A Three-Regime Model of Speculative Behaviour: 
Modelling the Evolution of Bubbles 
in the S&P 500 Composite Index

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Abstract

In this paper we examine whether a three-regime model that allows for dormant, explosive and collapsing speculative behaviour can explain the dynamics of the S&P 500 Composite Index for the period 1888-2001. We extend existing two-regime models of speculative behaviour by including a third regime that allows for a bubble to grow at a steady growth rate, and examine whether other variables, beyond the deviation of actual prices from fundamental values can help predict the level and the generating state of returns. We propose abnormal volume as an indicator of the probable time of the bubble collapse and thus include abnormal volume in the state and the classifying equations of the surviving regime in the explosive state. We show that abnormal volume is a significant predictor and classifier of returns. Furthermore, we find that the spread of the 6-month average actual returns above the 6-month average fundamental returns can help predict when a bubble will enter the explosive state. Finally, we examine the financial usefulness of the three-regime model by studying the risk-adjusted profits of a trading rule formed using inferences from it. Use of the three-regime model trading rule leads to higher risk adjusted returns and end of period wealth than those obtained from employing existing models or a buy and hold strategy.

Keywords: Stock market bubbles, fundamental values, dividends, regime switching, speculative bubble tests.

JEL Classifications: C51, C53, G12

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1. Introduction
The evolution of prices in equity markets during the 1920’s, the 1980’s and the late 1990’s, has puzzled economists and market participants. During these periods, market prices displayed significant growth followed by abrupt market collapses. It is hard to reconcile this behaviour with the variation of fundamentals during these periods and thus two alternative theories have been developed that try to explain why stock markets appear to be moving without fundamental justification. The first approach attributed these abrupt changes in the market trend to a non-linear relationship between actual prices and fundamental values. The second approach supports the view that self-fulfilling expectations and speculative bubbles caused the significant and increasing divergence of actual prices from fundamental values.

Initially, studies on the relationship between actual prices and fundamental values focused on the indirect identification of speculative bubbles in financial data (see Shiller (1981), Blanchard and Watson (1982), West (1987), Diba and Grossman (1988)). However, indirect tests of bubble presence suffered from potential problems of interpretation since bubble effects in stock prices could not be distinguished from the effects of unobservable market fundamentals. For this reason, direct bubble tests, which test directly for the presence of a particular bubble specification on stock market returns, where developed (see Flood and Garber (1980), Flood, Garber and Scott (1984), Summers (1986), Cutler, Potterba and Summers (1991), McQueen and Thorley (1994), Salge (1997), Van Norden and Schaller (1999)). Under these tests, researchers select the type of bubble that they suspect might be present in the data and then examine whether this form of speculative bubble has any explanatory power for stock market returns.

Although several different types of bubbles have been examined in the literature, periodically collapsing speculative bubbles, first proposed by Blanchard and Watson (1982), have attracted increasing attention, especially in the late 1990’s. Models of periodically collapsing speculative bubbles are able to capture several stylised characteristics of historical accounts of “manias and panics” (Kindleberger (1989)). The common characteristic of all such periods is that prices initially diverge from fundamental values in a deterministic fashion. As time passes, the rate of divergence accelerates and thus prices increase without bound and this exponential trend is followed by a sharp reversal of market prices to fundamental values when market participants realise that the current price level is unsustainable.

In order to empirically examine the presence of periodically collapsing bubbles, researchers in recent years have focused on the construction of direct bubble tests that can identify such stochastic bubble processes in financial and macroeconomic data. More specifically, Evans (1991) and Van Norden and
Schaller (1993) show how periodically collapsing speculative bubbles can induce regime switching behaviour in asset returns. Regime switching in asset returns and speculative behaviour have been linked in several studies. Van Norden and Schaller (1993) and Van Norden (1996) show that a two-regime speculative behaviour model has significant explanatory power for stock market and foreign exchange returns during several periods of observed market over- and under-valuations. Hall, Psaradakis and Sola (1998) test for the presence of collapsing speculative bubbles in Argentinean monetary data using a univariate Markov-switching ADF test and find evidence of a speculative bubble in consumer prices in the period June 1986 to August 1988.

Van Norden and Vigfusson (1998) compare the Van Norden and Schaller (1997) and the Hall, Psaradakis and Sola (1998) approaches for testing for the presence of speculative bubbles and find that both models have significant power in detecting periodically collapsing speculative bubbles. However, both approaches concentrate on the explosive phase of the speculative bubble since it is during this phase that asset prices display significant patterns of bubble behaviour. For this reason, both models are constructed to identify periods of explosive stock market growth followed by a sharp reversal. This specification of the bubble tests implicitly assumes that a speculative bubble will always display explosive growth, although this will be limited for small bubble sizes. This is an unrealistic assumption since there are periods during which asset prices display deterministic growth or mimic the behaviour of fundamentals. During such periods the speculative bubble can be assumed to be ‘dormant’ since it grows at a steady rate. A speculative bubble process that can replicate this switch from a dormant to an explosive state is described in Evans (1991). However, Evans uses this bubble process only in a simulation study and does not provide empirical evidence concerning the presence of such speculative bubbles in asset prices.

In what follows, we try to fill this gap by showing how the Van Norden and Schaller (1999) model can be extended to test for the presence of speculative bubbles of the form described in Evans. We do this by incorporating a third regime in which the bubble grows at the fundamental rate of return. We then examine whether the three-regime speculative behaviour model has explanatory power for U.S. stock market returns. Furthermore, we show that other variables, such as abnormally high volume, can be used in the Van Norden and Schaller framework in order to model the probability of a bubble collapse more effectively. We find that abnormal volume is a significant predictor of both the level and the generating state of returns in the next time period, thus helping in the identification of the time of the bubble collapse.

Furthermore, extant research has focused only on the issue of identifying the presence of speculative bubbles. Although the identification of a speculative bubble is useful in determining whether market
participants are rational and whether markets are efficient, it is also interesting to examine the financial usefulness of speculative bubble models by testing whether such models can be used to generate abnormal trading profits. To our knowledge, we are the first to examine the out of sample profitability of employing speculative bubble models. We do this by formulating a trading rule that exploits information about the implied probability of a stock market crash or rally derived from switching regime speculative bubble models. This allows us to evaluate the predictive ability of our three-regime model against the Van Norden and Schaller model in a financially intuitive manner: by determining which model can lead to higher risk adjusted returns. We also examine the market timing ability of speculative behaviour models by comparing the results of the trading rule to those of a buy and hold strategy and those of randomly generated trading rules. Finally, we employ a longer sample than in the original Van Norden and Schaller study, examining the returns on the S&P 500 for the period January 1888 to January 2001.

The rest of this paper is organised as follows. In section 2 we present the theory of periodically collapsing speculative bubbles and show how switching regression models can capture the characteristics of such bubbles. In section 3 we derive the three-regime speculative behaviour model. Section 4 presents the data and the methodology used to construct fundamental values. Section 5 presents the results of the speculative behaviour models while in section 6 we examine the out of sample forecasting ability and profitability of the speculative behaviour trading rules. Section 7 concludes.

2. Periodically Collapsing Bubbles and Regime Switching

Tests of rational speculative bubble presence rely on the present value model of stock prices which states that, under the assumptions of investor rationality and homogeneous expectations, the price of an asset must be equal to the present value of all its future cash flows. In a two-period setting, the price of a stock must thus be equal to:

$$p_t = \frac{E_t(p_{t+1} + d_{t+1})}{(1 + i)}$$  \hspace{1cm} (1)$$

where: $p_t$ is the stock price at time $t$, $d_t$ is the dividend paid in period $t$, $E_t(.)$ is the mathematical expectations operator, and $i$ is the constant discount rate. If the transversality condition holds, then in an infinite planning horizon setting, the stock price of period $t$ is equal to the present value of all future dividends:

$$p_t^f = \sum_{g=0}^{\infty} \frac{1}{(1 + i)^g} E_t(d_{t+g})$$  \hspace{1cm} (2)$$
In (2) the price of the stock is only a function of all future dividends and thus we refer to (2) as the fundamental price of the stock \( p_f^t \). However, equation (2) is not the only forward looking solution to (1). The actual price of a stock may deviate from the fundamental value if we assume that the size of the deviation satisfies the no arbitrage condition presented in equation (1). In this case, the actual price of a stock \( p^a_t \) will be equal to:

\[
p^a_t = p_f^t + b_t
\]

where the bubble term \( b_t \) satisfies the no arbitrage condition:

\[
b_t = \frac{E_t(b_{t+1})}{(1+i)}
\]

Note that in (4) we assume investor risk neutrality since the required rate of return on the bubble is equal to the required rate of return on the bubble free asset. If the assumption of rational expectations holds, then the expected bubble of the next time period should be equal to:

\[
E_t(b_{t+1}) = (1+i)b_t
\]

Equation (5) describes a bubble process in which bubble deviations are expected to grow for infinite time. This is not a realistic implication since such bubbles would cause actual prices to diverge infinitely from fundamental values. In order to examine a more plausible class of speculative bubbles, Blanchard (1979) and Blanchard and Watson (1982) formulate a two-regime stochastic bubble process in which the bubble deviation may continue to grow in the next time period with probability \( q \), or collapse to zero with probability \( 1-q \):

\[
\frac{(1+i)b_t}{q} \quad \text{with probability } \frac{q}{q} \\
0 \quad \text{with probability } 1-q
\]

In (6) \( q \) is the constant probability that the bubble will continue to exist in period \( t+1 \), \((0<q<1)\). Note that in this setting the bubble is expected to yield a higher return than \((1+i)\) in order to compensate investors for the positive probability of the bubble collapsing. If the probability of survival is equal to 1 then the model collapses to the deterministic bubble process described in (5).

However, the original Blanchard and Watson model imposes several unrealistic restrictions on the bubble process. More specifically, in their model, the value of the bubble term in the collapsing regime is assumed to be zero. If a bubble collapses to zero, then it cannot regenerate since its expected value will be zero (West (1986), Diba and Grossman (1988)). This in turn implies that there can only
be one observed bubble in a financial time series and this bubble must have been present from the first day of trading since bubbles cannot be self-generated in this setting.

Furthermore, in (6) the probability of a bubble collapse is constant \((1-q)\). Kindleberger (1989) argues that all historical episodes of bubbles and crashes share the characteristic that as the bubble grows larger, the probability that it will burst increases. Thus the probability of the bubble continuing to exist \((q)\) should be a negative function of the bubble’s size and thus time-varying. This behaviour of the bubble survival probability forces the bubble component to grow at an ever higher rate and so the bubble process is unbounded until the bubble bursts.

In addition to the above, equation (6) assumes that the bubble collapses in one period. This is a restrictive assumption since a bubble may deflate over several periods before it starts growing again. Van Norden and Schaller (1999) note that the strong correction of prices in the Tokyo stock exchange in the months following January 1990 was a gradual collapse of a speculative bubble. Moreover, the Blanchard and Watson model considers only positive (price increasing) speculative bubbles. This relies on the assumption of strict investor rationality. Negative bubbles have been frequently ruled out in the literature because of the consequent positive probability that prices will become negative if the bubble becomes sufficiently large. However, as noted in Blanchard and Fischer (1989), the probability of this event may be too small, or the event too far into the future, and thus investors may rationally choose to ignore it.

Finally, several episodes of bubble behaviour in stock markets show that bubbles may alternate between ‘dormant’ and explosive states. During the dormant state, the bubble grows at the required rate of return without explosive expectations since the probability of a crash is zero or negligible. However, once the bubble enters an explosive state, its behaviour changes and the bubble either grows at an ever higher rate with explosive expectations or collapses. This bubble behaviour is more consistent with a three-regime speculative bubble model. This switching between a dormant bubble state and an explosive bubble state can be observed in the early 1920’s, the early 1950’s, the 1960’s and the early 1990’s amongst other periods, when bubbles alternate between growing at a small steady growth rate and an increasingly explosive growth rate. This point will be analysed further in the following section.

In order to adjust for some of these restrictive assumptions, Evans (1991) and Van Norden and Schaller (1993) generalized the original Blanchard and Watson bubble model to allow for the size of collapses and the probability of the bubble surviving to be functions of the size of the bubble. In Evans’ model, the bubble process switches between a deterministic and an explosive state, where the
transition is governed by a predetermined threshold on the size of the bubble. In the deterministic regime \((N)\), the bubble grows at an average rate of \(1+i\) plus a random disturbance and the probability of the bubble collapsing in this regime is equal to zero. However, once the bubble size crosses the arbitrary threshold, it enters an explosive state in which the bubble is expected to continue to grow at a rate greater than \(1+i\), with probability \(q\) (regime \(S\)), or to collapse to a small positive value with probability \(1-q\) (regime \(C\)). The rate of growth in the surviving regime must be greater than the rate of growth in the deterministic regime since the bubble must compensate for the now positive probability of bubble collapse. Evans’ model can be expressed as:

\[
E_t(b_{t+1}) = \begin{cases} 
(1+i)b_t\epsilon_{t+1} & \text{if } b_t \leq a \\
\theta_{t+1}(1+i)b_t - \frac{\delta}{(1+i)} + \frac{\delta}{q} & \text{if } b_t > a 
\end{cases}
\]

where \(\epsilon_t\) is a positive i.i.d. random variable with \(E_t(\epsilon_{t+1}) = 1\), \(\theta_t\) is an i.i.d. Bernoulli process that takes the value 1 with probability \(q\) or zero with probability \(1-q\), \(\delta\) is a positive parameter that denotes the bubble size in the collapsing regime where \(0 < \delta \leq (1+i)a\), and \(a\) is the threshold beyond which the bubble enters the explosive state. In equation (7), the error term \(\epsilon_t\) is always positive and thus the bubble will never change signs and will never entirely collapse. Equation (7) allows for partial rather than total collapses since the expected value of the bubble in the collapsing regime is \(\delta\). The model also allows the probability of a collapse to be dependent on bubble size since if the bubble is smaller than \(a\), the probability of collapse is zero. This means that in the ‘dormant’ state, the probability of collapse is not taken into account in the expectation of the bubble growth rate and thus the bubble is growing at the mean rate \((1+i)\). Once the bubble size reaches the level \(a\), the bubble grows at the rate \((1+i)/q\) as long as the eruption continues or it may collapse with probability \(1-q\) to the mean value of \(\delta\).

At this point we must note that the deterministic state \((N)\) is not easily distinguishable from the mixture of the two explosive alternatives since the conditional expectation of the growth of the bubble in both the explosive and the deterministic regime is the required rate of return \((1+i)\). The deterministic state can only be distinguished from the explosive states by the distribution of the innovations of the bubble. In the deterministic state, the innovations of the bubble term will be less volatile than the innovations of the explosive states. Furthermore, in the surviving regime, bubble growth accelerates and thus yields increasing positive returns.
Evans (1991) and Van Norden and Vigfusson (1998) use equation (7) as a data generating process in Monte Carlo simulations in order to examine the size and power of bubble tests. However, to our knowledge there has been no direct examination of the presence of such bubbles in the literature. The main problem behind any test for speculative bubbles is that the type of bubbles described in equation (7) only exhibits characteristic bubble behaviour during the explosive state. For this reason, several researchers have focused on the construction of speculative bubble tests that can identify and model the behaviour of the bubble only in the explosive regime.

An alternative generalisation of the original Blanchard and Watson model that concentrates on the explosive state of the Evans bubble process is proposed by Van Norden and Schaller (1999)\(^1\). In their model, the size and probability of bubble collapses are dependent on bubble size and their model examines the presence of both positive and negative bubbles. Van Norden and Schaller consider the following bubble process:

\[
E_t(b_{t+1}) = \begin{cases} 
(1+i) b_t - \frac{(1 - q(B_t))}{q(B_t)} u(B_t) p_t^a & \text{with probability } q(B_t) \\
\frac{u(B_t) p_t^a}{1-q(B_t)} & \text{with probability } 1 - q(B_t)
\end{cases}
\]

where \( b_t \) is the relative size of the bubble in period \( t \) \( (B_t = b_t / p_t^a) \), \( u(B_t) \) is a continuous and everywhere differentiable function such that \( u(0) = 0 \) and \( 0 \leq \partial u(B_t) / \partial B_t \leq 1 \), \( q(B_t) \) is the probability of the bubble continuing to exist \( (q(B_t) = \Omega(\beta_{q,0} + \beta_{q,b} |B_t|)) \), which is assumed to be a negative function of the relative size of the bubble \( \partial q(B_t) / \partial |B_t| < 0 \), \( \Omega \) is the standard normal cumulative density function, \( \Omega(\beta_{q,0}) \) is the mean probability of being in the collapsing regime and \( \beta_{q,b} \) is the sensitivity of the probability of collapse to the absolute relative size of the bubble.

In the Van Norden and Schaller model, the probability of the bubble surviving \( (q(B_t)) \) is a negative function of the absolute size of the bubble, since they consider both positive and negative bubbles. In order to restrict the estimates of the probability of survival between zero and one, they use a probit specification. From equation (7) it is evident that the model allows for partial bubble collapses\(^2\) since the expected bubble size in the collapsing regime is a function of the relative bubble size \( (u(B_t)) \).

\(^1\) An alternative approach to testing for the presence of speculative bubbles of the form described by Evans (1991) is based on a switching stationarity test proposed by Hall, Psaradakis and Sola (1999), while Bohl (2000) suggests the use of a threshold autoregressive model in a cointegration framework to separate periods of deterministic from explosive bubble growth.

\(^2\) Evans (1991) and Hall and Sola (1993) also consider partial bubble collapses.
The restrictions imposed on this function ensure that the expected bubble in the collapsing regime is smaller than the expected bubble in the surviving regime and not bigger than the bubble in period $t$. The assumption of a continuous and everywhere differentiable function is required so that Van Norden and Schaller may linearise the model in order to estimate it. This functional form is not imposed on the data but is required in order to derive empirically testable implications from the bubble model.

Van Norden and Schaller use equation (8) in order to specify gross stock market returns as a state dependent process in which the state is unobservable. They show that under certain assumptions about the dividend process, the security’s gross returns are given by the following non-linear switching model:

$$E(r_{t+1} | S) = \left[ M + \frac{1 - q(B_t)}{q(B_t)} (MB_t - u(B_t)) \right] \text{ with probability } q(B_t)$$

$$E(r_{t+1} | C) = \left[ M + u(B_t) - MB_t \right] \text{ with probability } 1 - q(B_t)$$

$$q(B_t) = \Omega (\beta_1 + \beta_2 | B_t |)$$

where $(r_{t+1} | S)$ denotes the gross return of period $t+1$ conditioning on the fact that the bubble survives (state $(S)$) or the bubble collapses (state $(C)$) and on all other available information at time $t$, and $M$ is the gross fundamental return on the security. In order to estimate the model, Van Norden and Schaller linearise equations (9) and (10) and arrive at a linear regime-switching model for gross stock market returns with a single state-independent probability of switching regimes ($q(B_t)$).

Van Norden and Schaller estimate the model using data on the value-weighted index of all stocks from the Centre for Research on Security Prices (CRSP) database for the period January 1926 to December 1989. The estimation is performed using maximum likelihood and their results show that there is non-linear predictability in stock market returns and that the deviations of actual prices from fundamental values are a significant factor in predicting both the level and the generating state of returns. Furthermore, using the point estimates of the switching speculative bubble model, Van Norden and Schaller calculate the conditional probabilities of a crash and of a rally in the next time period. They find that the speculative behaviour model has explanatory power for several periods of apparently speculative behaviour of the data, since the probability of a crash increases significantly prior to large stock market declines while the probability of a rally is high prior to large stock market declines.

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advances. However, they note that some of the observed crashes can be explained better by a model of regime switching in fundamental values.

Although the Van Norden and Schaller approach can be used directly to test for the presence of periodically partially collapsing speculative bubbles, it has several weaknesses. Firstly, Van Norden and Schaller do not provide any information on the out of sample forecasting ability of their model. Furthermore, their model provides no information as to the likely time of a bubble collapse, since the probability of a collapse is solely dependent on the bubble size. This leads to long periods of high probabilities of collapse, when the bubble deviation is persistently high. Moreover, although the power of the speculative behaviour model to detect bubbles of the form described by Evans (1991) has been examined by Van Norden and Vigfusson (1998), the financial usefulness of this model has not, to our knowledge, been examined. It would thus be interesting to determine whether this model can be used in order to time large market falls.

Finally, the Van Norden and Schaller model examines only the explosive state of a speculative bubble since it is during this state that asset prices display apparent bubble behaviour. This implies that the Van Norden and Schaller model assumes that the bubble will always induce explosive behaviour in asset prices; the asset price will either grow with explosive expectations or will reverse to fundamental values. However, it is logical to assume that there are periods during which speculative bubbles grow deterministically at the fundamental rate of return, as in the Evans (1991) model. In such periods, the actual returns on the asset are equal to the returns on the fundamentals, plus a random disturbance and thus actual prices mimic the behaviour of fundamental values. In this ‘deterministic’ state, the probability of a bubble collapse is not significant in the pricing of the asset since it is negligible. However, as seen from the results of Van Norden and Schaller, their model will always assign a small but positive probability of the bubble collapsing ($\Omega(\beta_{q,0})$), and this probability will affect the expected returns on the asset. This will cause a positive bias in the estimates of the probability of collapse at every point in time and especially during periods when the bubble displays deterministic growth. It is therefore preferable to construct a speculative behaviour model that explicitly allows for deterministic and for explosive bubble growth, without the predetermined threshold contained in the Evans (1991) model.

In what follows, we will show how the Van Norden and Schaller model can be extended to a three-regime speculative bubble model that can directly examine the presence of positive and negative bubbles of the form described by Evans. In this three-regime model, we explicitly allow for periods of deterministic and of explosive bubble growth, and the bubble can switch between this ‘dormant’ state and the explosive state at any point in time and not according to a predetermined threshold.
Furthermore, we show that other variables can be used to forecast both the level and the generating state of returns more efficiently, and finally we will examine and compare the out of sample forecasting ability and the financial usefulness of the Van Norden and Schaller model and of the three-regime model.

3. A Three-Regime Speculative Behaviour Model

The three-regime speculative bubble model describes a bubble process in which the expected size of the bubble in the next time period can be generated from one of three regimes: a deterministic (or ‘dormant’) regime \( N \), a surviving-explosive regime \( S \), and a collapsing-explosive regime \( C \). The bubble of the next time period can be generated from any of the three regimes. In order to classify the behaviour into one of the three regimes, several variables may be significant, although the relative size of the bubble is expected to play a predominant role.

Following Evans, in regime \( N \), the bubble size is small and thus investors believe that the bubble will continue to grow almost certainly at a constant mean rate \((1 + i)\):

\[
E_t (b_{t+1} | N) = (1 + i)b_t
\]

(12)

This regime implies that investors believe that the bubble has a negligible probability of collapse and thus they do not expect to be rewarded with an excess return for this probability. Evans (1991) assumes that once a bubble crosses a certain threshold, it erupts into an explosive regime in which the bubble continues to grow or collapses to a smaller value. Nevertheless, as stated above, Evans arbitrarily assumes a threshold value, whereas we model the probability of being in regime \( N \). The probability of being in regime \( N \) in \( t+1 \) is denoted \( q_t \) and the collapsing regime that occurs with probability \( 1 - q_t \). In the collapsing regime \( C \), the size of the bubble is given by:

\[
E_t (b_{t+1} | C) = g(B_t) p_t a
\]

(13)
where \( g(B_t) \) is a continuous and everywhere differentiable function such that, \( g(0) = 0 \) and \( 0 \leq \frac{\partial g(B_t)}{\partial B_t} \leq 1 \). The size of the bubble in this regime is a function of the relative size of the bubble in period \( t \). This function is only for theoretical use and will not be imposed on the data. From (12) and (13) and since:

\[
E_t(b_{t+1}) = n_t\left( E_t(b_{t+1} | N) \right) + (1 - n_t)\left[ q_t\left( E_t(b_{t+1} | S) \right) + (1 - q_t)\left( E_t(b_{t+1} | C) \right) \right]
\]

we can show that the expected size of the bubble in the surviving regime will be:

\[
E_t(b_{t+1} | S) = \frac{(1 + i)}{q_t} b_t - \frac{(1 - q_t)}{q_t} g(B_t) p_t^a
\]

In the surviving regime, investors know that the bubble has a positive probability of collapsing and therefore they expect to be rewarded with an extra return to compensate for this. Furthermore, the expectation for the bubble size in the next time period is adjusted downwards compared with the original Blanchard and Watson (1982) model, since there is a positive payoff in the collapsing regime. Note that in the explosive state, the probability of being in the surviving regime in the next time period is given by \( q_t \) whereas the probability of being in regime \( C \) is \( 1 - q_t \). Therefore, at any point in time, the conditional probability of observing the surviving regime in the next time period is given by \( (1 - n_t) q_t \) and the probability of observing the collapsing regime is \( (1 - n_t)(1 - q_t) \). Grouping together (12), (13) and (15), the expected bubble of the next time period will be generated by the following stochastic process:

\[
E_t(b_{t+1} | S) = \begin{cases} 
M b_t & \text{with probability } n_t \\
\frac{M}{q_t} b_t - \frac{(1 - q_t)}{q_t} g(B_t) p_t^a & \text{with probability } (1 - n_t) q_t \\
g(B_t) p_t^a & \text{with probability } (1 - n_t)(1 - q_t)
\end{cases}
\]

At this point, it is useful to note that the main differences between this model and the Evans (1991) data generating process is that we allow the bubble of the next period to be generated by any of the three regimes conditional on the probability of the given regime, and we allow for the existence of negative (price decreasing) bubbles. In Evans, a strictly positive bubble must cross the arbitrary threshold in order to enter the explosive regime. Furthermore, the probability of collapse in state \( N \) is assumed to be zero.

\[\text{Note that in the original Van Norden and Schaller model the bubble size in the collapsing regime is } u(B_t) p_t^a \text{ where } 0 \leq \frac{\partial u(B_t)}{\partial B_t} \leq 1. \text{ This implies that in the collapsing regime the original Van Norden and Schaller model states that the bubble in period } t+1 \text{ is expected to be equal to or smaller than the bubble in period } t. \text{ We use a slightly different notation and assume that the bubble size is } g(B_t) p_t^a \text{ where } 0 \leq \frac{\partial g(B_t)}{\partial B_t} \leq (1 + i) B_t. \text{ This implies that the bubble in the collapsing regime is smaller than the bubble in the } 'dormant' \text{ state therefore, when the bubble does not yield the required rate of return, it is considered to be in the collapsing state.}\]
In (16) we assume that when the bubble is in the steady growth regime, the probability of collapse is negligible and therefore investors do not take it into account. Although this violates the rule of strict rationality, the probability of observing a collapse when the bubble is in regime $N$ might be so small that investors decide to ignore it. This is also the argument of Blanchard and Fischer (1989) concerning the rationality of non-fundamental solutions. Note that our model produces a very small probability of being in regime $C$ in the next time period $((1-n_t)(1-q_t))$ if the probability of being in regime $N$ is very high.

Let us now examine the economic significance of our model. The classic Blanchard and Watson (1982) bubble model describes asset prices as always being affected by a bubble. It is logical to assume that for small bubble sizes, investors react to changes in the bubble size differently than when actual prices are far from fundamental values. Our model takes this logical assumption directly into account in the bubble process and in the modelling of the probabilities of being in a given regime. When the bubble is small, the probability of being in the collapsing regime is very small, since the probability of being in the steady growth state is very large and the probability of being in the collapsing state given the explosive regime is very small. This is because the probability of the bubble surviving if the explosive regime is observed is close to one and therefore $1 - q_t$ is close to zero. Since $1 - n_t$ is also close to zero for small bubble sizes, $((1-n_t)(1-q_t))$ will be arbitrarily small.

The probability of the bubble collapsing is very small for small bubble sizes, so we can show that the expected gross return of the next period, if the bubble is in the deterministic regime ($N$), is \(^5\):

$$ E(r_{t+1}|N) = M $$  \hspace{1cm} (17)

Equation (17) states that in the deterministic state (regime $N$), the required gross return on the asset in the next time period is equal to the required rate of return on the bubble-free asset and thus in this state investors do not require additional compensation for the probability of collapse. However, as the bubble grows, the probability of being in the steady growth regime diminishes and thus the probability of being in the explosive state (surviving or collapsing regime) increases.

In this explosive state, as the bubble increases, the probability of being in the surviving state diminishes, causing the probability of collapse to increase geometrically. In the explosive state, investors will take into account the probability of a crash and thus the expected return in the surviving regime is:

\(^5\) Proofs of the equations are not presented here in the interest of brevity but are available in an appendix from the authors upon request.
In equation (18), investors adjust their expectations for the next period return to take into account the probability of collapse. Nevertheless, they adjust their expectation downwards to incorporate the payoff of the collapsing regime, taking into account the probability of this outcome. If the bubble collapses, the gross expected return is given by:

\[ E_t(r_{t+1}|S) = M + \frac{(1-q_t)}{q_t}(MB_t - g(B_t)) \]  

(18)

In equation (19), the gross return in the collapsing regime is a function of the required return on the bubbly asset and the relative size of the bubble in the collapsing regime. The magnitude of the return depends on the function \( g(B_t) \), a function that does not require specification since the model will subsequently be linearised.

\[ E_t(r_{t+1}|C) = M + g(B_t) - MB_t \]  

(19)

It is straightforward to see that the above bubble model collapses to the original Van Norden and Schaller model if we set the probability of being in the constant growth regime equal to zero. Furthermore, if we fix the probability of survival to a constant value and set the bubble size in the collapsing regime equal to zero then the model collapses to the original Blanchard and Watson (1982) model. Finally, the conditional expected return on the asset at any point in time satisfies the no arbitrage condition and ex ante the bubble is expected to grow at the rate \( M \).

Nothing yet has been said about the probabilities \( n_t \) and \( q_t \) apart that they are negative functions of the size of the bubble. Let us examine these probabilities in more detail. From the original Evans (1991) model, as the bubble grows, the probability of being in the deterministic regime \( (N) \) decreases. Here we consider both positive and negative bubbles and therefore we formulate the probability \( n_t \) as a function of the absolute size of the bubble. However, the size of the bubble may not be the only variable investors examine in order to decide whether the bubble is in the deterministic or the explosive state.

Furthermore, Harvey and Siddique (2000) and Chen Hong and Stein (2001), find that when returns have been high in the recent past, the skewness in future returns is more negative. This implies that high average returns will be followed by large negative returns. From equation (17) it is obvious that in the deterministic regime the bubble returns are indistinguishable from the fundamental returns. However, when the bubble enters the explosive state and survives, we know from (18) that it yields increasingly larger returns than the bubble-free asset. For this reason, it is logical to assume that when investors observe larger average actual returns than average fundamental returns in the near past they will conclude that the bubble must have entered the explosive state. This means that large positive
returns imply a smaller probability of being in the ‘dormant’ regime in the near future. Chen, Hong and Stein find that the predictive power of past returns for skewness is larger if one considers the last six months’ returns. They consider actual returns, however, and we want to partially separate bubble returns from fundamental returns.

For this reason, we include the spread of the average 6-month actual gross returns over the average 6-month fundamental gross returns as a variable in the probability of being in the deterministic regime. It should be expected that the larger the value of the spread, the lower the probability of the bubble continuing to be in the constant growth regime. Nevertheless, we examine both positive and negative bubbles. We thus take the absolute values of the averages before we calculate the spread. This ensures that when a negative bubble is entering the explosive state, we correctly identify the explosive ‘growth’ of such a bubble. This is equivalent to assuming that large negative past returns cause an increase in the skewness of the distribution of future returns. Furthermore, this variable will help to identify periods of steady bubble growth when the estimated bubble deviation is arbitrarily large. This point will be elucidated shortly. In order to ensure that estimates of the probability of the deterministic state are bounded between zero and one, we follow Van Norden and Schaller in using a probit specification.

Under this setting, the probability of being in the ‘dormant’ regime in the next time period is given by:

\[
P(r_{t+1}|N) = \Omega(\beta_{n,0} + \beta_{n,1}B_t + \beta_{n,S}S_{t}^{f,a})
\]

where \( S_{t}^{f,a} \) is the absolute value of the average 6-month actual gross returns minus the absolute value of the average 6-month returns of the estimated fundamental values.

Turning now to the probability of the bubble surviving in the explosive state \((q_t)\), we showed in the previous section that Van Norden and Schaller model this probability as a function of only the absolute size of the bubble. As in most of the speculative bubble models, the Van Norden and Schaller approach assumes that a bubble collapse is a random event that may or may not be fuelled by the arrival of news. In effect, most rational speculative bubble models implicitly assume that investors randomly organise and decide to sell at the same time thus causing the bubble to collapse. However, although investors can estimate the probability of a bubble collapse from the size of the bubble, they still face uncertainty about the time of the collapse. It is logical to assume that investors observe other, non-price, variables in an effort to draw inferences about the probable time of collapse, and thus identify to the optimal time to exit from the market.
Our suggestion is that bubble collapses occur because investors observe a signal that leads them to the conclusion that the market is no longer expecting the bubble to continue to exist. Once they observe this signal, they start to liquidate their holdings and thus cause the bubble to collapse. We consider abnormal trading volume as a possible signal and thus as a predictor of the time of the bubble collapse.

The relationship between trading volume and stock returns has been extensively researched in the literature (see Karpoff (1987) for a meticulous survey of the literature until 1987). Ying (1966) and Morgan (1976) find that large increases in volume are usually followed by increased variance of returns, a result that leads Morgan (1976) to conclude that volume is associated with systematic risk. One possible explanation for this finding is that trading volume is a proxy for the degree of disagreement in the stock market. Although Karpoff (1986) claims that abnormal volume is not proof of investor disagreement (either ex-ante or ex-post) about information that is available, Shalen (1993) claims that volume and return volatility have a positive relationship with the ex-ante dispersion of expectations about future prices. Furthermore, He and Wang (1995) claim a high degree of uncertainty about fundamental values leads to an increase in observed volume in the market.

Moreover, Marsh and Wagner (2000) state that, especially for the U.S. stock market, abnormal volume can help explain increases in the size of both the negative and the positive tails of the returns distribution. Although they find that this effect is symmetric, Hong and Stein (1999) and Chen, Hong and Stein (2001) claim that divergence of information, approximated by abnormal trading volume, only causes an increase in the negative tail of the future returns distribution.

Based on the above, we propose that abnormally high trading volume is a signal of changing market expectations about the future of a speculative bubble. This implies that abnormal volume signals an increase in the probability of the bubble collapsing, thus causing an increase in the probability of observing a large negative (positive) return if a positive (negative) speculative bubble is present. The main difference between our model and the models of price–volume relationship described above is that the latter state that volume increases both tails of the distribution of expected returns or it signifies a decrease in the skewness of future returns. We consider abnormal volume as a sign that other investors are selling the bubbly asset.

For abnormal volume to signal an imminent bubble collapse would require the assumption that investors have different endowments implying that there are agents in the economy that do not hold

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6 The reader is referred to Varian (1989), Harris and Raviv (1993), Kandel and Pearson (1995) and Odean (1998) for other models with the same feature.
equity and may decide to do so at a future date. The effect would be the same if the number of investors changes over time. Furthermore, we assume that some investors face short selling constraints and that in the short run, the supply of equity from firms, through IPOs and SEOs is limited. The latter assumption combined with the unwillingness of investors to sell the bubbly asset because of the expectation of high returns in the surviving regime, cause the supply curve to be relatively inelastic. Under this setting, speculative bubbles are a form of ‘demand side inflation’ in stock prices.

As the bubble continues to grow, the probability of a crash increases and thus some investors will decide to liquidate their holdings for profit taking or because they perceive a crash to be imminent. If a sufficiently large number of investors decide to sell the bubbly asset, supply will increase significantly and thus volume will be abnormally high while the rate of increase in prices will slow. This abnormally high volume will signal that a bubble collapse is imminent.

Under this setting we model the probability of the bubble continuing to be in the surviving regime ($S$) as a negative function of both the absolute relative size of the bubble deviation, and a measure of abnormal volume:

$$P(r_{rt} | S) = q_t = \Omega (\beta_{q,0} + \beta_{q,b} |B_t| + \beta_{q,v} V^x_t)$$

(21)

where $V^x_t$ is a measure of unusual volume in period $t$.

Grouping all the equations together yields the following non-linear switching model of gross stock market returns:

$$E(r_{rt} | N) = M$$
$$E(r_{rt} | S) = M + \frac{(1 - q_t)}{q_t} (MB_t - g(B_t))$$
$$E(r_{rt} | C) = M + g(B_t) - MB_t$$
$$P(r_{rt} | N) = n_t$$
$$P(r_{rt} | S) = (1 - n_t)q_t$$
$$P(r_{rt} | C) = (1 - n_t)(1 - q_t)$$

(22.1) to (22.6)

The model described by equations (22.1) through (22.6) is highly non-linear, so in order to estimate it, we linearise it by taking the first order Taylor series expansion of equations (22.1), (22.2) and (22.3) around an arbitrary $B_0$ and $V^x_0$. The resulting linear switching regression model of gross returns is:

$$r^N_{rt} = \beta_{N,0} + u^N_{rt}$$

(23.1)

7 Again proof of equations is available from the authors upon request.
\[ r_{t+1}^S = \beta_{S,0} + \beta_{S,B} B_t + \beta_{S,V} V_t^x + u_{t+1}^S \quad (23.2) \]
\[ r_{t+1}^C = \beta_{C,0} + \beta_{C,B} B_t + u_{t+1}^C \quad (23.3) \]
\[ P(r_{t+1} | N) = \Omega(\beta_{n,0} + \beta_{n,B} B_t + \beta_{n,s} S_t^{f,a}) \quad (23.4) \]
\[ P(r_{t+1} | S) = \left(1 - \Omega(\beta_{n,0} + \beta_{n,B} B_t + \beta_{n,s} S_t^{f,a})\right) \Omega(\beta_{q,0} + \beta_{q,B} B_t + \beta_{q,V} V_t^x) \quad (23.5) \]
\[ P(r_{t+1} | S) = \left(1 - \Omega(\beta_{n,0} + \beta_{n,B} B_t + \beta_{n,s} S_t^{f,a})\right) \left(1 - \Omega(\beta_{q,0} + \beta_{q,B} B_t + \beta_{q,V} V_t^x)\right) \quad (23.6) \]

We estimate the augmented model under the assumption of disturbance normality using maximum likelihood. Note that the probability density function of an observation conditional on it being generated by a given regime, assuming normality of residuals can be expressed as:

\[ \phi_S(u_{t+1}^S) = \frac{\omega \left(r_{t+1} - \beta_{S,0} - \beta_{S,B} B_t - \beta_{S,V} V_t^x \right)}{\sigma_S} \quad (24.1) \]
\[ \phi_C(u_{t+1}^C) = \frac{\omega \left(r_{t+1} - \beta_{C,0} - \beta_{C,B} B_t \right)}{\sigma_C} \quad (24.2) \]
\[ \phi_N(u_{t+1}^N) = \frac{\omega \left(r_{t+1} - \beta_{N,0} \right)}{\sigma_N} \quad (24.3) \]

where \( \omega \) is the standard normal probability density function (pdf), \( \sigma_N, \sigma_S, \sigma_C \) is the standard deviation of the disturbances in the steady growth, surviving and collapsing regimes respectively. The probabilities of observing a given regime in \( t+1 \) are given by equations (23.4), (23.5) and (23.6). This implies that the unconditional probability density function of each observation is:

\[ \ell(r_{t+1} | \xi) = (1 - n_s) q_t \phi_S(u_{t+1}^S) + (1 - n_s)(1 - q_t) \phi_C(u_{t+1}^C) + n_s \phi_N(u_{t+1}^N) \quad (25) \]

The log likelihood function for a set of \( T \) observations is thus:

\[ L(r_{t+1} | \xi) = \sum_{t=1}^{T-1} \ell(r_{t+1} | \xi) \quad (26) \]

In order to estimate the model, we directly maximise the above log-likelihood function using a constrained optimisation algorithm. The only constraint imposed is that the standard deviations of the residuals are positive, thus bounding the log likelihood function away from infinity\(^8\).

\(^8\) Note that the log-likelihood function is unbounded if the standard deviations of the error terms in the three regimes become zero.
From the Taylor series expansion, several testable implications can be derived concerning the sign and or relevant size of the coefficients of the state equations. These conditions should be satisfied if the three-regime model of speculative bubbles has explanatory power for gross returns, and are:

\[
\begin{align*}
\beta_{N,0} &\neq \beta_{S,0} \neq \beta_{C,0} \\
\beta_{C,B} &< 0 \\
\beta_{S,B} &> \beta_{C,B} \\
\beta_{S,V} &> 0 \\
\beta_{q,B} &< 0 \\
\beta_{q,V} &< 0 \\
\beta_{n,B} &< 0 \\
\beta_{n,S} &< 0
\end{align*}
\]

Restriction (i) implies that the mean return across the three regimes should be different, so that three distinct regimes exist. Restriction (ii) states that as the bubble increases in size, the expected returns in the collapsing regime should decrease (increase) if a positive (negative) bubble is present, since the bubble must collapse in regime \(C\). Furthermore, restriction (iii) ensures that the bubble yields higher returns in the surviving regime than in the collapsing regime. Restriction (iv) states that the expected returns in the surviving regime must increase if abnormal volume is observed in the market since abnormal volume signals an increase in the probability of a bubble collapse. Restrictions (v) and (vi) must hold by construction since the probability of the bubble surviving must decrease when the absolute size of the bubble or abnormal volume in the market increase. The same holds for restrictions (vii) and (viii) because the probability of being in the deterministic regime \(N\) must decrease as the bubble grows larger in absolute size or the returns of the market in the last 6 months are larger than the returns on the fundamental values.

In order to test the power of the model to capture bubble effects in the returns of the S&P 500, we follow Van Norden and Schaller (1999) and test the three-regime speculative bubble model against two alternative models that are nested within the model of speculative behaviour. These alternative models capture stylised facts of stock market returns and therefore we examine whether our model has any explanatory power beyond these simpler specifications. Firstly, we examine whether the effects captured by the switching model can be explained by a more parsimonious model of volatility regimes. In order to test this, we examine the following conditions:

\[
\begin{align*}
\beta_{N,0} &= \beta_{S,0} = \beta_{C,0} \\
\beta_{N,B} &= \beta_{S,B} = \beta_{C,B} = \beta_{S,V} = \beta_{q,B} = \beta_{q,V} = \beta_{n,S} = 0 \\
\sigma_{N} &\neq \sigma_{S} \neq \sigma_{C}
\end{align*}
\]
Restriction (27.1) implies that the mean return across the three regimes is the same while restriction (27.2) states that the bubble deviations, the measure of abnormal volume and the measure of the spread of actual returns above the fundamental returns have no explanatory power for the returns of period $t+1$ or for the probability of switching regimes. The later point suggests that there is a constant probability of switching between a low, medium and high variance regime as this is stated in restriction (27.3).

The volatility regimes model examines the joint hypothesis that the mean returns are the same across the three regimes and that returns and the generating state of returns are unpredictable if we use the variables under consideration. It is interesting to separate the two hypotheses and to examine whether returns can be characterised by a simple mixture of normal distributions model that allows both returns and variances to differ across the two regimes. This mixture of normals model implies the following restrictions:

$$\beta_{N,B} = \beta_{S,B} = \beta_{C,B} = \beta_{S,V} = \beta_{q,B} = \beta_{q,V} = \beta_{n,S} = 0$$

Finally, we augment the original VNS model into a three-regime model of speculative behaviour that does not include the measure of abnormal volume and the spread of returns in the state or transition equations (hereafter referred to as the AVNS model). If these volume and spread variables have explanatory power for the next period’s returns, the test should reject this simpler specification. The restrictions of this last test are:

$$\beta_{S,V} = \beta_{q,V} = \beta_{n,S} = 0$$

### 4. DATA AND FUNDAMENTAL VALUES

The data we use to test for the presence of speculative bubbles are 1357 monthly observations on the S&P 500 for the period January 1888 – January 2001. In order to calculate fundamental values and real gross returns, we use data taken from Shiller\(^9\). The measure of monthly abnormal volume is calculated as the sum of daily share volume, reported by the NYSE for the period January 1888 – January 2001\(^10\), and then the percentage deviation of last month’s volume from the 6 month moving average is taken\(^11\). The monthly dividend and price series are transformed into real variables using the monthly U.S. Consumer Price - All Items Seasonally Adjusted Index reported in Shiller.

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\(^9\) Data available at: [http://www.econ.yale.edu/~shiller/data.htm](http://www.econ.yale.edu/~shiller/data.htm). For a description of the data used see also Shiller (1999) and the description online. Shiller’s sample ends in January 2000, but we update his sample until January 2001 using data obtained from Datastream. In order to verify that the two datasets are consistent, we compare Shiller’s data from January 1965 to January 2000 with the values from Datastream and find no differences.


\(^11\) We also examined unusual trading volume measures using 3, 12 and 18 month moving averages but found that the deviation from the 6-month moving average has the highest explanatory power in predicting both the level and the
In order to construct fundamental values, we examine two different specifications that use only information on past prices and dividends. The models used are the dividend multiple model of Schaller and Van Norden (1999), and a mathematical manipulation of Campbell and Shiller’s (1987) VAR model of dividend components of prices. The dividend multiple model assumes a constant dividend-price ratio whereas the Campbell and Shiller measure allows for predictable variation in the dividend growth rate. Both models assume constant discount rates.

4.1. **DIVIDEND MULTIPLE MEASURE OF FUNDAMENTALS**

Schaller and van Norden (1999) show that if the discount rate is constant, then stock market prices follow the period-by-period arbitrage condition:

\[
p_t = \frac{E_t(p_{t+1} + d_{t+1})}{(1 + i)}
\]  

(30)

Assuming that dividends follow a geometric random walk, i.e. that log dividends follow a random walk with a drift, it can be shown that the fundamental price of a stock will be equal to a multiple of current dividends:

\[
p_t^f = \rho d_t
\]  

(31)

where: \( \rho = \frac{1 + r}{e^{(a + \frac{s^2}{2})} - 1} \)

We use the sample mean of the price-dividend ratio to approximate \( \rho \). In this setting, the relative bubble size is given by:

\[
B_t = \frac{b_{t}}{p_{t}^a} = \frac{p_{t}^a - p_{t}^f}{p_{t}^a} = 1 - \frac{\rho d_{t}}{p_{t}^a}
\]  

(32)

4.2. **CAMPBELL AND SHILLER FUNDAMENTAL VALUES**

The dividend multiple measure of fundamental values assumes that the expected dividend growth rate is constant. In order to allow for predictable variation in the dividend growth rate, we estimate fundamental values based on the Campbell and Shiller (1987) dividend component of prices. The Campbell and Shiller model states that the spread between stock prices and a constant multiple of current dividends is the optimal forecast of a multiple of the discounted value of all future dividend changes:

\[
S_t \equiv p_t^f - \frac{1+i}{i}d_t = \frac{1+i}{i} \left( \sum_{t=1}^{\infty} \frac{1}{(1+i)^t} E_t(\Delta d_{t+e}) \right)
\]  

(33)

generating state of returns. The results for the other measures of abnormal volume are not presented for brevity and are available upon request from the authors.

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Using the VAR methodology created by Campbell and Shiller, we examine whether changes in dividends can be forecasted by the spread between prices and the multiple of current dividends. If the changes in dividends cannot be forecasted by the spread, this would imply that investors use only past dividends to form expectations about future dividends. If, on the other hand, investors include other variables in their information set then this information will be reflected in past prices and thus past realisations of the spread. This would imply that the spread has power to forecast future dividend changes. We examine this relationship using the following VAR:

\[
\begin{bmatrix}
\Delta d_t \\
S_t
\end{bmatrix} = \begin{bmatrix}
a(L) & b(L) \\
c(L) & d(L)
\end{bmatrix} \begin{bmatrix}
\Delta d_{t-1} \\
S_{t-1}
\end{bmatrix} + \begin{bmatrix}
u_{1t} \\
u_{2t}
\end{bmatrix}
\]

(34)

where both \( \Delta d_t \) and \( S_t \) are de-meaned. Equation (34) can be rewritten more compactly as:

\[ z_t = A z_{t-1} + v_t \]

(35)

Equation (35) can be used to forecast future dividend changes conditional on the information set \((H_t)\) described above that contains data on past dividend growth rates and realisations of the spread. Since

\[ E_t(z_{t+1}|H_t) = A' z_t \]

(36)

then the present value of future dividend changes, equal to the fundamental spread, can be forecasted from equation (36) using the following equation:

\[ S_t^* = E_t(S_t^{|H_{t-1}}) = \left( \frac{1+i}{i} \right) \phi' \left( \frac{1}{1+i} \right) A \left( I - \left( \frac{1}{1+i} \right) A \right)^{-1} z_t \]

(37)

where \( \phi \) is a row vector that picks out \( \Delta d_{t-1} \). The fundamental price can then be calculated by solving equation (33):

\[ p_t^i = S_t^* + \left( \frac{1+i}{i} \right) d_{t-1} \]

(38)

where \( i \) is the average real total return over the entire period. We construct fundamental values using equation (36). The second measure of relative bubble size is thus given by:

\[ B_t = 1 - \frac{S_t^* + \left( \frac{1+i}{i} \right) d_{t-1}}{p_t} \]

(39)

Figure 1 presents the bubble deviations calculated from equations (22) and (37) for the entire sample. Note that both bubble deviations are increasingly large in 1929, 1987 and the late 1990’s. The bubble deviations are significantly negative in 1917, 1932, 1938, 1942 and 1982. The Campbell and Shiller measure of bubble deviations (dotted line) displays significantly more short term variability whereas the dividend multiple measure has larger and more persistent broad swings.
5. RESULTS

The results of the three-regime model of speculative behaviour, for the dividend multiple measure of fundamental values, are presented in the first part of Table 1 alongside the results of the VNS model, and the AVNS model for comparison. The AVNS model is a 3-regime extension of the Van Norden and Schaller model that is equivalent to our three-regime model but does not contain any additional variables other than the bubble deviations in the state and transition equations. The second panel of the table contains the results of the likelihood ratio (LR) tests of the restrictions on the coefficients implied by the speculative bubble model while the third panel of Table 1 presents the results of the likelihood ratio tests of the volatility regimes, mixture of normals, and the extended Van Norden and Schaller alternative models.

From the first part of Table 1, all the coefficients of the three-regime model have the expected sign and a financially meaningful magnitude. More specifically, the estimate of the mean return in the deterministic regime \( \beta_{N,0} \) is 1.0054, and it is highly significant as seen from the \( p \)-value of the \( t \)-test. This implies that the expected return in the deterministic regime, which is equal to the required fundamental return, is 0.54% per month (6.67% on an annual basis). Although this could be considered quite high for real returns, there are periods during which the S&P 500 displays significant growth, and these periods cannot be associated with explosive behaviour since returns display a positive mean and very low standard deviation. Such periods include the early 1920’s and the mid-1990’s. The corresponding value of this coefficient for the AVNS model is larger (1.0063), implying a mean return in the surviving regime of 7.68% on an annual basis.

However, when the bubble enters the explosive regime, the behaviour of the bubble becomes more extreme. The mean gross return in the surviving regime \( \beta_{S,0} \) is significantly higher than in the dormant regime, 1.16% per month (14.84% on an annualised basis) for the three-regime model compared with 13.08% on an annual basis for the AVNS model. The mean return in the surviving regime is significant in both equations and is higher than the mean return produced by the VNS model (10.69% on an annualised basis). Note that the mean return in the surviving state in the VNS model is between the mean returns of the deterministic and the surviving regimes in the three-regime models. This could be evidence that the mean return in the two-regime model is the result of the mixture of the deterministic and explosive growth regimes.

\(^{12}\) See Campbell and Shiller (1987) and Van Norden and Schaller (1999) for more details concerning the methodology.
Turning now to the mean return in the collapsing state, we can see that the expected mean return in state \( C \) for the three-regime model is -4.75% per month (-44.23% on an annualised basis). This is consistent with the theory of speculative bubbles since the expected return in the collapsing state should be negative. The point estimate of this coefficient is relatively close to the point estimates of the extended and the original Van Norden and Schaller model. Although the values of these coefficients are difficult to interpret in the case of negative bubbles, which collapse yielding positive returns, there are only a few negative bubbles observed in the sample and only one of these bubbles is very large in size (1932 - see Figure 1). We will return to this point shortly.

According to restriction (i), the periodically collapsing speculative bubble model implies that the mean returns across the three regimes must be statistically different from each other \((\beta_{N,0} \neq \beta_{S,0} \neq \beta_{C,0})\). From the second panel of Table 1, we see that there are three distinct return regimes since the null hypothesis that the mean returns across the three regimes are the same, is rejected at the 5% level \((p\text{-value 0.0214})\).

Turning to the slope coefficients in the state equations, we note that the coefficient on the relative bubble size in the surviving regime \((\beta_{S,b})\) is positive but statistically insignificant (coefficient estimate 0.0109 with \(p\text{-value 0.2162}\)), while the bubble coefficient in the collapsing regime \((\beta_{C,b})\) is negative and statistically significant at the 10% level. The speculative bubble model requires the return in the collapsing regime to be a negative function of the size of the bubble \((\beta_{C,b} < 0)\) while the coefficient of the bubble size in the surviving regime must be greater than the corresponding coefficient in the collapsing regime \((\beta_{S,b} > \beta_{C,b})\). From the second panel of Table 1, we can see that both of these conditions for model plausibility are supported by the data. The null hypothesis that the bubble coefficient in the collapsing regime is equal to zero is rejected at the 5% level \((p\text{-value of LR test 0.0334})\) implying that as the bubble size increases, the returns in the collapsing regime decrease. Furthermore, we can see that restriction (iii) is satisfied since \(\beta_{S,b} > \beta_{C,b}\) at the 5% level (in the second panel of Table 1 the \(p\text{-value of the LR test is 0.0346}\)).

The three-regime model also incorporates abnormal volume in the surviving state equation. The point estimate of the abnormal volume coefficient in the surviving state \((\beta_{S,v})\) is statistically significant \((p\text{-value 0.0109})\). It is not possible to derive an expected sign for this coefficient and the speculative bubble model only implies that it should be greater in value than the bubble coefficient in the collapsing regime. Nevertheless, we should expect that as the bubble increases in size, investors demand a higher return to compensate them for the increased risk of bubble collapse.
value 0.0064), and has the expected sign according to condition (iv) ($\beta_{SV} > 0$). This implies that as volume increases, expected returns for the next period increase. This result is consistent with our conjecture that increased abnormal volume signals increased risk and thus investors demand a higher return. For example, in September 1929, the dividend multiple bubble deviation measure was equal to 23.77% and the total volume for this month was 18.30% higher than the 6-month moving average. The expected return in the surviving regime for the next time period, given these values, is 2.01% for the three-regime model compared to 0.78% for the VNS and 1.19% for the AVNS model. The expected return in the collapsing regime for the same month is -6.45% for three-regime model, and -7.99% and -4.27% for the AVNS and the VNS model respectively. This difference in expected returns is a direct result of the inclusion of abnormal volume in the surviving state equation. The real difference of our model, however, lies the modelling of the classifying equation that gives the probability of switching regimes.

The coefficient estimates of the equation for the probability of being in the steady growth regime, equation (23.4), are in favour of the three-regime speculative behaviour model. For the three-regime model, the intercept coefficient ($\beta_{n,0}$) implies that there is a 17.08% mean probability of switching to the explosive state. This probability is calculated as $1 - \Omega(\beta_{n,0})$ using the point estimates shown in Table 1. The corresponding probability for the AVNS model is 14.84%. However, as the bubble grows, the probability of being in the deterministic state in period $t+1$ decreases since the coefficients on the absolute bubble size and the spread of returns are negative. More specifically, the point estimate of the coefficient on the absolute bubble size in the equation for the probability of the dormant regime ($\beta_{n,B}$) is -0.7023 and is significant at the 5% level. This implies that as the bubble grows in size, the probability of being in the explosive regime increases. The corresponding coefficient estimate for the AVNS model is similar (-0.8397), although it is quite small in value for both models. The original Evans (1991) model implies that as the bubble grows in size, the probability of being in the deterministic regime should diminish rapidly. However, here a bubble size in excess of 200% is required for the probability of being in the deterministic state to become negligible. This could be caused by the fact that the estimated bubble deviations display deterministic behaviour even when they are quite large in size. Furthermore, the estimated bubble deviations become persistently high in the 1990’s and this could be causing the model to assign small average probabilities of being in the explosive state.

14 The opposite holds for negative bubbles since they collapse by yielding positive abnormal returns. However, it would require a negative bubble size of -70% in order for the returns of the collapsing regime to become positive. This is probably due to the large negative bubble deviation in the early 1930’s.

15 Expected returns are calculated from the point estimates of the coefficients in Table 1.
For this reason, our model incorporates the spread of the average six-month actual returns over the 6-month average of fundamental returns in the transition equation. This variable helps to distinguish periods of explosive growth, especially when the estimated bubble deviations are large. The coefficient estimate on the measure of the spread \( (\hat{\beta}_{n,S}) \) is negative, as expected, and significant at the 1% level. Furthermore, the LR test results show that the conditions on the signs of the coefficients in the transition equation (vii) and (viii), are satisfied at the 1% level as seen from part two of Table 1. The LR test rejects the hypothesis that the spread of actual returns does not affect the probability of being in the dormant regime and confirms that the probability of deterministic growth decreases as the bubble growth accelerates. These findings are consistent with Evans’ assertions concerning bubble behaviour and Chen, Hong and Stein’s results about the relationship between past returns and future skewness.

Turning now to the probability of being in the surviving regime, the coefficient estimates of the classifying equation (23.5) for the three-regime model and for the AVNS model are in favour of the presence of periodically collapsing speculative bubbles. As the bubble grows, the probability of being in the surviving regime in period \( t+1 \) decreases since the coefficient on the absolute bubble size is negative (-2.0217) and statistically significant (p-value 0.0171). Note that the constant coefficient \( (\hat{\beta}_{q,0}) \) is quite high, implying that when a bubble is small and the volume in the market is normal, the probability of being in the collapsing state is negligible. This is consistent with Evans’ (1991) model and our assumption that when the bubble is relatively small, the probability of the bubble collapsing is small and thus investors do not take into account in the pricing of the asset.

As the bubble grows and or abnormal volume increases, the probability of the bubble collapsing increases geometrically. The point estimate of the bubble coefficient for the three-regime model in the classifying equation \( (\hat{\beta}_{q,0}) \) is -2.0217 and is highly significant. Furthermore, the LR test shows that condition (v), which states that this coefficient should be negative \( (\hat{\beta}_{q,0} < 0) \), is satisfied at the 10% level. Moreover, the point estimate of the abnormal volume coefficient in the equation for the probability of survival \( (\hat{\beta}_{q,V}) \) is negative, as expected, and statistically significant (-1.5604 with p-value 0.0687). The LR test rejects the null hypothesis that abnormal volume does not affect the probability of collapse and confirms that the probability of the bubble continuing to grow with explosive expectations is a negative function of abnormal volume. From the above, the probability of

\[ P(r_{t+1} | C) = \left(1 - P(r_{t+1} | V)\right)\left(1 - P(r_{t+1} | S)\right). \]

\[ P(r_{t+1} | C) = (1 - P(r_{t+1} | V))(1 - P(r_{t+1} | S)). \]

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\[ 16 \text{ Remember that the probability of being in the collapsing regime is given by } P(r_{t+1} | C) = \left(1 - P(r_{t+1} | V)\right)\left(1 - P(r_{t+1} | S)\right). \]
the bubble collapsing should increase significantly prior to a bubble collapse if our three-regime model is superior at forecasting regime changes.

Indeed, in August 1982 when a large negative bubble is present, the probability of the bubble collapsing, estimated from the three-regime model, increases by 300% to a value of 3.79%. The AVNS model, predicts, for the same month, a probability of collapse of 1.23%, which is only 2.8% higher than for the previous month. The VNS model predicts a much higher probability of a bubble collapse (12.18%), although the value of this probability is persistently high for a period of 16 months prior to the collapse of the bubble in September and October 1982. Furthermore, the VNS model’s probability of a collapse increases by only 1.4% in August 1982. The S&P 500 increased by 11.87% in the next month and by 48.88% during the next year.

The conditional probabilities of being in a given regime calculated from the point estimates of the three-regime model are presented in Figure 2. The corresponding probabilities from the AVNS model are presented in Figure 3. It is apparent that the three-regime model yields a more variable probability of being in the steady growth regime, which decreases significantly during periods of significant market advances or declines. Furthermore, the probability of being in the collapsing regime increases significantly before several bubble collapses, namely August 1929, June 1932, August 1982 and October 2000\textsuperscript{17}. This suggests that the three-regime model that incorporates abnormal volume and the spread of bubble returns is helpful in timing bubble collapses.

The standard deviations of the residual terms from the three-regime model are consistent with the theoretical predictions since the standard deviation of the error term in the in the collapsing regime are greater than in the surviving regime. This is because bubbles often collapse by yielding extreme negative returns (or positive returns in the case of price decreasing bubbles). Furthermore, according to Evans (1991) and Van Norden and Vigfusson (1998), the standard deviation of the errors in the deterministic regime should be small, since it is periodically collapsing speculative bubbles that cause an increase in the variance of returns. The standard deviation of the residuals in the steady growth regime is 2.75%, in the surviving regime it is 5.12%, while in the collapsing regime it is 17.57% on a monthly basis.

The third panel of Table 1 presents the results of the LR tests of the augmented model against simpler models that capture well-documented properties of stock market returns. The LR test result rejects the

\textsuperscript{17} Note that some of these periods were followed by market rallies. This is because we are also examining price decreasing bubbles, which collapse yielding positive returns. The probabilities of collapse for both models are also high in other periods that were not followed by bubble collapses. This could be evidence against the speculative bubble model. However, we will
volatility regimes alternative specification at the 1% level implying either that the mean returns are different across the three regimes, or that speculative bubbles, abnormal volume and the spread of actual returns have predictive power for the returns of period \( t+1 \) or for the probability of switching regimes. Alternatively, both of the restrictions may not be supported by the data. In order to separate the two restrictions, we test the speculative behaviour model against a mixture of normal distributions model and the result of the LR test shows that the data reject the mixture of three normal distributions alternative in favour of the three-regime periodically collapsing speculative bubble model. This suggests that the measure of bubble deviations, the measure of abnormal volume and the spread of actual returns have significant forecasting ability for next period returns and for the probability of switching regimes. The LR test statistic signifies a rejection of the null of the mixture of normal distributions at the 1% significance level.

We also examine the three-regime model against the AVNS model in order to see whether abnormal volume and the spread of actual returns should be used in order to forecast the level and the generating state of returns. Again, our model rejects this simpler specification. This shows that abnormal volume is significant in explaining expected returns and that it can be used to forecast the regime of the next time period. Furthermore, the spread of actual returns over fundamental returns can help to separate periods of deterministic bubble growth from periods of explosive growth.

Until now, we have considered fundamental values that assume constant dividend growth rates and thus assign all of the variation in the price-dividend ratio to speculative bubbles. In order to insure that our model is robust against alternative specifications of fundamentals, we re-estimate the model using the bubble deviations calculated from the Campbell and Shiller measure of fundamental values. These fundamental values allow for predictable variation in the dividend growth rate and thus in the dividend-price ratio. The results of all three models under this alternative fundamental specification are presented in Table 2.

The constant coefficient estimates across the three regimes (\( \beta_{N,0}, \beta_{S,0} \) and \( \beta_{C,0} \)) for the Campbell and Shiller fundamental values are 1.0043, 1.0128 and 0.9140 respectively compared to 1.0054, 1.0116 and 0.9525 for the dividend multiple measure of fundamentals. The differences in the relative value of these coefficients can be attributed to the fact that the Campbell and Shiller measure of fundamental values displays significantly more variation and is smaller on average. However, the results of the LR tests presented in the second panel of Table 2 show that restriction (i) is again supported by the data (that is, the null hypothesis that \( \beta_{N,0} = \beta_{S,0} = \beta_{C,0} \) is rejected) at the 1% level.
Furthermore, the significance of the slope coefficients, with the exception of the bubble coefficient in the surviving regime, appears to be unaffected if we allow for predictable variation in the dividend growth rate. The point estimate of the bubble coefficient in the collapsing equation ($\beta_{C,B}$) is smaller than zero (-0.1459) and is statistically significant (p-value 0.0006). The bubble coefficient in the surviving regime ($\beta_{S,B}$) is positive but now statistically insignificant. However, the restriction that $\beta_{S,B}$ should be greater than $\beta_{C,B}$ is still supported by the data. Note that in the VNS model, the bubble coefficient in the surviving equation is never significant, regardless of the specification of fundamentals, and the significance of the bubble coefficient in the collapsing regime diminishes under the Campbell and Shiller measure of fundamental values. Moreover, the size and significance of the abnormal volume coefficient in the surviving equation ($\beta_{S,V}$) is roughly unchanged. The results for the AVNS model show that the statistical significance of the bubble coefficients in the state equations diminishes if we allow for time variation in the dividend growth rate. Overall, for the three-regime model, the coefficient estimates for the state equations and their significances appear to be unaffected by the consideration of this alternative measure of fundamental values.

The results of the transition equations show that the difference between actual and fundamental values still has significant power in predicting the generating state of returns. More specifically, the coefficients in the ‘dormant’ probability equation are similar in value for the two specifications of fundamentals. However, the sizes of the coefficients for the surviving regime probability equation are quite different, although $\beta_{P,B}$ is still significant at the 5% level and is smaller than zero, as shown by the result of the LR test. Furthermore, the measure of abnormal volume and the spread of actual returns over fundamental returns are still significant and thus help to classify returns into the three regimes.

Finally, the LR tests of the robustness of the speculative bubble models against stylised alternatives presented in the third panel of Table 2 show again that the three-regime speculative bubble model captures effects that are not captured by the other, more parsimonious, models. More significantly, the LR test results of the three-regime Van Norden and Schaller show that the spread of actual returns and the deviation of volume from the six month moving average have predictive ability for the generating state and the level of next period’s returns. The test rejects the hypothesis that volume and the spread reversals.
do not affect the generating state (or the level in the case of abnormal volume) of future returns at the 1% level\textsuperscript{18}.

6. Predictive and Profitability Analysis

Although in the previous section we showed that the three-regime speculative bubble model has significant explanatory power for the S&P 500 returns, the ability of this model to forecast historical bubble collapses has not yet been examined. In a previous study, Van Norden and Vigfusson (1998) examined the size and the power of bubble tests based on regime switching models, and found that the tests are conservative, but have significant power in detecting periodically collapsing speculative bubbles of the form described in Evans (1991). However, their technique only examines the econometric reliability of the switching speculative behaviour model developed by Van Norden and Schaller.

In this section, we will examine the out of sample forecasting ability of the three-regime model and compare it with the forecasting ability of the VNS and the AVNS models. We will then investigate whether regime switching speculative bubble models can be used to determine optimal market entry and exit times. We do this by creating trading rules based on inferences from the speculative behaviour models and test whether such rules can yield abnormal trading profits. Until now, all of the bubble tests that have been created have, to our knowledge, only addressed the problem of bubble identification. We follow a different approach and examine whether inferences from speculative bubble tests can be used in order to make financially meaningful forecasts. This approach will also help examine the predictive ability of our model against that of Van Norden and Schaller’s models, in a financially intuitive way.

In the formulation of the trading rules, we ensure that only information that would have been available to investors at the time that trades were made is used. For this reason, we cut our sample approximately in half, and estimate the three speculative behaviour models using data from January 1888 to December 1945. This provides us with initial estimates of the apparent bubble deviations, the expected returns and the probabilities of being in the three regimes for January 1946. Using the point estimates of the three models, we can then calculate the conditional probability of an unusually low

\textsuperscript{18} However, Van Norden and Schaller estimate their model using part of our sample (January 1926 - December 1989). In order to directly examine the validity and statistical significance of our model, we re-estimate our model and using the original VNS sample. Again, in this sub-sample the abnormal volume measure is highly significant both as a risk factor in the surviving state equation and as a classifying variable in the transition equation for both measures of fundamental values. Furthermore, the spread between actual and fundamental returns has significant power in separating the explosive state from the deterministic state. We have also examined our model for different sub samples (namely: 1888-1926, 1888-1948, 1926-1954, 1954-1974, 1974-1989, 1974-2001, 1948-2001) in order to examine the robustness of the model, and the results are roughly unchanged. The results for these sub-samples are not presented here for brevity, but are available from the authors upon request.
and of an unusually high return in the next period (January 1946). The sample is then updated by one observation with the models and the probabilities of a crash and of a rally re-estimated. We continue in this fashion and update the sample period used by one month until the end of the sample (January 2001) is reached.

We consider both the probability of a crash and of a rally to allow for positive and negative bubbles. Using this rolling estimation with a fixed starting point, we form forecasts for the conditional probabilities of a crash and of a rally using only information that is available to investors up to that point in time. The conditional probability of a crash is calculated using the following equation:

\[
\Pr \left( r_{t+1} < K \right)_i = n_i \omega \left( \frac{K - \beta_{n,0,i} - \beta_{n,0,i} B_1 - \beta_{n,0,i} V^X}{\sigma_{n,i}} \right) + \left( 1 - n_i \right) \left[ q_i \omega \left( \frac{K - \beta_{n,0,i} - \beta_{n,0,i} B_1 - \beta_{n,0,i} V^X}{\sigma_{n,i}} \right) + \left( 1 - q_i \right) \omega \left( \frac{K - \beta_{C,B,i} - \beta_{C,B,i} B_1}{\sigma_{C,i}} \right) \right]
\]

where \( n_i \) is the logarithm of the real S&P 500 index and the logarithm of the dividend multiple measure of fundamental values. From Figure 3, it is evident that the probability of a crash increases during several periods when a bubble is suspected to be present but more importantly it is high before several of the 20 largest 1-month declines of the S&P 500. The probability of a crash estimated from the three-regime model increases by 100% in September 1987 to a value of 6.64%, suggesting that a bubble collapse was likely in October 1987. The corresponding probability from the VNS model remains high for an extended period prior to the October crash and actually decreases in September 1987 to a value of 4.24%. This decrease is attributable to the fact that the market declined between August and September 1987 by 3.75%. The average probability of a

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19 At this point, it should be noted that we have allowed for partial collapses in the specification of the bubble model and therefore a bubble may partially collapse for several periods before starting to grow again. For this reason, we also examine the probability of a crash (rally) against the top 20 3-month negative returns as well as the top 20 draw-downs. A draw-down is defined as the cumulative return from the last local maximum to the next local minimum of the S&P 500 Index and thus refers to cumulative continuous losses.
crash estimated using the three-regime model for the previous year is 3.18% while for the VNS model the average probability of a crash is 3.62%. In the following month, the market declined by 12.3% while the cumulative losses an investor would have sustained if he invested in the S&P 500 until the end of the year would have been 24.64%. The behaviour of the probability of a crash from the three-regime model can be taken as evidence that it is able to time bubble collapses more accurately than the VNS model. Note that the probability of a crash is also high in several other periods including 1959, 1961, 1970, 1974 and 2000. All of these periods are followed by a strong correction in stock price levels.

In Figure 5 we present the conditional probability of a rally calculated from the three-regime model and from the VNS model with markers to signify the 20 largest 1-month, 3-month and consecutive market advances. It is clear that the probability of a rally increases dramatically during several periods when a negative bubble appears to be present, especially in 1949-1950 and 1982. Again, the probability of a rally estimated from the three-regime model is significantly more variable than the corresponding probability derived from the VNS model. The same conclusions can be drawn by examining the probabilities of a crash and of a rally produced by the Campbell and Shiller measure of fundamental values presented in Figure 6 and Figure 7. The probability of a crash spikes prior to market corrections and the probability of a rally increases significantly before negative bubble collapses.

However, there are several periods during which both probabilities increase simultaneously, thus damping the effects we seek to observe, namely the conditional probability of a crash and of a rally in the next period. This is because the conditional distribution of expected returns is a mixture of a low variance (dormant state), medium variance (surviving state) and a high variance (collapsing state) distribution. As the relative size of the bubble increases, the weight of the high variance distribution increases and thus both tails increase at the same time. Furthermore, we note several spikes in the probabilities of a crash and of a rally that where not immediately followed by a market collapse or a market rally. These spikes are not present in the original two-regime Van Norden and Schaller model.

Although this could be taken as evidence against the forecasting ability of the three-regime model, the speculative behaviour model can only estimate the probability of a bubble collapse at each point in time and not the exact time of collapse. The late 1990’s are characterised by large probabilities attached to both tails of the expected returns distributions. However the large bubble in the end of the sample does not crash inside our sample period. This is what we would expect to see from a model of speculative behaviour, since if the time of the crash could be forecasted with great accuracy then this

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20 The 20 largest consecutive market advances (‘draw-ups’) are defined as the 20 largest consecutive positive returns, defined as the return from the last local minimum to the next local maximum.

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would violate the assumption of investor rationality, since if an investor knows that the bubble will
collapse in a month he will sell now or at any point until the time of collapse. This would rule out
speculative bubbles all together since actual prices could never deviate from fundamental values in
this setting.

We proceed to form a trading rule based on all three models and to calculate the risk and return of
these rules at each point in time. We then evaluate the trading rules’ results by calculating the profits
(or losses) an investor would have made if he was using the three-regime model in an effort to time
large market movements from January 1946 to January 2001 and compare the risk adjusted returns to
the results of the VNS model and with returns generated by random trading rules. The trading rule
states that when the probability of a crash (rally) crosses the upper 90% percentile of its historical
values, the investor should sell (buy) the index, investing entire wealth in a risk free asset (equities),
and maintain this position until the probability of a crash (rally) becomes lower than its historical
median value, i.e. until the bubble deflates. When the appropriate probability becomes lower than its
historical median value, entire wealth should be invested in the S&P 500 Index. We include the
probability of a rally in the strategy since an investor should buy if there is a negative bubble and the
probability of a rally is greater than the 90% percentile of its historical values. In order to insure that
we are not using any information that is not available to the investor at time \( t \), we calculate the median
value and the top 90th% value using a rolling window with a fixed starting point in January 1888.

We compare the three-regime model with the VNS model by calculating the total holding period
return for every month and examine the mean, standard deviation, skewness and kurtosis of each
trading rule’s return distribution. We note the number of trades that the trading rule has generated
over the trading period in order to adjust trading profits for transaction costs, and we also note the
percentage of time that an investor following the rule would have invested in equities. To take into
account transaction costs, we assume a 4% round trip cost. We then compare the trading performance
of the three-regime model with the results of the VNS model and with the results of a simple buy and
hold strategy.

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21 Focusing on the 90th percentile is somewhat arbitrary, but represents a trade-off between using too high a cut-off which
will encourage the investor to remain in the market when the bubble has a historically high probability of collapse, while
using too low a cut-off will lead the investor out of the market too frequently, resulting in missed bull market opportunities.
Our results are not qualitatively altered if an 80% or 95% cut-off is employed instead.
22 The risk free rate for the period January 1948 to January 2001 is taken to be the monthly continuously compounded yield
on 3-Month Treasury Bills. The data are taken from the Federal Reserve Bank of St Louis web site
23 We use the median in order to avoid any unwanted influence from extraordinarily large probabilities of a crash and of a
rally observed during the sample period (especially 1929-1933).
Furthermore, in order to examine the statistical significance of the profits generated from the trading rule, we form 10,000 long random trading rules created by randomly generating series of zeros and ones, the length of which is equal to the number of months in our trading sample (January 1946–January 2001, or 661 months) using a binomial distribution. The probability of success (i.e. of a binomial draw of one) is set equal to the percentage of time that trading rule would suggest the investor to be in the market. We use this probability of success because it yields random trading rules with comparable average holding periods to our trading rules. In order to test for the statistical significance of the bubble rules, we compare the risk adjusted returns and the other moments of the returns’ distributions with those of the random rules and if our model yields a risk adjusted profit larger than 90%, 95% or 99% of the random trading rules we can conclude that our abnormal profits are statistically significant at the 10%, 5% and 1% levels respectively. Finally, for every trading rule generated from the bubble models and the random rules, we calculate the wealth that an investor would have accumulated by January 2001 from an initial investment of one dollar in January 1948.

Table 3 contains the trading rule results for the three-regime and for the VNS models, and also presents the results of the buy and hold strategy. The values in parentheses are the percentage of random rules that lead to higher average returns, lower standard deviation, higher skewness etc. Lower values inside the parentheses denote a more superior relative performance of the speculative behaviour model trading rule against the random trading rules.

From the results for the dividend multiple measure of fundamental values, we see that the three-regime model trading rule yields higher risk adjusted returns and a larger end of period wealth compared to the VNS model. More specifically, if an investor used the three-regime speculative behaviour model with the dividend multiple measure of fundamental values throughout the period examined, he would have received an average return of 0.74% per month (9.25% on an annualised basis) with a standard deviation of 2.55%. If, however, the investor used the VNS model, he would only receive an average return of 0.46% per month with a standard deviation of 2.13%. Furthermore, the end of period wealth for the three-regime model is significantly higher than that of the original VNS model, and the buy and hold strategy. This difference in average performance across the two models persists even if the investor was using the Campbell and Shiller model to estimate fundamental values.

The three-regime model also clearly outperforms 99% of the randomly generated trading rules in risk adjusted returns and end of period wealth, signifying that the three-regime model has significant market timing ability. Application of the VNS model only manages to beat 93.66% of the randomly generated trading rules in end of period wealth and 90.2% of the random strategies in risk adjusted.
returns. Moreover, the end of period wealth and the Sharpe ratio of the three-regime model trading rule is higher than the Sharpe ratio of the buy and hold strategy, and this superiority does not fade if we examine the total wealth and the Sharpe ratio after we take into account the transaction costs involved. Note that the three-regime model requires considerably higher number of trades than the original Van Norden and Schaller model.

This higher end of period wealth of the three-regime model is achieved with higher skewness and lower kurtosis coefficients than the VNS model. Both of these higher moments of the distribution of three-regime model returns would be more desirable to investors under some fairly weak assumptions concerning the shape of investor utility functions (see Scott and Horvath (1980) for higher moment preferences and Kraus & Litzenberer (1976) and references therein for skewness preference in asset pricing).

The above results show that if an investor used the three-regime model to identify optimal market entry and exit points, he would have earned higher risk adjusted returns and end of period wealth in January 2001 than if he was using the VNS model or following a passive buy and hold strategy. In Figure 8 we plot the real S&P 500 Composite Index and the net excess wealth of the three-regime model trading rule as a percentage of the wealth of the buy and hold strategy. On the plot of the S&P 500, we place markers that signify entry and exit times that the three-regime model trading rule has generated using the dividend multiple measure of fundamentals. In this figure, investor wealth has been adjusted for transaction costs assuming a 4% round trip cost paid upon exit from the market.

From the figure, it is evident that the three-regime model, estimated using the dividend multiple measure of fundamentals, leads to profitable trades although sometimes it forces the investor to be out of the market for long periods of time (especially in the 1950’s and early 1980’s). Nevertheless, it produces higher wealth than the buy and hold strategy until March 1999. At the end of the sample, however, the buy and hold strategy yields higher wealth since the large observed bubble deviation does not reverse during the sample period employed. Furthermore, the three-regime model underperforms the buy and hold strategy in the 1950’s and 1960’s however this is attributable to the fact that the model issues a sell order in the early 1950’s and thus the investor would have missed a large percentage of the run up in prices until 1954 when the model issues a buy order. However, we

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24 We assume a 4% round trip transaction cost paid upon exit from the market. This could be considered high, especially in recent years since the trades involve large (S&P500) companies. However, this study examines a long period during which transactions costs have declined significantly. An investor following this strategy would have either to buy and sell all of the member stocks of the S&P500 with appropriate weights, or to buy an index fund that tracks the S&P500. Although trading index funds typically implies far lower transactions costs (for example, the Vanguard S&P500 tracker index fund currently has an expenses ratio of 0.18%), index funds only became available in the 1970’s. Therefore we assume higher transactions costs for the whole sample.
can see that in the late 1960’s and 1970’s our model issues a sell order in advance of many market declines and this leads to a higher end of wealth than the buy and hold strategy in 1974. This superiority is maintained until March 1999 when the model orders the investor to leave the market. However, in the end of the sample our model again outperforms the buy and hold strategy. This performance would have been even better if our sample included the large S&P 500 decline in 2001.

If we examine the trading rule results for the three-regime model using the Campbell and Shiller fundamental values, we note that the three-regime model trading rule produces a persistent sell signal for shorter periods of time (see Figure 9) and produces a higher number of trades. Again, the three-regime model trading rule yields a higher wealth than the buy and hold strategy from September 1974 to May 1997. The three-regime model trading rule is unable to generate higher returns than a buy and hold strategy post-1997, and there are a number of possible reasons for this. First, it is possible that a bubble of the form assumed was not present in the data at the end of the sample period or it is possible that the model used to estimate fundamental values is not adequate and does not capture fundamentals precisely. An alternative explanation is that this result is the consequence of a manifestation of the “peso problem”, where the speculative bubble models suggest that a market crash is imminent, but do not suggest a precise date when this will occur. Such an event did not occur during the sample period, but this does not mean that it was wrong to predict that it would; indeed, the market was subject to substantial falls over much of the subsequent 1-year period after the end of the sample.

7. CONCLUSIONS

In this study we have presented a three-regime switching speculative bubble model that is able to capture speculative dynamics present in the S&P 500 Composite Index. The model is based on the theory of periodically collapsing speculative bubbles and on the models developed by Van Norden and Schaller (1999) and Evans (1991). We show how the original Van Norden and Schaller model can be augmented to include a third regime in which the bubble grows at the fundamental rate of return. In this dormant state, the bubble behaviour is very different than in the explosive state since the probability of collapse is negligible. For this reason the bubble grows at a steady growth rate. However, when the bubble enters an explosive state, its behaviour changes and the bubble either grows with explosive expectations at an ever higher growth rate or collapses to a percentage of its value. We show that, as in Van Norden and Schaller, the magnitude of the difference between actual prices and fundamental values is a significant predictor and classifier of returns. Furthermore, we show that other variables like abnormally high volume and the difference of average actual returns from average fundamental returns can be used to classify future returns into the three regimes.
Using data on the S&P 500 for the period January 1888 – January 2001, we find that the speculative behaviour model has significant explanatory power for the next months returns. More specifically, as the bubble grows in size and/or yields higher returns, the probability of being in the explosive regime in the next time period increases, and thus the expectation of bubble growth is adjusted to compensate for the now higher probability of a bubble collapse. The significance of past period returns is consistent with models of conditional skewness. In the explosive state, the size of the bubble deviation and the measure of abnormal volume help in identifying the probability of observing a bubble collapse. We also find that abnormal volume indirectly affects future expected returns as a risk factor, a result that is consistent with previous research. We find that the probability of observing an extreme negative return, of at least two standard deviations below the historical mean of returns, increases significantly when a positive bubble is present and abnormal volume is high.

We also examine the robustness of our model against different specifications of fundamental values and find that our model is robust to fundamental values that allow for predictable variation in the dividend growth rate and thus predictable variation in the fundamental price dividend ratio. When the three-regime model is tested against more parsimonious and established alternatives such as mixtures of normal distributions, volatility regimes and a three-regime extension of the original model of Schaller and Van Norden, it appears that our model’s specification captures additional information present in the data and classifies returns in the switching regime framework more effectively.

In a previous study, Van Norden and Vigfusson (1998) examined the statistical power and reliability of regime switching bubble models. We follow a different approach and test the out-of-sample forecasting ability of the three-regime model and the Van Norden and Schaller model in a financially intuitive way. We construct trading rules based on inferences about the conditional probability of a crash and of a rally, and analyse the risk-adjusted returns obtained with the use of the original two regime speculative behaviour model and of our three-regime extension. In order to ensure that we have not used any data that is not available to investors, we estimate both models using rolling regressions with a fixed starting point. We examine the timing ability of the bubble models by comparing the returns of the speculative bubble model trading rules with the returns on 10,000 randomly generated trading rules that have the same average proportion of the sample period invested in equities. We find that the three-regime model can consistently lead to higher risk adjusted returns than the VNS model, the randomly generated trading rules, and the buy and hold strategy, although some of this superiority fades when we take into account the high transactions costs involved. Nevertheless, we must note that there are several limitations of the switching regime speculative bubble models employed here and elsewhere, which provide scope for further research.
First, although the speculative behaviour models presented are considered as direct tests for the presence of periodically collapsing speculative bubbles, in effect they examine the presence of a particular form of regime switching behaviour dictated by simple models of collapsing speculative bubbles. It is possible that this regime switching behaviour may be caused by expected shifts in the evolution of fundamentals that are not realised or any other factor that causes regime switching in asset returns (Van Norden and Vigfusson (1998)). However, this is the problem of any parametric econometric model: the behaviour of a dependent variable that the econometrician attributes to a particular variable may be caused by another unobservable variable.

Furthermore, although we allow for the time variation of the dividend growth rate, we assume that the discount rate is constant in the construction of fundamental values. However, the time variation of expected returns has been well documented and thus it would be interesting to determine whether speculative behaviour models are robust against specifications of fundamentals that allow for predictable variation in the discount rate. Moreover, our fundamental values are dividend based and there is evidence that companies in the 1990’s have shifted to low dividend payout policies. This could be a contributory factor in the development of the apparently large bubble deviation estimated in the late 1990’s. It is also worth noting that anecdotal evidence suggests that when a new sector appears in the economy, stock markets display significantly higher growth relative to dividend-based fundamentals. However, when these new sectors become mature, stock market prices appear to revert back to their dividend-based fundamentals.

8. References
9. TABLES AND FIGURES

Table 1:

\[
\begin{align*}
\hat{r}_{t+1}^N &= \beta_{N,0} + \hat{u}_{t+1}^N \\
\hat{r}_{t+1}^S &= \beta_{S,0} + \beta_{S,B} \hat{B}_t + \beta_{S,V} \hat{V}_t + \hat{u}_{t+1}^S \\
\hat{r}_{t+1}^C &= \beta_{C,0} + \beta_{C,B} \hat{B}_t + \hat{u}_{t+1}^C
\end{align*}
\]

\[
P(r_{t+1}|N) = \Omega(\beta_{n,0} + \beta_{n,B} |B_t| + \beta_{n,S} \hat{S}_t^{/a})
\]

\[
P(r_{t+1}|S) = (1 - P(r_{t+1}|N)) \Omega(\beta_{q,0} + \beta_{q,B} |B_t| + \beta_{q,V} \hat{V}_t^{/a})
\]

\[
P(r_{t+1}|C) = (1 - P(r_{t+1}|N))(1 - \Omega(\beta_{q,0} + \beta_{q,B} |B_t| + \beta_{q,V} \hat{V}_t^{/a}))
\]

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The three-regime extension of the VNS (Van Norden and Schaller) model is the same as the three-regime model but is estimated without the abnormal volume and the spread variables. The original Van Norden and Schaller model results are the results of the methodology described in their paper (Van Norden and Schaller (1999)) using our data and sample period. The \(t\)-test statistics are calculated using standard errors estimated from the inverse of the Hessian matrix at the optimum. The volatility regimes test imposes the restrictions described in equations (25.1), (25.2) and (25.3). The mixture of normals imposes restriction (26) while the 3 regime VNS model imposes restriction (27).
Table 2:

\[
\begin{align*}
  r_{t+1}^N &= \beta_{N,0} + u_{t+1}^N \\
  r_{t+1}^S &= \beta_{S,0} + \beta_{S,B} B_t + \beta_{S,Y} V_{t}^s + u_{t+1}^S \\
  r_{t+1}^C &= \beta_{C,0} + \beta_{C,B} B_t + u_{t+1}^C
\end{align*}
\]

\[
P(r_{t+1} \mid N) = \Omega \left( \beta_{n,0} + \beta_{n,B} \mid B_t \right) + \beta_{n,S} S_{t}^{\alpha} \\
P(r_{t+1} \mid S) = \left( 1 - P(r_{t+1} \mid N) \right) \Omega \left( \beta_{y,0} + \beta_{y,B} \mid B_t \right) + \beta_{y,V} V_{t}^s \\
P(r_{t+1} \mid C) = \left( 1 - P(r_{t+1} \mid N) \right) \left( 1 - P(r_{t+1} \mid S) \right)
\]

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See notes of Table 1

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<th>Skewness</th>
<th>Excess Kurtosis</th>
<th>End of Period Wealth</th>
<th>Sharpe Ratio</th>
<th>% of Time in the Market</th>
<th>Number of Trades</th>
<th>Adjusted Sharpe Ratio</th>
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Dividend Multiple Measure of Fundamentals

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<th>Excess Kurtosis</th>
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<td>10</td>
<td>0.1601</td>
<td>$15.18</td>
</tr>
</tbody>
</table>

Campbell and Shiller Measure of Fundamentals

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Mean Return</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Excess Kurtosis</th>
<th>End of Period Wealth</th>
<th>Sharpe Ratio</th>
<th>% of Time in the Market</th>
<th>Number of Trades</th>
<th>Adjusted Sharpe Ratio</th>
<th>Adjusted End of Period Wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 Regime Model</td>
<td>0.66%</td>
<td>2.41%</td>
<td>0.45</td>
<td>2.88</td>
<td>$62.13</td>
<td>0.2504</td>
<td>57.73%</td>
<td>23</td>
<td>0.2076</td>
<td>$31.01</td>
</tr>
<tr>
<td>Original VNS Model</td>
<td>0.41%</td>
<td>2.58%</td>
<td>-0.73</td>
<td>5.34</td>
<td>$12.11</td>
<td>0.1395</td>
<td>55.45%</td>
<td>19</td>
<td>0.1189</td>
<td>$9.21</td>
</tr>
</tbody>
</table>

Trading rules are formed based on the conditional probability of a crash when a positive bubble is present and the conditional probability of a rally when a negative bubble is present. The investor either places his entire wealth in the S&P 500 Composite Index or in the 3-month U.S. Treasury Bill. The end of period wealth is the real value of the investor’s portfolio in January 2001 if the initial value of the portfolio in January 1946 was $1.00. All the numbers and the returns are in real terms. Figures in parentheses show the percentile ranking of the trading rule relative to 10,000 random trading rules with an equal percentage of time invested in the index. The mean return is the average monthly real total return and the standard deviation of returns is the standard deviation of total returns. The Sharpe ratio is the ratio of the mean excess return of a given trading rule over the corresponding standard deviation of returns. The percentage of time in the market is the percentage of months the trading rule produced a hold signal out of the 661 months in the sample. The number of trades is the total number of buy and sell orders produced by a given trading rule. The adjusted end of period wealth shows the end of period wealth net of transaction costs. Transaction costs are assumed to be 4% per round trip on the total value of the trade.
Figure 1:

Figure 2:
Figure 3:

Figure 4:
Figure 5:
Probabilities of a Rally from the 3 Regime and the Van Norden and Schaller Models.
Dividend multiple Measure of Fundamental Values: January 1946 – January 2001

Figure 6:
Probabilities of a Crash from the 3 Regime and the Van Norden and Schaller Models.
Campbell and Shiller Measure of Fundamental Values: January 1946 – January 2001
Figure 7:

Figure 8:
Figure 9: