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Sectoral Disturbances and Aggregate Economic Activity

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A key topic in the literature on business cycles concerns the origins of shocks underlying fluctuations in economic activity. One dimension of this topic focuses on whether we should think of aggregate economic fluctuations as being driven by disturbances that affect all areas of the economy simultaneously, or whether these movements are instead better thought of as arising from shocks to different sectors that affect economic activity by way of production complementarities such as input-output linkages. To the extent that sources of fluctuations include sectoral shocks, another key consideration then is the manner in which sectoral shocks potentially become amplified and propagate throughout the economy for a given degree of disaggregation.

A conventional wisdom argues that shocks to different sectors of the economy are unlikely to matter for aggregate fluctuations because they tend to average out in aggregation. Thus, positive shocks in some sectors will generally be offset by negative shocks in other sectors. This notion has in part led the bulk of the literature on business cycles to concentrate on the effects of different types of aggregate shocks. However, whether or not idiosyncratic sectoral shocks do average out in aggregation depends on various aspects of the economic environment. In particular, Gabaix (2011) describes how, when the economy comprises a handful of very large sectors, sectoral disturbances will not average out and contribute nontrivially to aggregate fluctuations. Horvath (1998) also makes the point that because of input-output linkages, shocks to particular sectors feed back into other sectors in a way that leads to significant

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amplification and propagation of those shocks. This idea is further developed and analyzed from a network perspective in Carvalho (2007), and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2010).

This article provides an overview of some key dimensions related to the effects of sectoral shocks on aggregate economic activity. It describes how the entire distribution of sectoral shares, or the weight of different sectors in aggregate activity, generally matters for the measured contribution of a sector to aggregate variability. It also illustrates how intersectoral linkages affect the propagation and amplification of sectoral idiosyncratic shocks. In particular, it summarizes sufficient conditions, carefully articulated in Dupor (1999), under which aggregate outcomes are invariant to sectoral disturbances, even in the presence of input-output linkages across sectors. A key condition requires that the matrix describing input-output linkages satisfies a particular structure according to which all sectors serve as equally important material providers to all other sectors.

To the degree that input-output linkages descriptive of U.S. production depart from Dupor's (1999) sufficient conditions for the irrelevance of sectoral shocks, it is generally not straightforward to characterize how this departure translates into sectoral contributions to aggregate variability.[1] As shown in Foerster, Sarte, and Watson (2011), the contribution of sectoral shocks to aggregate fluctuations is generally model-dependent and cannot be analytically characterized. Thus, using an actual input use matrix obtained from the Bureau of Economic Analysis (BEA) for 1997, and a two-digit level disaggregation of gross domestic product (GDP), this article describes key aspects of this calculation and provides estimates of the relative contribution of different sectors to aggregate variations given each sector's share in aggregate output. By and large, the manufacturing sector and the sector related to real estate, rental, and leasing tend to contribute the most to aggregate variations.

Given this article's emphasis on sectoral shocks, it also examines how these shocks propagate to other sectors and become amplified as a result of feedback effects resulting from intersectoral linkages. Using two canonical multisector growth models in the literature, specifically the foundational work of Long and Plosser (1983) and, its descendant, Horvath (1998), it illustrates how the propagation and amplification of sectoral shocks depend importantly on the details of the economic environment in which intersectoral linkages operate. Thus, it explains why using the share of the sector in which a disturbance occurs as a gauge of its effect on aggregate output constitutes, in general, a poor approximation. The article also shows that the effects of a given sectoral shock both on other sectors and on aggregate output will typically extend well beyond the life of the shock itself. In some sectors, because

[1] Carvalho (2007), as well as Acemoglu, Ozdaglar, and Tahbaz-Salehi (2010) make considerable progress along this dimension.

of feedback effects, things can get worse before improving even though the shock has already dissipated.

This article is not strictly concerned with accounting for the actual volatility of output because of sectoral co-movement (that is the main thrust of Foerster, Sarte, and Watson [2011]). Rather, this article deals with the way in which sectoral weights and input-output considerations affect the amplification and propagation of shocks. Therefore, unless otherwise noted, we use a stylized version of sectoral shocks, namely i.i.d. and uncorrelated across industries.

The rest of the article proceeds as follows. In Section 1, we describe how sectoral size relates to aggregate variability absent any complementarities in production. Section 2 provides an overview of how sectoral linkages influence the effects of sectoral shocks using the canonical multisector growth models of Long and Plosser (1983) and Horvath (1998). In Section 3, we use an unanticipated one-time decline in manufacturing total factor productivity (TFP) to illustrate how sectoral shocks propagate to other sectors, as well as their ultimate impact on aggregate output. Section 4 concludes.

1. SECTORAL SIZE AND AGGREGATE VARIABILITY

As a first approximation, it is natural to conjecture that sectoral shocks should not matter for aggregate economic activity because they will "average out." However, Gabaix (2011) carefully articulates the idea that this intuition does not hold if some sectors play a large role in economic activity, which he refers to as the "granular" hypothesis. In this view, idiosyncratic shocks to sectors with large shares have the potential to generate nontrivial disturbances in aggregate output. In particular, Gabaix (2011) shows that idiosyncratic i.i.d. shocks fail to average out in aggregation when the size distribution of sectors is sufficiently leptokurtic, or has "fat tails," as characterized for instance by the power law distribution. The nature of Gabaix's (2011) arguments relies on asymptotic calculations where the number of sectors, N, is large. In practice, however, N may not necessarily be very large if we think, for example, that real estate or manufacturing as a whole are being disrupted. The question then becomes: How do sectoral shares affect aggregate variability in practice?

Table 1 gives the two-digit sectoral decomposition of GDP with the industry code in the first column. The second column of Table 1 gives the value-added shares of each sector, as a percent of GDP, associated with this decomposition. To get an idea of how sectoral shares, or weights ω_i, affect aggregate variability, observe that aggregate output growth, denoted Δy_t at date t, can be (approximately) written as the following weighted average of

Table 1 Sectoral Shares and Contributions to the Variability of GDP, 1988–2010

Industry Name	NAICS Code	GDP Share, ω_i (Percent)	λ_i (Percent)	$\overline{\lambda}_i$ (Percent)
Agriculture, Forestry, Fishing, and Hunting	11	1.22	0.18	1.37
Mining	21	1.34	0.22	1.88
Utilities	22	2.05	0.51	1.55
Construction	23	4.29	2.23	6.08
Manufacturing	31–33	14.27	24.72	43.81
Wholesale Trade	42	5.98	4.33	5.83
Retail Trade	44–45	6.76	5.54	7.57
Transportation and Warehousing	48–49	2.99	1.09	2.42
Information	51	4.37	2.31	4.03
Finance and Insurance	52	7.26	6.40	9.35
Real Estate, Rental, and Leasing	53	12.44	18.79	5.21
Professional, Scientific, and Technical Services	54	6.32	4.85	4.04
Management of Companies and Enterprises	55	1.58	0.30	0.40
Administrative and Support Management	56	2.55	0.79	1.73
Educational Services	61	0.87	0.09	0.03
Health Care and Social Assistance	62	6.35	4.89	0.76
Arts, Entertainment, and Recreation	71	0.90	0.10	0.23
Accommodation and Food Services	72	2.73	0.91	1.17
Other Services (except Public Administration)	81	2.60	0.82	1.26
Government	92	13.13	20.92	1.28

sectoral output growth,

$$\Delta y_t = \sum_{i=1}^{N} \omega_i \Delta y_{it}, \tag{1}$$

where Δy_{it} represents output growth in sector i at t, N is the number of sectors, and $\sum_{i=1}^{N} \omega_i = 1$.[2] Suppose for now that output growth in each sector results directly from cross-sectionally unrelated i.i.d. shocks, ε_{it}, with identical variance, σ_ε^2, so that

$$\Delta y_{it} = \varepsilon_{it}, \text{ where } \Sigma_{\varepsilon\varepsilon} = \sigma_\varepsilon^2 I, \tag{2}$$

and $\Sigma_{\varepsilon\varepsilon}$ denotes the variance-covariance matrix of sectoral shocks. What can we say about the contribution of a given sector to the variance of aggregate output growth in this case?

Under the maintained assumptions, the variance of aggregate output is $\sigma_\varepsilon^2 \sum_{i=1}^{N} \omega_i^2$. Let λ_i denote the contribution to aggregate variance from sector i. Then, it follows that

$$\lambda_i = \frac{\omega_i^2}{\sum_{i=1}^{N} \omega_i^2}. \tag{3}$$

Observe that the size of the denominator in the above equation depends on the distribution of the ω_i's. Therefore, while we have assumed away the role of idiosyncratic volatility by assuming that all sectors are characterized by the same shocks, the entire sectoral size distribution nevertheless matters for the contribution of a given sector to aggregate volatility. The denominator in (3) is minimized when $\omega_i = 1/N \ \forall i$, so that the closer the ω_i's are to being evenly distributed, the lower the denominator will be. When $\omega_i = 1/N \ \forall i$, all sectors play an equally important role in aggregate output, $\lambda_i = 1/N \ \forall i$, and each sector's contribution to aggregate variance is equal to its share. In that case, sectoral disruptions will not be important for aggregate considerations as N becomes large.

Given the data in Table 1, where $N = 20$, we have that $\sum_{i=1}^{N} \omega_i^2 = 0.082$. The third column of Table 1 gives the contribution to aggregate variance of each sector under the assumption that idiosyncratic shocks are identically and independently distributed across sectors. For example, in the case of the construction sector, denoted by λ_c, we have that

$$\lambda_c = \frac{0.043^2}{0.082} = 0.022. \tag{4}$$

Therefore, under the maintained assumptions, construction contributes about 2 percent to the variability of aggregate GDP. For comparison, if all sectors

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[2] To be specifc, ω_i in this case represents the mean share of sector i output as a percent of GDP over a given sample period.

were the same size, the contribution to aggregate variability from any one sector would be

$$\lambda_i = \frac{(1/20)^2}{20(1/20)^2} = \frac{1}{20} = 0.05. \tag{5}$$

Although construction is actually close to $\frac{1}{20}$ of GDP, its contribution to aggregate variability in this example is less than half of its share in GDP.[3] Put another way, the actual size distribution of sectors is such that it reduces the importance of construction relative to a distribution where all sectors have the same size. The reverse will be true for sectors that have large shares in GDP. For example, in the manufacturing sector, the contribution to aggregate variability, λ_m, implied by the share in Table 1 is

$$\lambda_m = \frac{0.143^2}{0.082} = 0.25. \tag{6}$$

Hence, although manufacturing represents 14 percent of GDP, when all sectors are subject to the same shocks, its contribution to aggregate variability is almost double its share. This gives one measure of the sense in which manufacturing might represent a key component of an economic recovery.

The basic calculations we have just outlined have ignored two important considerations. First, the size of sectoral shocks may be sector-dependent. Second, idiosyncratic shocks may be correlated across sectors. When the size of idiosyncratic shocks differs across sectors, the contribution of a given sector to aggregate variability also takes into account the volatility of that sector's output, $\sigma^2_{\varepsilon_i}$, relative to that of all other sectors,

$$\overline{\lambda}_i = \frac{\omega_i^2 \sigma^2_{\varepsilon_i}}{\sum_{i=1}^{N} \omega_i^2 \sigma^2_{\varepsilon_i}}. \tag{7}$$

Given equation (2), we have that $\sigma^2_{\varepsilon_i}$ =var(Δy_{it}). The fourth column of Table 1 then gives the contribution to aggregate variability from each sector implied by equation (7). Importantly, this calculation continues to assume that idiosyncractic shocks are uncorrelated across sectors. Note that the contribution of the construction sector to aggregate variability now almost triples, from 2.2 percent to 6.1 percent. This contribution now exceeds construction's share of GDP. Similarly, manufacturing sees its contribution to aggregate variability jump from 25 percent to 44 percent. At the other extreme, the government

[3] Table 1 distinguishes between construction and real estate, rental, and leasing. The construction sector is comprised of establishments that are primarily engaged in the construction of buildings or engineering projects. Construction work may include new work, additions, alterations, or maintenance and repairs. The real estate, rental, and leasing sector is comprised of establishments that are primarily engaged in leasing and renting, and establishments providing related services. Also included are establishments primarily engaged in appraising real estate and the management of real estate for others (e.g., renting, selling, or buying real estate), as well as owner-occupied real estate.

sector's contribution to aggregate volatility falls dramatically from 21 percent, in the third column of Table 1, to just 1.3 percent in the fourth column. This result stems from the fact that while government is a relatively large share of GDP, its output is very smooth relative to that of other sectors.

While we have thus far ignored the fact that sectoral shocks may be cross-sectionally correlated, it is important to recognize that the presence of input-output linkages between sectors is likely to create some degree of cross-sectional dependence. In Table 1 for example, mining is likely to use the output of manufacturing, utilities, and construction as inputs. In general, the effect of a shock to a given sector on aggregate output will reflect not only that sector's share, ω_i, but also its degree of connection to all other sectors. In particular, all else equal, a shock to a sector that produces inputs for many other sectors will have a larger effect on aggregate output. Put differently, the presence of input-output linkages creates additional propagation from sectoral disturbances that amplify their effect on aggregate output. The next section addresses key aspects of the mechanisms by which this additional amplification and propagation takes place.

2. SECTORAL SHOCKS AND SECTORAL LINKAGES: IMPLICATIONS FOR AGGREGATE ACTIVITY

This section explores the role of sectoral linkages in amplifying and propagating sector-specific shocks. In other words, these linkages may, effectively, transform shocks that are specific to a particular sector into shocks that affect all sectors and, therefore, amplify variations in aggregate output. Because this analysis requires a model that incorporates linkages between sectors, this section uses two canonical models in the literature. The first model reflects the foundational work of Long and Plosser (1983), which explicitly considers each sector as potentially using materials produced in other sectors. The second model is that of Horvath (1998), also discussed in Dupor (1999), which allows the effects of sectoral shocks to be propagated over time through capital accumulation. A key lesson in this section is that, conditional on a given set of sectoral linkages, conclusions about the effects of sectoral shocks may differ depending on other aspects of the model in which these linkages operate.

Long and Plosser (1983)

Consider an economy composed of N distinct sectors of production indexed by $j = 1, ..., N$. Each sector j produces the quantity $Y_{j,t}$ of good j at date t using labor, $L_{j,t-1}$, and materials produced in sector $i = 1, ..., N$, $M_{ij,t-1}$, according to the Cobb-Douglas technology

$$Y_{j,t} = A_{j,t} L_{j,t-1}^{\alpha_j} \Pi_{i=1}^{N} M_{ij,t-1}^{\gamma_{ij}}, \tag{8}$$

where $A_{j,t}$ is a productivity index for sector j. Note that the technology features a version of time-to-build in the sense that production is subject to a one-period lag.

The fact that each sector potentially uses materials produced in other sectors represents a source of interconnectedness in the model. An input-output matrix for this economy is an $N \times N$ matrix Γ with typical element γ_{ij}. The column sums of Γ give the degree of returns to scale in materials in each sector. The row sums of Γ measure the importance of each sector's output as materials to all other sectors. Put simply, one can think of the rows and columns of Γ as "sell to" and "buy from," respectively, for each sector.

Let $A_t = (A_{1,t}, A_{2,t}, ..., A_{N,t})^T$ denote a vector of productivity indices that follow a random walk,

$$\ln \mathbf{A}_t = \ln \mathbf{A}_{t-1} + \varepsilon_t, \tag{9}$$

where $\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t}, ..., \varepsilon_{N,t})^T$ has covariance matrix $\Sigma_{\varepsilon\varepsilon}$.

A representative household derives utility from the consumption of these N goods and leisure, Z_t, according to

$$E_t \sum_{t=0}^{\infty} \beta^t \left\{ \theta_0 \ln Z_t + \sum_{i=1}^{N} \theta_j \ln(C_{jt}) \right\}. \tag{10}$$

In addition, each sector is subject to the following resource constraints,

$$Z_t + \sum_{j=1}^{N} L_{j,t} = 1 \tag{11}$$

$$C_{jt} + \sum_{i=1}^{N} M_{ji,t} = Y_{j,t}, \ \ j = 1, ..., N. \tag{12}$$

Let Δy_t denote the vector of sectoral output growth, $(\Delta y_{1,t}, \Delta y_{2,t}, ..., \Delta y_{N,t})^T$. Then, Long and Plosser (1983) show that the solution to the planner's problem is given by

$$\Delta \mathbf{y}_t = \Gamma^T \Delta \mathbf{y}_{t-1} + \varepsilon_t. \tag{13}$$

Letting $\boldsymbol{\omega} = (\omega_1, \omega_2, ..., \omega_N)^T$ represent the vector of sectoral shares in Table 1, an expression for aggregate output growth is

$$\Delta y_t = \boldsymbol{\omega}^T \Delta \mathbf{y}_t. \tag{14}$$

Let σ_y^2 denote the variance of aggregate output growth. Then, given equation (14), we have that

$$\sigma_y^2 = \boldsymbol{\omega}^T \Sigma_{yy} \boldsymbol{\omega}, \tag{15}$$

where Σ_{yy} is the variance-covariance matrix of sectoral output growth.

For given N, and given equation (13), an analytical expression for the variance of output growth in the Long and Plosser (1983) model is given by (15) where[4]

$$\text{vec}(\Sigma_{yy}) = (I_{N^2} - \Gamma^T \otimes \Gamma^T)^{-1}\text{vec}(\Sigma_{\varepsilon\varepsilon}). \tag{16}$$

For the purpose of calibration, the matrix Γ in this article is based on estimates of the 1997 Input-Output use table constructed by the Bureau of Economic Analysis (BEA). The BEA constructs the use table based on data from the Economic Census conducted by the Bureau of the Census every five years. The table shows the value of commodities (given by commodity codes) used as inputs by intermediate and final users (represented by industry codes). By matching commodity and industry codes for the 20 industries, we create an input use table showing the value of commodities from each industry used by all other industries. A row sum of the use table represents the total value of materials provided by a given industry to all 20 industries. A column sum of the use table shows the total expenses of a given industry on the inputs from all sectors. Input shares, γ_{ij}, are the payments from sector j to sector i as a fraction of the total value of production in sector j.

We saw earlier that when sectoral shocks have unit variance, the variance of aggregate output growth absent sectoral linkages is $\sigma_y^2 = 0.082$, slightly larger that $N^{-1} = (\frac{1}{20})$ predicted under uniform sectoral shares. When sectoral linkages are taken into account in the model of Long and Plosser (1983), and using the input-output matrix corresponding to the sectoral decomposition in Table 1, the variance of aggregate output growth is approximately 0.12 or about one and a half times larger.

One can also obtain some measure of the contribution of individual sectors to aggregate variability. To calculate the relative effect of sector i on σ_y^2, let $\widetilde{\Sigma}_{\varepsilon\varepsilon}$ denote a diagonal matrix whose diagonal is (0, 0, ..., 1, ..., 0) where the "1" is located in the i^{th} position. Then, we can calculate what the variance of output growth would be with sectoral linkages if the model were driven exclusively by shocks to sector i:

$$\widetilde{\sigma}_y^2 = \boldsymbol{\omega}^T \widetilde{\Sigma}_{yy} \boldsymbol{\omega},$$

where

$$\text{vec}(\widetilde{\Sigma}_{yy}) = (I_{N^2} - \Gamma^T \otimes \Gamma^T)^{-1}\text{vec}(\widetilde{\Sigma}_{\varepsilon\varepsilon}). \tag{17}$$

In that case, the contribution of sector i to aggregate variability, λ_i, is

$$\widetilde{\lambda}_i = \frac{\widetilde{\sigma}_y^2}{\sigma_y^2}. \tag{18}$$

[4] This result follows from the fact that for any matrices A, B, and C, such that the product ABC exists, $\text{vec}(ABC) = (C^T \otimes A)\text{vec}(B)$.

Table 2 Input-Output and Contributions to the Variability of GDP, 1988–2010

Industry Name	**NAICS Code**	$\widetilde{\lambda}_i^{LP}$ **(Percent)**	$\widetilde{\lambda}_i^{HD}$ **(Percent)**
Agriculture, Forestry, Fishing, and Hunting	11	0.32	0.60
Mining	21	0.33	0.51
Utilities	22	0.50	0.56
Construction	23	1.71	0.69
Manufacturing	31–33	35.31	42.77
Wholesale Trade	42	3.69	2.66
Retail Trade	44–45	4.18	1.49
Transportation and Warehousing	48–49	1.34	1.48
Information	51	2.11	1.87
Finance and Insurance	52	6.16	6.06
Real Estate, Rental, and Leasing	53	15.77	25.51
Professional, Scientific, and Technical Services	54	5.80	6.80
Management of Companies and Enterprises	55	0.63	0.72
Administrative and Support Management	56	1.15	1.37
Educational Services	61	0.07	0.02
Health Care and Social Assistance	62	3.66	1.02
Arts, Entertainment, and Recreation	71	0.08	0.05
Accommodation and Food Services	72	0.72	0.38
Other Services (except Public Administration)	81	0.71	0.53
Government	92	15.68	4.90

Notes: $\widetilde{\lambda}_i^{LP}$ is computed using Long and Plosser (1983) and uncorrelated sectoral shocks with unit variance. Similarly, $\widetilde{\lambda}_i^{HD}$ is computed using Horvath (1998) or Dupor (1999).

Table 2 shows $\widetilde{\lambda}_i$ for the sectors considered in this paper using both the Long and Plosser (1983) and the Horvath (1998) frameworks. Under the maintained assumptions, the only difference between the third column in Table 1 and the second column in Table 2 relates to input linkages across sectors. In both cases, shares are taken into account in the calculations and sectors have homogenous variances.[5] Although input-output linkages generally increase overall variance by slowing down the averaging that takes place in aggregation, Table 2 indicates that the relative importance of any one sector may increase or decrease depending in part on how important it is as an input provider to other sectors. For example, manufacturing contributes 25 percent of aggregate variability absent input-output linkages. However, when input-output linkages are taken into account, this contribution increases to 35 percent using the Long and Plosser (1983) framework. Manufacturing, therefore, plays an important

[5] This calculation highlights the importance of input-output linkages only. As shown in Foerster, Sarte, and Watson (2011), in practice, the relative magniture of shocks across sectors also matters.

role as an input provider to other sectors. In contrast, retail trade explains roughly 6 percent of aggregate variations based solely on its share in total output. Once input-output linkages are considered, the contribution of retail trade to aggregate variability falls to 4 percent. Thus, linkages of retail trade to other sectors play somewhat minor roles relative to those of other sectors.

Horvath (1998)

The model in Horvath (1998) is very similar to that of Long and Plosser (1983) but adds sectoral capital. Specifically, production in sector j now takes the form

$$Y_{j,t} = A_{j,t} K_{j,t}^{\alpha_j} (\Pi_{i=1}^{N} M_{ij,t}^{\gamma_{ij}}) L_{j,t}^{1-\alpha_j - \Sigma_{i=1}^{N} \gamma_{ij}}, \tag{19}$$

while each sector's resource constraint now reads as

$$C_{jt} + \sum_{i=1}^{N} M_{ji,t} + K_{jt+1} = Y_{j,t},\ j = 1, ..., N. \tag{20}$$

Horvath's (1998) model makes two key concessions to realism for the sake of analytical tractability. First, capital is assumed to depreciate entirely within the period. In that sense, the distinction between materials and capital is more one of timing than any other consideration. Second, each sector produces its own capital. Under these assumptions, the solution for sectoral output growth is now given by

$$\Delta \mathbf{y}_t = Z^T \boldsymbol{\alpha}_d \Delta \mathbf{y}_{t-1} + Z^T \varepsilon_t, \tag{21}$$

where $\boldsymbol{\alpha}_d$ is a diagonal matrix with the vector of sectoral capital shares, $(\alpha_1, \alpha_2, ..., \alpha_N)$ along its diagonal and $Z = (I - \Gamma)^{-1}$. This vector is based on the estimates of other value added (rents on capital) from the BEA's use table.

Similar to Long and Plosser (1983), an analytical expression for the variance of aggregate output growth is given by equation (15):

$$\sigma_y^2 = \boldsymbol{\omega}^T \Sigma_{yy} \boldsymbol{\omega},$$

where Σ_{yy} now satisfies

$$\text{vec}(\Sigma_{yy}) = (I_{N^2} - Z^T \boldsymbol{\alpha}_d \otimes Z^T \boldsymbol{\alpha}_d)^{-1} \text{vec}(Z^T \Sigma_{\varepsilon\varepsilon} Z). \tag{22}$$

There are two key differences that distinguish Horvath's (1998) framework from that of Long and Plosser (1983). First, a shock to sector i immediately propagates to other sectors by way of input-output linkages, as captured by the term $Z^T \varepsilon_t$ in (21) rather than just ε_t in (13). This follows from the fact that Horvath's (1998) model loses the one-period time-to-build feature of Long and Plosser (1983). Second, sectoral shocks propagate through time by way of capital accumulation and thus are scaled by the matrix of capital shares, as captured by the autoregressive coefficient $Z^T \boldsymbol{\alpha}_d$. Both of these features will change the variance decompositions carried out earlier as well as the nature of the propagation of sector-specific shocks.

Recall that under Long and Plosser (1983) and unit variance sectoral shocks, aggregate variability was amplified one and a half times relative to the case without sectoral linkages. Under Horvath (1998), aggregate output variance increases to 0.42 or a five-time increase relative to the case without sectoral linkages. The third column of Table 2 shows $\widetilde{\lambda}$ the contribution from different sectors to aggregate variability using the Horvath (1998) model. By and large, the sectors that contribute most to aggregate variability are the same as those in the first column of the table using the Long and Plosser (1983) framework. However, the importance of the sectors with extensive sectoral linkages is amplified in Horvath (1998). Thus, manufacturing's share of aggregate variability increases from 35 percent to 43 percent. Similarly, real estate, rental, and leasing sees its contribution to aggregate variance increase from 16 percent to roughly 26 percent. As we shall see below, intersectoral linkages take on a greater role in Horvath (1998) because sectoral shocks get propagated by way of not only the input-output matrix but also internal capital accumulation, $Z^T \boldsymbol{\alpha}_d$, where $\boldsymbol{\alpha}_d$ is the matrix of capital shares.

Some Key Assumptions and the Irrelevance of Sectoral Shocks

We suggested earlier that sectoral shocks can fail to average out as N becomes large when the distribution of sectoral shares is sufficiently leptokurtic. Aside from this consideration, one might also ask whether sectoral linkages necessarily prevent sectoral shocks from being irrelevant at the aggregate level. To that end, Dupor (1999) uses Horvath's (1998) framework to analyze the conditions under which sectoral shocks average out even in the presence of sectoral linkages. In particular, Dupor's work relies on three key conditions:

(A1) $\boldsymbol{\omega} = N^{-1}\mathbf{h}$, where $\mathbf{h}$ is a vector of ones, $(1, 1, ..., 1)^T$.

(A2) $\Gamma\mathbf{h} = \kappa\mathbf{h}$, where κ is a positive scalar. Put another way, $\mathbf{h}$ is an eigenvector of the $N \times N$ matrix Γ with corresponding eigenvalue, κ. This assumption implies that all rows of Γ sum up to κ, so that all sectors serve as equally intense material input providers to all other sectors.

(A3) $\Sigma_{\varepsilon\varepsilon} = I$.

It turns out that under these assumptions, the role of sectoral shocks vanishes in aggregation not only in the environment studied by Dupor (1999), but also in other canonical versions of the multisector growth model including Long and Plosser (1983) and, more recently, Carvalho (2007).

In the case of Long and Plosser (1983), assumption (A1) and equation (13) imply that the variance of aggregate output growth (15) can be expressed as

$$\begin{aligned}\sigma_y^2 &= N^{-2}\mathbf{h}^T\Sigma_{yy}\mathbf{h} \\ &= N^{-2}\mathbf{h}^T\Gamma^T\Sigma_{yy}\Gamma\mathbf{h} + N^{-2}\mathbf{h}^T\Sigma_{\varepsilon\varepsilon}\mathbf{h}.\end{aligned}$$

When (A2) and (A3) hold, the first term on the right-hand side of this last equation simplifies as follows,

$$N^{-2}\mathbf{h}^T\Gamma^T\Sigma_{yy}\Gamma\mathbf{h} = N^{-2}\kappa^2\mathbf{h}^T\Sigma_{yy}\mathbf{h},$$

while the second term becomes

$$\begin{aligned}N^{-2}\mathbf{h}^T\Sigma_{\varepsilon\varepsilon}\mathbf{h} &= N^{-2}\mathbf{h}^T\mathbf{h} \\ &= N^{-1}.\end{aligned}$$

It immediately follows that

$$\sigma_y^2 = N^{-1}(1-\kappa^2)^{-1}, \tag{23}$$

which indeed converges to zero at rate N.

Using the same assumptions, and by following similar steps, the variance of aggregate output growth in Horvath (1998) becomes

$$\sigma_y^2 = N^{-1}[(1-\kappa-\alpha)(1-\kappa+\alpha)]^{-1}, \tag{24}$$

which also converges to zero at rate N.

Several observations are worth noting with respect to equations (23) and (24). Under the maintained assumptions, the irrelevance of sectoral shocks holds in the limit. Hence, a question arises as to what the relevant level of disaggregation is in practice. To us, this question is mainly one that relates to technology and the nature of shocks under consideration. In particular, it is likely befitting that manufacturing and retail trade should be thought of as characterized by fundamentally different technologies, and hence affected by fundamentally different shocks, but it may also be the case that within manufacturing, "iron and steel products" should be treated differently than "metalworking machinery."

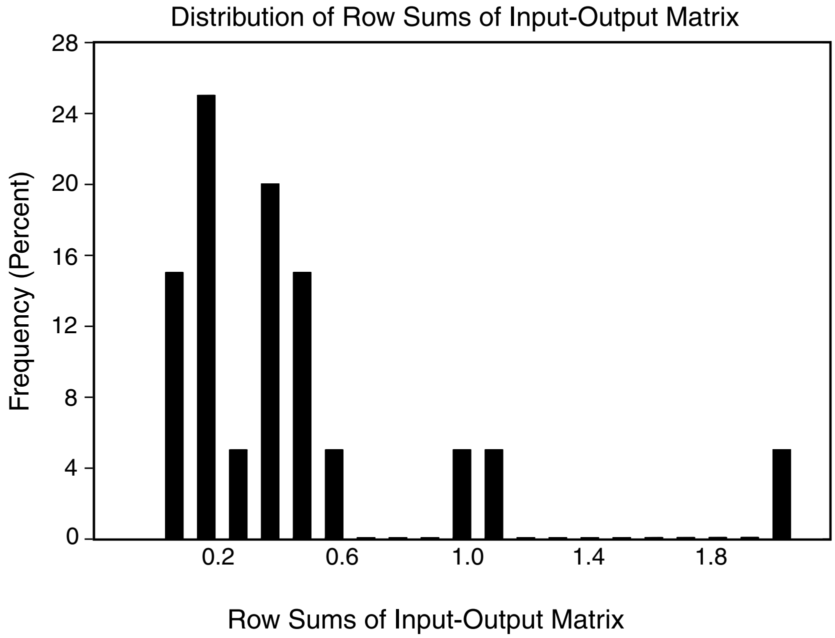
In general, the Census uses two criteria for making industry classifications. The first is the economic significance of the industry, which refers to the size of an industry at the highest level of disaggregation relative to the average size of industries in its particular division. For example, breaking up "iron and steel products" within manufacturing into two separate industries, "iron products" and "steel products" would involve comparing the size of each industry individually to the average manufacturing industry size. The notion of size, or economic significance, considers five main characteristics: the number of establishments in the industry, the industry's number of employees, its payroll, its value added, and its value of shipments. A weighted average is then constructed from these five measurements, which are expressed relative to a similarly computed measure of the average size of existing industries in the pertinent division. Once a given economic significance score is reached, the industry potentially qualifies as a new classification at the highest level of disaggregation. The second criterion is based on specialization and coverage ratios. These ratios combine to measure the share of production and shipments of an industry's primary products in the economy. Conditional on meeting the first criterion, an industry is then recognized only if each of these ratios reaches a threshold level.

In an exercise that focuses on U.S. industrial production, Foerster, Sarte, and Watson (2011) consider up to 117 sectors, the highest level of disaggregation for which input-output tables from the BEA can be matched to sectoral output data. Interestingly, the authors find that the relevance of sectoral shocks for aggregate variability appear robust to the level of disaggregation. This finding arises in part because, as an empirical matter, sectoral variability increases with the level of disaggregation, thus offsetting the "averaging out" effect of N^{-1} in equations (23) and (24).

The conclusions in this section rely crucially on assumption (A2), $\Gamma\mathbf{h} = \kappa\mathbf{h}$, so that all rows of Γ must sum up to the same scalar. Put differently, this condition requires the input-output matrix to be such that all sectors serve as equally important material input providers to all other sectors. Figure 1 shows the row sums of the input-output matrix, Γ, associated with our two-digit decomposition. The figure indicates that the row sums, $\Gamma\mathbf{h}$, can differ considerably from one another in practice. Using a four-digit decomposition of industrial production, Foerster, Sarte, and Watson (2011) show that when output is disaggregated further, $\Gamma\mathbf{h}$ further displays pronounced skewness. This skewness is consistent with the notion emphasized in Carvalho (2007) that a few sectors play crucial roles as input providers. Thus, the key step that allows for aggregation despite sectoral linkages, assumption (A2), does not appear to be consistent with our sectoral data.

That said, one should be clear that the assumptions outlined in this section represent sufficient conditions for the asymptotic irrelevance of sectoral shocks. To the degree that $\Gamma\mathbf{h}$ differs from $\kappa\mathbf{h}$, so that not all sectors serve as equally important material providers to other sectors, the implications of this difference for the contribution of sectoral shocks to aggregate variability is not immediately clear. In particular, the way in which a given sectoral shock becomes amplified and propagates to other sectors, and thus affects aggregate output, generally needs to be computed numerically for a given input-output matrix, Γ, and sectoral shares, $\boldsymbol{\omega}$. It is to this consideration that we next turn our attention.

Figure 1 Importance of Sectors as Material Input Providers to Other Sectors



3. SECTORAL SHOCK PROPAGATION WITH SECTORAL LINKAGES

Given the Long and Plosser (1983) model solution in (13), the effects of sectoral shocks arising at t, ε_t, on sectoral output growth at date $t+j$ are given by

$$\frac{\partial \Delta \mathbf{y}_{t+j}}{\partial \varepsilon_t} = (\Gamma^T)^j, \tag{25}$$

and the resulting change in aggregate output growth is

$$\boldsymbol{\omega}^T \frac{\partial \Delta \mathbf{y}_{t+j}}{\partial \varepsilon_t} = \boldsymbol{\omega}^T (\Gamma^T)^j.$$

Consider the effects of a negative shock to a given sector, say manufacturing, denoted by ε_{mt}, so that $\varepsilon_t = (0, 0, ..., \varepsilon_{mt}, ..., 0)^T$. Two noteworthy observations arise.[6]

[6] Observe that under conditions (A1) and (A2) in the previous section, $\omega^T \frac{\partial \Delta \mathbf{y}_{t+j}}{\partial \varepsilon_t} = N^{-1}\kappa$ $\forall j$, which goes to zero as N becomes large.

First, because $(\Gamma^T)^0 = I$, the shock to manufacturing will only affect itself in the period in which the shock occurs. In other words $\partial \Delta y_t / \partial \varepsilon_{it} = 0$ for all sectors that are not manufacturing. There is no propagation of the shock to other sectors in the period in which the shock occurs. Hence, the contemporaneous effect of the manufacturing shock on aggregate output growth is simply $\omega_m \partial \Delta y_t / \partial \varepsilon_{mt}$, where ω_m is the share of the manufacturing sector in GDP. This result may be interpreted as the formal justification for the notion that the aggregate effects of sectoral shocks can be judged from the output share of the sector in which the shock occurs. As we shall see shortly, however, this is generally not the case.

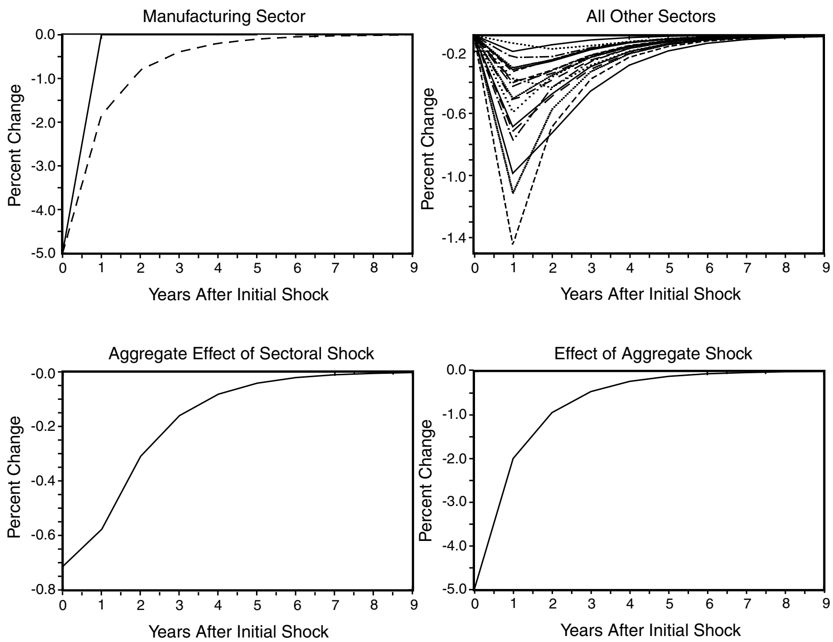
Second, the effect of the shock to manufacturing on any other sector j in the following period is given by

$$\frac{\partial \Delta y_{j,t+1}}{\partial \varepsilon_{m,t}} = \gamma_{mj}. \tag{26}$$

In other words, in Long and Plosser (1983), a shock to the manufacturing sector begins to propagate to another sector j, in the period after the shock, by exactly γ_{mj}, the amount that sector j spends on materials produced in the manufacturing sector as a fraction of sector j's total spending on inputs. Therefore, the less sector j spends on materials from sector m, the lower will be the effect of a shock to sector m on sector j.

Figure 2 depicts impulse responses to an unanticipated exogenous one-time 5 percent fall in manufacturing total factor productivity (TFP) in the Long and Plosser (1983) economy. The solid line in the top left-hand panel of Figure 2 shows the time path of the shock. The dashed line in that panel shows the response in output growth in the manufacturing sector. By design, the shock to manufacturing TFP has dissipated after one period. As discussed above, because the shock does not propagate to other sectors contemporaneously, thus also preventing a feedback effect from those sectors back into manufacturing, the initial decline in manufacturing output growth is exactly equal to the size of the decline in TFP, or 5 percent. Moreover, we can see that the effect of the shock on manufacturing output growth is considerably longer lived than the shock itself. The top right-hand panel of Figure 2 explains why. That panel depicts the effects of the fall in TFP in manufacturing on all other sectors. As suggested above, the initial effect of the manufacturing TFP decline on all other sectors is zero. However, in the period after the manufacturing disturbance occurs, the shock has spread to all other sectors by way of input-output linkages so that these all experience a decline in output growth. The size of this decline in the different sectors reflects the degree to which they rely on manufacturing as an input provider, $\gamma_{mj} > 0$, which then feeds back into manufacturing in so far as it uses the output of those sectors as inputs, $\gamma_{im} > 0$. In the top right-hand panel of Figure 2, the largest decline in output in the period following the manufacturing TFP shock occurs in the construction sector at -1.4 percent.

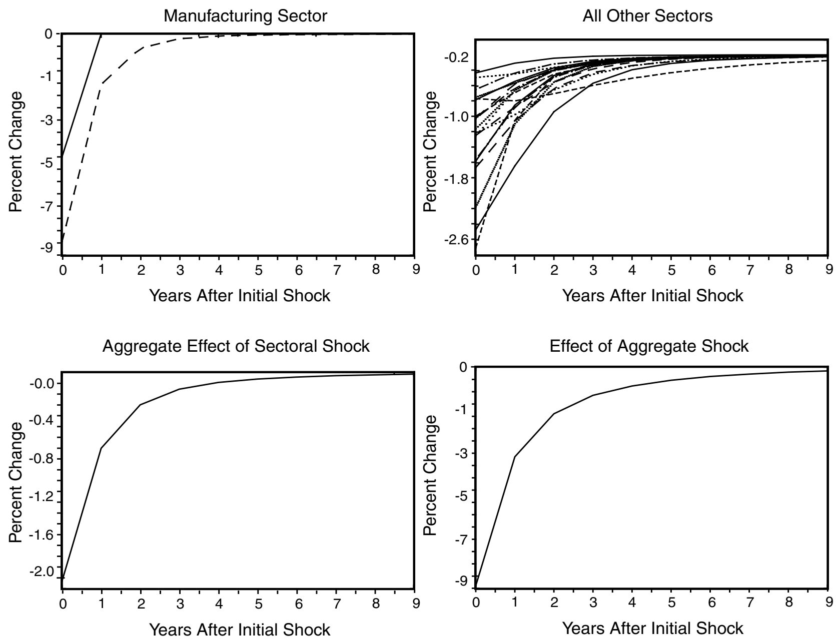
Figure 2 Effects of One-Time Unanticipated Shocks in Long and Plosser (1983)



The bottom left-hand panel of Figure 2 illustrates the aggregate effect of the manufacturing shock on output. As indicated above, because the share of manufacturing in GDP is approximately 0.14, the initial effect of the shock on aggregate output in the Long and Plosser (1983) model is roughly 0.14×5 or a 0.7 percent decline in GDP. In addition, note that this aggregate effect is considerably more persistent than the initial one-time decline in manufacturing TFP. As before, this feature arises mainly from the propagation of the shock to other sectors which, by way of a feedback effect, induces persistence in the output decline of all sectors and, therefore, at the aggregate level as well.[7] For comparison, the bottom right-hand panel of Figure 2 shows the effect on aggregate output of an unanticipated one-time decline in TFP in all sectors, which may be interpreted as an unanticipated aggregate TFP shock. As in the case of sectoral shocks, the effect on aggregate output is considerably

[7] Recall that the model in Long and Plosser (1983) contains no internal propagation mechanism, such as might occur through capital accumulation, other than the one-period delivery lag in materials. Strictly speaking, the induced persistence in the impulse responses to a one-time sectoral shock in Figure 2 stems from the combination of that lag with sectoral linkages.

Figure 3 Effects of One-Time Unanticipated Shocks in Horvath (1998)



longer lived than the shock itself. However, because the 5 percent fall in TFP now applies to all sectors, the size of the output decline is considerably more pronounced.

In Horvath (1998), the effects of sectoral shocks arising at t, ε_t, on sectoral output growth at date $t + j$ are given by

$$\frac{\partial \Delta \mathbf{y}_{t+j}}{\partial \varepsilon_t} = (Z^T \boldsymbol{\alpha}_d)^j Z^T. \tag{27}$$

In this case, a negative shock to the manufacturing sector immediately propagates to other sectors by way of input-output linkages, as embodied in $Z^T = (I - \Gamma^T)^{-1}$, because materials are used within the period. This is the source of the notable amplification of sectoral shocks in Horvath (1998) relative to one without input-output linkages. In particular, the variance-covariance matrix of sectoral output growth (absent any propagation) is $Z^T \Sigma_{\varepsilon\varepsilon} Z$ rather than just $\Sigma_{\varepsilon\varepsilon}$. In addition, sectoral shocks further propagate over time through their effects of capital accumulation by way of input-output linkages, $Z^T \boldsymbol{\alpha}_d$. In other words, the model contains an internal propagation mechanism that potentially extends the life of the original shock on aggregate economic activity.

Analogous to Figure 2, Figure 3 shows the effects of a 5 percent unanticipated one-time decline in manufacturing TFP, but this time in the Horvath

(1998) economy. The impulse responses in Figure 3 highlight several key differences with those that obtain in the Long and Plosser (1983) model. First, the effect of the fall in manufacturing TFP is immediately amplified through input-output linkages. In particular, while TFP falls by 5 percent in the top left-hand panel of Figure 3, manufacturing output growth falls by 9 percent or nearly double the size of the shock. This stems from the fact that materials are used within the period in Horvath (1998). As pointed out earlier, in the solution for sectoral output growth (21), output growth at time t reflects the effects of contemporaneous sectoral links, $(I - \Gamma^T)^{-1}\varepsilon_t$, instead of the effects of sectoral disturbances alone, ε_t, in the solution to the Long and Plosser (1983) model, (13). The top right-hand panel of Figure 3 illustrates this feature and, unlike Figure 2, shows that output growth falls in all sectors at the time that manufacturing TFP declines. In addition, observe that the output decline in all sectors is considerably larger than that in the period after the shock in the Long and Plosser (1983) economy in Figure 2. Second, the top right-hand panel of Figure 3 suggests that impulse responses in some sectors are slightly non-monotonic so that, in those sectors, the outlook gets worse before it gets better even though the manufacturing TFP shock itself has already dissipated.[8] Third, and related to this last observation, the effects of the one-time decline in TFP is somewhat more persistent than in the Long and Plosser (1983) framework. Finally, because of contemporaneous intersectoral linkages, the effect of the decline in manufacturing TFP on aggregate output is now noticeably more pronounced than in Figure 2. Specifically, aggregate output growth declines by 2 percent on impact and continues to be below its steady state well after the shock has dissipated. We saw earlier that the contribution of manufacturing's share to the fall in aggregate output is roughly 0.7 percent in the bottom left-hand panel of Figure 2. Therefore, in this case, contemporaneous input-output links add about 1.3 percent to the decline in aggregate output on impact.

The basic lesson of this section is that input-output linkages, and potentially other forms of complementarities in production, propagate and amplify the effects of sectoral disturbances. Therefore, using the share of the sector in which a disturbance occurs as a gauge of its effect on aggregate output constitutes, in general, a relatively poor approximation. However, the extent of the amplification and propagation mechanism that results from intersectoral linkages depends on the particular economic environment in which these linkages operate. In their recent article, Foerster, Sarte, and Watson (2011) extend the analysis in this section to include intersectoral linkages in investment goods (so that some sectors produce new capital goods for other sectors), less than full capital depreciation within the period, and allow for aggregate

[8] Given our calibration of Γ and α_d based on the input use tables from the BEA, $Z^T\alpha_d$ has complex eigenvalues.

shocks. They find that the importance of sectoral disturbances in explaining aggregate fluctuations has noticeably increased over time and that, over the period 1984–2007, these disturbances explain half the variation in U.S. industrial production. However, although the nature of intersectoral production has changed over time, the authors also find that changes in the input-output matrix reflecting new sectoral links has not led to greater propagation of shocks.

4. CONCLUDING REMARKS

This article has provided an overview of some key aspects of the effects of sectoral shocks on aggregate economic activity. It discussed the role of sectoral shares in determining each sector's contribution to aggregate variations. It also illustrated how input-output linkages in production influenced the amplification and propagation of sectoral shocks. The mechanisms by which this amplification and propagation take place depend importantly on the details of the economic environment in which intersectoral linkages operate. In general, because of input-output linkages across sectors, using the share of the sector in which a disturbance occurs as a gauge of its effect on aggregate output constitutes a poor approximation. In addition, the key condition required of the input-output matrix that lead sectoral shocks to average out in aggregation, carefully articulated in Dupor (1999), does not appear to apply in practice. Using an input use matrix obtained from the BEA for 1997, as well as a two-digit level disaggregation of GDP, suggests that manufacturing and real estate, rental, and leasing contribute the most to aggregate variations.

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