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CENTRAL BANK LEARNING, TERMS OF TRADE SHOCKS & CURRENCY RISKS: SHOULD ONLY INFLATION MATTER FOR MONETARY POLICY?

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Central Bank Learning, Terms of Trade Shocks & Currency Risks: Should Only Inflation Matter for Monetary Policy?

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Abstract

This paper examines the role of interest rate policy in a small open economy subject to terms of trade shocks, and time-varying currency risks. The private sector makes optimal decisions in an intertemporal non-linear setting with rational, forward-looking expectations. In contrast, the monetary authority practices "least-squares learning" about the evolution of inflation, output growth, and exchange rate depreciation in alternative policy scenarios. Interest rates are set by linear quadratic optimization, with the objectives for inflation, output growth, or depreciation depending on current conditions. The simulation results show that the prefered stance is one which targets inflation only. Including other targets such as growth and exchange rate changes significantly increases output variability, and unambiguously decreases welfare.

Key words: Currency risks, learning, parameterized expectations, policy targets

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1 Introduction

This paper examines the role of interest rate policy in a small open economy subject to terms of trade shocks, and time-varying currency risks. A central bank committed to low inflation controls neither the terms of trade nor the evolution of currency risk, both of which condition the response of inflation to its policy instruments. In this context, the best the central bank can do is to "learn" the effects indirectly, by frequently "updating" estimates of inflation dynamics and "re-adjusting" its policy rules accordingly.

Of course, central banks, even those with explicit inflation targets, adjust their policy stance from time to time to stimulate growth. And when the exchange rate depreciates rapidly, due to adverse external shocks, it should not be surprising if a central bank also comes under strong pressure to incorporate exchange rate volatility targets in its policy objectives. But, should growth and exchange rate changes be included as monetary policy targets?

Much of the discussion of monetary policy is framed by the well-known Taylor (1993, 1999) rule, whereby interest rates respond to their own lag, as well as to deviations of inflation and output from respective targets. Taylor (1993) points out that this "rule" need not be a mechanical formula, but something which can be operated "informally", with recognition of the "general instrument responses which underlie the policy rule". Not surprisingly, the specification of this rule, which reflect the underlying objectives of monetary policy, has been the subject of considerable controversy.[1]

In a closed-economy setting, Christiano and Guest (2000), for example, argue that only the inflation variable should appear as a target. Rotemberg and Woodford (1998) concur, but they argue that a higher average rate of inflation is required for monetary policy to do its job over the medium to long term. They base their argument on the zero lower bound for the nominal interest rate, since at very low inflation rates there is little room for this instrument to manoeuvre.[2]

In an open economy setting, McCallum (2000) takes issue with the Rotemberg and Woodford "policy ineffectiveness" argument under low inflation and zero "lower bounds" for nominal interest rates. McCallum argues that the central bank always has at its disposal a second tool, the exchange rate, so if the economy is stuck at a very low interest rate, there is the option of currency intervention. Christiano (2000) disagrees: McCallum's argument

[1]Recent technical papers on all aspects of the Taylor rule may be found on the web page, http://www.stanford.edu/~johntayl/PolRulLink.htm#Technical%20articles

[2]Erceg, Henderson and Levin (2000) argued that output deviations should also appear in the Taylor rule, but the output measure should be deviations of actual output from the level of output generated by a flexible-price economy.

rests on the assumption that currency depreciation is effective. Furthermore, the Central Bank must be willing to undermine public "confidence" that it stands ready to cut interest rates in the event of major adverse shocks.

For small emerging market economies, Taylor (2000) contends that policy rules that focus on a "smoothed inflation measure and real output" and which do not "try to react too much" to the exchange rate might work well. However, he leaves open the question of a role for the exchange rate. Ball (1999) argues that inflation targeting "can be dangerous" in an open economy setting because exchange rate changes have a direct effect on inflation via changes in import prices. Hence, adoption of a strict inflation targeting stance can result in large output variations.

However, practically all of these studies are based on *linear* stochastic and dynamic general equilibrium representations, or *linearized* approximations of nonlinear models. The Taylor-type feedback rules are either imposed or derived by linear quadratic optimization. While these approaches may be valid if the shocks impinging on the economy are indeed "small" and "symmetric" deviations from a steady state, they may be inappropriate if the shocks are large, persistent, and asymmetric, as they are in many highly open economies.

Furthermore, few if any of these studies incorporate "learning" on the part of the monetary authority itself. Bullard and Metra (2001) incorporate private sector "learning" of the specific Taylor rules used by the central bank in the Rotemberg-Woodford closed economy framework. They argue for Taylor rules based on *expectations* of *current* inflation and output deviations from target levels, rather than rules based on lagged values or forecasts further into the future.

In contrast to Bullard and Metra (2001), we assume that the private sector uses the true, stochastic dynamic, nonlinear model for formulating its own "laws of motion" for consumption, investment, and trade, with forward-looking rational expectations. In this analysis, the monetary policy authority learns the "laws of motion" of inflation dynamics from past data, through continuously-updated least squares regression. From the results of these regressions, the monetary authority obtains an optimal interest rate feedback rule based on linear quadratic optimization, using weights in the objective function for inflation which can vary with current conditions. The monetary authority is thus "boundedly rational", in the sense of Sargent (1999), with "rational" describing the use of least squares, and "bounded" meaning model misspecification.

Our results show that if the central bank decides to incorporate, in addition to inflation, growth and exchange rate dynamics in its learning and policy objectives, it does so at high welfare costs. In a learning environment, there is always the risk that the "perceived" laws of motion lag behind the actual laws of motion. Hence expanding the range of policy objectives may increase overall volatility and reduce welfare. For this reason, targeting only inflation dominates monetary policy based on multiple targets.

The next section describes the theoretical structure of the model for the private sector and the nature of the monetary authority "learning". The third section discusses the calibration as well as the solution method, while the fourth section analyzes the simulation results of the model. The last section concludes.

2 The Model

2.1 Consumption

The objective function for the private sector "representative agent" is given by the following utility function:

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma} \tag{1}$$

where C is the aggregate consumption index and γ is the coefficient of relative risk aversion. Unless otherwise specified, upper case variables denote the levels of the variables while lower case letters denote logarithms of the same variables. The exception is the interest rate denoted as i.

The representative agent as "household/firm" optimizes the following intertemporal welfare function, with an endogenous discount factor:

$$W = \mathbf{E}\left[\sum_{t=0}^{\infty} \vartheta_t U(C_t)\right] \tag{2}$$

$$\vartheta_{t+1} = [1+\overline{C}_t]^{-\beta} \cdot \vartheta_t \tag{3}$$

where β approximates the elasticity of the endogenous discount factor ϑ_{t+1} with respect to the average consumption index, $\overline{C}_t$. Endogenous discounting is due to Uzawa (1968) and Mendoza (2000) states that "endogenous discounting" is needed for the model to produce well-behaved dynamics with deterministic stationary equilibria.[3]

The specification used in this paper is due to Schmitt-Grohé and Uribe (2001). In our model, an individual agent's discount factor does not depend on their own consumption, but rather their discount factor depends on the

[3]Endogenous discounting also allows the model to support equilibria in which credit frictions may remain binding.

average level of consumption. Schmitt-Grohé and Uribe (2001) argue that this simplification reduces the equilibrium conditions by one Euler equation and one state variable, over the standard model with endogenous discounting, it greatly facilitates the computation of the equilibrium dynamics, while delivering "virtually identical" predictions of key macroeconomic variables as the standard endogenous-discounting model.[4] In equilibrium, of course, the individual consumption index and the average consumption index are identical. Hence,

$$C_t = \overline{C}_t \tag{4}$$

The consumption index is a composite index of non-tradeable goods N and tradeable goods T :

$$C = \left[(C^N)^{\alpha_1} (C^T)^{1-\alpha_1}\right] / \left[(\alpha_1^{\alpha_1}) (1-\alpha_1)^{1-\alpha_1}\right] \tag{5}$$

where α is the proportion of non-traded goods.

Given the aggregate consumption expenditure constraint,

$$PC = P^N C^N + P^T C^T \tag{6}$$

and the definition of the real exchange rate,

$$Z = \frac{P^T}{P^N} \tag{7}$$

the following expressions give the demand for traded and non-traded goods as functions of aggregate expenditure and the real exchange rate Z:

$$\begin{aligned} C^N &= \alpha_1 Z^{1-\alpha_1} C && (8) \\ C^T &= (1-\alpha_1) Z^{-\alpha_1} C && (9) \end{aligned}$$

while the domestic price index may be written as the geometric average of non-traded and non-traded goods:

$$P = (P^N)^{\alpha_1} (P^T)^{1-\alpha_1} \tag{10}$$

Similarly, consumption of traded goods is a composite index of export goods, X, and import goods F:

$$C^T = F^{\alpha_2} X^{1-\alpha_2} \tag{11}$$

[4] Schmitt-Grohé and Uribe (2001) argue that if the reason for introducing endogenous discounting is solely for introducing stationarity, "computational convenience" should be the decisive factor for modifying the standard Uzawa-type model. Kim and Kose (2001) reached similar conclusions.

where α_2 is the proportion of imported goods. The aggregate expenditure constraint for tradeable goods is given by the following expression:

$$P^T C^T = EF + EP^x X \tag{12}$$

where E is the nominal exchange rate, and P^x is the ratio of foreign export prices to foreign import prices, the terms of trade index (with $P^m = 1$)

The demand for export and import goods are functions of the aggregate consumption of traded goods as well as the terms of trade index:

$$\begin{aligned} F &= \alpha_2 (P^x)^{1-\alpha_2} C^T & (13) \\ X &= (1-\alpha_2)(P^x)^{-\alpha_2} C^T & (14) \end{aligned}$$

Similarly the price of traded goods may be expressed as a geometric average of the price of imported and export goods:

$$P^T = E(P^x)^{1-\alpha_2} \tag{15}$$

2.2 Production

Production of exports and imports is by the Cobb-Douglas technology:

$$\begin{aligned} Q^x &= A^x P^x \epsilon^x (K^x)^{1-\alpha_x} & (16) \\ Q^f &= A^f \epsilon^f (K^f)^{1-\alpha_f} & (17) \end{aligned}$$

where ϵ^x, ϵ^f represent productivity shocks for export and import-producing firms, while $(1-\alpha_x), (1-\alpha_f)$ are the capital coefficients, and A^x, A^f the total factor productivity effects.

Total capital is simply the sum of capital in each sector. Hence:

$$K = K^x + K^f \tag{18}$$

The production of non-traded goods is given by the interaction of an exogenous productivity shock with a fixed productive resource, $\overline{L}$:

$$Q^n = \epsilon^n \overline{L} \tag{19}$$

2.3 Aggregate Budget Constraint

The combined household/firm faces the following budget constraint, in terms of domestic purchasing power:

$$
\begin{aligned}
C_t &= \frac{E}{P}\left[A^f \epsilon^f (K_t^f)^{1-\alpha_f}\right] + \frac{EP^x}{P}\left[A^x P^x \epsilon^x (K_t^x)^{1-\alpha_x}\right] \\
&+ \frac{P^n}{P}\epsilon^n \overline{L} - K_{t+1} + K_t(1-\delta) - \frac{\phi}{2}\left[K_{t+1} - K_t\right]^2 \\
&+ \frac{E}{P}L^*_{t+1} - \frac{E}{P}L^*_t[1 + i^*_t + ({}_t e_{t+1} - e_t) + \theta_t - \pi_t] \\
&- B_{t+1} + B_t(1 + i_t - \pi_t)
\end{aligned} \tag{20}
$$

The aggregate resource constraint shows that the firms producing tradeable goods face quadratic adjustment costs when they accumulate capital, with these costs given by the term $\frac{\phi}{2}\left[K_{t+1} - K_t\right]^2$. For both firms, capital depreciates at a fixed rate δ.

Firms and households can borrow internationally and accumulate international debt L^* at the fixed rate i^*, but face a cost of borrowing in domestic currency which includes not only the expected rate of depreciation, $({}_t e_{t+1} - e_t)$ but also a time-varying risk premium θ_t, less expected inflation. The variable e is the logarithm of the nominal exchange rate E, and ${}_t e_{t+1}$ the expected logarithmic rate at time t.

The evolution of currency risk θ_t depends on the time-varying volatility of the rate of depreciation, here proxied by the absolute value of the lagged annualized rate of depreciation,as well as on its own lag:

$$
\theta_t = \xi_0 + \xi_1 \theta_{t-1} + \xi_2 \left| e_{t-1} - e_{t-4} \right| + \eta_t \tag{21}
$$

$$
\eta \sim N(0, \sigma_\eta^2) \tag{22}
$$

The higher the volatility of the rate of depreciation, the higher the level of the risk premium demanded by international lenders. Hence $\xi_2 > 0$.

The consolidated household/firm may also lend to the domestic government and accumulate bonds B which pay the nominal interest rate i.

2.4 Euler Equations

The consolidated household and firm solves the following intertemporal welfare optimization problem by choosing the path of "controls" $\{\nu_t\}$, representing consumption $\{C_t\}$, aggregate capital $\{K_{t+1}\}$, capital in the import-competing industries $\{K^f_{t+1}\}$, foreign borrowing $\{L^*_{t+1}\}$, and government bonds $\{B_{t+1}\}$:

$$\begin{aligned} W_t(\nu_t) &= \max_{\nu_t} U(C_t) + \vartheta_{t+1} W_{t+1}(\nu_{t+1}) \\ \nu_t &= \{C_t, K_{t+1}, K^f_{t+1}, B_{t+1}, L^*_{t+1}\} \end{aligned} \tag{23}$$

subject to the budget constraint, given in equation (20), as well as the following inequality restrictions:

$$C_t > 0 \tag{24}$$

$$K^f_t > 0 \tag{25}$$

$$K^x_t > 0 \tag{26}$$

The first order conditions are given by the following equations, representing the derivatives of constrained intertemporal optimization with respect to K_{t+1}, K^f_{t+1}, B_{t+1}, L^*_{t+1}:

$$\lambda_t = C_t^{-\gamma} \tag{27}$$

$$\begin{aligned} \frac{\lambda_t[1 + (\phi K_{t+1} - K_t)]}{\vartheta_{t+1}\lambda_{t+1}} &= \frac{EP^x_t}{P} \cdot \left[(1-\alpha_x)A^x P^x_{t+1}\epsilon^x_{t+1}(K_{t+1} - K^f_{t+1})^{-\alpha_x}\right] + \\ & (1-\delta) + \phi\left[K_{t+2} - K_{t+1}\right] \end{aligned} \tag{28}$$

$$(K_{t+1} - K^f_{t+1})^{-\alpha_x} = \frac{(1-\alpha_f)\left[A^f \epsilon^f_{t+1}(K^f_{t+1})^{-\alpha_f}\right]}{(1-\alpha_x)A^x P^x_{t+1}\epsilon^x_{t+1}} \tag{29}$$

$$\frac{\lambda_t}{\vartheta_{t+1}\lambda_{t+1}} = (1 + i_{t+1} - \pi) \tag{30}$$

$$\frac{\lambda_t}{\vartheta_{t+1}\lambda_{t+1}} = \left[1 + i^*_{t+1} + ({}_te_{t+2} - e_{t+1}) + \theta_{t+1} - \pi\right] \tag{31}$$

The first Euler equation is the familiar condition that the marginal utility of wealth is equal to the marginal utility of income.

The second equation relates to the marginal productivity of capital. Capital should be accumulated until the gross marginal productivity of capital, adjusted for depreciation and transactions costs is equal to the marginal utility of consumption today divided by the discounted marginal utility tomorrow.

The third equation simply states that the marginal utility of capital in each sector should be equal.

The last two equations tell us that the gross real returns on domestic or foreign assets should also be equal to the marginal utility of consumption today divided by the discounted marginal utility tomorrow.

The first equation may be combined with the fourth equation to solve for current consumption as a function of next period's expected marginal utility:

$$C_t = (\vartheta_{t+1}\lambda_{t+1} \cdot \{1 + i_{t+1} - \pi\})^{-\frac{1}{\gamma}} \tag{32}$$

The last two equations may be combined to give the interest arbitrage condition:

$$e_t = (i_t^* + \theta_t - i_t) +_t e_{t+1} \tag{33}$$

Both current consumption and the logarithm of the exchange rate depend on their expected future values.

The solution of the investment equation for aggregate capital and for capital in the two sectors takes place by equating the marginal productivity with the real returns of either domestic or foreign assets:

$$\begin{aligned}(1 + r_{t+1})[1 + (\phi K_{t+1} - K_t)] &= \frac{EP^x_{t+1}}{P}\left[(1-\alpha_x)A^x\epsilon^x_{t+1}(K_{t+1} - K^f_{t+1})^{-\alpha_x}\right] + \\ &\quad (1-\delta) + \phi\left[K_{t+2} - K_{t+1}\right] \end{aligned} \tag{34}$$

To solve for the capital stock, one first solves for K^x_{t+1} as a function of the real interest rate and the expected aggregate capital stocks, K_{t+1} and K_{t+2}, used to compute the costs of adjustment:

$$K^x_{t+1} = \left(\frac{(1 + r_{t+1})[1 + (\phi K_{t+1} - K_t)] - (1-\delta) - \phi\left[K_{t+2} - K_{t+1}\right]}{\frac{EP^x_{t+1}}{P}(1-\alpha_x)A^x\epsilon^x_{t+1}}\right)^{-\frac{1}{\alpha_x}} \tag{35}$$

Aggregate investment is the change in the total capital stock:

$$\Delta K_t = \Delta K^x + \Delta K^f \tag{36}$$

Investment in the capital stock takes place with imported goods, F.

2.5 Macroeconomic Identities and Market Clearing Conditions

From the above equations, the trade balance, TB, expressed in domestic currency, is simply net exports less net imports, inclusive of goods used for investment:

$$TB_t = \frac{EP^x}{P}[Q_t^x - X_t] - \frac{E}{P}[Q_t^f - \Delta K_t - F_t] \tag{37}$$

while the current account balance, CAB, is simply the trade balance plus interest on international debt:

$$CAB_t = TB_t - \frac{E}{P}L_t^*[i_t^* + ({}_te_{t+1} - e_t) + \theta_t - \pi_t] \tag{38}$$

Under flexible exchange rates, net capital inflows are simply the mirror image of the current account:

$$\frac{E}{P}L_{t+1}^* = \frac{E}{P}L_t^* - CAB_t \tag{39}$$

While the exchange rate is determined by the forward-looking interest parity relation, and the terms of trade are determined exogenously, the price of non-traded goods adjusts in response to demand and supply in this sector. To capture more realistic conditions of "sticky prices" in this sector, this model assumes that the price of non-tradeables follows a partial adjustment process to conditions of excess demand or supply:

$$\ln(P_{t+1}^N) - \ln(P_t^N) = (1 - \psi)[N_t - Q^n] \tag{40}$$

where ψ represents the degree of price stickiness.

2.6 The Consolidated Government Sector

2.6.1 Fiscal Authority

The government is bound by the following budget constraint:

$$B_{t+1} - B_t = (i - \pi)B_t + D_t \tag{41}$$

For simplicity, it is assumed that the government deficit D is usually zero. However, the fiscal authority will exact lump sum taxes from non-traded goods sector in order to run a surplus and "buy back" domestic debt if it grows above a critical domestic debt/gdp ratio or threshold, $\widetilde{B}$. Similarly, if the external debt grows above a critical external debt/gdp ratio, $\widetilde{L^*}$, the fiscal authority will levy taxes in the traded-goods sector in order to reduce or buy-back external debt.

With these assumptions about fiscal policy the usual no-Ponzi game applies to the evolution of real government debt:

$$\begin{aligned} \lim_{t\to\infty} B_t \exp^{-(i-\pi)t} &= 0 && (42) \\ \lim_{t\to\infty} L_t^* \exp^{-(i^i+\theta+\Delta e_{t+1}-\pi)t} &= 0 && (43) \end{aligned}$$

2.6.2 Monetary Authority

The monetary authority does not know the "correct" model for the evolution of inflation. We assume three different policy scenarios. In the pure inflation target case, the monetary authority estimates the evolution of inflation as a function of its own lag as well as of changes in the interest rate. In the inflation/growth scenario, the central bank estimates the evolution of inflation and growth as functions of their own lags and of changes in the interest rate. Finally, in the inflation/growth/depreciation scenario, the central bank estimates the evolution of all three as functions of their own lags as well as of changes in the interest rate. "Least squares learning" is used to forecast the future values of these "state" variables in each scenario.

$$\Psi_t = \Gamma_{1t}\Psi_{t-1} + \Gamma_{2t}\Delta i_t \qquad (44)$$

where $\Psi_t = [\pi_t]$ in the first scenario, $\Psi_t = [\ \pi_t,\ \Delta y_t]$ in the second scenario, and $\Psi_t = [\pi_t\ \Delta y_t\ \Delta e_t]$ in the third scenario.

Corresponding to each scenario, the government optimizes the following loss function Λ,

$$\begin{aligned} \Lambda_1 &= \lambda_{1t}(\pi_t - \pi^*)^2 && (45) \\ \Lambda_2 &= \lambda_{1t}(\pi_t - \pi^*)^2 + \lambda_{2t}(\Delta y_t - \psi)^2 && (46) \\ \Lambda_3 &= \lambda_{1t}(\pi_t - \pi^*)^2 + \lambda_{2t}(\Delta y_t - \psi)^2 + \lambda_{3t}(\Delta e_t - \chi)^2 && (47) \end{aligned}$$

where π^*, ψ, and χ represent the targets for the inflation, output growth and depreciation rates under alternative policy scenarios.

At time t, depending on the scenario, the monetary authority specifies the weights on the loss function, $\lambda_t = \{\lambda_{1t}, \lambda_{2t}, \lambda_{3t}\}$, and estimates the state-space system. From the parameter set $\widehat{\Gamma}_t$, the policy maker sets the systematic part of the interest rate as an optimal feedback function of the state variables Ψ_t :

$$\begin{aligned} i_{t+1} &= i_t + h(\widehat{\Gamma}_t, \lambda_t)\Psi_t + \varepsilon_t && (48) \\ \varepsilon &\sim N(0, \sigma_\varepsilon^2) && (49) \end{aligned}$$

where $h(\widehat{\Gamma}_t, \lambda_t)$ is the solution of the optimal linear quadratic "regulator" problem, with control variable Δi solved as a feedback response to the state variables, and ε_t is a random, non-systematic component of the interest rate at time t.

In formulating its optimal interest-rate feedback rule, the government acts as time t as if its estimated model for the evolution of inflation and output growth is true "forever", and that its relative weights for inflation, or growth or depreciation in the loss function are permanently fixed.

However, as Sargent (1999) points out in a similar model, the monetary authority's own procedure for re-estimation "falsifies" this pretense as it updates the coefficients $\{\Gamma_{1t}, \Gamma_{2t}\}$, and solves the linear quadratic regulator problem for a new optimal response "rule" of the interest rate to the evolution of the state variables Ψ_t.

The weights for inflation, output growth, and depreciation in the respective loss functions Λ_i depend on the conditions at time t.

In the pure anti-inflation scenario, if inflation is below the target level π^* then the government does not optimize. The interest rate $i_t = i_{t-1}$. This is the "no intervention" case. However, if inflation is above the target rate, the monetary authority puts greater weight on inflation. In this case, $\lambda_{1t} = 0.9$. Table I illustrates this scenario.

Table I: Policy Weights	
Inflation Only Target	
$\pi < 0.02$	do nothing
$\pi \geqq 0.02$	$\lambda_1 = 0.9$

In the second scenario, if inflation is below the target level π^* and output growth is positive, then the government does not optimize. If inflation is above the target rate, with positive growth, the monetary authority puts greater weight on inflation than on output growth. In this case, $\lambda_{1t} = 0.9$. If growth is negative but inflation is above its target, the inflation weight dominates but somewhat more weight is given to output. Finally, if inflation is below its target but output growth is negative, the central bank puts strong weight on the output target. The weights for this policy scenario are summarized in Table II.

Table II: Policy Weights		
Inflation and Growth Targets		
Inflation	Growth	
	$\Delta y < 0$	$\Delta y \geqq 0$
$\pi < 0.02$	$\lambda_1 = 0.1$ $\lambda_2 = 0.9$	do nothing
$\pi \geqq 0.02$	$\lambda_1 = 0.8$ $\lambda_2 = 0.2$	$\lambda_1 = 0.9$ $\lambda_2 = 0.1$

In the third scenario, targets for the depreciation rate are also taken into account for formulating the policy feedback rule. The relative weights are summarized in Table III.

Table III: Policy Weights			
Inflation, Growth and Exchange Rate Targets			
Inflation	Exchange Rate	Growth	
		$\Delta y \leqq 0$	$\Delta y > 0$
$\pi < 0.02$	$\Delta e < 0.02$	$\lambda_1 = 0.1$ $\lambda_2 = 0.8$ $\lambda_3 = 0.1$	do nothing
	$\Delta e \geqq 0.02$	$\lambda_1 = 0.1$ $\lambda_2 = 0.7$ $\lambda_3 = 0.2$	$\lambda_1 = 0.1$ $\lambda_2 = 0.1$ $\lambda_3 = 0.8$
$\pi \geqq 0.02$	$\Delta e < 0.02$	$\lambda_1 = 0.7$ $\lambda_2 = 0.2$ $\lambda_3 = 0.1$	$\lambda_1 = 0.8$ $\lambda_2 = 0.1$ $\lambda_3 = 0.1$
	$\Delta e \geqq 0.02$	$\lambda_1 = 0.5$ $\lambda_2 = 0.3$ $\lambda_3 = 0.2$	$\lambda_1 = 0.7$ $\lambda_2 = 0.1$ $\lambda_3 = 0.2$

3 Calibration and Solution

The section discusses the calibration of parameters, initial conditions, and stochastic processes for the exogenous variables of the model as well as the specification of the policy rules and risk premia "reaction function". Then it briefly discuses parameterized expectations algorithm (PEA) for solving the model.

3.1 Parameters and Initial Conditions

The parameter settings for the model appear in Table IV.

Table IV: Parameters

Consumption	$\gamma = 3.5, \quad \beta = 0.009$
	$\alpha_1 = 0.5, \quad \alpha_2 = 0.5,$
Production	$\alpha_f = 0.7, \quad \alpha_x = 0.5$
	$\delta = 0.1, \quad \phi = 0.028$
	$A^f = 0.1881, \quad A^x = 0.3115, \quad A^n = 1.0$
Price Coefficient	$\sigma = 0.7$
Debt Thresholds	$\tilde{b} = 0.5, \quad \tilde{l}^* = 0.5$

Many of the parameter selections follow Mendoza (1995, 2001). The constant relative risk aversion is set at 3.5, somewhat below the value of 5 usually set for developing countries. The shares of non-traded goods in overall consumption is set at 0.5, while the shares of exports and imports in traded goods consumption is 50 percent each. The production function coefficients Q^f and Q^x, along with the initial values of capital for each sector, are chosen to ensure that the marginal product of capital in each sector is equal to the real interest plus depreciation, while the level of production meets demand in each sector. In particular the values for α_f and α_x reflect the assumption that the production of commodity exports is more capital intensive than manufactured imports.

The initial values of the variables appear in Table V.

Table V: Initial Conditions

Consumption	$C = 2.02, \quad N = 1.01, \quad X = F = 0.505$
Production	$Q^f = Q^x = 0.505, \quad Q^n = 1.01$
Capital	$K^f = 4.33, \quad K^x = 7.21$
Prices	$E = P^n = Z = 1$
Interest Rates	$i = 0.04$
Debt	$B = L^* = 0.404$

The initial value of consumption is given by the steady-state value implied by the interest rate and the endogenous discount factor. The values of F, X, and N are then calculated on the basis of the preference parameters in the sub-utility functions. Similarly, the initial values of production are chosen on the basis of the initial steady state values.

Since the focus of the study is on the effects of time-varying currency risk and terms of trade shocks, the domestic productivity coefficients as well as the foreign interest rate were fixed at unity through the simulations. Table VI gives the values of these fixed variables.

Table VI: Fixed Values	
Foreign Interest rate	$i^* = 0.04$
Productivity shocks	$\epsilon^n = \epsilon^x = \epsilon^f = 1$

3.2 Terms of Trade and Currency Risk

The evolution of the terms of trade is specified to mimic the data generating processes estimated for several countries, namely that the variable is an $I(1)$ process:

$$\begin{aligned} \ln(P_t^x) &= \ln(P_{t-1}^x) + \eta_t^{P^x} \\ \eta_t^{P^x} &\sim N(0, 0.01) \end{aligned}$$

The parameter values for the evolution of currency risk appear in Table VII.

Table VII: Currency Risk Parameters	
Constant	$\xi_0 = 0.0$
Lag	$\xi_1 = 0.5$
Change in Exchange Rate	$\xi_2 = 1.0$
Variance of Shock	$\sigma_\eta^2 = 0.0$

These are set to sharpen the focus on the feedback effects of changes in the exchange rate on risk. Thus, there is no long-run constant "currency risk" and no other source of risk, except changes in the exchange rate, so that $\xi_0 = 0$ and $\sigma_\eta^2 = 0$.

3.3 Solution Algorithm

Following Marcet (1988, 1993), Den Haan and Marcet (1990, 1994), and Duffy and McNelis (2001), the approach of this study is to "parameterize" the forward-looking expectations in this model, with non-linear functional forms ψ^E, ψ^c:

$$\mathbf{E}_t\left(\vartheta_{t+1}\lambda_{t+1}\cdot\{1+i_{t+1}-\pi\}\right)=\psi^c(x_{t-1};\Omega_\lambda) \tag{50}$$

$$\mathbf{E}_t e_{t+1}=\psi^E(x_{t-1};\Omega_E) \tag{51}$$

where x_t represents a vector of observable variables at time t: consumption of imported and export goods, F and X, the marginal utility of consumption λ , the real interest rate r, and the real exchange rate, Z,

$$x_t=\{F,X,\lambda,r,Z\} \tag{52}$$

while Ω_λ,Ω_E represent the parameters for the expectation function.

Judd (1996) classifies this approach as a "projection" or a "weighted residual" method for solving functional equations, and notes that the approach was originally developed by Williams and Wright (1982, 1984, 1991). These authors pointed out that the conditional expectation of the future grain price is a "smooth function" of the current state of the market, and that this conditional expectation can be used to characterize equilibrium.

The function forms for ψ^E,ψ^c are usually second-order polynomial expansions [see, for example, Den Haan and Marcet (1994)]. However, Duffy and McNelis (2001) have shown that neural networks have produced results with greater accuracy for the same number of parameters, or equal accuracy with few parameters, than the second-order polynomial approximation.

The model was simulated for repeated parameter values for $\{\Omega_\lambda,\Omega_E\}$ until convergence was obtained for the expectational errors. A description of the solution algorithm appears in the appendix.

4 Simulation Results

To evaluate the effects of the alternative policy scenarios, 500 simulations of sample length 200 quarters were generated. Table VIII shows the means and standard deviations of the volatility of inflation, output growth, depreciation, and the change in the interest rate.

What is most startling about Table VIII is that incorporating growth as a target actually increases the volatility of growth, over the case of pure inflation targeting. Furthermore, incorporating exchange rate depreciation as a target, actually increases the volatility of inflation. Not surprisingly, the volatility of interest-rate changes increase as more targets are incorporated into the policy objectives and learning mechanism.

Table VIII: Volatility Measures for Alternative Policy Scenarios Means and Standard Deviations (in Parenthesis)			
	Policy Targets		
Variable	π	$\pi, \Delta y$	$\pi, \Delta y, \Delta e$
Volatility of Inflation (π)	0.0066 (0.0012)	0.0063 (0.0011)	0.0087 (0.0017)
Volatility of Growth (Δy)	0.1313 (0.0207)	0.2128 (0.0386)	0.1768 (0.0430)
Volatility of Exchange Rate Depreciations (Δe)	0.0012 (0.0001)	0.0012 (0.0008)	0.0018 (0.0015)
Volatility of Interest Rate Changes (Δi)	0.0001 (0.0002)	0.0012 (0.0010)	0.0020 (0.0019)
Intertemporal Welfare Index	0.3627 (0.0040)	0.2818 (0.0156)	0.2772 (0.0208)

The Epanechnikov kernel estimators for the distribution of inflation under the three policy scenarios appears in Figure 1. The Figure shows the reduction in mean (and spread) of volatility as the central bank changes its targets from the narrow pure inflation scenario to the broader inflation/growth scenario. However, incorporating depreciation increases the mean and spread of inflation volatility.

Figure 1: Kernel Estimates for Inflation
Under Alternative Policy Scenarios

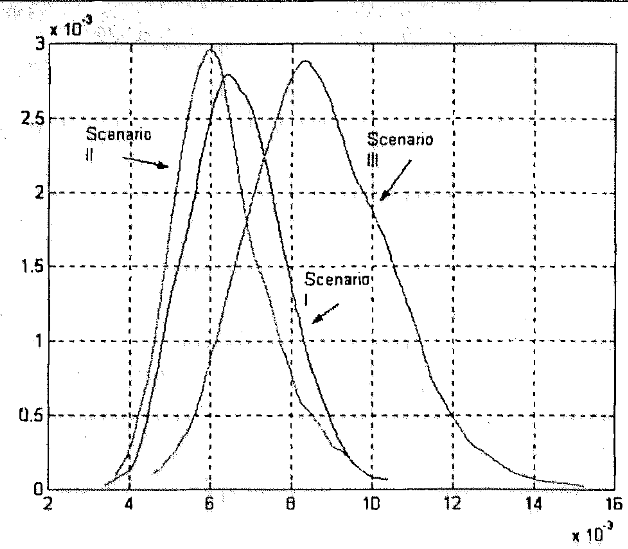


Figure 2 shows the distribution of the volatility estimates for economic growth under the three policy scenarios. Incorporating the additional growth target, or the additional growth and depreciation targets, markedly increase the expected volatility of growth.

Figure 2: Kernel Estimates for GDP Growth
Under Alternative Policy Scenarios

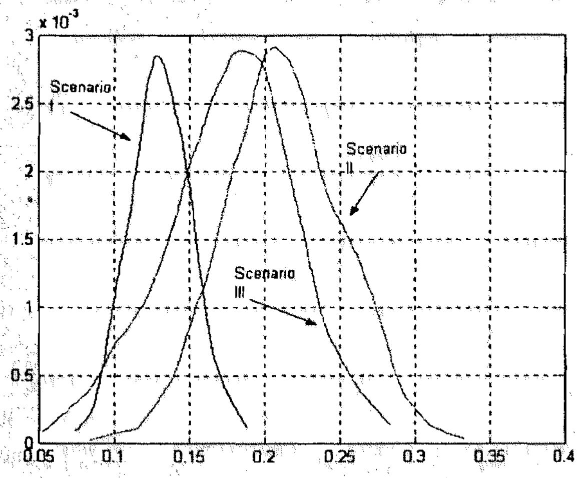
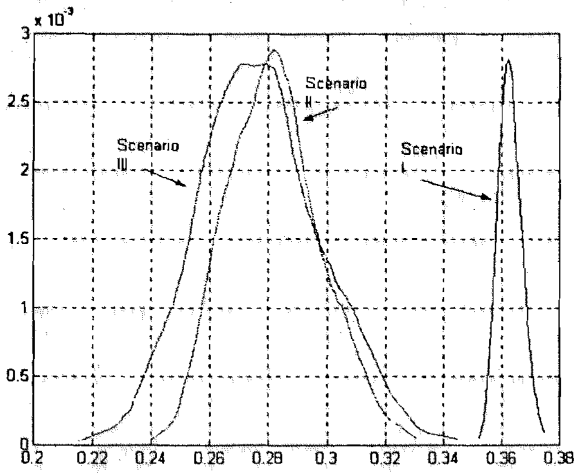


Figure 3 shows that the distribution of the welfare index is significantly higher under pure a inflation targeting scenario than under a growth-inflation or growth-inflation-depreciation targeting framework.

Figure 3: Kernel Estimates for Welfare
Under Alternative Policy Scenarios



5 Conclusions

This paper has compared three alternative policy scenarios for a central bank facing terms of trade shocks and time-varying currency risk. Unlike the private sector, the central bank has to learn the laws of motion for its key target variables in order to set the interest rate according to a feedback rule.

The results show that including growth or both growth and exchange rate changes in its learning and policy targeting framework will significantly increase output variability with the possibility of only small reductions in inflation variability. Adopting such a framework will unambiguously reduce welfare. The policy implication is that central banks which are already targeting inflation, should resist pressures to adopt growth or growth and exchange-rate targets.

Of course, the results of this paper may be conditioned by several key assumptions. One is the learning mechanism. Central banks may indeed have more sophisticated knowledge of underlying inflation dynamics than that which is implied by linear least squares learning. However, linear least squares learning is a good "tracking" mechanism for more complex dynamic processes and our recursive method serves as an approximation to the Kalman filtering method for updating and learning.

The other strong assumption of this paper is the evolution of currency risk. This variable may well be conditioned by changes in public internal debt and, external debt as well as inflation and exchange rate changes. How robust our policy results are to the way in which currency risk evolves is an open question. But assuming that currency risk first and foremost responds to past changes in the exchange rate is a sensible first approximation, and would bias the case, if at all, in favor of exchange rate targeting for the central bank.

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APPENDIX

Intertemporal Lagrangean Representation

Below is the equivalent Lagrangean expression of the intertemporal optimization problem of the representative household-firm:

$$\begin{aligned}
\underset{\langle \nu_t \rangle}{\text{Max}} V &= U(c_t) - \lambda_t \{ \frac{E}{P} \left[A^f \epsilon_t^f (K_t^f)^{1-\alpha_f} \right] + \frac{Ep^x}{P} \left[A^x p_t^x \epsilon_t^x (K_t^x)^{1-\alpha_x} \right] + \\
& \frac{P^n}{P} \epsilon_t^n \overline{L} - K_{t+1} + K_t(1-\delta) - \frac{\phi}{2} \left[K_{t+1} - K_t \right]^2 + \\
& \frac{E}{P} L_{t+1}^* - \frac{E}{P} L_t^* [1 + i_t^* + ({}_t e_{t+1} - e_t) + \theta_t - \pi_t] - \\
& \frac{P^n}{P} def_t - B_{t+1} + B_t(1 + i_t - \pi_t) - c_t \} + \\
& U(c_{t+1}) - \vartheta_{t+1} \lambda_{t+1} \{ \frac{E}{P} \left[A^f \epsilon_{t+1}^f (K_{t+1}^f)^{1-\alpha_f} \right] + \\
& \frac{Ep_{t+1}^x}{P} \left[A^x \epsilon_{t+1}^x (K_{t+1} - K_{t+1}^f)^{1-\alpha_x} \right] + \\
& \frac{P^n}{P} \epsilon_{t+1}^n \overline{L} - K_{t+2} + K_{t+1}(1-\delta) - \frac{\phi}{2} \left[K_{t+2} - K_{t+1} \right]^2 + \\
& \frac{E}{P} L_{t+2}^* - \frac{E}{P} L_{t+1}^* [1 + i_{t+1}^* + ({}_t e_{t+2} - e_{t+1}) + \theta_{t+1} - \pi_{t+1}] + \\
& \frac{P^n}{P} def_{t+1} - B_{t+2} + B_{t+1}(1 + i_{t+1} - \pi_{t+1}) - c_{t+1} \} \qquad (53)
\end{aligned}$$

$$\nu_t = \{ c_t, K_{t+1}, K_{t+1}^f, B_{t+1}, L_{t+1}^* \} \qquad (54)$$

Solution Algorithm

The solution algorithm for parameterized expectations makes use of neural network specification for the expectations, and a genetic algorithm for the iterative solution method, as well as the quasi-Newton method.

The specification of the functional forms $\psi^E(x_t; \Omega_E)$ and $\psi^c(x_t; \Omega_c)$ according to the neural network approximation, is done in the following way:

$$\begin{aligned}
n_{k,t} &= \sum_{j=1}^{J^*} b_j x_{j,t} \qquad (55) \\
N_{k,t} &= \frac{1}{1 + e^{-n_{i,t}}} \\
\widehat{\psi}_t &= \sum_{k=1}^{K^*} \kappa_k N_{k,t}
\end{aligned}$$

where J^* is the number of exogenous or input variables, K^* is the number of neurons, n_t is a linear combination of the input variables, N_t is a logsigmoid or logistic transformation of n_t, and $\widehat{\psi}_t$ is the neural network prediction at time t for either (e_{t+1}) or $\exp[-V(C_t)]\lambda_{t+1} \cdot \{1 + i_{t+1} - \pi\}$.

As seen in this equation, the only difference from ordinary non-linear estimation relating "regressors" to a "regressand" is the use of the hidden nodes or neurons, N. One forms a neuron by taking a linear combination of the regressors and then transforming this variable by the logistic or logsigmoid function. One then proceeds to thus one or more of these neurons in a linear way to forecast the dependent variable $\widehat{\psi}_t$.

Judd (1996) notes that the neural networks provide us with an "inherently nonlinear functional form" for approximation, in contrast with methods based on linear combinations of polynomial and trigonometric functions.

Both Judd (1996) and Sargent (1997) have drawn attention to the work of Barron (1993), who found that neural networks do a better job of "approximating" any non-linear function than polynomials, in that sense that a neural network achieves the same degree of in-sample predictive accuracy with fewer parameters, or achieves greater accuracy, using the same number of parameters. For this reason, Judd (1996) concedes that neural networks may be particularly efficient at "multidimensional approximation".

The main choices that one has to make for a neural network is J^*, the number of regression variables, and K^*, the number of hidden neurons, for predicting a given variable ψ_t. Generally, a neural network with only one hidden neuron closely approximates a simple linear model, whereas larger numbers of neurons approximate more complex non-linear relationships. Obviously, with a large number of "regressors" x and with a large number of neurons N, one approximates progressively more complex non-linear phenomena, with an increasingly larger parameter set.

The approach of this study is to use relatively simple neural networks, between two and four neurons, in order to show that even relatively simple neural network specifications do well for approximating non-linear relations implied by forward-looking stochastic general equilibrium models.

Since the parameterized expectation solution is a relatively complex nonlinear function, the optimization problem is solved with a repeated hybrid approach. First a global search method, genetic algorithm, similar to the one developed by Duffy and McNelis (2001), is used to find the initial parameter set, then a local optimization, the BFGS method, based on the quasi-Newton algorithm, is used to "fine tune" the genetic algorithm solution.

De Falco (1998) applied the genetic algorithm to nonlinear neural network estimation, and found that his results "proved the effectiveness" of such algorithms for neural network estimation. The main drawback of the genetic algorithm is that it is slow. For even a reasonable size or dimension of the coefficient vector, the various combinations and permutations of the coefficients which the genetic search may find "optimal" or close to optimal, at various generations, may become very large. This is another example of the well-known "curse of dimensionality" in non-linear optimization. Thus, one needs to let the genetic algorithm "run" over a large number of generations—perhaps several hundred—in order to arrive at results which resemble unique and global minimum points.

Quagliarella and Vicini (1998) point out that hybridization may lead to better solutions than those obtainable using the two methods individually. They argue that it is not necessary to carry out the quasi-Newton optimization until convergence, if one is going to repeat the process several times. The utility of the quasi-Newton BFGS algorithm is its ability to improve the "individuals it treats", so "its beneficial effects can be obtained just performing a few iterations each time" [Quagliarella and Vicini (1998): 307].

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