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Secondary products and the measurement of productivity growth*

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The paper considers alternative treatments of secondary products in input–output systems and analyzes their implications for the measurement of productivity growth at both the sectoral and overall level. Two standard models of secondary products are used: (1) the commodity technology model and (2) the industry technology model. It is argued that the first model correctly relates sectoral and overall levels of productivity growth; the second model, though more conventional, aggregates sectoral levels to a biased estimate of overall productivity growth. Estimates of the two measures are provided using U.S. 85-sector input–output data for 1967, 1972, and 1977. The empirical results indicate that the alternative assumptions do not lead to significantly different estimates of commodity-level and industry-level productivity growth over this period for the full economy but do for several sectors. Moreover, changes in secondary production did not contribute significantly to the decline in productivity growth over this period but secondary production was found to have a much lower rate of productivity growth than primary production.

1. Introduction

In almost all recent studies on productivity, industry productivity is defined on the basis of the primary (or major) output of the industry. Productivity growth in the production of secondary (or by-product) output is commingled with that of the primary output. Almost all these studies implicitly assume that productivity growth of secondary products behaves in precisely the same way as that of primary products. Certain technological and market share assumptions are thus embedded in the analysis of productivity growth. As a result, changes in the level, mix, and technology of

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secondary production may potentially bias such estimates of productivity growth.

In this paper, we explicitly consider the role of secondary production in input–output systems for the measurement of productivity growth at both the sectoral and overall level. For this purpose, we formulate two models of secondary production: (i) the commodity technology model and (ii) the industry technology model.[1] Moreover, within each, productivity growth can be measured on either a commodity basis or an industry basis.

We make four contributions on the analytical level. First, we derive the relation between overall productivity growth and individual sectoral productivity growth in each of the models. In particular, we isolate the contribution of secondary output productivity growth to overall productivity growth. Second, special methodological problems are present for both the scrap sector and import sector, and we present solutions for their treatment. Scrap productivity is shown to be given by the rate of recycling, and that of the import sector by the terms of trade. Third, we show analytically how the change in overall productivity growth can be decomposed into several effects, including the change in productivity growth on the sectoral level and shifts in the composition of final output. Fourth, we prove that in the commodity technology model such a decomposition is unbiased, whereas in the industry technology model, a bias is introduced by this type of decomposition.

Empirical results are then presented for the U.S. economy for the period 1967–1977. This period has received particular attention in recent years, because it is one characterized by a sharp productivity slowdown. We make use of the so-called 'make' and 'use' tables provided by the Bureau of Economic Analysis (BEA) on the 85-order level for 1967, 1972, and 1977. These tables show, respectively, the commodities produced by each industry and the commodities consumed in production by each industry. There are three findings of particular interest. First, about 85 percent of the slowdown in overall productivity growth is attributable to reductions in sectoral rates of productivity growth, with changes in the terms of trade faced by the U.S. on the international level accounting for about a quarter of this, and the remaining 15 percent to shifts in the composition of output. This compositional effect is of the same order of magnitude as found in Wolff (1985) for a much longer period (that between 1947–1967 and 1967–1976).[2] Second, though we were able to separate out the secondary product effect, little of the slowdown can be ascribed to changes in secondary product total factor productivity (TFP) growth rates, but the levels of secondary product productivity growth rates are much lower than that of primary products

[1]Also, see ten Raa et al. (1984), Viet (1986), and Kop Jansen and ten Raa (1989) for more discussion of models of secondary production and the properties of such models.

[2]Also see Denison (1979, 1984) and Wolff (1985a) for a discussion of related findings on so-called 'compositional effects'.

throughout the period of analysis. Third, though the bias from using the industry technology model and industry-level measures of productivity growth is low overall, results on the sectoral level indicate that the bias is quite large for several sectors.

Our work seems particularly relevant to multiregional input–output analysis, where the presence of secondary products is more prevalent, and the decomposition of productivity growth into primary and secondary effects admits a natural interpretation. Moreover, the model developed in our paper can be directly adapted to apply to the decomposition of nation-wide productivity movements into regional effects. As a result, our paper appears to have several important implications for the construction of multiregional input–output models.

The remainder of the paper is divided into six parts. The methodological issues are dealt with in the next part, where we present the basic accounting framework and derive the various measures of overall and sectoral productivity growth. In section 3, we present basic results on the growth of secondary production over the period from 1967 to 1977. The treatment of the scrap sector presents special methodological difficulties, since it is exclusively a secondary product, and these are discussed in section 4. Methodological problems also exist for the treatment of imports in a productivity analysis, since they have no domestic inputs in their production, and these are dealt with in section 5. Productivity growth is studied in section 6, where results are shown on sectoral productivity growth over the period and the decomposition of the change in overall productivity growth into sectoral effects, compositional effects, and secondary product effects. Concluding remarks are made in the final section of the paper.

2. The accounting framework and derivation of productivity measures

We follow ten Raa et al. (1984) and Wolff (1985b) in the development of the accounting framework. Define:

U = an input or 'use' commodity-by-industry flow matrix, where u_{ij} shows the total amount of commodity i consumed by industry j;

V = an output or 'make' industry-by-commodity flow matrix, where v_{ij} shows the total output of commodity j produced by industry i;

$\mathbf{1}$ = vector with unit entries;

$X = V^{\mathrm{T}}\mathbf{1}$ = column vector, showing the gross output of each commodity, where a superscript T refers to the transpose of the indicated matrix, and

$X^{\mathrm{I}} = V\mathbf{1}$ is a vector whose elements are the row sums of V, showing the total 'output' of each industry.[3]

[3]We use the expression X^{I} for reasons that will become apparent in subsection 2.3.

For convenience, it is assumed that the number of industries is the same as the number of commodities (that is, each commodity has an industry in which it is primary, and conversely).[4] Moreover, let

L = a row vector, showing total employment by industry;
$\Lambda = L\mathbf{1}$, total employment in the economy;
K = a row vector, showing total capital stock by industry;
$\kappa = K\mathbf{1}$, total capital stock in the economy;
w = the annual wage rate, assumed constant across industries;
r = the rate of profit on the capital stock, assumed constant across industries.[5]

The net output matrix (in terms of commodities) is then given by: $V^{\mathrm{T}} - U$. Note that U, V, L, K, w, and r comprise the data of the system. All other symbols refer to derived constructs.

We can now derive what we shall call the 'standard' row vector of commodity prices, p.[6] Since pV^{T} is the total value of output by industry and pU is the total value of inputs by industry, total value added by industry is given by: $p(V^{\mathrm{T}} - U)$. In competitive equilibrium, value added accrues to labor and capital by industry:

$$p(V^{\mathrm{T}} - U) = wL + rK. \tag{1}$$

Hence,

$$p = (wL + rK)(V^{\mathrm{T}} - U)^{-1}. \tag{2}$$

It should be emphasized that this set of prices is defined by the condition that total value added by industry is equated to factor returns and is determined independently of the model of secondary production. In this case, prices are determined by the actual flow matrix, not the coefficient matrices as in a standard Leontief system, and thus depend on the composition of final or total output.[7] There are other possible choices of price vectors, which we shall comment on below.

[4]This is not exactly true, since scrap output is produced only as a by-product. See below for modifications to the standard models engendered by the treatment of scrap.

[5]It is implicitly assumed that the government sector receives a shadow rate of return r on its capital stock.

[6]It is assumed that each commodity has the same price, irrespective of the technology of production.

[7]In such a system, it is assumed that each sector produces only one output. Then, the price vector p^* is given by:

$$p^* = (wl + rk)(I - A)^{-1},$$

where l is the (row) vector of sectoral labor coefficients, k is the (row) vector of capital coefficients, and A is the standard interindustry technical coefficients. In this system, prices are determined by technology and are invariant with respect to changes in the composition of final or total output.

One other component is needed for the analysis of productivity growth, which is Y, the vector of final demand by commodity. This is simply equal to net output by commodity summed over industries of production or consumption:

$$Y=(V^{\mathrm{T}}-U)\mathbf{1}. \tag{3}$$

The aggregate rate of TFP growth, ρ, is then defined as

$$\rho\equiv(p\,\mathrm{d}Y-w\,\mathrm{d}\Lambda-r\,\mathrm{d}\kappa)y, \tag{4}$$

where $y=pY$ is the ratio of final output.

We can now consider the two models of secondary production.

2.1. Commodity technology model

In this model, it is assumed that each commodity is produced by the same technology, irrespective of the industry of production. In this case, industries are considered independent combinations of outputs j, each with their separate input coefficients (a^{c}_{ij}). As shown in ten Raa et al. (1984), the commodity technology requirements (coefficient) matrix is given by $A^{\mathrm{c}}=UV^{-\mathrm{T}}$, where a superscript of $-\mathrm{T}$ refers to the inverse of the transpose of the indicated matrix (or the transpose of the inverse, since the two operations are communicative). Row vectors of labor and capital stock coefficients can be derived in the same way. Then $l^{\mathrm{c}}=LV^{-\mathrm{T}}$ and $k^{\mathrm{c}}=KV^{-\mathrm{T}}$. Substitution into (1) and multiplication by $V^{-\mathrm{T}}$ yields

$$p(I-A^{\mathrm{c}})=wl^{\mathrm{c}}+rk^{\mathrm{c}}. \tag{5}$$

Thus, in the commodity technology model, the value added for each commodity unit is directly equal to factor costs. In other words, the national accounting identity between real product and income is fully decentralized on a sectoral basis. As we shall see below, this is not true for the industry technology model.[8] Also, prices depend directly on the technical coefficients and are invariant with respect to changes in final demand composition, as in a standard Leontief system (see footnote 5).

The commodity technology has the added feature that overall TFP growth can be shown to be a weighted sum of sectoral (in this case, commodity-level) rates of TFP growth. A further consequence of the 'decentralization' equation (5) is that[9]

$$\rho=-(p\,\mathrm{d}A^{\mathrm{c}}+w\,\mathrm{d}l^{\mathrm{c}}+r\,\mathrm{d}k^{\mathrm{c}})X/y. \tag{6}$$

[8]Nor is it true for most other models of secondary production. See Kop Jansen and ten Raa (1989) for more details.

[9]See Wolff (1985) for details of the proof.

Since each commodity has a separate technology in this model, the rate of TFP growth for commodity j can be defined as

$$\pi_j^c \equiv -(p\,\mathrm{d}a_{\cdot j}^c + w\,\mathrm{d}l_j^c + r\,\mathrm{d}k_j^c)/p_j, \tag{7}$$

where π^c is the corresponding row vector and $a_{\cdot j}^c$ is the jth column of matrix A^c. It then follows directly that

$$\rho = \pi^c \hat{p} X / y. \tag{8}$$

Thus, the commodity technological model preserves the exact decomposition of overall TFP growth into sectoral components. Moreover, we can also show that overall TFP growth is a function of the sectoral composition of final output. First, by definition of A^c,

$$X = (I - A^c)^{-1}(I - UV^{-\mathrm{T}})V^{\mathrm{T}}\mathbf{1} = (I - A^c)^{-1} Y.$$

In other words, the commodity technology model satisfies the material balance equation of Leontief. (This is also true in the industry technology model.) As a result, it follows that (8) can be rewritten as

$$\rho = \pi^c s^c \beta, \tag{9}$$

where $s^c = \hat{p}(I - A^c)^{-1}\hat{p}^{-1}$, the Leontief (value) inverse coefficient matrix, and $\beta = \hat{p} Y / y$, which shows the value composition of final output in terms of commodities.

2.2. Industry technology model

There are two assumptions that are made in this model. First, each industry k has the same input requirements per dollar of output for each commodity that it produces. Second, the market shares for each commodity are fixed among industries. Thus, to produce commodity j, industry k needs $u_{ik}/\sum_1 v_{k1}$ of input i per unit of output j, and its market share $v_{kj}/\sum_1 v_{1j}$ is fixed. Then, as shown in ten Raa et al. (1984), the industry technology requirements per unit of commodity output (coefficient) matrix is given by

$$A^{\mathrm{I}} = U[\hat{X}^{\mathrm{I}}]^{-1} V \hat{X}^{-1},$$

where a hat (^) denotes a diagonal matrix whose diagonal is equal to the vector. Row vectors of labor and capital stock coefficients can be derived in the same way. Then, $l^{\mathrm{I}} = L[\hat{X}^{\mathrm{I}}]^{-1} V \hat{X}^{-1}$ and $k^{\mathrm{I}} = K[\hat{X}^{\mathrm{I}}]^{-1} V \hat{X}^{-1}$. From price equation (2), value added by commodity is

$$p(I-A^{\mathrm{I}})=(wL+rK)(V^{\mathrm{T}}-U)^{-1}(I-U[\hat{X}^{\mathrm{I}}]^{-1}V\hat{X}^{-1}),$$

$$p(I-A^{\mathrm{I}})=(wL+rK)(V^{\mathrm{T}}-U)^{-1}(\hat{X}V^{-1}\hat{X}^{\mathrm{I}}-U)[\hat{X}^{\mathrm{I}}]^{-1}V\hat{X}^{-1}. \tag{10}$$

Factor cost by commodity is

$$wl^{\mathrm{I}}+rk^{\mathrm{I}}=(wL+rK)[\hat{X}^{\mathrm{I}}]^{-1}V\hat{X}^{-1}. \tag{11}$$

Value added by commodity is equal to factor costs by commodity only if the two middle factors in (10) cancel – that is, $V^{\mathrm{T}}=\hat{X}V^{-1}\hat{X}^{\mathrm{I}}$. The presence of secondary production invalidates this condition and hence the equality of value added and factor costs on a commodity basis. The equality does hold for the combination of commodities that make industries and, a fortiori, for the economy as a whole. The distortion at the commodity level is due to the industry technology model notion of industry output, $V\mathbf{1}$. One implication of this, as shown in ten Raa et al. (1984), is that there is no base year price invariance of technology. The invalidation of the commodity value equation between revenues and cost (that is, materials and valued added) is due to the same reasons.

For our present purposes, the most important defect of the industry technology model is that it is no longer possible to decompose overall TFP growth into a weighted average of commodity-level rates of productivity growth. Let us first define the rate of commodity TFP growth in this model as:

$$\pi_j^{\mathrm{I}}\equiv-(p\,\mathrm{d}a_{\cdot j}^{\mathrm{I}}+w\,\mathrm{d}l_j^{\mathrm{I}}+r\,\mathrm{d}k_j^{\mathrm{I}})/p_j. \tag{12}$$

It can be shown directly that the material balance equation holds, namely:

$$Y=(I-A^{\mathrm{I}})X. \tag{13}$$

Hence, from (4) and (13),

$$\rho=[p(I-A^{\mathrm{I}})\,\mathrm{d}X-p(\mathrm{d}A^{\mathrm{I}})X-wl^{\mathrm{I}}\,\mathrm{d}X-w(\mathrm{d}l^{\mathrm{I}})X$$
$$-rk^{\mathrm{I}}\,\mathrm{d}X-r(\mathrm{d}k^{\mathrm{I}})X]/y. \tag{14}$$

Now, however, since factor cost by commodity does not equal value added by commodity [that is, (10) and (11) differ], we cannot derive an equation analogous to (8), at least when secondary production is present. Instead, we obtain from (13):

$$\rho=-(p\,\mathrm{d}A^{\mathrm{I}}+w\,\mathrm{d}l^{\mathrm{I}}+r\,\mathrm{d}k^{\mathrm{I}})X/y+[p(I-A^{\mathrm{I}})-(wl^{\mathrm{I}}+rk^{\mathrm{I}})]\,\mathrm{d}X/y. \tag{15}$$

The commodity technology derivation of (9) and (8) holds here in analogous fashion and, therefore, applies to the first term on the right-hand side of (15). The second term can be considered a residual factor θ. It then follows that

$$\rho = \pi^{\mathrm{I}} s^{\mathrm{I}} \beta + \theta, \tag{16}$$

where $s^{\mathrm{I}} = \hat{p}(I - A^{\mathrm{I}})^{-1} \hat{p}^{-1}$, the Leontief inverse coefficient matrix in the industry technology model, β is the commodity composition of final output, and

$$\theta = [p(I - A^{\mathrm{I}}) - (wl^{\mathrm{I}} + rk^{\mathrm{I}})\,\mathrm{d}X/y. \tag{17}$$

2.3. Industry-level productivity growth

The two vectors π^{c} and π^{I} both refer to commodity-level TFP growth – i.e., the productivity growth by individual commodity. The first shows commodity-based productivity growth as calculated using the commodity technology model, while the latter shows commodity-based productivity growth as computed from the industry technology model.

For reasons of comparison, we are also interested in industry-level or industry-based productivity growth, which shows productivity growth by individual industry. The reason is that the traditional and most common method of calculating productivity growth is on an industry basis rather than a commodity basis.[10] Moreover, the use of an industry basis allows us to separate out a specific secondary product effect in decomposing the change in overall TFP growth.

We define industry-level productivity growth as a weighted average of the productivity growth of the individual commodities it produces, where the weights are value shares. To circumvent the independent issue of bias, we shall define industry productivity growth on the basis of the commodity technology model only. By definition, $X = \sum_j v^{\mathrm{T}}_{\cdot j}$, where $v^{\mathrm{T}}_{\cdot j}$ is the jth column of V^{T} – i.e., the jth row of V, showing the industry of production, j. Substituting into (8), we obtain

[10]See, for example Wolff (1985b). It should be noted that the results of this study are based on neither the commodity technology model nor the industry technology model but rather on the so-called BEA transfer method. In this method, the transaction matrix is constructed on an industry by industry basis. A secondary product produced by industry i which is primary to industry j is recorded as a purchase made by industry j from industry i. The actual sales of the secondary product produced in i are then 'transferred' to the sales row of industry j. This method creates artificial transactions and can distort the measurement of productivity growth in both industries i and j. Moreover, they can also affect the measurement of linkages between sectors. The reason for using this method was for consistency with earlier years in the analysis (in particular, 1947, 1958, 1963), for which it was impossible to construct a separate secondary product make matrix.

$$\rho = \sum_j \pi^c \hat{p} v_{\cdot j}^T / y. \tag{18}$$

Note that the coefficients π^c are independent of sector j, by the properties of the commodity technology model. Each term $\pi^c \hat{p} v_j^T$ represents a sectoral contribution to overall TFP growth ρ. Let us define industry-level TFP growth in the commodity technology model for industry j as a weighted average of the TFP growth of the commodities it produces:

$$\psi_j = \pi^c \hat{p} v_{\cdot j}^T / p v_{\cdot j}^T,$$

where the weights are the value shares of the commodity output in the total value of the industry output.

We can now relate industry-level productivity growth rates to overall TFP growth as follows. First, define a matrix of market shares, $M = V\hat{X}^{-1}$. We can now demonstrate that

$$\rho = \pi^c s^c \beta = \psi M s^c \beta. \tag{19}$$

In other words, π^c and ψM act the same way on $s^c \beta$ (though, it should be noted, the two are not generally equal). Since the latter is proportional to the total output vector (in value terms), $\hat{p}X = \hat{p}V^T\mathbf{1}$, it is now necessary to show that $\pi^c \hat{p} V^T \mathbf{1} = \psi M \hat{p} V^T \mathbf{1}$. Now, by the definitions of ψ and M, the right-hand side equals

$$\pi^c \hat{p} V^T \widehat{[pV^T]}^{-1} V \hat{X}^{-1} \hat{p} V^T \mathbf{1} = \pi^c \hat{p} V^T \widehat{[Vp^T]}^{-1} V p^T = \pi^c \hat{p} V^T \mathbf{1}, \tag{20}$$

which is the left-hand side and completes the demonstration.

As an independent line of decomposition, useful in assessing the role of secondary production, we can also define overall productivity growth for primary output as a weighted sum of the commodity-level productivity growth of primary output only. To do this, let matrix P be the diagonal of matrix V (primary products) and matrix S be the off-diagonal elements (secondary products). Then,

$$V = P + S.$$

Productivity growth of primary output is then given by

$$\rho^P = [\pi^c \hat{p} P \mathbf{1} / p P \mathbf{1}] \cdot (pX/y), \tag{21}$$

where the weights are the value shares of primary output in the value of total primary output and the last term is included to reweigh to a corresponding overall productivity growth level. In analogous fashion, secondary product productivity growth is defined as

$$\rho^{s}=\left[\sum_{j}\pi^{c}\hat{p}s_{\cdot j}^{T}/pS^{T}\mathbf{1}\right]\cdot(pX/y), \tag{22}$$

where the weights are the value shares of secondary output in the value of total secondary output. Let $\omega^{p}=pPe/pX$, the value share of primary output in total output, and $\omega^{s}=pS^{T}\mathbf{1}/pX=1-\omega^{p}$, the value share of secondary output in total output. Then,

$$\rho=\omega^{p}\rho^{p}+\omega^{s}\rho^{s}. \tag{23}$$

Finally, the change in overall TFP growth can be decomposed into a primary product and secondary product effect, as follows:

$$\Delta\rho=\omega^{p}\Delta\rho^{p}+\omega^{s}\Delta\rho^{s}+\Delta\omega^{s}(\rho^{s}-\rho^{p}), \tag{24}$$

where the first term shows the change in overall TFP growth attributable to the change in productivity growth among primary products, the second term the portion due to the change in productivity growth among secondary output, and the third term the portion due to the change of the share of secondary output in total output.

2.4. A comparison of the three models

From (9) and (16) we now obtain

$$\rho=\pi^{c}s^{c}\beta=\pi^{I}s^{I}\beta+\theta. \tag{25}$$

This now leads directly to another interpretation of θ. Following Wolff (1985b), we first present two alternative growth accounting decompositions of (25). The first of these uses the commodity technology model:

$$\Delta\rho=\pi^{c}s^{c}(\Delta\beta)+\pi^{c}(\Delta s^{c})\beta+(\Delta\pi^{c})s^{c}\beta. \tag{26a}$$

In this decomposition, the change in overall TFP growth is decomposed into three effects, corresponding to the three terms on the right-hand side of (26a). The first of these can be called the *final output effect*, the second the *interindustry multiplier effect*, and the third the *sectoral TFP growth effect.*[11] The second decomposition uses the industry technology model:

$$\Delta\rho = \pi^{\mathrm{I}} s^{\mathrm{I}}(\Delta\beta) + \pi^{\mathrm{I}}(\Delta s^{\mathrm{I}})\beta + (\Delta\pi^{\mathrm{I}})s^{\mathrm{I}}\beta + \Delta\theta. \tag{26b}$$

The first three terms on the right-hand side of (26b) are analogous to those in (26a) and may be interpreted in analogous fashion. The last term may be called the *secondary bias effect*, since it shows the bias in the decomposition of overall TFP growth that can be attributed to the presence of secondary products.[12]

Thus, the commodity technology decomposition is unbiased. However, the industry technology decomposition is biased. The bias is from the presence of secondary products and the consequent wedge between the values of net outputs and unit factor costs at the sectoral level when calculated from the industry technology model.

The third model, the industry-level productivity growth, leads to a still different decomposition of overall TFP growth. Thus, in accounting for changes in productivity growth, we essentially get a still further decomposition of the sectoral TFP growth effect into a market share shift effect and an industry-level productivity growth effect. More precisely, by (19),

$$(\Delta\pi^{\mathrm{c}})s^{\mathrm{c}}\beta = \psi(\Delta M)s^{\mathrm{c}}\beta + (\Delta\psi)Ms^{\mathrm{c}}\beta.^{13} \tag{26c}$$

In the empirical analysis of section 6, there are three points of particular

[11]Note that by (2), π^{c}, s^{c} and β are each a function of all basic data, U, V, L, K, w, and r. Although a change in TFP growth can be attributed only to changes in the data, U, V, L, K, w, and r, it can be decomposed formally into the three terms indicated above. It would be interesting to perform a similar decomposition by starting with flows and stocks in constant prices, as is assumed throughout this paper, and attributing TFP growth directly to the real data (U, V, L, K) or the nominal ones (w, r). This can be done analytically by partial differentiations of (8) and then empirical evaluation. However, such an analysis is beyond the scope of the present paper.

[12]This can be seen more formally as follows. From (17),

$$\theta = [(wL+rK)(V^{\mathrm{T}}-U)^{-1}(I-U[\hat{X}^{\mathrm{I}}]^{-1}V\hat{X}^{-1}) - (wL+rK)([\hat{X}^{\mathrm{I}}]^{-1}V\hat{X}^{-1})]\,\mathrm{d}X/(wL+rK)\mathbf{1}$$
$$= (wL+rKL)\{(V^{\mathrm{T}}-U)^{-1}(I-U[\hat{X}^{\mathrm{I}}]^{-1}V\hat{X}^{-1}) - ([\hat{X}^{\mathrm{I}}]^{-1}V\hat{X}^{-1})\}\,\mathrm{d}X/(wL+rK)\mathbf{1}.$$

If there is only one primary production, then $V=\hat{X}$ and the bracketed expression on the right-hand side of the last equation reduces to

$$\begin{aligned}(\hat{X}-U)^{-1}(I-U\hat{X}^{-1}\hat{X}\hat{X}^{-1}) - \hat{X}^{-1}\hat{X}\hat{X}^{-1} &= [(I-U\hat{X}^{-1})\hat{X}](I-U\hat{X}^{-1}) - \hat{X}^{-1}\\ &= \hat{X}^{-1}(I-U\hat{X}^{-1})^{-1}(I-U\hat{X}^{-1}) - \hat{X}^{-1}\\ &= \hat{X}^{-1} - \hat{X}^{-1} = 0.\end{aligned}$$

Thus, without secondary production, there is no residual term θ. This provides another reason for calling θ a secondary bias effect.

[13]This further decomposition can also be shown to hold in the framework of the industry technology model. In the previous section, we did not address the issue in order to circumvent the independent issue of bias.

interest. The first is the contribution to the change in overall productivity growth from shifts in the composition of final output. In Wolff (1985b), it was found that this accounted for between 17 and 22 percent of the decline in overall TFP between the 1947–1967 and the 1967–1976 periods. However, this computation was implicitly based on the Bureau of Economic Analysis (BEA) transfer model and was therefore biased (see footnote 9). For the mathematics of the transfer model, see Kop Jansen and ten Raa (1989). The bias can be established in precisely the same way as for the industry technology model. Eq. (26a) will allow us to redo this calculation using the unbiased commodity technology model, at least for the 1967–1977 period. The second is the contribution to the decline in TFP accounted for by shifts in the level and composition of secondary output. Since this factor has not received attention in the literature, it will add to our knowledge on the sources of the productivity slowdown in the U.S.

The third is to determine the direction and magnitude of the bias which results from the use of the industry technology model and from the use of the industry-level productivity growth model. Both sorts of biases could be important, particularly since the latter two models are most commonly used. In particular, is the compositional effect greater using a commodity-base model than one using an industry-base model? Is it greater using the commodity-base commodity technology model than the commodity-base industry technology model?

One final comment should be made. We have not said which of the two secondary product models, if either, is the 'true' model of the U.S. economy. Such an analysis is beyond the scope of the present paper.[14] However, the use of both the commodity technology and the industry technology models will provide us with a range of values for both the output composition and the secondary product effects.

3. Secondary output, 1967–1977

As noted in the Introduction, we use the BEA 85-order 1967, 1972, and 1977 'make' and 'use' input–output tables for our analysis.[15] The 1972 and

[14]It is also not possible for the U.S. economy, since we do not have annual input–output tables. However, see ten Raa et al. (1984) for a similar type of analysis for the Canadian economy for which annual input–output tables were available.

[15]These are the only three years for which such data are available. A description of the 1972 tables can be found in Ritz (1979) and Ritz et al. (1979), and documentation of the 1977 tables in U.S. Interindustry Economics Division (1984). The 1967 data were not published as separate make and use tables, but the raw data for them are available on computer tape, which Paula Young of BEA graciously supplied to us. A description of the 1967 total flow tables can be found in U.S. Interindustry Economics Division (1974). Sources and methods for the 1967 and 1972 labor coefficients are described in Wolff (1985b). Employment data for 1977 were obtained from Yuskavage (1985). Capital stock data for all three years were obtained from Gorman et al. (1985).

1977 tables use the same accounting conventions. However, there are four important changes between the 1967 tables and those of 1972 and 1977. First, two dummy sectors, business travel and entertainment and office supplies, are present in the 1967 table but were eliminated in the 1972 and 1977 tables. We follow the later convention and distribute the output of the two dummy sectors to the appropriate using industries. Second, in the 1972 and 1977 tables, the restaurant sector was separated from the trade sector, while in the 1967 table the two are aggregated into a single sector. It was not possible to separate the restaurant sector from the trade sector in the 1967 data. As a result, we have aggregated the two sectors in the 1972 and 1977 data for consistency with the earlier year.[16] Third, in the 1967 table, a portion of the wholesale and retail trade activity and real estate (rental) activity engaged in by the various sectors were recorded as a secondary product of these sectors, whereas in the later years these transactions were recorded as primary to the trade and real estate sectors, respectively. For consistency with the later years, we transferred these secondary outputs to their primary sector.[17] Fourth, in the 1967 table, comparable imports are recorded as if purchased by the industry producing the comparable domestic commodity and then added to that industry's output for distribution to the actual purchasing industries. In the later tables, comparable imports are recorded as directly purchased by the using industry from the comparable domestic industry. We follow the later convention in our work.[18]

The first three tables show some basic results on the change in the importance of secondary products over the three years. Unless otherwise noted, secondary production is defined on the 85-order level. In 1967, 3.9 percent of the total value of output, with the exclusion of scrap output, consisted of secondary products. In 1972, the ratio was somewhat lower, at 3.4 percent, and between 1972 and 1977 the ratio rose to 3.6 percent. In constant 1972 dollar terms, the ratio of secondary to total output fell from 4.0 percent in 1967 to 3.4 percent in 1972 and then rose to 3.9 percent in 1977 (last row of table 2). The importance of secondary output is increased somewhat when the scrap sector is included in the calculation of secondary output. With this definition, the ratio of secondary to total output in current dollars was 4.0 percent in 1967, 3.4 percent in 1972, and 3.7 percent in 1977 (last row of table 3). Though these ratios are rather small, it should be

[16]We refer to the aggregated sector (number 69) as the trade sector.

[17]To balance the flow tables, we adjusted the value added of the trade sector so that its total inputs equalled its new output total and adjusted both the value added of the real estate sector and the real estate input row so that the value of total output and inputs of the real estate sector matched.

[18]Another problem arose with the broadcasting sector, whose output is almost entirely secondary, since it does not sell its broadcasting 'output' to any other sector or to final users. Since its major secondary output is business services (advertizing), we aggregated the broadcasting sector (67) with business services (71) for all three years.

Table 1

Ratio of secondary to total output by industry of production, 10-sectors, current dollars, scrap sector excluded.[a]

	1967	1972	1977	Change 1967–1977
1. Agriculture	0.037	0.043	0.043	0.007
2. Mining	0.060	0.053	0.089	0.029
3. Construction	0.000	0.000	0.000	0.000
4. Non-durable manufacturing	0.063	0.063	0.068	0.004
5. Durable manufacturing	0.066	0.060	0.057	−0.010
6. Transportation, communications, utilities	0.037	0.036	0.033	−0.004
7. Wholesale and retail trade	0.000	0.000	0.000	0.000
8. Finance, insurance, real estate	0.010	0.002	0.002	−0.009
9. Other services	0.007	0.002	0.004	−0.003
10. Government	0.090	0.078	0.095	0.005
11. Total	0.039	0.034	0.036	−0.003

[a]Secondary production based on BEA 85-order classification scheme.

Table 2

Ratio of secondary to total output by industry of production, 10-sectors, constant (1972) dollars, scrap sector excluded.[a]

	1967	1972	1977	Change 1967–1977
1. Agriculture	0.036	0.043	0.046	0.010
2. Mining	0.065	0.053	0.079	0.014
3. Construction	0.000	0.000	0.000	0.000
4. Non-durable manufacturing	0.062	0.063	0.065	0.004
5. Durable manufacturing	0.067	0.060	0.057	−0.010
6. Transportation, communications, utilities	0.035	0.036	0.032	−0.003
7. Wholesale and retail trade	0.000	0.000	0.000	0.000
8. Finance, insurance, real estate	0.011	0.002	0.002	−0.009
9. Other services	0.007	0.002	0.004	−0.003
10. Government	0.106	0.078	0.111	0.005
11. Total	0.040	0.034	0.039	−0.001

[a]Secondary production based on BEA 85-order classification scheme.

stressed that the results on the importance of secondary output is very sensitive to level of aggregation. At more disaggregated levels, secondary output naturally comprises a higher percentage of total output.

There is considerable variation among sectors in the importance of secondary output. Tables 1 and 2 show the ratio of secondary output to total output by major industry of production. In 1972, this ratio varied from a low of zero percent in construction and trade to a high of 7.8 percent in the government sector. The ratio was over 4 percent in agriculture, over 5 percent in mining, and over 6 percent in manufacturing. The importance of secondary output in total production increased most notably in agriculture,

Table 3
Ratio of secondary to total output by commodity type produced, 10-sectors, current dollars.[a]

	1967	1972	1977	Change 1967–1977
1. Agriculture	0.012	0.011	0.014	0.002
2. Mining	0.012	0.010	0.011	−0.002
3. Construction	0.000	0.000	0.000	0.000
4. Non-durable manufacturing	0.041	0.040	0.053	0.013
5. Durable manufacturing	0.064	0.059	0.056	−0.008
6. Transportation, communications, utilities	0.068	0.064	0.068	−0.000
7. Wholesale and retail trade	0.010	0.008	0.008	−0.002
8. Finance, insurance, real estate	0.015	0.001	0.008	−0.006
9. Other services	0.094	0.083	0.066	−0.028
10. Government	0.000	0.000	0.000	0.000
11. Scrap sector	1.000	1.000	1.000	0.000
12. Total (excluding scrap)	0.039	0.034	0.036	−0.003
13. Total (including scrap)	0.040	0.034	0.037	−0.003

[a]Secondary production based on BEA 85-order classification scheme.

mining, and the government sector over the 1967–1977 period, but declined in durable manufacturing and in the finance, insurance, and real estate sector. On the 85-sector level of production, there is even greater variation in the importance of secondary output. Moreover, at this level of disaggregation, secondary output now assumes major importance for some sectors. In 1972, secondary output (excluding scrap) comprised 78 percent of the value of the output of the state and local government enterprise sector (79), 45 percent of the output of the printing and publishing sector (26), 37 percent of the output of chemicals and fertilizer mineral mining (10), 19 percent of the output of the government enterprise sector (78), 15 percent of plastics and synthetic material sector (28), 14 percent of the service industry machinery sector (52) and of the miscellaneous electrical machinery, equipment, and supplies sector (58), 11 percent of general industrial machinery and equipment sector (49), 10 percent of the electric wiring and equipment sector (55), of the electronics components and accessory sector (57), of the professional and scientific instrument sector (62), and of miscellaneous manufacturing (64), and 9 percent of the output of the ordnance sector (13). Moreover, in terms of the number of different commodities produced by a sector, secondary output is also quite important, particularly in manufacturing. In 1972, there were 9 manufacturing sectors which produced 30 or more commodities (excluding scrap), and 20 sectors which produced between 20 and 29 different commodities (excluding scrap).

Table 3 shows the ratio of secondary to total output on the basis of commodity type. In 1972, one percent of agriculture output was produced as another sector's secondary product. This ratio varied from zero percent for construction and government output to 100 percent for scrap output in 1972.

Four percent of non-durable manufactures, 5.9 percent of durables, 6.4 percent of transportation, communication, and utility output, and 8.3 percent of other service output was produced as a secondary output. The most notable changes over the 1967–1977 period were the increase in importance of secondary non-durable output and the decline in secondary other service output.

The last change is particularly noteworthy, since it indicates that many establishments which produced these services in addition to their primary output during the 1960s sloughed off this production during the 1970s. The most dramatic change was in business services (73), in which the proportion of total output accounted for by secondary production fell from 25 to 17 percent. These results suggests that many of these services switched from being produced internally in many establishments to being produced in specialized establishments and being purchased externally through market transactions. It is interesting that Carter (1970) found an increase in the total requirements of service output over the 1947–1967 period in the U.S., but could not decompose this into a real interindustry effect of greater specialization and a specious effect from the reclassification of such service activities from secondary to primary output. Such a distinction is important for pinning down the sources of technical change. The table confirms Carter's intuition that the shift in service output is important. Section 6 of the paper will address the decomposition issue raised but not resolved in Carter's work.

Of the 85-sector level, there were a number of commodities for which the proportion of their total output accounted for by secondary production exceeded 10 percent in 1972. Besides business services (73), these included forestry and fishery products (3), agricultural, forestry and fishery services (4), miscellaneous fabricated textile products (19), chemicals (27), plastics and synthetic materials (28), fabricated metal products (42), engines and turbines (43), metalworking machinery and equipment (47), household appliances (54), electronic components and accessories (57), professional and scientific instruments (62), and electrical, gas, water, and sanitary services (68). Of these, the most dramatic changes were in agricultural services, where the proportion of secondary production declined sharply from 19 to 12 percent, engines and turbines, where it fell from 18 to 123 percent, and miscellaneous textile products, where it declined from 21 to 16 percent. Reversing these trends were chemicals and plastics, in which secondary production grew from 16 to 20 percent and from 12 to 22 percent, respectively.

4. The treatment of the scrap sector

The treatment of the scrap sector, 81, poses a special methodological problem, since it is an important secondary product of many sectors and yet there is no primary output that corresponds to it and hence has no input structure in the use table. Unlike the other sectors, it provides no information. One price equation and one production vector equation are missing. It is impossible to allocate value added between net scrap output and net commodity output. Neither can material inputs be ascribed to scrap output vis-à-vis commodity output. As a result, we must make certain assumptions to fill the gaps.

As regards the price of scrap, the use value seems to be determinate. An engineering approach would be to estimate the equivalent metal ore content of scrap. For this purpose we would need time-series analysis, from which we shy away because of identification problems in the presence of technical change. A better way to determine the economic metal content of scrap is to use an additional bit of information. In this case, we can just as well make a shortcut by using an exogenous price of scrap. This is what we do.

As regards the input structure of scrap, the material components seem to be zero. Nevertheless, scrap is no bonus contributor to productivity. A factor cost is involved, namely capital or, more precisely, replacement investment. In our model, which is not dynamic but rather a sequence of static models, this cost is disguised in rK, the cost of capital. The latter is assumed to be proportional to output, both in the commodity and in the industry technology approaches, which is a reasonable reduced form of a full dynamic model, provided that capital decays exponentially. Intuitively, a high rate of scrap is unproductive, because of the replacement involved. This, however, is taken care of by the value of rK, or its change over time. The use of scrap, as a material input, is unambiguously productive. Under the capital decay assumption, scrap is most appropriately modeled as proportional to capital stock. However, the proportion may vary with the production process.

We can formalize these ideas as follows. The basic data of the system are U, V, L, K, w, r, plus p_{81}, the price of scrap. The vectors $u_{.81}$ and $v_{81.}$ for the scrap sector are zero. It is convenient to partition the use and make tables as follows:

$$U=\begin{pmatrix} U^\circ & 0 \\ u_{81.} & 0 \end{pmatrix} \quad \text{and} \quad V=\begin{pmatrix} V^\circ & v_{.81} \\ 0 & 0 \end{pmatrix}.$$

Here U° and V° are the 80-by-80 use and make tables, respectively, of the economy without the scrap vector; $u_{81.}$ is the 80-row vector of scrap inputs; and $v_{.81}$ is the 80-column vector of scrap outputs. Labor and capital are partitioned similarly:

$$L=(L^\circ \;\; 0) \quad \text{and} \quad K=(K^\circ \;\; 0).$$

This new formulation entails certain modifications of the original model. Eq. (1) still remains valid, though it can now be written as

$$p^{\circ}(V^{\circ\mathrm{T}}-U^{\circ})+p_{81}(v_{.81}^{\mathrm{T}}-u_{81.})=wL^{\circ}+rK^{\circ}. \tag{1'}$$

Eq. (2) must be rectified as follows:

$$p^{\circ}=[wL^{\circ}+rK^{\circ}+p_{81}(u_{81.}-v_{.81}^{\mathrm{T}})](V^{\circ\mathrm{T}}-U^{\circ})^{-1}. \tag{2'}$$

In effect, the exogenous value of the net scrap input is implicitly included in factor costs as a depreciation term. Eqs. (3) and (4), which define net output and overall TFP growth, respectively, remain intact. We are now prepared to reconsider the two models of secondary production.

4.1. Commodity technology model

In this new formulation, we now define:

$$A^{\circ\mathrm{c}}=U^{\circ}V^{\circ-\mathrm{T}},\quad l^{\mathrm{c}}=L^{\circ}V^{\circ-\mathrm{T}}\quad\text{and}\quad k^{\mathrm{c}}=K^{\circ}V^{\circ-\mathrm{T}}.$$

Similarly we have scrap input coefficients $a_{81.}^{\mathrm{c}}=u_{81.}V^{\circ-\mathrm{T}}$. In accordance with the assumptions of the commodity technology model, it is assumed that the proportion of capital stock scrapped per unit of commodity produced is the same for each sector that produces that commodity. To determine the scrap output coefficients, consider sector 1. It has stock $k_1^{\mathrm{c}}v_{1,1}+\cdots+k_{80}^{\mathrm{c}}v_{1,80}$ for its respective outputs. Let b_i^{c} be the fraction of the capital stock of output i that is scrapped for each commodity i. Then, sector 1 scraps a total of $b_1^{\mathrm{c}}k_1^{\mathrm{c}}v_{1,1}+\cdots+b_{80}^{\mathrm{c}}k_{80}^{\mathrm{c}}v_{1,80}$. This must match the observed output of scrap in sector 1, $v_{1,81}$. Similar equations can be derived for the other sectors, and we obtain $v_{.81}^{\mathrm{T}}=b^{\mathrm{c}}\hat{k}^{\mathrm{c}}V^{\circ\mathrm{T}}$. Hence, the scrap output coefficients are specified by

$$b^{\mathrm{c}}=v_{.81}^{\mathrm{T}}V^{\circ-\mathrm{T}}\hat{k}_{\mathrm{c}}^{-1}.$$

The price equation for the commodity technology model must now be modified. Substitution into (1′) and multiplication by $V^{\circ-\mathrm{T}}$ yields

$$p^{\circ}(I-A^{\circ\mathrm{c}})=wl^{\mathrm{c}}+rk^{\mathrm{c}}+p_{81}(a_{81.}^{\mathrm{c}}-b^{\mathrm{c}}\hat{k}^{\mathrm{c}}). \tag{5'}$$

Multiplication of both sides of eq. (5′) by the Leontief inverse, $(I-A^{\circ\mathrm{c}})^{-1}$, yields commodity prices as a function of the technical coefficients, factor prices, and the price of scrap. The material balance equation remains $Y=(I-A^{\mathrm{c}})X$, where

$$A^{c}=\begin{pmatrix} A^{oc} & 0 \\ a^{c}_{81.} & 0 \end{pmatrix}.$$

From (5′), it then follows that[19]

$$\rho=-(p\,\mathrm{d}A^{c}+w\,\mathrm{d}l^{c}+r\,\mathrm{d}k^{c})X^{\circ}/y-p_{81}b^{c}\hat{k}^{c}\,\mathrm{d}X^{\circ}+p_{81}\,\mathrm{d}x_{81}.$$

Recall that scrap output coefficients were derived from $v^{T}_{.81}=b^{c}\hat{k}^{c}V^{\circ T}$. By adding components and using the fact that sector 81 has zero output, we obtain $x_{81}=b^{c}\hat{k}^{c}X^{\circ}$.

Hence,

$$\mathrm{d}x_{81}=b^{c}\hat{k}^{c}\,\mathrm{d}X^{\circ}+(\mathrm{d}k^{c})\hat{b}^{c}X^{\circ}+(\mathrm{d}b^{c})\hat{k}^{c}X^{\circ}.$$

Substitution of this for $\mathrm{d}x_{81}$ into the previous equation now yields

$$\rho=-[p\,\mathrm{d}A^{c}+w\,\mathrm{d}l^{c}+\mathrm{d}k^{c}(r-p_{81}\hat{b}^{c})-p_{81}(\mathrm{d}b^{c})\hat{k}^{c}]X^{\circ}/y. \qquad (7')$$

In this equation, the rate of return on capital is now net of (scrap) depreciation, and the productivity gains from the recycling of scrap as an input in production has now been captured.[20]

4.2. *Industry technology model*

In accord with the assumptions of this model, we assume here that the rate of scrapping depends only on the sector of production, not the particular commodity that is produced. In particular, it is assumed that the amount of scrap produced per *dollar* of output is the same for all commodities produced by a given sector of production. As a result,

$$A^{oI}=U^{\circ}[\hat{X}^{oI}]^{-1}V^{\circ}[\hat{X}^{\circ}]^{-1},\quad l^{I}=L^{\circ}[\hat{X}^{oI}]^{-1}V^{\circ}[\hat{X}^{\circ}]^{-1}\quad \text{and}$$

$$k^{I}=K^{\circ}[\hat{X}^{oI}]^{-1}V^{\circ}[\hat{X}^{\circ}]^{-1},$$

where $X^{oI}=V^{\circ}\mathbf{1}$ and $X^{\circ}=V^{\circ T}\mathbf{1}$. Similarly, the scrap input coefficients are given by

$$a^{I}_{81.}=u_{81.}[\hat{X}^{oI}]^{-1}V^{\circ}[\hat{X}^{\circ}]^{-1}$$

[19] See Wolff (1985, eq. (7)) for details of the proof.

[20] Under the assumption that capital decays exponentially, total depreciation would equal total capital decay and hence the total value of scrap.

and scrap output coefficients by[21]

$$b^{\mathrm{I}}=v^{\mathrm{T}}_{.81}[\hat{X}^{\circ\mathrm{I}}]^{-1}V^{\circ}[\hat{X}^{\circ}]^{-1}[\hat{k}^{\mathrm{I}}]^{-1}.$$

We can now redefine π^{I}, the vector of sectoral rates of TFP growth in the industry model, as

$$\pi^{\mathrm{I}}=-[p\,\mathrm{d}A^{\mathrm{I}}+w\,\mathrm{d}l^{\mathrm{I}}+(\mathrm{d}k^{\mathrm{I}})(r-p_{81}\hat{b}^{\mathrm{I}})-p_{81}(\mathrm{d}b^{\mathrm{I}})\hat{k}^{\mathrm{I}}]\hat{p}^{\circ-1}. \tag{12'}$$

Then, eq. (16) remains as before:

$$\rho=\pi^{\mathrm{I}}s^{\mathrm{I}}\beta+\theta, \tag{16}$$

where, as before, $s^{\mathrm{I}}=\hat{p}(I-A^{\mathrm{I}})^{-1}\hat{p}^{-1}$, β is the commodity composition of final output, but now

$$\theta=[p^{\circ}(I-A^{\mathrm{I}})-(wl^{\mathrm{I}}+rk^{\mathrm{I}}+p_{81}(a^{\mathrm{I}}_{81.}-b^{\mathrm{c}}\hat{k}^{\mathrm{c}}))]\,\mathrm{d}X^{\circ}/y. \tag{17'}$$

As before,

$$\rho=\pi^{\mathrm{c}}s^{\mathrm{c}}\beta^{\mathrm{c}}=\pi^{\mathrm{I}}s^{\mathrm{I}}\beta^{\mathrm{I}}+\theta \tag{25}$$

and the comparison of the two models is identical to that presented in subsection 2.3.

5. The inclusion of international trade

The trade sector is modeled after Leontief (1941). Let non-competitive imports be arranged in a row vector, m. Competitive imports need no separate symbol, but are treated as a (negative) part of final demand.[22] To support the non-competitive imports, the trade sector needs some exports, say e, a column vector, where e could be called the vector of required or debt exports. Excess exports, on top of debt exports, need no separate symbol, but are treated as a (positive) part of final demand. The trade vector uses debt exports as inputs and yields non-competitive imports as output to be distributed over the other sectors. Total non-competitive imports are given by the scalar, $m\mathbf{1}$, which is simply the sum of the components of m. The augmented make table becomes

[21]This is essentially the same as the procedure recommended by the Bureau of Economic Analysis.

[22]Note that non-competitive imports are given by sector of purchase, but aggregated by commodity, while for competitive imports it is just the other way.

$$\tilde{V}=\begin{pmatrix} V & 0 \\ 0 & m\mathbf{1} \end{pmatrix}.$$

Non-competitive imports and debt exports are attached to the use table in the usual way:

$$\tilde{U}=\begin{pmatrix} U & e \\ m & 0 \end{pmatrix}.$$

The adjustment of final demand becomes automatic. Prior to the modeling of the trade sector, final demand was defined by

$$Y=(V^{\mathrm{T}}-U)\mathbf{1}.$$

That is, final demand is net output aggregated for each commodity across industries. It includes all exports and competitive imports. Non-competitive imports, m, are reported 'under the line', like a factor cost.

After this new treatment of the trade sector, the resulting mechanics remain the same. Final demand is net output aggregated over all sectors, including trade:

$$\tilde{Y}=(\tilde{V}^{\mathrm{T}}-\tilde{U})\mathbf{1}.$$

It is easy to check that substitution of the above expressions for $\tilde{V}$ and $\tilde{U}$ and of Y yields

$$\tilde{Y}=\begin{pmatrix} Y-e \\ 0 \end{pmatrix}.$$

In other words, not only non-competitive imports, but also debt exports are excluded from final demand in the model with endogenous trade. This completes the new accounting framework.

We can now analyze productivity growth. At the sectoral level, trade productivity growth is

$$\pi^{\mathrm{c}}_{\mathrm{trade}}=-p\,\mathrm{d}\tilde{A}^{\mathrm{c}}_{\cdot\mathrm{trade}}/p_{\mathrm{trade}},$$

where p_{trade} is the price of the international trade sector and $\tilde{A}^{\mathrm{c}}$ is the commodity technology coefficients matrix of the augmented interindustry flow matrices:

$$\tilde{A}^c = \tilde{U}\tilde{V}^{-T} = \begin{pmatrix} U & e \\ m & 0 \end{pmatrix} \begin{pmatrix} V^{-T} & 0 \\ 0 & (m\mathbf{1})^{-1} \end{pmatrix} = \begin{pmatrix} A^c & e/m\mathbf{1} \\ mV^{-T} & 0 \end{pmatrix}.$$

The industry technology trade coefficients are the same, since this sector has no secondary products. Moreover, since no other sector of the economy produces trade 'output', the treatment of the trade sector is an issue independent of the choice of the model of secondary production. For this reason, the treatment of the trade sector is the same in the industry technology model.

Sectoral productivity growth of the international trade sector reduces to

$$\pi^c_{\text{trade}} = -p\,\mathrm{d}(e/m\mathbf{1})/p_{\text{trade}}.$$

In this expression, $e/m\mathbf{1}$ is the export/import ratio in physical units. Because of the negative sign, the change in this ratio, valued at fixed prices, is the change in the terms of trade. Hence trade productivity growth equals the change in the terms of trade. In other words, the productivity of the trade sector is given by the terms of trade, a result that agrees with one's intuition.

For the economy as a whole, total productivity growth is given by

$$\tilde{\rho} = (\tilde{p}\,\mathrm{d}\tilde{Y} - w\,\mathrm{d}\Lambda - r\,\mathrm{d}\kappa)/\tilde{Y}.$$

As before, a tilde refers to the augmented flow matrices. In the case of labor and capital (Λ and κ, respectively) it is immaterial, since the trade sector does not use them, and hence the tilde may be omitted. Note that excess exports, which is included in $\tilde{Y}$, contribute to total factor productivity. The opposite is true of debt exports, as they are merely an input requirement for non-competitive imports.

Since the coefficients we have specified for the augmented matrices are based on the commodity technology model, the alternative expression for total factor productivity growth holds:

$$\tilde{\rho} = -[\tilde{p}\,\mathrm{d}\tilde{A}^c + w\,\mathrm{d}(l,0) + r\,\mathrm{d}(k,0)]\tilde{X}/\tilde{Y}.$$

Once more, it is illumination to substitute the special structure of the trade sector. The expression becomes

$$\tilde{\rho} = -\left[(p, p_{\text{trade}})\,\mathrm{d}\begin{pmatrix} A^{c} & e/m\mathbf{1} \\ mV^{-\mathrm{T}} & 0 \end{pmatrix} + w\,\mathrm{d}(l,0) + r\,\mathrm{d}(k,0)\right]\begin{pmatrix} X \\ m\mathbf{1} \end{pmatrix}\Big/ p(Y-e)$$

$$= -[(p\,\mathrm{d}A^{c} + p_{\text{trade}}\,\mathrm{d}(mV^{-\mathrm{T}}),\ p\,\mathrm{d}(e/m\mathbf{1})) + w\,\mathrm{d}(l,0) + r\,\mathrm{d}(k,0)]\begin{pmatrix} X \\ m\mathbf{1} \end{pmatrix}\Big/ p(Y-e)$$

$$= -[p(\mathrm{d}A^{c})X + p_{\text{trade}}\,\mathrm{d}(mV^{-\mathrm{T}})X + p\,\mathrm{d}(e/m\mathbf{1})m\mathbf{1} + \mathrm{w}(\mathrm{d}l)X + r(\mathrm{d}k)X]/p(Y-e).$$

A comparison with the usual total factor productivity growth formula for ρ that neglects the trade sector yields two new terms,

$$-p_{\text{trade}}\,\mathrm{d}(mV^{-\mathrm{T}})X - p\,\mathrm{d}(e/m\mathbf{1})m\mathbf{1}.$$

The latter term is basically π^{c}_{trade}, so that productivity growth of the international trade sector is additively separable from total factor productivity growth. This fact is due to the absence of circular flows within that sector. The first term is basically the factor productivity aspect of non-competitive imports. It is also separable, essentially since non-competitive imports are aggregated across commodities and a new physical dimension is created for this aggregate.

In many studies, non-competitive imports are modeled as a pure factor input without taking into account the exports needed to fund them. In such studies, only the first term arises. We prefer to include the productivity of the trade sector which turns out to be given by the terms of trade.

6. Productivity analysis

We begin the analysis by computing two measures of the overall rate of TFP growth in the economy. From expression (4), TFP growth consists of an amalgam of changes and weights. Changes of net outputs are added and changes of factor inputs are subtracted, each weighted by their respective relative prices. The formula holds exactly for continuous time estimates. However, the data, of course, are available only for discrete time periods, 1967–1972 and 1972–1977. Thus, an approximation to the formula must be made. A change over a period can be estimated only by taking the difference of the two observations made during the period, at the base year and at the end year. Thus, the problem of approximation is reduced to the choice of weights in the formula. The most common choice is to take the average of the base year value and the end year value of any weight. For any period, the ratios p/y, w/y, and r/y are approximated by the averages of their respective values at the base year and the end year. This constitutes the TFP growth measure based on the average relative price index.

Table 4
Annual rate of overall TFP growth.

	1967–1972	1972–1977	1967–1977	Change
1. Turnqvist–Divisia	0.73%	−0.26%	0.17%	−0.99%
2. Average period prices	0.74%	−0.24%	0.17%	−0.98%

This measure of TFP growth is the most natural one, based on the specification of changes and their weights, as given in expression (4). However, it is possible to transform the changes and the weights without altering the equation in continuous time. Then the same reasoning leads to another measure in discrete time. The most common transformation is to relative changes. If we define $\alpha = w\Lambda/y$ as the wage share in the national product, use rK/y as the profit share in view of eq. (1) after aggregation (postmultiplication by **1**), and recall that the definition of the value shares, $\beta = \hat{p}Y/y$, can be transformed into an equation for relative changes, then

$$\rho = \beta^{\mathrm{T}}\,\mathrm{d}(\ln Y) - \alpha\,\mathrm{d}(\ln \Lambda) - (1-\alpha)\,\mathrm{d}(\ln \kappa), \tag{4'}$$

where $\mathrm{d}(\ln Y)$ is the vector whose jth component is equal to $\mathrm{d}(\ln Y_j) = \mathrm{d}Y_j/Y_j$. If we now replace the differentials by finite differences and the weights by their respective averages over the period, we obtain the TFP growth measure based on the Tornqvist–Divisia index.

To streamline the presentation of our results, we present pairs of percentages, where the first component is based on the Tornqvist–Divisia index and the second component (in parentheses) on the average relative price index.[23] TFP growth over the 1967–1972 period is 0.73 (0.74) percent per year, while for the 1972–1977 period it averages −0.26 (−0.24) percent per annum (see table 4). Hence the change in annual TFP growth between the two periods is −0.99 (−0.98) percent. This result accords with previous studies that show about a one percentage point drop in annual productivity growth over this time span [see Wolff (1985a) for a survey]. Note also that the choice of index has a negligible influence on the measurement of TFP growth and its slowdown.

We next consider alternative decompositions of the change in overall TFP growth into its various effects. The first of these, from eq. (26a), is based on commodity-level measures of TFP growth computed from the commodity technology model. There are three components to this decomposition. The first of these is the sectoral TFP growth effect, resulting from the change in

[23]Under conditions of strictly-concave and continuously differentiable production functions, constant returns to scale, and perfect competition, the Tornqvist–Divisia index is the theoretically correct measure. However, if any of these conditions is violated, other measures may be preferred. [See Baumol and Wolff (forthcoming) for a discussion of this.]

Table 5

Percentage decomposition of the change in overall TFP growth between 1967–1972 and 1972–1977 into three effects (based on the commodity-level commodity technology model).[a]

		Percentage contribution			
	$\Delta\rho$	$(\Delta\pi^c)s^c\beta$	$\pi^c(\Delta s^c)\beta$	$\pi^c s^c(\Delta\beta)$	Sum
1. Turnqvist–Divisia	−0.99%	85.0%	3.1%	12.0%	100.0%
2. Average period prices	−0.98%	90.0%	−1.1%	11.1%	100.0%

[a]See eq. (26a) for decomposition.

Table 6

Primary and secondary product annual TFP growth, 1967–1977 and their percentage contribution to the change in overall TFP growth (all computations are based on the commodity technology model).[a]

	1967–1972	1972–1977	1967–1977	Change	Percentage contribution
1. Turnqvist–Divisia					
a. Primary product TFP	0.80%	−0.17%	−0.26%	−0.97%	94.5%
b. Secondary product TFP	−1.22%	−2.75%	−2.18%	−1.53%	5.9%
c. Secondary product weight	3.64%	3.50%	3.77%	−0.15%	−0.4%
d. Overall TFP growth	0.73%	−0.26%	0.17%	−0.99%	100.0%
2. Average period prices					
a. Primary product TFP	0.79%	−0.17%	0.27%	−0.96%	94.4%
b. Secondary product TFP	−0.67%	−2.28%	−2.02%	−1.61%	6.0%
c. Secondary product weight	3.64%	3.50%	3.77%	−0.15%	−0.3%
d. Overall TFP growth	0.74%	−0.24%	0.17%	−0.98%	100.0%

[a]See eq. (24) for decomposition.

sectoral rates of TFP. This accounts for 85.0% (90.0%) of the decline in overall TFP growth (see table 5). The second is the interindustry multiplier effect, from a change in matrix s. It is small, accounting for 3.1% (−1.1%) of the decline. The third is the final output or composition effect. It accounts for 12.0% (11.1%) of the slowdown. The composition effect is larger than those reported in Wolff (1985b) for the 1958–1976 period, even though the period under consideration here, 1967–1977, is shorter.[24]

The second decomposition of TFP growth, also based on the commodity technology model, involves separate results for primary output and secondary output (see table 6). Primary product TFP growth is 0.80% (0.79%) for the 1967–1972 period and −0.17% (−0.17%) for the 1972–1977 period, yielding a change of −0.97% (−0.96%). Secondary product TFP growth is

[24]Since the composition of final output tends to change slowly over time, the composition effect is usually greater the longer the period under consideration. These results suggest that the BEA transfer method for secondary output, which was used in Wolff (1985b), tends to bias downward the contribution of compositional shifts of final output to changes in overall productivity growth.

Table 7

Percentage decomposition of the change in overall TFP growth between 1967–1972 and 1972–1977 into three effects (based on the commodity-level industry technology model).[a]

	Percentage contribution					
	$\Delta\rho$	$(\Delta\pi^1)s^1\beta$	$\pi^1(\Delta s^1)\beta$	$\pi^1 s^1(\Delta\beta)$	$\Delta\theta$	Sum
1. Turnqvist–Divisia	−0.99%	85.4%	2.6%	11.5%	0.5%	100.0%
2. Average period prices	−0.98%	91.8%	−1.3%	10.7%	−1.1%	100.0%

[a]See eq. (26b) for decomposition.

−1.22% (−0.67%) for the first period and −2.75% (−2.28%) for the second, yielding a change of −1.53% (−1.61%). The most striking result is that productivity growth was considerably lower for secondary output than for primary output. Also, the decline in TFP growth was more severe for secondary output than for primary output. From eq. (24), the change in overall TFP growth is then decomposed into three effects. The first of these, from the change in primary product TFP growth, accounts for 94.5 (94.4) percent of the change in overall TFP growth – a result largely due to the fact that primary output comprises over 96 percent of total output, as the secondary product weights are 3.64% (3.64%) for the 1967–1972 period and 3.50% (3.50%) for the 1972–1977 period. The second, from the decline in secondary product TFP growth, accounts for the remaining 5.9 (6.0) percent. The third effect, from the change in the relative level and composition of secondary output, is of almost no importance: −0.4% (−0.3%). Thus, the change in overall TFP growth is dominated by the change in primary output TFP growth, because secondary output comprise a relatively small proportion of total output at this level of aggregation. Secondary product TFP growth, although starting at a negative level, declined further and thus contributed to the slowdown.

We next look to the bias that results from the use of the industry technology model. Eq. (26b) decomposes overall TFP growth into four effects. The relative importance of the effects is given by the following results: 85.4% (91.8%) for the sectoral TFP growth effect, 2.6% (−1.3%) for the interindustry multiplier effect, 11.5% (10.7%) for the final output or composition effect, and 0.5% (−1.1%) for the secondary bias effect (see table 7). The bias in computing the overall TFP slowdown from the industry technology model is insignificant. The distribution over the three other effects is not affected much either, as a comparison with the commodity technology model above shows. In short, the use of the industry technology model, though theoretically inferior to the commodity technology model for the decomposition of TFP change, is relatively harmless, at least for this level of aggregation and this period. The reason is that the relative level and composition of secondary output was stable over the period.

Table 8

Percentage decomposition of the change in overall TFP growth between 1967–1972 and 1972–1977 into two effects (based on the industry-level commodity technology model).[a]

	Percentage contribution					
	$\Delta\rho$	$\pi^c s^c(\Delta\beta)$	$\pi^c(\Delta s^c)\beta$	$(\Delta\psi)Ms^c\beta$	$\psi(\Delta M)s^I\beta$	Sum
1. Turnqvist–Divisia	−0.99%	12.0%	3.1%	82.1%	2.9%	100.0%
2. Average period prices	−0.98%	11.1%	−1.1%	86.9%	3.1%	100.0%

[a]See eqs. (26a) and (26c) for decomposition.

We next turn to the industry-level productivity growth effect. As was argued in the body of the text, the use of industry-level productivity growth rates leads to a further decomposition of the sectoral TFP growth effect into a market share effect and an industry-level productivity growth effect. Our result is that 97% (97%) of the sectoral TFP growth effect can be ascribed to the industry-level productivity growth effect, and the remainder to the market share shift effect (see table 8). Thus, in addition to the final output composition effect accounting for 12.0 (11.1) percent of the slowdown, another 2.9 (3.1) percent can be ascribed to changes of market shares among the industries. This result, in particular, indicates that so-called 'shift effects', embodying both final output compositional changes and shifts in industry market shares, were important in explaining the productivity slowdown of this period. Also, accounting for the interindustry multiplier effect, only 82.1 (86.9) percent of the overall productivity slowdown remains to be ascribed to the slowdown in industry-level productivity growth.

Finally, on the sectoral level, there are some rather interesting differences in the measurement of TFP growth based on commodity-level and industry-level indices derived from the commodity technology model. These are shown in table 9. Though most of the differences are small, there are several sectors in which the differences are quite large. The first of these is forestry and fisheries (sector 3), with a 1.7 percentage point difference in estimated rates of annual TFP growth; the second is agricultural services (4), with a difference of 0.7 percentage points; the third is plastics (28), also with a 0.7 percentage point difference; and the final set consists of chemical products (27), drugs and related products (29), and transportation and warehousing (65), each with a 0.3 percentage point difference. However, the mean square error over all 82 sectors is rather small, 0.1 percentage points.

The last column of table 9 shows the 'contribution' of each sector to overall TFP growth, where the contribution is defined as $\pi_j^c p_j X_j / y$ and is thus sectoral TFP growth multiplied by its normalized gross output weight. Sectors with large positive contributions are livestock (1), other agricultural products (2), transportation and warehousing (65), and wholesale and retail trade (69). Sectors with strong negative contributions are construction (11),

Table 9

Commodity-level and industry-level TFP growth by sector, 1967–1977 (based on the commodity technology model and Turnqvist–Divisia index).

	Commodity level TFP (π^c)	Industry level TFP (π^I)	Difference ($\pi^I - \pi^c$)	Weight ($p_j X_j / y$)	Contribution ($\pi_j^c p_j X_j / y$)
1. Livstock	2.08%	2.08%	0.00%	0.0164	0.034%
2. Agr prod	3.98	3.98	0.00	0.0125	0.050
3. For fish	−6.03	−4.34	1.69	0.0008	−0.005
4. Agr serv	−1.60	−0.87	0.73	0.0022	−0.003
5. Iron min	−3.58	−3.48	0.10	0.0006	−0.002
6. Nfer min	−0.66	−0.68	−0.02	0.0007	−0.000
7. Coal min	−6.08	−6.07	0.01	0.0019	−0.011
8. Gas petr	0.22	0.21	−0.01	0.0060	0.001
9. Ston min	3.14	2.85	−0.29	0.0009	0.003
10. Chm ming	−4.87	−4.71	0.17	0.0002	−0.001
11. New cons	−1.52	−1.52	0.00	0.0321	−0.049
12. Main&rep	−0.21	−0.21	0.00	0.0097	−0.002
13. Ordnance	−0.72	−0.58	0.14	0.0017	−0.001
14. Food pro	0.41	0.45	0.04	0.0379	0.016
15. Toba man	0.79	0.79	0.00	0.0015	0.001
16. Fabr&yrn	0.51	0.53	0.02	0.0061	0.003
17. Txt good	2.12	2.04	−0.09	0.0018	0.004
18. Apparel	1.16	1.15	0.00	0.0107	0.012
19. Misc txt	1.50	1.34	−0.16	0.0019	0.003
20. Lmb&wood	0.12	0.16	0.04	0.0060	0.001
21. Wood con	−2.36	−2.18	0.18	0.0002	−0.000
22. Hhld fur	1.09	1.08	−0.01	0.0022	0.002
23. Oth furn	0.47	0.50	0.03	0.0011	0.001
24. Papr&pro	0.20	0.21	0.01	0.0062	0.001
25. Papr con	1.16	1.14	−0.03	0.0023	0.003
26. Prnt&pub	0.28	0.28	0.00	0.0058	0.002
27. Chem pro	−2.11	−1.81	0.31	0.0094	−0.020
28. Plastics	2.48	1.82	−0.67	0.0036	0.009
29. Drugs et	2.02	1.77	−0.25	0.0040	0.008
30. Paint pr	0.63	0.59	−0.05	0.0010	0.001
31. Petr ref	−0.98	−0.94	0.04	0.0116	−0.011
32. Rbbr pro	0.12	0.18	0.06	0.0062	0.001
33. Leath in	2.23	2.21	−0.02	0.0004	0.001
34. Footwear	−0.01	0.01	0.02	0.0016	−0.000
35. Glass pr	−0.84	−0.80	0.04	0.0016	−0.001
36. Stn clay	−0.15	−0.16	0.00	0.0046	−0.001
37. Iron&stl	−1.18	−1.15	0.03	0.0109	−0.013
38. N-fr met	−0.75	−0.73	0.02	0.0061	−0.005
39. Met cont	−0.18	−0.17	0.01	0.0045	−0.000
40. Heat plb	0.42	0.40	−0.02	0.0045	0.002
41. Screw ma	0.89	0.85	−0.04	0.0032	0.003
42. Oth metl	−0.36	−0.37	−0.01	0.0044	−0.002

Table 9 (continued)

	Commodity level TFP (π^c)	Industry level TFP (π^I)	Difference ($\pi^I - \pi^c$)	Weight (p_jX_j/y)	Contribution ($\pi_j^c p_j X_j/y$)
43. Engines	0.47%	0.38%	−0.09%	0.0014	0.001%
44. Farm mag	1.36	1.27	−0.09	0.0016	0.002
45. C min&oi	−1.32	−1.21	0.11	0.0022	−0.003
46. Mat hndl	0.67	0.57	−0.10	0.0008	0.001
47. Met&wrk	0.08	0.09	0.01	0.0022	0.000
48. Spc ind	−0.76	−0.68	0.08	0.0015	−0.001
49. Gen ind	−0.35	−0.29	0.06	0.0024	−0.001
50. Maih sop	−0.34	−0.31	0.03	0.0014	−0.000
51. Offc mag	3.97	3.77	−0.20	0.0025	0.010
52. Serv ind	1.60	1.57	−0.03	0.0018	0.003
53. Elec ind	0.47	0.50	0.03	0.0030	0.001
54. Hhsld ap	2.47	2.37	−0.10	0.0017	0.004
55. Light&wi	0.71	0.71	0.00	0.0014	0.001
56. Radio&TV	0.16	0.20	0.04	0.0049	0.001
57. Elec com	3.41	3.17	−0.24	0.0028	0.010
58. Misc e m	0.52	0.55	0.03	0.0011	0.001
59. Motr veh	0.61	0.60	−0.01	0.0160	0.010
60. Aircrfts	−0.01	−0.01	0.00	0.0048	−0.000
61. Oth trns	0.65	0.64	−0.01	0.0034	0.002
62. Scientif	1.97	1.83	−0.15	0.0022	0.004
63. Opt phot	2.77	2.60	−0.17	0.0015	0.004
64. Misc man	1.43	1.39	−0.04	0.0034	0.005
65. Trnsp&wh	2.34	2.01	−0.33	0.0274	0.064
66. Communic	2.31	2.31	0.00	0.0110	0.025
67. Brodcast	0.00	0.00	0.00		0.000
68. Utility	−1.62	−1.72	−0.10	0.0312	−0.050
69. Trade-rt	1.73	1.72	−0.01	0.0849	0.147
70. Fin & in	0.18	0.17	−0.01	0.0225	0.004
71. Rl est r	0.33	0.21	−0.12	0.0140	0.005
72. Hotl rep	2.02	2.02	0.00	0.0140	0.028
73. Busn ser	0.16	0.23	0.07	0.0228	0.004
74. Auto rep	0.37	0.28	−0.08	0.0066	0.002
75. Amusemen	1.43	1.39	−0.04	0.0052	0.007
76. Med ed s	−0.48	−0.48	0.00	0.0383	−0.018
77. Fed govt	3.59	3.59	0.00	0.0039	0.014
78. State sr	−2.72	−2.71	0.00	0.0026	−0.007
79. Govt ind	−1.16	−1.16	0.00	0.0798	−0.093
80. Houshold	0.00	0.00	0.00		0.000
81. Scrap	0.00	0.00	0.00		0.000
82. Import/exp	−2.45	−2.45	0.00	0.0187	−0.046
83. Unwt ave	0.07	0.08	0.01		
84. Overall	0.17	0.17	0.00		0.172

utilities (68), and the government industry (79). The government sector shows a negative one percent per annum rate of TFP growth over the 1967–1977 period, largely due to the rapid growth in its capital stock. One sector in particular, the import–export sector (82), deserves special mention, since its 'rate of TFP growth' is equivalent to the annual rate of change in the terms of trade. The terms of trade deteriorated sharply against the U.S. over the 1967–1977 period, at an annual rate of 2.5 percent.

Table 10 shows calculations of the change in TFP growth between the 1967–1972 and the 1972–1977 periods based on the commodity-level and industry-level measures. Here, again, differences are generally small, with an overall mean square error of 0.12 percentage points. However, there are 13 sectors which show sizeable differences: forestry and fisheries (3), agricultural services (4), stone quarrying (9), ordnance (13), chemical products (27), plastics (28), drugs and related products (29), engine manufacturing (43), metal working machinery (47), specialized industrial machinery (48), miscellaneous machinery (50), service industry machinery (52), and business services (73).[25]

The fourth column of table 10 shows the 'contribution' of each sector to the change in overall TFP growth, where the contribution is defined as $(\Delta\pi)^c_j p_j X_j/y$ and is thus the change in sectoral TFP growth multiplied by its normalized gross output weight. There are no sectors with large positive contributions, except wholesale and retail trade (69). Sectors with strong negative contributions are construction (11), food processing (14), petroleum refining (31), and the government industry (79). The government sector shows an almost two percentage point decline in its rate of TFP growth between the 1967–1972 and the 1972–1977 period because of the rapid acceleration in the growth of its capital stock. The export–import sector (82) again deserves special mention. The results indicate that the terms of trade fell against the U.S. by 3.2 percentage points between the 1967–1972 and the 1972–1977 periods. Since non-competitive imports comprise about three percent of GDP, deterioration in the terms of trade between the two periods accounted for about a quarter ($-0.0026/-0.0099$) of the overall productivity slowdown.

7. Conclusion

By starting the productivity analysis with flow data of inputs and outputs,

[25]A sector-by-sector comparison of commodity-level TFP growth derived from the commodity technology model with that derived from the industry technology model shows a slightly higher degree of bias from the use of the latter. The mean square error over all 82 sectors in the computation of TFP growth over the 1967–1977 period from the two models is 0.20 percentage points, and that for the computation of the change in TFP growth between the 1967–1972 and the 1972–1977 periods is 0.33 percentage points.

Table 10

Change in commodity-level and industry-level TFP growth by sector between the 1967–1972 and the 1972–1977 periods (based on the commodity technology model and Turnqvist–Divisia index).

	Commodity level TFP ($\Delta\pi^c$)	Industry level TFP ($\Delta\pi^I$)	Difference ($\Delta\pi^I - \Delta\pi^c$)	Contribution ($\Delta\pi_j^c p_j X_j/y$)
1. Livstock	1.54%	1.54%	0.00%	0.11%
2. Agr prod	1.07	1.07	0.00	0.06
3. For fish	−3.72	−3.35	0.37	−0.01
4. Agr serv	2.76	2.36	−0.40	0.03
5. Iron min	−3.29	−3.30	−0.01	−0.01
6. Nfer min	−5.72	−5.70	0.02	−0.02
7. Coal min	0.06	0.05	−0.01	0.00
8. Gas petr	−4.76	−4.75	0.01	−0.12
9. Ston min	4.93	4.48	−0.45	0.02
10. Chm ming	3.75	3.65	−0.10	0.00
11. New cons	−2.25	−2.25	0.00	−0.31
12. Main&rep	−3.22	−3.22	0.00	−0.13
13. Ordnance	−2.22	−1.92	0.30	−0.02
14. Food pro	−1.15	−1.03	0.12	−0.19
15. Toba man	0.33	0.32	−0.01	0.00
16. Fabr&yrn	1.19	1.18	−0.01	0.03
17. Txt good	2.60	2.40	−0.20	0.02
18. Apparel	1.29	1.28	−0.01	0.06
19. Misc txt	0.99	1.03	0.04	0.01
20. Lmb&wood	−3.68	−3.61	0.07	−0.09
21. Wood con	−3.87	−3.80	0.07	−0.00
22. Hhld fur	1.54	1.50	−0.04	0.01
23. Oth furn	−0.32	−0.30	0.02	−0.00
24. Papr&pro	−1.03	−0.99	0.04	−0.03
25. Papr con	2.21	2.18	−0.03	0.02
26. Prnt&pub	1.76	1.73	−0.03	0.04
27. Chem pro	−4.12	−3.67	0.45	−0.16
28. Plastics	−1.13	−1.86	−0.73	−0.02
29. Drugs et	−0.56	−0.94	−0.38	−0.01
30. Paint pr	1.20	1.01	−0.19	0.01
31. Petr ref	−3.60	−3.64	−0.04	−0.18
32. Rbbr pro	−1.55	−1.49	0.06	−0.04
33. Leath in	2.27	2.25	−0.02	0.00
34. Footwear	1.32	1.33	0.01	0.01
35. Glass pr	2.04	1.99	−0.05	0.01
36. Stn clay	−0.87	−0.85	0.02	−0.02
37. Iron&stl	0.69	0.68	−0.01	0.03
38. N-fr met	1.38	1.34	−0.04	0.04
39. Met cont	2.92	2.82	−0.10	0.02
40. Heat plb	0.63	0.56	−0.07	0.01
41. Screw ma	−1.84	−1.66	0.18	−0.03
42. Oth metl	0.15	0.18	0.03	0.00

Table 10 (continued)

	Commodity level TFP ($\Delta\pi^c$)	Industry level TFP ($\Delta\pi^I$)	Difference ($\Delta\pi^I - \Delta\pi^c$)	Contribution ($\Delta\pi_j^c p_j X_j/y$)
43. Engines	−2.47%	−2.16%	0.31%	−0.02%
44. Farm mag	−1.26	−1.22	0.04	−0.01
45. C min&oi	−2.75	−2.64	0.11	−0.03
46. Mat hndl	−1.17	−1.14	0.03	−0.00
47. Met&wrk	2.41	2.07	−0.34	0.02
48. Spc ind	−3.98	−3.58	0.40	−0.03
49. Gen ind	−0.07	−0.11	−0.04	−0.00
50. Maih sop	3.49	3.29	−0.19	0.02
51. Offc mag	1.68	1.55	−0.13	0.02
52. Serv ind	−2.98	−2.56	0.42	−0.02
53. Elec ind	1.46	1.33	−0.13	0.02
54. Hhsld ap	1.27	1.03	−0.24	0.01
55. Light&wi	−0.80	−0.68	0.12	−0.00
56. Radio&TV	2.34	2.25	−0.09	0.05
57. Elec com	1.53	1.45	−0.08	0.02
58. Misc e m	−0.47	−0.44	0.03	−0.00
59. Motr veh	0.26	0.24	−0.02	0.02
60. Aircrfts	0.73	0.69	−0.04	0.01
61. Oth trns	0.68	0.64	−0.04	0.01
62. Scientif	0.46	0.43	−0.03	0.00
63. Opt phot	−1.73	−1.77	−0.04	−0.01
64. Misc man	1.18	1.12	−0.06	0.02
65. Trnsp&wh	0.07	−0.03	−0.10	0.01
66. Communic	1.50	1.50	0.00	0.07
67. Brodcast	0.00	0.00	0.00	0.00
68. Utility	−1.01	−0.90	0.11	−0.13
69. Trade-rt	0.64	0.63	−0.02	0.23
70. Fin & in	1.16	1.16	0.00	0.11
71. Rl est r	0.90	0.78	−0.12	0.05
72. Hotl rep	1.02	1.02	0.00	0.06
73. Busn ser	1.16	1.42	0.26	0.11
74. Auto rep	1.26	1.23	−0.03	0.04
75. Amusemen	2.64	2.58	−0.06	0.06
76. Med ed s	1.18	1.18	0.00	0.19
77. Fed govt	−1.76	−1.76	0.00	−0.03
78. State sr	−0.34	−0.33	0.01	−0.00
79. Govt ind	−1.83	−1.83	0.00	−0.62
80. Houshold	0.00	0.00	0.00	0.00
81. Scrap	0.00	0.00	0.00	0.00
82. Import/exp	−3.24	−3.24	0.00	−0.26
83. Unwt ave	−0.09	−0.10	−0.01	
84. Overall	−0.99	−0.99	0.00	−0.84

constructing input–output coefficients in the process, and setting up value relations simultaneously, we have shown that the presence of secondary products have both theoretical and empirical ramifications. With regard to the former, we have shown that in order to establish a theoretically correct relationship between sectoral and overall levels of productivity growth, we must adopt the so-called commodity technology model of secondary production in setting up the input–output relations. Since the literature has employed ready-to-use input–output coefficient matrices derived from the industry technology model, productivity growth decompositions based on them have been biased. We have proved that a decomposition of overall productivity growth into industry-level productivity growth rates involves changes not only in final demand and the Leontief inverse but also a matrix of market shares.

The empirical results indicate that, though the industry technology model bias is by itself insignificant, a portion of the sectoral TFP growth effect is captured by shifts in market shares. In particular, only 82.1 (86.9) percent of the overall productivity slowdown can be ascribed to the slowdown in industry-level productivity growth, particularly that of construction, food processing, petroleum refining, and the government industry, with the remaining 13 to 18 percent due to changes in the composition of final output and market shares, including the interindustry multiplier effect. This compositional effect is of the same order of magnitude as found in Wolff (1985) for a much longer period (that between 1947–1967 and 1967–1976). Though we were able to separate out the secondary product effect, little of the slowdown can be ascribed to changes in secondary product TFP growth rates, but the levels of secondary product TFP growth rates are extremely low throughout the period of analysis. Since our analysis allows a detailed commodity breakdown of these rates, the source of this problem can be identified as the high representation of some slow productivity growers among secondary products, particularly the following products: chemical products in the petroleum refining industry, non-ferrous metal products in the iron and steel industry, and business services provided by the printing and publishing industry.

Results on the sectoral level indicate that the bias from using industry-level measures of TFP growth instead of commodity-level indices, while small on average, is quite large for several sectors. Slightly larger biases were found on the sectoral level from using the industry technology model. Two special sectors in this study are the scrap sector and international trade. Inclusion of the scrap sector in our framework captures depreciation and the gains from recycling. In our modeling of international trade, its sectoral productivity growth is found to be identical to the change in the terms of trade, and non-competitive import savings in other sectors are captured as well. Changes in the terms of trade were found to be significant for the U.S. over the 1967–

1977 period and accounted for almost a fourth of the estimated slowdown in overall TFP growth.

Though the results reported in this paper do not indicate a major effect on overall TFP growth from changes in secondary output and composition, this may be due to the high order of aggregation. Even at the 85-sector order, this may not necessarily remain true in the future. In particular, the relative level and composition of secondary output may be changed more substantially over time, even at the 85-sector level. As a result, the model presented here may produce outcomes that differ more from standard factor productivity growth studies that ignore the correct specification of the input–output value relations between the sectors, including scrap and trade.

Finally, there are two major implications of our work for multiregional analysis. First, to avoid biased relationships between regional and national levels of productivity growth, we suggest the use of the commodity technology model in the construction of multiregional input–output models. Second, our accounting framework can be directly adapted to identify strong and weak regions in terms of productivity growth and, more particularly, to single out those areas of the country that contributed most to the productivity slowdown. We hope that such an analysis will be undertaken in the future.

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