Cross Hedging with Single Stock Futures*

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Abstract

This study evaluates the efficiency of cross hedging with single stock futures (SSF) contracts. We propose a new technique for hedging exposure to an individual stock that does not have options or exchange-traded SSF contracts written on it. Our method selects as a hedging instrument a portfolio of SSF contracts which are selected based on how closely matched their underlying firm characteristics are with the characteristics of the individual stock we are attempting to hedge. We investigate whether using cross-sectional characteristics to construct our hedge can provide hedging efficiency gains over that of constructing the hedge based on return correlations alone. Overall, we find that the best hedging performance is achieved through a portfolio that is hedged with market index futures and a SSF matched by both historical return correlation and cross-sectional matching characteristics. We also find it preferable to retain the chosen SSF contracts for the whole out-of-sample period while re-estimating the optimal hedge ratio at each rolling window.

1 Introduction

There are a variety of reasons why retail and institutional investors may have substantial undiversified exposures to single stocks. For example, an investment bank may acquire shares through syndication that are subject to a covenant restricting their sale. Similarly, an investor may hold stock options that are currently deep in the money but for which selling is not permitted for a prescribed period. Or, a fund manager may have a large exposure to a stock that he does not want to close out. In all of these cases, the investor may desire to hedge, rather than sell, his shares as protection against price falls.

One way that an investor could deal with such a problem is to enter into an offsetting short position. This position's associated margin requirements, up-tick trading restrictions, and loan interest, mean that it is likely to be a high cost tool. As an alternative, the investor could use stock options. This is often impractical, however, since the vast majority of listed stocks do not have exchange-traded options written on them and since over-the-counter options often have substantial initial premiums based on opaque "black box" pricing.

Futures contracts are likely to represent a much cleaner hedging tool. Futures contracts have no premium, low transaction costs, low margin requirements, and more transparent pricing than over-the-counter options. Hedging with stock index futures is certainly easy and cost-effective, but may provide an inadequate hedge if the returns profile of the stock exposure is significantly different to that of the index as a whole. As an alternative, one may consider hedging with single stock futures (SSF) contracts. Such a hedge is likely to work well if there is a traded future on the required stock. In cases for which the required SSF does not exist, the investor faces a choice: hedge with a stock index or cross-hedge using the futures contract of a closely related stock. Since cross-hedging efficiency is degraded by the inevitable 'basis risk', it is essential to select the appropriate futures contract carefully and to develop an effective cross-hedging model.

To this end, the objective of this study is to evaluate the efficiency of cross hedging with the new SSF contracts introduced in the U.S. in November 2002. At the end of May 2005, 175 individual U.S. stocks had SSF contracts written on them. To cross hedge other stocks, we propose using a technique that matches the spot stock with one or more of the available SSF contracts in a manner designed to reduce the basis risk of cross hedging and to obtain the most efficient hedging portfolio.

The benefits of hedging with futures have been well studied, and cross hedging with futures has been successfully used in various financial markets including commodity (Foster and Whiteman, 2002; Franken and Parcell, 2003), foreign exchange (Brooks and Chong, 2001) and equity markets. While there has been extensive testing of the various econometric models available to estimate the optimal hedge ratio, there has been little research on how to select optimally the hedging asset which minimizes the basis risk of cross hedging. If the futures contract for the specific individual stock does not exist, the effectiveness of the hedge may depend more crucially on the selected futures contract than on the optimality of the estimated hedge ratio.

The hedging efficiency of conventional estimation models of the optimal hedge ratio depends on the return covariance between the spot and hedging assets. As the estimated hedge ratio and resulting efficiency are contingent on the sample period and its length, there is no guarantee that an effective hedge will continue over a different time horizon. Unfortunately, there is no universally accepted objective decision criterion for the appropriate length of the sample period.

As an alternative, one could consider the common fundamental factors that affect the price movement of the spot asset and the hedging asset. In the context of cross hedging, if two assets have similar fundamental factors that determine their subsequent price movements, then the resulting hedge can be expected to be relatively effective. We would argue that fundamental characteristics are, by their very nature, likely to vary much less from one sample to another than returns, and should therefore lead to more stable and more accurate hedging ratios. We propose matching the spot asset with the ideal hedging asset(s) using nonparametric sample matching techniques that control those fundamental factors as the matching characteristics. The resulting hedged portfolio should minimize the basis risk. We show that using matching techniques to construct the hedged portfolio can provide efficiency gains over a hedged portfolio constructed purely according to the correlation between the futures and spot returns.

Our method is supported by two recent papers which provide strong evidence that individual stocks often move together, allowing one stock (or its associated single stock futures contract) to provide a natural hedge for another. First, Gatev, Goetzmann, and Rouwnhorst (2005) investigate the following "pairs trading" strategy: (i) The investor first finds two stocks that have historically moved together;

(ii) When their prices diverge the investor shorts the winner and buys the loser; (iii) Eventually, the prices (hopefully) converge again, generating profits. Gatev, et al. show that this simple strategy produces significant positive risk-adjusted returns.[2] Tookes (2004) also provides support for our methodology. She shows in the context of earnings announcements, that returns in the stocks on non-announcing competitors have information content for announcing firms.

For our empirical analysis, we construct four types of cross-hedged portfolios that are hedged with: i) single SSF only, ii) single SSF and market index futures, iii) multiple SSF contracts, and iv) multiple SSF contracts and market index futures. Each futures contract is chosen according to three different characteristic sets. The first matching characteristic set consists of only historical return correlations between spot and potential futures implied in the conventional cross hedging model. The second set consists of possible fundamental factors (industry, beta, market capitalization, and price to book ratio) that influence the price movements of stocks. The last set includes both return correlations and fundamental factors. Finally, we repeat the same analysis with the additional restriction that the selected SSF contracts are from the same industry as the spot stock.

To examine the hedging efficiency of each hedged portfolio, we consider the percentage reduction of the variance of the hedged portfolio relative to that of the unhedged portfolio. To compare the out-of-sample hedging efficiency of each model over time, we construct a hedged portfolio with a 1-day life and roll it over with fixed sized time windows. Overall, we find that the best hedging performance is achieved through a portfolio that is hedged with market index futures and a SSF matched by both historical return correlation and cross-sectional matching characteristics. We also find it preferable to retain the chosen SSF contracts for the whole out-of-sample period while re-estimating the optimal hedge ratio at each rolling window.

The paper is organised as follows. The next section describes the SSF markets. Section 3 outlines our methodology for estimating the hedge ratio and determining hedging efficiency. Section 4 outlines the various cross-hedging models based on different hedging strategies and explains the estimation procedure and rebalancing

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[2] Gatev, Goetzmann, Rouwnhorst (2005) form pairs over a 12-month period and trade them in the next 6-month period. They choose a matching partner for each stock by finding the security that minimizes the sum of squared deviations between the two normalized price series. They also present results by sector, where they restrict both matched stocks to belong to the same broad industry categories.

methods. Section 5 describes the data. Section 6 presents the estimation results and finally, Section 7 concludes.

2 The Single Stock Futures Market

Both market players and academics recognise the potential benefits of single stock futures, including that they: (i) enable easy short selling; (ii) reduce the cost of obtaining a leveraged long position; (iii) provide the opportunity for spread trading; (iv) enable a trader to isolate a stock from an index; and (v) provide a cleaner and more efficient hedging tool than options. Despite these benefits, SSFs have only recently been introduced in the U.S. and on futures exchanges in Hong Kong, London, Madrid, Warsaw, Helsinki, South Africa, Mexico and Bombay, among others (see Lascelles (2002) for a survey of exchanges trading SSF contracts). Given their lack of history, SSFs have been subject to little attention in published research. Exceptions include Dutt and Wein (2003), who suggest suitable margin requirements for the U.S. SSF market, and McKenzie, Brailford and Faff (2001), who examine the impact of SSF listing on the liquidity of the spot stock market in Australia.[3]

Prior to November 2002, SSF contracts were not permitted in the U.S. In part, this was because of regulatory concerns about the leverage effect of SSF and possible manipulation of the underlying spot stock price. The approval of listing standards and margin requirements by the Securities and Exchange Commission and the authorization of trading rules by the Commodity Futures Trading Commission paved the way for the November 8, 2002 launch of the first U.S.-based SSF markets: OneChicago and Nasdaq.LIFFE (NQLX). Soon after launch, OneChicago quickly became the dominant trading venue for SSF contracts in the US and is thus the focus of this study. NQLX ceased operations in December 2004.

OneChicago is a joint venture of the Chicago Board Options Exchange, the Chicago Mercantile Exchange, and the Chicago Board of Trade. For each SSF contract, a lead market maker provides continuous two-sided quote prices and ensures liquidity.[4] Contracts are settled by physical delivery. Since inception, OneChicago has enjoyed tremendous growth. For instance, during June 2005, trading volume was 336,823 contracts, representing an increase of 66% over June 2004 trading volume.

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[3] See also Ang and Cheng (2004, 2005), Hung, Lee, and So (2003), and Partnoy (2001).

[4] This market model contrasted with the combination of a multiple market maker system and a central order book adopted by the now defunct NQLX single stock futures trading system.

3 Methodology

3.1 Basis Risk

Minimizing basis risk is the most important criterion for improving the cross-hedging efficiency of hedging with futures contracts. Basis risk, defined as the difference in price between the spot and futures at maturity, arises because the quality and/or the quantity of the underlying spot assets usually differ from those of the futures contracts.

The payoff of a hedged portfolio with a hedge ratio of one can be written

$$P_{S,T} + P_{F,T-1} - P_{F,T} \quad (1)$$

where P_S indicates the spot price, and P_F indicates the futures price. At time T-1, the hedge is put in place, and at time T, the hedging position is closed. When we consider cross hedging, equation (1) can be rewritten

$$P_{F,T-1} + (P^*_{S,T} - P_{F,T}) + (P_{S,T} - P^*_{S,T}) \quad (2)$$

where the superscript * indicates that the underlying asset of the hedging futures is different from the spot asset exposed. Equation (2) illustrates that the basis from cross hedging consists of two components. The first component, $P^*_{S,T} - P_{F,T}$, represents the basis risk from the difference in price at clearing time between the futures and the spot asset, given that the spot is the same as the underlying asset of the futures contract. The second component, $P_{S,T} - P^*_{S,T}$, captures the difference between the spot and the underlying asset of the futures contract. Since the first component of the basis risk cannot be controlled, the main concern in cross hedging is the minimization of the second component of the basis risk. That means that we have to select the 'optimal futures' whose underlying asset has the most similar price movement to that of the spot asset.

3.2 Optimal Hedge Ratio

When the hedge ratio is defined as the ratio of the futures exposure to the spot exposure, the naive hedge ratio of one is only optimal when the spot and futures returns are perfectly correlated and constant over time. Clearly, this is not supported empirically. The key, therefore, is to estimate the 'optimal' hedge ratio. As Lien and Tse (2002) show, we can categorize the models for estimating the optimal hedge ratio by the purpose of hedging, by the asset manager's utility function, and by the assumptions about the distribution of the futures and spot returns.

The OLS Hedge Ratio

The optimal hedge ratio that minimizes the variance of the payoff of the hedged portfolio is analytically the same as the slope coefficient of an OLS regression of the spot returns ($r_{S,t}$) on the futures returns ($r_{F,t}$).[5] Thus, the optimal hedge ratio (HR_{OLS}) for each of the j futures contracts is found by estimating:

$$r_{S,t} = \alpha + \sum_j HR^j_{OLS} \cdot r^j_{F,t} + \varepsilon_t \tag{3}$$

where α is a constant and ε_t is a white noise term. The regression R^2 gives the in-sample hedging efficiency.

Notice that the error term of (3) represents the sum of the basis risk components of (2). Thus, minimizing the basis risk of (2) is equivalent to minimizing the variance of the error term of (3) (i.e., maximizing its R^2). If the underlying of the futures is exactly the same as the spot asset, then the correlation is likely to be close to unity. In this case, if the correlation is also constant over time and the amount of the spot asset is deterministic, then the OLS model will always produce efficient hedges. The extent to which reality deviates from these ideal conditions dictates how well the OLS model will perform in practice.

Other Approaches to Estimating the Hedge Ratio

An alternative approach to estimate the optimal hedge ratio is based on maximizing an expected utility function which incorporates the mean-variance of the hedged portfolio payoff.[6] This approach implies that the optimal hedge ratio sets the hedger's subjective marginal substitution ratio between risk and returns equal to that of the objective hedged portfolio.

Another approach to estimate the optimal hedge ratio is to use econometric models, such as the GARCH, which capture the time varying second moment of returns distributions. These models can be used to estimate a dynamic optimal hedge ratio that allows for time variation in the variance of future returns and in the

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[5] Ederington (1979) shows that the optimal hedge ratio to minimize the variance of the payoff of the hedged portfolio usually differs from 1. Anderson and Danthine (1980) extend the analysis to multiple hedging futures by considering the degree of risk aversion in the utility function, and prove that the optimal hedge ratio for each futures asset is analytically the same as the slope coefficients of each futures asset in a multiple regression.

[6] Anderson and Danthine (1981) prove that the optimal hedge ratio in a mean-variance context for the pure hedger is equal to the variance minimizing hedging ratio with predetermined spot position when the futures price follows a martingale (i.e. $E(\Delta F)=0$). Cecchetitti, Cumby and Fieglewski (1988) argue, through an empirical analysis of the U.S. Treasury bond market, that the optimal hedge ratio to maximize a log utility function is smaller than the risk-minimizing ratio.

covariance between spot and futures returns. For example, Baillie and Myers (1991) apply the bivariate GARCH model to commodity futures market data, and argue that a time-invariant hedge ratio is inappropriate and that a GARCH model performs better than the regression model, especially out-of-sample. Hedge ratio estimation based on variants on the GARCH model framework are proposed by Kroner and Sultan (1993), Brooks and Chong (2001), Brooks, Henry and Persand (2002), Poomimars, Cadle and Thebald (2003), and others.

All of these models, including the OLS hedge ratio, assume either that the best futures asset is optimally given to minimize the second component of equation (2) for cross hedging or that the hedging futures' underlying asset exists in the spot market. To the best of our knowledge, there is no existing literature which provides a theoretical method to minimize the second component of equation (2) or examines its effect on basis risk and hedging efficiency.

We develop a hedging model that reduces basis risk by selecting an optimal hedging futures asset as well as estimating the optimal hedge ratio. We adopt the variance minimizing hedge ratio estimated using OLS because a comparison of the efficiency of the hedge ratio is not the main focus of this paper. Empirically, Brooks *et al*. (2002), and others, have shown that there is often little difference in out-of-sample hedging efficiency between hedge ratios estimated using OLS and with other more complex models. Moreover, in practice, the OLS hedge ratio is widely used by market players thanks to its simplicity of understanding and estimation.

3.3 Hedging Efficiency

In the futures literature, the most commonly measure used to gauge hedging efficiency is related to the variance of the payoff of the hedged portfolio – either the level of the variance or the reduction ratio to that of an unhedged portfolio. This means that the smaller the variance of the hedged portfolio, the larger the probability that it has a lower basis risk. It is worth noting that the hedge ratio from a regression model analytically guarantees the minimum variance in-sample provided that the hedging futures series employed has the highest correlation with the spot asset during the in-sample period.

The payoff (π) of a hedged portfolio at time t is defined as

$$\pi_t = r_{S,t} - \sum_j HR_{OLS}^j \cdot r_{F,t}^j$$

For each spot stock, we compute the percentage reduction in variance (Var) of the payoff of the hedged portfolio ("hedge") against that of the corresponding unhedged portfolio ("no hedge"), as:

$$\left(1-\frac{\operatorname{Var}\left(\pi_{\text{hedge}}\right)}{\operatorname{Var}\left(\pi_{\text{no hedge}}\right)}\right)\times 100$$

In unreported results, we also considered measuring hedging efficiency with the mean of the negative payoffs of the hedged portfolio. Qualitatively similar results were obtained.[7]

4 The Cross Hedging Model

It is common to choose the futures asset for cross hedging based on only its historical return correlation, ρ, since the highest historical return correlation ensures the highest hedging efficiency (i.e. the minimum variance of the hedged portfolio payoff) during the *in-sample* period. However, as discussed above, this may not provide optimal out-of-sample performance.

Here, we consider hedging from another point of view. If the prices of two assets are influenced by similar fundamental factors, then clearly these two assets should have similar expected price movements. Thus, we could select our hedging asset based on the extent to which its price movements share the same common fundamental factors as the spot price. Our hope is that this approach will lead to better results since fundamental factors are likely to be less noisy and more stable through time than return correlations. In the following subsections, we describe various techniques designed to select a futures asset that has the closest matching characteristics to the spot asset we are attempting to hedge.

4.1 Matching Characteristics

We construct three sets of matching characteristics (X). The first set consists of only the historical return correlation. The second consists of only the fundamental factors, which are capital asset pricing model (CAPM) beta, market capitalization and the price to book ratio. The final set consists of both the correlation and the fundamental factors. For multiple matching characteristics, we measure the distance between spot and hedging futures, in terms of matching characteristics, using the

[7] The literature has proposed many other hedging criteria. Examples include the maximization of expected return given a specified risk tolerance level or criteria that incorporate an asymmetric impact of portfolio returns on utility (Lien 2001a, 2001b). However, as the degree of risk aversion is usually unobservable and given the abstract nature of the utility function framework, we instead focus on the standard variance reduction measure.

Mahalanobis metric:

$$\| X_S - X_F \| = (X_S - X_F)' \Lambda^{-1} (X_S - X_F)$$

where $\Lambda = [(N_S - 1)\Lambda_S + (N_F - 1)\Lambda_F]/(N_S + N_F - 2)$, N_D denotes the sample size, and Λ_D denotes the sample covariance for $D=S$, F. For each spot asset, we select as the corresponding hedging futures contract(s), the contract(s) which minimize(s) this distance metric over the set of matching characteristics. When the matching characteristic is correlation, $X_S = 1$ and X_F equals the correlation between the spot asset and futures contract.

4.2 Industry Classification

It is likely that, all other things being equal, firms within the same industry will have stock price movements that are more correlated than they are with those in other industries. This suggests that the hedger may primarily seek a hedging SSF that is in the same industry sector as the spot asset to minimize the industry effect. Hence, we examine whether classification of futures by industry can help to improve cross-hedging efficiency. The SSF contracts and spot stocks are classified according to their FTSE level 3 economic/industrial sector, and then spot stocks are matched with SSF contracts within the same industrial classification. We use the lowest level of FTSE industry classification to increase the likelihood that there exists a SSF in the same industry as each spot stock. When no SSF contract is available in the same industrial sector as the spot stock, we select a SSF in the most similar industry.

4.3 Hedging with multiple matched SSF contracts

In the context of currency futures, DeMaskey (1997) shows that hedging with multiple futures contracts performs better than hedging with a single futures contract. Furthermore, he finds that adding more than three futures is unlikely to improve performance further. In light of these results, it is reasonable to suppose that using multiple SSF contracts to hedge could result in better hedging efficiency relative to that of using a single SSF. We explore this possibility by using up to three SSF contracts of “nearby” stocks to hedge.

4.4 Hedging with SSF contracts and Market Index Futures

Hedging with market index futures is the most prevalent hedging tool for spot stocks having no derivatives, since it allows for a diversified portfolio to eliminate market risk with low trading costs. However, as index futures can only eliminate market risk, the residual basis risk can be substantial. In other words, index futures cannot remove firm specific risk. Thus, if we hedge the spot stocks’ exposures with market index futures in addition to the matched individual stock futures, the hedging efficiency may improve because this approach may mitigate both the market risk and the residual firm specific risk.

Thus, to summarize, we have with four types of cross-hedged models, hedged with: i) single matched futures only; ii) single matched futures and market index futures; iii) multiple futures; and iv) multiple futures and market index futures. The hedging SSF contracts are matched by: i) return correlation only; ii) cross-sectional fundamental factors only; and iii) both of them. All models are examined both with and without industry matching.

4.5 Estimation and Rebalancing

To determine the ex-ante hedging efficiency during the out-of-sample period, rolling windows of fixed length (1-day), corresponding to the supposed portfolio life, are employed until data are exhausted. The issue of the lengths of the in-sample and out-of-sample periods is addressed later. Hedging efficiency is measured in terms of variance reduction, assuming that each portfolio consists of one spot stock. Then, the hedging efficiency is estimated over the sample of spot stocks.

Assuming a short hedge and using the minimum-variance hedging ratios estimated by OLS, we consider three rebalancing procedures for hedging efficiency:

1. **Low effort and transaction costs:** Retain a single optimally chosen hedging SSF contract for a given spot stock position and use the same OLS hedge ratio over all rolling windows during the out-of-sample period. That is, there is a one-time matching and a one-time estimation of the hedge ratio at the start of the out-of-sample period.
2. **Medium effort and transaction costs:** Fix throughout the optimally chosen SSF contracts for a given stock, but re-estimate the OLS hedge ratio at every rolling window during the out-of-sample period as new price information becomes available.
3. **High effort and transaction costs:** Re-select, at each rolling window, the hedging SSF contracts for each spot stock according to the new information, and re-estimate the hedge ratios.

These three procedures impose different computation and transaction costs on the hedger and allow us to test whether increased hedging efficiency can be obtained by increasing the frequency of rebalancing. Note that the second and third methods both allow for the possibility of time-variation in the correlation between SSF and spot asset returns.

5 Data

We collect daily settlement prices, daily trade price ranges (open, close, high, low), trading volume, and open interest of each SSF contract listed on OneChicago from its website (www.onechicago.com) for the period September 2, 2003 to March 31, 2005 (396 trading days). To ensure sufficient observations to estimate the return correlations, we restrict our sample to SSF contracts written on US-based stocks that had a deliverable SSF contract written on them prior to September 2, 2003. Our final sample consists of SSF contracts written on 86 underlying stocks.

Typically, each stock has four SSF contracts written on it. Until July 19, 2004, the contracts followed the quarterly cycle of March, June, September and December. After this date, the contract expiration schedule was changed to include two front months and then two quarterly months listed, for a total of four expirations per product class. Thus, after the change, the expiries for the longest term contracts range from six to eight months, depending on the time of the year. For hedging purposes, we always focus on the nearby **quarterly** contract, rather than the nearby serial contract, since this contract is normally the most liquid.[8] We make standard adjustments for dividends and major corporate events, and exclude non-standard listings. Table 1 lists the final sample of SSF contracts and provides the average daily trading volume and average open interest of the nearby quarterly contract. While clearly some contracts are less frequently traded than others, we note that the arbitrage relation between the futures and underlying stock ensures that all contracts have intraday bid-ask spreads which remain very narrow.[9]

The criteria for the spot stocks included in our sample are that they must: i) not have corresponding derivatives – either SSF or options[10]; ii) be US-based firms; iii) be listed on a U.S.-based stock exchange before September 2, 2003; and iv) have matching characteristic data available. From the set of stocks satisfying these four criteria, we select the largest 350 stocks based on market capitalization on December

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[8] Bernhardt, Davies, Spicer (2005) provide a theoretical explanation for why liquidity concentrates in the nearby contract.

[9] As anecdotal evidence of how relatively new SSF markets can have narrow spreads, the *Futures Industry Magazine* reports that on the Spanish futures exchange MEFF, "Underlying shares in the cash market, which are generally priced between 10 euro and 25 euro, trade with bid-ask spreads of 0.01 euro to 0.02 euro. The market for single stock futures are seeing bid-ask spreads of only 0.02 euro to 0.03 euro." ("Spain's MEFF Scores Solid Success", Joshua Levitt, *Futures Industry Magazine*, Dec. 2001).

[10] This restriction ensures that hedging with the same futures asset as the underlying spot asset is not a possibility for any of the stocks in our sample.

31, 2003. For the spot stocks and the firms underlying the SSF contracts, we collect matching characteristics (industry, beta, market capitalization, and price to book ratio) from Datastream. Table 2 provides summary statistics of the sample. Notice that the firms underlying the SSF contracts are much larger, in general, than the sample of spot stocks. Our restriction that spot stocks have no exchange-traded derivatives written on them results in a sample of firms that is smaller and younger. To the extent that these firms are more difficult to match, our results will provide a conservative estimate of the true potential effectiveness of our hedging methods.

6 Results

First, we examine the issue of which rebalancing procedure shows the best hedging efficiency during the out-of-sample period. In Figure 1, the average variance reductions from different rebalancing methods are depicted over different lengths of out-of-sample period. Interestingly, our expectation that the most complicated rebalancing method would show the best performance is not supported. All three rebalancing methods are based on a hedge using a sole SSF matched by historical correlation only. Even though rebalancing according to the time varying hedge ratio performs better than the constant hedge ratio over the out-of-sample period, changing the SSF used for hedging according to the updated historical return correlation does not guarantee a better performance.

We have tested a total of 33 cases of hedging models – for example, hedging with multiple SSF contracts, matching SSF contracts with different matching characteristic sets, adding industrial classifications, and with market index futures. Even though Figure 1 is based on the simplest hedging model, for most of the hedging models, the second rebalancing procedure – re-estimating the hedge ratio and not re-selecting the SSF – is still preferred. Hence, the following sections focus on the results obtained from this second balancing method (that is, updating the hedge ratio but using the same SSF for a given stock for the whole out of sample period).

When conducting an out-of-sample evaluation of hedging efficiency, it is of interest to examine the sensitivity of the results to the portion of the total sample that is retained as the out-of-sample period. To this end, we conduct all estimation procedures for out-of-sample periods ranging from 15% to 50% of the total sample period. Based on the results presented in Figure 1, we choose 32.5% of the total sample (128 days) for the out-of-sample period, which also ties in with the loose "two-thirds, one-third" rule commonly used in empirical analysis.

Matching characteristics: In figure 2, we examine the effect of three different matching characteristics on the choice of optimal hedging asset. The first one consists of the historical return correlation only while the second consists of three cross-sectional matching characteristics (CAPM beta, market capitalization and price to book ratio). Both the historical correlation and the cross-sectional matching characteristics are combined in the last set. Note that in this cumulative probability distribution diagram we prefer the line to be further to the lower right. Here, we find that historical correlation is very important. For all three hedging models, the variance of the hedged portfolio returns is reduced for about 75% of the spot stocks.

Multiple SSF contracts: If there are benefits from diversification, hedging with multiple SSF contracts may improve hedging efficiency. In Figure 3, the variance reduction from hedging with multiple SSF contracts is compared with that of hedging with only one SSF for each stock. For hedging with three SSF contracts, half of the spot stocks show at least a 15% variance reduction and 297 stocks (80%) show a better performance than that of hedging with a single SSF.

Table 3 summarizes the average variance reduction across the sample of 350 stocks. We find that the best approach is to use three SSF contracts selected on the basis of both return correlation and firm characteristics, to adjust the hedge ratio throughout the sample, and to fix the SSF's employed (rebalancing method 2). In a few cases, the median is much larger the mean, indicating that there are a few large negative outliers. Such situations arise when the spot stock's price collapsed or rose spectacularly, but the hedging futures contract's price did not; or vice versa. For instance, during the out-of-sample period, three of the SSF contracts had very large price falls: SanDisk fell about 30% on October 14, 2004; AMD fell about 25% on November 1, 2004; and Biogen Idec fell almost 40% on February 28, 2005. Such events are quite rare, but are bound to happen in a sample of this size.

Industry classification: Table 3 also examines whether matching SSF contracts within the same industrial sectors as the spot stocks improves hedging efficiency. Classifying by industry improves the results when only one SSF is used for each spot stock hedged, but this improvement is less than that of moving from one to three SSF contracts without concern for industry. The problem is that for the 86 SSFs available, some industrial sectors contain no SSFs or very few. Comparing portfolios hedged with industrial classification but limited to sole SSF hedging and the portfolio hedged without industrial classification but unlimited as to the number of SSFs, the latter shows better hedging efficiency in this context.

Market index futures: While hedging with SSF may reduce firm specific risk because we hedge with a SSF similar to the spot asset, market risk will remain. Hence, it is possible that hedging efficiency can be further improved by hedging with market index futures. From Figure 4, it can be seen that controlling for market risk as well does indeed improve the variance reduction. Adding market index futures to the hedging model with three SSF, leads to an improvement in variance reduction for half of the spot stocks from at least 14% to at least 21%.

Figure 4 illustrates that hedging with only market index futures shows a better performance than hedging with both market index futures and three SSF contracts. Since this may arise from noise caused by the use of so many hedging contracts, we examine the hedging model with market index futures plus a single SSF matched by return correlation, firm characteristics and industry sector. Hedging with index futures and one SSF shows a slightly better performance than hedging with market index futures alone (p-value = 0.07, one-sided paired t-test).

Table 4 provides the average reduction in variance across the sample of stocks for the different hedging models. This table corresponds to Table 3, except that the hedging is now done with market index futures as well as the SSF contracts. We find that hedging with market index futures is effective, but that improvements can be made by using both index futures and one SSF contract from the same industry as the spot stock.

Optimal hedging model: To summarize, the best hedging performance is achieved through a portfolio that is hedged with market index futures and a SSF matched both by historical return correlation and by cross-sectional matching characteristics, keeping the chosen SSF contract for the whole out-of-sample period and using the optimal hedge ratio re-estimated for each rolling window. For the best performing model, half of the spot stocks show at least a 21% reduction in variance of returns and the best hedging model reduces the hedged portfolio variance for 94% of spot stocks relative to no hedging. For interest, in terms of variance reduction, *Commercial Federal Corp,* is the stock whose return movements can be hedged most effectively – the variance of payoff is reduced 54%. Its matched SSF is *Wells Fargo & Co*, which is in the same 'Financials' sector.

7 Conclusions

Investors holding positions in individual stocks may wish to hedge using futures contracts, but it would be necessary for them to cross hedge (or to hedge with a stock index) in the likely situation that there exists no futures contract on the spot stock(s) that they hold. But the appropriate method for selecting the optimal futures contract is not obvious. Thus, this study examines the use of sample matching techniques together with fundamental firm characteristics for cross hedging with single stock futures. Since individual stocks have very different characteristics from one another, the efficiency of cross-hedging using futures whose underlying asset differs from the spot stock may have been expected to be low.

We show that hedging efficiency can be improved by using industrial classification to control for industry-specific effects or by using additional SSF contracts to obtain additional diversification. Overall, matching the industry of the SSF and spot stock is more important than the use of multiple SSF for hedging efficiency. In addition, eliminating market risk is at least as important as eliminating firm specific risk. Thus, hedging with market index futures as well improves hedging effectiveness compared to hedging with only SSF contracts.

Our empirical results suggest that while single stock futures have much potential for hedging firm-specific risk, they still have far to go before they become a viable alternative to other traditional methods of hedging. Most of the SSF contracts currently in existence are written on larger blue chip stocks. But these stocks already have many viable hedging alternatives and are highly correlated with the market index. Our results suggest that writing exchange-traded SSF contracts on smaller, more diverse companies may be better suited for investor cross-hedging needs – specifically, the firms underlying these contracts will be closer matches (in terms of size and other firm characteristics) to the many small companies that lack other suitable derivative products. By aiming to "complete the market" rather than duplicate it, SSF exchanges may be better able to foster growth. To be fair, the results reported in this paper probably underestimate the true effectiveness of our methods in practice. There are now more than twice as many firms with SSF contracts written on them as used in this study. As the number of available SSF contracts increases, hedgers will be able to more closely match firm characteristics and thus further increase hedging efficiency.

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Table 1: Final sample of Single Stock Futures (SSF) contracts. The table lists our final sample of SSF contracts written on 86 underlying stocks, listed by abbreviated firm name and ticker symbol (in parenthesis). The table also reports the average daily open interest and the average daily volume in contracts (each contract represents 100 shares of the underlying security) of the nearest contract on a quarterly expiration cycle. Results are based on the period September 2, 2003 to March 31, 2005.

Underlying Stock	**Open Interest**	**Volume**	**Underlying Stock**	**Open Interest**	**Volume**
Alcoa Inc. (AA)	806	27	KLA-Tencor Corp. (KLAC)	586	36
American Inter. Group (AIG)	1,877	33	Coca-Cola Co. (KO)	1,570	72
Altera Corporation (ALTR)	1,260	47	Linear Technology (LLTC)	634	33
Applied Materials (AMAT)	1,703	77	McDonald's Corp. (MCD)	3,736	99
Amgen Inc. (AMGN)	1,755	71	Merrill Lynch & Co. (MER)	833	25
Amazon.com, Inc. (AMZN)	224	22	3M (MMM)	675	24
American Express (AXP)	2,333	58	Altria Group (MO)	1,808	78
Boeing Co. (BA)	1,095	23	Motorola, Inc. (MOT)	124	17
Bank of America Corp. (BAC)	2,369	87	Merck & Co., Inc. (MRK)	643	20
Bed Bath & Beyond (BBBY)	1,251	48	Microsoft Corp. (MSFT)	4,792	153
Best Buy Co., Inc. (BBY)	284	10	Micron Tech., Inc. (MU)	164	13
Biogen Idec Inc (BIIB)	82	12	Morgan Stanley (MWD)	1,233	25
Bristol-Myers Squibb (BMY)	1,240	35	Maxim Integ. Prod. (MXIM)	837	37
Brocade Comm. Sys. (BRCD)	258	15	Newmont Mining (NEM)	165	15
Broadcom Corp. (BRCM)	247	33	Northrop Grumman (NOC)	1,253	42
Citigroup, Inc. (C)	1,663	50	NVIDIA Corp. (NVDA)	991	41
Caterpillar, Inc. (CAT)	388	21	Novellus Systems (NVLS)	711	28
Cephalon, Inc. (CEPH)	127	14	Nextel Comm., Inc. (NXTL)	1,652	66
Comcast Corp. (CMCS)	2,295	52	Oracle Corp. (ORCL)	142	8
Comverse Tech., Inc. (CMVT)	1,058	43	Pepsico, Inc. (PEP)	779	31
Cisco Systems, Inc. (CSCO)	1,667	57	Pfizer, Inc. (PFE)	2,610	80
ChevronTexaco Corp. (CVX)	1,403	40	Procter & Gamble Co. (PG)	1,461	42
E.I. du Pont de Nemours (DD)	2,470	67	Qualcomm Inc. (QCOM)	1,002	52
Dell Inc. (DELL)	2,984	94	QLogic Corp. (QLGC)	604	24
Walt Disney Co. (DIS)	1,707	54	SBC Communications (SBC)	2,264	84
Dow Chemical Co. (DOW)	992	38	Starbucks Corp. (SBUX)	1,699	53
eBay Inc. (EBAY)	578	51	Siebel Systems, Inc. (SEBL)	59	5
Eastman Kodak Co. (EK)	1,286	73	Schlumberger N.V. (SLB)	2,003	70
Emulex Corp. (ELX)	1,225	30	SanDisk Corp. (SNDK)	193	23
Ford Motor Co. (F)	48	3	Sun Microsystems (SUNW)	870	44
General Electric Co. (GE)	2,909	99	Symantec Corp. (SYMC)	637	34
Genzyme General (GENZ)	317	12	AT&T Corp. (T)	43	2
General Motors Corp. (GM)	1,466	71	Time Warner Inc. (TWX)	60	4
Goldman Sachs (GS)	558	23	Texas Instruments (TXN)	695	33
Halliburton Co. (HAL)	2,736	73	Tyco International (TYC)	1,503	49
Home Depot, Inc. (The) (HD)	567	18	United Technologies (UTX)	673	32
Honeywell Inter. (HON)	2,972	76	Veritas Software (VRTS)	120	9
Hewlett-Packard Co. (HPQ)	2,012	57	Verizon Comm. (VZ)	2,563	84
IBM Corp. (IBM)	1,897	54	Wells Fargo & Co. (WFC)	2,721	69
Intel Corp. (INTC)	1,293	58	Wal-Mart Stores (WMT)	1,467	38
International Paper Co. (IP)	2,020	44	Xilinx, Inc. (XLNX)	1,189	45
Johnson & Johnson (JNJ)	2,250	59	Exxon Mobil Corp. (XOM)	2,695	68
J.P. Morgan Chase (JPM)	2,414	66	Yahoo! Inc. (YHOO)	362	30

Table 2. Matching characteristics of the final sample of spot stocks and single stock futures (SSF) contracts. The matching characteristics (market capitalization, CAPM Beta, and the price to book ratio) are obtained from Datastream on December 31, 2003.

		Spot stocks to be hedged	Underlying stocks of single stock futures contracts
Number of firms		350	86
Market Capitalization (millions of dollars)	**Max**	6,698	311,755
	Min	86	1,495
	Mean	959	64,461
CAPM Beta	**Max**	2.887	2.893
	Min	0.004	-0.213
	Mean	0.509	1.333
Price / Book	**Max**	68.05	18.73
	Min	-32.42	-61.85
	Mean	2.93	2.82

Table 3. Variance reduction of hedging models with using matched single stock futures (SSF) only. The cross-sectional matching characteristics used are beta, market capitalization, and price to book ratio. The three rebalancing methods are: 1. Keep both SSF and hedge ratio fixed during the whole out-of-sample period. 2. Keep the matched SSF, but re-estimate the hedge ratio and rebalance the portfolio every day during the out-of-sample period. 3. Re-match the SSF, re-estimate the hedge ratio and rebalance the portfolio every day during the out-of-sample period.

Model		**Rebalancing method 1**				**Rebalancing method 2**				**Rebalancing method 3**			
		1 SSF in any industry	**1 SSF in same industry**	**2 SSFs**	**3 SSFs**	**1 SSF in any industry**	**1 SSF in same industry**	**2 SSFs**	**3 SSFs**	**1 SSF in any industry**	**1 SSF in same industry**	**2 SSFs**	**3 SSFs**
A	**Matching with price correlation only**												
	Mean	8.88	8.79	12.51	13.70	9.46	9.24	13.06	14.21	8.51	8.75	12.35	13.73
	Median	9.72	9.38	13.43	14.52	9.74	9.10	13.50	14.86	8.55	8.56	13.00	14.10
	Max	40.35	38.98	45.47	47.35	41.79	40.86	47.47	49.30	41.79	40.05	49.22	51.38
	Min	-73.72	-73.93	-55.65	-55.43	-76.82	-35.27	-61.81	-62.19	-76.82	-27.41	-65.40	-62.94
B	**Matching with cross-sectional characteristics only**												
	Mean	2.11	8.81	4.99	8.35	1.98	8.91	4.85	8.66				
	Median	1.61	8.02	4.84	8.44	1.56	8.07	4.45	8.05				
	Max	20.25	38.98	25.54	30.37	19.95	40.86	24.30	29.35				
	Min	-28.95	-11.86	-20.61	-39.52	-27.16	-21.85	-21.17	-33.93				
C	**Matching with both correlation and cross-sectional characteristics**												
	Mean	8.30	9.14	12.39	14.14	9.00	9.33	12.85	14.48	7.76	7.96	12.11	13.74
	Median	9.59	9.64	13.47	14.67	9.27	9.38	13.58	14.61	7.74	7.88	12.01	14.23
	Max	38.98	38.98	45.47	47.35	40.86	40.86	47.47	49.30	42.32	39.20	47.47	49.30
	Min	-73.72	-34.76	-65.16	-58.24	-76.82	-22.81	-69.58	-64.73	-76.82	-66.70	-68.82	-58.02

Table 4. Variance reduction for hedging models with single stock futures (SSF) and market index futures. The cross-sectional matching characteristics used are beta, market capitalization, and price to book ratio. The three rebalancing methods are: 1. Keep both SSF and hedge ratio fixed during the whole out-of-sample period. 2. Keep the matched SSF, but re-estimate the hedge ratio and rebalance the portfolio every day during the out-of-sample period. 3. Re-match the SSF, re-estimate the hedge ratio and rebalance the portfolio every day during the out-of-sample period.

Model		**Rebalancing method 1**				**Rebalancing method 2**				**Rebalancing method 3**			
		1 SSF in any industry	**1 SSF in same industry**	**2 SSFs**	**3 SSFs**	**1 SSF in any industry**	**1 SSF in same industry**	**2 SSFs**	**3 SSFs**	**1 SSF in any industry**	**1 SSF in same industry**	**2 SSFs**	**3 SSFs**
A′	**Market index futures +SSF: matching with price correlation only**												
	Mean	19.07	19.95	18.56	17.89	20.19	20.72	19.96	19.42	19.61	20.40	19.10	18.46
	Median	20.46	20.50	19.77	18.53	20.74	20.68	20.62	19.90	20.26	20.25	19.21	19.11
	Max	53.64	53.64	52.29	52.00	54.39	54.39	53.58	53.38	54.05	54.05	56.28	55.86
	Min	-57.42	-17.29	-54.78	-54.82	-66.99	-15.32	-66.88	-68.17	-66.99	-15.32	-69.94	-68.94
B′	**Market index futures + SSF: matching with cross-sectional characteristics only**												
	Mean	19.61	20.30	19.35	18.93	20.22	20.74	19.96	19.60				
	Median	19.83	20.37	19.62	19.12	20.19	20.61	20.00	19.55				
	Max	52.51	54.30	52.44	52.80	52.83	54.12	52.56	52.80				
	Min	-6.21	-5.14	-9.21	-25.82	-5.05	-4.90	-5.95	-17.95				
C′	**Market index futures + SSF: matching with both correlation and cross-sectional characteristics**												
	Mean	19.50	20.24	19.13	18.63	20.57	20.93	20.26	19.84	19.81	20.53	19.48	18.92
	Median	20.34	20.37	20.15	19.27	20.71	20.78	20.76	20.08	20.09	20.48	19.64	18.77
	Max	54.30	54.30	52.14	53.03	54.12	54.12	52.85	53.86	53.86	51.69	54.07	53.97
	Min	-57.42	-17.29	-57.54	-55.41	-66.99	-15.32	-66.94	-68.40	-66.99	-15.32	-67.89	-63.57
α	**Market index futures only**												
	Mean	20.07				20.59							
	Median	20.25				20.74							
	Max	52.98				53.44							
	Min	-6.19				-5.36							

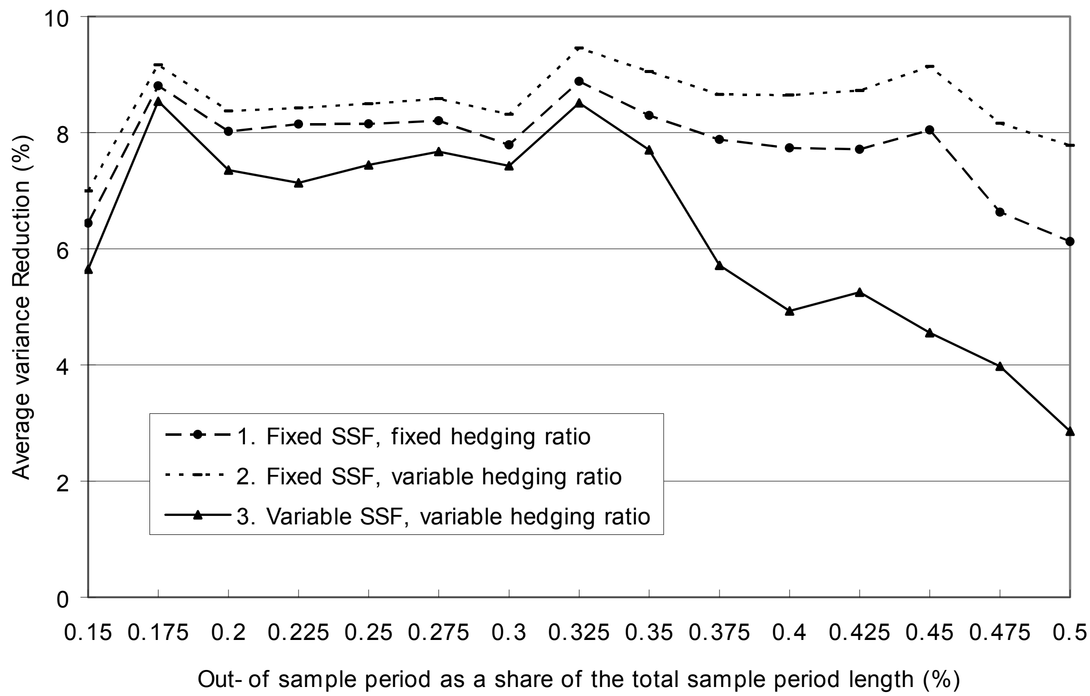

Figure 1. Average variance reduction: comparison of rebalancing procedures. Average reduction in variance of returns of 350 spot stocks hedged with a SSF against the unhedged case. Out-of-sample periods are presented on the x-axis as a portion of the total sample period with a fixed end point and a variable starting point. Key: 1. Keep both SSF and hedge ratio fixed during the whole out-of-sample period. 2. Keep the matched SSF, but re-estimate the hedge ratio and rebalance the portfolio every day during the out-of-sample period. 3. Re-match the SSF, re-estimate the hedge ratio and rebalance the portfolio every day during the out-of-sample period.

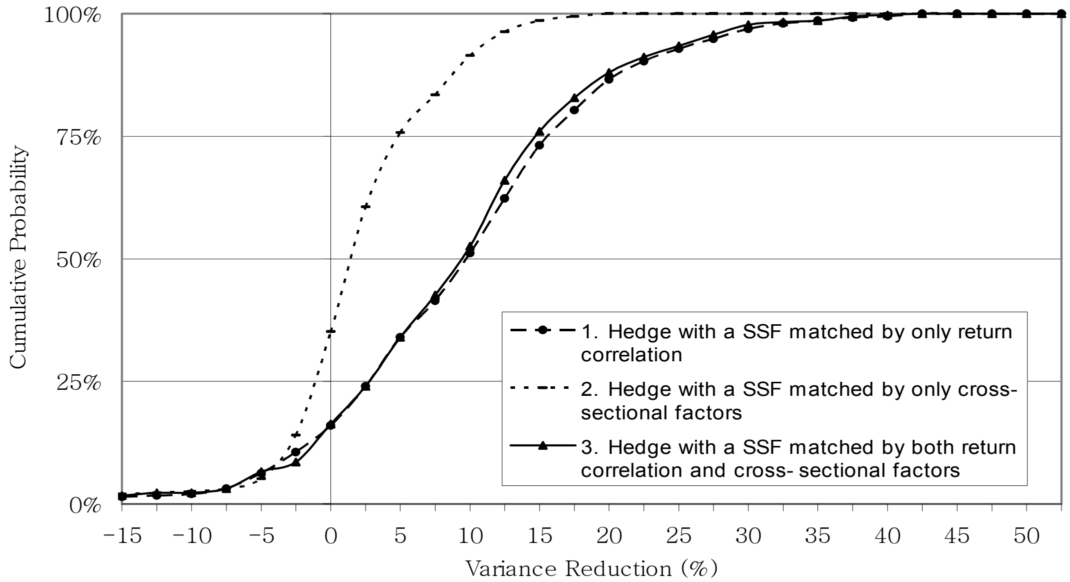

Figure 2. Variance reduction: comparison of matching characteristics. The figure shows the cumulative probability distribution of the reduction in variance of returns of 350 spot stocks hedged with a SSF against the unhedged case. Key: 1. Hedge with a SSF matched by only a historical return correlation. 2. Hedge with a SSF matched by only cross-sectional matching characteristics. 3. Hedge with a SSF matched by both a return correlation and cross-sectional matching characteristics.

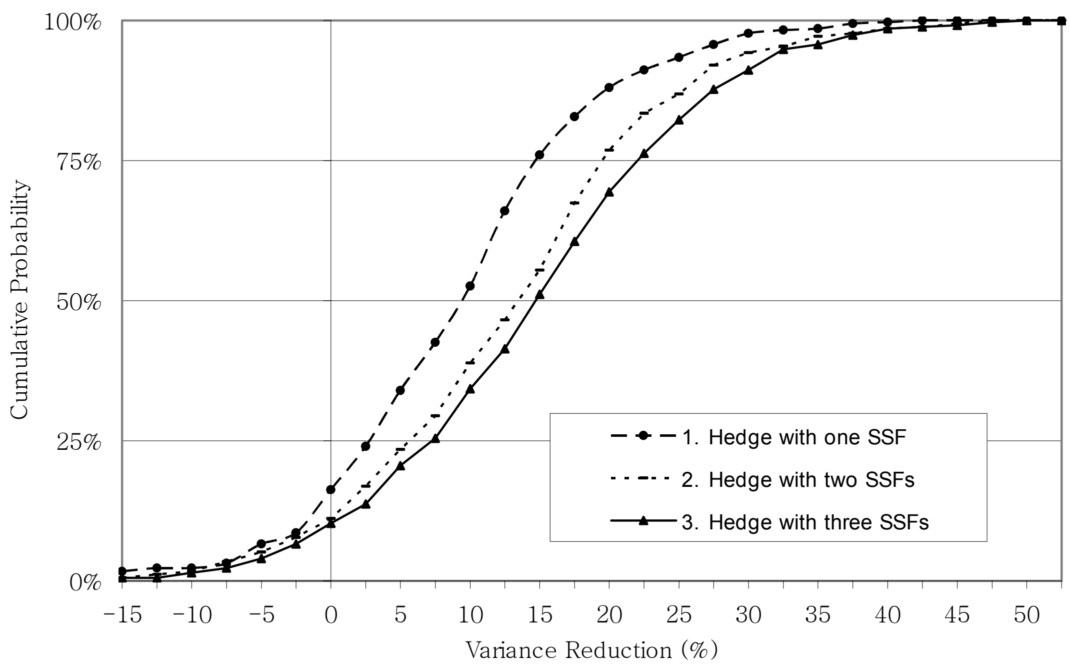

Figure 3. Average variance reduction and the number of SSF contracts. Cumulative probability distribution of reduction in variance of returns of 350 spot stocks hedged with SSF against the unhedged case. Each SSF is matched with both historical correlation and cross-sectional matching characteristics.

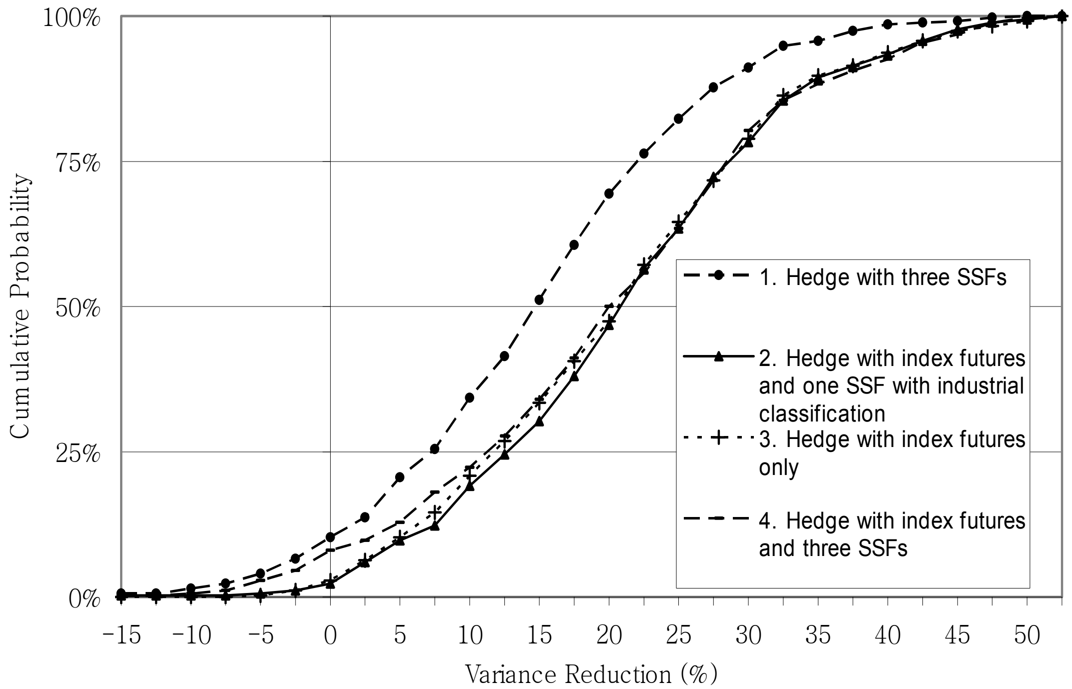

Figure 4. Average variance reduction: hedging with and without market index futures. Average of reduction in variance of returns for 350 spot stocks hedged with SSF against the unhedged case. Key: 1. Hedge with three SSF matched by both a historical return correlation and cross-sectional matching characteristics. 2. Hedge with market index futures and a SSF matched by both a historical return correlation and cross-sectional matching characteristics with industrial classification. 3. Hedge with market index futures only. 4. Hedge with market index futures and three SSF matched by both a historical return correlation and cross-sectional matching characteristics.