

## **Area interpolation in presence of measurement error and an application to German administrative data**

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## Data access

The data used in this Methodenreport relate to all available individual data of the research data center (IABS, IEBS, LIAB and BAP). For more information on the data, please have a look at <http://fdz.iab.de> (currently only in German).

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# Area interpolation in presence of measurement error and an application to German administrative data\*

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## Abstract

In many situations the applied researcher wants to combine different data sources without knowing the exact link and merging rule. This paper considers different interpolation methods for interpolating attributes from German labor office districts to German counties and vice versa. In particular, we apply dasymetric weighting as an alternative to simple area weighting both of which are based on estimated intersection areas. Since these estimates can be spurious, our theoretical framework extends the well-known Goodchild and Lam (1980) approach to the presence of measurement error in the underlying maps. We also present conditions under which the choice of interpolation method does not matter and confirm the theoretical results with a simulation study. Our application to German administrative data suggests robustness of estimation results of interpolated attributes with respect to the choice of interpolation method. We deliver weighting matrices for regional data sources of the two largest German data producers.

**Keywords:** area interpolation, spurious polygons, dasymetric mapping, German administrative data

**JEL:** C49, C89, R10

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# 1 Introduction

With a rising interest in research on the effects of recent German labor market reforms such as Hartz IV, researchers from both economics and social sciences alike have been increasingly concerned with combining information from different administrative data sources. In particular, researchers want to combine information collected by the federal statistical bureau (Statistisches Bundesamt) which is coded at the level of German counties with data from the federal employment office (Bundesagentur für Arbeit) which is reported for labor office districts. Yet, the two sets of regions are geographically incompatible, i.e. one set of regions does not in general respect the boundaries of the other set and the two sets are not nested hierarchically. In this case, transferring data from one set of regional objects to the other is non-trivial and proves to be an obstacle to current research on German labor market reforms. Thus, what is needed are appropriate weighting matrices in order to transfer attributes from the labor office districts to counties and vice versa. The purpose of this paper is to develop appropriate weighting matrices and thus provide a solution to this areal interpolation problem that may facilitate research based on data from both data sources in Germany.

Since this areal interpolation problem has often been encountered in all kinds of situations, there is a rich literature suggesting different types of interpolation in order to derive appropriate weighting matrices. Following the literature in this field, we refer to the regions for which an attribute is known as source region and the region to which the attributes have to be transferred to as target areas (Goodchild and Lam, 1980). One simple cartographic approach based on the intersections of source and target regions has already been suggested by Goodchild and Lam (1980). Assuming a uniform distribution of the attribute of interest, each entry in the weighting matrix corresponds to the share of the source region that lies within the target region. Clearly, this simple areal weighting method critically hinges on the assumption of uniform densities within the source region. Since in many cases, this assumption is not plausible, two different techniques have been proposed to relax this assumption: smoothing techniques and dasymetric weighting techniques.

Smoothing techniques try to estimate a continuous density surface based on some density information of the source region that may then be used for calculating target area densities. Tobler (1979) proposes the so called smooth pycnophylactic interpolation which minimizes curvature on the surface under the constraint that data from a source region can only be allocated to an intersecting target region ("pycnophylactic criterion"). An alternative method makes use of source zone centroids and a spreading function such as a radially symmetric

kernel function (Bracken and Martin, 1989; Bracken, 1994). Such smoothing methods critically rely on knowing the central location of the source area and an adequate spreading function. In cases in which no center-periphery structure can be assumed, using such a spreading function may thus be quite inappropriate.

Dasymetric mapping provides a more general approach by using auxiliary information on the source area in order to identify a non-uniform density distribution within the area. Based on satellite images of land use, it is, for example, possible to distinguish unpopulated from populated areas in order to refine density estimates within the regions before allocating attributes to the target regions (Fisher and Langford, 1995). As an extension to this binary approach, it is also possible to distinguish more than two types of land use. In this case dasymetric mapping is only straightforward if the densities of different land use classes are known or somehow pre-defined (Eicher and Brewer, 2001). Alternatively, a regression technique has been proposed to derive population density estimates based on regressing the population of the source region on the different areas of land use (Langford et al., 1991; Yuan et al., 1997).

While dasymetric techniques summarize all approaches that use additional information on the source areas in order to refine its density distribution, a related approach uses auxiliary information from either the target areas or another external set, called control zones. Flowerdew and Green (1989) refine the simple area weighting by assuming uniform densities for the target areas. In this case, population densities for the target zones can be estimated by using the observed attributes for the source area and the area of overlap. Goodchild et al. (1993) develop a more general approach by using an external set of control zones for which uniform densities can be assumed. In a first step, control zone densities are being estimated similar to the procedure described by Flowerdew and Green (1989). In a second step, the estimated control zone densities are used to estimate target zone densities.

All of the approaches based on regression techniques have to deal with a number of estimation issues such as the required non-negativity of estimated densities and meeting the pycnophylactic criterion. Moreover, frequency data such as population ("spatially extensive data") and proportional data such as average income or unemployment rates ("spatially intensive data") have to be treated differently (Goodchild and Lam, 1980). For spatially extensive data, Poisson regression has been proposed (Flowerdew and Green, 1989) as an alternative to constrained OLS regression (Judge and Yancey, 1986). In particular, Flowerdew and Green (1989) suggest an iterative Poisson regression using an EM algorithm to derive target area estimates. While this approach was first developed for spatially extensive

data and binary auxiliary information only, extensions to continuous auxiliary data and spatially intensive data followed (Flowerdew and Green, 1992). Recently, Bayesian hierarchical models have been used for modelling Poisson responses with covariates that are spatially misaligned and thus unknown. Unlike the earlier approaches, the Bayesian approach allows for full inference of the distributions of estimated target zone attributes (Mugglin and Carlin, 1998; Mugglin et al., 2000; Best et al., 2000).

Thus, ever more sophisticated methods have been applied to deal with the areal interpolation problem and to reduce the error involved in any interpolation exercise. Several authors have addressed the reliability of different methods and typically conclude that simple area weighting performs poorly compared to more sophisticated methods such as dasymetric mapping using regression frameworks (Goodchild et al., 1993; Fisher and Langford, 1995).

Despite the shortcomings that have been attributed to the simple area weighting, this paper proposes simple area weights as suggested by Goodchild and Lam (1980) as a feasible solution to the areal interpolation problem between German counties and labor office districts. Alternatively, we consider a specific form of dasymetric mapping that uses information on a control variable that is available for both source and target region and does not necessitate the use of regression techniques in order to derive at refined density estimates. While the areal weighting matrices differ quite substantially for some source and target regions, transferred target area attributes are remarkably similar. We therefore introduce the concepts of local homogeneity and local similarity to explain this finding. In fact, under a high degree of local homogeneity and/or similarity, the choice of interpolation used does not have much influence. In the context of interpolating data from German labor office districts to German counties, these conditions seem to be met such that differences between different types of interpolation are rather small. Thus, from a practitioners point of view, even using the simple area weighting seems a feasible solution in this case. A sensitivity analysis of the types of interpolation when using the interpolated attributes as covariates in an economic analysis confirms that estimation results are not strongly affected by the choice of interpolation.

Since intersection areas that form the basis of any intersection based weighting schemes are not readily available for German counties and German labor office districts, intersection areas were being estimated by the GIS procedure of polygon overlay using the software package ArcView. Since the map of labor office districts comes with a stronger generalization than the map of German counties, intersecting both maps by polygon overlay results in spurious polygons, i.e. nonzero entries in the weighting matrix that are spurious due to

digitizing errors and the degree of generalization. This measurement error has typically been neglected by erasing any entries below an arbitrary threshold. Due to the arbitrariness of this approach, we decided to keep spurious polygons and develop a general framework of areal interpolation in the presence of measurement error. We discuss theoretical conditions under which estimated weighting schemes are unbiased even in the presence of spurious polygons. Our paper has therefore a slightly different focus compared to the recent contributions in this field which address errors due to misspecified interpolation methods only, but neglect errors stemming from measurement errors of the underlying map intersection (see Fisher and Langford, 1995). Using a Monte Carlo simulation, we therefore demonstrate the effect of local homogeneity in presence of measurement error on the proposed methods of interpolation.

The paper is structured as follows: section 2 presents the theoretical framework of the estimation of map intersections and it suggests several interpolation methods with different weighting matrices. Section 3 contains the application to German communities and labor office districts. Section 4 summarizes the main findings.

## 2 Theory

### 2.1 Estimation of map intersections

This subsection introduces the theoretical framework for the estimation of map intersections. We have two maps  $R$  and  $D$ . Each map contains a different disjoint regional classification of the same country. Denote  $\{D_j\}_{j=1,\dots,n}$  and  $\{R_j\}_{j=1,\dots,m}$  as two sequences of disjoint regions.

Let us denote  $\mu$  as a measure of land area with the usual properties (Elstrodt, 1999, definition 4.1):  $\mu(\emptyset) = 0$ ,  $\mu(A) \leq \mu(B)$  for  $A \subset B$  (monotonicity). For a sequence of subregions  $R_j$  (or  $D_j$ ) we have

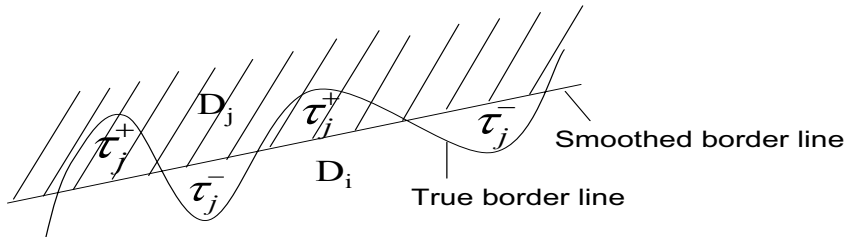
$$\mu\left(\bigcup_j R_j\right) \leq \sum_j \mu(R_j) \quad (\sigma\text{-additivity}).$$

The inequality holds with equality if  $R_j$  is a sequence of disjoint subregions. Then  $\mu(R) = \mu(\bigcup_j R_j) = \mu(\bigcup_j D_j) = \mu(D)$ . Our purpose is to determine  $\mu_{ij} = \mu(D_i \cap R_j)$ , the intersection size of regions  $D_i$  and  $R_j$ , for  $i = 1, \dots, n$  and  $j = 1, \dots, m$ .

Since we don't know the true  $\mu(R_j)$  and  $\mu(D_j)$  we have to estimate them by intersecting the two maps with a GIS procedure called polygon overlay based on the GIS software package ArcView. The estimated areas may be affected by the properties of the underlying maps. In particular, maps usually come with a certain degree of cartographic generalization. The

corresponding smoothing of the border lines generates a non-systematic error component whenever a part of region  $i$  is allocated to region  $j$  on the map. For the exposition of the theoretical framework, we assume the border lines of map  $R$  to be exact whereas the border lines of map  $D$  generate a non-systematic random error by smoothing the true border<sup>1</sup>. For this reason some part of  $D_j$  is falsely allocated to  $D_i$  ( $i \neq j$ ) and vice versa. Figure 1 shows the resulting spurious polygons.

Figure 1: Map generalization, random measurement error and spurious polygons



We assume that in expectation over two randomly chosen regions these errors balance out. Let us denote  $\epsilon_j$  as the error set associated with region  $j$ , i.e. some subset of  $D_j$  that is misleadingly allocated to  $D_i$ ,  $i \neq j$ , on the map. The error area  $\mu(\epsilon_j)$  is therefore a stochastic measurement error. Also, note that  $\mu(D_j \cap \epsilon_j) = 0$  since by definition  $D_j$  and  $\epsilon_j$  are disjoint subsets. Moreover,  $\epsilon_j$  is not necessarily a subset of  $D$  since at the outer border of the map  $\epsilon_j$  may lie outside the territory of map  $D$ . Denote  $D^C$  as the complementary set of  $D$ , i.e. the area surrounding  $D$ , and let us denote  $\tau_j^- = D_j \cap (\bigcup_i \epsilon_i \cup \epsilon^{D^C})$  and  $\tau_j^+ = (D \cup D^C) \cap \epsilon_j$ . The intersections with  $D^C$  and  $\epsilon^{D^C}$  are relevant at the outer border of  $D$  only. We make three assumptions about the outer border line and the aggregated error area:

**Assumption 1** *The measurement error does not systematically in- or decrease the area of any region, i.e.  $E\mu(\tau_j^+) = E\mu(\tau_j^-) \geq 0$ .*

**Assumption 2**  *$\mu(D^C \cap \bigcup_j \epsilon_j) = \mu(D \cap \epsilon^{D^C})$ , i.e. the error area at the outer border of  $D$  balances out.*

In this paper, expectations are always taken over the regions and over the nonsystematic smoothing error of the border lines.

Let us denote  $\hat{\mu}(A)$  as an estimate of  $\mu(A)$ .

<sup>1</sup>The theoretical framework carries over to the more complex case with both maps introducing a random error due to cartographic generalization.



**Theorem 1** *Suppose assumptions 1-2 hold, then  $\hat{\mu}(R_j)$  equals to  $\mu(R_j)$  and  $\hat{\mu}(D_j)$  is an unbiased estimator for  $\mu(D_j)$ .*

The first result is stable with respect to all unions of  $R_j$  and therefore also applies to  $\bigcup_j R_j$ . The second result is due to the observation  $\hat{\mu}(D_j) = \mu(D_j) - \mu(\tau_j^-) + \mu(\tau_j^+)$  and assumption 1. We need an additional lemma before we come to  $\hat{\mu}(D)$ .

**Lemma 1** *The error areas between the regions  $(\epsilon_j)$  perfectly balance out, i.e.  $\sum_j \mu(\tau_j^-) = \sum_j \mu(\tau_j^+)$ .*

**Proof.**

$$\begin{aligned}
\sum_j \mu(\tau_j^-) &= \sum_j \mu(D_j \cap \bigcup_i \epsilon_i) + \sum_j \mu(D_j \cap \epsilon^{D^C}) \\
&= \mu(D \cap \bigcup_i \epsilon_i) + \mu(D \cap \epsilon^{D^C}) \\
&= \mu(D \cap \bigcup_j \epsilon_j) + \mu(D^C \cap \bigcup_j \epsilon_j) \\
&= \mu(D \cup D^C \cap \bigcup_j \epsilon_j) \\
&= \sum_j \mu(D \cup D^C \cap \epsilon_j) \\
&= \sum_j \mu(\tau_j^+)
\end{aligned}$$

where we use the properties of  $\mu$  and assumption 2. ■

**Theorem 2** *Suppose assumption 2 holds, then  $\hat{\mu}(D)$  equals  $\mu(D)$ .*

**Proof.**

$$\begin{aligned}
\hat{\mu}(D) &= \hat{\mu}\left(\bigcup_j D_j\right) \\
&= \sum_j \mu(D_j) - \sum_j \mu(\tau_j^-) + \sum_j \mu(\tau_j^+) \\
&= \mu(D),
\end{aligned}$$

where lemma 1 immediately applies. ■

An interesting quantity is the relative bias of the size of  $D_j$ . Rewrite the previous equation for one particular area  $D_j$  as a fraction of its true area size  $\mu(D_j)$ :

$$\frac{\hat{\mu}(D_j)}{\mu(D_j)} = 1 + \frac{\mu(\tau_j^+) - \mu(\tau_j^-)}{\mu(D_j)}.$$

In expectation, the last term equals zero due to assumption 1. However in an application the distribution of this error may depend on the perimeter-size ratio of  $D_j$ .

A similar line of argument applies to the area size of the intersection of regions  $D_i$  and  $R_j$  if we make an additional assumption that slightly extends assumption 1.

**Assumption 3** *The measurement error does not systematically in- or decrease the area of any intersection between  $R_j$  and  $D_i$ , i.e.  $E\mu(\tau_i^- \cap R_j) = E\mu(\tau_i^+ \cap R_j) \geq 0$  for all  $i, j$ .*

This is a non crucial assumption if one considers that the partitioning of the regions into sub-regions as a result of the intersection between  $D_i$ 's and  $R_j$ 's does not systematically depend on the topology of the border lines. In the real world this is because administrative considerations typically form the basis of establishing border lines between sub-regions.

**Theorem 3** *Suppose assumptions 2-3 hold, then  $\hat{\mu}(D \cap R)$  equals to  $\mu(D \cap R)$  and  $\hat{\mu}(D_i \cap R_j)$  is an unbiased estimator for  $\mu(D_i \cap R_j)$ .*

**Proof.** The first part is shown by

$$\begin{aligned}
\hat{\mu}(D \cap R) &= \sum_{i,j} \hat{\mu}(D_i \cap R_j) \\
&= \sum_{i,j} \mu(D_i \cap R_j) - \sum_{i,j} \mu(\tau_i^- \cap R_j) + \sum_{i,j} \mu(\tau_i^+ \cap R_j) \\
&= \sum_{i,j} \mu(D_i \cap R_j) - \sum_i \mu(\tau_i^- \cap \bigcup_j R_j) + \sum_i \mu(\tau_i^+ \cap \bigcup_j R_j) \\
&= \sum_{i,j} \mu(D_i \cap R_j) - \sum_i \mu(\tau_i^-) + \sum_i \mu(\tau_i^+) \\
&= \mu(D \cap R),
\end{aligned}$$

where lemma 1 immediately applies. The second part follows from an application of the expectation operator to the second equality above together with assumption 3. ■

Again, rewrite the previous equation for one particular intersection area  $\hat{\mu}(D_i \cap R_j)$  as a fraction of its true area size  $\mu(D_i \cap R_j)$ :

$$\frac{\hat{\mu}(D_i \cap R_j)}{\mu(D_i \cap R_j)} = 1 - \frac{\mu(\tau_i^- \cap R_j)}{\mu(D_i \cap R_j)} + \frac{\mu(\tau_i^+ \cap R_j)}{\mu(D_i \cap R_j)}$$

where the last two terms balance out in expectations due to assumption 3. As argued above, these two terms may affect the estimated intersection area in an application and higher moments of the error distribution may depend on the perimeter-area ratio of any  $D_j$ .

## 2.2 Weighting schemes for area interpolation

In this section, we present different interpolation methods for transferring attributes from  $D_j$ , the source region, to  $R_i$ , the target region<sup>2</sup> and show how the required weighting matrices may be constructed based on the available estimates of areas  $\hat{\mu}(D_j)$ ,  $\hat{\mu}(R_i)$  and  $\hat{\mu}(R_i \cap D_j)$ . In particular, this section discusses how the measurement error of the map intersection affects such weighting matrices. We also consider a possible misspecification of these weighting schemes if the underlying assumptions regarding the density distributions of the source zone attributes do not hold. Moreover, we derive conditions under which such a misspecification does not affect the resulting area interpolation.

Before discussing several possible weighting schemes, note that there are two different kinds of attributes which have to be treated differently, i.e. for which different weighting matrices need to be used: frequencies (F) such as the number of job vacancies, participants in certain employment policies etc. and proportions (P) such as an unemployment rate<sup>3</sup>.

**Weighting Schemes** Without loss of generality, we focus on the case where we convert information from regions  $D_j$  to regions  $R_i$ . Let us denote  $f_{i,j}$  and  $p_{i,j}$  as weights with the usual properties:  $f_{i,j}$  and  $p_{i,j} \geq 0$ ,  $\sum_i f_{i,j} = 1$  and  $\sum_j p_{i,j} = 1$  for all  $i, j$ . The general rule for interpolating data from  $D_j$  to  $R_i$  is

$$F_{R_i} = \sum_j F_{D_j} f_{i,j} \quad \text{for } i = 1, \dots, n$$

where  $f_{i,j}$  is an appropriate weight for frequency  $F_{D_j}$ ,  $j = 1, \dots, m$  and

$$P_{R_i} = \sum_j P_{D_j} p_{i,j} \quad \text{for } i = 1, \dots, n$$

where  $p_{i,j}$  is an appropriate weight for proportion  $P_{D_j}$ ,  $j = 1, \dots, m$ . These merging schemes contain the special case of uniform weights  $f_{i,j} = f_i$  or  $p_{i,j} = p_i$  for all  $i$ . Uniform weights imply that  $F_{R_i}$  and  $P_{R_i}$  are simple averages over the  $F_{D_j}$  and  $P_{D_j}$  and corresponds to the simple area weighting proposed by Goodchild and Lam (1980).

**Construction of weights** There are several ways how the weights  $f_{i,j}$  and  $p_{i,j}$  can be constructed. Apart from simple area weights, we focus here on two alternative approaches:

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<sup>2</sup>Note that the vice versa case is not considered but our framework directly carries over.

<sup>3</sup>Goodchild and Lam (1980) have introduced the terms spatially extensive data for frequencies and spatially intensive data for proportions in the context of area interpolation.

naive binary weights<sup>4</sup> and some special form of dasymetric weighting that refines the simple area weights by using additional information on a region-specific attribute such as the population density that is known for both source and target regions.

First, consider naive binary weights. Region  $D_j$  is allocated to region  $R_i$  if they possess the largest intersection. In other words, we allocate a weight of one to the region  $D_j$  that shares the largest common area with  $R_i$  among all other intersecting regions. Obviously,  $w_{i,j} = f_{i,j} = p_{i,j}$ , where

$$w_{i,j} = \begin{cases} 1/\#_{i,j}(\mu(R_i \cap D_j) = \mu(R_i \cap D_l)) & \text{if } \mu(R_i \cap D_j) = \sup_{D_l} \mu(R_i \cap D_l) \\ 0 & \text{otherwise} \end{cases}$$

for all  $i, j$ , where  $\#_{i,j}(\mu(R_i \cap D_j) = \mu(R_i \cap D_l))$  is the number of sets  $D_l$  for which the equality holds. In an application we have typically  $\#_{i,j} = 1$  for all  $i, j$  and therefore we refer to these weights as binary weights. They may be considered as a rule of thumb and can be obtained by simple visual inspection. We include this naive binary weighting despite the much more sophisticated methods available because this rule of thumb is still being used by practitioners who are not familiar with the area interpolation literature. Therefore, it is worthwhile to compare these weights to more sophisticated methods for our application to German counties and labor office districts.

Secondly, we suggest a special form of dasymetric weighting that refines the simple area weights by using a region-specific attribute that is known for both source and target regions and which is denoted as  $S_{R_i}$  and  $S_{D_j}$ . Under the assumption that the distribution of this known attribute is highly correlated to the attribute to be interpolated to the target areas, one can use this information to re-estimate attribute densities of the intersection areas between source and target area<sup>5</sup>. For frequencies we suggest

$$f_{i,j} = \frac{\mu(R_i \cap D_j)S_{R_i}}{\sum_i \mu(R_i \cap D_j)S_{R_i}} \quad \text{for all } i, j$$

with an appropriately defined  $S_{R_i}$ . For the merger of proportions we suggest

$$p_{i,j} = \frac{\mu(R_i \cap D_j)S_{D_j}}{\sum_j \mu(R_i \cap D_j)S_{D_j}} \quad \text{for all } i, j$$

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<sup>4</sup>These weights are also considered by Goodchild and Lam (1980), see their equation (13).

<sup>5</sup>Thus, instead of using zones assumed to have equal densities in order to refine density estimates for the source regions (see e.g. Goodchild et al., 1993), we refine the source area density by using the known densities of intersecting target areas. In an application one may use any known region-specific information that is highly spatially correlated to the attributes to be interpolated. When using, for example, population,  $S_{R_i} = \text{pop}(R_i)/\mu(R_i)$  where  $\text{pop}(R_i)$  is the number of individuals in  $R_i$ .

with an appropriately defined  $S_{D_j}$ . These weights include the special case in which the region-specific variable does not contain any information, i.e.  $S_{R_i} = S_R$  or  $S_{D_j} = S_D$  for all  $i, j$ . In this case the information is uniformly distributed across area space<sup>6</sup> and the weights simplify to the simple area weights by Goodchild and Lam (1980) which is

$$f_{i,j} = \frac{\mu(R_i \cap D_j)}{\mu(D_j)} \quad \text{for all } i, j$$

in the case of frequencies and

$$p_{i,j} = \frac{\mu(R_i \cap D_j)}{\mu(R_i)} \quad \text{for all } i, j$$

in the case of proportions. These weights use information on the intersection and area size of  $R_i$  and  $D_j$  only.

**Estimation of weights** The above weights can be estimated by replacing the true area sizes  $\mu$  with their empirical counterparts  $\hat{\mu}$ . Naive weights can be estimated by

$$\hat{w}_{i,j} = \begin{cases} 1/\#_{i,j}(\hat{\mu}(R_i \cap D_j) = \hat{\mu}(R_i \cap D_l)) & \text{if } \hat{\mu}(R_i \cap D_j) = \sup_{D_l} \hat{\mu}(R_i \cap D_l) \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

for all  $i, j$ .

**Theorem 4** *Suppose assumptions 1-3 hold, then estimator (1) is unbiased, i.e.  $E\hat{w}_{i,j} = w_{i,j}$ .*

The proof is straightforward by taking expectations over  $\hat{\mu}(R_i \cap D_j)$ .

The estimator for the second continuous weight is given by

$$\hat{f}_{i,j} = \frac{\hat{\mu}(R_i \cap D_j)S_{R_i}}{\sum_i \hat{\mu}(R_i \cap D_j)S_{R_i}}, \quad (2)$$

for all  $i, j$  and for  $\hat{p}_{i,j}$  analogously. Note that for simplicity we assume here that  $S_{R_i}$  and  $S_{D_j}$  are known numbers. Furthermore we require:

**Assumption 4** *Assume that the measurement error of  $D_j$  intersected with any  $R_i$  is independent of the total measurement error of  $D_j$  for all  $i, j$ .*

From assumption 4 follows

$$E \left[ \frac{\mu(\tau_j^+ \cap R_i) - \mu(\tau_j^- \cap R_i)}{\mu(\tau_j^+) - \mu(\tau_j^-)} \right] = 0$$

for all  $i, j$ .

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<sup>6</sup>For a given region  $i$  this requirement could be relaxed since it is only necessary that  $S_{R_i}$  does not vary in the neighborhood of  $i$ .

**Theorem 5** *Suppose assumptions 1-4 hold, then estimator (2) is unbiased, i.e.  $E\hat{f}_{i,j} = f_{i,j}$ .*

The proof uses the results of the previous subsection and assumption 4. Note that  $S_{R_i}$  and  $S_{D_j}$  are constants. In an application, however,  $\hat{f}_{i,j}$  may be affected by the random measurement error of the map intersection.

We conclude that our proposed estimators have nice theoretical properties, i.e. they are unbiased. The estimates in an application are more precise if the underlying maps are exact. Note that the theorems directly carry over to the case of  $\hat{p}_{i,j}$ .

**Misspecification of area interpolation** Area interpolation based on the proposed weighting matrices may not only be affected by the random measurement error of the underlying map intersection. The construction of weights, i.e. interpolation method itself, may be misspecified if underlying assumptions do not hold. In particular, simple area weighting assumes a uniform density distribution within the source region while our dasymetric weighting approach assumes the distribution of a known attribute in intersecting target areas to reflect the density distribution within the source region. Clearly, none of the proposed interpolation methods need to be appropriate if there is further local heterogeneity within the source region. However, this case is not modelled here and we assume the dasymetric approach to yield the least misspecified interpolation results. The question thus arises under which conditions the misspecifications implied in naive binary weighting and simple area weighting result in large differences between the estimated frequencies  $F_{R_i}$  and proportions  $P_{R_i}$  across interpolation methods and under which conditions all methods yield very similar results. For this purpose we introduce the concept of local homogeneity and global heterogeneity with respect to information  $S$ .

**Definition 1** *Local homogeneity with respect to information contained in  $S_i$  induces that  $S_i \approx S_j$  for all  $i$  and all  $j$  in the direct neighborhood of  $i$ .*

**Definition 2** *Global  $c$ -heterogeneity corresponds to*

$$\sup_i \inf_j |S_i - S_j| \leq c$$

*for all regions  $i$  and all regions  $j$  in the direct neighborhood of  $i$  and any  $c \geq 0$ .*

It is then evident that a small  $c$  implies local homogeneity for all regions  $i$ . Having this in mind it is easy to show that local homogeneity implies that simple area weighting and dasymetric weighting using the region-specific information  $S$  yield very similar results.

**Definition 3** *Similarity of the regional entities  $R_i$  and  $D_j$  is defined by*

$$\sup_{R_i} |\mu(R_i) - \sup_{D_j} \mu(R_i \cap D_j)| < \epsilon$$

for all  $i, j$  and any  $\epsilon > 0$ .

Similarity of the regional entities suggests that weights are similar across all weighting schemes. Clearly, if for all intersections  $i, j$  there is one large intersection that almost completely covers the reference region, differences between the interpolation methods tend to be small. In practice, a combination of local homogeneity and similarity of the two regional entities may yield very similar results for all interpolation methods.

**Monte Carlo Evidence** It is interesting to investigate how the weighting schemes considered above affect the results in the presence of measurement error when the true value is known.<sup>7</sup> For this reason we perform a series of simulations for the prediction of frequency  $F_R$ . In order to make the simulation results comparable to our application in the following section we use here the same regional classification for  $R$  and  $D$ . The number of sets  $R_i$  and  $D_j$  and the set of intersections is therefore identical to the empirical framework. The remaining simulation framework is chosen as follows:

- maximum dissimilarity of regional entities conditional on the set of intersections. This implies equal intersection areas for a given  $R_i$ , i.e.  $\mu(R_i \cap D_j) = \mu(R_i \cap D_l)$  for all  $l$  s.t.  $\mu(R_i \cap D_l) > 0$ .
- $F_D \sim U(900, 1100)$  is a discrete and independently drawn random variable, i.e. no autocorrelation in  $F_{D_j}$ .
- the measurement error of the estimated intersection areas follows a normal distribution:  $\hat{\mu}(R_i \cap D_j) - \mu(R_i \cap D_j) = \epsilon_{i,j}$ , where  $\epsilon_{i,j} \sim N(0, \mu(R_i \cap D_j))$ . This error is resampled in each repetition of the 500 simulations.
- $S_R$  is drawn according to three different designs of spatial autocorrelation:
  - *i*)  $S_R = 1$ , no variation in the region-specific information.
  - *ii*)  $S_R$  is drawn element by element from  $N(5, 0.5)$ . If there is already a  $S_R$  assigned to the direct neighborhood of  $S_{R_i}$  we compute  $S_{R_i} = 0.2\epsilon_{R_i} + \bar{S}_{R_i}$ , where

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<sup>7</sup>See also Fisher and Langford (1995) for an extensive Monte carlo study for the comparison of different weighting schemes in the absence of measurement errors.

$\epsilon_R \sim N(0, 0.5)$  and  $\bar{S}_{R_i}$  is the average over all neighboring and already assigned  $S_{R_i}$ . This simulation design induces a weak spatial autocorrelation which is confirmed by a Moran's I statistic. Accordingly, there is significant clustering of similar values of the region-specific information  $S_{R_i}$ <sup>8</sup>.

- *iii*)  $S_R \sim N(5, 0.5)$ , random variation in the region-specific information,

Simulation designs *i-iii* allow us to evaluate the relevance of the information  $S_R$  in an application. Simulation results are presented in table 1, where we relate the resulting  $\hat{F}_R$  to their true values. The true values are computed with the exact  $\mu(R_i \cap D_j)$  and the correct interpolation method which is assumed to be the dasymetric weighting approach that uses the region-specific information. Any biases and higher moments of the distribution are therefore due to either measurement errors or due to the misspecification of the weighting scheme. In particular, the interpolation based on the dasymetric weights that use the region-specific information deviate from the true  $F_R$  only due to the measurement error, while the other weighting schemes may be affected by a combination of measurement errors and misspecification.

Table 1 clearly supports our theoretical framework that the measurement error does not bias estimation results if the weighting scheme is correctly specified. As expected for our simulation design, naive binary weighting performs poorly in our simulation framework. We also observe that ignoring region-specific information biases results and the variance increases slightly (see *ii*) and *iii*). Moreover, the misspecification is more severe in case of a random variation in  $S$  than in the case of spatial autocorrelation. Moreover, in case of spatial autocorrelation in  $F_D$  and similarity of the regional entities, all three interpolation methods produce similar results<sup>9</sup>.

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<sup>8</sup>We calculate Moran's I using different weights for the spatially lagged vector. Using a weight of one for regions within a 0.5 degree radius of the grid location of the county, we get a test statistic of 0.23 ( $z = 7.0$ ). Using a 1 degree radius the test statistic falls to 0.15 ( $z = 9.6$ ) but again is highly significant. 0.1 degree correspond to 11.1 km along the longitude and between 6.5 to 7.5 km along the latitude. Clearly, using the grid position for the weighting scheme is a somewhat crude but justifiable approach.

<sup>9</sup>These cases are not presented but results are available on request.



Table 1: Monte Carlo Evidence for the distribution of  $(\hat{F}_R - F_R)/F_R$

	Mean	Sd	MSE <sup>‡</sup>	MSE <sup>‡</sup> in % of $i$
<i>Simulation i</i>				
Naive weights	-0.2417	1.5350	2.4146	100%
Area weights, $S_{R_i} = 1$	-0.0001	0.0436	0.0019	100%
Dasymetric weights	-0.0001	0.0436	0.0019	100%
<i>Simulation ii</i>				
Naive weights	-0.2369	1.5166	2.3562	97.6%
Area weights, $S_{R_i} = 1$	0.0035	0.0682	0.0047	247.4%
Dasymetric weights	-0.0000	0.0436	0.0019	100%
<i>Simulation iii</i>				
Naive weights	-0.2331	1.5337	2.4066	99.7%
Area weights, $S_{R_i} = 1$	0.0085	0.0965	0.0094	494.7%
Dasymetric weights	-0.0000	0.0436	0.0019	100%

<sup>‡</sup> Mean squared error

We conclude that without any precise information on the spatial distribution of the data and the degree of similarity of the regional entities, there is no way to tell how strongly research results are affected by the choice of interpolation method. In empirical applications, a sensitivity analysis may be useful to investigate the robustness of research results based on different interpolation approaches. Our simulation results suggest that higher moments of the error distribution are also affected by the choice of the weighting scheme.

### 3 Empirical application

The purpose of the empirical application is to identify an appropriate interpolation method in order to transfer attributes from the German labor office districts to the counties. As has been discussed in the introduction, different administrative agencies report data for different sets of regions such that research is severely hampered. In particular, both agencies provide important data for researchers in labor economics, other fields of economics and social sciences alike. Typically, microdata are coded at the level of the German counties while important labor market characteristics are coded at the level of the federal employment office districts. Since current research on German labor market reforms often necessitates

combining both data sources, solving this areal interpolation problem is thus of some importance and urgency. We apply the three interpolation methods proposed in the previous section and perform a sensitivity analysis in order to test the robustness of estimation results with regard to the choice of method and discuss the results in light of the above theoretical considerations.

Figure 2: The German Communities (left) and the German federal employment office districts (right)

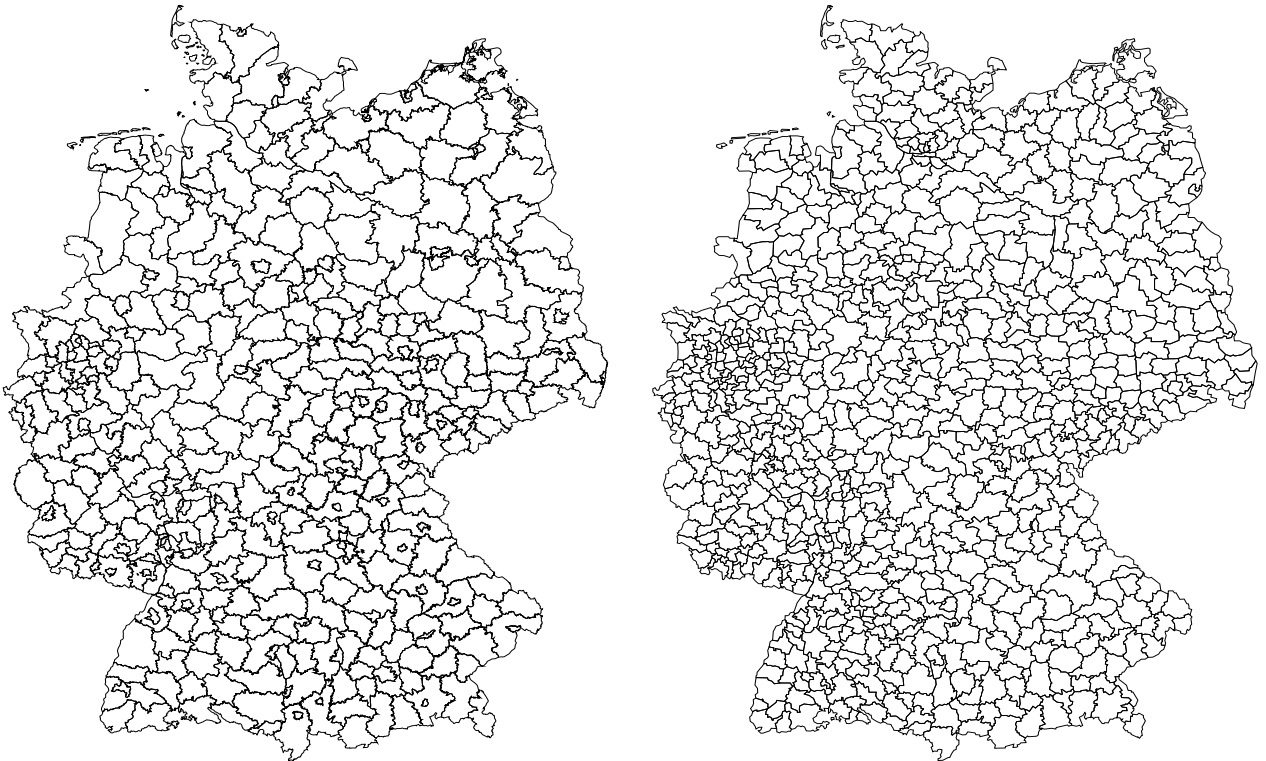
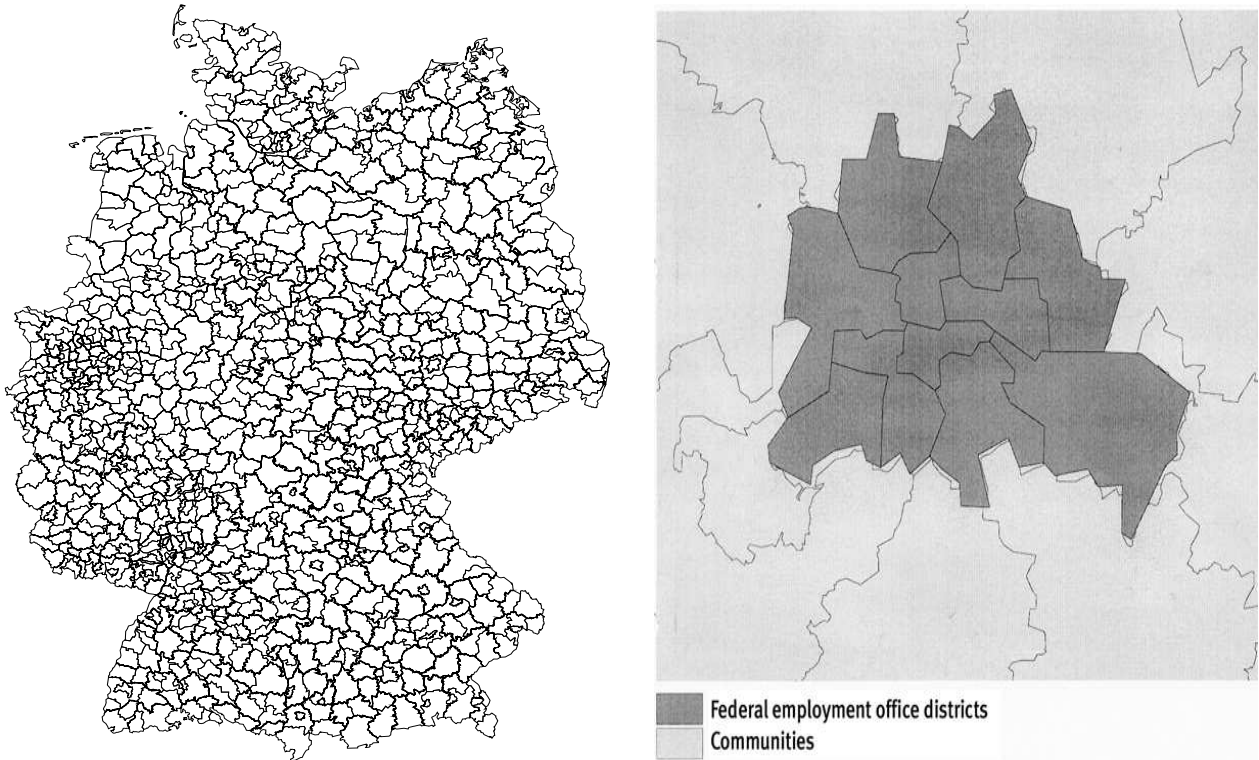


Figure 2 shows a map of German counties (Kreise) and a map of federal employment office districts (Arbeitsamtsdienststellen). Think of the German counties as the  $R_i$  target regions with  $i = 1, \dots, 440$  disjoint entities. The federal employment office districts correspond to the  $D_j$  source regions with  $j = 1, \dots, 840$ . In order to develop weighting schemes based on intersecting both regional classifications, we estimate the county areas  $R_i$ , district areas  $D_j$  and their intersections  $\hat{\mu}(R_i \cap D_j)$  using the GIS procedure of polygon overlay provided in the software package ArcView. Figure 3 to the right shows the resulting map from intersecting counties and districts. This intersection results in more than 3,600 subregions, some of which are certainly spurious due to the measurement errors involved in any intersection based on maps with some degree of cartographic generalization.

Figure 3: The intersection of German Communities and German federal employment office districts (left) and stochastic measurement error at the Berlin border lines (right)



In line with the theoretical framework, the district map  $D$  comes with a larger impreciseness than the county map  $R^{10}$ . However, both maps come with a scale that involves some smoothing of the border lines. This slightly extends the theoretical framework with two instead of one source of random noise, the border lines of  $D_j$  as well as the border lines of  $R_i$ . The spurious polygons resulting from the measurement error can be seen at the border line of the Berlin area (see figure 3 to the right). Moreover, the stochastic measurement error now is also relevant at the outer border of Germany. Still, the spirit of our theoretical framework directly carries over to this application.

In particular, we expect area estimates not to show any systematic biases, but to be very close to the true area sizes on average. Thus, we examine the measurement error involved in estimating regional area sizes by comparing  $\hat{\mu}(R_i)$  to its exact area size  $\mu(R_i)$  which are

<sup>10</sup>In our particular case, map  $D$  was not available electronically such that we scanned the map in a raster data format. Afterwards the raster data have been converted to vector data by means of digitizing. Thus, in addition to smoothing errors due to cartographic generalization, digitizing errors may be another source of measurement error. However, the conversion should not produce any systematic errors so that consistent with the theoretical framework, the measurement error along the border lines may be considered random.

officially released by the federal German statistical office (Statistik Regional, 1999). Table 2 shows the summary statistics of  $\hat{\mu}(R_i)$ ,  $\mu(R_i)$  and their percentage deviation.

Table 2: Comparing the estimated to the true area size of 440 German counties.

	Mean	Std. dev.	25th pct.	50th pct.	75th pct.	Min	Max
$\hat{\mu}(R_i)$	812.23	599.28	264.1	760.5	1186.5	35.7	3073.6
$\mu(R_i)$	811.15	596.97	262.1	759.5	1188.7	35.6	3058.2
$\frac{\hat{\mu}(R_i) - \mu(R_i)}{\mu(R_i)} * 100$	-0.075	2.180	-0.214	0.048	0.313	-19.764	10.275

Comparing the summary statistics for  $\hat{\mu}(R_i)$  and  $\mu(R_i)$ , suggests that, on average, the estimated and true areas are very similar with a percentage deviation of less than 0.1%. However, note that there are some rather extreme outliers in both directions. In particular, we find that some Eastern urban areas such as Chemnitz, Zwickau, Görlitz, Stollberg, Wartburgkreis and Leipzig are among these outliers. Apparently, there is a problem with some Eastern areas stemming from the fact that there have been reforms during the last decade to spatially restructure the county such that the  $\mu(R_i)$  reported in Statistik regional (1999) do not reflect the true area sizes of all Eastern areas. Consequently, excluding the Eastern areas eliminates some of the major outliers. The remaining outliers unsurprisingly tend to be coastal areas such as Lübeck and Bremerhaven. For coastal areas which typically possess a natural border line, the smoothing of the border lines may be expected to result in larger error components than for other regions. Apart from this aspect, no systematic relationship between the measurement error and any regional characteristic (e.g. perimeter-area ratio) can be found. Thus, as predicted by the theoretical framework, area estimates seem to be unbiased.

We conclude that for some (coastal) sub-regions the smoothing of the border lines results in pronounced under- or overestimation of the true area size due to the stochastic measurement error involved. However, on average, this stochastic component is very small. Moreover, no systematic influences could be detected. This suggests that, in line with the theoretical predictions, area estimates and the corresponding weighting schemes are unbiased.

**Sensitivity Analysis** Of course, having unbiased weighting schemes is only one necessary precondition for an appropriate interpolation of attributes from labor office districts to counties. However, due to the possibility of misspecifying the weighting schemes, even unbiased weights may produce invalid results. Put differently, unbiasedness does not tell us anything

about the best choice among the various interpolation methods. Ultimately, whether a particular method is preferable compared to an alternative method depends on the degree of similarity and local homogeneity in the underlying spatial context. As presented in section 2.2, a high degree of similarity between two types of regions as well as a high degree of local homogeneity render differences between merging schemes negligible. Under such conditions, even a naive merging scheme may be an appropriate choice. Otherwise, only a sensitivity analysis reveals whether estimation results are robust with respect to the interpolation method used.

Therefore, this section conducts a sensitivity analysis of the effect of certain regional labor market characteristics on the job-finding hazard of unemployed individuals in West Germany (excluding the Berlin area) between 1981 and 1997. The micro data set used for the analysis is the IAB Employment Subsample (IAB-Beschäftigtenstichprobe) 1975 to 1997. See Bender et al. (2000) for a detailed discussion of the data. The data set contains daily register data of about 500,000 individuals in West-Germany with information on their employment spells as well as on spells during which they received unemployment insurance. The data set is a representative sample of employment that is subject to social security taxation and excludes, for example, civil servants and self-employed individuals. All individual information is coded at the level of the so called micro-census regions. These regional sub-divisions lump together up to four communities. There are 270 micro-census regions in West Germany. Based on this data set, we want to test the effect of two regional labor market indicators, namely the unemployment rate ( $P_{D_j}$ ) and the ratio of unemployed individuals to vacancies in the region ( $F_{D_j}$ ) on the job-finding hazard of unemployed individuals. Both indicators are proxies for labor market tightness and may be expected to have a significant negative effect on the job-finding hazard of unemployed individuals in West Germany. More importantly, since these regional indicators are reported for labor office regions only, they need to be interpolated to micro-census regions. Labor office regions lump together three to four labor office districts. Thus, we can use the map intersection of German labor office districts and counties for an interpolation between the 270 microcensus regions and the 141 labor office regions by aggregating the estimated areas to the level of microcensus and labor office regions. Intersecting these two regional entities yields a total of 1.149 sub-regions.

There are two possible reasons why estimated weights might not differ substantially between alternative weighting schemes. First of all, there may be a high degree of local homogeneity in the region-specific information that is used for the dasymetric weighting approach. Here, we use regional labor force densities as the region-specific information  $S$

because the distribution of the labor force should be highly correlated to other labor-market related attributes. Using a Moran’s I statistic<sup>11</sup>, we find evidence in favor of positive spatial autocorrelation, i.e. areas with high (low) labor force densities tend to be close to other regions with high (low) densities. Apparently, there is a high degree of local homogeneity or a low level of c-heterogeneity in the underlying region-specific information  $S$  (see section 2.1). As a consequence, differences between area and dasymetric weighting should be rather small.

Secondly, we may also expect differences between the naive and the two continuous merging schemes to be rather small. This is because the intersected regional maps do show a high degree of similarity (see figure 2). In several cases, counties do not even intersect with a labor office region or only have small intersections with one additional labor office region. As a consequence, the naive merging scheme may be relatively close to the more sophisticated interpolation methods.

Indeed, we find that the resulting weights on average do not differ substantially. In fact, with an average value that differs only in the 10th decimal place, dasymetric weights show an extremely similar distribution to simple area weights that assume a uniform distribution of the region-specific information. Standard deviations, percentiles as well as minima and maxima are also quite similar. However, while on average both methods seem to be quite similar, weights differ substantially for some sub-regions for which there is a low degree of local homogeneity within the neighboring area. Table 3 looks at an extreme example to demonstrate this point.

Table 3: Weighting schemes  $\hat{f}_{i,j}$  for the Bremen labor office region

Labor office region	Micro census region	$\hat{S}_{R_i} = 1$	$\hat{S}_{R_i} = \frac{lf(R_i)}{\hat{\mu}_{R_i}}$	Naive
Bremen	Bremen	.31311	.83763	1
Bremen	Diepholz	.00189	.00039	0
Bremen	Wesermarsch	.01382	.00208	0
Bremen	Osterholz	.65232	.15748	1
Bremen	Rotenburg	.01576	.00169	0
Bremen	Verden	.00309	.00073	0

<sup>11</sup>See footnote on page 14 for details on the test statistic. Using a weight of one for regions within a 0.4 degree radius of the grid location of the county, we get a test statistic of 0.21 ( $z = 5.3$ ). Using a 0.8 degree radius the test statistic falls to 0.16 ( $z = 8.9$ ) but again is highly significant.

Bremen is a large city in the north of Germany with about 500,000 residents and a relatively high labor force density compared to the surrounding rural areas (Diepholz, Wesermarsch, Osterholz, Rotenburg, Verden). Thus, while around 31 % of the area of the Bremen labor office region intersects with the micro-census region of the same name, taking account of the fact that most of the labor force of the labor office region works in this intersecting area results in a weight of almost 84 %.

We conclude that, on average, weights do not differ substantially at all. Apparently, in most cases, labor force densities in neighboring and intersecting regions are relatively homogenous or the underlying regions are relatively similar so that all schemes result in very similar weighting matrices. However, for some selective regions with a high degree of heterogeneity in the region-specific information within the local neighborhood, the choice of merging rule may have an important influence. We therefore decide to look at two different samples for the sensitivity analysis, a full and a selective sample. The full sample includes all 255,100 unemployment spells<sup>12</sup> generated by 126,189 individuals and beginning between 1981 and 1997 in any West German micro-census region<sup>13</sup>. The selective sample includes only unemployment spells from those micro-census regions whose estimated weighting schemes differed substantially<sup>14</sup>. Given the above results, we expect the analysis based on the full sample to be more sensitive with respect to the chosen interpolation method than the heterogeneous subsample. However, even for the selective sample, estimation results may be quite robust if the regional data to be converted,  $F_{D_j}$  and  $P_{D_j}$ , does not vary significantly between adjacent and nearby regions. Indeed, a Moran's I statistic for both regional

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<sup>12</sup>Periods of registered unemployment cannot be identified easily given the data structure of the IAB employment subsample. This is because we only observe periods of dependent employment and periods of transfer payments from the labor office, but do not observe any information on the labor force status of the individuals during these spells or during the gaps between spells. For a detailed discussion of these problems see Fitzenberger and Wilke (2004). For our purpose, we define an unemployment spell as all episodes after an employment spell during which an individual continuously receives transfer payments. There may be interruptions of these transfer payments of up to four weeks - in the case of cut-off times up to six weeks. Moreover, the gap between employment and the beginning of transfer payments may not exceed 10 weeks. The gap between the end of transfer payments and the beginning of employment may not exceed 12 weeks. Otherwise, the unemployment spell is treated as censored when transfer payments end. This is a reasonable restriction because longer gaps may mean that individuals temporarily or permanently left the labor force or that they became self-employed in which case we do not observe them any longer in our sample.

<sup>13</sup>The sample has been restricted to individuals aged 18-52 at the beginning of the unemployment spell.

<sup>14</sup>A micro-census region belongs to the selective sample if either the absolute deviation between  $\hat{f}_{i,j}(S = const.)$  and  $\hat{f}_{i,j}(S \neq const.)$  or the absolute deviation between  $\hat{p}_{i,j}(S = const.)$  and  $\hat{p}_{i,j}(S \neq const.)$  is above the 99th or below the 1st percentile.

indicators finds significant spatial clustering of similar values<sup>15</sup>. As a consequence, even for a selective sample of regions for which weighting matrices differ significantly, the converted regional data  $F_{R_i}$  and  $P_{R_i}$  might be quite similar for different merging schemes.

Table 4: Summary statistics of unemployment rates for the full and the selective sample by merging scheme

Weights	$\hat{S}_{R_i}$	Obs.	Mean	Std. dev.	Min	Max
Full Sample						
$\hat{f}_{i,j}$	1	270	7.864	2.833	3.183	15.757
$\hat{f}_{i,j}$	$\frac{l f(R_i)}{\hat{\mu} R_i}$	270	7.890	2.848	3.172	15.760
Naive	-	270	7.873	2.864	3.167	15.767
Selective Sample						
$\hat{f}_{i,j}$	1	14	9.226	3.646	3.905	15.009
$\hat{f}_{i,j}$	$\frac{l f(R_i)}{\hat{\mu} R_i}$	14	9.245	3.679	3.917	14.769
Naive	-	14	9.227	3.809	3.933	14.333

Summary statistics of the converted unemployment rate  $P_{R_i}$  and the converted unemployment-vacancy ratio  $F_{R_i}$  at the level of micro-census regions (see table 4 and 5) confirm that differences between interpolation methods are levelled out. Even for the selective sample of 14 micro-census regions for which the weights differed most, there is not much variation across the weighting schemes. There is some more variation in the selective sample for the unemployment-vacancy ratio than for the unemployment rate. Still, summary statistics are quite similar across merging schemes. This suggests that estimated effects of the unemployment rate and the unemployment-vacancy ratio on the unemployment duration of West German job seekers should be very robust across merging schemes, even for the selective sample.

<sup>15</sup>Again (see footnote on page 14) we calculate Moran's I using different weights for the spatially lagged vector. Using a weight of one for regions within a 0.4 degree radius of the grid location of the county, we get a test statistic of 0.85 ( $z = 16.3$ ). Using a 0.8 degree radius the test statistic is 0.72 ( $z = 31.4$ ) which again is highly significant.



Table 5: Summary statistics of unemployment/vacancy ratio for the full and the selective sample by merging scheme

Weights	$\hat{S}_{D_i}$	Obs.	Mean	Std. dev.	Min	Max
Full Sample						
$\hat{p}_{i,j}$	1	270	8.371	5.121	1.723	30.803
$\hat{p}_{i,j}$	$\frac{l f(D_j)}{\hat{\mu} D_j}$	270	8.372	5.109	1.730	31.001
Naive	-	270	8.544	5.484	1.720	31.494
Selective Sample						
$\hat{p}_{i,j}$	1	14	9.833	6.788	1.727	25.023
$\hat{p}_{i,j}$	$\frac{l f(D_j)}{\hat{\mu} D_j}$	14	10.064	6.625	1.733	24.994
Naive	-	14	11.549	8.178	1.720	25.278

For the sensitivity analysis, we estimate a proportional hazard model where the baseline hazard includes a common fixed effects for individuals in the same labor market region<sup>16</sup>. This may be estimated using Cox’s partial likelihood estimator (Cox, 1972). Including location-fixed effects in this estimator removes a potential bias of individual and labor market related variables that may result from omitting important regional labor market characteristics (Kalbfleisch and Prentice, 1980; Ridder and Tunali, 1999). In addition to the location-specific fixed effects we also take account of the fact that some individuals have repeated unemployment spells. Thus, we use the modified sandwich variance estimator to correct for dependence at the level of the individual (Lin and Wei, 1989).

Table 6 summarizes estimation results for the unemployment rate and the unemployment-vacancy ratio for the full sample and the three interpolation methods. We control for education, sex, age, marital status, occupational status, economic sector, a set of year dummies as well as some indicators of prior employment history including total previous unemployment duration, tenure in the previous job and an indicator variable of whether there has ever been a recall from the previous employer. Summary statistics and estimation results using the full and the selective sample can be found in the appendix<sup>17</sup>.

<sup>16</sup>We use labor market regions instead of microcensus regions because labor market regions are likely to be the relevant regional context in which individuals mainly seek employment. There are a total of 180 West-German labor market regions.

<sup>17</sup>Since estimation results across the various specifications are very similar, the appendix only includes detailed results for the Cox model using the unemployment rate as the regional labor market variable in

Table 6: Cox PH model estimates for regional indicators by merging scheme and sample

Merging Scheme	Full Sample		Selective Sample	
	Haz.	Rat. Std. Err.	Haz.	Rat. Std. Err.
Unemployment-vacancy ratio				
$\hat{f}_{i,j}$ with $S_{R_i} = 1$	0.989**	0.000	0.986**	0.001
$\hat{f}_{i,j}$ with $S_{R_i} \neq 1$	0.989**	0.000	0.985**	0.001
Naive	0.989**	0.000	0.987**	0.001
Unemployment rate				
$\hat{f}_{i,j}$ with $S_{D_j} = 1$	0.967**	0.001	0.971**	0.005
$\hat{f}_{i,j}$ with $S_{D_j} \neq 1$	0.966**	0.001	0.973**	0.005
Naive	0.967**	0.001	0.976**	0.005
Significance levels : † : 10% * : 5% ** : 1%				

As expected from the above discussion, the effect of the unemployment rate and the unemployment-vacancy ratio on the job finding hazard is extremely robust across the different interpolation methods for the full and the selective sample. In our empirical application the effects of interpolated attributes on the estimated hazard ratios do not differ up to the 4th decimal place for the full and up to the 3rd decimal place for the selective sample. This even holds for naive binary weighting.

We conclude that, at least in the case of interpolating data between German districts and counties, the choice of interpolation method does not substantially affect our estimation results. In our specific application it even seems safe to take the simplest approach available to the researcher: an interpolation based on simple binary weights. However, due to a high degree of local homogeneity in  $S$ , a high degree of similarity of the regional entities and a strong positive spatial autocorrelation of the data to be interpolated, this is likely to be a result that is unique to this particular application. Thus, researchers applying the above approach to a different set of regional entities should be aware that these factors have an important effect on the robustness of their results. Also, they should check the degree of spatial autocorrelation of the spatially misaligned data. If there is spatial clustering of dissimilar values, interpolation is likely to be much more sensitive to the choice of interpolation method than in our particular application. Therefore, researchers are advised to examine

addition to the individual-specific characteristics. Moreover the estimation results only show the case of merging the unemployment rate based on a uniform distribution of the region-specific information.

the conditions of local homogeneity, similarity of regional entities and positive or negative spatial autocorrelation in detail before choosing an interpolation method. If there is evidence that even the dasymetric weighting approach may be seriously misspecified and no positive spatial autocorrelation of the attributes to be interpolated mitigates this misspecification, other more sophisticated methods might be necessary to derive at satisfactory results.

## 4 Conclusion

This paper presents several methods for interpolating spatially misaligned data from German labor office districts to German counties. We compare interpolation results from binary weighting, simple area weighting and a more sophisticated dasymetric weighting approach that makes use of additional regional information. In particular, we apply dasymetric weighting as an alternative to simple area weighting both of which are based on estimated intersection areas.

In a theoretical framework, we consider the attributes of these interpolation methods if estimated intersection areas come with a measurement error in the form of spurious polygons. Such spurious polygons results from intersecting maps that come with some degree of cartographic generalization and/or digitizing errors. Thus, our theoretical framework extends the well-known Goodchild and Lam (1980) approach to the presence of measurement error in the underlying maps.

Moreover, we identify conditions under which all interpolation methods including naive binary weighting yield comparable and reliable results. Under a high degree of local homogeneity in the region-specific information used for the dasymetric weighting approach and under a high degree of similarity between the two regional classifications, the choice of interpolation method does not matter. We confirm these theoretical results with a simulation study.

As a sensitivity analysis for the area interpolation between labor office districts and counties, we compare the effects of interpolated attributes on the job-finding hazard of unemployed individuals using all three interpolation methods. Our application suggests robustness of estimation results with respect to the choice of interpolation method. Apparently, local homogeneity in the attribute to be interpolated further mitigates any differences between the three methods. Thus, we conclude that in our particular application even a simple rule of thumb yields reliable results. The estimated weighting matrices for interpolating data from the two largest German data producers, the federal Employment Office and the federal

Statistical Office, are freely accessible to the research community and can be downloaded from *ftp : //ftp.zew.de/pub/zew - docs/div/arntz - wilke - weights.xls*

## 5 Appendix

Table 7: Summary statistics for the full and the selective sample of unemployment spells, IAB employment subsample, 1981-1997

	Full Sample		Selective Sample	
	Mean	Std. Err.	Mean	Std. Err.
Unemployment duration (in days)	293.24	443.86	285.55	407.46
Female	0.41	0.49	0.44	0.50
Married	0.46	0.50	0.44	0.50
Married female	0.21	0.41	0.21	0.41
Age < 21	0.08	0.28	0.07	0.26
Age 21-25	0.23	0.42	0.21	0.41
Age 31-35	0.14	0.35	0.14	0.35
Age 36-40	0.11	0.31	0.13	0.32
Age 41-45	0.10	0.30	0.11	0.31
Age 46-49	0.07	0.26	0.08	0.27
Age 50-53	0.08	0.27	0.08	0.27
Low education	0.38	0.49	0.36	0.48
Higher education	0.04	0.20	0.05	0.23
Low educ. x Sex	0.16	0.37	0.17	0.37
High. educ. x Sex	0.02	0.13	0.02	0.15
Apprenticeship	0.07	0.25	0.06	0.25
Low skilled worker	0.34	0.48	0.32	0.47
White collar worker	0.25	0.43	0.30	0.46
Parttime work	0.08	0.27	0.09	0.28
Agriculture	0.03	0.17	0.02	0.13
Inv. goods industry	0.20	0.40	0.17	0.38
Cons. goods industry	0.12	0.32	0.08	0.28
Construction	0.15	0.36	0.12	0.33
Services	0.31	0.46	0.38	0.49
Tenure in previous job (in months)	27.20	38.09	26.88	38.64
Previous recall	0.06	0.23	0.05	0.22
Total unemp. duration (in months)	8.43	15.03	8.29	14.70
1983-1987	0.32	0.47	0.32	0.47
1988-1991	0.19	0.39	0.19	0.39
1992-1997	0.34	0.47	0.34	0.47
Unemployment rate <sup>a</sup>	9.70	3.38	9.34	3.56
Number of spells	255,100		83,104	
Number of individuals	126,189		24,674	
Percentage right-censored	28.4		29.7	

<sup>a</sup> Regional information has been merged using the uniform distribution of the region-specific information  $S_{D_j} = 1$ .

Table 8: Cox PH model estimates using the full and the selective sample, IAB employment subsample, 1981-1997

Variable	Full Sample		Selective Sample	
	Hazard Ratio	(Std. Err.)	Hazard Ratio	(Std. Err.)
Female	1.112**	(0.011)	1.127**	(0.035)
Married	1.219**	(0.008)	1.227**	(0.031)
Married female	0.539**	(0.013)	0.583**	(0.022)
Age < 21	1.217**	(0.010)	1.281**	(0.045)
Age 21-25	1.103**	(0.008)	1.141**	(0.029)
Age 31-35	0.985†	(0.009)	1.001†	(0.029)
Age 36-40	1.000	(0.011)	1.002	(0.031)
Age 41-45	1.001	(0.011)	1.011	(0.035)
Age 46-49	0.968*	(0.013)	0.930*	(0.037)
Age 50-53	0.831**	(0.015)	0.823**	(0.037)
Low education	0.883**	(0.009)	0.847**	(0.023)
Higher education	0.792**	(0.020)	0.779**	(0.044)
Low educ. x Sex	0.968*	(0.013)	1.035*	(0.041)
High. educ. x Sex	1.149**	(0.030)	1.120**	(0.089)
Apprenticeship	1.082**	(0.013)	1.136**	(0.045)
Low skilled worker	0.798**	(0.009)	0.845**	(0.023)
White collar worker	0.752**	(0.010)	0.805**	(0.023)
Parttime work	0.806**	(0.016)	0.829**	(0.035)
Agriculture	1.317**	(0.020)	1.333**	(0.092)
Inv. goods industry	0.927**	(0.010)	0.926**	(0.027)
Cons. goods industry	0.925**	(0.011)	1.029**	(0.036)
Construction	1.221**	(0.010)	1.347**	(0.041)
Services	0.984†	(0.009)	1.024†	(0.025)
Tenure in previous job	0.995**	(0.000)	0.994**	(0.000)
Previous recall	0.781**	(0.012)	0.756**	(0.031)
Total unemp. duration	0.995**	(0.000)	0.997**	(0.001)
1983-1987	1.245**	(0.008)	1.252**	(0.033)
1988-1991	1.332**	(0.009)	1.365**	(0.039)
1992-1997	1.085**	(0.009)	1.122**	(0.032)
Unemployment rate	0.967**	(0.001)	0.971**	(0.005)
Log-likelihood	-1,217,399.365		-121,004	
$\chi^2_{(30)}$	18,724.774		1,961.91	

Significance levels : † : 10% \* : 5% \*\* : 1%

Using the merging scheme with  $S_{D_j} = 1$ .

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