A NEW APPROACH TO THE CONSTRUCTION OF FAMILY EQUIVALENCE SCALES*

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Received April 1975, revised version received November 1975

The question of the money compensation which should be given to families of different sizes in order that they enjoy equal welfare levels is considered. By comparison of individual welfare functions, estimated for 3,000 individuals in the Netherlands, family welfare equivalence scales are derived. The obtained equivalence scale depends on family size and the ages of the family members. There are considerable 'economies of scale'. The method employed may be used to derive money compensations for other situational differences. Evidence was found that people adapt their needs to situational changes. That effect was quantitatively assessed. Results are obtained and compared for various social subgroups.

1. Introduction

It is generally felt that an increase in family size decreases the material welfare of the family under ceteris paribus conditions. An increase in family size may be caused by an increase in number or by a virtual increase in the sense that family members grow older. We shall speak in both cases of an increase in family size.

As early as the previous century the problem was posed how much family B with say 6 children had to spend in order to be as happy as family A with 2 children. The solution to this problem consists in the construction of a family equivalence scale.

There are many of such scales. In order of increasing content of the underlying theories we mention scales based on:

(1) aprioristic judgment,
(2) normative budgets,
(3) nutritional needs,
(4) the proportion of income (or total expenditures) spent on food or necessities,
(5) systems based on all expenditure categories simultaneously.

*Earlier drafts of this study were presented at the Colloque d'Econométrie 1973 at Lyons and at the meeting of the Econometric Society at Oslo, 1973. We thank the discussants and the referees for their valuable comments. Responsibility for the remaining errors is ours. The research, reported in this article, has been made possible by a grant from The Netherlands Organization for the Advancement of Pure Research (Z.W.O.) and by the kind cooperation of the Consumer Union in The Netherlands. The authors are greatly indebted to these organizations.

Examples of the scales (1), (2) and (3) can be found in Presvelou (1968) and Cramer (1969). Examples of the fourth approach can be found in Jackson (1968), Seneca and Taussig (1971). The fifth approach has been adopted and discussed by, among others, Prais and Houthakker (1955), Blokland and Somermeyer (1970), Singh and Nagar (1973).

The theoretical basis for the first three approaches is not very clear [of. Cramer (1969, p. 164)]. The fourth approach entails some arbitrariness because the choice of the basket of food and necessities can be done in a variety of ways. A theoretical justification appears to require very restrictive assumptions [cf. Habib (1973)]. The theory underlying the fifth method has been developed

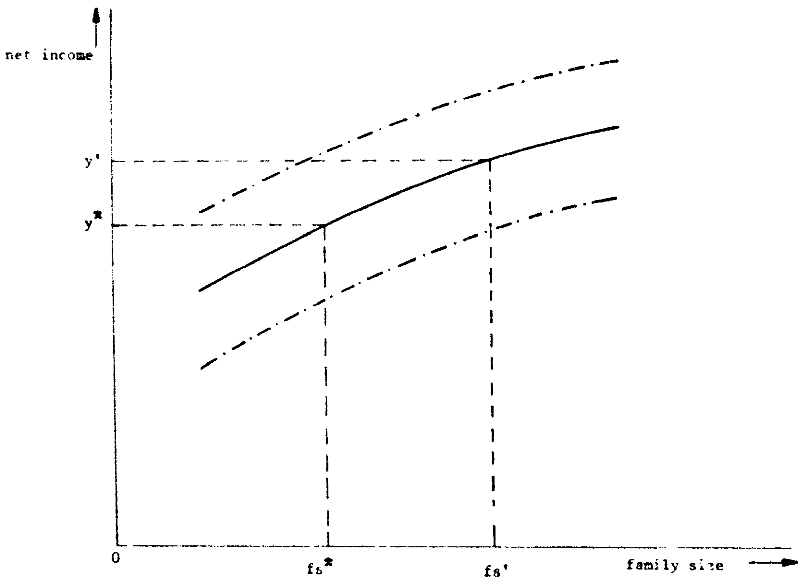

Fig. 1. Iso-welfare curves between net income and family size.

by Barten (1964) and Muellbauer (1974, 1975).[1] Muellbauer has pointed out some drawbacks of the latter method. The empirical applications appear to imply very strong assumptions about the underlying utility functions and generally some arbitrary assumptions are necessary to attain identifiability of the scales [see also Cramer (1969, p. 167 ff.)]. Because of these problems an alternative approach seems worthwhile to consider.

In this paper we discuss and apply such an alternative. We do not leave from observed market behaviour like methods (4) and (5) but from evaluation questions with respect to income levels. The evaluation questions serve for the

[1]More information on this method is provided by Cramer (1969) and Brown and Deaton (1972).

measurement of the *individual welfare function of income* introduced and elaborated by Van Praag (1968, 1971) and Van Praag and Kapteyn (1973). The individual welfare function of income describes the relationship between income levels and the welfare evaluations of these income levels on a [0, 1]-scale. We provide more details on the individual welfare function in section 2.

Intuition tells us that if under ceteris paribus conditions we want to keep a family's welfare evaluation of its net income constant when the size of the family increases, then the income of the family has to rise with the size of the family.[2] Fig. 1 visualizes this intuitive idea. The curves in fig. 1 represent combinations of net income and family size which generate equal welfare evaluations of income. We call these curves *iso-welfare curves*. They resemble, for example, the well-known indifference curves between leisure and income.

Suppose that fs^* denotes the *family size* of a standard household. The quest for family equivalence scales now amounts to the problem of how much income y' a family of size fs' needs in order to be equally happy as the standard family with its income y^*. Clearly the income y' has to be such that the household of size fs' is on the same iso-welfare curve as the standard family, assuming that the field of iso-welfare curves is the same for all households. Thus the iso-welfare curves completely determine the family equivalence scale system which we are looking for. The ratio y'/y^* may be looked upon as the ratio of *costs of living* of families of size fs' to families of size fs^*. It is the income compensation needed to keep welfare constant if fs^* changes into fs'. This cost definition conforms to the Hicksian cost concept [Klein and Rubin (1947)].

In this paper we derive iso-welfare curves between income and family size basing our calculations on the empirical findings in Van Praag (1971) and Van Praag and Kapteyn (1973), where individual welfare functions of income of about 3,000 Belgians and about 3,000 Dutchmen have been estimated. These results are partly summarized in section 2. In Van Praag (1971) and Van Praag and Kapteyn (1973) it was found that the evaluation of income depends primarily on two parameters: *actual net income* and *family size*, where family size is defined in a naive way as the *number* of family members, adults and children counting alike.

In this paper the effect of a change in family size on welfare is more closely analyzed. We distinguish between a *short-term effect* and a *long-term effect* that remains after the family has adapted its standards to the new circumstances. This is considered in section 3. In section 4 a more sophisticated family size concept is developed, in which a member of the family is characterized by his *age* and his *rank* in the family, the children being ordered according to decreasing

[2]We assume tacitly that the welfare of a household is represented by the welfare perception expressed by the head of that household (usually the main breadwinner). Therefore we shall use the words: family, household, individual, person, etc. interchangeably. The word 'welfare' is an abbreviation of, 'the welfare evaluation of income'. By 'income', always net family income will be meant.

age. Each member gets a weight which depends on his age and rank. The age variable takes into account the fact that an older person may have greater or smaller wants than a young one, while the rank variable allows for the introduction of economies of scale which may be present in large families. The weights are added to get the constructed 'family size'. Finally this family size is transformed in a simple way to obtain the factor by which the family income has to be multiplied in order to compensate for family composition changes. In sections 5 and 6 the results are presented.

2. The individual welfare function of income

Suppose, we confront an individual with evaluations 'good', 'sufficient', 'bad', etc., and ask him which income levels correspond to these evaluations. Suppose moreover that these income levels can be translated unambiguously into numbers on a numerical scale, say the [0, 1]-interval. Then we come fairly close to the measurement of the *individual welfare function of income*. A theoretical basis for the 'translation problem' and a theoretical justification for the functional specification of the individual welfare function of income has been provided by Van Praag (1968, 1971).

The empirical experiment described above has been performed for several large scale samples. On some of the outcomes was reported in this Review [Van Praag (1971), Van Praag and Kapteyn (1973)].

Summarizing the theoretical and empirical results we have gathered evidence in favour of the following thesis:

> An individual is able to evaluate net-income levels on a bounded numerical scale. The evaluation function is called the individual welfare function of income. The evaluation function is unique up to a positive linear transformation. An individual evaluates a net-income level z approximately by a lognormal distribution function,
>
> $$U(z) = \Lambda(z;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}\int_0^z \frac{1}{t}\exp\left\{-\frac{1}{2}\left(\frac{\ln(t)-\mu}{\sigma}\right)^2\right\}\mathrm{d}t, \tag{1}$$
>
> after normalization of the evaluations to a [0, 1]-scale.

With respect to the lognormal distribution function $\Lambda(.;\mu,\sigma)$ on the right-hand side of (1) there holds

$$\Lambda(z;\mu,\sigma) = N(\ln(z);\mu,\sigma),$$

where $N(\ ;\mu,\sigma)$ is the normal distribution function with mean μ and variance σ^2.

The parameters μ and σ of the individual welfare function of income are individually determined, i.e., they vary between individuals. In figs. 2a and 2b

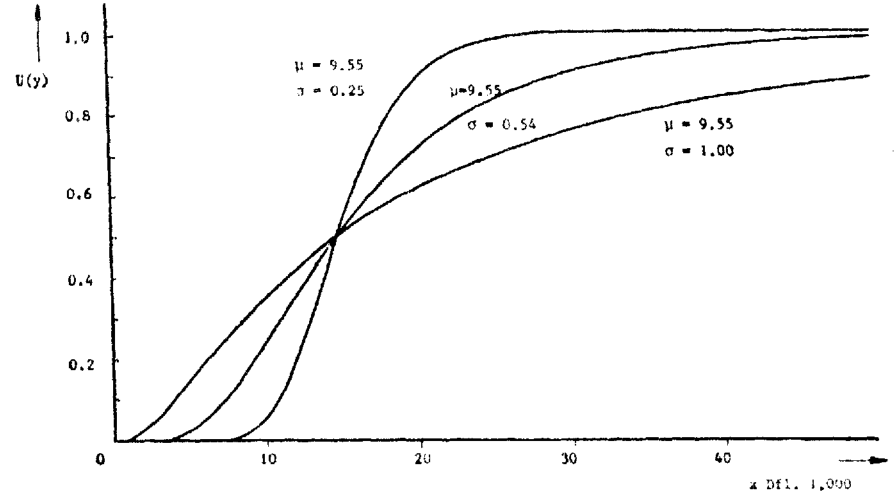

Fig. 2a. The welfare function of income for different values of σ. Vertical axis: $U(y)$, horizontal axis: $y \times$ Dfl. 1 000.

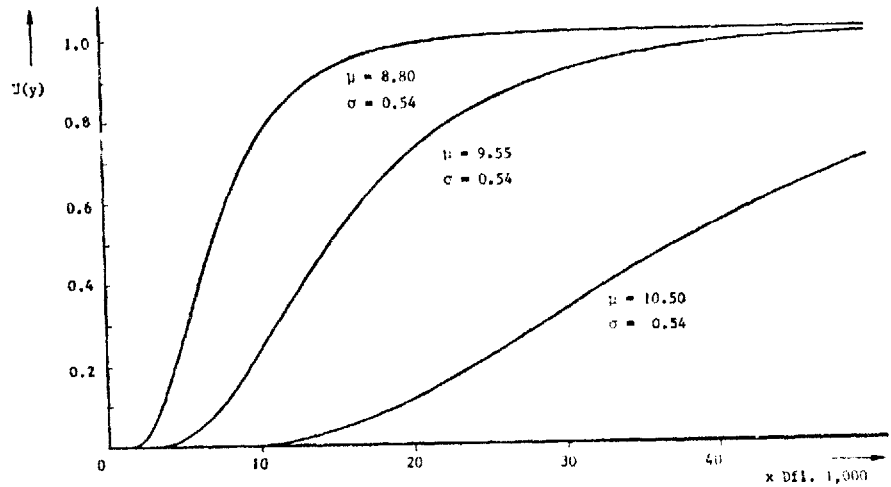

Fig. 2b. The welfare function of income for different values of μ. Vertical axis: $U(y)$, horizontal axis: $y \times$ Dfl. 1 000.

the individual welfare functions of individuals with different parameter values have been sketched.

The interpretation of μ and σ is of interest for the subsequent analysis. An individual with 'welfare parameter' μ assigns to the income level $\exp(\mu)$ the evaluation 0.5. When μ is large, a person needs a large net-income to be content. When μ is small, a small net-income will suffice to acquire a high welfare evaluation (see fig. 2b). The quantity $\exp(\mu)$ has been called the *natural unit* (of income). For a motivation for this term, see Van Praag (1968).

Fig. 2a shows individual welfare functions of income of persons with equal μ but different σ. When σ is small, only a narrow income range is evaluated substantially different from zero or one. When σ is large, a broad income range is evaluated substantially different from zero or one. The parameter σ has been called the *welfare sensitivity* (of income) [Van Praag (1968)].

Up to now, there have been conducted six surveys in Belgium and the Netherlands, from which individual μ's and σ's have been estimated for about 12,000 individuals. We reported on two of them in Van Praag (1971) and Van Praag and Kapteyn (1973). In this paper we use the same Dutch sample that was considered in Van Praag and Kapteyn (1973).

In this sample drawn from the (Dutch) Consumer Union membership in 1971, the estimates of the individual μ's varied about the average 9.55 with sample standard deviation 0.49. The estimates of σ varied about the average 0.54 with sample standard deviation 0.25. The value of μ depends on the money unit chosen, σ is dimensionless. The variation of σ among the people in the sample appeared unexplainable by socio-economic factors like income, family size, job, etc. Therefore σ has been held to be a reflection of a genuinely individual psychological trait and will be assumed exogenous in the following analysis. On the other hand explanation of μ was successful. In the following sections a further explanation of μ will be pursued.

3. A naive model of family costs

In Van Praag and Kapteyn (1973) we attempted to explain the variation of the parameter μ over the individuals in the sample by personal characteristics, like actual net income, family size, education, etc. The most successful regression specification was

$$\mu = \beta_1 \ln(fs) + \beta_2 \ln(y) + \beta_3 + \varepsilon, \tag{2}$$

where fs stands for the number of individuals in a family, y stands for the family's *net* income (in guilders), β_3 is a constant, and ε represents a random disturbance term with constant variance and zero expectation. For the complete Dutch sample consisting of about 3,000 individuals we obtained

$$\mu = \underset{(0.01)}{0.13} \ln(fs) + \underset{(0.01)}{0.64} \ln(y) + \underset{(0.11)}{3.02}, \qquad R^2 = 0.60,$$

where R^2 is the multiple correlation coefficient; the estimated standard errors have been added in parentheses.

The interpretation of β_1 and β_2 is of interest. We start with β_2. Let there be an individual with net-income y and let him expect his income to increase by a factor $(1+\alpha)$. Ex ante he will evaluate his future income by

$$N\{\ln(y)+\ln(1+\alpha)-\beta_2 \ln(y)-\beta_1 \ln(fs)-\beta_3; 0, \sigma\}.$$

After the increase has been realized, μ will rise according to (2) (setting ε equal to its expected value 0). The ex-post evaluation of the new income level will be

$$N\{\ln(y)+\ln(1+\alpha)-\beta_2 \ln(y)-\beta_2 \ln(1+\alpha)-\beta_1 \ln(fs)-\beta_3; 0, \sigma\} =$$
$$N\{\ln(y)+(1-\beta_2) \ln(1+\alpha)-\beta_2 \ln(y)-\beta_1 \ln(fs)-\beta_3; 0, \sigma\}.$$

This evaluation corresponds with the evaluation on the old welfare scale of a net income level $\{y(1+\alpha)^{(1-\beta_2)}\}$. In other words: the welfare scale *shifts* with income. This has been called the *preference drift* effect and β_2 has been called the *preference drift rate* [Van Praag (1971)].

The dependence of μ on family size and net income provides the iso-welfare curves introduced in section 1.

The evaluation of net income y by a family of size fs equals

$$N(\ln(y)-\mu; 0, \sigma) = N\{\ln(y)-\beta_1 \ln(fs)-\beta_2 \ln(y)-\beta_3; 0, \sigma\},$$

after substitution of (2) – setting ε equal to its expected value 0. In order to keep the welfare of a household constant for varying family size, net income y has to satisfy the equation:

$$\ln(y)-\beta_1 \ln(fs) -\beta_2 \ln(y)-\beta_3 = \text{constant}. \quad (3)$$

The iso-welfare curves described by (3) have been sketched in the $(\ln(fs), \ln(y))$-space in fig. 3.

From (3) we infer

$$\partial \ln(y)/\partial \ln(fs) = \beta_1/(1-\beta_2),$$

welfare being constant; $\beta_1/(1-\beta_2)$ has been called *the family size elasticity* [Van Praag and Kapteyn (1973)].

Now we may give a neat answer to the question which income y' the household of size fs' needs to be equally happy with its income as the standard household of size fs^* with net income y^*. If $fs' = (1+\alpha)fs^*$ then y', according to (3), is given by

$$y' = y^*(1+\alpha)^{\beta_1/(1-\beta_2)}.$$

We observe:

(1) The derived family equivalence scale system depends neither on the family size nor on the income level of the standard household.

(2) From the previous analysis one might expect that an increase in family size by a factor $(1+\alpha)$ would cause μ to rise to μ', the difference being the logarithm of the compensating family allowance, i.e.,

$$\mu' = \mu + [\beta_1/(1-\beta_2)] \ln(1+\alpha).$$

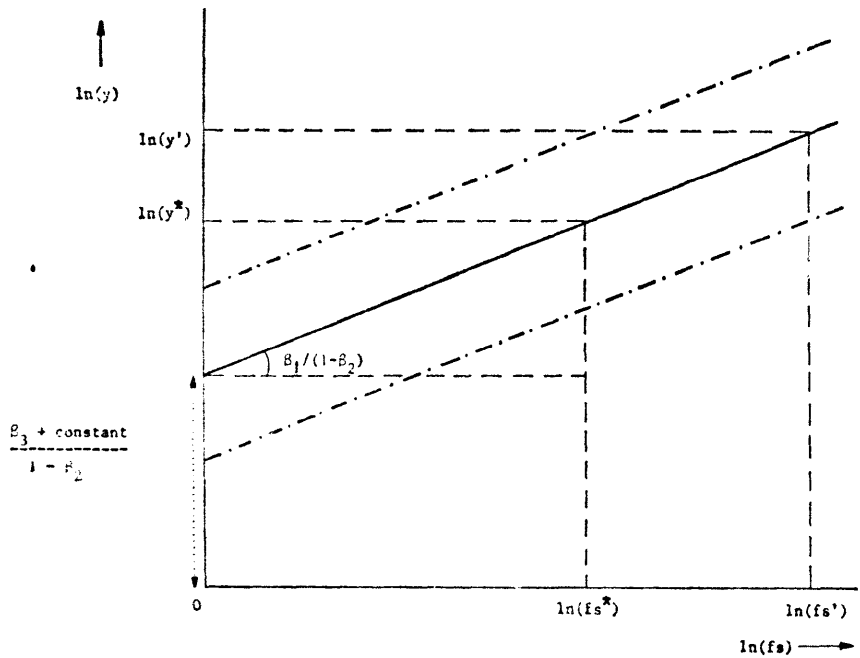

Fig. 3. Iso-welfare curves between net income and family size in the (ln(*fs*), ln(*y*))-space.

However eq. (2) implies that the *observed* difference only amounts to $\beta_1 \ln(1+\alpha)$, which is smaller (provided, of course, that $0 < \beta_2 < 1$). Obviously the difference is due to the preference drift.

One may interpret this outcome as follows. If, after an increase in family size by a factor $(1+\alpha)$, no family allowance is given, the family will partly adapt its standards to the new situation. Only a difference $\beta_1 \ln(1+\alpha)$ remains. This may be seen as a *long-term* effect. Correspondingly we call β_1 the *long-term* family size elasticity. One may decompose the long-term effect $\beta_1 \ln(1+\alpha)$ into two separate effects:

$$\textit{True Cost Effect}: \quad \left(\frac{\beta_1}{1-\beta_2}\right) \ln(1+\alpha),$$

and

$$\textit{Adaptation Effect}: \; -\beta_2\left(\frac{\beta_1}{1-\beta_2}\right)\ln(1+\alpha).$$

The Adaptation Effect is identical to the change that would result from an income *de*crease by a factor $(1+\alpha)^{\beta_1/(1-\beta_2)}$.

(3) In Van Praag and Kapteyn (1973, p. 52), we have hinted at the possibility that μ does not depend on own *actual* income, but rather on some kind of permanent income. It is probable that μ is not affected by every incidental income change. Only changes in income which can be considered to be *permanent* are likely to influence μ. It is well-known that, if permanent income is the correct explanatory variable instead of actual income, β_2 will be underestimated by the regression of μ on actual income [Cramer (1969, pp. 138, 183, 184)]. Fortunately the net-income concept defined in the questionnaire used in the Dutch survey leaves room for interpretation by the respondent in such a way that windfall gains and other transitory income components are presumably largely neglected. The actual income level stated may be identified with a long-term perception of income, which in its turn may be equated to permanent income.

4. A generalized model

We called model (2) a 'naive' model for obvious reasons. We want to get rid of the simplification that all family members would have equal weights with respect to the family's cost of living. It is generally felt that there is considerable difference between adults, children and babies.

Denote the age of the mother by a_1, the age of the father by a_2, and the ages of the children by $a_3, a_4, \ldots,$ in *de*creasing order of magnitude. Then we may consider a generalized 'family size function' for a family consisting of n persons, namely,[3]

$$fs = \sum_{i=1}^{n} f_i(a_i). \tag{4}$$

In the naive model

$$f_i(a_i) = 1, \qquad i = 1, 2, \ldots, n.$$

In this specification we leave room for the possibility that older people need more income than children to be equally happy. In addition we presume the existence of an 'economies of scale' effect which explains, for example, that a

[3]We tried a number of more sophisticated non-separable specifications which did not improve the results.

three-year-old child seems to cost less if he is the third child in a family than if he is the second one. This effect is accounted for by the distinction of the age functions with respect to rank. The simplest form is

$$f_i(a_i) = \alpha(i) f(a_i), \tag{5}$$

where the age effect and the rank effect are separated. The $\alpha(i)$'s account for the possible 'economies of scale' when the number of children increases. Most likely $\alpha(i)$ decreases with increasing i ($i > 2$). On the other hand the age function $f(a)$ may be expected to increase with rising a.

Some preliminary estimation experiments with fourth- and fifth-degree polynomials led us to the following specification of *the age function*,

$$f(a) = \Lambda(a; \mu_2, \sigma_2) + C, \tag{6}$$

that is, a lognormal distribution function *plus* a constant that denotes the value of the age function when $a = 0$. An intuitively evident restriction on C is that C has to be non-negative. Consequently C has been specified as $C = \exp(\gamma)$ in order to avoid non-negativity constraints on the parameter to be estimated. There is no theoretical reason to select the lognormal distribution function in (6). We chose this function because it is one of the most flexible functions with only two parameters. In the relevant region ($a \in [0, 100]$) the function may be convex, concave, flat on the zero-level, flat on the one-level, the function may reveal an inflection point, etc. All these possible forms depend on the values of the parameters μ_2 and σ_2.

The estimation of the unrestricted set of parameters $\alpha(i)$, $i = 1, \ldots, 8$, suggested a uni-modal density function of the lognormal or Γ-type. Henceforth we specified the $\alpha(i)$ by

$$\alpha(i) = \Lambda(i; \mu_1, \sigma_1) - \Lambda(i-1; \mu_1, \sigma_1). \tag{7}$$

We call eq. (7) the *rank function*. For the same reasons as with the age function, also the rank function is very flexible.

Thus model (2) is replaced by

$$\mu = \beta_1 \ln\Bigg[\sum_{i=1}^{n} \{\Lambda(i; \mu_1, \sigma_1) - \Lambda(i-1; \mu_1, \sigma_1)\} \times \{\Lambda(a_i; \mu_2, \sigma_2) + \exp(\gamma)\} \Bigg] + \beta_2 \ln(y) + \beta_3 + \varepsilon. \tag{8}$$

5. The estimated family equivalence scale

Model (8) has been estimated by means of least squares, using the data gathered in the survey among the members of the Dutch Consumer Union.[4] We excluded from our observation the 'incomplete' families which did not include at least a married couple. Accordingly, bachelors, widows and divorced people are excluded. The number of observations for these categories is too small to guarantee reliable estimates, when dealt with in isolation. Inclusion of these categories endangers the homogeneity of the set of observations. The exclusion of these categories diminishes the number of observations to 2573. The estimates are presented in table 1, where the corresponding estimates of the standard-deviations are given in parentheses.[5]

Since re-estimation of the 'naive' model on the sub-sample of 2573 observations did not alter the outcomes we may compare the results in table 1 with the estimates in the 'naive' model of section 3. We see that β_1 has increased considerably and that the preference drift has decreased slightly. As may be expected the explanation has improved, R^2 rises from 0.60 to 0.65.

Table 1
Parameter estimates for the complete sample.

Long-term family size elasticity	β_1 = 0.41 (0.27)	
Rank function parameters	μ_1 = 0.32 (1.06),	σ_1 = 1.04 (0.03)
Age function parameters	μ_2 = 3.52 (0.09),	σ_2 = 0.24 (0.11)
	γ = 0.73 (0.86),	$C = \exp(\gamma)$ = 2.07
Preference drift rate	β_2 = 0.56 (0.01)	
Regression constant	β_3 = 3.80 (0.66)	
Number of observations	2573	
Coefficient of determination	0.65	

The estimated standard errors are rather large for μ_1 and γ. Those large standard errors are presumably caused by considerable multicollinearity between the explanatory variables. This is due to the fact that the sample had not been designed for the estimation problem of this paper. More reliable estimates could be obtained from an experiment where the sample would have been designed in such a way that the variation of family composition is as large as possible. For instance, the fact that all included households consist of at least both husband *and* wife who are usually of about the same age, makes it impossible to discriminate sharply between husband and wife with respect to their contribution to the cost of living of the family. As a consequence the estimate of μ_1 is inaccurate and one should not attach much meaning to the difference between rank weights

[4]In order to minimize the sum of squares corresponding to the non-linear model (8), a numerical procedure was needed. Both the Fletcher–Powell Descent Method (1963) and the Marquardt Procedure (1963) were tried out. The latter procedure needed less iterations and required less computer time to reach the minimum. This finding agrees with other research [e.g. Heuts and Rens (1972)].

[5]The standard deviations were computed from the asymptotic variance–covariance matrix of the parameter estimates [see Goldfeld and Quandt (1972, pp. 58, 63, 70 ff.) and Jennrich (1969)].

of husband and wife. We shall see from a simulation experiment (to be described in footnote 7) that in spite of the inaccuracy of some estimated parameter values the constructed family equivalence scale appears to be rather reliable.

In section 4 we redefined the family size variable fs as

$$fs = \sum_{i=1}^{n} \alpha(i) f(a_i), \tag{9}$$

where n is the unweighted family size. The rank function, defining the $\alpha(i)$, and the age function have been specified in (6) and (7). Hence for any family composition the expression fs can be computed in a simple way by using fig. 4, where the functions $\alpha(i)f(a)$ have been sketched as a function of a for $i = 1, \ldots, 7$. The functions have been normalized (after the estimation) in such a way that $\alpha(1)f(0) = 1$.[6]

The first thing that strikes us when looking at fig. 4 is that welfare is not influenced by the ages of the children; only their number counts. The younger child counts less than the older one. This is not due to the age difference but it is caused by the rank effect only.

It seems that children need more when they grow older. This appears to be caused by the fact that, when the children grow up, the parents grow older as well and pass through the sensitive age bracket between 24 and 48 years; in that bracket the parents' requirements appear to grow considerably while the children's needs measured as a percentage of family income remain constant.

The reader may wonder to what extent these results are imposed by the specification of the age function. In section 4, we mentioned already that the log-normal function is very flexible. Moreover, we tried a more complicated model with two separate age functions for children and parents. We found the same results, so the flatness of the age function for low ages does not seem to be imposed by our specification of the age function.

Other studies [e.g. Blokland and Somermeyer (1970) and McClements (1975)] have found an increase of total expenditures with rising ages of the children. This indicates that more income is needed to attain a certain welfare level when the children grow older. However, these studies have not taken into account the ages of the parents. Given the positive correlation between the ages of parents and the ages of children this implies that an increase of expenditure which is due to the parents' growing older, is almost automatically ascribed to the increasing ages of the children, when the parents' ages are not in the model.

The fact that the children's needs as a percentage of family income remain constant when the children (and consequently the parents) grow older, does not imply that these needs do not rise in money terms. When the parents' ages rise, the family income tends to rise as well according to the so-called age–

[6]In fact the absolute value of fs is immaterial. Only the ratios of the terms $\alpha(i)f(a_i)$ are of interest, and one is free to normalize fs to any reasonable unit.

income profiles [cf. Fase (1969)]. So a constant proportion of family income means a growing amount of money. In the next section we return to the relationship between the age function and age-income profiles.

Consider a 4-person family consisting of a husband, 37 years old, a wife, 35 years old, and two children, 12 and 10 years of age. We derive the family size by looking at fig. 4. From the wife's curve we find that the housewife counts for 1.28; the husband counts for 0.91 and the children for 0.35 and 0.20 respectively. Summing these weights, we find 2.74. We call this family the *standard family*, with family size $fs^* = 2.74$.

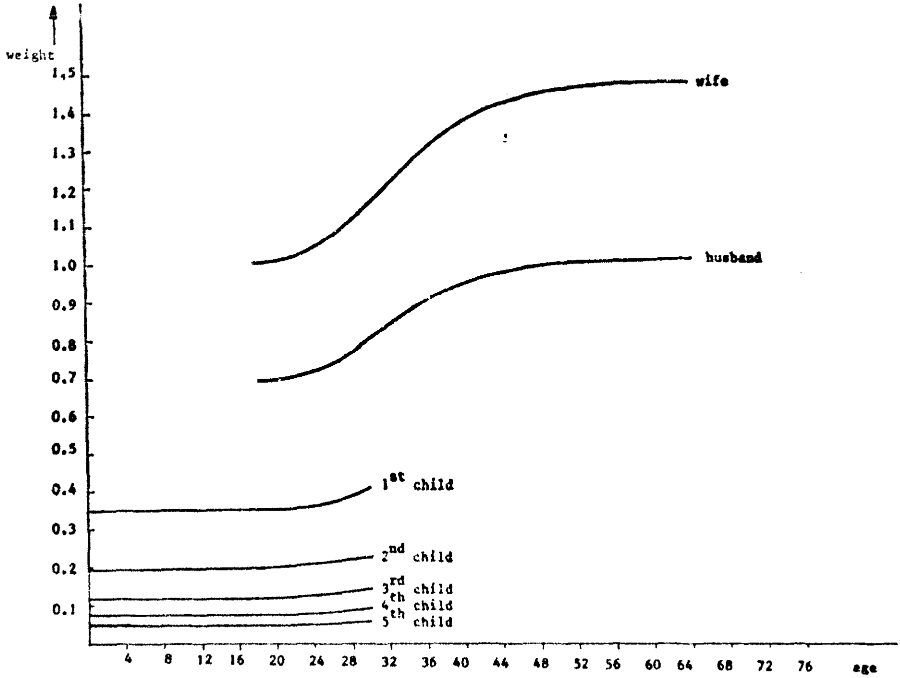

Fig. 4. Nomograph for the construction of family equivalence scales.

In section 3, it was shown that a family of size fs' needs an income y', with

$$y' = y^*(fs'/fs^*)^{\beta_1/(1-\beta_2)}, \tag{10}$$

to be equally happy with its income as the standard family.

The ratio

$$y'/y^* = (fs'/fs^*)^{\beta_1/(1-\beta_2)} \tag{11}$$

is called the 'true' or short-term family equivalence scale value of the household of size fs', relative to the standard family.

If no income compensation is given, the family will adapt its standards. In the long run the family of size fs' believes that an income compensation to

$$y'' = y^*(fs'/fs^*)^{\beta_1} \tag{12}$$

would be sufficient to attain the same welfare level as the family of size fs^*.

Table 2
Family equivalence scale values for some family types.[a]

Number of persons in the family	Ages a_1 (mother)	a_2 (father)	a_3	a_4	a_5	a_6	Perceived scale values[b] [cf. (12)]	True scale values [cf. (11)]
2	25	27					0.84 (0.11)	0.67 (0.29)
2	25	40					0.88 (0.07)	0.74 (0.14)
2	50	52					0.96 (0.06)	0.90 (0.13)
2	55	57					0.96 (0.06)	0.91 (0.13)
3	25	27	2				0.91 (0.05)	0.80 (0.13)
3	50	52	22				1.01 (0.03)	1.03 (0.07)
4	25	27	2	1			0.94 (0.03)	0.86 (0.07)
4	50	52	22	20			1.04 (0.03)	1.09 (0.06)
5	25	27	4	2	1		0.96 (0.04)	0.91 (0.08)
5	50	52	24	22	20		1.06 (0.04)	1.14 (0.09)
5	50	52	24	20	12		1.06 (0.04)	1.14 (0.09)
6	25	27	6	4	2	1	0.97 (0.05)	0.93 (0.11)
6	50	52	26	24	22	20	1.07 (0.05)	1.17 (0.13)
6	50	52	26	24	22	12	1.07 (0.05)	1.17 (0.12)
6	50	52	26	20	16	12	1.07 (0.05)	1.17 (0.12)
6	50	52	20	20	16	12	1.07 (0.05)	1.16 (0.12)
4	35	37	12	10			1.00	1.00

[a]The standard errors of the estimated scale values have been added in parentheses.
[b]Without compensation.

In table 2 we present family equivalence scale values for a number of household compositions. The estimates of the corresponding standard errors, obtained by simulation, are given in parentheses. The standard errors appear to be of moderate size.[7]

[7]The variance of the family equivalence scale is assessed by a Monte-Carlo experiment. The parameter vector, the estimate of which has been presented in table 1, has asymptotically a multivariate normal distribtuon. The variance–covariance matrix can be calculated by applying the well-known results of large-sample theory [see Goldfeld and Quandt (1972) and Jennrich (1969)]. We simulated a sample of 3,000 values of the parameter vector. Subsequently for each household in table 2 we obtained a frequency distribution of the family equivalence scale values according to (11) and calculated its mean and variance. The resulting distribution appeared to be more peaked than the corresponding normal distribution. An interval of one standard deviation about the mean contains approximately 80 percent of the density mass. Hence the tabulated standard deviations may be interpreted in a more optimistic manner than in the 'normal' case. We preferred the simulation approach to the well-known non-linear approximation of variances [cf. Cramer (1969, p. 96)], because very little can be said of the accuracy of the latter procedure.

One observes, for instance, that a small young family (25, 27) needs only $0.67/1.17 = 58$ percent of the income of a large old family (50, 52, 26, 20, 16, 12) to be equally happy. However, when the net incomes of both households are equal, the perceived cost-difference between both family types only amounts to 21.5 percent ($1-0.84/1.07$), instead of 42 percent which is the 'true' difference.

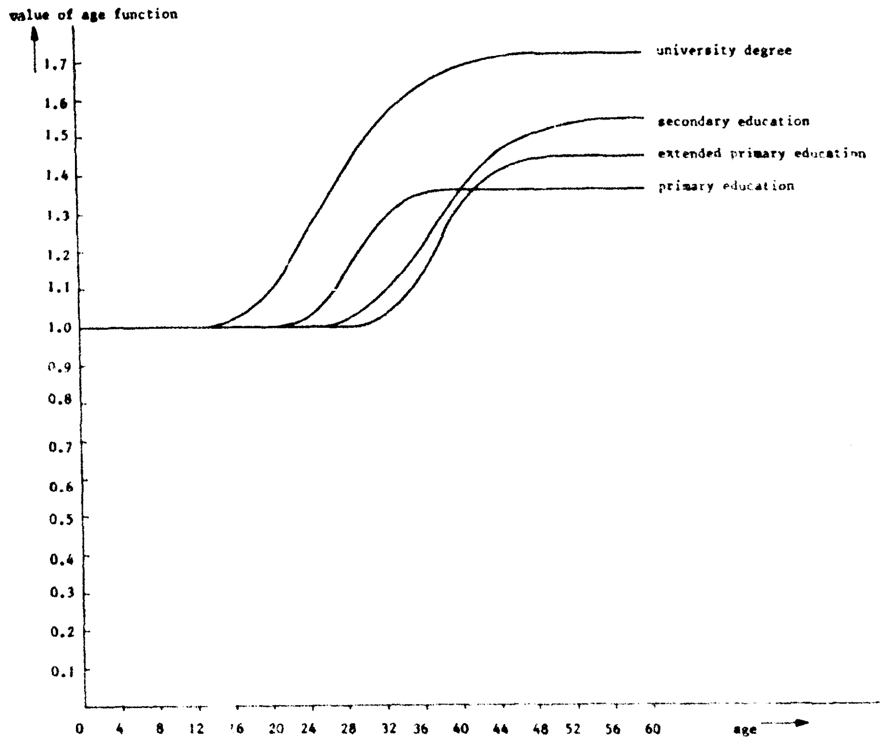

Fig. 5a. Age functions of education groups.

6. Social and geographical differences

In addition to the outcomes for the complete sample we present estimates based on subclasses of the sample defined according to the following characteristics of the head of the family:

(a) education (primary, extended primary, secondary, university),
(b) urbanization (living in a large town or in the country),
(c) wife's activities (both partners have a paid full-time job or only the husband has one).

The estimates are given in table 3. Since the sample is not completely representative for the Dutch population, the following interpretations have a tentative

Table 3
Parameter estimates for several subclassifications.

Parameter estimates	β_1	μ_1	σ_1	μ_2	σ_2	C (exp (γ))	β_2	β_3	R^2	No. of observations
Education										
Primary	0.40	−0.08	1.18	3.39	0.12	2.80	0.50	4.14	0.46	176
Extended primary	0.26	0.96	1.09	3.56	0.10	2.29	0.55	4.03	0.60	600
Secondary	0.41	0.39	1.10	3.65	0.16	1.85	0.55	4.02	0.62	1215
University	0.64	0.11	1.08	3.31	0.25	1.34	0.52	4.23	0.65	578
Urbanization										
Country	0.18	0.58	1.03	3.64	0.13	0.77	0.61	3.70	0.64	275
Large cities (over 100,000)	0.25	1.06	1.15	3.29	0.06	1.20	0.56	4.14	0.56	470
Wife's activities										
Full-time paid	0.52	0.42	1.08	3.81	0.01	2.75	0.57	3.55	0.61	362
No paid job	0.22	0.84	1.11	3.56	0.05	1.74	0.61	3.57	0.67	1654

character. The age functions and the rank functions have been sketched in figs. 5, 6 and 7, respectively. Instead of $\Lambda(a; \mu_2, \sigma_2)+C$ (see section 4), the expression $1+C^{-1}\Lambda(a; \mu_2, \sigma_2)$ has been sketched, in order to allow each age function to start at level 1.

6.1. Educational differences

Fig. 5a shows that age differences weigh more heavily, the more education one has. The age functions start increasing approximately at the age of marriage

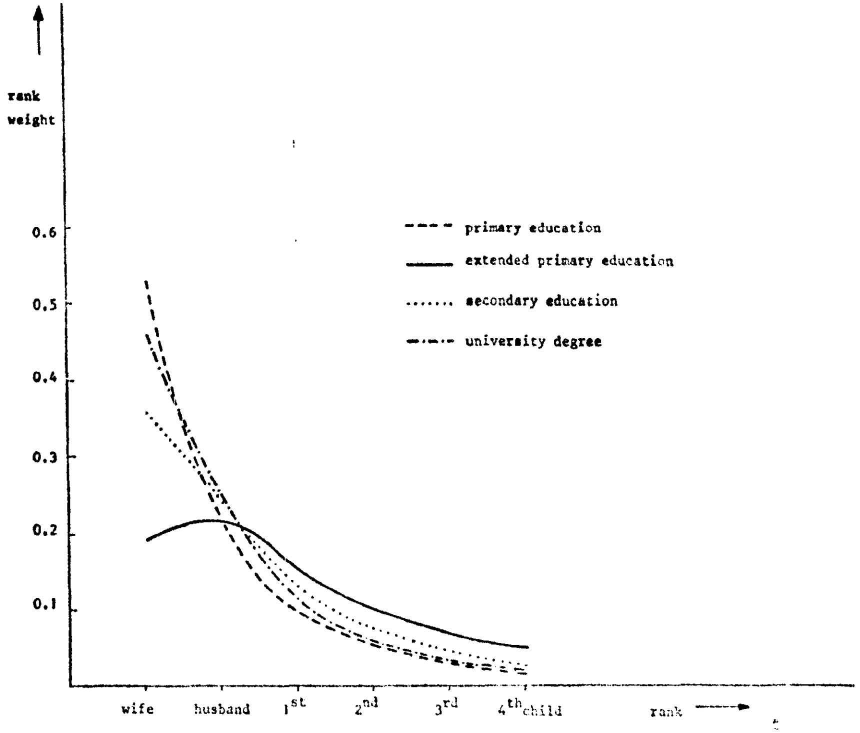

Fig. 5b. Rank functions of education groups.

(22 through 29) except for the class with university education. In this class the age function starts its upswing at the age of about 15. This is the only category in which a real difference exists between older and younger children.

With respect to the range of increase, we notice that it ends much earlier for the class with primary education, namely at about 38, than for the other categories. It is interesting to observe that in the class with university education the age function becomes flat at about 48 at a very high level, compared to the other classes. This pattern of age functions resembles age–income profiles per education category [cf. Fase (1969)]. In other words: the age functions seem to reflect the average behaviour of incomes over age in the various education categories. Why is this so? An obvious answer is: because people refer to their social environment. When people in the social environment of an individual (i.e., people of the same education and age) get higher incomes then the individual under consideration wants a higher income as well.

The age-income profile depends on the course of the career. Therefore the range of increase of the age function may be interpreted as the period in life during which one is making his career. We call that period one's 'career span'. Summarizing we find by chart-reading on fig. 5a the following 'career spans'.

Primary education	22–38	years of age;
Extended primary education	29–50	years of age;
Secondary education	25–54	years of age;
University education	14–48	years of age.

From fig. 5b one sees that the rank functions differ as well. In order to evaluate these differences one should also take into account the values of β_1 and β_2. For example, consider the compensation in net income for the birth of a second child in families of different educational background. Assume that the previous composition of the families had been (32, 35, 4). Denote the corresponding weighted family size by fs' and the size after the happy event by fs''. Then we construct by chart-reading from figs. 5a, 5b and using table 3:

	fs''/fs'	$\beta_1/(1-\beta_2)$	$(fs''/fs')^{\beta_1/(1-\beta_2)}$
Primary education	1.05	0.80	1.04
Extended primary education	1.18	0.58	1.10
Secondary education	1.10	0.91	1.09
University education	1'05	1.33	1.07

The compensation for an identical family increase varies from 4 percent to 10 percent. The family with primary education needs the smallest compensation in net-income. Notice that, if the additional child were to be adopted at an age of over 14 years, the compensation for the university family would increase while this would not hold for the other families.

6.2. Urbanization

Considering the difference between countrymen and large-city inhabitants, we see from fig. 6a that the region of increase of the age function for a large-city inhabitant ranges from 24 to 32. The country-dweller seems to be much more sensitive to age differences. With respect to the rank effect we notice that in the large cities an additional child has more influence on the cost of living than in the country. For example, a (32, 35, 4)-family living in the country needs only

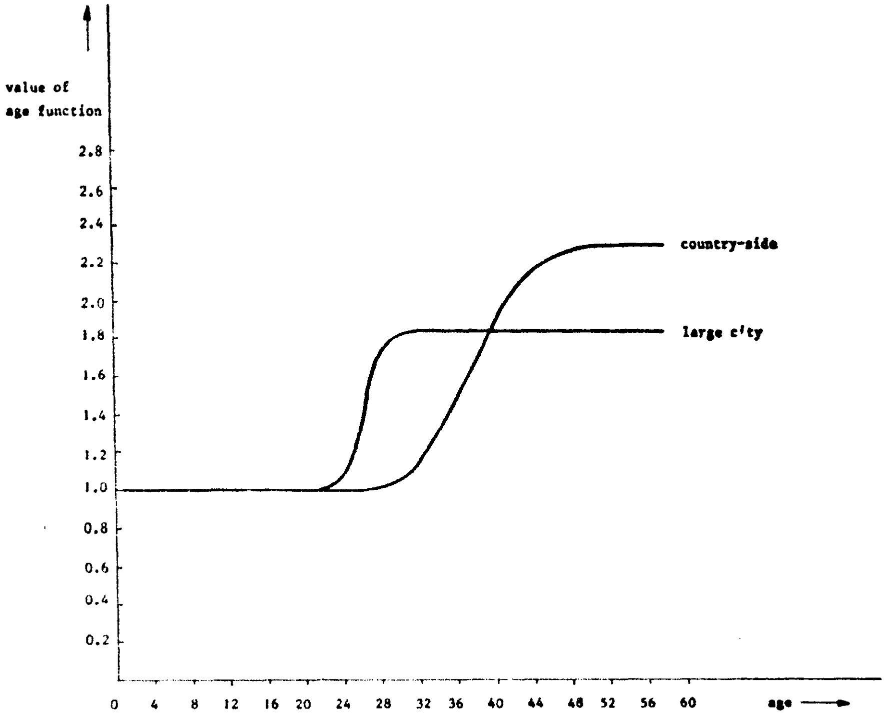

Fig. 6a. Age functions of families in the country and of families in large cities.

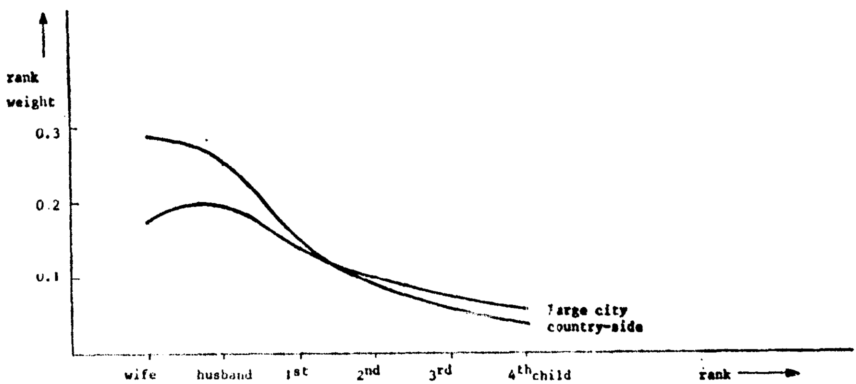

Fig. 6b. Rank functions of families in the country and of families in large cities.

8 percent increase of net-income when a second child is born. If the same family were living in a large city, the increase would have to be 12 percent. For example, in the case of a net-income of US$ 15,000 before the birth of the child, this implies a cost difference between town and country of about US$ 600 per annum.

6.3. *Wife's activities*

Finally, we consider the dichotomy between couples where both partners work in a paid full-time job and those where only the husband earns the income.

From fig. 7a we see that aging is quite abrupt in the case of the working wife, while it is more gradual in the other case.

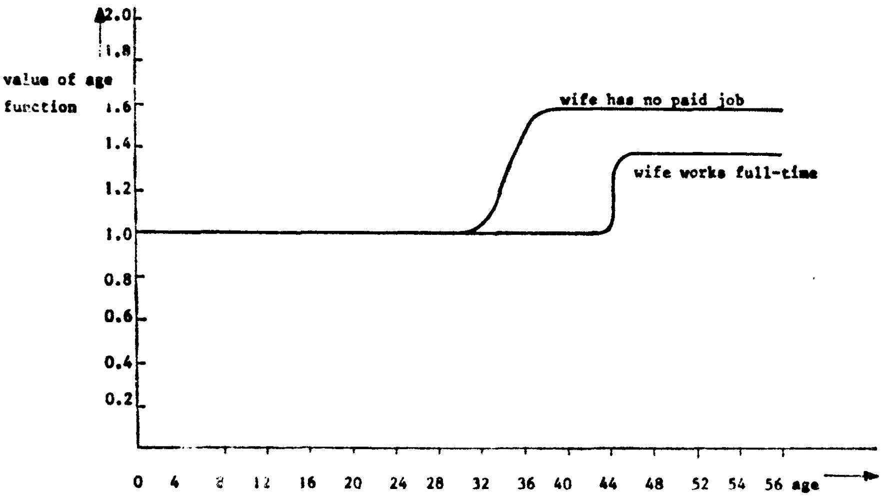

Fig. 7a. Age functions of families where the female partner has a full-time paid job and of families where the female partner has no paid job.

Doing the same exercise as before, we find that a (35, 32, 4)-family needs a compensation of about 17 percent for a second child, if the wife works, and only 8 percent, if the wife stays at hime. For example, in the case of a net-income of US$ 15,000 per annum the cost difference amounts to US$ 1,300 per annum. which may be seen as a reward for the wife's child-care function.

7. Conclusion

In this paper we developed a fairly complicated model to assess the influence of the family composition on the family's well-being as measured by the individual welfare function of income. We distinguished a rank effect, representing the 'economies of scale' inherent to a large family, and an age effect representing the fact that older persons have more needs. The sample had not been expressly designed for the kind of research reported in this paper, nor is the sample completely representative of the Dutch population. Nevertheless, the impression is gained that family composition is an important determinant of well-being under ceteris paribus conditions and that its impact varies substantially between social subclasses.

Apart from the results with respect to the family equivalence scale problem, we feel that three methodological features of our approach, which may have a wider applicability, should be stressed.

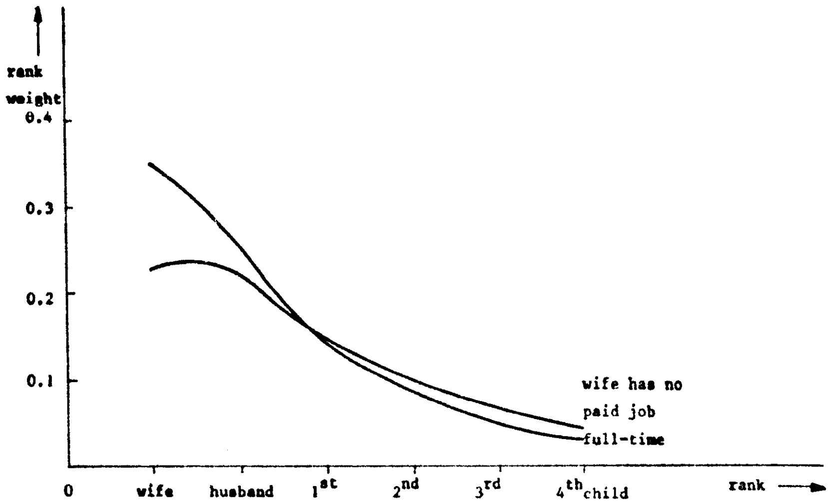

Fig. 7b. Rank functions of families where the female partner has full-time paid work and of families where the female partner has no paid job.

(A) An individual adapts his welfare function to his own income. This effect has been discussed earlier [Van Praag (1971), Van Praag and Kapteyn (1973)] and has been called the *preference drift* effect. In this paper the concept has been extended to a change in family size. In our opinion there is no barrier to prevent generalization of this concept still further in order to make it applicable to changes in any situational characteristic, relevant for welfare evaluation.

(B) The difference between *ex-ante evaluations* and *ex-post evaluations* has been operationalized. Among other things the effect may account for seemingly inconsistent behaviour of individuals that cannot be explained by the assumption of *constant* preferences.

(C) Differences in material circumstances (i.e., family composition) were *translated* into *money* amounts by comparing the individual welfare functions of income of individuals who differ with respect to those circumstances.

This method is not necessarily limited to family composition effects. In principle the method may be used to transform any situational difference into differences in required net-income. Thus many, hitherto non-measurable, effects – e.g. environmental changes – may be measured in money terms by the method adopted.

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