to generate the four moments: cross-sectional mean, standard deviation and autocorrelation of investment rates, and fraction of total investment financed externally [8]. The corresponding problem with costless external finance is also re-solved to compute the ratios of aggregate productivity, aggregate capital stock and aggregate output to their costless counterparts. Smaller ratios imply more severe adverse effects of costly external finance. Table 6 summarizes the results. The middle column of each panel refers to the baseline calibration.

(Insert Table 6 here)

The first panel of Table 6 shows that the adverse effects of costly external finance on aggregate productivity, aggregate capital accumulation and output increase with the return to scale parameter, α. Notice from the table that as α increases, the average investment rate and the standard deviation of investment rates both increase, implying that more firms are likely to resort to external funds to finance their investment needs. This is reflected in the higher external finance ratio as α increases. As a result, costly external finance imposes more severe adverse impacts on the economy. Since α indicates market power, as pointed out in Cooper and Ejarque (2001), this result implies that the adverse effects of costly external finance are more severe for an economy where there are more competition among firms (α is closer to 1).

The second panel shows that the adverse effects of costly external finance increase with the variability in idiosyncratic productivity shocks. Higher σ implies greater heterogeneity among firms. This result suggests that the more diversified the productions units are, the greater the loss in aggregate productivity and output through resource misallocation due to costly external finance. Considering that some studies based on more comprehensive data set for the U.S. manufacturing give a higher estimate for σ (for example, $\sigma = .64$ in Cooper and Haltiwanger (2006), which uses LRD.), our quantitative result may underestimate the impact of costly external finance on aggregate productivity and output. This will be further discussed in Section 5.

The third panel shows how the impacts of costly external finance change with the persistence of productivity shocks. If the shock process is more persistent (higher ρ), the adverse impacts are more severe. At the first look, this may seem confusing. But note that the standard deviation of the productivity shock z is given by $\frac{\sigma}{\sqrt{1-\rho^2}}$. For a given σ, higher ρ

implies higher variability in the productivity shocks. So the results here are consistent with the comparative statics with respect to σ.

The last two panels consider how the impacts vary with the external finance costs. Not surprisingly, either higher fixed cost or higher unit cost of external finance leads to more severe adverse effects in aggregate productivity, aggregate capital accumulation and aggregate output. This property may provide another source of underestimation of our quantitative results, since Compustat is mainly composed of large mature firms, while large mature firms tend to face lower external finance costs than young small firms.

5 Discussion

The baseline model described in Section 2 is a partial equilibrium model that excludes firm entry and exit. In this section, we discuss whether our result concerning the quantitative impact of costly external finance on aggregate productivity would change if a general equilibrium analysis is adopted and if firm entry and exit are considered. In addition, several sources of underestimation of this result are identified in the comparative static analysis due to the nature of the Compustat data used to calibrate the model. A brief discussion is also given on the robustness of the result to a richer data set.

5.1 Considering A General Equilibrium Analysis

A key element of the partial equilibrium analysis we adopted for simplicity is that the price of capital is kept unchanged when we solve for the problem with costless external finance. In a general equilibrium setting, this price (still a constant in a stationary equilibrium) is determined by equating the demand and supply of capital. So it may differ across the two cases, external finance is costly and costless. However, this difference does not matter for our quantitative result. We show this by proving that the output-weighted aggregate productivity with costless external finance is independent of the price of capital.

If external finance is costless, the stationary equilibrium is characterized by a uniform distribution over 10 types of firms, each with a fixed productivity level and capital stock. The capital stock corresponding to a given productivity level z, denoted by $k(z)$ is given by

$$\beta(E_{z'|z}e^{z'})\alpha k(z)^{\alpha-1} = p(1-\beta(1-\delta)). \tag{3}$$

So the output-weighted aggregate total factor productivity is given by

$$TFP = \sum_{j=1}^{10} \frac{e^{z_j}k(z_j)^\alpha}{\sum_{l=1}^{10} e^{z_l}k(z_l)^\alpha} e^{z_j}. \tag{4}$$

It follows from (3) that $k(z)^\alpha = \left(\frac{p(1-\beta(1-\delta))}{\alpha\beta}\right)^{\frac{\alpha}{\alpha-1}} (E_{z'|z}e^{z'})^{\frac{\alpha}{1-\alpha}}$. So (4) simplifies to

$$TFP = \sum_{j=1}^{10} \frac{e^{z_j}(E_{z'|z_j}e^{z'})^{\frac{\alpha}{1-\alpha}}}{\sum_{l=1}^{10} e^{z_l}(E_{z'|z_l}e^{z'})^{\frac{\alpha}{1-\alpha}}} e^{z_j}.$$

Notice that the output-weighted aggregate TFP does not depend on the price of capital p.

Therefore our results regarding the quantitative impacts of costly external finance on aggregate productivity and output through reallocation do not hinge on the partial equilibrium analysis undertaken.

5.2 Considering Firm Entry and Exit

The Compustat data set we use to estimate the model does not exhibit a lot of firm entry and exit, but entry and exit are a common behavior of the U.S. manufacturing industry. According to Dunne, Roberts and Samuelson (1988), on average approximately 4.5% of firms entered the U.S. manufacturing industry every year during the period of 1963 to 1982 and similar percentage of firms exited every year. Empirical studies also find a significant role of entry and exit of production units in accounting for aggregate productivity growth. This section presents a brief discussion of how the quantitative impact of costly external finance on aggregate productivity would change if adding firm entry and exit to the model. Rather than doing a comprehensive analysis, we consider some simple cases of firm entry and exit.

Assume that the firm's exit is exogenous: every period, the firm has a probability of η to exit, where $\eta = 0.045$ [9]. Upon exit, the firm secures a zero exit value. Now the firm's problem is given by

$$
\begin{aligned}
V(k,z) &= \max_{k' \geq 0} \quad \pi(k,z) - p\, i(k,k') - \lambda_0 I\{p\, i(k,k') > \pi(k,z)\} \\
&\quad -\lambda_1 \max\{p\, i(k,k') - \pi(k,z), 0\} + \beta(1-\eta) E_{z'|z} V(k',z'). \qquad (5)
\end{aligned}
$$

In the data, there are high-productivity entrants and low-productivity entrants. So we consider two extreme cases of firm entry to infer the impact of entry and exit. First, as in Cooley and Quadrini (2001), new entry firms are of the highest productivity, and second, new entry firms are of the lowest productivity [10]. Upon entry, a new firm chooses its initial capital stock, which is financed all by external funds, to maximize its expected continuation value. The entry problem is formulated as.

$$
V_0(z_0; p) \equiv \max_{k_0} \quad \int V(k_0, z') P(z_0, dz') - \lambda_0 - p(1+\lambda_1) k_0, \qquad (6)
$$

where $z_0 = \bar{z}$ for the first case, and $z_0 = \underline{z}$ for the second case. Free entry condition implies that

$$
V_0(z_0; p) = c_e, \qquad (7)
$$

where c_e is a fixed entry cost.

Then for each case, we re-calibrate the model following the same procedure as discussed in Section 3, i.e., we re-estimate parameters δ, α, ρ, σ and λ_0 to match the five data moments as described in Section 3 using simulated annealing algorithm. In each case, c_e is chosen such that the free entry condition (7) is satisfied. To solve the corresponding costless problem, we let $\lambda_0 = \lambda_1 = 0$ in problem (5) and (6), and choose the price of capital good, p^c such that (7) is satisfied. The new parameter estimates, matched moments, and productivity measures for the two cases are reported in Table 7. Note that the matched moments in both cases are reasonably close to the data moments in Table 3, except the standard deviation of investment rates in Case 1 [11]. The results here show that the ratio of output-weighted aggregate productivity to its costless counterpart is 0.9982 if new firms are of the highest productivity, and 0.9922 if new firms are of the lowest productivity. In both cases, the loss in output-weighted aggregate productivity due to costly external finance is less than 1 percent.

One may argue that the re-calibration here is not very appropriate since a model that considers firm entry and exit should be calibrated to a richer data set that exhibits a lot of firm entry and exit. For a robustness check, we also consider a crude calibration of the model to LRD. Of course, due to the lack of access to LRD and the lack of financial data in LRD, we are not able to compute the exact moments of LRD that we need for calibration. So we use the moments of plant level investment rates reported in Cooper and Haltiwanger (2006) (LRD, 1972-88, mean: 12.2%, standard deviation: 33.7%, autocorrelation: 5.8%) as the moments of firm level investment rates, and the fraction of external funds in the sources of funds reported in Fazarri, Hubbard and Peterson (1988) (U.S. manufacturing, 1970-84, 28.9%) as the external finance ratio. Note that values of these moments are quite different from what we use in the previous calibrations. In particular, standard deviation of investment rates and external finance ratio are much higher, which is probably true for LRD firms as compared to the Compustat firms. We let $\delta = 0.1$, β and λ_1 be the same as before, and estimate α, ρ, σ and λ_0 to match the four moments. Table 8 reports the results under the new calibration. As expected, the loss in aggregate productivity due to costly external finance is larger than reported in Table 7, but it's clear that the magnitude is comparable with what we get from the baseline model.

These results suggest that we may safely conclude that adding firm entry and exit is unlikely to change the magnitude of the impact of costly external finance on aggregate productivity dramatically.

5.3 Robustness of the Result to a Richer Data Set

The comparative static results described in Section 4.3 suggest that the adverse impact of costly external finance increase with the variability of productivity across firms and external finance costs. Since the Compustat data we use to calibrate the model excludes non-publicly traded manufacturing firms, it may exhibit less variability in firm level TFPs. On the other hand, compared with the Compustat firms, these non-publicly traded firms tend to face higher external finance costs since they are typically younger and smaller. Consequently, our results may underestimate the quantitiative impact of costly external finance on aggregate productivity and aggregate output through reallocation. A recalibration of the model to a richer data set like LRD for the U.S. manufacturing would be desirable. However, as discussed in the Introduction, the LRD lacks important financial information that is crucial to calibrate the model, which restricts a rigorous recalibration to check the robusteness of our results. Several other existing databases for the U.S. manufacturing such as small business database (SBDB) are less representative than the Compustat data file.

However, we argue that the result would not change dramatically if we are able to re-calibrate the model to a richer data set incorporating finance and performance information. First, the aggregate productivity measure we consider is output-weighted aggregate productivity. The Compustat firms in our data sample account for 82% of the total employment of the U.S. manufacturing during the sample period. Their output share in the total output of the U.S. manufacturing must be of a comparable magnitude. This implies a small output share for the non-publicly traded firms. As a result, the contribution of those non-publicly traded firms to output-weighted aggregate productivity is small. Second, the results from the crude calibration of the model with entry and exit to LRD show that although the moments and parameter estimates are quite different from those obtained in the baseline model, the quantitative magnitude of the impact of costly external finance on aggregate productivity is comparable. This also gives us confidence about the robustness of the quantitative results.

6 Conclusion

This paper studies the quantitative impact of costly external finance on aggregate productivity by incorporating an external finance cost function into a firm's capital accumulation problem with idiosyncratic productivity shocks. Our main result is that costly external finance leads to a reallocation of output shares from high productivity firms to low productivity firms such that the output-weighted aggregate productivity is 1 percent smaller than it would be if external finance is not costly. This constitutes a significant loss to aggregate productivity. We show that this result does not hinge on the partial equilibrium analysis undertaken, and considering firm entry and exit is unlikely to change the results significantly. A consequence of this reduced aggregate productivity is that it decreases aggregate output by 0.3 percent. This is an indirect impact of costly external finance, in addition to the direct impact through reducing aggregate capital accumulation.

We abstract from entry and exit in the main analysis. As entry and exit plays an important role in aggregate productivity growth, an interesting extension of the paper is to model how costly external finance affects firms' entry and exit decisions and quantitatively evaluate this impact on aggregate productivity. In addition, we adopt a homogeneous external finance cost function and a stationary analysis. There is empirical evidence suggesting that firms differ in external finance costs along a lot of dimensions, such as firm size, age, credit worthiness, and etc. It's not clear whether such heterogeneity matters a lot for the question outlined here. These questions are open for future research.

Notes

⋆ : I thank my Ph.D. supervisors, Prof. Russell Cooper and Dean Corbae, for their valuable guidance. I also thank Prof. Ken Hendricks, Prof. Hong Yan, and seminar participants at the University of Texas for helpful comments. All errors are my own.

1. Petrin and Levinsohn (2004) argue that the popular measurement of industry productivity growth adds a "reallocation" term to the growth accounting measure and fails to use the correct weights in the aggregation such that they call into question the literature's interpretation of "reallocation" as productivity growth. Instead, they propose a new method for separating real productivity growth from reallocation effects and find that such reallocation effects are reasonably stable within industries and almost always positively impact aggregate productivity growth.

2. The Compustat data records the year a firm is deleted from the file and the reason for deletion. Among the reasons for deletion, bankruptcy and liquidation are regarded as closely related to firm exit from operation. During the period 1989 to 2003, which is the sample period of the data set we use to calibrate the model, firm deletion rate due to bankruptcy and liquidation is about 0.5%.

3. Currently, the Center for Economic Studies of the Bureau of the Census is linking the LRD to many other data sets, including public financial databases.

4. The profit function can be regarded as a reduced form that has optimized out inputs other than capital, as in Cooper and Haltiwanger (2006).

5. There is belief that the high external finance ratios for small firms as shown in Table 1 are due to the fact that a lot of small firms in Compustat are young high-tech firms which are recently publicly listed and have very high equity financing. Since firm age information is not available in Compustat, we are not able to re-examine this relationship by controlling for firm age. But we re-calculate the external finance ratios by asset class for each of the 20 manufacturing industries and find that the negative relationship between external finance ratio and firm asset size holds for most industries and is particularly remarkable for some high-tech industries such as Chemicals & Allied Products (SIC code 2800), Industrial and Commercial Machinery and Computer Equipment (SIC code 3500), Electric and Electronic Equipment and Exchange Components (SIC code 3600), Measurement Instrument, Photo Goods and Watches (SIC code 3800). When we exclude these industries from the data sample, the negative relationship between firm size and external finance ratio still holds but is less remarkable than shown in Table 1.

6. For our data sample, equity finance is about 10% of total external finance.

7. Data on total expenses of external finance is not available in Compustat. Otherwise, the cost function of external finance could be directly estimated.

8. Aggregate investment rate is mainly determined by the depreciation rate of capital. It is about 0.17 in all these scenarios and hence is skipped in Table 6.

9. There is evidence that firm exits are related to low productivity, and also impacted by external financing issues. Some recent literature on firm dynamics has explicitly modeled these links, see Jovanovic (1982), Hopenhayn (1992), and Clementi and Hopenhayn (2006) for examples. Modeling these issues here is beyond the scope of the paper. Instead we assume exogenous firm exit.

10. A more realistic way to model firm entry is to let new firms' productivity follow some distribution. We avoided this complication because results from the two simple extreme cases would somehow provide a range for the quantitative impact of external finance on aggregate productivity with firm entry and exit (exogenous firm exit and new entry firms' productivities ranging from the lowest level to the highest level), and in our view this is sufficient for us to get some insights on the sensitivity of the results to firm entry and exit.

11. The estimation routine finds that there is a tension in the two moments: standard deviation of investment rates and external finance ratio. Since we put more emphasis on external finance ratio, as we do for the baseline model, the estimation yields a low standard deviation of investment rates than in the data.

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Tables and Figures

Table 1. External Finance Ratio by Asset Class, Compustat Manufacturing Firms, 1989-2003

	external funds [a]/ sources of funds	external funds/ uses of funds
All firms	0.1077	0.1123
$<$ \$250 million	0.9337	0.9660
\$250 million - \$ 1 billion	0.2593	0.2974
\$ 1-2 billion	0.1691	0.1844
$>$ \$ 2 billion	0.0784	0.0800

[a]: For definitions of external funds and sources and uses of funds, see Appendix A.1.2.

Table 2. Identification of parameters

Parameters	(1)	(2)		(3)		(4)		(5)		(6)	
δ	0.1	0.11	0.09	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
α	0.975	0.975	0.975	0.986	0.966	0.975	0.975	0.975	0.975	0.975	0.975
ρ	0.95	0.95	0.95	0.95	0.95	0.96	0.94	0.95	0.95	0.95	0.95
σ	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.011	0.009	0.01	0.01
λ_0	1000	1000	1000	1000	1000	1000	1000	1000	1000	1100	900
Moments											
I/K	0.1004	0.1106	0.0906	0.1020	0.1007	0.1008	0.1008	0.1006	0.1005	0.1002	0.1009
Avg ^{b}. of i/k	0.1118	0.1223	0.1010	0.1365	0.1079	0.1143	0.1106	0.1149	0.1094	0.1032	0.1117
Std ^{c}. of i/k	0.1396	0.1419	0.1342	0.2982	0.1113	0.1563	0.1280	0.1606	0.1248	0.0725	0.1391
Corr ^{d}. of i/k	0.1320	0.1351	0.1359	0.0917	0.1380	0.1304	0.1353	0.1361	0.1329	0.1275	0.1350
Extfin $^e/I$	0.0491	0.0492	0.0471	0.1401	0.0195	0.0692	0.0360	0.0743	0.0323	0.0155	0.0493

^{b}: Average; ^{c}: Standard deviation; ^{d}: Autocorrelation; ^{e}: External finance.

Table 3. Baseline Calibration

	Parameter	Value
Price of capital	p	1
Discount factor	β	0.95
Returns to scale	α	0.8993
Depreciation rate	δ	0.17
Persistence of shock	ρ	0.8767
Variability of shock	σ	0.0393
Fixed cost of external finance	λ_0	608.4139
Unit cost of external finance	λ_1	0.028
Matched Moments	Data	Model
I/K	0.17	0.1703
Avg. of i/k	0.22	0.1868
Std. of i/k	0.19	0.1784
Corr. of i/k	0.21	0.1632
Extfin. $/I$	0.072	0.0724

Table 4. Quantitative Impacts of Costly External Finance on Aggregate Productivity, Capital Accumulation and Output

	costly ext. finance	costless ext. finance	ratio (costly/costless)
Average productivity	1	1	1
Output-weighted productivity	1.0395	1.0496	0.9904
Aggregate capital stock ^{f}	$1.2290 \cdot 10^6$	$1.3255 \cdot 10^6$	0.9272
Aggregate output	$3.0562 \cdot 10^5$	$3.2814 \cdot 10^5$	0.9314

^{f}: The aggregates are based on a unit measure of firms in both cases.

Table 5. Quantitative Impacts of Costly External Finance on Aggregate Productivity and Output

	costly ext. finance	costless ext. finance	ratio (costly/costless)
Price of capital good	1	1.0076	0.9925
Average productivity	1	1	1
Output-weighted productivity	1.0395	1.0496	0.9904
Aggregate output	$3.0562 \cdot 10^5$	$3.0657 \cdot 10^5$	0.9969

Table 6. Comparative Statics

	$\alpha = 0.85$	$\alpha = 0.8993$	$\alpha = 0.95$
Average investment rate:	0.1787	0.1868	0.2377
Std. of investment rate:	0.1214	0.1784	0.4317
Autocorrelation of inv. rate:	0.1177	0.1632	0.1217
External finance ratio:	0	0.0724	0.2849
Aggregate capital stock [g]	0.9678	0.9272	0.736
Aggregate output	0.9706	0.9314	0.7429
Aggregate productivity	0.9938	0.9904	0.9838
	$\sigma = 0.03$	$\sigma = 0.0393$	$\sigma = 0.05$
Average investment rate:	0.1796	0.1868	0.2015
Std. of investment rate:	0.1294	0.1784	0.2597
Autocorrelation of inv. rate:	0.1469	0.1632	0.1400
External finance ratio:	0.0217	0.0724	0.1623
Aggregate capital stock	0.9559	0.9272	0.8972
Aggregate output	0.9582	0.9314	0.9034
Aggregate productivity	0.9937	0.9904	0.9878
	$\rho = 0.84$	$\rho = 0.8767$	$\rho = 0.9$
Average investment rate:	0.1833	0.1868	0.1906
Std. of investment rate:	0.1553	0.1784	0.2011
Autocorrelation of inv. rate:	0.1482	0.1632	0.1581
External finance ratio:	0.0480	0.0724	0.0982
Aggregate capital stock	0.9361	0.9272	0.9193
Aggregate output	0.9395	0.9314	0.9243
Aggregate productivity	0.9911	0.9904	0.9901
	$\lambda_0 = 0$	$\lambda_0 = 608.4139$	$\lambda_0 = 1000$
Average investment rate:	0.1875	0.1868	0.1868
Std. of investment rate:	0.1790	0.1784	0.1795
Autocorrelation of inv. rate:	0.1757	0.1632	0.1528
External finance ratio:	0.0809	0.0724	0.0717
Aggregate capital stock	0.9273	0.9272	0.9264
Aggregate output	0.9315	0.9314	0.9305
Aggregate productivity	0.9907	0.9904	0.9903
	$\lambda_1 = 0.02$	$\lambda_1 = 0.028$	$\lambda_1 = 0.035$
Average investment rate:	0.1906	0.1868	0.1845
Std. of investment rate:	0.2052	0.1784	0.1629
Autocorrelation of inv. rate:	0.1378	0.1632	0.1598
External finance ratio:	0.1126	0.0724	0.0510
Aggregate capital stock	0.9346	0.9272	0.9237
Aggregate output	0.9388	0.9314	0.9277
Aggregate productivity	0.9923	0.9904	0.9891

[g]: Figures in the second part of each panel are ratios to their costless counterparts.

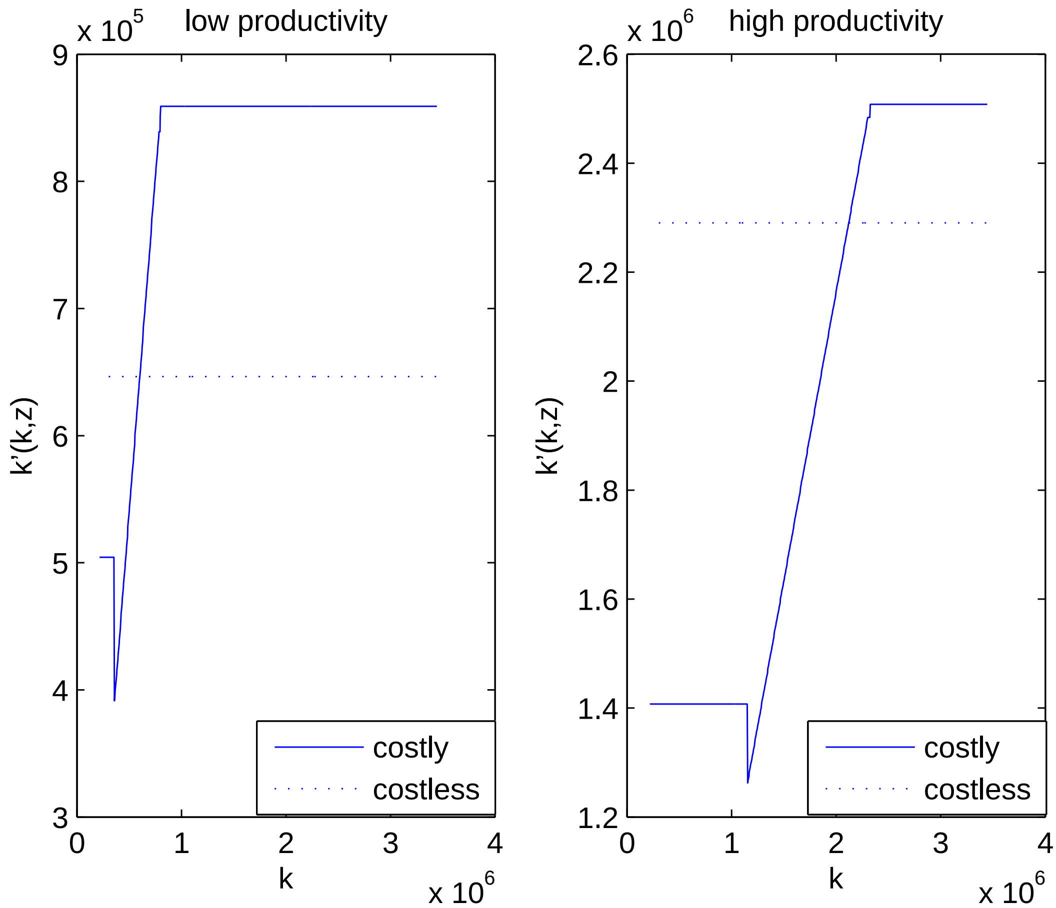
Table 7. Aggregate Productivity and Moments with Firm Entry and Exit

	Case 1: new firms are of the highest productivity	Case 2: new firms are of the lowest productivity
Output weighted productivity	1.0031	1.0096
Ratio to costless counterpart	0.9982	0.9922
Re-calibration		
δ	0.17	0.17
α	0.9451	0.9028
ρ	0.5922	0.8958
σ	0.013	0.0324
λ_0	4551.6822	1399.4129
Matched Moments		
Aggregate investment rate	0.1698	0.1703
Average investment rate	0.1972	0.2247
Std. of investment rate	0.0909	0.1732
Autocorrelation of inv. rate	0.1253	0.1912
External finance ratio	0.1299	0.0718

Table 8. Aggregate Productivity and Moments with Firm Entry and Exit (a crude calibration to LRD)

	Case 1	Case 2	
Output weighted productivity	1.0992	1.0396	
Ratio to costless counterpart	0.9902	0.9860	
Re-calibration			
δ	0.1	0.1	
α	0.7576	0.8762	
ρ	0.7502	0.9151	
σ	0.0963	0.0486	
λ_0	2129.2396	1237.0818	
Matched Moments			data
Aggregate investment rate	0.0998	0.1004	
Average investment rate	0.1114	0.1678	0.122
Std. of investment rate	0.1878	0.3036	0.337
Autocorrelation of inv. rate	0.0588	0.0791	0.058
External finance ratio	0.2684	0.2833	0.289

Figure 1. Decision rule for $k'(k, z)$ [h]



^{h}: In the computation, the productivity shock process is approximated with a 10-state Markov chain. Here, the low productivity refers to the third state, and the high productivity refers to the 9^{th} state. Similar patterns hold for other choices of productivity levels.

Figure 2. Firm Distribution over Productivity Types

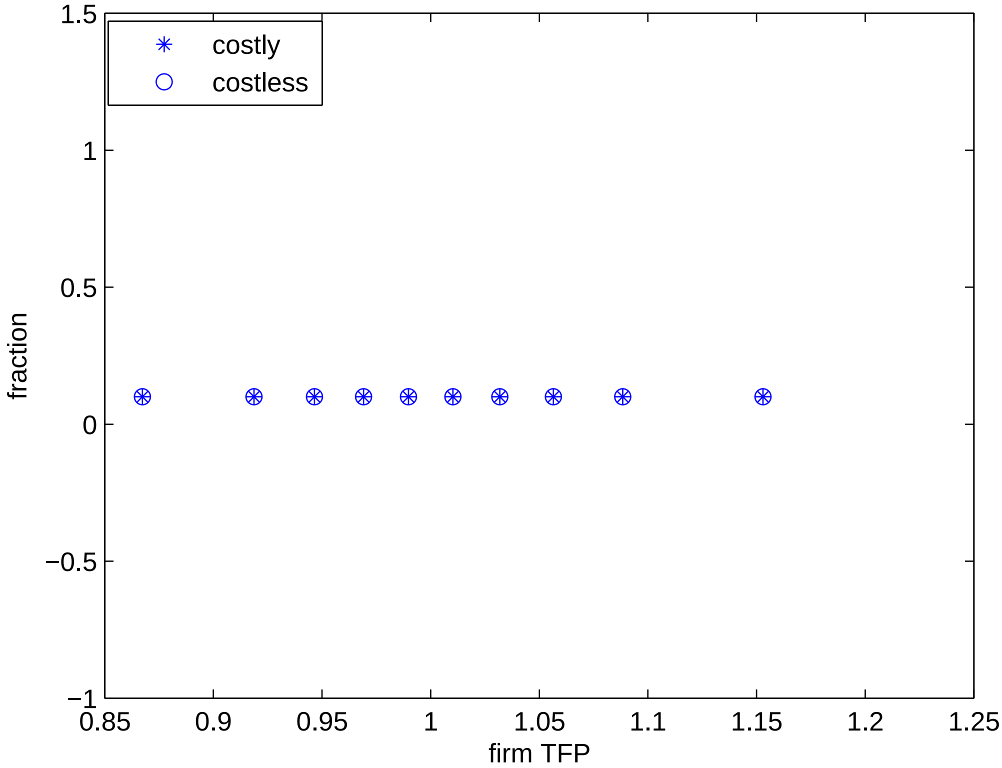


Figure 3. Distribution of Output Shares over Productivity Types

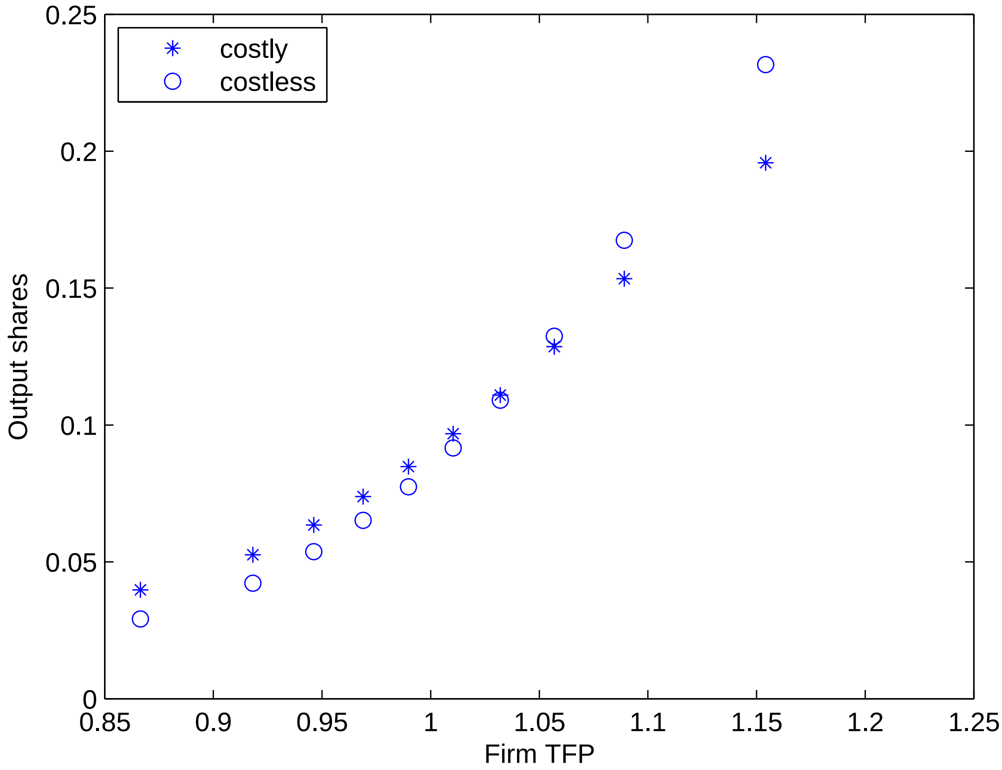
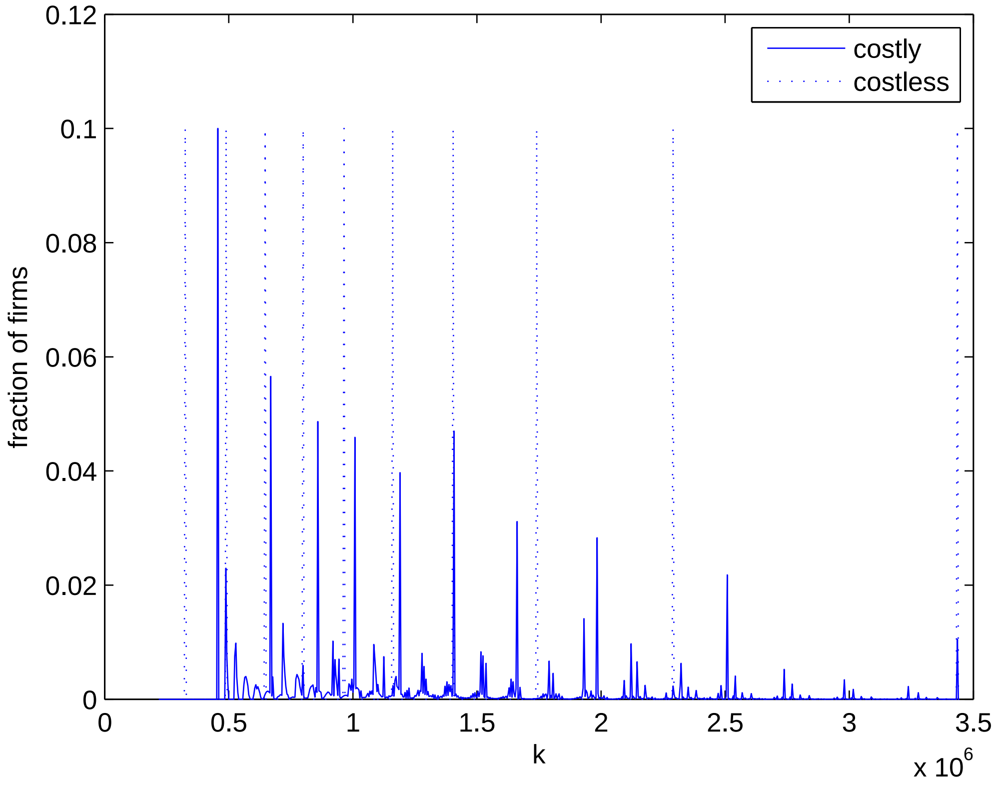


Figure 4. Firm Distribution over Capital Stock



Appendix

A.1 Data Description

A.1.1 Rule for deleting major capital changes

We exclude observations for which

$$|Gk_{i,t} - Gk_{i,t-1} - i_{i,t} + Retr_{i,t}| > 0.15 \cdot Gk_{i,t-1},$$

where $GK_{i,t}$ denotes book value of gross plant, property and equipment (DATA7), and $Retr_{i,t}$ denotes retirements (DATA184). In the instances where the retirement number is missing, we assume it is zero unless the discrepancy was negative. In this case, a value of $0.1 \cdot Gk_{i,t-1}$ is substituted for $Retr_{i,t}$.

A.1.2 Variables

- Investment: reported capital expenditure on property, plant and equipment (DATA30).
- Gross PPE: book value of gross plant, property and equipment (DATA7).
- Depreciation: reported value of depreciation and amortization (DATA14).
- External finance: sum of net debt issuance, net equity issuance and net changes in current debt (DATA313+DATA127).
- Sources of funds: sum of operating net cash flow and net cash flow from financing activities (DATA308+DATA313).
- Uses of funds: sum of capital expenditures, acquisitions and increases in financial assets (-DATA311).
- Debt: sum of long-term debt and debt in current liabilities (DATA9+DATA34).
- Interest expenses on total debt (DATA15).

A.1.3 Procedure for Constructing Investment Rates

A major work for constructing investment rates for each firm at each year involves converting the book value of capital stock into its replacement value. Denote $k_{i,t}$ as the replacement value of firm i's capital stock at the beginning of period t (or at the end of period $t-1$). It is constructed by the perpetual inventory method described in Salinger and Summers (1983).

- First, set the replacement value of the initial capital stock equal to the book value of gross PPE for the first year the firm appears on Compustat file if it is later than 1979 or for year 1979 otherwise (using years earlier than 1979 as the base year does not change the results significantly), i.e., $k_{i,0} = Gk_{i,0}$, where $Gk_{i,t}$ is the reported value of gross PPE at the end of period t.

- Then estimate the useful life of capital goods in any year using the formula $L_{i,t} = \frac{Gk_{i,t-1}+i_{i,t}}{Depr_{i,t}}$, where $Depr_{i,t}$ is the reported value of depreciation and amortization. Take the time average of $L_{i,t}$, denoted by L_i.
- Define the replacement value of the capital stock using the double declining balance method of depreciation.

$$k_{i,t} = \left[k_{i,t-1}\frac{P_t^k}{P_{t-1}^k} + i_{i,t}\right](1 - 2/L_i), t = 1, 2, \cdots,$$

  where P_t^k is the deflator for non-residential investment, which can be downloaded from the BEA website.

In calculating the cross sectional mean, standard deviation and autocorrelation of investment rates, observations with investment rates over 300% are excluded.

A.2 Proof of Proposition 2

To characterize $k'(k, z)$, we rewrite the problem (2) as

$$\begin{aligned} V(k,z) \quad = \quad \max\Big\{ & \max_{k'>(1-\delta)k+\frac{\pi(k,z)}{p}} \pi(k,z) - p\,(k' - (1-\delta)k) - \lambda_0 \\ & -\lambda_1[p(k' - (1-\delta)k) - \pi(k,z)] + \beta E_{z'|z}V(k',z'), \\ & \max_{k'\leq(1-\delta)k+\frac{\pi(k,z)}{p}} \pi(k,z) - p\,(k' - (1-\delta)k) + \beta E_{z'|z}V(k',z')\Big\}. \end{aligned} \tag{8}$$

The firm can choose to use external finance or not. The first inner maximization problem is the decision of the firm if external funds are needed to finance the investment, and the second inner maximization problem is the decision if investment can be fully financed by the firm's operating profits. The first order condition for the first inner maximization problem is given by

$$\beta E_{z'|z}V_1(k',z') = p(1+\lambda_1), \quad \text{if} \quad k' > (1-\delta)k + \frac{\pi(k,z)}{p}. \tag{9}$$

Note that for given z, (9) determines a unique k', denoted by $k'_e(z)$. Equating $k'_e(z) = (1-\delta)k + \frac{\pi(k,z)}{p}$ gives a unique k, denoted by $k_1(z)$. Then if $k < k_1(z)$, $k'(k,z) = k'_e(z)$.

The first order condition for the second inner maximization problem is

$$\begin{aligned} \beta E_{z'|z}V_1(k',z') = p, \quad \text{if} \quad k' < (1-\delta)k + \frac{\pi(k,z)}{p}, \\ \text{otherwise}, k' = (1-\delta)k + \frac{\pi(k,z)}{p}. \end{aligned} \tag{10}$$

Note that there is a unique k' satisfying $\beta E_{z'|z}V_1(k',z') = p$, which is the unconstrained level, denoted by $k'_u(z)$. Since $\lambda_1 > 0$ and $V(k,z)$ is concave in k, $k'_u(z) > k'_e(z)$. Let $k = k_2(z)$ satisfy $k'_u(z) = (1-\delta)k + \frac{\pi(k,z)}{p}$. Then $k_2(z) > k_1(z)$. For $k > k_2(z)$, $k'(k,z) = k'_u(z)$. For $k_1(z) \leq k \leq k_2(z)$, $k'(k,z) = (1-\delta)k + \frac{\pi(k,z)}{p}$.

Note that the policy function $k'(k,z)$ may be discontinuous at the cutoff point $k_1(z)$ due to the presence of a fixed external finance cost.

A.3 Estimation Procedure

The basic idea of the estimation routine is to choose values of $\Theta \equiv (\alpha, \delta, \rho, \sigma, \lambda_0)$ to match the five moments described in Section 3, i.e., to solve the minimization problem

$$\min_{\Theta} \left[\Psi^d - \Psi^s(\Theta)\right]' W \left[\Psi^d - \Psi^s(\Theta)\right],$$

where Ψ^d denotes the vector of data moments, $\Psi^s(\Theta)$ denotes the moments implied by the model for given Θ, W is a weighting matrix. For simplicity, we didn't use the optimal weighting matrix, instead we specify W as a diagonal matrix, with equal weights to aggregate investment rate, average investment rate, standard deviation of investment rates and autocorrelation of investment rates, while a higher weight to external finance ratio (Since our focus is on the quantitative impact of costly external finance, we put more emphasis on this moment condition).

Simulated annealing algorithm, as described in Goffe, Ferrier and Rogers (1994), is applied to perform the optimization. For arbitrary values of model parameters Θ, the model moments $\Psi^s(\Theta)$ is computed as follows.

1. Solve the firm's problem by value function iteration:
   (a) Approximate the productivity shock process by a 10-state Markov chain, as described in Tauchen (1986);
   (b) Let the state space for k be $[10^{-6}, \bar{k}_0]$, where $\bar{k}_0$ is the steady state capital stock in a deterministic problem with productivity being the highest level. Discretize $[10^{-6}, \bar{k}_0]$ into 301 equally spaced points, and do value function iteration until convergence is obtained;
   (c) Refine the state space for k as $[\underline{k}, \bar{k}]$, where $\underline{k} = \min_{(k,z)} k'(k, z)$, $\bar{k} = \max_{(k,z)} k'(k, z)$, and $k'(k, z)$ is the policy function obtained in (b). Discretize $[\underline{k}, \bar{k}]$ into 801 equally spaced points, take the value function obtained in (b) as the initial value function, and do value function iteration until convergence.
2. Starting from a uniform distribution over (k, z) and using the decision rule $k'(k, z)$ obtained in 1(c), do another function iteration to obtain the stationary firm distribution, $\mu(k, z)$;
3. Generate 20,000 firms from the stationary firm distribution and carry out the simulation for 15 periods, compute the moments using the simulated panel data set.

The resulted model moments are used to compute the weighted distance from the data moments. If the termination criteria is not met, Θ is updated by the simulated annealing algorithm, and steps 1 to 3 are repeated. This process continues until the stopping criteria is met.

The algorithm is written in Matlab (The program can be made available to readers on request). The number of updates of Θ is 400 times and the resulted weighted distance is 0.0035. The computer we use is Intel(R) Xeon (TM), double cpu 2.80GHz and 2.79GHz, 1.00GB of RAM. The estimation routine takes roughly 60 hours.