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ANU Working Papers in Economics and Econometrics
555

October, 2011
JEL: C72, D43, D81, L10, L13, R30, R39.

ISBN: 086831 555 9

Optimal Product Variety in a Hotelling Model*

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June 9, 2011

Abstract

In Hotelling style duopoly location games the product variety (or firm locations) is typically not socially optimal. This occurs because the competitive outcome is driven by the density of consumers at the margin while the socially optimal outcome depends on the whole distribution of consumer locations/tastes. We consider a natural extension of the standard model in which firms are imperfectly informed about the distribution of consumers, in particular firms are uncertain about the consumer mean. In the uniform case, as the aggregate uncertainty about the mean becomes large relative to the dispersion of consumers about the mean, competitive locations become socially optimal. A limit result on prices for discontinuous, log-concave densities shows the result will hold in a range of cases.

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Keywords: location, product variety, pricing, price discrimination, uncertainty.

1 Introduction

In Hotelling style duopoly location games, the product variety is typically not socially optimal. This occurs because the competitive outcome is driven by the density of consumers at the margin while the socially optimal outcome depends on the whole distribution of consumer locations/tastes. We will show how, in a natural generalization of the Hotelling model, product variety/locations become approximately optimal.

We consider a generalization of the standard model in which there is uncertainty over the mean of the distribution of consumer types/locations, so-called *demand location uncertainty* (see Casado-Izaga, F.J., 2000, Harter, 1996, Meagher and Zauner 2004, 2005). This is a natural generalisation of the standard model, since in many real-world situations it is unlikely that firms are perfectly informed about consumer preferences. Indeed the very existence of market research proves that firms are not perfectly informed about demand conditions.

*Thanks to Simon Anderson, John Stachurski and seminar participants at UNSW and the University of Sydney

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We first show that in the uniform version of the demand location uncertainty model the competitive locations limit to the socially optimal locations, i.e. variety becomes optimal.

The second result shows the underlying economics at work — why firms internalize total transport costs — and hence why the limiting optimal variety result can hold for equilibria in non-uniform cases. The general result relies on uncovering a relationship between price discrimination (in which socially optimal locations are a competitive equilibrium) and demand location uncertainty. Price discrimination and competitive prices (a single price for each firm) are normally thought of as distinct entities in spatial competition. Indeed, price discrimination necessarily implies a schedule of prices, and hence a function, while competitive pricing requires a single price per firm, which is just a number. However if one introduces demand location uncertainty into the competitive pricing game then the firm specific prices become functions of the uncertainty and limit to price discrimination prices when the aggregate uncertainty over the mean becomes large compared to the idiosyncratic uncertainty of consumers around the mean.

2 The Demand Location Uncertainty Model

Consumers, x , are distributed on $[a_\theta(M), b_\theta(M)] \in \mathbb{R}$, with cumulative density function $F_\theta(x; M)$. M is the mean of the consumer density and is a random variable with density given by $g(M)$, referred to as the uncertainty distribution. Both the density functions are common knowledge.

The two firms, $i = 1, 2$ have locations $x_i \in \mathbb{R}$. Without loss of generality $x_1 \leq x_2$. Production costs are normalized to zero.

Each consumer demands either one or zero units of the good and has sufficient income, y , to buy one unit of the good. A consumer located at x consuming the good of firm i has indirect utility

$$V_i(x, x_i, p_i) = y - p_i - (x_i - x)^2, \quad (1)$$

where p_i is the (mill) price of good i and $(x_i - x)^2$ is the standard quadratic transport cost/disutility of distance term.

Firms simultaneously choose locations, uncertainty is resolved, and then firms simultaneously choose prices under full information. Since prices are more easily adjusted than the characteristics

of a good it is natural to consider the polar case in which firms set the optimal ex post prices in the price subgame.

Given prices and locations, there is a unique location ξ at which a consumer is indifferent between the two firms: $V_1(\xi, x_1, p_1) = V_2(\xi, x_2, p_2)$. Consumers buy one unit from the firm that gives them the highest (net) utility, thus firms' profits are: $\pi_1 = p_1 F_\theta(\xi; M)$ and $\pi_2 = p_2(1 - F_\theta(\xi; M))$.

3 Results

3.1 An Example of Asymptotically Efficient Locations

Proposition 1. *If consumers are uniformly distributed on $[M - \frac{1}{2\theta}, M + \frac{1}{2\theta}]$ with M uniformly distributed on $[-\frac{1}{2}, \frac{1}{2}]$ then the unique location-price equilibrium of the demand location uncertainty game is asymptotically efficient, i.e.*

$$\lim_{\theta \rightarrow \infty} x_2^* = - \lim_{\theta \rightarrow \infty} x_1^* = \frac{1}{4}. \quad (2)$$

Proof. Meagher and Zauner (2005) characterise the unique location-price equilibrium of a demand location uncertainty game in which consumers are uniformly distributed about a mean which is also uniformly distributed. Our parameterization is different, but applying some basic algebra to their result yields the unique equilibrium locations for our game:

$$-x_1^* = x_2^* = \frac{5 + \theta^2 - 2\theta}{4\theta(\theta - 1)} \quad \text{if } 3 < \theta. \quad (3)$$

The limit result follows immediately. The socially optimal locations minimise the total transport costs which is well known in the literature to occur when the firms are at the quartiles in the uniform case, i.e. $\pm \frac{1}{4}$. \square

When demand location uncertainty about the mean is large relative to the idiosyncratic uncertainty the equilibrium locations under mill pricing will be approximately socially optimal. The question of the social optimality of equilibrium locations has been the key focus of the location literature. Typically, in mill pricing models product/location differentiation is excessive and

in a few cases insufficient, but not optimal. As McFadden and many others have observed the most realistic description of competition in differentiated products is for there to be significant uncertainty about consumer tastes. Our results show that the presence of large aggregate uncertainty yields a completely different conclusion to the previous decades of analysis which focused exclusively on idiosyncratic uncertainty.

Although technically a straight forward extension of the existing literature, the proof of Proposition (1) is not very intuitive and it is not clear if the social optimality is merely a coincidence. In the next section we take a deeper look at the underlying economic forces at work.

3.2 A General Limit Result

The Hotelling location model is a discrete choice model and hence has inelastic demand at the individual level. Individually inelastic demand and zero production costs mean that firm payoffs are determined by prices and the split of customers. Similarly, social optimality simplifies to minimizing aggregate transport costs. Thus our approach to understanding social optimality is to uncover how aggregate transport costs are internalised, at the limit, by firms through equilibrium prices. We show the missing link between competitive prices and minimising aggregate transport costs is their common link to spatial price discrimination.

Lederer and Hurter (1986) show, under a spatial price discrimination policy, each point of the consumer distribution is essentially a separate market. Thus, in their equilibrium the more distant firm charges a price of zero and the closer firm charges a price to extract the consumer surplus arising from the difference in transport costs — and this result is independent of specific distributional assumptions. Thus if 1 is the closer firm, equilibrium (discrimination) prices for location x , denoted p_i^d , will be

$$p_1^d(x; x_1, x_2) = (x - x_2)^2 - (x - x_1)^2 \tag{4}$$

and $p_2^d(x) = 0$. The symmetric result holds if the roles are reversed.

We now establish a limit result between spatial price discrimination and competitive pricing under demand location uncertainty.

Proposition 2. Suppose $f_\theta(x; M)$ is a the density of consumers, satisfying the two technical conditions of Anderson et al (1997), with support $[a_\theta(M), b_\theta(M)]$ such that

1. f is twice differentiable and log concave in x , continuous in θ ;
2. $f_\theta(a_\theta; M), f_\theta(b_\theta; M) \neq 0$; and
3. $\lim_{\theta \rightarrow \infty} a_\theta = \lim_{\theta \rightarrow \infty} b_\theta = M$

then the limit of the competitive price when consumers have mean M is the spatial price discrimination price for location M , that is:

$$\lim_{\theta \rightarrow \infty} p_i^*(x_1, x_2; M, \theta) = p_i^d(M; x_1, x_2), \quad i = 1, 2. \quad (5)$$

Proof. By the symmetry between the firms we need only consider firm 1. From Anderson et al (1997, p107) a unique competitive price equilibrium exist where p_1^* is defined in three pieces. The interval and expressions defining the three cases of equilibrium prices below, are also from Anderson et al (1997, p107) and are not derived again here.

Case 1: $(x_1 + x_2) \geq 2(b_\theta + 1/f_\theta(b_\theta; M))$. Firm 1 is the closer firm for all consumers, indeed there is sufficient asymmetry that firm 1 captures the whole market with an equilibrium price of $p_1^* = (x_2 - x_1)(x_1 + x_2 - 2b_\theta)$. Rearranging gives $(b_\theta - x_2)^2 - (b_\theta - x_1)^2 = p_1^d(b_\theta, x_1, x_2)$. Now $\lim_{\theta \rightarrow \infty} b_\theta = M$ so $\lim_{\theta \rightarrow \infty} p_1^* = p_1^d(M; x_1, x_2)$.

Case 2: $(x_1 + x_2)/2 \in [(a_\theta - 1/f_\theta(a_\theta, M, \theta)), (b_\theta + 1/f_\theta(b_\theta, M, \theta))]$ yields an interior indifferent consumer, i.e. $\xi \in [a_\theta, b_\theta]$ and $p_1^* = 2(x_2 - x_1)F_\theta(\xi)/f_\theta(\xi)$. Now $\lim_{\theta \rightarrow \infty} a_\theta = \lim_{\theta \rightarrow \infty} b_\theta = M$ so $\lim_{\theta \rightarrow \infty} p_1^* = 2(x_2 - x_1) \lim_{\theta \rightarrow \infty} (F_\theta(M)/f_\theta(M)) = 0$. The interval which defines this case also has a limit: $(x_1 + x_2)/2 \in \lim_{\theta \rightarrow \infty} [(a_\theta - 1/f_\theta(a_\theta, M, \theta)), (b_\theta + 1/f_\theta(b_\theta, M, \theta))] = [M, M]$, which is equivalent to requiring in the limit that the firms be symmetric either side of M , but in that case $p_1^d = 0$ establishing the result.

Case 3: $(x_1 + x_2) \leq 2(a_\theta - 1/f_\theta(a_\theta; M))$. Here firm 1 is the more distant firm for all consumers and there is sufficient asymmetry that firm 1 bids down to marginal cost, i.e. 0. Thus $p_1^* = 0$ which is also $p_1^d(M, x_1, x_2)$ in this case.

□

Socially optimal locations require transport costs to be minimized. Equation (4) shows, that under spatial price discrimination, prices and hence payoffs for a firm are determined by the negative of the transport costs of the consumers who purchase from a firm. But as Proposition 2 shows the competitive payoffs under demand location uncertainty are approximately equal to the price discrimination payoffs and hence similar forces, pushing towards socially optimal varieties/locations, will be at work asymptotically in the demand location uncertainty game.

Of course, none of this establishes the existence of a location equilibrium in the general case, or that any location equilibrium would be well behaved in θ . But if a well behaved location equilibrium exists, as it does in the uniform example, then we will see asymptotically efficient locations in the location uncertainty game, as aggregate uncertainty becomes large relative to idiosyncratic uncertainty, for the same reason we get efficiency under spatial price discrimination: firm profits are driven by the aggregate of the difference in transport costs.

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