1. Introduction

Several empirical studies have rejected the restrictions implied by Hall's version of the life-cycle model. Liquidity constraints and preference interactions between goods and leisure have alternatively been put forward as likely explanations of this failure.

As Zeldes (1985) and others point out, the Euler equation for consumption with borrowing restrictions involves an additional unobservable variable $\mu_r$, the Kuhn-Tucker multiplier associated with the net wealth constraint. Different methods have been proposed in the literature to tackle the observation problem of $\mu_r$. Most of these methods are not quite satisfactory, because they rely on very simple rules of thumb or on usually unavailable sample separation information about the liquidity constrained status of the household [cf. Zeldes (1985)].

In a theoretical paper Alessie, Melenberg and Weber (1988) (AMW from now on) show that if borrowing restrictions depend on earnings, preferences are non-separable between goods and leisure, and individuals are employed, one can derive an Euler equation involving observable variables only. In section 2 we will briefly review this study. It appears that, in contrast with the non-earnings dependent liquidity constraint case, for this model the well-known two stage budgeting rule in terms of 'full expenditures', i.e. the sum of consumption expenditures and expenditures on leisure [cf. Blundell and Walker (1986)] is not valid any more. However, a two stage budgeting rule in terms of the pure consumption goods, conditional upon the choice of leisure can be obtained. In our model we exploit this property by specifying a

*The authors thank G. Kock for research assistance and M. Kerkhofs for helpful comments.
rationed intratemporal indirect utility function of the RAIDS type [cf. Deaton (1981) and Ioannides (1986)] with total consumption expenditures, leisure and consumption prices as arguments.

Our estimation model consists of two parts. The first part corresponds to the Euler equation mentioned above, describing the first stage allocation. For estimation the availability of panel data is required. The parameters of the Euler equation are estimated by means of the Generalized Method of Moments (GMM) proposed by Hansen and Singleton (1982). The second part deals with the RAIDS demand system describing the within period allocation of total consumption expenditures to the different commodity groups as a function of the within period consumption prices and leisure. The conditioning variable leisure appears because preferences are weakly non-separable in consumption and leisure [e.g., Pollak (1969, 1971)]. This observation also suggests a straightforward test of separability, see Meghir and Browning (1988) for details. Since total consumption expenditures and leisure are decision variables, this demand system needs to be estimated by means of instrumental variables methods.

2. The model

Consider a single consumer (or household), who has to plan consumption and labor supply from the present period \( t \) up to a terminal period \( L \) in an uncertain environment. Like AMW we assume that the individual faces liquidity constraints which depend on earnings. Consequently the consumer chooses leisure and a consumption bundle by solving the following problem:

\[
\max E, \sum_{t=1}^{L} \frac{1}{(1+r)^t} u(q_t, l_t) \tag{1a}
\]

s.t.
\[
A_t = (1+r)A_{t-1} + m_t + w_t(T-l_t) - p_t q_t, \quad \tau = t,...,L \tag{1b}
\]
\[
A_t \geq M_t = \Phi_0 + \Phi_1 w_t(T-l_t), \quad \tau = t,...,L-1 \tag{1c}
\]
\[
l_t \leq T, \quad \tau = t,...,L \tag{1d}
\]
\[
A_{t-1} \text{ given, } \quad A_L \geq 0. \tag{1e}
\]

where \( u_t(q_t, l_t) \) is the intratemporal utility function in period \( t \), strictly concave and monotonically increasing in its arguments; \( q_t \) is a bundle of commodities in period \( t \); \( l_t \) is leisure in period \( t \); \( p_t \) is the price vector in period \( t \); \( w_t \) is the wage rate in period \( t \); \( m_t \) is non-labor income in period \( t \); \( A_t \) is the value of assets at the end of period \( t \); \( r \) is the interest rate. One
expects the borrowing limit to be inversely related to current earnings, i.e., $\Phi_1 < 0$.

One can derive a two-stage budgeting result in terms of the pure consumption goods, conditional upon $l_\tau$, $\tau = t, \ldots, L$, as follows. Rewrite (1):

$$\max_{x_t} E_t \sum_{\tau=t}^L \frac{1}{(1+\rho)^{t-\tau}} \Psi_t(x_t, l_\tau, p_\tau)$$

s.t. $A_t = (1+r)A_{t-1} + m_t + w_t(T-l_\tau) - x_t$

and (1c), (1d), (1e), where

$$\Psi_t(x_t, l_\tau, p_\tau) = \max_{q_t} \{ u_t(q_t, l_t); p'_t q_t = x_t \}$$

is the rationed indirect utility function in period $\tau$, strictly concave in $x_t$ and $l_\tau$, and where $x_t$ is total consumption expenditures in period $\tau$. In the discussion below, the cardinal period specific indirect utility function is parameterized as

$$\Psi_t(x_t, l_\tau, p_\tau) = F_t(\Psi^*_t(x_t, l_\tau, p_\tau), l_\tau).$$

where $F_t(\cdot)$ is a monotonically increasing function in both its arguments and $\Psi^*_t(\cdot)$ possesses all the conventional properties of a utility function. The choice of the monotonic transformation is irrelevant in static analysis. However, in case of models such as (1) the dynamic properties of the model (e.g. the value of the elasticity of intertemporal substitution) crucially depend on the functional form of $F_t(\cdot)$.

The first order conditions for period $t$ are

$$\frac{\partial \Psi_t(x_t, l_\tau, p_\tau)}{\partial x_t} = \lambda_t,$$

$$\frac{\partial \Psi_t(x_t, l_\tau, p_\tau)}{\partial l_\tau} = \lambda_t w_t - \Phi_1 \mu_t w_t + \nu_t,$$

$$\lambda_t - \mu_t = E_t \frac{(1+r)}{(1+\rho)} \lambda_{t+1},$$

$$\mu_t(A_t - M_t) = 0; \quad \nu_t(T - l_t) = 0; \quad \mu_t \geq 0; \quad \nu_t \geq 0.$$

The variables $\lambda_t, \lambda_{t+1}$ denote the Lagrange multipliers associated to (1b'),
whereas \( \mu_t \) and \( \nu_t \) are the Kuhn-Tucker multipliers corresponding to the borrowing and the time constraints, (1c) and (1d), respectively.

We can rewrite the Euler equation (5) by using (3). The result is

\[
\frac{(1+r) \cdot \partial \Psi_t(x_{t+1}, l_{t+1}, p_{t+1})}{\partial x_t} = \frac{\partial \Psi_t(x_t, l_t, p_t)}{\partial x_t} - \mu_t + \epsilon_{t+1},
\]

(7)

where the error \( \epsilon_{t+1} \) has zero mean conditional on all information available in period \( t \).

For estimation purposes, eq. (7) is unsatisfactory in that it contains the unobservable, endogenous variable \( \mu_t \). In general we do not observe when the constraint is binding, i.e. when \( \mu_t \) is non-zero. However, until now we have not used the information that the borrowing limit is earnings dependent. This information allows us to get an Euler equation in terms of observable variables as follows: use the first order conditions (3) and (4) to obtain an expression for \( \mu_t \), and then substitute this expression into (7) to obtain

\[
\frac{(1+r) \cdot \partial \Psi_t(x_{t+1}, l_{t+1}, p_{t+1})}{\partial x_t} = \left( -\frac{1}{\phi_t} \right) \frac{\partial \Psi_t(x_t, l_t, p_t)}{\partial x_t} + \frac{1}{\phi_t} \frac{\partial \Psi_t(x_t, l_t, p_t)}{\partial l_t} - \nu_t + \epsilon_{t+1}.
\]

(8)

We have thus obtained an Euler equation wherein \( \mu_t \) does not appear. In its place, we now have the Kuhn-Tucker multiplier on leisure, \( \nu_t \), which is going to be positive when a corner solution obtains in the labour market, and zero otherwise.

If panel data on individual households are available, we can estimate the parameters of eq. (8) by GMM, by restricting the sample to the employed in period \( t \); this does not cause selection bias, because the error \( \epsilon_{t+1} \) is orthogonal to the selection rule (as \( \nu_t \) belongs to the relevant information set).

Next to the Euler equation (8) we want to estimate the following conditional demand system which explains the within-period allocation of the total consumption expenditures to the different commodities:

\[
q_t = g_t(x_t, l_t, p_t) = \frac{-\partial \Psi_t^*(x_t, l_t, p_t)/\partial l_t}{\partial \Psi_t^*(x_t, l_t, p_t)/\partial x_t},
\]

(9)

where the last equality follows from Roy’s identity.

In order to identify all parameters of interest, one generally needs to estimate both (8) and (9). It would be efficient to estimate (8) and (9) jointly. Since the Euler equation (8) is in general highly non-linear, there are
substantial computational advantages to the following procedure. First estimate the second stage demand system (9). This identifies all parameters of the ordinal utility function. Next the remaining parameters are estimated by using (8). Although this is not fully efficient, it has the merit that the second stage parameter estimates are not affected by possible misspecification of the borrowing constraints or of the cardinal specification $F_c$.

3. Specification of the model

Suppose $\Psi^*(\cdot)$ in formula (2) can be described by the Rationed Almost Ideal Demand System of Deaton (1981)

$$\Psi_t^*(x_t, l_t, p_t) = \frac{\ln x_t - \ln a_t(p_t, l_t)}{b(p_t)}, \text{ where}$$

$$\ln a_t(p_t, l_t) = \alpha_0 + \sum_{i=1}^I (\alpha_i + \eta_i h(l_i)) \ln p_{it} + \frac{1}{2} \sum_{i=1}^I \sum_{j=1}^J \gamma_{ij} \ln p_{it} \ln p_{jt},$$

$$b(p_t) = \prod_{i=1}^I p_{it}^{\beta_i},$$

$h(\cdot)$ is some function of leisure (we only consider the logarithm and a linear specification), $I$ is the number of consumption goods. On the parameters rest well-known symmetry and adding-up restrictions [e.g., Deaton and Muellbauer (1980)].

The demand system for period $t$ now has the following form:

$$s_{it} = \alpha_i + \eta_i h(l_i) + \sum_{j=1}^J \gamma_{ij} \ln p_{jt} + \beta_i (\ln x_t - \ln a_t(p_t, l_t)), \quad (11)$$

where $s_{it}$ is the budget share of consumption good $i$ in period $t$.

For the functional form of the monotonic transformation $F_t(\Psi_t^*(\cdot), l_t)$ we consider the following specification:

$$\Psi_t(x_t, l_t, p_t) = F_t(\Psi_t^*(\cdot), l_t) = \exp[(1 - \gamma)(\Psi_t^*(\cdot) + \theta_1 \ln l_t)]. \quad (12)$$

This intratemporal utility function is basically a constant relative risk aversion utility function. In this case the Euler equation (8) is given by

$$\Psi_{t+1}(x_{t+1}, l_{t+1}, p_{t+1})(1+r)$$

$$x_{t+1} b(p_{t+1})(1+r)$$
where \( h' \) is the derivative of the function \( h \).

4. Estimation of the second stage

The data used come from the so-called 'Intomart consumer expenditure panel', which is a panel of households in The Netherlands for which consumption expenditures of all members over 12 years of age are registered continually and for which income, demographics, labor supply, etc. are measured once a year. In this paper we use annual aggregates, so that the time unit on which the utility function is defined is a year. The data pertain to the period April 1984–April 1987. The numbers of observations in the respective years are: 1984: 265, 1985: 302, 1986: 304. The limited number of periods on which observations are available makes it impossible to estimate the \( y_s \) in the second stage model (11), because there is not enough price variation in three years time. The terms involving the \( y_s \) are lumped together in a year-specific intercept. In the theoretical framework sketched above no allowance for durables has been made: we have assumed that the intratemporal preferences are additively separable between durables and non-durables. Hence durables do not enter into the equations for non-durables. Below ‘total expenditures’ are defined as expenditures on non-durables only. ‘Leisure’ is defined as leisure of the head of the household, because in the borrowing constraint the partner’s leisure is not expected to be very important, in keeping with the institutional framework in The Netherlands.

In the estimation, the labor market behavior of the partner (if any) of the head of household is taken exogenous and the partner’s income is part of unearned income of the household. The price index \( a_t \) is replaced by a consumer price index. The variables total expenditures and leisure have been instrumented linearly by the following variables: logarithm of unearned income; the same variable multiplied by log-family size; five education dummies; log-family size; the variables representing the number of children in various age brackets (0–6, 6–12, 12–18); log-age of head of household; a dummy for the age of the head of household being over 65; dummies for the size of the town of residence; log-squared of age of head of household. The parameters \( \alpha_t \) are parameterized by making them dependent on some of the same variables (cf. table 1) and an additive error term. The instrument equations allow for random individual effects and have been estimated by GLS. Given the parameterization of the \( \alpha_t \), eq. (11) represents a system of seemingly unrelated regressions. Also here we have allowed for the possibility
of random individual effects. Taking into account the correlation of the errors resulting from this and the endogeneity of leisure and total expenditures, the estimation method basically amounts to 3SLS. In deriving the estimators one has to take account of the fact that not all households have participated in the panel during the whole period [cf. Hsiao (1986)]. We have assumed that data are missing randomly, so that no correction for selectivity bias is necessary. Estimates of the parameters of most interest in (11) are given in table 1. For reasons of space we only give results for the specification with $\log l$; the results for $l$ being similar.

A test for homotheticity of preferences amounts to a test of all $\beta$s being zero. This can be tested straightforwardly by a Wald test. We find $\chi^2(6) = 45.5$ which indicates rejection of the null at any reasonable level of significance. Similarly, weak separability of preferences for leisure and consumption can be investigated by testing for joint null-ness of the coefficients of $\log$-leisure in all share equations. The $\chi^2(6)$-statistic comes out at 23.4, which also indicates decisive rejection of the null.

The parameter estimates are very much according to expectation, showing for instance that food, clothing and housing are necessities. Of particular interest of course are the estimates for $\log$-leisure. An increase in leisure (i.e. a reduction of time spent on market work) leads to a rather sizable reduction of the budget share of food. Since food expenditures also include eating out this may mean that people who work many hours in a paid job eat out more. We also observe that those who do not work as much in a paid job, spend a larger proportion of their budget on medical or legal expenses or on education. This may reflect the presence in the sample of students working

<table>
<thead>
<tr>
<th>Goods</th>
<th>log/$s$</th>
<th>DK1</th>
<th>DK2</th>
<th>DK3</th>
<th>log-age</th>
<th>log-exp.</th>
<th>log-leisure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>0.12</td>
<td>-0.04</td>
<td>-0.03</td>
<td>-0.01</td>
<td>0.10</td>
<td>-0.17</td>
<td>-0.14</td>
</tr>
<tr>
<td></td>
<td>(7.3)</td>
<td>(-3.2)</td>
<td>(-2.7)</td>
<td>(-0.7)</td>
<td>(5.4)</td>
<td>(-5.7)</td>
<td>(-2.6)</td>
</tr>
<tr>
<td>Clothing/footwear</td>
<td>0.02</td>
<td>-0.03</td>
<td>-0.004</td>
<td>0.01</td>
<td>0.003</td>
<td>-0.01</td>
<td>-0.08</td>
</tr>
<tr>
<td></td>
<td>(1.7)</td>
<td>(-4.2)</td>
<td>(-0.6)</td>
<td>(1.3)</td>
<td>(0.3)</td>
<td>(-0.7)</td>
<td>(-2.5)</td>
</tr>
<tr>
<td>Housing</td>
<td>-0.07</td>
<td>0.08</td>
<td>0.03</td>
<td>-0.01</td>
<td>-0.08</td>
<td>-0.03</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>(-3.4)</td>
<td>(4.8)</td>
<td>(1.9)</td>
<td>(-0.7)</td>
<td>(-3.6)</td>
<td>(-0.7)</td>
<td>(-2.3)</td>
</tr>
<tr>
<td>Recreation/pets</td>
<td>-0.02</td>
<td>-0.02</td>
<td>0.01</td>
<td>-0.004</td>
<td>-0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(-1.9)</td>
<td>(-3.0)</td>
<td>(1.0)</td>
<td>(-0.6)</td>
<td>(-2.7)</td>
<td>(1.3)</td>
<td>(-0.5)</td>
</tr>
<tr>
<td>Insurance prem.</td>
<td>-0.01</td>
<td>0.01</td>
<td>0.002</td>
<td>-0.01</td>
<td>0.04</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>(-0.08)</td>
<td>(1.4)</td>
<td>(0.4)</td>
<td>(-1.0)</td>
<td>(3.3)</td>
<td>(3.7)</td>
<td>(1.9)</td>
</tr>
<tr>
<td>Med. exp. etc.</td>
<td>-0.03</td>
<td>0.003</td>
<td>-0.003</td>
<td>0.01</td>
<td>-0.02</td>
<td>0.11</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>(-1.9)</td>
<td>(0.2)</td>
<td>(-0.3)</td>
<td>(1.0)</td>
<td>(-1.0)</td>
<td>(3.9)</td>
<td>(3.1)</td>
</tr>
</tbody>
</table>

Number of observations: 871

*Explanation: DK1 is the logarithm of $1 +$ (the number of children under 6); DK2 and DK3 are defined similarly, but now with the number of children 6-12, and over 12 respectively.
part-time or people with failing health who have higher medical expenses and are not able to work as much as healthy individuals.

Although for reasons of space we do not present the estimates of the variance components, it should be noted that the individual effects have variances that are on average five times larger than the variances of the white noise error terms. This not only indicates substantial efficiency gains of our estimation method, but it also shows considerable individual variation not captured by the explanatory variables in the model.

5. The first stage

Given the parameters obtained in the estimation of the second stage model, there remains only a limited number of parameters to be estimated in the first stage model. Considering (13), we see that only the parameters \( \Phi_1 \), \( \theta_1 \), \( \gamma \), and \( (1 + r)/(1 + \rho) \) remain unknown.

For the GMM-estimation of the first stage model [see Hotz, Kydland and Sedlacek (1988) for details] we need observations of households that have participated in the panel for at least two consecutive years. Given that we have three periods of observation, there are Euler equations to be estimated for two ‘transitions’: from period 1 to 2 and from period 2 to 3. We use as instruments: deflated total expenditures, wage rate, leisure, unearned income, log-family size, education level, log-age, and the right hand side variables in eq. (13).

To allow for correlated forecast errors across individuals a period specific dummy is added to the equation. The total number of observations used in the first stage estimation is equal to 124. This low number is due to the requirement that heads of households had to have a job in period \( t \). The resulting parameter estimates are as follows (with \( t \)-values in parentheses). For the specification with log \( l \): \( 1/\Phi_1 = -0.026 \) (\( -1.9 \)), \( \theta_1 = 0.007 \) (\( 2.4 \)), \( \gamma = 0.002 \) (\( 0.2 \)). For the specification with \( l \): \( 1/\Phi_1 = 0.02 \) (\( 1.0 \)), \( \theta_1 = 0.007 \) (\( 2.17 \)), \( \gamma = 0.023 \) (\( 2.43 \)). The parameter \( (1 + r)/(1 + \rho) \) has in both cases been restricted to one. The reason is that without this restriction its estimate tended to values considerably above one. This would imply a value of \( \rho \) less than zero. Although this is not an uncommon finding [see, for instance Hotz, Kydland and Sedlacek (1988)], it seems to be unacceptable on a priori grounds; also, in that case the other parameters tended to unacceptable values. Given the restriction, the estimates of the other parameters look plausible, with a correct sign for the liquidity constraints in case of the log \( l \)-specification and a slightly concave intertemporal utility function. The \( t \)-values of the first stage estimates have to be viewed with care because we have not corrected for the fact that the second stage parameters on which the first stage estimates are conditioned are themselves estimates. For the same reason specification
tests of the model can only be ascribed approximate value. Yet, a general specification test suggested by Hansen and Singleton (1982) yields a value for $\chi^2(5)$ equal to approximately 645 for the log l-specification and 738 for the l-specification. Together with the mentioned tendency of $(1 + r)/(1 + p)$ to attain unacceptable values, this indicates misspecification of the model.

6. Conclusions

Preferences are not homothetic, nor are they separable between consumption and leisure. These findings have been reported in the literature many times, and are corroborated by our analysis. The importance of liquidity constraints has been investigated less frequently. Our results do not provide unambiguous evidence as to the existence of limits on borrowing. Among other things, this may be due to the fact that in the estimation of the first stage model we have only used households with an employed head, for which constraints may be less often binding than for other households. The use of panel data has been quite essential in our analysis. Not only did we need longitudinal data for individual households to be able to estimate the Euler equations, the allowance for individual effects has contributed substantially to the accuracy of the second stage estimates. Yet, to investigate the importance of liquidity constraints, it would be useful to have more observations. Furthermore, additional specification analysis is required.

References


Meghir, C. and M. Browning, 1988, A simple test for separability between goods and leisure using conditional demand functions, Mimeo. (UCL-IFS McMaster University).

