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TESTING FOR ASYMMETRY IN INTEREST RATE VOLATILITY IN THE PRESENCE OF A NEGLECTED LEVEL EFFECT

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Testing for Asymmetry in Interest Rate Volatility in the Presence of a Neglected Level Effect*

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Abstract

Empirical evidence documents a level effect in the volatility of short term rates of interest. That is, volatility is positively correlated with the level of the short term interest rate. Using Monte-Carlo simulations this paper examines the performance of the commonly used Engle-Ng (1993) tests which differentiate the effect of good and bad news on the predictability of future short rate volatility. Our results show that the tests exhibit serious size distortions and loss of power in the face of a neglected level effect.

Keywords: Level Effects; Asymmetry; Engle-Ng Tests
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1 Testing for Asymmetry in Volatility

Asymmetry in volatility may be detected using the Engle and Ng (1993) sign and size bias tests. These tests are commonly used to differentiate the effect of good and bad news on the predictability of stock returns volatility (see Engle and Ng, 1993, Henry, 1998, Kroner and Ng, 1998, Brooks and Henry, 2002, *inter alia*). Brenner, Harjes and Kroner (1996), Bali (2000 a,b), *inter alia,* report evidence of asymmetry in US short-term interest rates. It is not clear whether the Engle-Ng tests may be used as a diagnostic tool for interest rates which display a level effect. The aim of this note is to determine whether the Engle-Ng tests provide reliable inferences regarding the sign and size bias in short rate volatility.

Engle and Ng (1993) (Engle-Ng, hereafter) develop a test for size and sign bias in conditionally heteroscedastic models. Consider a GARCH (1,1) model of the form

$$x_t = \mu + \varepsilon_t, \quad \varepsilon_t|\Omega_{t-1} \backsim N(0, h_t) \tag{1}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1}.$$

Define I_{t-1}^- as an indicator dummy that takes the value of 1 if $\varepsilon_{t-1} < 0$ and the value zero otherwise. The test for sign bias is based on the significance of ϕ_1 in

$$\upsilon_t^2 = \phi_0 + \phi_1 I_{t-1}^- + e_t \tag{2}$$

where υ_t^2 is the squared standardised residuals and e_t is a white noise error term. If positive and negative innovations to ε_t impact on the conditional variance of x_t differently to the prediction of the model, then ϕ_1 will be statistically significant. It may also be the case that the source of bias is caused not only by the sign, but also the magnitude or the size of the shock. The negative size bias test is based on the significance of the slope coefficient ϕ_1 in

$$\upsilon_t^2 = \phi_0 + \phi_1 \varepsilon_{t-1} I_{t-1}^- + e_t \tag{3}$$

The test statistics for the individual sign and size bias tests are distributed asymptotically with a t-distribution. Likewise, define $I_{t-1}^+ = 1 - I_{t-1}^-$, then the Engle-Ng joint test for asymmetry in variance is based on the regression

$$\upsilon_t^2 = \phi_0 + \phi_1 I_{t-1}^- + \phi_2 \varepsilon_{t-1} I_{t-1}^- + \phi_3 \varepsilon_{t-1} I_{t-1}^+ + e_t \tag{4}$$

where e_t is a white noise disturbance term. Significance of parameter ϕ_1 indicates the presence of sign bias. That is positive and negative realisations of ε_{t-1} affect future volatility differently to the prediction of the model. Similarly significance of ϕ_2 and ϕ_3 would suggest size bias, where not only the sign, but also the magnitude of innovation in x_t is important. A joint test for sign and size bias, based upon the Lagrange Multiplier Principle, may be performed as $T \cdot R^2$ from the estimation of (4) where T is the number of observations in the regression and R^2 is the coefficient of determination of the regression.

2 Short Term Interest Rate Models

Chan, Karolyi, Longstaff and Sanders (1992) (CKLS, hereafter) propose the general non-linear process for short-term interest rates, $\{r_t, t \geq 0\}$, written as

$$dr = (\mu + \lambda r)\, dt + \phi r^{\delta} dW. \tag{5}$$

Here r represents the level of the short-term interest rate, W is a Brownian motion and μ, λ and δ are parameters. The drift component of short-term interest rates is captured by $\mu + \lambda r$ while the variance of unexpected changes in interest rates equals $\phi^2 r^{2\delta}$. The parameter ϕ is a scale factor and δ controls the degree to which the interest rate level influences the volatility of short-term interest rates. By placing restrictions on δ, the CKLS model nests many of the existing interest rate models. For example, when $\delta = 0$ then (5) reduces to the Vasicek (1977) model, while $\delta = 1/2$ yields the Cox, Ingersoll and Ross (1985) model, see CKLS, Bekaert et al. (2002), *inter alia* for further details.

Brenner, Harjes and Kroner (1996) (BHK, hereafter) argue that by allowing ϕ^2 to be a time varying function of the information set, Ω, it gives rise to a superior conditional characterisation of short term interest rate changes. CKLS and BHK, *inter alia,* consider the Euler-Maruyama discrete time approximation to (5) written as

$$\Delta r_t = \mu + \lambda r_{t-1} + \varepsilon_t. \tag{6}$$

Here Ω_{t-1} represents the information set available at time $t-1$ and $E\left(\varepsilon_t | \Omega_{t-1}\right) = 0$. Letting h_t represent the conditional variance of the short-term interest rate then $E\left(\varepsilon_t^2 | \Omega_{t-1}\right) \equiv h_t = \phi^2 r_{t-1}^{2\delta}$. The sole source of conditional heteroscedasticity in (6) is through the level of the interest rate and thus excludes the information arrival process.

One common approach to capturing the effect of news is the GARCH(1,1) model

$$h_t = \alpha_0 + \beta h_{t-1} + \alpha_1 \varepsilon_{t-1}^2. \tag{7}$$

The innovation ε_t represents a change in the information set from time $t-1$ to t and can be treated as a collective measure of news. In (7) only the magnitude of the innovation is important in determining h_t. BHK extend (6) to allow for volatility clustering caused by information arrival using

$$\begin{aligned} \Delta r_t &= \mu + \lambda r_{t-1} + \varepsilon_t. \\ E(\varepsilon_t|\Omega_{t-1}) &= 0,\ E\left(\varepsilon_t^2|\Omega_{t-1}\right) \equiv h_t = \phi_t^2 r_{t-1}^{2\delta} \\ \phi_t^2 &= \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta\phi_{t-1}^2 \end{aligned} \tag{8}$$

Equation (8) defines the multiplicative level effect model given that the conditional volatility of the short-rate change is multiplicatively dependent on the short rate levels. In high information periods, when the magnitude of ε_t is largest then the sensitivity of volatility to the level of short term interest rates is highest. Under the restriction $\alpha_1 = \beta = 0, (8)$ collapses to (6) and volatility depends on levels alone. BHK generalise both the conditional variance specifications of the multiplicative and additive level models by adopting Glosten *et al.* (1993) model of asymmetry. The asymmetric multiplicative level model is

$$\phi_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\eta_{t-1}^2 + \beta\phi_{t-1}^2 \tag{9}$$

where $\eta_{t-1} = \min(\varepsilon_{t-1}, 0)$. For $\alpha_2 > 0$ bad news (negative shocks) has a larger impact on the volatility than good news (positive shocks).

An alternative approach to modelling volatility clustering and levels effects is the additive level effect model

$$h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta h_{t-1} + br_{t-1}^{\delta}. \tag{10}$$

The additive level model (10) also nests both the CKLS model ($\alpha_0 = \alpha_1 = \beta = 0$) and the GARCH(1,1) model ($\delta = b = 0$). Likewise, the asymmetric additive level model is

$$h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\eta_{t-1}^2 + \beta h_{t-1} + br_{t-1}^{\delta}. \tag{11}$$

3 Experimental Design

Brooks and Henry (2000) present evidence that the Engle-Ng tests are undersized, but that the downward bias in the empirical size diminishes as the sample size increases. The Monte Carlo study consists of two parts. The first part examines the simulated size of the Engle-Ng test in the presence of level effects.

Three different degrees of persistence in the GARCH (1,1) structure are considered using the parameters values provided by Engle-Ng. They are as follows:

1. model H (for high persistence), where $(\alpha_0, \beta, \alpha_1) = (0.01, 0.9, 0.09)$ and $\alpha_1 + \beta = 0.99$
2. model M (for medium persistence), where $(\alpha_0, \beta, \alpha_1) = (0.05, 0.9, 0.05)$ and $\alpha_1 + \beta = 0.95$
3. model L (for low persistence), where $(\alpha_0, \beta, \alpha_1) = (0.2, 0.75, 0.05)$ and $\alpha_1 + \beta = 0.80$

These sets of parameter values are used in both the multiplicative and additive level models (8) and (10), their asymmetric counterparts defined by equations (9) and (11) respectively. The parameter δ is set to $(0.5, 1.0, 1.5)$ in accordance with the various theoretical short rate models. For the additive level model, b takes on the values $(0.01, 0.5, 0.99)$ to control for the persistence in the level effect.

The second part of the Monte Carlo experiment determines the simulated power of the tests to detect neglected asymmetries. This is obtained by augmenting the conditional variance of the data generating processes (8) and (10) with two types of asymmetric GARCH processes, namely the EGARCH and GJR models. Specifically, the asymmetric conditional variance specifications are as follows:

EGARCH Model

$$\log(h_t) = -0.23 + 0.9 \cdot \log(h_{t-1}) + 0.25 \cdot \left[v_{t-1}^2 - 0.3 \cdot v_{t-1}\right] \qquad (12)$$

GJR Model

$$h_t = 0.005 + 0.7 \cdot h_{t-1} + 0.28 \cdot \left[|\varepsilon_{t-1}| - 0.23 \cdot \varepsilon_{t-1}\right]^2 . \qquad (13)$$

Here $\varepsilon_t = \sqrt{h_t} \cdot v_t$ and $v_t \backsim i.i.d.N(0,1)$. We discard the initial 500 observations to mitigate the effect of start-up values yielding samples of 500, 1000 and 3000 observations, drawn with 10,000 replications. Once the data has been generated, a GARCH (1,1) specification is estimated by maximizing the log-likelihood function using the Broyden, Fletcher, Goldfarb and Shanno (BFGS) algorithm. The Engle-Ng test statistics are then calculated on the resulting standardised residuals.

4 Results and Implications

Table 1 summarizes the simulated size of the various Engle-Ng test statistics in the presence of an additive level effect (Panel A) and a multiplicative level effect (Panel B).

-Table 1 about here-

The additive level model (10) allows for different degrees of persistence in the GARCH structure with the level effects given by $\delta = 1.0$ and $b = 0.01$. For a large sample of 3000 observations, the joint and size bias test statistics are oversized across the different degrees of persistence in the conditional variance. The empirical size of the test statistics, however, are relatively robust to the level effects for a sample of 1000 observations. The empirical size of the sign bias test is free from upward bias for all the samples and varying degree of persistence in the conditional variance. The magnitude of the distortion in the empirical size of the test statistics is more apparent in the case of multiplicative level model with parameterised level effects governed by $\delta = 1.0$ (see Table 1 Panel B). The distortion occurs even for a smaller sample of 1000 observations. The impact of multiplicative level effects on the test statistic is more pervasive; with the exception of the joint test statistic, the simulated sizes of the tests statistics exceed the true significance level. Moreover the simulated sizes decrease with the persistence of the conditional variance.

-Tables 2 and 3 about here-

The effect of variation in the strength of the level effects on the simulated size of the test statistics are reported in Table 2 for an additive level effect and in Table 3 for a multiplicative level effect. [1] In the case of an additive level effects, the strength of the level effects is governed by the parameters b and δ. Overall, the upward bias of the simulated sizes increases as b and/or δ increase in value. The degree of bias, however, decreases as the persistence in the conditional variance (measured by the sum $\alpha_1 + \beta$) falls. Similarly, there are severe size distortions when the DGP exhibits highly persistent conditional variance and a multiplicative level effects model.

-Table 4 about here-

Table 4 summarizes the simulated power of the various Engle-Ng test statistics in the presence of no level effect (Panel A), an additive level effect

[1] To conserve space, we only report the results for a sample size of 3000. The results for sample sizes of 500 and 1000 concur with those reported herein and can be obtained upon request from the authors.

(Panel B) and a multiplicative level effect (Panel C). We consider GJR and EGARCH processes in this experiment. The power of the Engle-Ng tests appears to increase for both types of asymmetric DGP as the sample size increases. When we introduce additive level effects the power of the Engle-Ng tests fall, irrespective of the type of asymmetry displayed by the DGP. However, the reduction in the power of the tests is larger in the GJR model than the EGARCH model for all sample sizes. The negative size bias test and the joint test do not suffer from as large a reduction in their power as the positive size bias and the negative sign bias tests.

Contrary to the impact of the additive level effects on the power of the tests, multiplicative level effects either slightly reduce the power as in the GJR case, or enhance it in the case of the EGARCH model.

-Table 5 about here-

Table 5 displays simulated power for the Engle-Ng tests, with reasonable results for the case where $b = 0.01$ and $\delta = 0.5$ in a GJR model with additive level effects. However, upon varying the strength of the additive level effect, by holding one of b or δ constant and varying the other parameter, the power of the test diminishes rapidly. Table 9 reports the power of the test statistics in presence of a multiplicative level effect. There is a sharp contrast between the results of the additive and the multiplicative level effects; unlike the results in the additive level model, the power of the tests remains impressive as the multiplicative level effect increases.

5 Conclusion

This paper examines the usefulness of the Engle-Ng tests as diagnostic tool for detecting neglected asymmetries in the conditional variance models of short rate. The results show that the presence of a neglected additive or multiplicative level effect impacts on the reliability of inference based upon the sign bias, size bias and joint tests. Independent of the persistence in the conditional variance, the tests spuriously detect sign and size bias in the conditional volatility of the short rate when the level effect is strong. The presence of multiplicative level effects in the DGP exacerbates the degree of distortion in the empirical size of the test statistics.

The power of the Engle-Ng test statistics is also sensitive to the type of asymmetric structure present in the data and exhibits significant downward bias in the presence of neglected additive or multiplicative level effects. The power of the tests falls as the strength of the additive level effect is increased.

Consistent with the findings of Brooks and Henry (2000), the results also point to the importance of the sample size on the tests' performance. In particular, it appears that for reliable inference a minimum sample of 3000 observations is necessary. Overall, the findings of this study caution against relying on Engle-Ng tests alone to make inference about asymmetric volatility without first validating the presence of a level effect in short rates.

References

[1] Bali, Turan G. (2000a), "Modeling the conditional mean and variance of the short rate using diffusion, GARCH and moving average models", *Journal of Futures Markets*, **20**, 717-751.

[2] Bali, Turan G. (2000b), "Testing the empirial performance of stochastic volatility models of the short-term interest rate", *Journal of Financial and Quantitative Analysis*, **35**, 191-215.

[3] Bekaert, Geert, Robert J. Hodrick, and David A. Marshall (2001), "Peso Problem Explanations for Term Structure Anomalies", *Journal of Monetary Economics*, **48**, 241-270.

[4] Brenner R.J., R.H. Harjes, and K.F. Kroner (1996), "Another look at models of the short-term interest rate", *Journal of Financial and Quantitative Analysis,* **31**, 85-107.

[5] Brooks, C. and Ó. T. Henry (2000), "Can portmanteau nonlinearity tests serve as general misspecification tests? Evidence from symmetric and asymmetric GARCH models", *Economics Letters*, **67**, 245-251.

[6] Brooks, C. and Ó. T. Henry (2002) "The impact of news on measures of undiversifiable risk: Evidence from the UK stock market", *Oxford Bulletin of Economics and Statistics* **64**, 487-507.

[7] Chan, K.C., G.A. Karolyi, F.A. Longstaff, and A.B. Sanders (1992), "An empirical comparison of alternative models of the short-term interest rate", *Journal of Finance,* **47**, 1209-1227.

[8] Cox, John C., J.E. Ingersoll and S.A. Ross (1985), "A theory of the term structure of interest rates", *Econometrica,* **53**, 385-407.

[9] Engle, R.F. and V. K. Ng (1993), "Measuring and testing the impact of news and volatility", *Journal of Finance*, **48**, 1749-1778.

[10] Glosten, Lawrence R., R. Jagannathan, and D.E. Runkle (1993), "On the relation between the expected value and the volatility of the nominal excess return on stocks", *Journal of Finance*, **48(5)**, 1779-1801.

[11] Henry, Ólan T.(1998), "Modelling the asymmetry of stock market volatility", *Applied Financial Economics*, **8**, 145-153.

[12] Kroner, Kenneth F and V.K. Ng (1998), "Modeling asymmetric comovements of asset returns", *Review of Financial Studies*, **11**, 817-44.

[13] Vasicek, O. (1977), "An equilibrium characterization of the term structure", *Journal of Financial Economics*, **5(2)**, 177-188.

Table 1: Impact of level effects on the Simulated Size of the Engle Ng Tests

Rejection Frequencies When the Null is True

Panel A: Additive level effects

$$\Delta r_t = \varepsilon_t \ , \ \varepsilon_t = \sqrt{h_t} \cdot v_t \quad where \ v_t \backsim i.i.d.N(0,1)$$
$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1} + 0.01 r_{t-1}$$

Persistence		*H* (α_o,β,α_1)=(0.01,0.9,0.09)			*M* (α_o,β,α_1)=(0.05,0.9,0.05)			*L* (α_o,β,α_1)=(0.2,0.75,0.05)		
Sample Size		500	1000	3000	500	1000	3000	500	1000	3000
N sign Test	1%	0.00	0.29	0.44	0.00	0.35	0.40	0.00	0.39	0.81
	5%	0.00	3.37	3.11	0.00	3.37	2.83	0.00	3.57	4.41
	10%	0.00	7.89	7.09	0.00	7.84	6.78	0.00	8.17	9.16
N size bias	1%	0.00	1.36	**3.52**	0.00	1.21	**3.38**	0.00	**1.81**	**5.48**
	5%	0.47	3.80	**10.13**	1.62	3.33	**9.05**	0.00	**7.28**	**14.81**
	10%	29.42	7.22	**16.17**	48.17	6.91	**14.64**	0.00	**14.21**	**22.81**
P size bias	1%	0.00	0.83	**2.82**	0.00	0.72	**3.26**	0.00	0.62	**4.19**
	5%	0.00	3.75	**8.33**	0.00	3.76	**8.68**	0.00	2.14	**12.27**
	10%	0.00	8.11	**13.25**	0.00	8.41	**13.91**	0.00	4.46	**19.75**
Joint test	1%	0.00	**1.81**	**7.5**	0.02	**1.50**	**7.78**	0.00	**1.38**	**16.74**
	5%	0.00	3.49	**12.36**	0.55	3.17	**12.47**	4.87	3.77	**34.5**
	10%	17.14	5.74	**16.88**	26.10	5.67	**16.35**	8.80	7.43	**45.87**

Panel B: Multiplicative level effects

$$\Delta r_t = \varepsilon_t \ , \ \varepsilon_t = \phi_t \cdot v_t \cdot r_{t-1}^{1.0} \quad where \ v_t \backsim i.i.d.N(0,1)$$
$$\phi_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta \phi_{t-1}^2$$

Persistence		*H* (α_o,β,α_1)=(0.01,0.9,0.09)			*M* (α_o,β,α_1)=(0.05,0.9,0.05)			*L* (α_o,β,α_1)=(0.2,0.75,0.05)		
Sample Size		500	1000	3000	500	1000	3000	500	1000	3000
N sign Test	1%	69.54	**78.63**	**83.55**	57.66	**61.01**	**62.80**	50.29	**54.06**	**55.82**
	5%	74.79	**82.12**	**86.12**	64.43	**67.03**	**68.25**	57.06	**60.85**	**61.69**
	10%	77.58	**84.22**	**87.46**	68.05	**70.28**	**71.14**	60.75	**64.39**	**65.16**
N size bias	1%	46.60	**57.23**	**70.33**	16.69	**17.67**	**18.89**	13.81	**14.70**	**15.13**
	5%	55.66	**64.82**	**75.66**	23.54	**23.70**	**25.35**	19.50	**20.57**	**20.50**
	10%	60.49	**68.86**	**78.85**	28.18	**28.44**	**30.32**	23.98	**25.52**	**24.49**
P size bias	1%	48.07	**58.42**	**68.60**	18.37	**19.33**	**19.74**	17.49	**18.27**	**18.39**
	5%	56.14	**65.70**	**74.01**	24.24	**25.65**	**26.07**	22.93	**23.85**	**23.79**
	10%	60.83	**69.98**	**76.93**	28.68	**30.80**	**30.84**	27.23	**28.15**	**27.95**
Joint test	1%	85.01	**91.38**	**95.58**	78.30	**80.19**	**87.14**	77.43	**79.24**	**79.20**
	5%	88.53	**93.12**	**96.43**	80.57	**85.42**	**89.20**	78.77	**83.41**	**86.36**
	10%	90.00	**94.06**	**96.95**	82.70	**86.50**	**90.34**	80.95	**84.55**	**88.45**

Table 2: Simulated Size and Variation of Additive Level Effects (Sample Size of 3000):

$$\Delta r_t = \varepsilon_t \quad , \quad \varepsilon_t = \sqrt{h_t} \cdot v_t \quad \text{where } v_t \backsim i.i.d.N(0,1)$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta h_{t-1} + b r_{t-1}^{\delta}$$

		Panel A: δ=0.5								
Persistence		*H* $(\alpha_o,\beta,\alpha_1)$=(0.01,0.9,0.09)			*M* $(\alpha_o,\beta,\alpha_1)$=(0.05,0.9,0.05)			*L* $(\alpha_o,\beta,\alpha_1)$=(0.2,0.75,0.05)		
b		0.01	0.5	0.99	0.01	0.5	0.99	0.01	0.5	0.99
		Actual Rejection Frequencies (%)								
N sign Test	1%	0.52	0.46	0.48	0.55	0.41	0.50	0.47	0.42	0.38
	5%	3.30	3.06	3.18	3.46	3.19	3.14	3.11	3.03	3.07
	10%	7.28	6.81	6.91	7.21	6.91	6.86	7.04	7.05	6.79
N size bias	1%	1.27	1.97	2.31	1.03	1.74	2.25	1.08	2.59	2.72
	5%	3.82	6.44	6.83	3.16	5.68	6.52	3.31	7.49	7.67
	10%	7.58	11.31	11.75	6.55	10.52	11.31	6.42	12.74	12.95
P size bias	1%	1.26	2.36	2.52	0.88	1.99	2.57	1.09	2.29	2.59
	5%	4.07	7.40	7.96	3.61	7.16	7.86	3.10	7.35	7.59
	10%	7.79	13.03	13.75	7.41	12.54	13.63	6.29	12.80	13.40
Joint test	1%	1.80	5.48	5.86	1.15	4.53	5.71	1.77	5.70	6.19
	5%	4.04	10.26	10.64	2.93	8.96	10.57	3.25	10.72	11.47
	10%	6.95	14.48	15.15	5.46	13.09	15.11	5.22	15.45	15.97
		Panel B: δ=1.0								
N sign Test	1%	0.44	1.52	1.66	0.40	1.15	1.15	0.81	2.58	2.02
	5%	3.11	5.71	5.91	2.83	4.33	4.42	4.41	7.78	5.38
	10%	7.09	10.49	10.78	6.78	8.49	8.55	9.16	13.21	10.10
N size bias	1%	3.52	7.94	8.07	3.38	8.89	9.17	5.48	6.84	7.54
	5%	10.13	16.15	16.31	9.05	16.64	16.98	14.81	15.13	14.65
	10%	16.17	22.46	22.73	14.64	21.82	22.15	22.81	22.34	20.71
P size bias	1%	2.82	5.84	6.02	3.26	8.06	8.02	4.19	4.79	6.86
	5%	8.33	12.09	12.00	8.68	17.74	17.37	12.27	9.98	14.75
	10%	13.25	17.58	17.57	13.91	25.46	25.35	19.75	15.29	21.95
Joint test	1%	7.50	14.55	14.63	7.78	18.24	18.11	11.74	12.21	15.11
	5%	12.36	20.15	20.28	12.47	24.19	24.47	14.50	18.25	21.42
	10%	16.88	24.51	24.52	16.35	29.64	29.80	16.87	23.13	26.93
		Panel C: δ=1.5								
N sign Test	1%	4.14	15.13	15.09	3.94	12.32	15.11	1.63	21.79	21.94
	5%	7.47	19.57	19.58	7.33	16.21	19.81	4.67	27.99	27.85
	10%	11.38	23.86	24.10	11.05	20.38	24.14	9.04	33.24	33.05
N size bias	1%	5.33	9.31	9.57	5.23	8.39	9.73	4.17	18.06	17.70
	5%	10.68	16.66	16.91	10.17	15.16	17.16	10.81	32.84	32.41
	10%	15.24	23.43	23.50	14.74	21.55	23.75	16.94	44.60	44.00
P size bias	1%	8.83	17.04	17.50	8.54	14.49	17.64	3.71	21.42	21.46
	5%	16.54	26.53	26.71	16.65	23.43	26.75	8.02	30.28	30.61
	10%	23.83	35.03	35.31	24.40	31.81	35.32	13.15	39.03	39.06
Joint test	1%	13.75	25.62	25.84	13.14	21.39	26.17	17.10	36.22	36.18
	5%	20.56	35.19	35.71	20.21	31.21	35.92	22.27	52.58	52.04
	10%	26.52	43.43	43.74	26.85	39.56	43.79	27.62	63.59	63.37

Table 3: Simulated Size and Variation of Multiplicative Level Effects (Sample Size of 3000):

$$\Delta r_t = \varepsilon_t \ , \ \varepsilon_t = \phi_t \cdot v_t \cdot r_{t-1}^{\delta} \quad \text{where} \quad v_t \backsim i.i.d.N(0,1)$$

$$\phi_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta \phi_{t-1}^2$$

Persistence		*H* $(\alpha_o,\beta,\alpha_1)$=(0.01,0.9,0.09)	*M* $(\alpha_o,\beta,\alpha_1)$=(0.05,0.9,0.05)	*L* $(\alpha_o,\beta,\alpha_1)$=(0.2,0.75,0.05)
		Panel A: δ=0.5		
		Actual Rejection Frequencies (%)		
N sign Test	1%	**11**	**10.01**	**8.07**
	5%	**20.14**	**18.32**	**16.11**
	10%	**27.21**	**22.18**	**20.31**
N size bias	1%	**9.72**	**9.33**	**7.21**
	5%	**18.16**	**17.09**	**15.13**
	10%	**25.7**	**20.01**	**19.17**
P size bias	1%	**12.21**	**11.98**	**10.15**
	5%	**21.98**	**20.32**	**18.91**
	10%	**29.87**	**23.14**	**22.65**
Joint test	1%	**19.53**	**18.02**	**17.14**
	5%	**27.58**	**25.23**	**24.20**
	10%	**33.43**	**30.59**	**27.54**
		Panel B: δ=1.0		
N sign Test	1%	**83.55**	**62.8**	**55.82**
	5%	**86.12**	**68.25**	**61.69**
	10%	**87.46**	**71.14**	**65.16**
N size bias	1%	**70.33**	**18.89**	**15.13**
	5%	**75.66**	**25.35**	**20.5**
	10%	**78.85**	**30.32**	**24.49**
P size bias	1%	**68.6**	**19.74**	**18.39**
	5%	**74.01**	**26.07**	**23.79**
	10%	**76.93**	**30.84**	**27.95**
Joint test	1%	**95.58**	**20.14**	**19.20**
	5%	**96.43**	**31.20**	**27.79**
	10%	**96.95**	**34.34**	**31.45**
		Panel C: δ=1.5		
N sign Test	1%	**62.73**	**64.25**	**27.24**
	5%	**70.08**	**71.31**	**32.31**
	10%	**74.67**	**73.56**	**35.22**
N size bias	1%	**48.11**	**18.25**	**8.08**
	5%	**57.65**	**25.61**	**10.8**
	10%	**64.71**	**31.97**	**13.29**
P size bias	1%	**52.13**	**21.13**	**14.54**
	5%	**62.15**	**27.71**	**17.48**
	10%	**67.05**	**31.89**	**19.82**
Joint test	1%	**76.25**	**24.23**	**20.52**
	5%	**84.45**	**32.30**	**30.72**
	10%	**87.17**	**35.42**	**31.86**

Table 4: Simulated Power for Asymmetric GARCH Models

Panel A: No Level Effect

$$\Delta r_t = \varepsilon_t \ , \varepsilon_t = \sqrt{h_t} \cdot v_t \quad where\ v_t \backsim i.i.d.N(0,1)$$
$$EGARCH: \ \log(h_t) = -0.23 + 0.9 \cdot \log(h_{t-1}) + 0.25 \cdot \left[v_{t-1}^2 - 0.3 \cdot v_{t-1}\right]$$
$$GJR: \ h_t = 0.005 + 0.7 \cdot h_{t-1} + 0.28 \cdot \left[|\varepsilon_{t-1}| - 0.23 \cdot \varepsilon_{t-1}\right]^2$$

		EGARCH			GJR		
Sample Size		500	1000	3000	500	1000	3000
N sign Test	1%	2.83	5.85	23.70	0.00	35.92	81.93
	5%	10.21	18.12	46.57	0.00	70.31	94.90
	10%	18.19	28.04	60.11	0.00	83.48	97.73
N size bias	1%	5.71	13.35	45.39	100	80.45	95.82
	5%	15.32	29.12	67.93	100	96.69	99.41
	10%	23.74	39.42	77.58	100	99.1	99.92
P size bias	1%	2.26	5.74	24.96	0.00	58.18	95.45
	5%	10.54	19.16	49.13	0.00	89.41	99.33
	10%	19.64	30.05	61.99	100	96	99.8
Joint test	1%	3.71	8.93	35.66	100	62.49	93.86
	5%	11.28	21.53	56.33	100	88.19	98.87
	10%	18.18	31.1	67.41	100	94.93	99.67

Panel B: Additive Level Effect

$$\Delta r_t = \varepsilon_t \ , \varepsilon_t = \sqrt{h_t} \cdot v_t \quad where\ v_t \backsim i.i.d.N(0,1)$$
$$EGARCH: \ \log(h_t) = -0.23 + 0.9 \cdot \log(h_{t-1}) + 0.25 \cdot \left[v_{t-1}^2 - 0.3 \cdot v_{t-1}\right] + 0.01 r_{t-1}$$
$$GJR: \ h_t = 0.005 + 0.7 \cdot h_{t-1} + 0.28 \cdot \left[|\varepsilon_{t-1}| - 0.23 \cdot \varepsilon_{t-1}\right]^2 + 0.01 r_{t-1}$$

		EGARCH			GJR		
Sample Size		500	1000	3000	500	1000	3000
N sign Test	1%	4.85	9.38	20.52	0.00	18.92	28.97
	5%	11.61	19.31	39.85	0.00	47.88	54.46
	10%	18.63	27.64	51.76	0.00	63.52	67.30
N size bias	1%	6.80	14.76	49.23	0.00	40.67	79.34
	5%	15.77	29.20	67.82	73.97	69.99	91.58
	10%	22.95	38.68	76.03	96.77	81.88	95.06
P size bias	1%	5.43	9.66	17.85	0.00	16.62	22.77
	5%	12.46	19.88	35.35	0.00	45.88	42.23
	10%	19.74	28.33	46.92	15.49	62.20	52.11
Joint test	1%	8.88	17.21	40.83	2.37	20.37	67.41
	5%	16.10	27.92	59.77	49.80	46.64	84.30
	10%	23.07	36.74	69.41	87.95	61.68	90.22

Panel C: Multiplicative Level Effect

$$\Delta r_t = \varepsilon_t \ , \ \varepsilon_t = \phi_t \cdot v_t \cdot r_{t-1}^{0.5} \quad where v_t \backsim i.i.d.N(0,1)$$
$$EGARCH: \ \log(\phi_t^2) = -0.23 + 0.9 \cdot \log(\phi_{t-1}^2) + 0.25 \cdot \left[v_{t-1}^2 - 0.3 \cdot v_{t-1}\right]$$
$$GJR: \phi_t^2 = 0.005 + 0.7 \cdot \phi_{t-1}^2 + 0.28 \cdot \left[|\varepsilon_{t-1}| - 0.23 \cdot \varepsilon_{t-1}\right]^2$$

		EGARCH			GJR		
Sample Size		500	1000	3000	500	1000	3000
N sign Test	1%	66.91	75.53	88.74	66.66	1.25	2.41
	5%	73.50	80.16	91.16	72.94	8.14	11.18
	10%	76.84	82.55	92.36	76.18	16.83	19.93
N size bias	1%	50.73	61.66	76.27	53.72	80.26	89.59
	5%	58.82	68.71	80.95	61.65	84.05	91.61
	10%	63.45	72.71	83.32	66.40	86.20	92.73
P size bias	1%	52.12	59.43	75.56	48.10	76.88	88.75
	5%	60.26	66.96	80.62	56.35	82.43	91.32
	10%	65.09	71.03	83.07	61.31	85.31	92.60
Joint test	1%	90.48	92.62	98.04	89.77	97.16	98.86
	5%	93.33	94.22	98.59	92.55	97.89	99.16
	10%	94.53	95.23	98.73	93.80	98.30	99.28

Table 5: Simulated Power for a GJR Model:

Panel A; Additive Level Effects

$\Delta r_t = \varepsilon_t \ , \ \varepsilon_t = \sqrt{h_t} \cdot v_t \quad where v_t \backsim i.i.d.N(0,1)$

$h_t = 0.005 + 0.7 \cdot h_{t-1} + 0.28 \cdot [|\varepsilon_{t-1}| - 0.23 \cdot \varepsilon_{t-1}]^2 + br_{t-1}^{\delta}$

		$\delta = 0.5$			$\delta = 1.0$			$\delta = 1.5$		
b		0.01	0.5	0.99	0.01	0.5	0.99	0.01	0.5	0.99
		Actual Rejection Frequencies (%)								
N sign Test	1%	70.82	1.08	0.65	28.97	1.93	2.91	9.76	22.55	21.52
	5%	89.16	5.59	3.91	54.46	5.69	8.17	24.92	28.71	27.51
	10%	94.27	10.98	8.30	67.30	10.44	13.40	35.07	33.97	32.60
N size bias	1%	88.29	9.00	3.39	79.34	7.20	6.55	54.96	22.12	21.03
	5%	97.56	21.21	10.63	91.58	13.85	14.65	69.08	39.00	38.44
	10%	99.03	30.78	17.39	95.06	20.04	21.28	75.20	51.76	51.49
P size bias	1%	88.96	2.19	1.88	22.77	7.19	4.73	5.74	22.48	20.20
	5%	97.54	7.49	7.35	42.23	15.00	10.20	15.79	33.43	30.55
	10%	98.98	13.32	12.99	52.11	21.98	15.55	24.22	43.11	39.84
Joint test	1%	85.37	7.62	5.41	67.41	15.39	11.88	49.70	41.46	39.25
	5%	96.01	17.45	11.25	84.30	21.20	17.91	67.13	60.67	58.25
	10%	98.12	26.57	17.19	90.22	26.06	22.51	74.89	72.43	70.65

Panel B; Multiplicative Level Effects

$\Delta r_t = \varepsilon_t \ , \ \varepsilon_t = \phi_t \cdot v_t \cdot r_{t-1}^{\delta} \quad where v_t \backsim i.i.d.N(0,1)$

$\phi_t^2 = 0.005 + 0.7 \cdot \phi_{t-1}^2 + 0.28 \cdot [|\varepsilon_{t-1}| - 0.23 \cdot \varepsilon_{t-1}]^2$

		$\delta = 0.5$	$\delta = 1.0$	$\delta = 1.5$
		Actual Rejection Frequencies (%)		
N sign Test	1%	31.78	72.42	71.86
	5%	49.60	81.18	76.53
	10%	59.36	87.93	79.23
N size bias	1%	63.42	89.59	73.46
	5%	77.30	91.61	78.98
	10%	88.55	92.73	81.86
P size bias	1%	30.70	88.75	61.32
	5%	52.47	91.32	68.15
	10%	63.67	92.60	71.98
Joint test	1%	60.40	98.86	89.22
	5%	74.32	99.16	92.19
	10%	80.98	99.28	93.69