Economics Letters 27 (1988) 101–104
North-Holland

CONSUMPTION, LEISURE AND EARNINGS-RELATED LIQUIDITY CONSTRAINTS
A Note *

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Received 16 December 1987
Accepted 19 January 1988

The life-cycle model with liquidity constraints produces a Euler equation with unobservable Kuhn–Tucker multipliers. If borrowing restrictions depend on earnings, preferences are non-separable between goods and leisure, and individuals are employed, we derive a Euler equation involving only observable variables.

1. Introduction

Recent studies of intertemporal behaviour in a risky environment have adopted the Euler equation approach introduced by Hall (1978). Assuming perfect capital markets, strongly separable preferences between goods and leisure and rational expectations, Hall showed that the marginal utility of consumption is a random walk.

However, several empirical papers have rejected the restrictions implied by Hall's version of the life-cycle model [Flavin (1981), Hall and Mishkin (1982), Hayashi (1985) and Weber (1987)]. Liquidity constraints and preference interactions between goods and leisure have alternatively been put forward as likely explanations of this failure.

As Muellbauer (1983), Zeldes (1985) and others point out, the Euler equation for consumption with borrowing restrictions contains an additional (unobservable) explanatory variable, the Kuhn–Tucker multiplier associated with the net wealth condition. In this note, we argue that a simple analytical framework for incorporating both earnings-dependent liquidity constraints and non-separable preferences is readily available, and we show that under weak conditions it produces an Euler equation involving only observable variables.

This note is organized as follows: in section 2 we establish our notation, and derive the standard Euler equation. Section 3 presents our proposed extension, and discusses its empirical implications. Section 4 draws some conclusions.

* The results in this note were independently derived by G. Weber and by R. Alessie and B. Melenberg. R. Alessie and B. Melenberg wish to thank A. Kapteyn and Th. Nijman for valuable comments and R. Alessie gratefully acknowledges the Netherlands Organization for the Advancement of Pure Research and the NMB Bank in the Netherlands for financial support. G. Weber is grateful for helpful discussions with M. Browning and F. Schiantarelli.

2. The life-cycle model with liquidity constraints

We assume that goods and leisure are choice variables, and that capital markets are imperfect. Consequently, consumers solve the following optimization problem:

$$\max \quad \mathrm{E}_t \sum_{\tau=t}^{L} (1+\rho)^{t-\tau} u_\tau(c_\tau, l_\tau), \tag{1}$$

$$\text{s.t.} \quad A_\tau = (1+r_{\tau-1})A_{\tau-1} + m_\tau + w_\tau(T-l_\tau) - p_\tau c_\tau \tau = t,\ldots,L, \tag{2}$$

$$A_\tau \geqslant M_\tau, \qquad \tau = t,\ldots,L, \tag{3}$$

$$l_\tau \leqslant T, \qquad \tau = t,\ldots,L, \tag{4}$$

$$A_{t-1} \quad \text{given}, \quad A_L = 0,$$

where

$u_\tau(.,.)$:= intratemporal utility function, [1]
ρ, $r_{\tau-1}$:= time preference and interest rates, respectively,
c_τ, l_τ := goods and leisure in period τ, respectively,
A_τ := non-human wealth at the end of period τ,
m_τ := non labour income in period τ,
p_τ, w_τ := goods price and wage rate in period τ,
L, T := length of life and time endownment.

The first-order conditions for period t are

$$\frac{\partial u_t(c_t, l_t)}{\partial c_t} = \lambda_t p_t, \tag{5}$$

$$\frac{\partial u_t(c_t, l_t)}{\partial l_t} = \lambda_t w_t + \nu_t, \tag{6}$$

$$\lambda_t - \mu_t = \mathrm{E}_t(1+r_t)/(1+\rho)\lambda_{t+1}, \tag{7}$$

$$\mu_t(A_t - M_t) = 0; \qquad \nu_t(T - l_t) = 0, \tag{8}$$

$$\mu_t \geqslant 0; \qquad \nu_t \geqslant 0. \tag{9}$$

The variables λ_t, λ_{t+1} denote the Lagrange multipliers associated to (2), whereas μ_t and ν_t are the Kuhn–Tucker multipliers corresponding to the borrowing and the time constraints, (3) and (4), respectively.

By using (5) we can rewrite the Euler eq. (7) as

$$\frac{(1+r_t)}{(1+\rho)} \cdot \frac{1}{p_{t+1}} \cdot \frac{\partial u_{t+1}(c_{t+1}), l_{t+1}}{\partial c_{t+1}} = \frac{1}{p_t} \frac{\partial u_t(c_t, l_t)}{\partial c_t} - \mu_t + \epsilon_{t+1}, \tag{10}$$

where the error has zero conditional mean.

[1] We assume c_t and l_t to be strictly positive, for example, by imposing enough regularity conditions on $u_\tau(.,.)$.

For estimation purposes, eq. (10) is unsatisfactory in that it contains the unobservable, endogenous variable μ_t. Only in special cases can a closed form solution be found; moreover, in general, no information can be gleaned from eq. (10) as to whether liquidity constraints are operating, i.e., whether μ_t is positive or zero.

The only cases where eq. (10) can be used to assess the importance of liquidity constraints is where prior information is available: if we know that for some observations μ_t is zero, then we can estimate (10) on this subsample and thus compute predicted μ_t for the remaining observations. We can then check whether μ_t has a positive mean for the liquidity constrained as the model predicts [Zeldes (1985)]. The trouble with the method is its absolute reliance on usually unavailable sample separation information.

3. An alternative approach: Earnings-related liquidity constraints

Let us now assume and earnings-related liquidity constraint [as in Muellbauer (1983)]:

$$A_\tau \geqslant \Psi_0 + \Psi_1 w_\tau (T - l_t), \qquad \tau = 1, \dots, L, \tag{3'}$$

where Ψ_0 and Ψ_1 are parameters. We expect the borrowing limit to be inversely related to current earnings, i.e., $\Psi_1 < 0$.

In this setting the first-order condition (6) becomes

$$\frac{\partial u_t(c_t, l_t)}{\partial l_t} = \lambda_t w_t - \Psi_1 \mu_t w_t + \nu_t, \tag{6'}$$

and we can use (5) and (6′) to obtain an expression for μ_t:

$$\mu_t = \frac{1}{\Psi_1}\left[\frac{1}{p_t}\frac{\partial u_t(c_t, l_t)}{\partial c_t} - \frac{1}{w_t}\frac{\partial u_t(c_t, l_t)}{\partial l_t} + \frac{1}{w_t}\nu_t\right]. \tag{11}$$

Finally, we substitue eq. (11) into (10) and get

$$\frac{(1+r_t)}{(1+\rho)} \cdot \frac{1}{p_{t+1}} \cdot \frac{\partial u_{t+1}(c_{t+1}, l_{t+1})}{\partial c_{t+1}} = \left[1 - \frac{1}{\Psi_1}\right]\frac{1}{p_t}\frac{\partial u_t(c_t, l_t)}{\partial c_t}$$
$$+ \frac{1}{\Psi_1}\frac{1}{w_t}\left[\frac{\partial u_t(c_t, l_t)}{\partial l_t} - \nu_t\right] + \epsilon_{t+1}, \quad \text{with} \quad \mathrm{E}_t \epsilon_{t+1} = 0. \tag{12}$$

We have thus obtained an Euler equation where μ_t does not appear. In its place, we now have the Kuhn–Tucker multiplier on leisure, ν_t, which is going to be positive when a corner solution obtains in the labour market. Once again, a closed form solution for this endogenous variable is unlikely to exist; however, contrary to the case of capital markets, in the case of the labour market sample separation imformation is readily available.

If panel data on individual households are available, we can estimate the parameters of eq. (12) by the generalized method of moments [Hansen and Singleton (1982)] by restricting the sample to the employed in period t: no selection bias will arise, because the error term is orthogonal to the selection rule (as ν_t belongs to the relevant information set). We can then formally test for the absence of liquidity constraints (i.e., by setting $1/\Psi_1 = 0$ in eq. (12)) by standard statistical methods.

Some further remarks on eq. (12) are in order:

(i) Eq. (12) holds whether a consumer is liquidity constrained or not and whether he is rationed in his labour supply choice in period $t+1$ or not.

(ii) Eq. (12) can only be derived if Ψ_1 differs from zero. The limit case where Ψ_1 equals zero is discussed in section 2: in this case *standard* rules for within period allocation of full expenditure into goods and leisure are unaffected by the presence of a binding constraint in the capital market [Blundell and Walker (1986)].

(iii) Consistent estimates of the parameters in Eq. (12) can be obtained by truncating the sample to include only the workers. Such estimates can then be used to compute ν_t for non-workers, which should be positive (this prediction provides a simple specification check).

(iv) We can use eq. (10) or (11) and the parameter estimates from (12) to compute μ_t for each individual household.

The relationship between the formal test for $(1/\Psi_1) = 0$ and the informal computation of μ_t is worth exploring. Inspection of eq. (12) and consideration of the underlying model suggest that rejection of the null in the formal test does not imply that the estimated μs should be zero: it is in fact possible for consumers not to be bound by the liquidity constraint in period t even though the earnings-related constraint exists in a non trivial form (i.e., $\Psi_1 > -\infty$).

4. Conclusions

In this paper we started from the well-known result that the life-cycle model with liquidity constraints produces an Euler equation with unobservable Kuhn–Tucker multipliers. We then showed that if borrowing restrictions depend on earning and leisure is a choice variable, the Euler equation involves only observable variables as long as we only select those consumers who are employed in period t.

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