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Spatial Economics: Density, Potential, and Flow

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Mathematical economics studies economies, usually markets, with a finite number of commodities in the standard case. If the commodities are differentiated by time of delivery, one enters the special field of dynamic economics. If they are differentiated by location, one is in the realm of spatial economics. Standard mathematical economics is mature now. Dynamic economics, particularly growth theory, has achieved a high standard in a relatively short time. Spatial economics, particularly location theory, has progressed slowly and is still in its infant stage. The book reviewed now fills a gap.

Spatial economists have the unfortunate tendency to substitute models at an even higher rate than macro economists and the present authors are in good standing with their peers. To their credit, however, it must be said that a considerable part of the book is about a single model, the so called market model, which will be reviewed in detail.

The market model consists of two equations for the commodity flow vector field, ϕ, and the price distribution, λ, plus boundary conditions. Given are excess demand, $q(\lambda, x)$, where x is location in a two dimensional area A, and local transportation cost, $k(x)$. q is assumed to be local in that it depends on local price, $\lambda(x)$. k is assumed to be independent of direction. The first equation is presented as a noneconomic, physical constraint (p. 16), but nonetheless reflects the assumption that inventory investment is not done, so that local excess supply (the negative of demand) must be matched by a net outflow (divergence):

$$-q(\lambda, x) = \operatorname{div} \phi(x).$$

The second equation is very stringent as it represents, in addition to the profit maximization condition imposed on all goods, the free entry condition of zero profits (p. 17):

$$k\phi = \|\phi\| \operatorname{grad} \lambda.$$

The direction of this vector equation indicates that commodities flow in the direction of steepest price increase (gradient); the modulus shows that the price increase matches transportation cost. The boundary condition reflects the assumption that no flow enters or leaves area A.

Initially, in Chapter 2, the dependence of excess demand on price is suppressed. Walras' Law, that values of excess demand are nonpositive (because of budget constraints), is not observed, but some insights in the working of the model are obtained. Basically, commodities flow from sources of excess supply to sinks of excess demand according to the divergence law and prices direct them passively according to the gradient law. It is shown that the model is equivalent to the mathematical program of minimizing total transportation costs subject to the divergence and boundary constraints. The Lagrangean multipliers and the first order conditions turn out as the prices and the gradient law. The Kuhn–Tucker minimax theorem is 'extended' to this case with a continuum of linear constraints by casual discussion. Reference to the John (1948) classic is missing. Existence and a very partial uniqueness result are established. Alternative boundary conditions and transportation cost simplifications are shown to reproduce the elementary boundary value problems of potential theory. In full ordain, spatial economics is more difficult.

The next chapter (3) restores price dependence of excess demand. The first theorem claims that for negative dependence any equilibrium price distribution is unique (p. 73). "One possible solution is that where markets are balanced locally at prices whose gradients do not permit profitable arbitrage. Then there is no integrated spatial market but merely a system of locally autarchic markets. As transportation cost declines through technical progress such a system may become an integrated spatial market." I am afraid though that the autarchic markets solution persists. (Simply put $\phi = 0$ and solve the standard economic problem $q(\lambda, x) = 0$ for equilibrium price $\lambda(x)$.) Thus it seems that either the uniqueness theorem is false or integrated spatial markets may not emerge. The proof of the theorem offers no clue due to sloppiness (in the transition from (3a) to the next equation). Equivalence to a mathematical program is again established, generalizing a result of Samuelson. A number of special cases with supply derived from Leontief or Cobb–Douglas technologies is investigated. For such constant returns to scale technologies, a linear programming result generalizes a land use specialization result of von Thünen.

To analyze the stability of solutions, the authors propose adjustment rules for price as well as flow. The latter adjustment is an unjustified augmentation to the ordinary market model in which quantities (including shipments) are set by price-takers and the economy is equilibrated purely by the price mechanism. Consequently the order of Beckmann–Puu's adjustment equations is one too high. Revision of the specification of the process would reduce their wave equation to a diffusion process.

Chapters 4 and 5 redo the equilibrium analysis and its planning counterpart for excess demand derived from specific production and utility functions and introduce a rationale for trade through one single mapping that associates locations of households to locations of property. Structurally stable flows and land use specialization are discussed in an interesting synopsis, loosely related to the specification of the equations at hand though. Chapters 6 and 7 consolidate a number of location and traffic theoretic problems in the authors' framework. As in linear programming, corner solutions are often established and interpreted in the infinite dimensional setting. The last chapter is the only one on a truly dynamic model, presenting a spatial version of the Samuelson-Hicksian multiplier-accelerator model of income and trade. Net exports have a multiplier effect on income and the income gradient of the latter accelerates net exports. A partial differential equation of the second order is derived and standing eigensolutions are determined. Incidentally, a full analysis would reveal that the equation is essentially Klein–Gordon and yield the complete solution to the model.

Spatial Economics specifies economic laws hastily, organizes theorems poorly and derives them sloppily. This is the price we have to pay. The return is a wealth of insightful ideas to develop the subject in a unified manner. Beckmann and Puu's book should be on the shelf of every spatial economic theorist, as well as of students of vector analysis and other methods of mathematical physics who want to probe an unknown field.

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Logistiksysteme: Betriebswirtschaftliche Grundlagen

Logistik in Industrie, Handel und Dienstleistungen, Springer, Berlin, 1985, viii + 250 pages, DM42.00

A more than 10 year old development of the conception and notion of "logistics" (physical distribution management, Materialwirtschaft) is summarized in this book. "Logistics" covers:

- storing, handling, transporting (the main processes of product flow),
- packing and marking (support processes),
- order transacting (information processes).

These actions ensure in their cooperation that the right product in the right condition at the right time at the right place to minimal costs is available (the four "r").

Part A explains the idea of logistics and its consequences: system thinking, total cost thinking, service thinking, efficiency thinking. Here many aspects, hitherto considered isolated, are being considered in their total connection and new problems are formulated for OR specialists. That is the industrial management frame for the upper models (Obermodelle) von Schneeweiss [1]. Part B states the requirements of the subsystems: order transacting, storing (when and how much to order?), warehouse organization, packing, transporting. Part C discusses possible management structures for logistic processes.

The book is to be recommended for everybody who intends to overcome the limited view of classical OR-models for inventory, transport and lot size problems and wants to get to know industrial management requirements from a total view. Many of these requirements are still day-dreams; the OR