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2007

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Current Version: December 31, 2007

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Abstract

Evidence indicates that people fear change and the unknown. We offer a model of familiarity bias in which individuals focus on adverse scenarios in evaluating defections from the status quo. The model explains the endowment effect, portfolio underdiversification, home and local biases. Equilibrium stock prices reflect an unfamiliarity premium. In an international setting, our model implies that the absolute pricing error of the standard CAPM is positively correlated with the amount of home bias. It also predicts that a modified CAPM holds wherein the market portfolio is replaced with a portfolio of the stock holdings of investors not subject to familiarity bias.

1 Introduction

People fear change and the unknown. Experimental evidence on judgment and decisionmaking documents that individuals prefer familiar goods and people, status quo choices, and gambles which seem unambiguous and that individuals feel competent to evaluate. These effects are also manifested in both goods and capital markets. Individuals favor investments that they are more familiar with, and that are geographically and linguistically proximate (familiarity, local, or home bias); investors are reluctant to trade away from their current ownership positions, and are biased in favor of choice options made salient as default choices (*status quo* bias); and hold strongly to past choices or to goods or investments that they currently hold (the endowment effect, the sunk cost effect, inertia).[1]

We provide a model that captures a range of experimental phenomena and capital markets evidence based upon two psychological forces. The first is the tendency for individuals to use a focal choice alternative as a benchmark for comparison in evaluating other possible choices. An alternative becomes focal by being familiar, cognitively simple to analyze, salient, and assigned by default.[2] We refer to such a focal choice option as the *status quo*. The other force is the tendency to evaluate skeptically choice alternatives that deviate from the status quo. This force reflects fear of change and of the unfamiliar.

We model these forces as arising from egocentrically pessimistic guesses about how the world works in the presence of uncertainty. In our approach individuals do *not* penalize the status quo choice option for the uncertainty associated with its outcomes. Pessimistic beliefs are primed only by contemplation of an action that deviates from the status quo choice (a familiar, endowed, or default choice option). This linkage between contemplated action and pessimism captures fear of change and of the unfamiliar—*familiarity bias*.

Specifically, in our approach the decisionmaker acts as if he thinks that any choice that deviates from the status quo is likely to be countered by a structure of the world that minimizes his welfare. In other words, we model an inclination of individuals who

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[1]We discuss the evidence of these various effects in more detail in Section 2.

[2]Individuals' choices in our model depend upon salient benchmarks, but in a fashion different from prospect theory (Kahneman and Tversky (1979)). In our approach decisionmakers fear deviations from a salient *choice alternative*. In contrast, under prospect theory, individuals are averse to deviations from a benchmark *payoff* level. Further, in our setting it is pessimistic beliefs rather than the shape of the utility function that induce investors' conservatism.

are faced with model uncertainty to focus on worst-case (or at least, bad-case) scenarios when contemplating deviations from the status quo. An individual selects a strategy over the status quo only if the strategy provides higher expected utility over a sufficiently large probability mass of possible models of the world.

In this framework unfamiliarity induces anomalies relating to the unwillingness to trade or to shift investment policy. A natural status quo option when evaluating investment or consumption alternatives is the investor's current position. Endowment effects arise because an investor evaluates purchases under a probability distribution that is adverse to buying (i.e., one in which the expected utility from the good or security is low), while he evaluates a possible sale under a distribution that is adverse to selling (i.e., one in which the expected utility from the good or security is high).

If investors are endowed with portfolios that include only a subset of available goods or securities, then pessimism about trades provides a quantifiable model of various puzzles of non-participation in securities markets and investors' limited diversification across stocks and asset classes. Special cases include the under-diversification puzzle, the home bias puzzle, the preference of individuals to invest in own company stock, and an attachment to investing "styles" (such as preferences for size, industry, and value/growth categories, for example). These effects follow naturally from our model. More importantly, our analysis provides empirical implications about the circumstances under which these effects are stronger, weaker, or nonexistent; and about the effect of familiarity bias on equilibrium asset prices and returns.

In calibration analysis we find that with a reasonable degree of uncertainty about the mean stock returns, our model can explain why many investors hold poorly diversified portfolios. Familiarity bias substantially reduces the perceived certainty equivalent gains from diversification. Modest levels of model uncertainty induce underdiversification and a home bias comparable to the observed magnitude.

Our paper differs in several ways from previous studies relating ambiguity aversion and model uncertainty to underdiversification and home bias (e.g., Dow and Werlang (1992), Uppal and Wang (2003), Epstein and Miao (2003), and Cao, Wang and Zhang (2005)). The fundamental difference arises from our focus on fear of change and of the unfamiliar,

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so that in our model the aversion to uncertainty is *conditional.* In most models investors dislike uncertainty of all choice options symmetrically, so that there is no special role in the status quo.[3] In contrast, we consider agents who are influenced by their status quo options (typically, their initial endowments).

This basic difference in motivation and modeling leads to different implications. First, some of the important findings of previous studies are driven by the assumption that some assets have greater model uncertainty than others—an assumption that is often reasonable. However, our main findings apply even when investors are equally uncertain about the returns of different assets. For example, home bias in our model does not derive from differences in uncertainty, but from the fact that the pessimism induced by a given level of uncertainty is greater in an unfamiliar asset than in a familiar one.

Second, in previous models of home bias, investors always hold non-zero quantities of stocks (including foreign stocks). In our model, some investors can reach an autarkic outcome; if they start out not holding some asset class such as foreign stocks, they continue to hold zero of that asset class. Thus, our model explains not just home bias, but frequent zero holdings in foreign stocks by many investors.[4]

The dynamic and policy implications of our approach for underdiversification also differ from previous papers on ambiguity aversion and model uncertainty. In our approach, if investors can be induced to purchase a new asset class, this asset class will become more familiar and will be more likely to remain in the portfolio. In contrast, in previous models where endowment does not matter, such a change has no lasting effect.[5] Similarly, our analysis suggests that privatizations of government owned firms in which shares are allocated to individual investors can permanently increase their stockholdings by making the holding of these shares (and to some extent, the holding of stocks in general) more familiar.

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[3]This leads to important differences. For example, suppose that model uncertainty increases for all risky assets, but the increase is greater for the status quo choice than for alternative choices. In previous models which make no distinction between status quo versus other choices, investors will reduce their holdings of the choice whose uncertainty has increased the most. In contrast, in our approach the individual sticks *more strongly* to the status quo choice, since he evaluates the alternatives more pessimistically.

[4]In reality, holding positive amounts of the domestic stock market and zero foreign stocks is quite common for many investors in many countries and time periods.

[5]For example, if an employer pays an employee with stock, the stock becomes familiar as part of the portfolio, so the employee may voluntarily retain a lot of own-company stock even though, if he had not been paid in stock, he would not have purchased it. Or, if a stock market boom and heavy media publicity increase the familiarity of stock investing and induce individuals to start participating in the stock market, they are likely to continue to hold stocks later even if favorable publicity about holding stocks declines.

More importantly, we derive implications of familiarity bias for equilibrium asset pricing. We consider stock markets in two countries, each populated by both rational and familiarity-biased investors. The degree of uncertainty and the fraction of familiarity-biased investors are the same across the two countries. Given an endowed portfolio, there is an interval of prices within which the familiarity-biased investors do not trade. In equilibrium, familiarity-biased investors' trade depends on how the amount of uncertainty compares to some endogenously determined country-specific uncertainty thresholds. When the degree of uncertainty is either very low or very high relative to the thresholds for both countries, the effects of familiarity bias on stock demand and supply offset. In these cases, equilibrium asset prices are not affected by the presence of familiarity-biased investors, and the standard CAPM relation holds with respect to the world market portfolio, although no one holds the world market portfolio.

In contrast, when investors' uncertainty is between the thresholds of uncertainty of the two countries, familiarity-bias affects equilibrium stock prices. In this case, familiarity-biased investors in the high-uncertainty-threshold country sell some of their endowed shares to rational investors; familiarity-biased investors in the other country remain at their endowed portfolio positions. The stock price in the high-uncertainty-threshold country is lower than that when there are no familiarity-biased investors in either country. The difference in the stock prices between the two economies without and with familiarity-biased investors captures an unfamiliarity premium—an extra return to compensate familiarity-biased investors for deviating from their endowment positions. The unfamiliarity premium increases with the fraction of familiarity-biased investors and decreases with the degree of uncertainty.

Under this circumstance, the standard CAPM with respect to the world market portfolio no longer holds. Further, the absolute pricing errors of the standard CAPM in both countries increase with the fraction of familiarity-biased investors. Since familiarity-biased investors are more likely to hold only domestic equity, the absolute pricing errors of the standard CAPM are predicted to be positively correlated with the amount of home bias. However, we show that a modified CAPM holds when the world market portfolio is replaced by the aggregate stock holdings of the rational investors.

Our analysis shows that familiarity bias can explain the failure of empirical tests of the international CAPM. Further, our investigation offers a new approach for testing the

international CAPM with respect to the aggregate stock portfolio of rational investors, given measures for the degree of uncertainty and the fraction of rational investors. For example, Anderson, Ghysels and Juergens (2007) use the data on professional forecasters to extract a measure of uncertainty, while the fraction of rational investors can be proxied by the fraction of investors participating in foreign (world) stock markets. With these measures, a proxy can be formed for the aggregate portfolio of rational investors, which permits testing the familiarity-based version of the CAPM using the modified market portfolio.

Our findings on the pricing effects of familiarity bias are related to the asset pricing implications of the incomplete information model of Merton (1987). Assuming that each investor is 'uninformed' about a subset of stocks (i.e., exogenously unable to take a position in these stocks), Merton shows that for a given stock, the price discount increases with the fraction of uninformed investors on this stock. In our model, a familiarity-biased investor is 'informed' about all stocks (there is no exogenous constraint on participation), but he may choose not to participate in an unfamiliar stock when the uncertainty is sufficiently high. From that perspective, the fraction of familiarity-biased investors in our model endogenizes the fraction of uninformed investors in Merton's model. By doing so, our analysis provides stronger predictions about the form of deviations from the CAPM and suggests the construction of a modified market portfolio with respect to which the CAPM relation still holds.

2   Motivating Evidence

We begin by summarizing the evidence relating to human attitudes toward the familiar and toward deviations from salient benchmark choice alternatives.

One of the rationality axioms underlying von-Neumann Morgenstern expected utility is the Independence of Irrelevant Alternatives, which says that whether choice A or B is better should not be changed by the availability of an irrelevant third alternative. However, extensive experimental evidence shows that individuals usually focus attention on an irrelevant alternative as a benchmark for evaluating other alternatives.[6] For example, Simonson and

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[6]See, e.g., Tversky (1972), Huber, Payne, and Puto (1982), Simonson (1989), Simonson and Tversky (1992).

Tversky (1992) find that when offered a choice between three alternatives, people tend to choose the intermediate alternative.

Often, the focal choice alternative is to do nothing, and remain at the status quo. An individual who is subject to the status quo bias prefers either the current state or some choice alternative that has been made salient as the default option that will apply should no alternative be selected explicitly (Samuelson and Zeckhauser (1988), Fox and Tversky (1995)). For example, in a set of experiments on portfolio choices following a hypothetical inheritance, Samuelson and Zeckhauser (1988) find that an option becomes significantly more popular when it is designated as the status quo while others are designated as alternatives.[7]

Sharing some of the flavor of the status quo bias is the endowment effect (Thaler (1980)). It has been well documented that people often demand a higher price to give up an object than they would be willing to pay to acquire it (e.g., Knetsch and Sinden (1984), Kahneman, Knetsch, and Thaler (1991)). This difference between the willingness to pay and the willingness to accept is called the endowment effect.

When neither choice alternative is made salient as a passive or default choice, sometimes the focal choice alternative is the one that is easier to process.[8] The greater comfort that individuals have with easily processed choice alternatives probably lies in the fact that people prefer choices about which they can feel expert and competent. As shown by Heath and Tversky (1991), individuals prefer to bet in a decision domain within which they feel expert than on another gamble with an identical distribution of payoff outcomes.

Another principle that emerges from the experimental studies is that when there is a single clearcut focal choice alternative, people evaluate skeptically the possible outcomes of choice alternatives that deviate from the focal choice. For example, when the focal choice is the status quo, individuals tend to dislike risks that derive from active choices more than risks that result from remaining passive. Psychologists have referred to this as the omission bias (Ritov and Baron (1990), Josephs et al. (1996)). For example, individuals are reluctant

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[7]As another illustration of status quo bias and inertia, Madrian and Shea (2001) and Ameriks and Zeldes (2004) find that people stick for long periods of time to the default option offered by their firm and make no change in composition of their retirement portfolios.

[8]For example, in the Ellsberg (1961) experiments establishing ambiguity aversion, a straight gamble with a 50:50 probability is much easier to evaluate than one that involves compounding of gambles, even though there is a logical line of reasoning indicating that the overall probability is still 50:50.

to take seemingly risky actions such as getting vaccinated, often preferring to bear the much bigger risks associated with remaining passive.

More broadly, there is a general tendency for people to like things more which they are more familiar with. Starting with Zajonc (1968), psychologists have documented a strong and robust *mere exposure effect*: individuals tend to like stimuli that are more familiar, even when this familiarity is not associated with substantive resolution of uncertainty about the desirability of the stimulus (e.g., Bornstein and Dagostino (1992), Moreland and Beach (1992)). Advertisers try to take advantage of this by repeatedly exposing consumers to the name of a brand hoping that consumers will use their brand as default choice when the consumers face uncertainty about many brands offered on similar products.

In general, things which we are familiar with, choice alternatives that fall within our domain of competence, or default choice alternatives involving continuing to do what we were doing previously, will tend to be less risky than new initiatives into unknown territory. So it makes sense as a general heuristic to favor familiar or status quo choice alternatives. Such a heuristic can go astray, however, when in fact we possess other clear sources of information about the distributions of payoffs of different alternatives.

There is a great deal of evidence suggesting that these two psychological forces—the tendency to evaluate choices relative to a focal choice benchmark, and the tendency to be unduly skeptical about the non-focal choice alternatives relative to the focal one— operate in capital market decisions as well.

In corporate finance, a manager's reluctance to invest or to terminate investment, like the endowment effect, involves holding insistently to a previously selected or endowed choice alternative. Investment is the exchange of cash for a project, and termination is the opposite exchange. Previous studies document that firms commonly use hurdle rates that exceed the cost of capital, thereby discouraging new projects (e.g., Poterba and Summers (1995), Graham and Harvey (2001)). A related phenomenon is the sunk-cost effect (Arkes and Blumer (1985)), wherein an initial investment in a project creates reluctance to terminate the project.

In stock investments, individuals prefer familiar choices. For example, U.S. investment managers invest disproportionately in locally headquartered firms (Coval and Moskowitz

(1999)). Similar evidence is found in Finland and Sweden (Grinblatt and Keloharju (2001), Massa and Simonov (2006)). Ackert et al. (2005) find that investors have a greater perceived familiarity with local and domestic securities and, in turn, invest more in such securities. U.S. individual investors experience lower return performance for their local stock picks than for their remote stock trades, suggesting that individuals trade local stocks despite a lack of information about these stocks (Seasholes and Zhu (2005)).

In pension fund investments, many people invest a significant fraction of their discretionary contributions in their own company stock. For example, Mitchell and Utkus (2002) and Meulbroek (2002) find that the percentage of assets in company stock in defined contribution plans is around 29 percent. In a sample of S&P 500 firms, Benartzi (2001) finds that about one third of the assets in retirement plans are invested in company stock, and of the discretionary contributions, about a quarter are invested in company stock.

In international financial markets, investors tend to hold domestic assets instead of diversifying across countries, a puzzle known as home bias (e.g., French and Poterba (1991)). Although various explanations such as transaction costs, differential taxes, political risk, exchange rate risk, asymmetric information, purchasing power parity, and non-tradable assets have been offered, none has been shown to explain the magnitude of observed home bias (e.g., Lewis (1999), Stulz (1999)). A related phenomenon is that firms tend to cross list their stocks in countries where investors are more familiar with the firms to be listed (e.g., Pagano, Roell, and Zechner (2002), Sarkissian and Schill (2004)).

The preference for the familiar goes above and beyond motivations based upon lower true risk or higher returns. Both individuals and portfolio managers have more pessimistic expectations about foreign stocks than about domestic stocks (Shiller, Konya, and Tsutsui (1996), Strong and Xu (2003), and Kilka and Weber (2000)). This is consistent with our modeling approach.

3   The Model

To highlight the intuition of the model, we consider a two-date setting in which investment decisions are made at date 0, and consumption takes place at date 1. We consider a preference

relation that reflects aspects of the preferences described by Gilboa-Schmeidler (1989), but which emphasizes fear of the unfamiliar as reflected in a reluctance to deviate from a specified status quo action.

The unique subjective probability distribution used in standard expected utility calculation is replaced by a set of probability distributions $\mathcal{P}$ which capture investors' uncertainty about the distribution of asset payoffs. A larger set $\mathcal{P}$ corresponds to a higher degree of model uncertainty.

Each individual has a twice differentiable and concave utility function $U(W)$ defined over the end-of-period wealth, W. Let $W(x)$ denote the wealth random variable for an investor following a given strategy x. The following definition describes a preference relation that captures fear of change and unfamiliar choices.

Definition 1 Status Quo Deviation Aversion
Let x be a feasible strategy and s be the status quo strategy. Then x is strictly preferred to s if and only if x gives higher expected utility than s under any probability distribution Q in $\mathcal{P}$.

$$x \succ s \quad \text{iff} \quad \min_{Q \in \mathcal{P}} \left\{ \mathrm{E}^Q[U(W(x))] - \mathrm{E}^Q[U(W(s))] \right\} > 0.$$

Status Quo Deviation Aversion (SQDA) gives a privileged position to the status quo strategy. A strategy is preferred to the status quo strategy only if it provides higher expected utilities under all probability models in $\mathcal{P}$.

Status quo deviation aversion is an incomplete preference relation, as it does not specify how to compare two non-status-quo alternatives. The following definition gives one way to complete the preference ordering:

Definition 2 Strong Status Quo Deviation Aversion
Let x and y be any two strategies and s be the status quo strategy. Then

$$x \succ y \quad \text{iff} \quad \min_{Q \in \mathcal{P}} \left\{ \mathrm{E}^Q[U(W(x))] - \mathrm{E}^Q[U(W(s))] \right\} > \min_{Q \in \mathcal{P}} \left\{ \mathrm{E}^Q[U(W(y))] - \mathrm{E}^Q[U(W(s))] \right\}.$$

It is easy to show that SSQDA implies SQDA. Our results on the endowment effect in Section 4.1 require only the milder SQDA. The analysis in the later sections apply SSQDA.

Status Quo Deviation Aversion, both in its basic form and its strong form, assigns a privileged role to a status quo alternative. This familiar option is chosen unless there exists an alternative that is preferred for all possible beliefs within the set $\mathcal{P}$. Thus, a familiar choice option acts as an anchor from which deviations are pessimistically considered. When there is uncertainty, deviations from more familiar choices will be scrutinized with skepticism and suspicion. This results in a tendency to prefer more familiar choices, or choices that seem to preserve the status quo.

SSQDA implies that when there are choices that dominate the status quo option, the investor chooses among them according to a procedure similar to that described by Gilboa and Schmeidler (1989), i.e., the investor evaluates each strategy under the scenario that is most adverse to that strategy. Thus, if the status quo action is dominated by an alternative strategy x, then strategy x is evaluated according to the minimum gains in expected utility, and the alternative strategy with the highest minimum gains in expected utility is selected.

To derive closed-form solutions for portfolio choice and equilibrium analyses in later sections, we assume a constant absolute risk aversion (CARA) utility function and normally distributed stock payoffs. Further, investors have precise knowledge of the variance of stock payoff but do not know the mean. This is motivated by the fact that it is much easier to obtain accurate estimates of the variances and covariances than that of the expected values. Thus, in our model, fear of the unfamiliar derives from aversion to model uncertainty about the mean payoffs of unfamiliar choice alternatives. Investors will consider a set of probability distributions with different means when making their investment decisions.

When specifying the set $\mathcal{P}$, we consider a reference distribution P (e.g., obtained from econometric analysis) and form the set $\mathcal{P}$ around P based on the log likelihood ratio. We define $\mathcal{P}$ as the collection of all probability distributions Q satisfying $\mathrm{E}^Q[-\ln(dQ/dP)] < \beta$ for a preselected positive value β. Intuitively, $\mathcal{P}$ can be viewed as a confidence region around P. To gain more insights on $\mathcal{P}$, consider the case of a single stock.[9] If $\mathcal{P}$ is chosen to be the set of normal distributions with a common known variance, Kogan and Wang (2002) show that the confidence region can always be described by a set of quadratic inequalities. In our case, it takes the form of $\mu + v$ where v measures the adjustment made to the estimated

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[9]In Sections 4.4 and 5, we describe the set $\mathcal{P}$ when there are multiple stocks.

mean μ (e.g., sample average) under probability measure Q and satisfies

$$Tv^2\sigma^{-2} \leq \beta^2, \tag{1}$$

where β is a parameter that captures the investor's uncertainty about the mean of the payoff of the stock, and T is the number of periods for which data on the stock are available. We therefore define the set $\mathcal{P}$ as the collection of all normal distributions with mean $\mu + v$ and variance σ^2 such that v satisfies (1). The higher is β, the wider is the range for the mean payoff. We refer to β as the level of investors' uncertainty.

4 Implications of Familiarity Bias for Individual Decisions

In this section we examine the implications of familiarity bias for individuals' decisionmaking. We demonstrate that familiarity bias can induce the endowment effect, the underdiversification in risky asset holdings, and the home bias. We also quantify the magnitude of the effect of familiarity bias.

4.1 The Endowment Effect

Consider the case of acquiring more shares of a stock whose random payoff is denoted r. We assume that the individual perceives making no trade as the default or status quo choice option. Let W_0 denote the initial wealth in the risk-free bond, e denote the endowment in the stock, c denote the dollar amount the individual pays for the additional shares of the stock, and d denote the dollar amount the individual receives for giving up the additional shares of the stock under measure Q. For small additional shares in the stock Δe, let ΔC_P denote the greatest amount of cash an investor would be willing to give up in exchange for the additional quantity of the asset,

$$\Delta C_P \equiv \sup_c \{c \mid \min_{Q \in \mathcal{P}} \mathrm{E}^Q[U(W_0 + (e + \Delta e)r - c)] - \mathrm{E}^Q[U(W_0 + er)] > 0\}. \tag{2}$$

Similarly, we let ΔC_A denote the least amount of cash required to induce an individual to give up a small amount of the stock,

$$\Delta C_A \equiv \inf_d \{d \mid \min_{Q \in \mathcal{P}} \mathrm{E}^Q[U(W_0 + (e - \Delta e)r + d)] - \mathrm{E}^Q[U(W_0 + er)] > 0\}. \tag{3}$$

11

The willingness to accept (WTA) and willingness to pay (WTP) are defined as

$$\text{WTA} = \lim_{\Delta e \to 0} \frac{\Delta C_A}{\Delta e}, \tag{4}$$

$$\text{WTP} = \lim_{\Delta e \to 0} \frac{\Delta C_P}{\Delta e}. \tag{5}$$

Proposition 1 *Under Status Quo Deviation Aversion, there is an endowment effect, i.e., WTA is always greater than or equal to WTP.*

We prove this proposition for all concave utility function in the appendix. To derive closed-form solutions for WTA and WTP, we assume a CARA utility with risk aversion γ and the normal distributions for stock payoff. Under the assumption of normal distributions for the set $\mathcal{P}$, the worst distribution for holding additional shares of stock is a normal distribution with mean payoff $(\mu - \beta\sigma/\sqrt{T})$ (the mean stock payoff adjusted downward by $-\beta\sigma/\sqrt{T}$). It is straightforward to show that

$$\Delta C_P = \Delta e(\mu - \beta\sigma/\sqrt{T}) - \frac{\gamma\sigma^2}{2}[(e + \Delta e)^2 - e^2].$$

Letting Δe approach zero, we obtain the marginal willingness to pay:

$$WTP = \mu - \beta\sigma/\sqrt{T} - \gamma e\sigma^2.$$

Similarly, the worst case scenario for selling stock is a normal distribution with a mean payoff $(\mu + \beta\sigma/\sqrt{T})$ (the mean stock payoff adjusted upward by $\beta\sigma/\sqrt{T}$). Therefore, the amount that an investor requires to reduce stock holding from e to $e - \Delta e$ units is

$$\Delta C_A = \Delta e(\mu + \beta\sigma/\sqrt{T}) - \frac{\gamma\sigma^2}{2}[(e - \Delta e)^2 - e^2].$$

The marginal willingness to accept is given by

$$WTA = \mu + \beta\sigma/\sqrt{T} - \gamma e\sigma^2.$$

The difference between WTP and WTA is

$$WTA - WTP = 2\beta\sigma/\sqrt{T}. \tag{6}$$

This gap is proportional to the product of the degree of uncertainty, measured by β, and the degree of risk, measured by σ. When there is no model uncertainty ($\beta = 0$), the gap approaches zero. The gap occurs because when an investor purchases a share of stock, he considers the scenario that is most adverse to buying, and when he sells a share of stock, he considers the scenario that is most adverse to selling. The disparity in WTA and WTP comes from the difference in perceived outcome distribution.

In the approach above, investors who exhibit familiarity bias focus on the worst case scenarios associated with contemplated deviations from status quo choices. Similar results can be obtained under a less extreme assumption: investors focus on bad cases instead of worst cases.

In order to define "bad cases" specifically, we consider an investor who is uncertain about which model of the world is valid. Let s be the status quo strategy and x be an alternative strategy that the investor is contemplating. We rank the probability distributions Q in $\mathcal{P}$ by $\mathrm{E}^Q[U(W(x))] - \mathrm{E}^Q[U(W(s))]$. The individual may pessimistically select a probability distribution Q at the $1-\delta$ quantile of this ranking ($\delta > 0.5$). This yields a quantile utility gain relative to the status quo choice, for any given degree of pessimism δ. We can then define status quo deviation aversion based on this more moderate skepticism about deviations. This condition is milder, making it easier for individuals to choose alternatives for the status quo. Even if a choice alternative could conceivably pay off worse than the status quo, the alternative might be preferred if this possibility is sufficiently unlikely. Similar results of autarky on the part of individuals (the endowment effect), and quantification of the circumstances under which this occurs, can be derived under the more moderate familiarity bias described by this quantile approach.[10]

The endowment effect is commonly interpreted as the result of loss aversion. The loss aversion explanation to the endowment effect is purely preference based. The idea is that if a good is evaluated as a loss when it is given up, and as a gain when it is acquired, loss aversion will, on average, induce a higher dollar value for owners than for potential buyers. In contrast, in our approach, the endowment effect arises without loss aversion. Instead, it derives from skepticism about the desirability of giving up the object by virtue of the fact that retaining the object is the focal, status quo choice.

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[10]Formal derivation based on the quantile utility gain is available upon request.

In a capital budgeting context, the willingness to pay can be interpreted as the amount the manager values the payoff distribution from a new investment project. A lower willingness to pay implies a higher implicit discount rate, and therefore, an excessively high hurdle rate for new investment. For an ongoing project, the willingness to accept can be interpreted as the level of the liquidation value that would make the manager just willing to terminate the project. A willingness to accept that is too high indicates that the implicit hurdle rate for continuing the project is too low. Thus, our specification of familiarity bias can explain the use by managers of excessively high hurdle rates in investment choices, and also reluctance to terminate on the part of managers in their existing investments.

4.2   Portfolio Choice under Familiarity Bias

We now consider the optimal risky portfolio of an investor who has CARA utility but is subject to Strong Status Quo Deviation Aversion (SSQDA). There are two stocks whose returns are normally distributed. Let $\mu = (\mu_1, \mu_2)^\top$ be the vector of mean returns, and Σ be the covariance matrix of the returns, say, estimated from historical data of T observations. Familiarity-biased investors make adjustments v to the mean stock returns satisfying[11]

$$v^\top \Sigma^{-1} v \leq \beta^2/T, \tag{7}$$

where β represents the investor's degree of uncertainty on the expected return (common for both stocks). These adjustments form the probability set $\mathcal{P}$, which the investors utilize to evaluate their investment strategies.

Let $e \equiv (\omega, 1 - \omega)^\top$ denote the investor's endowed equity portfolio and $(\omega + \Delta D, 1 - \omega - \Delta D)^\top$ be a contemplated new portfolio. The investor's initial wealth is set to be one. Given the CARA utility considered here, this normalization does not affect the investor's portfolio choice. Under the SSQDA preference, the perceived certainty-equivalent gains of the trade from the endowment portfolio to the contemplated new portfolio is

$$G(\Delta D, e) \equiv \min_v \{-e^{-\gamma[(\Delta D u + e)^\top(\mu+v) - \frac{\gamma}{2}(\Delta D u + e)^\top \Sigma (\Delta D u + e)]} + e^{-\gamma e^\top(\mu+v) + \frac{\gamma^2}{2} e^\top \Sigma e}\},$$

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[11]Under the normal distribution assumption, this implies that the ratio of the log-likelihood function of the joint stock returns under the distributions in $\mathcal{P}$ relative to that under the reference probability lies above some threshold level.

where γ is the risk aversion coefficient and $u \equiv (1, -1)^\top$. The investor chooses ΔD to maximize $G(\Delta D, e)$.

The following proposition summarizes the optimal risky portfolio choice under familiaritybias. For ease of exposition, we use superscript "fb" to refer to a familiarity-biased investor and superscript "R" to refer to a rational investor.

Proposition 2 *Suppose the familiarity-biased investor's endowed equity portfolio is $e \equiv (\omega, 1 - \omega)^\top$. Then his optimal risky portfolio $(\omega^{fb}, 1 - \omega^{fb})^\top$ is*

$$\omega^{fb} = \begin{cases} \frac{\mu_1 - \mu_2 - v_m - \gamma u^\top \Sigma (1-u)/2}{\gamma u^\top \Sigma u} & \text{if } \mu_1 - \mu_2 - \gamma u^\top \Sigma e > v_m \\ \omega & \text{if } |\mu_1 - \mu_2 - \gamma u^\top \Sigma e| \le v_m \\ \frac{\mu_1 - \mu_2 + v_m - \gamma u^\top \Sigma (1-u)/2}{\gamma u^\top \Sigma u} & \text{if } \mu_1 - \mu_2 - \gamma u^\top \Sigma e < -v_m, \end{cases}$$

where $u \equiv (1, -1)^\top$ and $v_m = \beta \sqrt{u^\top \Sigma u / T}$.

Proposition 2 implies that the familiarity-biased investor's optimal trade from endowed equity position satisfies

$$\Delta D = \begin{cases} \frac{\mu_1 - \mu_2 - v_m - \gamma u^\top \Sigma e}{\gamma u^\top \Sigma u} & \text{if } \mu_1 - \mu_2 - \gamma u^\top \Sigma e > v_m \\ 0 & \text{if } |\mu_1 - \mu_2 - \gamma u^\top \Sigma e| \le v_m \\ \frac{\mu_1 - \mu_2 + v_m - \gamma u^\top \Sigma e}{\gamma u^\top \Sigma u} & \text{if } \mu_1 - \mu_2 - \gamma u^\top \Sigma e < -v_m. \end{cases} \tag{8}$$

Intuitively, in the first case of (8), the difference between the expected returns of stock 1 and stock 2 is sufficiently high to overcome investor's fear of change and uncertainty, so that he increases the weight on stock 1 relative to the endowment. On the other hand, in the third case of (8), stock 2 is sufficiently more attractive so that the investor buys more of stock 2 and sell some shares of stock 1. The second case of (8) corresponds to the region of no trade, which occurs when the degree of uncertainty is sufficiently high.

The optimal risky portfolio $(\omega^R, 1 - \omega^R)$ for a rational investor with CARA utility maximizes

$$\mathrm{E}\left[-e^{-\gamma[\omega r_1 + (1-\omega) r_2]} \right].$$

This leads to

$$\omega^R = \frac{\mu_1 - \mu_2 - \gamma u^\top \Sigma (1 - u)/2}{\gamma u^\top \Sigma u},$$

which coincides with the special case of Proposition 2 when $\beta = 0$ (i.e., no influence of familiarity bias). Unlike the rational investor's optimal risky portfolio which is determined by the expected returns of stocks and their covariances, the familiarity-biased investor's equity portfolio also depends on his endowment and the degree of uncertainty about expected stock returns. For a given degree of uncertainty β and endowment portfolio $e = (\omega, 1 - \omega)^\top$, the difference between the optimal equity portfolio of a familiarity biased investor and an otherwise identical rational investor is

$$
\omega^{fb} - \omega^R = \begin{cases} \frac{-v_m}{\gamma u^\top \Sigma u} & \text{if } \mu_1 - \mu_2 - \gamma u^\top \Sigma e > v_m \\ \omega - \omega^R & \text{if } |\mu_1 - \mu_2 - \gamma u^\top \Sigma e| \leq v_m \\ \frac{v_m}{\gamma u^\top \Sigma u} & \text{if } \mu_1 - \mu_2 - \gamma u^\top \Sigma e < -v_m. \end{cases}
$$

Thus, even when the familiarity-biased investor trades away from his endowment in the direction of the stock having superior risk-return tradeoff, he is more conservative than the rational investor as he underweighs the more attractive stock. This conservatism is a direct consequence of the familiarity-biased investor's pessimism when deviating from the status quo position.

Similar approach used to establish Proposition 2 can be applied to solve the portfolio choice of a risky stock and the riskfree asset. In fact, the latter problem can be viewed as a special case of Proposition 2 when the variance of one of the stock returns degenerates to zero. In particular, for every initial stock endowment e, there is a price interval $[\mu - \gamma\sigma^2 e - \beta\sigma/\sqrt{T}, \mu - \gamma\sigma^2 e + \beta\sigma/\sqrt{T}]$ within which the investor does not deviate from his status quo position. In this sense, the effect of familiarity bias on an investor's portfolio choice resembles that of transaction cost. In the case of a single stock with 50% sample standard deviation based on 100 observations, a risk aversion $\gamma = 1$, and an uncertainty parameter $\beta = 1$, the effect of familiarity bias on the investor's portfolio choice is similar to a setting in which there is a 5% proportional transaction cost without familiarity bias.

4.3   Underdiversification

Blume and Friend (1975) find that investors hold highly undiversified portfolios. Using more recent data from a major discount brokerage firm, Barber and Odean (2000) find that investors, on average, hold 4.3 stocks at this brokerage firm, with the median being only 2.6

stocks; see also Goetzmann and Kumar (2006). This phenomenon is in sharp contrast to the recommendation of traditional portfolio theory, and especially puzzling prior to the rise of mutual funds in recent decades. We illustrate here that when deviating from the status quo choice triggers investor aversion to model uncertainty, investors may remain at poorly diversified endowment positions and do not perceive further diversification to be beneficial.

Consider the case of N stocks with identically distributed returns. Assume that asset returns are jointly normally distributed, with sample variance σ^2 and correlation ρ estimated from historical data of T observations. The amount of uncertainty β is assumed to be the same for each stock. We define a portfolio p_e as *undominated* if a familiarity-biased investor who starts with p_e as his status quo prefers to hold p_e. Thus, a portfolio p_e is undominated if, for any arbitrary portfolio p,

$$\min_{Q \in \mathcal{P}}[CE(p) - CE(p_e)] \leq 0,$$

where $CE(\cdot)$ represents the certainty equivalent value for a portfolio.

Given the symmetry of the model, all risk-averse investors would hold equally weighted portfolios. For any positive integer K, let e_K denote an equally weighted portfolio of K stocks. We now characterize the minimum number of stocks K so that e_K is undominated. For this purpose, we examine the conditions under which a familiarity-biased investor endowed with e_K would not want to combine e_K with any e_{M-K}, $K < M \leq N$, where e_{M-K} denotes the equally weighted portfolio of $M - K$ of the $N - K$ stocks not contained in e_K.

Let v_K and v_{M-K} be the adjustments to the mean returns of portfolios e_K and e_{M-K}, respectively. As in Section 4.2, familiarity-biased investors contemplating a trade make adjustments $v = (v_K, v_{M-K})^\top$ to perceived mean portfolio returns satisfying

$$v^\top \Sigma_M^{-1} v \leq \beta^2 / T,$$

where Σ_M, the variance-covariance matrix of returns of e_K and e_{M-K}, is given by

$$\Sigma_M = \begin{pmatrix} \frac{1+(K-1)\rho}{K} & \rho \\ \rho & \frac{1+(M-K-1)\rho}{M-K} \end{pmatrix} \sigma^2.$$

Applying Proposition 2, familiarity-biased investors would hold onto the endowment portfolio e_K if

$$v_M - \gamma u^\top \Sigma_M (1, 0)^\top \geq 0, \tag{9}$$

17

where $u = [1 \ -1]^\top$, and $v_M = \beta\sqrt{u^\top \Sigma_M u/T} = \beta\sigma\sqrt{\frac{M(1-\rho)}{K(M-K)T}}$. It is straightforward to show that (9) implies

$$K \geq \frac{\gamma^2\sigma^2(1-\rho)M}{M\beta^2/T + \gamma^2\sigma^2(1-\rho)}. \tag{10}$$

Thus, given the uncertainty about mean stock returns described by β, a familiarity-biased investor who holds the portfolio e_K with K stocks will not want to diversify further, as long as (10) holds for all M such that $K < M \leq N$. Since the right-hand side of (10) increases with M, the minimum number of stocks K so that a familiarity-biased investor endowed with e_K will not diversify further is

$$K^* = 1 + \text{Int}\left[\frac{\gamma^2\sigma^2(1-\rho)N}{N\beta^2/T + \gamma^2\sigma^2(1-\rho)}\right],$$

where $\text{Int}[x]$ represents the largest integer below x.

When $\beta = 0$ (i.e., no influence of familiarity bias), the investor holds all N stocks. However, when $\beta > 0$, a familiarity-biased investor holds on to portfolios with a much fewer number of stocks. For example, take $N = 500, \rho = 0.5, \gamma = 1, \sigma = 0.5, T = 100$. With the model uncertainty associated with deviation from the status quo at $\beta = 2$, an equally weighted portfolio with only four stocks ($K^* = 4$) is undominated. When there are $T = 100$ observations, the amount of uncertainty corresponding to $\beta = 2$ implies that the investor adjusts the sample mean stock return up and down by, at most, one-fifth of the standard deviation. Thus, our model can generate empirically observed underdiversification with reasonable parameter values.

Figure 1 plots the minimum number of stocks needed to construct an undominated portfolio for different degree of uncertainty. It illustrates the tradeoff between the benefit of risk reduction through diversification and the fear of the uncertainty associated with deviating from the initial endowment. As uncertainty increases, the minimum number of stocks needed to construct an undominated portfolio decreases monotonically, reflecting the investor's desire to reduce the overall uncertainty in his portfolio. Furthermore, as the investor's risk aversion increases, the gains from diversification are higher and the investor increasingly desires to hold a well diversified portfolio. Holding fixed the amount of uncertainty, the number of stocks in an investor's undominated portfolio is uniformly larger for higher values of risk aversion.

18

Our finding that limited diversification occurs due to fear of unfamiliar choice options suggests that mutual funds (and especially index funds) provide a social benefit for a reason different from standard explanations. A standard argument is that mutual funds reduce the transaction cost needed for investors to diversify. However, for a long-term buy-and-hold investor, it is not really all that costly to form a reasonably diversified portfolio on an individual account. In our model, investors stop adding stocks to their portfolios because a large diversification gain is needed to offset the aversion to buying an unfamiliar stock. A mutual fund can address this issue in two ways. First, the individual needs to add just a single new asset to his portfolio, the mutual fund. Second, by focusing on marketing to investors, mutual funds can make their product more familiar to investors. In other words, where corporations specialize in making profits, mutual funds can specialize in being invested in. Our approach suggests that there is a socially valuable complementarity between being good at marketing that assuages investor fears about stocks, and providing a diversified portfolio of securities in which individuals can invest.[12]

4.4 A Calibration Analysis of the Home Bias Puzzle

A well known puzzle in international finance is that investors in aggregate tend to hold mostly the assets of the country they reside in, rather than diversifying internationally–home bias. Since domestic assets start out being owned by domestic investors (i.e., firms that are born in a given country are typically owned first by domestic entrepreneurs), domestic stocks tend to initially be part of the endowment of domestic individuals. Thus, home bias could be viewed as a more general version of the endowment effect. For historical reasons, domestic stocks start out domestically held, and there is a reluctance to shift from this initial position.[13]

Our contribution here is not simply showing the possible existence of home bias, but to

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[12]In a study that examines the role of advertising in the mutual fund industry, Cronqvist (2006) finds that funds took advantage of investor familiarity in their advertisements (e.g., Absolut Strategi Fund associated itself with the Vodka brand Absolut). Fund advertising is shown to affect investors' choices, although it provides little information (e.g., about fund fees or manager ability). In particular, advertising induces more home bias.

[13]Of course, in a dynamic setting with heterogeneous investors, there can be movement over time to a situation in which some investors hold foreign stocks and some do not. Those investors who become familiar with the foreign asset class may become more willing to increase their investments in the future. Nevertheless, the basic fact that domestic assets start out domestically owned suggests that home bias may be the result of fear of change rather than an active effort to sell off foreign stocks. This is the possibility that our analysis captures.

calibrate quantitatively whether home bias occurs under plausible parameter values, evaluating how much uncertainty is needed for investors to hold mostly local assets; and to provide an analysis that is consistent with some aspects of home bias not reflected in previous studies.

We calibrate the model to the data for four countries, including Germany, Japan, United Kingdom, and the United States. Table 1 shows the summary statistics of annual stock market returns for the four countries and the world market portfolio, based on data from 1975-2006.[14] To facilitate comparison, we use value-weighted dollar returns for all four countries and the world market portfolio. Investor initial endowment is assumed to be 100 percent in domestic stock market. This offers the highest level of certainty equivalent gains for diversifying into the world equity market. It therefore creates the most challenging situation for home bias. The risk aversion is set at $\gamma = 2$.

For each country we calculate the optimal combination of the domestic portfolio and the world equity portfolio for the familiarity-biased investor as given by Proposition 2, at different levels of model uncertainty. We also quantify the perceived certainty equivalent gains of moving from the endowment to the more diversified optimal combination of the domestic and world equity portfolios.

The portfolio chosen by investors in each country reflects the fear of unfamiliar (though low risk) associated with defecting from the endowment in order to invest more globally. Figure 2 plots the domestic equity proportion perceived as optimal in investor's total portfolio as a function of the model uncertainty for the four countries. At low levels of model uncertainty, the perceived optimal portfolios for investors in all four countries fall below their respective initial domestic endowments, suggesting that it is beneficial for these investors to shift from entirely domestic equity to the world market portfolio.[15]

On the other hand, with sufficiently high levels of model uncertainty about stock returns, Figure 2 shows that familiarity-biased investors in all four countries perceive their endowment (which is 100 percent domestic equity) as optimal. This is consistent with empirically observed home bias. There is also cross-sectional variation in the amount of uncertainty

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[14]We thank Kenneth French for making available on his webpage http://mba.tuck.dartmouth.edu /pages /faculty/ken.french the data used in our analysis.

[15]Based on the sample estimates, the optimal weight on the Japanese stock market is negative when the degree of uncertainty is low. Since it is costly to sell short in international markets, the weight is set to be zero in such cases in Figure 2.

needed to induce investors to hold on to their endowed (100 percent domestic equity) portfolio. In Germany, the model uncertainty needs to be above two. In Japan, the required uncertainty is slightly under three before investors find it unattractive to add world stock market exposure to their portfolio. In contrast, UK and U.S. investors stop diversifying into world stock market at much lower levels of uncertainty (about one-half).

When there is no familiarity bias, the perceived certainty equivalent gain in moving from the initial all domestic equity position to the rational optimal combination of the domestic portfolio and the world equity portfolio is about 6.2 percent for Germany, 10.7 percent for Japan, 0.6 percent for the UK, and 0.5 percent for the United States. Figure 3 shows that the perceived certainty equivalent gain decreases to zero as the model uncertainty increases. The decline is faster for the UK and U.S., mirroring the finding that UK and U.S. investors stop diversifying into the world stock market at low levels of uncertainty.

5   Capital Market Equilibrium with Familiarity Bias

We have analyzed the strategies perceived to be optimal by investors who have familiarity bias. We now turn to the question of how familiarity bias affects prices in an endogenously determined market equilibrium. There are two stock markets, domestic and foreign.[16] The population size of each country is normalized to one, and the proportion of rational investors (who are not subject to familiarity bias) in each country is denoted m. Thus, there are four groups of investors: domestic and foreign rational investors, as well as domestic and foreign investors who are subject to familiarity bias. All investors have CARA utility function with risk aversion coefficient γ. We use subscript "d" to denote home country and subscript "f" to denote foreign country.

The payoffs V of the stocks in the two countries are assumed to be joint normally distributed with mean vector $\mu = (\mu_d, \mu_f)^\top$. The variance-covariance matrix of the payoffs, Σ, has diagonal elements of σ_d^2 and σ_f^2. The correlation of stock payoffs is ρ. Σ is known to all investors. For the rational investors, the expected payoff is μ. For familiarity-biased investors, the perceived mean payoff is denoted by a set $\mu + v$, where the adjustments to

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[16]Although we consider international stock markets, our results also apply to a cross-section of stocks in one market, e.g., to settings where investors have preferred habitats or styles.

perceived mean payoffs (v) are uniformly distributed on a rectangular set given below

$$v \in [-\alpha\sigma_d, \alpha\sigma_d] \times [-\alpha\sigma_f, \alpha\sigma_f], \tag{11}$$

where $\alpha = \beta/\sqrt{T}$. Thus in the rest of the paper α represents investor's degree of uncertainty on the expected stock payoffs after analyzing historical data of sample size T.

The per capita supplies of the domestic and foreign stocks are denoted x_d and x_f, respectively. We assume that the entire supply of domestic stocks is initially endowed among domestic investors evenly, while the entire supply of foreign stocks is endowed among foreign investors evenly. Besides the risky stocks, there is a risk-free asset in zero net supply with zero rate of return.

Consider an investor's optimal portfolio choice corresponding to a given price vector $P = (P_d, P_f)^\top$. Let W_0 denote his initial wealth in the risk-free asset, $e \equiv (e_d, e_f)^\top$ denote his initial share endowment in the stock markets, and $\Delta D \equiv (\Delta D_d, \Delta D_f)^\top$ denote deviation of his share holdings from the initial endowment. The optimal portfolio holdings of domestic rational, domestic familiarity-biased, foreign rational, and foreign familiarity-biased investors are denoted respectively by D_{dr}, D_{db}, D_{fr}, and D_{fb}.

The rational investors maximize $\mathrm{E}\left[e^{-\gamma W_1}\right]$, where W_1 is the wealth next period,

$$W_1 = W_0 + (e + \Delta D)^\top V - (\Delta D)^\top P.$$

It follows that

$$D_{dr} = D_{fr} = \frac{1}{\gamma}\Sigma^{-1}(\mu - P). \tag{12}$$

The familiarity-biased investor's optimal trade ΔD^* can be computed in two steps. First, for each proposed demand deviation ΔD, evaluate the certainty equivalent gains $G(\Delta D, e)$ of deviating from endowment e by ΔD. Second, choose ΔD^* to maximize $G(\Delta D, e)$ for a given endowment e.[17] The following proposition provides the equilibrium stock returns.

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[17]The details on finding the optimal trade are given in the proof of Proposition 3.

Proposition 3 *(1) When $\alpha < \min\{(1-\rho)\gamma\sigma_d x_d/2, (1-\rho)\gamma\sigma_f x_f/2\}$, rational and familiarity-biased investors trade internationally. The equilibrium stock returns are*

$$\begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix} = \left(\frac{\gamma}{2}\right)\Sigma\begin{pmatrix} x_d \\ x_f \end{pmatrix}. \tag{13}$$

(2) When $\alpha \geq \max\{(1-\rho)\gamma\sigma_d x_d/2, (1-\rho)\gamma\sigma_f x_f/2\}$, the equilibrium stock returns are also given by (13). Rational investors trade internationally, whereas familiarity-biased investors remain at their endowment positions.

(3) When α is between $(1-\rho)\gamma\sigma_d x_d/2$ and $(1-\rho)\gamma\sigma_f x_f/2$, rational investors trade internationally, while familiarity-biased investors from the country with a higher uncertainty threshold trade in their home market, and familiarity-biased investors from the other country remain at their endowment positions. Suppose the domestic country has a higher uncertainty threshold, the equilibrium stock returns are

$$\begin{pmatrix} \mu_d - P_d \\ \mu_f - P_f \end{pmatrix} = \Sigma\begin{pmatrix} \frac{1}{1+m}\gamma x_d - \frac{(1-m)\alpha}{(1+m)(1-\rho^2)\sigma_d} \\ \frac{1}{2}\gamma x_f \end{pmatrix}. \tag{14}$$

The case in which the foreign country has a higher uncertainty threshold is symmetric.

Case (2) here is the equilibrium analog of the no-trade case in Proposition 2. Further, even when a familiarity-biased investor trades away from his endowment position, he does not move all the way to the rational optimal position. The equilibrium holdings of a familiarity-biased investor differ more from those of a rational investor when the uncertainty is higher, and when the correlation between domestic and foreign stock payoffs is higher.

The equilibrium stock returns in Cases (1) and (2) of Proposition 3 coincide with that when everyone is rational. In Case (1), this is because the effects of familiarity bias on domestic and foreign investors offset each other, leaving the rational investors holding the same optimal portfolios as when there are no familiarity-biased investors. In Case (2), uncertainty is too high so that only rational investors participate in the markets and determine the prices. Familiarity-biased investors stay at their endowment positions and do not affect the equilibrium prices.

To better understand the effect of familiarity bias on equilibrium asset prices in Case (3), without loss of generality, suppose that domestic uncertainty threshold is higher than foreign uncertainty threshold (i.e., $\sigma_f x_f < \sigma_d x_d$). Proposition 3 implies that the equilibrium price for domestic stock P_d is lower than the fully rational price P_d^R:

$$P_d^R - P_d = \left(\frac{1-m}{1+m}\right) [\gamma \sigma_d x_d / 2 - \alpha/(1-\rho^2)]\sigma_d > 0. \tag{15}$$

This occurs because in equilibrium domestic familiarity-biased investors sell some domestic shares, but foreign familiarity-biased investors do not buy domestic shares. To clear the market, rational investors have to hold more shares of the domestic stock than the optimal amount when everyone is rational. Thus, the equilibrium price P_d has to be lower relative to the rational benchmark P_d^R to induce risk-averse rational investors to hold more shares. The equilibrium expected return for the domestic stock is higher when there are familiarity-biased investors than when everyone is rational. The difference increases with $1 - m$, the fraction of familiarity-biased investors.

Our model shares some similar implications as the incomplete information model of Merton (1987). In Merton (1987), each investor is endowed with an information set about a subset of all stocks. In the equilibrium under incomplete information, stocks are priced lower and expected returns are higher compared to the full information case (see equations (18), (20) and (21) of Merton (1987)). For a given stock, the lower price and higher expected return induced by incomplete information increase with the fraction of uninformed investors on this stock, which is the analog of the fraction of familiarity-biased investors in our model. Based on a background information-cost story, Merton assumes that an investor does not know about stocks outside his information set and does not use them in constructing his optimal portfolio. In our model, although a familiarity-biased investor knows about all stocks, he chooses not to participate in an unfamiliar stock when the uncertainty is sufficiently high. Thus, familiaritybias provides another justification for the assumed investor behavior in Merton (1987).

The difference in the stock price between the fully rational economy and that with familiarity-biased investors $(P_d^R - P_d)$ captures an unfamiliarity premium. Correspondingly, the expected stock return $\mu_d - P_d$ in our model can be decomposed into two components: the standard rational risk premium and the unfamiliarity premium. Equation (15) shows that

the unfamiliarity premium increases with the fraction of familiarity-biased investors $(1-m)$. It also decreases with the degree of uncertainty (α). Intuitively, when uncertainty is higher, domestic familiarity-biased investors sell less of domestic stock because the perceived gains of deviating from their endowment positions are smaller. This leads to reduced supply of shares in the domestic market, and thus a higher equilibrium price and a lower unfamiliarity premium.

The stock market of the country with low uncertainty threshold is less affected by familiarity bias. When $(1-\rho)\gamma\sigma_f x_f/2 < \alpha < (1-\rho)\gamma\sigma_d x_d/2$, only rational investors participate in the foreign stock market. Familiarity bias affects foreign stock price only indirectly through its correlation with the domestic stock:

$$P_f^R - P_f = \left(\frac{1-m}{1+m}\right)[\gamma\sigma_d x_d/2 - \alpha/(1-\rho^2)]\rho\sigma_f.$$

The foreign stock price is lower (higher) relative to the rational benchmark if the domestic and foreign stock returns are positively (negatively) correlated. When the domestic and the foreign stock markets are uncorrelated, the stock with low uncertainty threshold is unaffected by familiarity bias.

Since familiarity bias affects the expected equity premium, it will have an effect on the CAPM which characterizes the relation between expected stock returns and the systematic risk of stocks. Our next proposition concerns the validity of international CAPM when some investors are subject to familiarity bias. In Cases (1) and (2) of Proposition 3, equilibrium stock returns are the same as in the case when everyone is rational. It is not surprising that the CAPM holds in these cases. What is interesting is that CAPM holds although no one is holding the market portfolio.[18] In Case (3), familiarity bias affects stock returns, and the traditional CAPM fails. Interestingly, in this case, a modified CAPM holds with respect to the rational investors' aggregate stock portfolio rather than the world market portfolio.

Proposition 4 *(1) When* $\alpha < \min\{(1-\rho)\gamma\sigma_d x_d/2, (1-\rho)\gamma\sigma_f x_f/2\}$ *or* $\alpha \geq \max\{(1-\rho)\gamma\sigma_d x_d/2, (1-\rho)\gamma\sigma_f x_f/2\}$,

$$\mathrm{E}[r_i] = \beta_i \mathrm{E}[r_M], \tag{16}$$

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[18]Using evidence from large-scale experimental financial markets, Bossaerts and Plott (2004) find that financial assets are priced by the CAPM even though the subjects participating in the experiments do not hold the market portfolio.

where r_i and r_M are the return of country i's stock market ($i = d$ or f) and the value-weighted world stock market M, β_i is the beta of stock i's return with respect to the world market return.

(2) When α is between $(1 - \rho)\gamma\sigma_d x_d/2$ and $(1 - \rho)\gamma\sigma_f x_f/2$, the uncertainty thresholds for the two countries,

$$\mathrm{E}[r_i] = \tau_i \beta_i \mathrm{E}[r_M], \tag{17}$$

where τ_i is greater (smaller) than one for the country with the high (low) uncertainty threshold. The absolute pricing error of the standard CAPM with respect to the market portfolio increases with the fraction of familiarity-biased investors. Furthermore, a modified CAPM holds:

$$\mathrm{E}[r_i] = \beta_i' \mathrm{E}[r_{M'}], \tag{18}$$

where M' is the rational investors' aggregate stock portfolio, and β_i' is the beta of stock i's return ($i = d$ or f) with respect to M'.

Our results above suggest that the failure of the empirical testing of the international CAPM may be caused by familiarity bias on the part of some investors. We find that the absolute pricing error of the standard CAPM increases with the fraction of familiarity-biased investors. Given that familiarity-biased investors are more likely to hold only domestic equity, the absolute pricing error of the standard CAPM is expected to be positively correlated with the amount of home bias.

Proposition 4 presents a testable hypothesis on the modified international CAPM. Given measures for the degree of uncertainty and the fraction of rational investors, we show how to construct the aggregate stock portfolio held by rational investors in the proof of Proposition 4 in the Appendix. In practice, the uncertainty can be measured according to Anderson, Ghysels and Juergens (2007) using the data on professional forecasters. The fraction of investors that participate in foreign (world) stock markets can serve as a proxy for the fraction of rational investors. The empirical test of the modified international CAPM is left for future studies.

We conclude this section by examining the equilibrium home bias in our model. The measure of home bias for domestic investors is the ratio of their domestic holdings in the total risky portfolio relative to the weight of domestic stock market value in the world market

portfolio:

$$H_d \equiv \frac{y_d P_d}{y_d P_d + y_f P_f} - \frac{x_d P_d}{x_d P_d + x_f P_f} = \frac{(y_d x_f - y_f x_d) P_d P_f}{(y_d P_d + y_f P_f)(x_d P_d + x_f P_f)}, \tag{19}$$

where y_d is the total holdings of domestic stock by domestic investors, and y_f is the total holdings of foreign stock by domestic investors. By Proposition 3, the total holdings of domestic stock by domestic investors and the total holdings of foreign stock by domestic investors are given by:

(1) when $\alpha < \min\{(1-\rho)\gamma\sigma_d x_d/2, (1-\rho)\gamma\sigma_f x_f/2\}$,

$$y_d = \frac{x_d}{2} + \frac{\alpha(1-m)}{(1-\rho)\gamma\sigma_d},$$

$$y_f = \frac{x_f}{2} - \frac{\alpha(1-m)}{(1-\rho)\gamma\sigma_f}.$$

(2) when $\alpha \geq \max\{(1-\rho)\gamma\sigma_d x_d/2, (1-\rho)\gamma\sigma_f x_f/2\}$,

$$y_d = \left(1 - \frac{m}{2}\right) x_d,$$

$$y_f = \left(\frac{m}{2}\right) x_f.$$

(3) when $(1-\rho)\gamma\sigma_f x_f/2 < \alpha < (1-\rho)\gamma\sigma_d x_d/2$,

$$y_d = \left(\frac{1}{1+m}\right) x_d + \frac{m(1-m)}{1+m} \frac{\alpha}{(1-\rho^2)\gamma\sigma_d},$$

$$y_f = \left(\frac{m}{2}\right) x_f.$$

(4) when $(1-\rho)\gamma\sigma_d x_d/2 < \alpha < (1-\rho)\gamma\sigma_f x_f/2$,

$$y_d = \left(1 - \frac{m}{2}\right) x_d,$$

$$y_f = \left(\frac{m}{1+m}\right) x_f - \frac{m(1-m)}{1+m} \frac{\alpha}{(1-\rho^2)\gamma\sigma_f}.$$

27

If all domestic investors are rational ($m = 1$), then $y_d/x_d = y_f/x_f = 1/2$, there is no home bias, and the home bias measure H_d takes a value of 0. If there is a positive fraction of investors subject to familiarity bias ($m < 1$), then in all of the cases above, $y_d/x_d \geq 1/2$ and $y_f/x_f \leq 1/2$. Thus, $y_d x_f - y_f x_d > 0$, and the home bias measure H_d in (19) is positive.

Figure 4 plots the equilibrium home bias ratio as a function of the model uncertainty for Germany, Japan, the United Kingdom, and the United States. In all four panels, the home bias ratio initially increases rapidly with the degree of model uncertainty. At sufficiently high levels of model uncertainty, familiarity-biased investors choose not to trade. In this case, only rational investors trade the risky assets, and the home bias ratio stays at the peak level.

6 Conclusion

Experimental and capital market evidence indicates that individuals favor geographically and linguistically proximate and more familiar investments; are biased in favor of staying at current consumption/investment positions or strategies and in favor of choice alternatives made salient as default options; and are averse even to small gambles when presented as increments relative to an endowed certainty position. More generally, individuals are more reluctant to take actions that impose risk than to bear risk associated with remaining passive; tend to like stimuli they have been exposed to more, tend to like people they are located close to, and are prone to malice toward outsiders.

These effects have generally been discussed separately, as reflected by a variety of labels: familiarity, local or home bias; the endowment effect; status quo bias; sunk cost effects, inertia; omission bias; the mere exposure effect; xenophobia; proximity bias in international cross listings, and propinquity effects. We offer a unified explanation for these effects based upon fear of *change* and of *the unfamiliar*.

Endowment effect arises endogenously in our setting. The model also offers an explanation for limited diversification of investors across stocks and asset classes; special cases include the under-diversification puzzle, the home bias puzzle and the preference of individuals to invest in company stock. We calculate the minimum number of stocks in a portfolio such that defection-induced fear of uncertainty deters individuals from diversifying further.

We find that for plausible parameter values, investors settle for very undiversified portfolio with just a few stocks. In calibration analysis, we find that the observed magnitude of home bias is consistent with a reasonable level of model uncertainty.

More importantly, we find that with familiarity-biased investors in the world equity markets, the standard CAPM with respect to the world market portfolio sometimes does not hold. The absolute pricing errors of the CAPM using the world market portfolio increase with the fraction of familiarity-biased investors and is positively correlated with the amount of home bias.

Interestingly, however, a modified CAPM holds when the world market portfolio is replaced by the aggregate stock holdings of rational investors. Our findings suggest a new reason for the empirical failures of the standard capital asset pricing model, that the market portfolio includes shares held by investors who are subject to familiarity bias. Our analysis predicts that the CAPM risk-return relation applies for the aggregate portfolios of those stocks held by rational investors (who are not subject to familiarity bias), and suggests how this portfolio can be identified empirically.

Our approach is consistent with the evidence that stocks that receive greater publicity or have greater news arrival tend to be purchased more heavily (even if the news is, on average, neutral). Stocks whose names are prominently mentioned in the media or by other individuals become more familiar. In our approach, investors therefore perceive the uncertainty as smaller in highly-publicized firms. Less familiarity bias with respect to a stock that is not currently part of an investor's portfolio will therefore encourage the investor to add the stock to his portfolio. Increased publicity about a stock expands breadth of ownership, increases net demand for the stock, and thereby induces a positive stock price reaction. Thus, our approach is consistent with the fact that firms sometimes make non-substantive advertisements prominently emphasizing the name of their firm, apparently aimed at attracting potential investors. Stocks that receive heavy publicity are purchased disproportionately by otherwise-non-participating investors.

We have argued that the emotions of fear and suspicion are directed to the unfamiliar and toward potential change, and that this phenomenon explains several biases in individual psychology as well as economic and financial decisions. One issue we have not addressed is the effect of these feelings on decisions made in response to the arrival of new information. Such

news will occasionally stimulate new uncertainty about the economic environment, thereby making individuals reluctant to trade. For example, it seems likely that extreme economic news could raise doubts among investors about whether their beliefs about how the world is structured are correct. In such circumstances of heightened uncertainty, familiarity bias effects could become especially strong, leading to reduction in trade.[19] Fear of the unfamiliar deserves further study as a possible explanation for the dynamics of market participation, liquidity, and prices.

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[19]See Routledge and Zin (2003) on how ambiguity aversion can lead to fluctuations in liquidity, such as the extreme illiquidity and "flight to quality" that occurred in international bond markets during the Russian debt crisis of August 1998.

References

Ackert, L. F., B. K. Church, J. Tompkins, and P. Zhang, 2005, "What's in a Name? An Experimental Examination of Investment Behavior," *Review of Finance*, 9, 281-304.

Ameriks, J., and S. P. Zeldes, 2004, "How Do Household Portfolio Shares Vary with Age?" Working paper, Columbia University.

Anderson, E. W., E. Ghysels, and J. L. Juergens, 2007, "The Impact of Risk and Uncertainty on Expected Returns," Working paper, Arizona State University.

Arkes, H. R., and C. Blumer, 1985, "The Psychology of Sunk Costs," *Organizational Behavior and Human Decision Processes*, 35, 124-140.

Barber, B., and T. Odean, 2000, "Trading is Hazardous to Your Wealth: The Common Stock Investment Performance of Individual Investors," *Journal of Finance*, 55, 773-806.

Benartzi, S., 2001, "Excessive Extrapolation and the Allocation of 401(k) Accounts to Company Stock?" *Journal of Finance*, 56, 1747-1764.

Bornstein, R. F., and P. R. D'Agostino, 1992, "Stimulus recognition and the mere exposure effect." *Journal of Personality and Social Psychology*, 63, 545-552.

Bossaerts, P. L., and C. Plott, 2004, "Basic principles of asset pricing theory: Evidence from large-scale experimental financial markets," *Review of Finance*, 8, 135-169.

Cao, H. H., T. Wang, and H. H. Zhang, 2005, "Model Uncertainty, Limited Market Participation, and Asset Prices," *Review of Financial Studies*, 18, 1219-1251.

Chen, Z., and L. Epstein, 2001, "Ambiguity, risk and asset returns in continuous time," *Econometrica*, 70, 1403-1443.

Coval, J., and T. Moskowitz, 1999, "Home Bias at Home: Local Equity Preference in Domestic Portfolios." *Journal of Finance*, 54, 1695-1704.

Cronqvist, H., 2006, "Advertising and Portfolio Choice," Dice Center Working Paper, Ohio State University.

Dow, J., and S. Werlang, 1992, "Uncertainty Aversion, Risk Aversion, and the Optimal

Choice of Portfolio," *Econometrica*, 60, 197-204.

Ellsberg, D., 1961, "Risk, Ambiguity, and the Savage Axioms," *Quarterly Journal of Economics*, 75, 643-669.

Epstein, L., and J. Miao, 2003, "A two-person dynamic equilibrium under ambiguity," *Journal of Economics and Dynamic Control*, 27, 1253-1288.

Epstein, L., and T. Wang, 1994, "Intertemporal Asset Pricing under Knightian Uncertainty," *Econometrica*, 62, 283-322.

Epstein, L., and T. Wang, 1995, "Uncertainty, Risk-Neutral Measures and Security Price Booms and Crashes," *Journal of Economic Theory*, 67, 40-80.

Fox, C. R., and A. Tversky, 1995, "Ambiguity Aversion and Comparative Ignorance," *Quarterly Journal of Economics*, 110, 585-603.

French, K., and J. Poterba, 1991, "Investor Diversification and International Equity Markets," *American Economic Review*, 81, 222-226.

Gilboa, I., and D. Schmeidler, 1989, "Maxmin Expected Utility Theory with Non-Unique Prior," *Journal of Mathematical Economics*, 18, 141-153.

Gneezy, U., J. A. List, and G. Wu, 2006, "The Uncertainty Effect: When a Risky Prospect is Valued Less than its Worst Possible Outcome," *Quarterly Journal of Economics*, 121, 1283-1309.

Goetzmann, W., and A. Kumar, 2006, "Why Do Individual Investors Hold Under-Diversified Portfolios?" Working Paper, University of Texas at Austin.

Graham, J., and Harvey, C., 2001, "The theory and practice of corporate finance: Evidence from the field," *Journal of Financial Economics*, 60, 2-3, 187-243.

Grinblatt, M., and M. Keloharju, 2001, "How Distance, Language and Culture Influence Stockholdings and Trades," *Journal of Finance*, 56, 1053-1073.

Hansen, L.P. and T.J. Sargent (2001): "Robust Control and Model Uncertainty," *American Economic Review*, 91, 60-66.

Heath C., and A. Tversky, 1991, "Preferences and Beliefs: Ambiguity and Competence in Choice under Uncertainty," *Journal of Risk and Uncertainty*, 4, 1991, 5-28.

Huber, J., J.W. Payne, and C. Puto, 1982, "Adding asymmetrically dominated alternatives: Violations of the regularity hypothesis," *Journal of Consumer Research*, 9, 90-98.

Josephs, R., R. P. Larrick, C. M. Steele, and R. E. Nisbett, 1992, "Protecting the self from the negative consequences of risky decisions," *Journal of Personality and Social Psychology*, 62, 1996, 26-37.

Kahneman, D., J. L. Knetsch, and R. H. Thaler, (1991), "The Endowment Effect, Loss Aversion, and Status Quo Bias: Anomalies," *Journal of Economic Perspectives*, 5(1), 193-206.

Kahneman, D., and A. Tversky, 1979, "Prospect Theory: An analysis of decision under risk," *Econometrica*, 47, 263-291.

Kilka, M., and Weber, M., 2000, "Home Bias in International Stock Return Expectations," *Journal of Psychology and Financial Markets*, 1(3&4), 176-192.

Knetsch, J. L., and J. A. Sinden, 1984, "Willingness to Pay and Compensation Demanded: Experimental Evidence of an Unexpected Disparity in Measures of Value," *Quarterly Journal of Economics*, 99(3), 507-521.

Lewis, K., 1999, "Trying to explain home bias in equities and consumption," *Journal of Economic Literature*, 37 (2), 571-608.

Madrian, B., and D. Shea, 2001, "The Power of Suggestion: Inertia in 401(k) Participation and Savings Behavior," *Quarterly Journal of Economics*, 116, 1149-1525.

Massa, Massimo, and Andrei Simonov, 2006, "Hedging, Familiarity and Portfolio Choice," *Review of Financial Studies*, 19, 633-685.

Merton, R.C., 1987, "A Simple Model of Capital Market Equilibrium with Incomplete Information," *Journal of Finance*, 42, 483-510.

Meulbroek, L. K., 2002, "Company Stock in Pension Plans: How Costly Is It?," Harvard Business School Working Paper 02-058.

Mitchell, O. S., and S. P. Utkus, 2002, "Company Stock and Retirement Plan Diversification," Pension Research Council Working Paper No. 2002-4.

Moreland, R. L., and R. Beach, 1992, "Exposure Effects in the Classroom: The Development of Affinity Among Students," *Journal of Experimental Social Psychology*, 28, 255-276.

Pagano, M., A. A. Roell, and J. Zechner, 2002, "The Geography of Equity Listing: Why Do Companies List Abroad?" *Journal of Finance*, 57, 2651-2694.

Poterba, J., and L. Summers, 1995, "A CEO Survey of U.S. Companies' Time Horizons and Hurdle Rates," *Sloan Management Review*, 37, 43-53.

Ritov, I., and J. Baron, 1990, "Reluctance to vaccinate: omission bias and ambiguity," *Journal of Behavioral Decision Making*, 3, 263-277.

Routledge, B., and S. Zin, 2003, "Model Uncertainty and Liquidity," working paper, Carnegie Mellon University.

Samuelson, W., and R. Zeckhauser, 1988, "Status Quo Bias in Decision Making," *Journal of Risk and Uncertainty*, 1, 7-59.

Sarkissian, S. and M. J. Schill, 2004, "The Overseas Listing Decision: New Evidence of Proximity Preference," *Review of Financial Studies*, 17, 769-809.

Seasholes, M. S., and N. Zhu, 2005, "Is there Information in the Local Portfolio Choices of Individuals?" Working Paper, University of California, Davis.

Shiller, R., F. Konya, and Y. Tsutsui, 1996, "Why did the Nikkei crash? Expanding the scope of expectations data collection," *Review of Economics and Statistics*, 78, 156-164.

Simonson, I., 1989, "Choice based on reasons: The case of attraction and compromise effects," *Journal of Consumer Research*, 16, 158174.

Simonson, I., and A. Tversky, 1992, "Choice in Context: Tradeoff Contrast and Extremeness Aversion," *Journal of Marketing Research* 29, 281-295.

Strong, N. C. and Xu, X. 2003, "Understanding the Equity Home Bias: Evidence from Survey Data," *Review of Economics and Statistics*, 85, 307-312.

Stulz, R. M., 1999, "International Portfolio Flows and Security Markets," in *International Capital Flows*, edited by Martin Feldstein, University Chicago Press, 257-293.

Thaler, R., 1980, "Toward a positive theory of consumer choice," *Journal of Economic Behavior and Organization,* 1, 39-60.

Tversky, A., 1972, "Elimination by aspects: A theory of choice," *Psychological Review*, 79, 281-299.

Uppal, R. and T. Wang, 2003, "Model Misspecification and Under-Diversification," *Journal of Finance*, 58, 2465-2486.

Zajonc, R. B., 1968, "Attitudinal effects of mere exposure," *Journal of Personality and Social Psychology Monograph Supplement*, 9, 1-27.

APPENDIX

Proof of Proposition1 :

Because U is concave, the first order condition for (2) and (3) implies that:

$$\min_{Q\in\mathcal{P}} \mathrm{E}^Q\left[\left(r - \frac{\Delta C_P}{\Delta e}\right)U'(W_0 + er)\right] = 0,$$

$$\min_{Q\in\mathcal{P}} \mathrm{E}^Q\left[\left(\frac{\Delta C_A}{\Delta e} - r\right)U'(W_0 + er)\right] = 0.$$

Letting $\Delta e \to 0$, we get

$$\min_{Q\in\mathcal{P}} \mathrm{E}^Q[(r - WTP)U'(W_0 + er)] = 0, \tag{20}$$

$$\min_{Q\in\mathcal{P}} \mathrm{E}^Q[(WTA - r)U'(W_0 + er)] = 0. \tag{21}$$

Equation (21) is equivalent to

$$\max_{Q\in\mathcal{P}} \mathrm{E}[(r - WTA)U'(W_0 + er)] = 0. \tag{22}$$

Equations (20) and (22) indicate that there is a kink around the endowed stock position. When determining his willingness to pay, an individual considers the scenario most adverse to buying the stock. On the other hand, when determining his willingness to accept, he contemplates the best case scenario for holding on to the stock.

We must have $WTP \leq WTA$. To show this, suppose that $WTP > WTA$. Then

$$0 = \min_{Q\in\mathcal{P}} E[(r - WTP)U'] = \max_{Q\in\mathcal{P}} E[(WTP - r)U'] > \min_{Q\in\mathcal{P}} E[(WTA - r)U'] = 0,$$

a contradiction. Q.E.D.

Proof of Proposition 2: Under SSQDA preferences, the perceived certainty-equivalent gains of moving from endowment portfolio $(\omega, 1 - \omega)^\top$ to a portfolio $(\omega + \Delta D, 1 - \omega - \Delta D)^\top$ is

$$G(\Delta D, e) \equiv \min_v\{-e^{-\gamma[(\Delta Du+e)^\top(\mu+v) - \frac{\gamma}{2}(\Delta Du+e)^\top\Sigma(\Delta Du+e)]} + e^{-\gamma e^\top(\mu+v) + \frac{\gamma^2}{2}e^\top\Sigma e}\},$$

where γ is the risk aversion coefficient and $u \equiv (1, -1)^\top$. The certainty equivalent gain can be expressed as

$$G(\Delta D, e) \approx \gamma C(\Delta D, e),$$

$$
\begin{aligned}
C(\Delta D, e) &= \min_v \{ (\Delta D u)^\top (\mu + v) - \frac{\gamma}{2} [(\Delta D u + e)^\top \Sigma (\Delta D u + e) - e^\top \Sigma e] \} \\
&= \Delta D [u^\top \mu - \text{sign} (\Delta D) v_m] - \frac{\gamma}{2} [\Delta D^2 u^\top \Sigma u + 2 \Delta D u^\top \Sigma e],
\end{aligned}
$$

where v satisfies (7) and $v_m \equiv - \min_{Q \in \mathcal{P}} u^\top v$. It is straightforward to show that

$$v_m = \beta \sqrt{u^\top \Sigma u / T}.$$

Given initial endowment e, the optimal trade ΔD maximizes the certainty equivalent gain $C(\Delta D, e)$. The unconstrained first order condition is:

$$u^\top \mu - \text{sign} (\Delta D) v_m - \gamma \Delta D u^\top \Sigma u - \gamma u^\top \Sigma e = 0.$$

There are two scenarios: (1) No trading is perceived to be optimal, i.e., $\Delta D = 0$; (2) Trading is perceived to be optimal and satisfies the first order condition above, which implies

$$\Delta D = \frac{u^\top \mu - \text{sign} (\Delta D) v_m - \gamma u^\top \Sigma e}{\gamma u^\top \Sigma u}.$$

The no trade scenario occurs if and only if

$$-v_m < u^\top \mu - \gamma u^\top \Sigma e < v_m.$$

Otherwise, ΔD is positive when $u^\top \mu - \gamma u^\top \Sigma e > v_m$, and is negative when $u^\top \mu - \gamma u^\top \Sigma e < -v_m$. Q.E.D.

Proof of Proposition 3:

Under SSQDA preferences, the perceived certainty equivalent gains of moving from endowment portfolio e to a portfolio $e + \Delta D$ is

$$G(\Delta D, e) \approx \gamma C(\Delta D, e),$$

where v is the adjustments to perceived mean stock payoffs as in (11), and

$$C(\Delta D, e) \equiv \min_v \{ \Delta D^\top (\mu + v - P) - \left(\frac{\gamma}{2} \right) [\Delta D^\top \Sigma \Delta D + 2 \Delta D^\top \Sigma e] \}.$$

The familiarity-biased investor evaluates any deviation in the worse case scenario among the possible probability distributions. For $i = 1$ (corresponding to domestic stock) or $i = 2$ (corresponding to foreign stock), if $\Delta D_i > 0$ (buy more shares), the worse case scenario mean adjustment is $-\alpha\sigma_i$; if $\Delta D_i < 0$ (sell some shares), the worse case scenario mean adjustment is $\alpha\sigma_i$. Thus,

$$C(\Delta D, e) = \Delta D^\top [\mu - P - \text{sign}(\Delta D)v_m] + \left(\frac{\gamma}{2}\right) [\Delta D^\top \Sigma \Delta D + 2\Delta D^\top \Sigma e], \qquad (23)$$

where $\text{sign}(\Delta D)$ is a vector that gives the sign of each component of the vector ΔD, and v_m is a vector defined as $v_m \equiv \alpha(\sigma_d, \sigma_f)^\top$.

The optimal trade ΔD_b^* for a familiarity biased investor corresponding to a given endowment e maximizes $G(\Delta D, e)$. If it is nonzero, then it necessarily satisfies the first order condition derived from (23),

$$\mu - P - \text{sign}(\Delta D_b^*)v_m - \gamma\Sigma\Delta D_b^* - \gamma\Sigma e = 0, \qquad (24)$$

which implies that familiarity biased investor's optimal holding is

$$\Delta D_b^* + e = \left(\frac{1}{\gamma}\right) \Sigma^{-1} \left[\mu - P - \text{sign}(\Delta D_b^*)v_m\right]. \qquad (25)$$

This applies to both domestic and foreign familiarity biased investors, with $e = (x_d, 0)$ and $e = (0, x_f)$ respectively.

There are several possibilities for the familiarity biased investors' demand in equilibrium. In the first case when the amount of uncertainty is sufficiently low in both countries, we will show that familiarity biased investors would sell some of their own country's stock and buy some of the other country's stock. In the second case when the amount of uncertainty is sufficiently high in both country, the familiarity biased investors would keep their endowment. We also consider a third case where the amount of uncertainty is too high in only one country.

Case (1): when uncertainty is low for both domestic and foreign stock markets, so that familiarity biased investors in both countries sell some of their own country's stock and buy some of the other country's stock. Then by (25), the optimal demand of domestic familiarity biased investor is

$$D_{db} = \left(\frac{1}{\gamma}\right) \Sigma^{-1} \begin{pmatrix} \mu_d - P_d + \alpha\sigma_d \\ \mu_f - P_f - \alpha\sigma_f \end{pmatrix}, \qquad (26)$$

and the optimal demand by the foreign familiarity biased investor is

$$D_{fb} = \left(\frac{1}{\gamma}\right) \Sigma^{-1} \begin{pmatrix} \mu_d - P_d - \alpha\sigma_d \\ \mu_f - P_f + \alpha\sigma_f \end{pmatrix}. \qquad (27)$$

Aggregating the rational investors' demand in (12) and familiarity biased investors' demand in (26) and (27), the market clearing condition is

$$\left(\frac{2m}{\gamma}\right)\Sigma^{-1}\left(\begin{array}{c} \mu_d - P_d \\ \mu_f - P_f \end{array}\right) + \left(\frac{1-m}{\gamma}\right)\Sigma^{-1}\left(\begin{array}{c} \mu_d - P_d + \alpha\sigma_d \\ \mu_f - P_f - \alpha\sigma_f \end{array}\right)$$

$$+ \left(\frac{1-m}{\gamma}\right)\Sigma^{-1}\left(\begin{array}{c} \mu_d - P_d - \alpha\sigma_d \\ \mu_f - P_f + \alpha\sigma_f \end{array}\right) = \left(\begin{array}{c} x_d \\ x_f \end{array}\right).$$

This simplifies to

$$\left(\frac{2}{\gamma}\right)\Sigma^{-1}\left(\begin{array}{c} \mu_d - P_d \\ \mu_f - P_f \end{array}\right) = \left(\begin{array}{c} x_d \\ x_f \end{array}\right),$$

which implies that the equilibrium stock prices in the first case satisfy

$$\left(\begin{array}{c} \mu_d - P_d \\ \mu_f - P_f \end{array}\right) = \left(\frac{\gamma}{2}\right)\Sigma\left(\begin{array}{c} x_d \\ x_f \end{array}\right),$$

just as claimed in Case (1) of Proposition 3 (see equation (13)). The equilibrium stock prices in Case 1 coincides with the equilibrium stock prices when all investors are rational.

We need to check that familiarity biased investors in both countries sell some of their own country's stock and buy some of the other country's stock. For this to obtain the model parameters must satisfy:

$$\frac{1}{2}x_d + \frac{\alpha}{(1-\rho)\gamma\sigma_d} < x_d,$$

$$\frac{1}{2}x_d - \frac{\alpha}{(1-\rho)\gamma\sigma_d} > 0,$$

$$\frac{1}{2}x_f - \frac{\alpha}{(1-\rho)\gamma\sigma_f} > 0,$$

$$\frac{1}{2}x_f + \frac{\alpha}{(1-\rho)\gamma\sigma_f} < x_f.$$

The necessary and sufficient condition for the above to hold is

$$\alpha < min\left\{\left(\frac{1-\rho}{2}\right)\gamma\sigma_d x_d, \left(\frac{1-\rho}{2}\right)\gamma\sigma_f x_f\right\}.$$

Case (2): $\alpha > max\left\{(\frac{1-\rho}{2})\gamma\sigma_d x_d, (\frac{1-\rho}{2})\gamma\sigma_f x_f\right\}$. In this case, both domestic and foreign familiarity biased investors choose to stay at the endowment because the perceived amount of uncertainty is

39

too high. The market clearing condition is

$$\left(\frac{2m}{\gamma}\right)\Sigma^{-1}\left(\begin{array}{c}\mu_d - P_d \\ \mu_f - P_f\end{array}\right) + (1-m)\left(\begin{array}{c}x_d \\ 0\end{array}\right) + (1-m)\left(\begin{array}{c}0 \\ x_f\end{array}\right) = \left(\begin{array}{c}x_d \\ x_f\end{array}\right),$$

which implies that the equilibrium stock prices satisfy (13).

Case (3): The amount of uncertainty is too high in one country but not the other. Without loss of generality, assume the parameters are such that $(\frac{1-\rho}{2})\gamma\sigma_f x_f < \alpha < (\frac{1-\rho}{2})\gamma\sigma_d x_d$. In this case, the domestic familiarity biased investor sells some of his endowment but he does not buy any shares of the foreign stock. The foreign familiarity biased investor stays at his endowed foreign stock shares and does not invest in the domestic stock. The market clearing condition is

$$\left(\frac{2m}{\gamma}\right)\Sigma^{-1}\left(\begin{array}{c}\mu_d - P_d \\ \mu_f - P_f\end{array}\right) + \left(\frac{1-m}{(1-\rho^2)\gamma\sigma_d^2\sigma_f^2}\right)\left(\begin{array}{c}(\mu_d - P_d + \alpha\sigma_d)\sigma_f^2 - \rho\sigma_d\sigma_f(\mu_f - P_f) \\ 0\end{array}\right)$$

$$+ (1-m)\left(\begin{array}{c}0 \\ x_f\end{array}\right) = \left(\begin{array}{c}x_d \\ x_f\end{array}\right).$$

This is equivalent to the following system of linear equations for $\mu_d - P_d$ and $\mu_f - P_f$:

$$\left(\frac{1}{(1-\rho^2)\sigma_d^2\sigma_f^2}\right)\left(\begin{array}{cc}\sigma_f^2 & -\rho\sigma_d\sigma_f \\ -\rho\sigma_d\sigma_f & \sigma_d^2\end{array}\right)\left(\begin{array}{c}\mu_d - P_d \\ \mu_f - P_f\end{array}\right) = \left(\begin{array}{c}\frac{1}{1+m}\left(\gamma x_d - \frac{(1-m)\alpha}{(1-\rho^2)\sigma_d}\right) \\ \frac{1}{2}\gamma x_f\end{array}\right).$$

But

$$\Sigma^{-1} = \frac{1}{(1-\rho^2)\sigma_d^2\sigma_f^2}\left(\begin{array}{cc}\sigma_f^2 & -\rho\sigma_d\sigma_f \\ -\rho\sigma_d\sigma_f & \sigma_d^2\end{array}\right),$$

so the equilibrium stock prices satisfy (14) as claimed in the case (3) of Proposition 3.

Proof of Proposition 4: The world stock market M consists of x_d shares of the domestic stock and x_f share of the foreign stock. Its payoff next period is normally distributed as

$$V_M \sim \mathcal{N}\left(x_d\mu_d + x_f\mu_f, (x_d\ x_f)\Sigma\left(\begin{array}{c}x_d \\ x_f\end{array}\right)\right).$$

The value of the world stock market P_M is $x_d P_d + x_f P_f$. Stock returns are

$$r_d = \frac{V_d - P_d}{P_d}, \ r_f = \frac{V_f - P_f}{P_f}.$$

$$r_M = \frac{V_M - P_M}{P_M} = \frac{x_d(V_d - P_d) + x_f(V_f - P_f)}{P_M}.$$

It follows that

$$\mathrm{E}[r_M] = \left(\frac{1}{P_M}\right)(x_d \ x_f)\left(\begin{array}{c}\mu_d - P_d \\ \mu_f - P_f\end{array}\right),$$

$$\mathrm{Var}(r_M) = \left(\frac{1}{P_M^2}\right)(x_d \ x_f)\Sigma\left(\begin{array}{c}x_d \\ x_f\end{array}\right),$$

$$\mathrm{Cov}(r_i, r_M) = \left(\frac{1}{P_i P_M}\right)\left[\Sigma\left(\begin{array}{c}x_d \\ x_f\end{array}\right)\right]_i,$$

where $[\cdot]_i$ denotes the *ith* component of a vector, $i = 1$ (respectively $i = 2$) corresponds to the domestic (foreign) stock. Thus, the beta of the domestic (respectively foreign) stock return with respect to the world market return β_d (respectively β_f) is

$$\beta_d = \left(\frac{P_M}{P_d}\right)\frac{\left[\Sigma\left(\begin{array}{c}x_d \\ x_f\end{array}\right)\right]_1}{(x_d \ x_f)\Sigma\left(\begin{array}{c}x_d \\ x_f\end{array}\right)},$$

$$\beta_f = \left(\frac{P_M}{P_f}\right)\frac{\left[\Sigma\left(\begin{array}{c}x_d \\ x_f\end{array}\right)\right]_2}{(x_d \ x_f)\Sigma\left(\begin{array}{c}x_d \\ x_f\end{array}\right)}.$$

It follows that

$$\beta_i \mathrm{E}[r_M] = \left(\frac{\beta_i}{P_M}\right)(x_d \ x_f)\left(\begin{array}{c}\mu_d - P_d \\ \mu_f - P_f\end{array}\right), \quad i = d \text{ or } f. \tag{28}$$

CAPM holds if and only $\mathrm{E}[r_i] = \beta_i \mathrm{E}[r_M]$ in the equilibrium (the riskfree rate is zero in our economy).

For Case 1 and Case 2 of Proposition 3, equilibrium prices P_d and P_f satisfy (13). Substituting (13) into (28),

$$\begin{aligned}
\beta_d \mathrm{E}[r_M] &= \left(\frac{\gamma \beta_d}{P_M}\right)(x_d \ x_f)\Sigma\left(\begin{array}{c}x_d \\ x_f\end{array}\right) \\
&= \left(\frac{\gamma}{2P_d}\right)\left[\Sigma\left(\begin{array}{c}x_d \\ x_f\end{array}\right)\right]_1 \\
&= \frac{\mu_d - P_d}{P_d} \\
&= \mathrm{E}[r_d]
\end{aligned}$$

41

Thus, CAPM holds for the domestic stock. Similarly, CAPM holds for the foreign stock in these cases as well.

For Case 3 of Proposition 3, to conserve space we consider only the case $\left(\frac{1-\rho}{2}\right)\gamma\sigma_f x_f < \alpha < \left(\frac{1-\rho}{2}\right)\gamma\sigma_d x_d$ (the case $\left(\frac{1-\rho}{2}\right)\gamma\sigma_d x_d < \alpha < \left(\frac{1-\rho}{2}\right)\gamma\sigma_f x_f$ can be dealt with in the same manner.) The equilibrium prices P_d and P_f satisfy (14). It follows that

$$
\begin{aligned}
\beta_d \mathrm{E}[r_M] &= \left(\frac{1}{P_d}\right) \frac{\left[\Sigma\begin{pmatrix} x_d \\ x_f \end{pmatrix}\right]_1}{(x_d\ x_f)\Sigma\begin{pmatrix} x_d \\ x_f \end{pmatrix}} (x_d\ x_f)\Sigma\begin{pmatrix} \frac{1}{1+m}\left(\gamma x_d - \frac{(1-m)\alpha}{(1-\rho^2)\sigma_d}\right) \\ \frac{1}{2}\gamma x_f \end{pmatrix} \\
&= k_d \mathrm{E}[r_d],
\end{aligned}
$$

where

$$
k_1 = \frac{(x_d\ x_f)\Sigma\begin{pmatrix} \frac{1}{1+m}\left(\gamma x_d - \frac{(1-m)\alpha}{(1-\rho^2)\sigma_d}\right) \\ \frac{1}{2}\gamma x_f \end{pmatrix}}{(x_d\ x_f)\Sigma\begin{pmatrix} x_d \\ x_f \end{pmatrix}} \frac{\left[\Sigma\begin{pmatrix} x_d \\ x_f \end{pmatrix}\right]_1}{\left[\Sigma\begin{pmatrix} \frac{1}{1+m}\left(\gamma x_d - \frac{(1-m)\alpha}{(1-\rho^2)\sigma_d}\right) \\ \frac{1}{2}\gamma x_f \end{pmatrix}\right]_1}.
$$

Similarly, for the foreign stock market,

$$
\beta_f \mathrm{E}[r_M] = k_f \mathrm{E}[r_f],
$$

where

$$
k_2 = \frac{(x_d\ x_f)\Sigma\begin{pmatrix} \frac{1}{1+m}\left(\gamma x_d - \frac{(1-m)\alpha}{(1-\rho^2)\sigma_d}\right) \\ \frac{1}{2}\gamma x_f \end{pmatrix}}{(x_d\ x_f)\Sigma\begin{pmatrix} x_d \\ x_f \end{pmatrix}} \frac{\left[\Sigma\begin{pmatrix} x_d \\ x_f \end{pmatrix}\right]_2}{\left[\Sigma\begin{pmatrix} \frac{1}{1+m}\left(\gamma x_d - \frac{(1-m)\alpha}{(1-\rho^2)\sigma_d}\right) \\ \frac{1}{2}\gamma x_f \end{pmatrix}\right]_2}.
$$

The constants τ_i's in the Proposition 4 are

$$
\tau_d = 1/k_d, \quad \tau_f = 1/k_f.
$$

They are not equal to one in general. Thus, the CAPM does not hold when the degree of uncertainty is between the uncertainty thresholds of the two countries. In fact, $\tau_d > 1$ and $\tau_f < 1$ when

$\left(\frac{1-\rho}{2}\right)\gamma\sigma_f x_f < \alpha < \left(\frac{1-\rho}{2}\right)\gamma\sigma_d x_d$. It is straightforward to verify that $\tau_d - 1$ can be expressed as a fraction whose denominator is positive, with numerator given by

$$\left(\frac{1-m}{1+m}\right)\sigma_d\sigma_f^2 x_f^2[(1-\rho^2)\gamma\sigma_d x_d/2 - \alpha].$$

The numerator of $\tau_d - 1$ is also positive because $\alpha < (1-\rho)\gamma\sigma_d x_d/2$, and $1 + \rho > 1$. Similarly, $\tau_d - 1$ can be expressed as a fraction whose denominator is positive, with numerator given by

$$-\left(\frac{1-m}{1+m}\right)\sigma_d\sigma_f^2 x_d x_f[(1-\rho^2)\gamma\sigma_d x_d/2 - \alpha].$$

The numerator of $\tau_f - 1$ is negative, thus $\tau_f < 1$ when $\left(\frac{1-\rho}{2}\right)\gamma\sigma_f x_f < \alpha < \left(\frac{1-\rho}{2}\right)\gamma\sigma_d x_d$. The expected pricing errors of the standard CAPM under our model are given by $(\tau_d - 1)\mathrm{E}[r_M]$ and $(\tau_f - 1)\mathrm{E}[r_M]$. The absolute pricing errors are proportional to $|\tau_i - 1|$, which increase with $1 - m$, the fraction of familiarity-biased investors.

Finally, we show that a modified version of CAPM holds. Suppose $\left(\frac{1-\rho}{2}\right)\gamma\sigma_f x_f < \alpha < \left(\frac{1-\rho}{2}\right)\gamma\sigma_d x_d$. The rational investors' optimal holdings are given by

$$\left(\frac{1}{\gamma}\right)\Sigma^{-1}\begin{pmatrix}\mu_d - P_d \\ \mu_f - P_f\end{pmatrix}$$

Substituting the equilibrium stock returns given by (14), the rational investors' portfolio M' consist of $n_1 x_d$ shares of the domestic stock and $n_2 x_f$ shares of the foreign stock, where

$$n_1 = \frac{1}{1+m} - \left(\frac{1-m}{1+m}\right)\frac{\alpha}{(1-\rho^2)\gamma\sigma_d x_d}, \quad n_2 = \frac{1}{2}.$$

Note that $n_1 > n_2$, and the difference increases with the amount of uncertainty α.

The expected return of the portfolio M' is:

$$\mathrm{E}[r_M] = \left(\frac{1}{P_{M'}}\right)(n_1 x_d \; n_2 x_f)\begin{pmatrix}\mu_d - P_d \\ \mu_f - P_f\end{pmatrix}.$$

The beta of stock i with respect to the portfolio M' is ($i = 1$ for the domestic stock, $i = 2$ for the

foreign stock)

$$\beta_i = \left(\frac{P_{M'}}{P_i} \right) \frac{\left[\Sigma \left(\begin{array}{c} n_1 x_d \\ n_2 x_f \end{array} \right) \right]_i}{(n_1 x_d \ n_2 x_f) \Sigma \left(\begin{array}{c} n_1 x_d \\ n_2 x_f \end{array} \right)}.$$

By the definition of n_1 and n_2, and the equilibrium return relation (14),

$$\left(\begin{array}{c} \mu_d - P_d \\ \mu_f - P_f \end{array} \right) = \Sigma \left(\begin{array}{c} n_1 x_d \\ n_2 x_f \end{array} \right).$$

Using equations (6) and (6), it follows that for the domestic stock,

$$\begin{aligned} \beta_d \mathrm{E}[r_{M'}] &= \left(\frac{1}{P_d} \right) \frac{\left[\Sigma \left(\begin{array}{c} n_1 x_d \\ n_2 x_f \end{array} \right) \right]_1}{(n_1 x_d \ n_2 x_f) \Sigma \left(\begin{array}{c} n_1 x_d \\ n_2 x_f \end{array} \right)} (n_1 x_d \ n_2 x_f) \left(\begin{array}{c} \mu_d - P_d \\ \mu_f - P_f \end{array} \right) \\ &= \left(\frac{1}{P_d} \right) \left[\Sigma \left(\begin{array}{c} n_1 x_d \\ n_2 x_f \end{array} \right) \right]_1 \\ &= \frac{\mu_d - P_d}{P_d} \\ &= \mathrm{E}[r_d]. \end{aligned}$$

Thus, the CAPM holds for the domestic stock with respect to the modified market portfolio M'. The case for the foreign stock is similar.

Table 1. Summary statistics of annual stock market returns for various countries.

	Mean	Std. Dev.	Correlation
Germany	0.1356	0.2431	0.5679
Japan	0.1434	0.3017	0.8508
UK	0.1890	0.2504	0.6076
US	0.1478	0.1569	0.5004
World	0.1495	0.2084	1.0000

The reported statistics are for the annual value-weighted dollar returns from January 1975 to December 2006. "Correlation" measures the sample correlation between the stock market return in each country and the return on the world market portfolio. The original datasets are obtained from Kenneth French's website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french.

Figure 1: Under-diversification and uncertainty
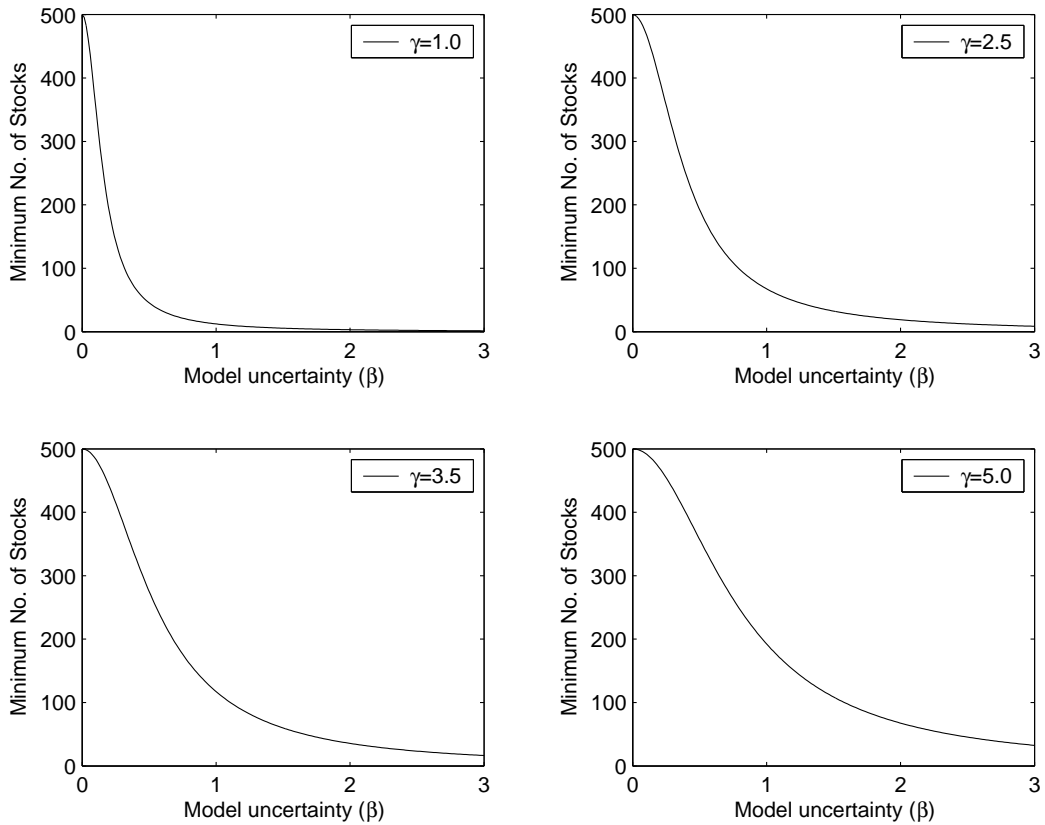
This figure plots the minimum number of stocks in an investor's portfolio when defection from the endowment induces aversion to model uncertainty, for various risk aversion coefficients. The total number of available stocks is 500. Assume the investor uses $T = 100$ data points and estimates that the annual standard deviation of stock return is $\sigma = 0.3$, and the pairwise correlation is 0.5.

Figure 2: Optimal weight on domestic equity for familiarity biased investors
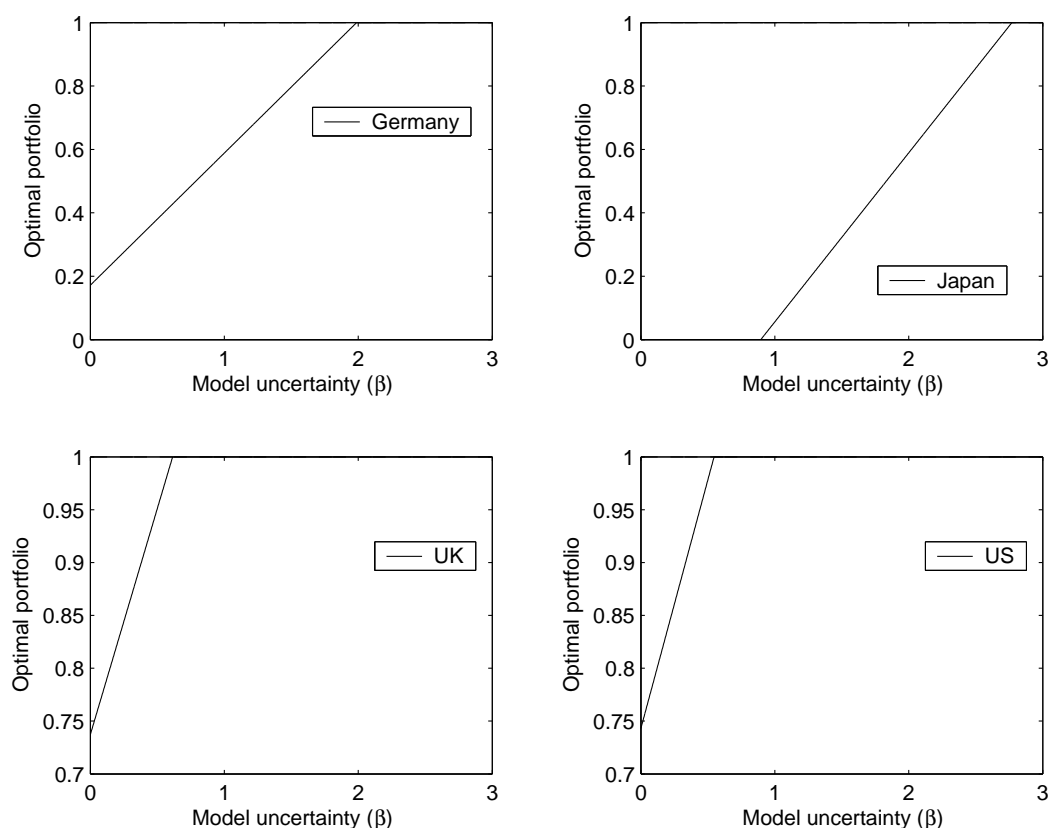
This figure plots the perceived optimal domestic equity proportion as a function of model uncertainty (β) for Germany, Japan, United Kingdom, and the United States. Investors are allowed to hold their domestic market portfolio and the world market portfolio. Their initial endowment is 100% in domestic equity. The adjustments v for the mean stock returns for familiarity biased investors satisfies $v^{\top}\Sigma^{-1}v \leq \beta^2/T$, where Σ is the covariance matrix of return of a country with the world market using the annual return data from January 1975 to December 2006 ($T = 32$). The risk aversion coefficient is set to $\gamma = 2$ for all four panels.

Figure 3: Perceived certainty equivalent gains
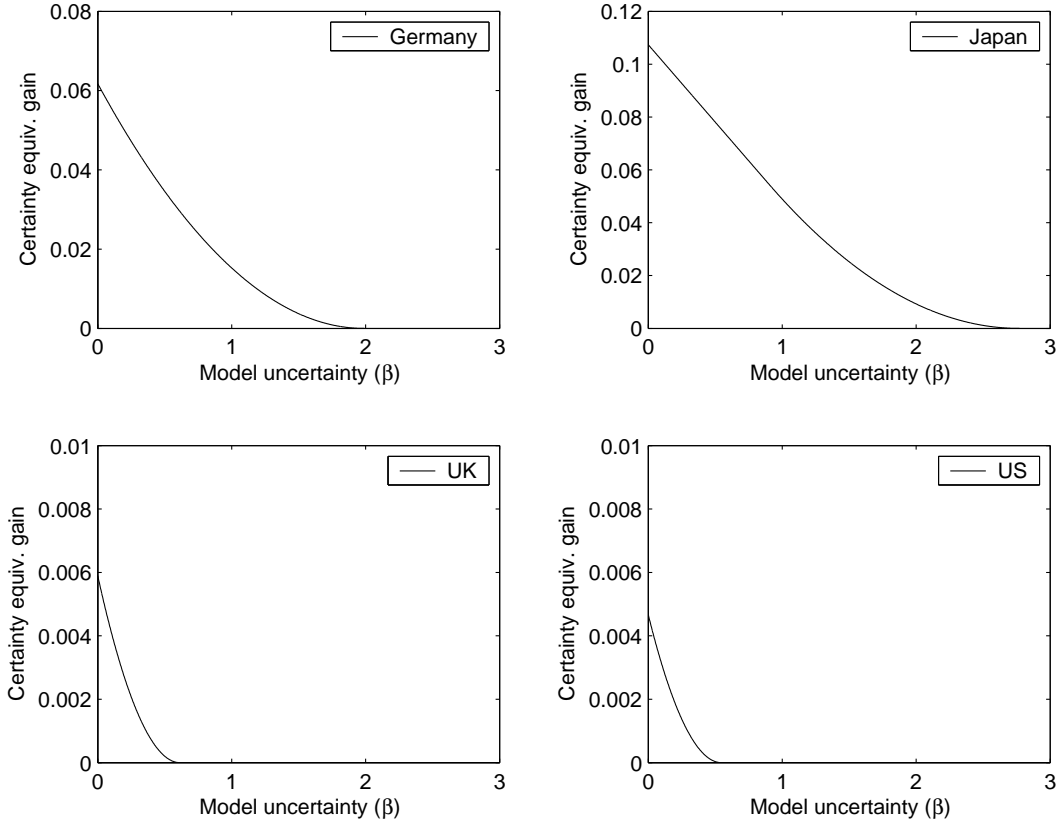
This figure plots the certainty equivalent gains as a function of model uncertainty (β) for Germany, Japan, United Kingdom, and the United States when familiarity biased investors move from their 100% domestic equity initial endowment to the optimal combination of domestic and world market (see Figure 2). The adjustments v for the mean stock returns for familiarity biased investors satisfies $v^\top \Sigma^{-1} v \leq \beta^2/T$, where Σ is the covariance matrix of return of a country with the world market using the annual return data from January 1975 to December 2006 ($T = 32$). The risk aversion coefficient is set to $\gamma = 2$ for all four panels.

Figure 4: Home bias ratio
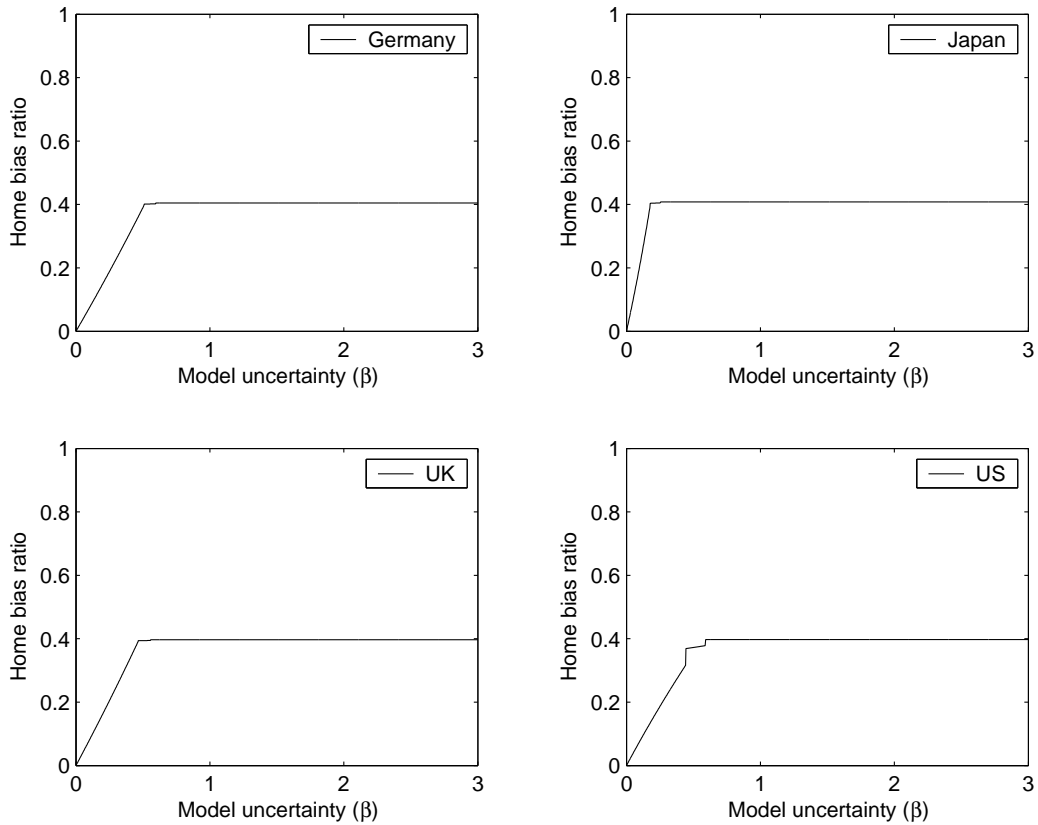
This figure plots the home bias ratio as a function of model uncertainty (α) for Germany, Japan, United Kingdom, and the United States in a general equilibrium model with both rational and familiarity-biased investors. The home bias ratio is defined as the ratio of each country's domestic holdings in the total risky portfolio relative to that of the weighted domestic holdings in the world market portfolio. The proportion of rational investors is set at 20 percent. The risk aversion coefficient is set at $\gamma = 2$.