A statistical approach to the problem of negatives in input–output analysis

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The purest and theoretically superior method for the construction of input–output coefficients is given by the commodity technology model. The commodity technology based input–output coefficients have one shortcoming, however. Some of them turn out negative, which is economically not meaningful. This paper presents a methodology to deal with the problem of negatives in input–output analysis. It allows a statistical assessment of the problem. We are led to reject the commodity technology model. This conclusion is surprising, at least to us, in view of the theoretical appeal of the model and the empirical smallness of the negatives.

Keywords: Input–output analysis; Re-estimation; Construction of coefficients

The construction of input–output coefficients matrices is complicated by the presence of secondary outputs. Sectors produce not only own or *primary* output, but also each others' or *secondary* outputs. In textbook input–output analysis coefficients are determined by dividing inputs by primary output, while secondary output is assumed away. In reality we must account for secondary products and a number of methods are available for the construction of technical coefficients (ten Raa, Chakraborty and Small [7], Fukui and Seneta [2] and Viet [11]).

The purest and theoretically superior method is given by the *commodity technology* model. This model simply postulates input–output coefficients, calculates the consequent direct requirements for the outputs of each sector and equates the sum to the observed inputs. Thus, for each sector we have a commodity vector equation. These equations can be solved simultaneously for the technical coefficients. The solution is simple: the input–output coefficients matrix is basically the input matrix divided by the output matrix, and has nice properties, such as scaling invariance.

The input–output coefficients based on the commodity technology model have one shortcoming, however. Some of them turn out negative, which is economically not meaningful. This paper presents a methodology to deal with the problem of negatives in input–output analysis; it allows a statistical assessment of the problem. We will be led to reject the commodity technology model. This conclusion is surprising, at least to us, in view of the theoretical appeal of the model and the empirical smallness of the negatives.

The paper is organized as follows. The second section reviews the commodity technology model and shows how it may generate negative input–output coefficients. The third section presents a diagnosis of the negatives for UK data to provide some intuition. The fourth section applies a re-estimation procedure to eliminate the negatives; results are presented and discussed in the fifth section. They confirm the established practice of dealing with negatives by simply setting them zero, but must, none the less, reject the model that underlies the construction of coefficients, as the last section concludes.

The commodity technology model

The system of national accounts (UN [8]) includes

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an input or 'use' table U and an output or 'make' table V. Entry u_{ij} is the amount of commodity i consumed by industry j. v_{jk} is industry j's amount of product k. The commodity technology model postulates technical coefficients a_{ik} for all sectors (van Rijckeghem [10]). In particular, industry j requires $a_{ik}v_{jk}$ of input i for output k. Its consumption of input i equals the requirements summed over outputs: $u_{ij} = \Sigma_k a_{ik} v_{jk}$. Hence $U = AV^{\tau}$ or $A = UV^{-\tau}$ where $^{\tau}$ denotes transposition and $^{-1}$ inversion. (Since the latter two operations commute, their compositions may be denoted $^{-\tau}$ without confusion.)

It is instructive to consider the example of a two-sector economy with one sector, say the first one, producing some secondary output:

$$U = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix} v_{11} & v_{12} \\ 0 & v_{22} \end{pmatrix}$$

Then

$$A = UV^{-\tau} = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix} \begin{pmatrix} v_{11} & 0 \\ v_{12} & v_{22} \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix} \begin{pmatrix} 1/v_{11} & 0 \\ -v_{12}/(v_{11}v_{22}) & 1/v_{22} \end{pmatrix}$$

$$= \begin{pmatrix} \left(u_{11} - \dfrac{u_{12}}{v_{22}} v_{12}\right) \Big/ v_{11} & \dfrac{u_{12}}{v_{22}} \\ \left(u_{21} - \dfrac{u_{22}}{v_{22}} v_{12}\right) \Big/ v_{11} & \dfrac{u_{22}}{v_{22}} \end{pmatrix}$$

For sector 2 we have the usual coefficients, $a_{12} = u_{12}/v_{22}$ and $a_{22} = u_{22}/v_{22}$, but for sector 1 we obtain

$$a_{11} = (u_{11} - a_{12}v_{12})/v_{11}$$

and

$$a_{21} = (u_{21} - a_{22}v_{12})/v_{11} \tag{1}$$

In other words, the technical coefficients are *net input* over *net output* where net output is total output net of secondary products and net input is input net of the associated secondary product requirements. In theory the input requirements of secondary products cannot exceed the total input of the sector, so the coefficients of the input–output table, A, cannot be negative. However, the theory may not be valid in its pure form, or at least the use and make tables are observed with measurement errors. Therefore the input requirements of secondary products may exceed the observed input of the sector. Then the subtraction yields a negative net input and hence we observe a negative technical coefficient, a_{11} or a_{21}, in this case. Alternatively, if the use and make data are measured without error and negatives nevertheless arise in the construction of input–output tables, the basic assumption of the commodity technology model must be wrong. This incompatibility between theory and empirical outcome is the subject of this study.

Diagnosis of negative input–output coefficients

The data used in this study are in the system of national accounts for 1975 of the UK (Barker, van der Ploeg and Weale [1]). The use and make tables are square tables; the size is the number of sectors, 39. (The 'Unallocated' sector is omitted.) U and V are reproduced in Tables 1 and 2. The unit of measurement is million pounds. The derived technical coefficients matrix, $A = UV^{-\tau}$ is in Table 3. They are multiplied by a factor of 100, so that the unit is pennies per pound. All tables are in the appendix.

There are three negatives on digit level two, namely $a_{4,10} = -0.007$, $a_{28,31} = -0.015$ and $a_{28,32} = -0.005$. (They are multiplied by a factor of 100 in Table 3.) The biggest one, $a_{28,31}$, is the only one that persists when indirect requirements are taken into account through the Leontief inverse $(I - A)^{-1}$. No other negatives on digit level two are created in the inverse.

It is well known why the commodity technology model produces negatives. Each commodity is assumed to have its own input structure, irrespective of the sector where it is fabricated. To identify input structures, sectors are purified of secondary activities by subtraction. Negative net inputs are created if secondary products have input components that, in sum, exceed the actual inputs of the sector at hand, as reported by the use table, U (recall Equation (1)).

In each of the cases listed above a single secondary product accounts for the negative value of the input–output coefficient. Each one will be taken up in turn. First, $a_{4,10} = -0.007$. Sector 10 (chemicals) produces one secondary output with a sizable petroleum and natural gas (commodity 4) requirement, namely $v_{10,9} = 78.9$ (petroleum products). None the less, sector 10 itself uses no petroleum and natural gas. The petroleum and natural gas requirement amounts to $a_{4,9}v_{10,9} = 0.62 * 78.9 = 48.6$ which, after division by primary output $v_{10,10} = 6\,928.0$, accounts precisely for the negative value of $a_{4,10}$. How can the chemical sector produce petroleum products without petroleum? In theory, there are three possible answers: vertical integration, throughput or alternative technology. If the chemical sector were vertically integrated into the petroleum sector, then it could produce petroleum products from petroleum and natural gas inputs. The latter inputs are not well represented in the chemical sector, so that vertical integration is not the answer in this case. The second possibility, throughput, turns out to be the right answer. The chemical sector produces petroleum products out of petroleum products. It has a sizable petroleum products output, $v_{10,9} = 78.9$, *as well as* input, $u_{9,10} = 494.9$. Thus, the first negative, in the chemical sector, is due to the problem associated with products having much *own* input (ten Raa, Chakraborty and Small [7], p 93). It can be considered as an alternative technology instance, namely one with own input coefficient one and all others zero. (It will not be so extreme in practice, but one petroleum product may be turned into another, which is essentially an aggregation problem.)

Next take the second negative, $a_{28,31} = -0.015$. Sector 31 (water) produces one secondary output with a sizable construction (commodity 28) requirement, namely $v_{31,28} = 73.3$ (construction). The requirement amounts to $a_{28,28}v_{31,28} = 0.18*73.3 = 13.3$ which, after division by primary output $v_{31,31} = 654.5$, accounts precisely for the reduction of $a_{28,31}$ to its negative value. How can the water department produce construction with relatively little construction? This is the mirror image of the first case. Now we have the problem of products with much *own* input, not in the sector at hand (31), but in the sector of *reference* of the secondary input structure (28). So the answer is that construction use of construction in its own sector, $u_{28,28} = 2836.3$, is big. The third and last negative, $a_{28,32} = -0.005$, is similar. The construction secondary output, $v_{32,28}$, is again the source of the problem: its commodity 28 (construction) requirement accounts for the reduction of $a_{28,32}$ to its negative value.

Our diagnosis of negative input–output coefficients can now be summarized. The source of the trouble is the presence of much throughput of secondary products, either in the sector under consideration ($u_{9,10} \rightarrow v_{10,9}$ which causes negativity of $a_{4,10}$), or in the sector of reference of the secondary product ($u_{28,28}$ which causes negativity of $a_{28,31}$ and $a_{28,32}$).

Throughput typically remains within a firm and its statistics are considered worthless relative to inter-industry data for reasons of definition of transactions as well as confidentiality. Thus, our diagnosis of the problem of negatives directs attention to the reliability of the data (the use and make tables).

The re-estimation procedure

The negatives generated in the process of constructing an input–output coefficients matrix are clearly a nuisance. Something must be wrong. Either the model underlying the construction is misspecified or the data must be erroneous because of measurement error and so on. We begin by exploring the latter case. Our null hypothesis is that the model is correct. Data (U, V) fail to observe non-negativity of input–output coefficients,

$$UV^{-\tau} \geqslant 0 \tag{2}$$

but this constraint may hold for the true values of the inputs and the outputs. The wedge between data and true values is error. The question is if, given our null hypothesis, the errors take probable values. If not, we must reject the commodity technology model.

The situation is reminiscent of accounting theory. This is easily explained by incorporating the value-added vector of the system of national accounts, y, in our presentation. For each sector, the value of input and value-added must add to the value of output

$$U^{\tau}e + y = Ve \tag{3}$$

where e is the vector with all entries equal to one. Data (U, V, y) typically fail to meet this balance constraint. Accountants proceed to adjust the data until constraint (3) is observed. For this purpose a re-estimation procedure has been designed by Stone, Champernowne and Meade [6] and extended by van der Ploeg [9]. We adopt the idea and will re-estimate U and V so that constraint (2) instead of (3) is observed.

We need more precise notation. From now on, u_{ij} and v_{jk} refer to *true* values of inputs and outputs of sector j. Attached to them are error terms δ_{ij} and ε_{jk}. Errors can be positive due to over-reporting and negative in the case of under-reporting. True value plus error makes the datum: observed *data* are indexed by a superscript $^{\circ}$: u_{ij}° and v_{jk}°. It follows that the data equal

$$u_{ij}^{\circ} = u_{ij} + \delta_{ij}$$

and

$$v_{jk}^{\circ} = v_{jk} + \varepsilon_{jk}$$

These data are sectoral statistics which are obtained by adding establishment figures. Assume that establishments report with errors which are independent and identically distributed. Then, by the central limit theorem, sectoral errors δ_{ij} and ε_{jk} are distributed normally. We also assume that these errors are independent, across cells $(i, j, k = 1, \ldots, 39)$. The first assumption is natural, the second less so. However, the presence of correlations (for example between inputs and outputs within sectors) would modify the re-estimation procedure in a straightforward way (van der Ploeg [9]) without affecting our conclusions.

In mainstream econometrics we need many observations u_{ij}° and v_{jk}° for each i, j and k to infer the mean and variance of δ_{ij} and ε_{jk}. In input–output analysis, on the contrary, we typically have only one observation. This hampers the application of sound statistical analysis. None the less, we have used subjective information on the accuracy of the data as furnished by the statisticians who gather them. We believe that this direct method of estimating errors in measurement is a good substitute for inference.

As regards the mean of the errors, we assume that in the absence of accounting or economic constraints, statisticians have compiled data without systematic bias. Hence the means are zero. With the variances the specification is more delicate. Sir Richard Stone has pushed for revelation of such error information. All that we know is available are the standard deviations reported as percentages of the sectoral statistics underlying Barker, van der Ploeg and Weale [1]. For self-containedness we publish the sectors and the percentages in Table 4 (in the appendix).

So the variance of the first datum, u_{11}°, is $\sigma_{11}^2 = (5\%$ of $1\,420.2)^2 = 5\,042.4201$. The second one is similar, but the third is more complicated, since u_{31}° is not confined to sectors of the same reliability. Its accuracy will be neither 5% nor 10%, but some average. The reporting of errors as percentages suggests that mixed data have geometric mean accuracy. Hence, it is natural to set the variance of u_{31}° equal to $\sigma_{31}^2 = (\sqrt{0.05*0.10}u_{31})^2 = 0.05*0.10*3.5^2 = 0.06125$. The variances of all other data are determined in the same way.

We are now in a position to write down the likelihood of real values (U, V). Its logarithm is

$$L(U,V) = -\frac{1}{2}\sum_{i,j}\sigma_{ij}^{-2}(u_{ij}-u_{ij}^{\circ})^2 - \frac{1}{2}\sum_{j,k}\tau_{jk}^{-2}(v_{jk}-v_{jk}^{\circ})^2 - \sum_{i,j}\log(\sigma_{ij}^2) - \sum_{j,k}\log(\tau_{jk}^2) - \frac{1}{2}(2*39^2)\log(2\pi) \quad (4)$$

where σ_{ij}^2 is the variance of u_{ij} and τ_{jk}^2 is the variance of v_{jk}. The basic idea is to find the most likely (U, V) that is consistent with non-negativity of input–output coefficients, (2). Since the variances are assumed to be known, maximizing L is equivalent to minimizing f defined by

$$f(U,V) = \sum_{i,j}\sigma_{ij}^{-2}(u_{ij}-u_{ij}^{\circ})^2 + \sum_{j,k}\tau_{jk}^{-2}(v_{jk}-v_{jk}^{\circ})^2 \quad (5)$$

The constraints, A, are given by

$$A(U,V) = UV^{-\tau} \geq 0 \quad (6)$$

The use of (6) instead of (3) complicates the application of mathematical statistics, not so much by the inequality sign, but by the non-linearity of the constraint in at least one set of variables, namely V. The best linear unbiased estimate property of Stone, Champernowne and Meade's [6] or van der Ploeg's [9] re-estimator is lost if some of the constraints are binding. Furthermore, if the initial estimates are normally distributed, then the adjusted estimates are not necessary normally distributed. This means that it is difficult to calculate the variances of the re-estimated data. However, it is always possible to use the likelihood ratio test (Silvey [5], sections 7.1 and 7.2) to investigate whether any binding non-negativity constraints are consistent with the prior covariance matrices of the unadjusted data (see the next section). Since our conclusion will be negative, we do not really need the optimality properties mentioned above.

The objective function, f, is exceedingly simple. It has linear first order and constant second order derivatives. The function of constraints, A, is linear in U, but complicated in V. We can nevertheless write down the sensitivity of the input–output coefficients with respect to inputs and outputs.

Lemma 1. $\frac{\delta a_{ij}}{\delta u_{rs}} = 0$ if $i \neq r$, $\frac{\delta a_{rj}}{\delta u_{rs}} = w_{sj}$, and $\frac{\delta a_{ij}}{\delta v_{rs}} = -a_{is}w_{rj}$, where w_{ij} $(i, j = 1, \ldots, 39)$ are the elements of $W = V^{-\tau}$. See appendix for proof.

We have also been able to calculate the second order derivatives.

Lemma 2. $\frac{\delta^2 a_{ij}}{\delta u_{kl}\delta u_{rs}} = 0$, $\frac{\delta^2 a_{ij}}{\delta u_{kl}\delta v_{rs}} = 0$ $(i \neq k)$, $\frac{\delta^2 a_{kj}}{\delta u_{kl}\delta v_{rs}} = -w_{ls}w_{rj}$ and $\frac{\delta^2 a_{ij}}{\delta v_{kl}\delta v_{rs}} = a_{il}w_{ks}w_{rj} + a_{is}w_{rl}w_{kj}$ where w_{ij} $(i, j = 1, \ldots, 39)$ are the elements of $W = V^{-\tau}$. See appendix for proof.

We turn to a routine for non-linear constrained optimization that exploits analytical knowledge of first order and second order derivatives: E04WAF of the Numerical Algorithms Group [4]. The computation is complicated by the prohibitive size of the second order derivatives matrix, the non-convexity of the constraint set and the presence of stationary points that do not solve the constrained optimization problem, (5) and (6), globally. To keep it manageable, we aggregate the data. Aggregation usually blurs the analysis, but here it accentuates the problem and the nature of the solution.

Aggregation is by the rather traditional scheme, specified in Table 8 of the appendix (p 19). The constraint set, (6), remains unchanged. The objective function, (5), must be reinterpreted. The coefficients – the variances – are now variances of the aggregated flows. Now, as the data are independently normal distributed, the variances of sums are equal to the sums of variances. In short, the aggregation also applies to the objective function coefficients.

Results

Table 5 (see appendix, p 17) presents the aggregated inputs, the square roots of their variances as percentages (that is standard deviations) and the re-estimates. Table 6 (p 18) presents the same for the outputs. The percentages are basically weighted averages of the disaggregated percentage standard deviations. If the flows are zero so that no weights can be determined, then a blank enters. This is no problem, since zero flows remain zero in the maximum likelihood adjustment procedure for finite percentage standard deviations. The standard deviations percentages are sometimes smaller than in the disaggregated case, because of the cancelling out of errors.

We wish to draw the reader's attention to two, related results. First, the maximum likelihood estimation involves the setting of some secondary outputs equal to zero. Second, the adjustment sets some data off the 'true' values by more than two standard deviations. We will elaborate on both of these.

The solution features zero values of some variables. This is easily explained through the example given in the introduction. Non-negativity of the input–output coefficients of sector 1, (1), requires that its inputs exceed the secondary output requirements. But, if such an input, say u_{11}, is zero, then, since standard deviations are given as percentages so that zeros remain zero, the secondary output requirements, $a_{12}v_{12}$, must be zero. Hence u_{12} or v_{12} must be set zero to meet non-negativity of a_{11}. In short, if an input is zero, then the corresponding input requirements of the secondary products of that sector must also be zero. The maximum likelihood readjustment brings this about by setting to zero the secondary outputs with such an input requirement.

In this study, Table 7 shows that secondary outputs v_{24}, v_{27}, v_{28}, v_{29} and v_{59} are set to zero. Clearly these constitute significant adjustment steps. They are independent of the standard deviations of the variables and may exceed them by multiples. For example, if a flow belongs to a sector of which data are accurate up to 5%, then a readjustment towards zero corresponds to 20 standard deviations. This holds for the mining and gas sector, 2. In other words, the data have errors that have much less than even 1% probability of being observed. This is, of course, very unlikely. Statisticians reject unlikely outcomes. In our context, we shall be forced to reject the model that underlies the re-estimation procedure, that is constraint (6) or the commodity technology model for input–output coefficients.

The raw input–output coefficients, $UV^{-\tau}$ based on the aggregated data, as well as the adjusted input–output coefficients, $UV^{-\tau}$ stemming from the constrained optimization problem (5, 6), are reported in Table 7 of the appendix (p 18). They are multiplied by a factor of 100, so that the unit is pennies per pound. It is interesting to note that, basically, our adjustment procedure sets the negatives equal to zero up to digit level 3. That is the common practice in dealing with the problem. Routine practice is thus given a statistical foundation. Table 6 also confirms that the coefficient adjustments are minor. However, coefficients are derived constructs. Any change must be conceived as the result of a change in data. Although the change in coefficients is small, the underlying change in data must be large. Large data must be reduced all the way to zero. This involves many standard deviations and, therefore, a large leap in terms of likelihood. So although the common practice of ignoring the input–output coefficients seems justified at first sight, statistical analysis raises doubts.

One way of obtaining insight into this question is the use of the likelihood ratio test (Silvey [5] sections 7.1 and 7.2). Since the variances in the unadjusted data are assumed to be known from the Central Statistical Office, twice times the difference in the log likelihood, (4), equals (minus) the difference in the 'sum of squares', (5), and this is the test statistic of the likelihood ratio test. It is distributed as a $\chi^2(r)$ variate, where r is the number of *binding* non-negativity constraints. In our case $r = 9$ and the test statistic is 1914.2. Since the critical value of χ^2 (9) at the 5% significance level is 16.92, the non-negativity constraints are violated at the 5% level. This leaves no room for other than for an empirical rejection of the commodity technology model.

Conclusion

We find that the magnitude of the adjustments to the use and make data which are required to ensure the non-negativity of the input–output coefficients, based on the commodity technology model, are inconsistent with the distribution of the unadjusted data. This means that we have a statistical basis for rejecting the commodity technology model. This rejection is particularly surprising given the high level of aggregation we used in our exercise. At such a high level of aggregation there are only a few negative input–output coefficients and their magnitude is tiny; but the adjustments required to satisfy non-negativity are nevertheless sweeping and inconsistent with the data.

It follows that we must accept that different industries have different technologies for producing the same commodity. This is clear when some industries produce more efficiently than others, but it may hold even in a perfectly competitive world. The A matrix is limited to material inputs, and apparent comparative disadvantages may be offset by lower direct factor costs (fixed capital or labour). Since Kop Jansen and ten Raa [3] reject the alternatives to the commodity technology model for other reasons, we must abandon the very linear framework of deriving technical unit coefficients (A) from the black box of input and output flows (U, V). We must account for the output destination of inputs within sectors. In the absence of such information we may continue to compute the pure commodity technology input–output matrix, but limit its application to final demand or value-added vectors of which the proportions are close to the ones in the year on which the construction of the technical coefficients is based. We can still suppress the negatives as usual, since their magnitude is small, but within the just described class of admissable scenarios industrial output or price projections will be positive anyway and the zero setting yields modifications which are redundant. In short, adjustments, even when based on information about reliabilities, make projections along trends worse instead of better. We should either determine the within sector commodity destination of inputs or limit the applications to scenarios proportioned close to the structure of the economy in the years of construction and leave the negatives as they are.

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Appendix

Proof of lemma 1. $A = UV^{-\tau} = UW$, hence $dA = dU.W + UdW$. To determine $\frac{\delta a_{ij}}{\delta u_{rs}}$, put $(dU)_{rs} = \delta u_{rs}$ and zeros elsewhere and put $dW = 0$ as W depends on V only. Non-r rows of dU being zero, it follows that $\frac{\delta a_{ij}}{\delta u_{rs}} = 0$ for $i \neq r$. The rth row of the equation reads $\delta a_{rj} = \delta u_{rs}.w_{sj}$, $j = 1, \ldots, 39$. To determine $\frac{\delta a_{ij}}{\delta v_{rs}}$, put $dU = 0$. Now, differentiating $WV^{\tau} = I$, we have $(dW)V^{\tau} + WdV^{\tau} = 0$ or $dW = -W(dV^{\tau})V^{-\tau} = -W(dV^{\tau})W$. Hence $dA = -UW(dV^{\tau})W = -A(dV^{\tau})W$. We must put $(dV)_{rs} = \delta v_{rs}$ and zeros elsewhere. Then $\delta a_{ij} = -a_{is}\delta v_{rs}.w_{rj}$ or $\frac{\delta a_{ij}}{\delta v_{rs}} = -a_{is}w_{rj}$.

Proof of lemma 2. Lemma 1 shows that the first order derivatives with respect to U depend only on W hence V. Consequently, the second order derivatives with respect to U vanish. The cross partials vanish for $i \neq k$ by the first part of lemma 1. If $i = k$ we have $\frac{\delta^2 a_{kj}}{\delta u_{kl}\delta v_{rs}} = \frac{\delta w_{lj}}{\delta v_{rs}}$ by lemma 1. To evaluate this, note that $dW = -W(dV^{\tau})W$ (proof of lemma 1). Pit $(dV)_{rs} = \delta v_{rs}$ and zeros elsewhere, then the (l,j)th component reads $\delta w_{lj} = -w_{ls}\delta v_{rs}w_{rj}$ or $\frac{\delta w_{lj}}{\delta v_{rs}} = -w_{ls}w_{rj}$. It follows that $\frac{\delta^2 a_{kj}}{\delta u_{kl}\delta v_{rs}} = -w_{ls}w_{rj}$. It remains to determine the second order derivatives with respect to V. By lemma 1 and the product rule, $\frac{\delta^2 a_{ij}}{\delta v_{kl}\delta v_{rs}} = \frac{-\delta}{\delta v_{kl}}(a_{is}w_{rj}) = -\frac{\delta a_{is}}{\delta v_{kl}}w_{rj} - a_{is}\frac{\delta w_{rj}}{\delta v_{kl}}$. By lemma 1 and the above expression for the partials of W with respect to V, we obtain $\frac{\delta^2 a_{ij}}{\delta v_{kl}\delta v_{rs}} = a_{il}w_{ks}w_{rj} + a_{is}w_{rl}w_{kj}$.

Table 1. Use table *U*.

	Agriculture etc	Coal mining	Mining	Petroleum and gas	Food manufacturing	Drink	Tobacco	Coal products	Petroleum products	Chemicals	Iron and steel	Non-ferrous metals	Mechanical engineering
Agriculture etc	1 420.2	0.0	0.0	0.0	2 623.1	177.4	146.7	0.0	0.0	36.0	0.0	0.0	1.6
Coal mining	0.1	0.0	0.1	0.0	3.7	0.7	0.0	172.6	0.0	2.3	133.3	1.4	1.2
Mining	3.5	0.0	12.2	0.0	9.2	0.3	0.0	0.0	0.0	90.0	177.0	174.5	5.3
Petroleum and gas	0.0	0.0	0.0	47.0	0.0	0.0	0.0	0.0	3 104.4	0.0	0.0	0.0	0.0
Food manufacturing	912.4	0.0	0.0	0.0	2 456.5	83.3	0.0	0.0	3.4	129.8	0.0	0.0	0.0
Drink	16.0	0.0	0.0	0.0	15.1	236.7	0.0	0.0	0.5	2.6	0.0	0.0	0.0
Tobacco	8.3	0.0	0.0	0.0	0.0	0.0	3.0	0.0	0.0	0.0	0.0	0.1	0.0
Coal products	0.2	0.0	0.1	0.0	1.3	0.0	0.0	14.2	0.0	20.0	327.1	3.5	5.5
Petroleum products	43.6	17.2	79.3	5.2	187.6	60.6	3.3	1.8	631.8	494.9	131.2	20.8	106.0
Chemicals	349.0	8.6	12.9	5.5	178.9	23.6	5.9	6.7	137.8	2 468.5	53.3	13.0	99.0
Iron and steel	1.6	46.6	0.2	5.6	19.6	0.4	0.0	0.1	2.0	12.7	850.3	5.4	1 017.6
Non-ferrous metals	1.3	0.0	0.0	0.0	24.9	0.5	6.3	0.0	0.0	59.2	159.9	522.9	213.4
Mechanical engineering	41.5	106.6	27.8	17.1	45.7	11.0	1.9	1.8	11.5	51.3	113.9	20.4	1 221.2
Instrument engineering	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.4	0.8	2.9	25.7
Electrical engineering	1.0	26.3	1.4	0.0	0.7	0.3	0.0	0.1	0.0	1.8	18.1	12.0	223.4
Ship building	16.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.1	0.0	0.0
Motor vehicles	3.6	1.7	1.7	0.3	10.5	3.6	0.1	0.0	0.4	2.9	26.9	0.7	27.0
Aerospace equipment	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.1	0.0	0.0
Other vehicles	20.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.2
Metal goods	15.7	10.1	5.0	15.4	197.4	92.7	7.5	7.7	28.2	198.4	179.3	77.9	485.7
Textiles	28.0	1.5	0.1	0.0	7.0	1.0	0.1	0.2	0.1	30.1	0.7	0.5	13.2
Leather, clothing etc	1.7	1.5	0.0	0.0	4.4	1.7	0.2	0.7	0.6	12.7	3.7	1.1	5.1
Bricks	15.0	4.2	5.1	0.0	36.7	49.2	0.0	0.0	0.3	32.8	68.9	0.9	34.6
Timber and furniture	20.4	21.0	1.1	0.0	15.6	5.9	0.7	0.2	1.6	16.1	4.2	1.9	29.6
Paper and board	14.1	0.6	4.3	0.0	295.6	57.5	90.6	0.0	1.1	160.1	9.0	11.3	42.2
Printing and publishing	1.7	0.3	0.0	0.0	18.9	13.9	23.7	0.0	0.4	33.1	0.8	1.0	29.0
Other manufacturing	16.2	5.8	6.3	0.0	118.7	27.5	2.7	0.0	1.5	86.8	12.7	1.5	94.8
Construction	129.6	87.5	0.1	18.4	9.9	7.5	0.6	0.0	0.9	7.5	8.0	2.4	106.8
Gas	2.4	0.1	0.3	0.0	15.2	3.8	0.6	16.3	1.7	70.8	30.3	8.0	18.4
Electricity	33.1	76.2	21.7	0.0	83.4	16.1	2.7	6.1	13.5	179.7	170.1	36.6	74.5
Water	10.5	1.1	0.1	0.0	13.6	6.4	0.1	0.6	4.0	23.2	6.4	1.8	7.2
Rail	6.4	6.5	36.1	0.0	7.8	3.6	2.3	4.7	1.5	17.2	68.3	1.0	8.1
Road	62.7	8.7	2.6	12.0	192.4	29.9	4.1	11.1	5.8	95.2	139.1	28.4	157.5
Other transport	21.7	7.3	3.8	62.0	178.5	18.7	10.9	9.3	371.9	133.9	55.4	56.6	61.7
Communication	16.7	2.8	2.0	0.0	18.0	5.2	1.3	0.2	1.4	20.3	6.4	2.7	42.1
Distribution	222.8	12.8	10.9	7.6	661.5	90.2	11.1	2.4	142.5	231.9	364.1	162.5	335.6
Business services	119.4	7.4	14.0	3.9	84.1	20.2	3.8	2.8	39.6	71.0	48.2	15.3	117.2
Professional services	19.9	18.0	5.8	19.5	51.7	20.6	7.2	4.2	8.8	61.1	11.6	9.5	36.6
Miscellaneous services	76.8	122.2	35.5	92.8	362.5	168.1	52.7	20.8	17.4	434.8	56.4	47.6	201.2

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Table 1 continued.

	Instrument engineering	Electrical engineering	Ship building	Motor vehicles	Aerospace equipment	Other vehicles	Metal goods	Textiles	Leather, clothing etc	Bricks	Timber and furniture	Paper and board	Printing and publishing
Agriculture etc	0.0	0.8	0.1	0.0	0.0	0.0	0.1	109.0	87.4	0.9	53.7	3.2	0.0
Coal mining	0.1	0.6	0.1	9.4	0.6	0.8	0.5	4.7	0.4	26.1	0.2	4.5	0.1
Mining	0.0	0.6	0.2	4.5	0.0	0.4	0.6	1.4	0.0	104.6	0.3	5.3	0.0
Petroleum and gas	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Food manufacturing	0.0	0.0	0.0	0.0	0.0	0.0	1.5	3.6	9.8	0.8	4.3	6.7	0.0
Drink	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.6	0.7	0.1	0.0
Tobacco	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.3	0.0
Coal products	0.0	0.1	0.4	4.6	0.0	0.1	1.5	0.0	0.0	4.9	0.2	0.0	0.0
Petroleum products	9.9	53.3	13.1	108.0	30.5	10.2	68.0	74.7	19.5	173.9	65.9	57.4	28.8
Chemicals	15.9	168.4	11.4	54.9	12.2	6.0	71.4	341.2	38.9	83.3	43.1	145.3	65.9
Iron and steel	17.8	205.7	78.5	732.9	38.0	123.8	824.1	1.1	0.6	9.6	19.4	3.7	0.9
Non-ferrous metals	26.6	352.9	14.2	123.3	53.1	5.8	259.8	0.7	0.0	4.1	7.9	11.6	11.8
Mechanical engineering	2.3	147.5	88.3	62.4	53.5	33.4	69.5	38.0	9.9	46.4	19.3	26.0	23.3
Instrument engineering	64.4	23.2	3.0	3.8	5.0	0.9	2.8	0.0	0.0	0.0	0.0	0.0	0.0
Electrical engineering	95.1	895.7	66.0	218.5	124.0	28.0	10.9	0.0	0.0	0.8	6.4	0.1	0.1
Ship building	0.0	0.0	168.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Motor vehicles	0.2	4.2	7.9	1 004.0	7.3	71.0	9.3	5.3	0.8	5.3	3.8	1.1	1.1
Aerospace equipment	0.0	0.0	0.0	0.0	388.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Other vehicles	0.0	0.0	0.0	10.5	0.0	118.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Metal goods	68.4	392.6	22.4	328.6	120.3	43.4	673.5	49.0	28.6	72.3	89.9	28.0	8.8
Textiles	3.7	9.6	2.6	38.5	1.1	5.9	13.4	1 554.8	608.5	24.5	102.3	17.1	13.5
Leather, clothing etc	6.9	2.7	2.3	6.0	0.9	1.7	3.2	7.0	273.0	2.4	3.2	0.8	0.2
Bricks	6.8	72.0	1.3	36.4	1.1	1.3	26.2	2.3	0.8	167.4	25.4	1.7	0.3
Timber and furniture	4.4	49.2	11.9	22.5	1.4	3.7	26.2	2.2	3.1	15.6	683.2	21.4	2.3
Paper and board	12.3	82.4	2.0	31.6	7.9	3.8	40.7	78.6	31.2	62.1	39.5	1 003.9	472.3
Printing and publishing	9.6	28.0	1.6	33.0	2.7	5.1	9.0	7.3	12.2	10.1	15.3	24.6	349.1
Other manufacturing	38.9	119.9	4.5	279.9	12.2	28.8	45.4	28.0	76.0	46.7	78.3	30.2	13.4
Construction	1.7	9.2	3.3	7.6	6.2	1.4	9.8	8.0	5.4	6.1	6.8	4.8	10.9
Gas	1.7	10.0	1.5	15.7	2.7	2.4	20.1	6.4	1.7	25.9	1.8	7.4	2.4
Electricity	8.5	51.4	15.1	56.1	20.7	9.1	58.2	78.5	12.6	90.2	21.5	36.1	16.1
Water	1.4	6.8	2.0	5.8	2.0	0.9	4.6	9.3	2.1	3.3	0.2	3.9	0.6
Rail	2.4	5.9	1.5	13.1	1.2	1.3	5.5	4.2	2.5	18.5	1.2	3.1	20.9
Road	6.5	62.7	6.9	95.6	6.9	8.4	98.9	30.8	13.6	135.4	36.8	48.0	20.3
Other transport	10.3	50.2	6.6	43.3	20.1	9.0	101.5	62.1	26.4	30.9	64.6	68.6	41.2
Communication	8.2	31.0	2.2	24.4	9.0	2.4	17.5	11.0	6.3	9.5	11.2	8.3	47.3
Distribution	38.3	208.8	28.6	231.2	35.5	32.5	169.9	201.9	98.9	68.2	51.2	66.7	37.1
Business services	18.6	83.6	16.7	51.9	43.3	7.6	55.8	50.8	35.6	40.2	55.2	29.8	50.2
Professional services	4.3	54.5	4.7	16.8	6.7	4.4	15.0	27.8	9.0	19.2	14.2	13.5	27.2
Miscellaneous services	23.2	322.3	19.2	78.9	22.5	3.2	78.8	169.6	45.1	119.3	66.9	80.6	179.9

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Table 1 continued.

	Other manufacturing	Construction	Gas	Electricity	Water	Rail	Road	Other transport	Communication	Distribution	Business services	Professional services	Miscellaneous services
Agriculture etc	56.1	4.1	0.0	0.0	0.0	0.0	0.0	15.3	0.0	1.5	0.0	0.0	157.6
Coal mining	1.1	0.4	0.2	1 078.0	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Mining	3.6	294.0	0.0	0.0	0.0	4.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Petroleum and gas	0.0	0.0	235.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Food manufacturing	1.3	2.9	0.0	0.0	0.0	0.0	0.0	57.5	0.0	124.5	0.0	0.0	638.0
Drink	0.1	0.0	0.0	0.0	0.0	0.0	0.0	22.2	0.0	0.0	0.0	0.0	74.3
Tobacco	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.2	0.0	0.0	0.0	0.0	0.0
Coal products	0.0	0.0	0.9	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Petroleum products	44.3	107.2	56.3	493.8	11.0	54.0	219.1	889.3	7.6	111.9	26.8	26.3	149.6
Chemicals	522.3	233.5	0.5	34.1	6.6	3.7	3.1	12.1	1.3	29.8	2.1	20.3	224.4
Iron and steel	14.0	411.8	25.7	5.0	14.3	22.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Non-ferrous metals	16.0	171.5	0.8	4.4	3.4	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0
Mechanical engineering	62.5	692.1	28.7	25.9	43.2	0.0	0.0	1.0	0.0	26.4	10.8	4.7	26.5
Instrument engineering	0.0	0.0	3.5	0.8	0.2	0.0	0.0	2.1	0.0	0.0	0.0	26.3	18.7
Electrical engineering	2.1	202.6	0.0	91.1	0.6	19.4	25.4	11.5	173.2	65.4	2.4	1.4	207.0
Ship building	0.0	0.0	0.0	0.0	0.0	0.0	0.0	157.6	0.0	0.0	0.0	0.0	3.2
Motor vehicles	1.5	35.3	1.7	1.7	0.5	0.0	117.2	22.7	3.1	8.7	0.0	0.0	448.7
Aerospace equipment	0.0	0.0	0.0	0.0	0.0	0.0	0.0	94.1	0.0	0.0	0.0	0.0	0.0
Other vehicles	0.0	0.0	0.0	0.0	0.0	145.3	0.0	2.0	0.0	0.0	0.0	0.0	0.0
Metal goods	106.2	372.9	53.2	23.8	3.4	12.1	8.8	5.0	13.2	17.5	4.1	0.2	63.1
Textiles	120.7	31.1	0.0	0.0	0.1	0.0	0.0	1.1	0.0	227.9	3.7	1.7	42.8
Leather, clothing etc	6.0	4.4	0.6	1.1	0.7	4.5	4.1	0.8	4.8	22.8	0.0	0.0	11.5
Bricks	13.1	1 318.1	0.0	11.3	3.5	18.0	0.0	0.0	3.3	34.2	0.0	1.2	35.2
Timber and furniture	30.8	662.7	0.5	3.3	0.5	6.0	0.0	16.6	3.3	50.3	36.3	10.5	169.8
Paper and board	92.1	9.2	3.1	5.1	0.6	8.7	26.3	5.8	0.0	199.1	70.5	62.2	52.4
Printing and publishing	8.6	36.1	4.1	2.9	1.0	10.6	4.3	36.4	32.6	329.3	424.6	134.9	30.3
Other manufacturing	131.3	266.4	5.1	2.9	3.7	11.4	119.5	16.5	17.5	118.6	21.6	6.5	129.4
Construction	4.3	2 836.3	68.9	111.6	3.6	3.3	2.4	14.2	12.0	78.5	326.9	43.7	42.1
Gas	8.3	3.9	0.2	48.7	0.0	4.6	1.4	5.9	6.7	55.8	10.6	14.9	42.1
Electricity	52.0	42.6	8.0	30.3	27.5	48.7	3.0	16.6	22.9	221.3	61.8	45.0	127.8
Water	2.4	0.0	0.8	13.4	10.8	0.0	0.0	2.6	3.3	30.9	17.2	5.6	11.0
Rail	3.6	6.1	1.8	0.1	0.0	0.0	0.0	1.0	56.9	27.0	1.9	2.6	8.1
Road	42.9	250.9	2.8	23.2	0.7	2.0	27.2	18.4	1.9	1 235.3	0.0	36.3	120.1
Other transport	33.5	68.3	1.8	18.2	0.3	11.1	2.3	1 646.1	44.4	767.8	18.3	30.2	117.7
Communication	12.6	53.4	13.4	15.4	1.9	18.2	33.2	61.2	243.1	342.4	575.7	290.3	182.5
Distribution	94.4	456.3	17.1	41.0	3.9	10.5	42.6	11.1	10.1	67.4	17.8	38.5	210.9
Business services	53.0	155.9	13.5	26.4	4.1	2.5	167.9	210.1	60.2	504.2	2 006.5	259.1	912.4
Professional services	22.4	40.2	18.5	34.1	0.0	9.2	22.5	86.1	31.4	10.3	261.8	203.6	549.9
Miscellaneous services	133.7	74.2	136.0	260.6	4.3	24.2	3.0	146.0	102.6	30.0	40.0	363.4	512.8

Table 2. Make table *V*.

	Agriculture etc	Coal mining	Mining	Petroleum and gas	Food manufacturing	Drink	Tobacco	Coal products	Petroleum products	Chemicals	Iron and steel	Non-ferrous metals	Mechanical engineering
Agriculture etc	5 616.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Coal mining	0.0	1 855.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.0
Mining	0.0	0.3	469.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Petroleum and gas	0.0	0.0	0.0	297.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Food manufacturing	0.0	0.0	0.0	0.0	9 463.8	17.4	0.0	0.0	0.1	16.8	0.0	0.0	0.0
Drink	0.0	0.0	0.0	0.0	2.0	1 732.0	0.0	0.0	0.0	7.9	0.0	0.0	0.0
Tobacco	0.0	0.0	0.0	0.0	0.0	0.0	629.1	0.0	0.0	0.0	0.0	0.0	0.0
Coal products	0.0	0.0	0.0	0.0	0.0	0.0	0.0	328.9	0.0	3.1	0.0	0.0	0.0
Petroleum products	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5 040.2	33.9	0.0	0.0	0.0
Chemicals	0.0	0.0	0.0	0.0	21.8	0.0	0.0	0.2	78.9	6 928.0	0.3	7.7	17.4
Iron and steel	0.0	0.0	0.0	0.0	0.0	0.0	0.0	248.0	0.0	5.8	4 333.3	7.0	28.9
Non-ferrous metals	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	10.6	6.6	1 679.9	5.6
Mechanical engineering	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4	40.2	4.4	7 701.6
Instrument engineering	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	7.3	0.5	0.0	10.3
Electrical engineering	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.4	0.0	41.0
Ship building	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.8	0.0	48.8
Motor vehicles	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	32.6	3.3	142.0
Aerospace equipment	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	6.3
Other vehicles	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.0	36.6
Metal goods	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.1	19.3	27.6	79.4
Textiles	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	6.0	0.3	0.0	2.2
Leather, clothing etc	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Bricks	0.0	0.0	33.9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.8
Timber and furniture	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.0	1.3
Paper and board	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	16.2	0.3
Printing and publishing	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.3
Other manufacturing	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	46.5	0.0	0.7	0.3
Construction	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Gas	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Electricity	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Water	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Rail	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Road	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Other transport	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Communication	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Distribution	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Business services	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Professional services	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Miscellaneous services	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

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Table 2 continued.

	Instrument engineering	Electrical engineering	Ship building	Motor vehicles	Aerospace equipment	Other vehicles	Metal goods	Textiles	Leather, clothing etc	Bricks	Timber and furniture	Paper and board	Printing and publishing
Agriculture etc	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Coal mining	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Mining	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	36.0	0.0	0.0	0.0
Petroleum and gas	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Food manufacturing	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.0
Drink	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	4.4	0.0	0.0
Tobacco	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Coal products	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Petroleum products	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4	0.0	0.0	0.0
Chemicals	5.8	0.0	0.0	0.0	0.0	0.0	3.6	3.3	0.0	12.5	0.0	5.2	0.0
Iron and steel	0.0	0.3	0.8	4.9	0.0	12.4	30.7	0.0	0.0	0.8	0.0	0.0	0.0
Non-ferrous metals	0.0	0.0	0.5	0.0	0.0	0.4	42.3	0.0	0.0	0.0	0.0	0.0	0.0
Mechanical engineering	34.3	13.0	52.3	59.1	10.5	13.8	83.2	0.0	0.0	0.4	0.3	0.0	0.0
Instrument engineering	828.1	74.3	0.0	1.4	6.1	0.0	4.6	0.0	1.9	0.0	0.0	0.7	0.0
Electrical engineering	66.7	5 572.3	33.7	6.3	23.2	7.8	7.5	0.0	0.0	1.3	0.0	0.0	0.0
Ship building	0.0	0.1	1 058.8	0.0	0.0	0.0	0.4	0.0	0.0	4.1	0.0	0.0	0.0
Motor vehicles	0.0	16.5	7.8	4 729.9	2.8	128.6	30.4	0.0	0.0	0.0	0.0	0.0	0.0
Aerospace equipment	0.0	22.3	2.1	2.0	1 653.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Other vehicles	0.0	0.0	0.0	1.6	0.0	721.5	0.3	0.0	0.0	0.0	0.0	0.0	0.0
Metal goods	2.8	18.0	0.5	20.3	4.5	1.1	4 215.9	0.0	0.0	0.3	2.7	1.7	0.2
Textiles	0.0	0.0	0.0	2.2	0.0	0.0	0.1	4 274.7	8.1	0.4	8.2	0.0	0.0
Leather, clothing etc	0.4	0.0	0.0	0.0	0.0	0.0	0.0	1.5	2 400.7	0.0	0.0	0.0	0.0
Bricks	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	2 360.5	1.3	0.0	0.0
Timber and furniture	0.2	0.0	0.0	0.7	0.0	0.0	31.1	1.6	0.0	1.4	2 382.3	12.4	0.1
Paper and board	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.9	2 459.4	37.1
Printing and publishing	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.7	0.0	0.5	85.5	2 677.4
Other manufacturing	0.3	0.2	0.1	0.0	0.1	0.4	4.7	7.4	15.8	3.0	10.7	31.9	0.2
Construction	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Gas	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Electricity	0.0	36.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Water	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Rail	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Road	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Other transport	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Communication	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Distribution	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Business services	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Professional services	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Miscellaneous services	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

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Table 2 continued.

	Other manufacturing	Construction	Gas	Electricity	Water	Rail	Road	Other transport	Communication	Distribution	Business services	Professional services	Miscellaneous services
Agriculture etc	0.0	30.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2
Coal mining	0.0	11.7	0.0	2.0	0.0	0.0	17.5	0.0	0.0	0.0	0.0	0.0	0.0
Mining	0.0	0.4	0.0	0.0	0.0	0.0	50.1	0.0	0.0	0.0	0.0	0.0	0.0
Petroleum gas	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Food manufacturing	0.2	10.8	0.0	0.0	0.0	0.0	352.8	0.0	0.0	145.7	0.0	0.0	0.0
Drink	0.0	7.3	0.0	0.0	0.0	0.0	111.4	0.0	0.0	129.5	0.0	0.0	0.0
Tobacco	0.8	1.0	0.0	0.0	0.0	0.0	5.5	0.0	0.0	−13.9	0.0	0.0	0.0
Coal products	0.0	0.4	3.3	0.0	0.0	0.0	0.1	0.0	0.0	0.9	0.0	0.0	0.0
Petroleum products	0.0	2.8	0.0	2.6	0.0	0.0	9.7	0.0	0.0	1.6	0.0	0.0	0.0
Chemicals	27.5	52.9	0.1	10.8	0.0	0.0	73.7	0.0	0.0	175.1	0.0	0.0	0.0
Iron and steel	0.0	6.2	8.6	7.2	0.0	0.0	45.3	0.0	0.0	8.9	0.0	0.0	3.8
Non-ferrous metals	0.0	2.0	0.0	0.0	0.0	0.0	12.1	0.0	0.0	9.2	0.0	0.0	0.0
Mechanical engineering	5.0	12.2	0.0	0.0	0.0	0.0	126.1	0.0	0.0	167.6	0.0	0.0	0.0
Instrument engineering	2.3	2.4	0.0	0.0	0.0	0.0	14.8	0.0	0.0	22.1	0.0	0.0	0.1
Electrical engineering	8.7	32.7	0.0	0.0	0.0	0.0	63.2	0.0	0.0	49.9	0.0	0.0	15.9
Ship building	0.0	9.0	0.0	0.0	0.0	0.0	5.3	0.0	0.0	2.6	0.0	0.0	0.1
Motor vehicles	5.5	17.5	0.0	0.0	0.0	0.0	68.1	0.0	0.0	151.3	0.0	0.0	0.0
Aerospace equipment	0.0	0.6	0.0	0.0	0.0	0.0	12.0	0.0	0.0	−4.0	0.0	0.0	0.0
Other vehicles	0.0	0.2	0.0	0.0	0.0	0.0	3.0	0.0	0.0	2.4	0.0	0.0	0.0
Metal goods	32.7	5.2	0.0	0.0	0.0	0.0	81.4	0.0	0.0	41.2	0.0	0.0	0.0
Textiles	19.4	11.4	0.0	0.0	0.0	0.0	52.2	0.0	0.0	25.0	0.0	0.0	0.0
Leather, clothing etc	11.0	0.7	0.0	0.0	0.0	0.0	34.7	0.0	0.0	31.3	0.0	0.0	0.0
Bricks	3.6	8.4	0.0	0.0	0.0	0.0	69.8	0.0	0.0	23.6	0.0	0.0	0.0
Timber and furniture	12.4	1.5	0.0	0.0	0.0	0.0	96.5	0.0	0.0	62.5	0.0	0.0	0.0
Paper and board	36.8	1.7	0.0	0.0	0.0	0.0	40.2	0.0	0.0	26.8	0.0	0.0	0.0
Printing and publishing	0.9	1.5	0.0	0.0	0.0	0.0	56.0	0.0	0.0	30.1	0.0	0.0	0.0
Other manufacturing	2614.9	7.7	0.0	0.0	0.0	0.0	44.1	0.0	0.0	64.9	0.0	0.0	0.0
Construction	0.0	15669.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	21.9	0.0	0.0	0.0
Gas	0.0	51.5	1287.4	0.0	0.0	0.0	66.4	0.0	0.0	32.4	0.0	0.0	0.0
Electricity	0.0	335.9	0.0	3583.7	0.0	0.0	101.5	0.0	0.0	55.5	0.0	0.0	3.3
Water	0.0	73.3	0.0	0.0	654.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Rail	0.0	46.7	0.0	3.6	0.0	1099.9	32.8	0.0	0.0	0.0	11.5	0.0	29.8
Road	0.0	0.0	0.0	0.0	0.0	0.0	2832.7	0.0	0.0	0.0	6.9	0.0	6.5
Other transport	0.0	0.0	0.0	0.0	0.0	0.0	0.0	6074.1	0.0	0.0	53.4	0.0	6.5
Communication	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3529.2	0.0	0.0	0.0	0.0
Distribution	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	15300.0	0.0	0.0	60.0
Business services	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	7481.2	0.0	0.0
Professional services	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5867.0	0.0
Miscellaneous services	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	180.2	0.0	0.0	13207.3

Table 3. Technical coefficients matrix *A*.

	Agriculture etc	Coal mining	Mining	Petroleum and gas	Food manufacturing	Drink	Tobacco	Coal products	Petroleum products	Chemicals	Iron and steel	Non-ferrous metals	Mechanical engineering
Agriculture etc	25.28	−0.00	−0.00	0.00	27.70	10.20	23.32	−0.00	−0.00	0.42	−0.00	−0.00	0.02
Coal mining	0.00	−0.03	−0.06	0.00	0.04	0.04	−0.00	52.48	−0.02	−0.02	0.02	0.08	0.01
Mining	0.05	−0.01	2.26	0.00	0.09	0.00	−0.00	−0.01	−0.01	1.27	4.06	10.36	0.04
Petroleum and gas	0.00	0.00	0.00	15.82	0.00	0.00	−0.00	−0.18	61.60	−0.70	−0.03	0.00	0.00
Food manufacturing	16.24	−0.00	−0.00	0.00	25.93	4.71	0.00	−0.02	0.06	1.77	−0.01	−0.02	−0.02
Drink	0.28	−0.00	−0.00	0.00	0.13	13.67	0.00	−0.00	0.01	0.04	−0.00	−0.00	0.00
Tobacco	0.15	0.00	0.00	0.00	0.00	0.00	0.48	0.00	0.00	−0.00	−0.00	0.01	−0.00
Coal products	0.00	−0.00	0.01	0.00	0.01	−0.00	0.00	4.31	−0.00	0.29	7.30	0.18	0.03
Petroleum products	0.77	0.83	15.54	1.75	1.66	2.90	0.45	0.44	12.47	6.84	2.85	1.08	1.17
Chemicals	6.20	0.44	2.47	1.85	1.82	1.17	0.91	1.70	2.49	35.48	1.05	0.50	1.21
Iron and steel	0.01	2.49	0.01	1.89	0.20	0.01	−0.00	0.01	0.04	0.12	19.33	−0.29	12.70
Non-ferrous metals	0.02	−0.01	−0.01	0.00	0.26	0.02	1.00	−0.01	−0.01	0.80	3.57	30.95	2.61
Mechanical engineering	0.72	5.72	5.77	5.76	0.47	0.60	0.29	0.51	0.22	0.64	2.46	1.11	15.75
Instrument engineering	−0.00	−0.00	0.00	0.00	−0.00	−0.00	0.00	−0.00	−0.00	0.03	0.01	0.17	0.29
Electrical engineering	0.01	1.40	0.20	0.00	−0.03	−0.08	−0.01	0.03	−0.00	−0.02	0.36	0.69	2.71
Ship building	0.28	−0.00	0.00	0.00	0.00	0.00	0.00	−0.00	−0.00	0.00	0.05	−0.00	−0.11
Motor vehicles	0.06	0.05	−0.09	0.10	−0.04	−0.06	−0.02	−0.00	−0.00	−0.00	0.52	0.01	0.10
Aerospace equipment	0.00	−0.00	−0.00	0.00	−0.00	−0.00	0.00	−0.00	−0.00	0.00	0.03	0.00	−0.03
Other vehicles	0.36	−0.00	−0.00	0.00	−0.00	−0.00	0.00	−0.00	−0.00	0.00	−0.04	−0.00	−0.02
Metal goods	0.27	0.52	0.80	5.19	2.05	5.29	1.18	2.27	0.54	2.77	3.80	4.18	5.97
Textiles	0.50	0.08	−0.06	0.00	0.05	−0.07	0.01	0.05	−0.00	0.36	0.00	0.01	0.12
Leather, clothing etc	0.03	0.08	−0.02	0.00	0.04	0.08	0.03	0.21	0.01	0.18	0.07	0.06	0.05
Bricks	0.22	0.17	0.54	0.00	0.37	2.78	−0.01	−0.01	−0.00	0.39	1.57	0.02	0.41
Timber and furniture	0.34	1.10	0.18	0.00	0.15	0.23	0.10	0.05	0.03	0.18	0.08	0.09	0.35
Paper and board	0.25	0.02	0.62	0.00	3.06	3.15	14.39	−0.03	0.00	2.21	0.18	0.62	0.48
Printing and publishing	0.03	0.01	−0.05	0.00	0.16	0.63	3.77	−0.01	0.00	0.42	0.01	0.04	0.31
Other manufacturing	0.28	0.26	0.75	0.00	1.08	1.24	0.38	−0.02	0.01	1.14	0.21	0.02	1.05
Construction	2.21	4.60	−0.02	6.19	0.07	0.31	0.07	−0.07	0.01	−0.05	0.14	0.11	1.34
Gas	0.04	0.00	−0.02	0.00	0.15	0.18	0.09	4.94	0.03	1.00	0.40	0.45	0.22
Electricity	0.59	4.10	4.32	0.00	0.85	0.80	0.43	1.82	0.25	2.52	3.79	2.10	0.87
Water	0.19	0.06	0.01	0.00	0.14	0.35	0.02	0.18	0.08	0.33	0.13	0.10	0.08
Rail	0.11	0.35	7.64	0.00	0.08	0.19	0.37	1.43	0.03	0.24	1.49	0.05	0.09
Road	1.11	0.45	0.02	4.04	1.87	1.04	0.64	3.34	0.10	1.11	2.95	1.56	1.79
Other transport	0.38	0.39	0.71	20.88	1.80	0.68	1.73	2.80	7.36	1.69	1.07	3.27	0.64
Communication	0.30	0.14	0.28	0.00	0.11	0.06	0.20	0.04	0.02	0.22	0.12	0.13	0.46
Distribution	3.95	0.65	1.94	2.56	6.91	5.04	1.74	0.68	2.80	3.19	8.25	9.49	4.14
Business services	2.12	0.34	2.25	1.31	0.62	0.53	0.55	0.83	0.77	0.84	0.96	0.81	1.30
Professional services	0.35	0.96	1.09	6.57	0.51	1.13	1.14	1.25	0.17	0.86	0.17	0.54	0.45
Miscellaneous services	1.36	6.57	7.17	31.25	3.80	9.64	8.37	6.16	0.30	6.19	0.86	2.74	2.53

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Table 3 continued.

	Instrument engineering	Electrical engineering	Ship building	Motor vehicles	Aerospace equipment	Other vehicles	Metal goods	Textiles	Leather, clothing etc	Bricks	Timber and furniture	Paper and board	Printing and publishing
Agriculture etc	−0.02	0.01	0.01	−0.00	−0.00	−0.00	−0.02	2.53	3.63	0.03	2.24	0.10	−0.01
Coal mining	0.01	0.01	0.00	0.19	0.04	0.11	0.01	0.11	0.02	1.11	0.01	0.18	−0.00
Mining	−0.02	−0.00	−0.02	0.05	−0.00	0.05	−0.08	0.02	−0.00	4.39	0.01	0.14	−0.01
Petroleum and gas	0.01	−0.00	0.00	0.00	0.00	0.00	0.00	0.00	−0.00	−0.00	−0.00	0.00	−0.00
Food manufacturing	−0.04	−0.02	−0.00	−0.02	0.00	−0.00	0.03	0.08	0.40	0.03	0.16	0.26	−0.02
Drink	−0.00	−0.00	−0.00	0.00	0.00	0.00	0.00	−0.00	0.00	0.03	0.03	0.00	−0.00
Tobacco	−0.00	−0.00	0.00	−0.00	0.00	0.00	0.00	0.00	0.00	−0.00	−0.00	0.01	−0.00
Coal products	−0.01	−0.00	0.03	0.05	−0.00	0.01	0.00	−0.00	0.00	0.21	0.01	−0.00	0.00
Petroleum products	0.86	0.81	1.11	2.04	1.77	1.31	1.38	1.62	0.68	6.90	2.39	2.16	0.84
Chemicals	1.24	2.94	0.99	1.04	0.69	0.76	1.48	7.83	1.52	3.45	1.64	5.56	2.27
Iron and steel	1.51	3.44	6.78	14.38	2.18	16.46	19.12	−0.00	0.02	0.38	0.55	0.14	0.03
Non-ferrous metals	2.55	6.24	1.20	2.40	3.11	0.66	5.84	0.01	−0.00	0.17	0.25	0.25	0.43
Mechanical engineering	−0.20	2.43	7.57	0.67	3.13	3.82	1.29	0.85	0.40	1.86	0.76	1.00	0.83
Instrument engineering	7.74	0.32	0.27	0.07	0.30	0.11	0.05	−0.00	−0.00	−0.00	−0.00	−0.00	0.00
Electrical engineering	9.94	15.83	6.09	4.33	7.26	3.73	0.07	−0.02	−0.02	−0.01	0.22	−0.02	−0.02
Ship building	0.01	−0.10	15.93	−0.02	−0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Motor vehicles	−0.09	−0.03	0.72	20.89	0.39	9.77	0.03	0.06	−0.03	0.10	−0.02	−0.02	−0.05
Aerospace equipment	−0.16	−0.10	0.00	−0.01	23.52	0.00	−0.02	0.00	0.00	0.00	0.00	0.00	0.00
Other vehicles	0.00	−0.02	0.00	−0.22	0.00	16.37	−0.00	0.00	0.00	0.00	0.00	0.00	0.00
Metal goods	7.37	6.81	1.80	6.43	7.15	5.69	15.70	1.10	1.17	3.02	3.52	1.04	0.28
Textiles	0.32	0.14	0.23	0.73	0.06	0.80	0.26	36.29	25.28	1.01	4.20	0.60	0.46
Leather, clothing etc	0.79	0.03	0.21	0.11	0.05	0.23	0.07	0.14	11.37	0.10	0.12	0.02	−0.00
Bricks	0.67	1.22	0.00	0.70	0.04	0.15	0.58	0.02	0.03	7.05	1.04	0.05	0.00
Timber and furniture	0.42	0.83	1.07	0.42	0.07	0.49	0.57	−0.02	0.12	0.62	28.65	0.84	0.05
Paper and board	1.23	1.43	0.15	0.57	0.45	0.49	0.87	1.80	1.26	2.58	1.35	40.49	16.31
Printing and publishing	1.04	0.46	0.13	0.59	0.15	0.68	0.17	0.15	0.48	0.40	0.57	0.77	12.99
Other manufacturing	4.35	2.00	0.33	5.67	0.67	3.90	0.88	0.56	3.07	1.82	3.05	1.07	0.37
Construction	0.11	0.04	0.09	0.03	0.36	0.12	0.18	0.13	0.21	0.19	0.25	0.17	0.38
Gas	0.16	0.17	0.13	0.30	0.16	0.32	0.46	0.14	0.06	1.09	0.05	0.29	0.08
Electricity	0.85	0.86	1.36	1.04	1.23	1.20	1.29	1.81	0.49	3.74	0.82	1.40	0.54
Water	0.15	0.12	0.18	0.11	0.12	0.12	0.10	0.22	0.08	0.14	0.00	0.15	0.02
Rail	0.27	0.10	0.13	0.25	0.07	0.17	0.12	0.10	0.10	0.67	0.04	0.11	0.78
Road	0.40	1.00	0.51	1.62	0.39	1.04	2.16	0.64	0.44	5.62	1.24	1.81	0.59
Other transport	0.98	0.82	0.57	0.67	1.20	1.19	2.30	1.41	1.03	1.24	2.52	2.67	1.39
Communication	0.85	0.50	0.17	0.40	0.53	0.30	0.35	0.23	0.21	0.34	0.36	0.26	1.71
Distribution	4.11	3.58	2.46	4.49	2.06	4.27	3.74	4.66	4.07	2.79	1.99	2.54	1.26
Business services	1.87	1.32	1.46	0.81	2.55	0.95	1.11	1.08	1.35	1.46	1.96	1.02	1.68
Professional services	0.40	0.94	0.41	0.30	0.38	0.58	0.31	0.63	0.36	0.77	0.55	0.50	0.98
Miscellaneous services	2.15	5.69	1.67	1.53	1.27	0.31	1.72	3.92	1.85	4.93	2.73	3.08	6.61

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Table 3 continued.

	Other manufacturing	Construction	Gas	Electrical	Water	Rail	Road	Other transport	Communication	Distribution	Business services	Professional services	Miscellaneous services
Agriculture etc	2.10	0.03	−0.00	−0.00	−0.00	−0.03	−0.00	0.25	0.00	0.01	0.00	0.00	1.19
Coal mining	0.04	0.00	0.02	30.08	0.08	−0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Mining	0.10	1.88	−0.08	−0.18	−0.21	0.34	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Petroleum and gas	0.01	0.00	18.32	0.00	0.00	−0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Food manufacturing	−0.01	0.02	−0.02	−0.02	−0.00	−0.13	−0.01	0.94	0.00	0.79	0.00	0.00	4.82
Drink	0.00	0.00	0.00	−0.00	−0.00	−0.02	−0.00	0.36	0.00	−0.00	0.00	0.00	0.56
Tobacco	−0.00	0.00	0.00	0.00	0.00	−0.00	0.00	0.05	0.00	0.00	0.00	0.00	0.00
Coal products	−0.01	0.00	0.07	0.02	0.00	−0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Petroleum products	1.37	0.68	3.93	13.48	1.60	4.57	7.73	14.64	0.22	0.73	0.36	0.45	1.12
Chemicals	19.22	1.49	−0.03	0.77	0.84	0.22	0.11	0.20	0.04	0.19	0.03	0.35	1.70
Iron and steel	0.48	2.63	1.89	−0.14	1.89	1.91	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Non-ferrous metals	0.57	1.09	0.02	−0.04	0.40	−0.05	0.00	0.00	0.00	0.01	0.00	0.00	−0.00
Mechanical engineering	2.33	4.42	2.05	0.28	6.11	−0.20	−0.00	0.01	0.00	0.17	0.14	0.08	0.20
Instrument engineering	−0.00	0.00	0.27	0.02	0.03	−0.00	−0.00	0.03	0.00	−0.00	0.00	0.45	0.14
Electrical engineering	0.05	1.29	−0.11	2.23	−0.05	1.63	0.89	0.19	4.91	0.42	0.03	0.02	1.56
Ship building	−0.00	0.00	0.00	0.00	−0.00	−0.00	−0.00	2.59	0.00	−0.00	0.00	0.00	0.02
Motor vehicles	−0.02	0.23	−0.09	−0.09	0.05	−0.22	4.13	0.37	0.09	0.04	0.00	0.00	3.40
Aerospace equipment	−0.00	0.00	0.00	0.00	0.00	−0.00	0.00	1.55	0.00	0.00	0.00	0.00	0.00
Other vehicles	−0.00	0.00	0.00	0.00	0.00	13.21	0.00	0.03	0.00	0.00	0.00	0.00	0.00
Metal goods	3.92	2.38	4.02	0.36	0.25	0.98	0.31	0.08	0.37	0.11	0.05	0.00	0.48
Textiles	4.29	0.20	−0.05	−0.04	−0.01	−0.02	−0.00	0.02	0.00	1.49	0.05	0.03	0.30
Leather, clothing etc	0.15	0.03	0.03	0.02	0.10	0.40	0.14	0.01	0.14	0.15	0.00	0.00	0.09
Bricks	0.45	8.41	−0.34	−0.49	−0.41	1.27	−0.00	−0.00	0.09	0.22	0.00	0.02	0.26
Timber and furniture	1.02	4.23	−0.14	−0.32	−0.40	0.33	−0.00	0.27	0.09	0.32	0.49	0.18	1.28
Paper and board	2.92	0.06	0.16	0.08	0.09	0.74	0.93	0.09	0.00	1.30	0.94	1.06	0.38
Printing and publishing	0.25	0.23	0.25	0.02	0.13	0.89	0.14	0.55	0.92	2.15	5.68	2.30	0.20
Other manufacturing	4.85	1.70	0.09	−0.23	0.38	0.81	4.22	0.27	0.50	0.77	0.29	0.11	0.97
Construction	0.09	18.10	4.61	1.41	−1.48	−0.53	0.07	0.19	0.34	0.51	4.37	0.74	0.31
Gas	0.28	0.02	0.00	1.35	−0.00	0.40	0.05	0.10	0.19	0.36	0.14	0.25	0.31
Electricity	1.87	0.27	0.57	0.79	4.17	4.38	0.10	0.26	0.65	1.44	0.83	0.77	0.95
Water	0.08	−0.00	0.06	0.37	1.65	−0.01	−0.00	0.04	0.09	0.20	0.23	0.10	0.08
Rail	0.13	0.04	0.13	−0.00	−0.00	−0.00	−0.00	0.02	1.61	0.18	0.03	0.04	0.06
Road	1.36	1.59	−0.10	0.34	−0.07	0.06	0.96	0.30	0.05	8.07	0.00	0.62	0.80
Other transport	1.06	0.43	−0.01	0.38	−0.00	0.96	0.08	27.10	1.26	5.02	0.24	0.51	0.82
Communication	0.39	0.34	0.91	0.32	0.25	1.49	1.15	0.94	6.89	2.23	7.69	4.95	1.35
Distribution	3.42	2.91	1.12	0.78	0.27	0.74	1.50	0.18	0.29	0.43	0.24	0.66	1.59
Business services	1.79	0.99	0.63	0.41	0.52	−0.46	5.85	3.22	1.71	3.27	26.82	4.42	6.86
Professional services	0.81	0.26	1.38	0.89	−0.03	0.65	0.78	1.38	0.89	0.05	3.50	3.47	4.16
Miscellaneous services	4.91	0.47	10.54	7.16	0.60	2.04	0.10	2.40	2.91	0.18	0.53	6.19	3.88

Table 4. Reliabilities.

Index	Sector	Accuracy (%)	Index	Sector	Accuracy (%)
1	Agriculture etc	5	21	Textiles	5
2	Coal mining	5	22	Leather, clothing etc	5
3	Mining	10	23	Bricks	5
4	Petroleum and natural gas	5	24	Timber and furniture	5
5	Food manufacturing	5	25	Paper and board	5
6	Drink	5	26	Printing and publishing	5
7	Tobacco	5	27	Other manufacturing	5
8	Coal products	5	28	Construction	15
9	Petroleum products	5	29	Gas	5
10	Chemicals	5	30	Electricity	5
11	Iron and steel	5	31	Water	15
12	Non-ferrous metals	5	32	Rail	5
13	Mechanical engineering	5	33	Road	40
14	Instrument engineering	5	34	Other transport	40
15	Electrical engineering	5	35	Communication	5
16	Ship building	5	36	Distribution	50
17	Motor vehicles	5	37	Business services	60
18	Aerospace equipment	5	38	Professional service	60
19	Other vehicles	5	39	Miscellaneous services	60
20	Metal goods	5			

Table 5. Aggregated U, accuracies, and re-estimates.

	Agriculture	Mining	Food, drink and tobacco	Mining, gas and petroleum	Metals	Heavy manufacturing	Light manufacturing	Construction	Services
Agriculture	1 420.20	0.00	2 947.20	36.00	2.40	0.20	310.30	4.10	174.40
	5.0%	–	4.5%	5.0%	3.7%	3.5%	2.6%	8.7%	15.7%
	1 420.00	0.00	6 753.00	37.08	2.10	0.24	269.90	4.15	0.11
Mining	3.60	59.30	13.90	3 369.30	494.00	17.10	152.30	294.40	1 319.10
	6.9%	4.5%	4.9%	4.6%	3.8%	3.3%	4.9%	12.2%	4.2%
	3.60	61.20	14.65	3 292.00	494.00	17.10	152.30	292.70	1 288.00
Food, drink	936.70	0.00	2 794.60	136.30	0.10	1.70	28.30	2.90	919.70
and tobacco	4.9%	–	4.4%	4.8%	5.0%	4.4%	2.3%	8.7%	12.3%
	936.70	0.00	2 884.00	10.55	0.10	1.77	28.24	2.88	601.30
Mining, gas	392.80	128.80	461.20	3 775.70	1 007.00	392.30	1 709.60	340.70	2 385.40
and petroleum	4.5%	4.5%	2.9%	3.4%	2.1%	2.1%	2.0%	6.5%	5.9%
	392.80	128.80	461.20	3 776.00	1 007.00	392.30	1 710.00	340.70	2 385.00
Metals	45.40	231.60	111.30	142.90	6 239.10	3 023.50	336.30	1 478.00	893.00
	4.6%	2.7%	2.6%	2.8%	1.7%	2.0%	1.5%	5.0%	4.5%
	45.40	231.60	111.30	142.90	6 239.00	3 023.00	336.30	1 478.00	893.00
Heavy	55.40	34.20	311.80	237.60	1 266.40	2 973.70	401.70	408.20	1 210.90
manufacturing	2.7%	2.9%	3.5%	4.2%	2.6%	2.2%	2.1%	7.9%	7.0%
	55.40	34.20	311.80	237.60	1 266.00	2 974.00	401.70	408.20	1 211.00
Light	97.10	51.80	771.60	378.40	813.10	715.80	6 314.40	2 328.00	2 629.50
manufacturing	2.2%	2.5%	2.2%	2.5%	1.4%	2.1%	1.7%	5.6%	4.4%
	97.10	51.80	771.60	378.40	813.10	715.80	6 314.00	2 328.00	2 630.00
Construction	129.60	106.00	18.00	8.40	128.10	28.30	46.30	2 836.30	707.20
	8.7%	7.3%	6.0%	7.8%	7.3%	4.4%	3.4%	15.0%	14.5%
	129.60	106.00	18.00	8.76	128.00	28.30	46.29	2 421.00	683.20
Services	592.40	593.70	2 148.30	2 025.70	3 397.00	1 615.40	3 059.60	1 151.80	14 390.30
	7.5%	5.2%	6.2%	5.2%	3.6%	3.6%	2.6%	13.0%	12.2%
	592.40	593.70	2 148.00	2 026.00	3 397.00	1 615.00	3 060.00	1 152.00	14 390.00

Table 6. Aggregated V, accuracies, and re-estimates.

	Agriculture	Mining	Food, drink and tobacco	Mining, gas and petroleum	Metals	Heavy manufacturing	Light manufacturing	Construction	Services
Agriculture	5616.90	0.00	0.00	0.00	0.00	0.00	0.00	30.20	0.20
	5.0%	–	–	–	–	–	–	8.7%	17.3%
	5617.00	0.00	0.00	0.00	0.00	0.00	0.00	30.20	0.20
Mining	0.00	2622.40	0.00	0.80	0.00	0.00	36.00	12.10	69.60
	–	4.0%	–	5.0%	–	–	7.1%	8.4%	14.8%
	0.00	2622.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Food, drink	0.00	0.00	11 844.30	24.80	0.00	0.00	5.60	19.10	731.00
and tobacco	–	–	4.1%	3.7%	–	–	4.0%	5.9%	8.3%
	0.00	0.00	11 540.00	24.43	0.00	0.00	5.60	19.07	657.80
Mining, gas	0.00	0.00	21.80	12 413.20	31.20	3.60	48.90	56.10	277.90
and petroleum	–	–	5.0%	3.5%	3.2%	5.0%	3.2%	8.2%	10.7%
	0.00	0.00	22.58	12 560.00	31.20	3.60	48.89	53.87	288.40
Metals	0.00	0.00	0.00	272.20	20 450.70	401.50	21.40	55.50	554.80
	–	–	–	4.6%	2.6%	1.7%	2.5%	5.6%	6.3%
	0.00	0.00	0.00	273.80	20 780.00	401.50	21.42	55.35	0.00
Heavy	0.00	0.00	0.00	1.10	457.10	12 582.00	47.20	32.50	363.40
manufacturing	–	–	–	5.0%	2.0%	2.7%	3.6%	5.4%	8.0%
	0.00	0.00	0.00	1.09	452.60	12 580.00	15.73	31.95	307.50
Light	0.00	33.90	0.10	53.60	23.80	40.40	19 482.90	32.90	657.70
manufacturing	–	7.1%	5.0%	4.4%	3.5%	3.9%	1.9%	4.3%	4.3%
	0.00	33.90	0.10	53.61	23.80	40.40	20 650.00	32.90	658.40
Construction	0.00	0.00	0.00	0.00	0.00	0.00	0.00	15 669.70	21.90
								15.0%	27.4%
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	19 230.00	21.98
Services	0.00	0.00	0.00	0.00	36.20	0.00	0.00	507.40	61 567.30
				–	5.0%			6.2%	20.6%
	0.00	0.00	0.00	0.00	36.20	0.00	0.00	507.90	106 100.00

Table 7. Technical coefficients and their re-estimates.

	Agriculture	Mining	Food, drink and tobacco	Mining gas and petroleum	Metals	Heavy manufacturing	Light manufacturing	Construction	Services
Agriculture	25.28	−0.03	24.86	0.23	−0.00	−0.01	1.58	0.03	0.28
	25.28	0.00	58.50	0.18	0.01	0.00	1.31	0.02	0.00
Mining	0.05	2.18	−0.07	27.08	1.99	−0.01	0.63	1.88	2.13
	0.06	2.33	0.00	26.17	2.03	0.03	0.62	1.52	1.21
Food, drink	16.68	−0.04	23.50	1.02	−0.05	−0.03	0.09	0.02	1.49
and tobacco	16.68	0.00	24.94	0.03	0.00	0.00	0.12	0.01	0.57
Mining, gas	6.98	4.67	3.59	30.27	4.35	2.81	8.54	2.17	3.85
and petroleum	6.98	4.91	3.80	29.96	4.38	2.89	8.11	1.77	2.24
Metals	0.76	8.73	0.84	0.99	29.98	22.87	1.56	9.43	1.36
	0.77	8.83	0.90	1.00	29.55	22.93	1.49	7.68	0.79
Heavy	0.97	1.21	2.50	1.83	5.65	23.36	1.93	2.60	1.94
manufacturing	0.97	1.30	2.63	1.83	5.61	23.40	1.85	2.12	1.13
Light	1.65	1.35	6.21	2.74	3.65	5.28	32.22	14.85	4.15
manufacturing	1.66	1.98	6.51	2.77	3.71	5.43	30.46	12.10	2.42
Construction	2.21	3.93	0.06	−0.04	0.55	0.13	0.17	18.10	1.00
	2.24	4.04	0.10	0.00	0.58	0.16	0.18	12.59	0.58
Services	10.51	21.78	16.65	15.63	15.51	11.53	14.78	7.32	23.30
	10.51	22.64	17.79	15.67	15.88	11.90	14.26	5.97	13.53

Table 8. Aggregation.

1 Agriculture etc	1 Agriculture etc
2 Mining and gas	2 Coal mining
	3 Mining
	4 Petroleum and natural gas
3 Food, drink and tobacco	5 Food manufacturing
	6 Drink
	7 Tobacco
4 Mining and gas products	8 Coal products
	9 Petroleum products
	10 Chemicals
5 Metals	11 Iron and steel
	12 Non-ferrous metals
	13 Mechanical engineering
	14 Instrument engineering
	15 Electrical engineering
6 Heavy manufacturing	16 Ship building
	17 Motor vehicles
	18 Aerospace equipment
	19 Other vehicles
	20 Metal goods
7 Light manufacturing	21 Textiles
	22 Leather, clothing etc
	23 Bricks
	24 Timber and furniture
	25 Paper and board
	26 Printing and publishing
	27 Other manufacturing
8 Construction	28 Construction
9 Services	29 Gas
	30 Electricity
	31 Water
	32 Rail
	33 Road
	34 Other transport
	35 Communication
	36 Distribution
	37 Business services
	38 Professional services
	39 Miscellaneous services