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The Forward Premium Puzzle and Latent Factors Day by Day

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The Forward Premium Puzzle and Latent Factors Day by Day[*]

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Abstract   We use futures instead of forward rates to study the complete maturity spectrum of the forward premium puzzle from two days to six months. At short maturities the slope coefficient is positive, but these turn negative as the maturity increases to the monthly level. Futures data allow us to control for the influence of an unobserved factor that can be decomposed into a contract-specific and a time-to-maturity effect. Once we do this, we find that the coefficients on the forward premium are much closer to one. The latent factor is shown to be related to conventional proxies of risk.

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1 Introduction

According to the Expectations Hypothesis (EH), forward exchange rates should be efficient predictors of future spot exchange rates. The hypothesis, which assumes rational expectations and risk neutral speculators, is an important building block of models of international macro and finance. Its empirical support, however, is weak at best. Fama (1984) first reported that, in a regression of monthly foreign exchange returns on monthly forward premiums, the estimated slope coefficient is negative instead of being one. This result has become known as the 'forward premium puzzle' (FPP) and is the subject of a large body of empirical research; see e.g. Froot and Frankel (1990) and Engel (1996) for overviews. Froot and Frankel (1990) found that the average estimate of the slope coefficient in over 75 published empirical studies was (-0.88). This suggests that market participants may not even be able to predict the direction of exchange-rate changes correctly.

Various explanations of the FPP have been offered. One branch of the literature argues that the forward premium contains a time-varying risk premium which is negatively correlated with the expected change in the exchange rate (e.g. Fama, 1984; Hodrick and Srivastava, 1987; and Hsieh, 1984). Another branch argues that the forward premium contains a systematic forecast error due to learning about regime shifts, Peso problems, or irrational information processing, see e.g. Bilson, (1981); Mark and Wu, (1998); Krasker, (1980); Evans and Lewis, (1995); Lewis, (1995); Gourinchas and Tornell, (2004). Following McCallum (1994), Meredith and Ma (2002) argue that the estimate of the slope coefficient is biased due to the response of monetary policy to output and inflation which in turn are correlated with the exchange rate. Recently, Bacchetta and van Wincoop (2007, 2010) have argued that infrequent portfolio revisions and incomplete information processing can explain the FPP.

In this paper, we present new tests of the EH based on exchange rate futures rather than forward contracts. While data for forward contracts are available only for fixed maturity *horizons*, futures contracts have fixed delivery *dates*. Since futures contracts are traded in secondary markets, we can take futures rates from the first to the last trading day of a given contract and construct the full maturity spectrum of futures premiums in daily units. While the differences in trading mechanisms, default, or liquidity premiums between forward and futures contracts might cloud the comparability of the two, empirical studies suggest that this is not the case; see e.g. Cornell and

Reinganum, 1981; Hodrick and Srivastava, 1987; Polakoff and Grier, 1991; Chang and Chang, 1990. Moreover, at monthly maturities our futures data generate the same results as the forward data.

To our knowledge, no other study has used futures data for this purpose.[1] The nature of our data allows us to vary the time horizon over which expectations are formed from two days all the way up to six months. We find that the slope coefficient is decreasing in the length of the maturity horizon. For maturity horizons shorter than one month, it is generally positive, and, for horizons up to the three weeks, the EH is not rejected by the data. For maturity horizons longer than one month, we find that the slope coefficients become negative. Thus, we find that FPP emerges only gradually as the time horizon over which expectations are formed increases.

A few other studies also confirm that the strength of the FPP may be a function of the maturity. Using forward data with various maturities including spot/next exchange rate rats as a proxy for one-day forwards, Yang and Shintani (2006) find that the slope coefficient is initially around zero for the shortest maturity and estimate declining slope coefficients as the length of the contract increases. This result is similar to empirical tests of uncovered interest parity (UIP).[2] Using very short end of the day interest rate differentials, Chaboud and Wright (2005) find that already after a few hours the slope coefficient turns from a positive to a negative value due to the ex-interest effect. Thus, our futures based data results suggest that the evidence from intra-day interest differentials cannot be simply extrapolated to futures data.[3]

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[1]Hodrick and Srivastava (1987) use data from three-months futures contracts to test a hypothesis related to the EH, i.e., that the futures rate at time t from a contract expiring in $t + k$ is an efficient predictor of the futures rate at time $t + 1$ from the same contract. They present evidence rejecting this hypothesis.

[2]UIP holds that the difference between the interest rate on two otherwise identical assets denominated in different currencies equals the expected relative change in the exchange rate over the holding period. Under UIP, the slope of a regression of foreign exchange returns on interest rate differentials should also be one. Like the EH, UIP has been strongly rejected. Several studies testing UIP for assets with multi-year maturities confirm that the slope coefficient depends on the length of the holding period of the asset, see Alexius (2001), Chinn and Meredith (2004), Fama (2006) and Fama and Bliss (1987).

[3]Chaboud and Wright (2005) use the interest differential around the time of the daily ex-interest rate moment and observe uncovered interest rate parity (UIP), while the UIP already fails at the 12 hour horizon. Since we use futures rates, the ex-dividend effect is not in our data. Moreover, futures rates reflect market views about the specific future spot rate. Therfore, these rates may incorporate other information than short interest

Furthermore, we show that the slope coefficient can be represented as a quadratic function of the length of this time horizon with an intercept equal to one, a negative linear term, and a positive quadratic term. Thus, although this is not covered by our sample, our results are compatible with the hypothesis that the FPP disappears for very long maturity horizons. This is consistent with recent findings in the empirical literature on UIP; see e.g. Chinn and Meredith (2004) and Chinn (2006).

Using futures data has another advantage apart from being able to complete the maturity spectrum of the premium puzzle estimates. The wealth of data over the maturity horizon and across the contracts facilitates the extraction of a latent factor that is potentially correlated with the futures premium. The common factor model explains foreign exchange returns as the sum of the futures premium and an unobserved factor that varies over time and with maturity. We use Pesaran's (2006) common correlated effects (CCE) estimator. Once one corrects for the presence of the unobserved factor, the slope coefficient on the futures premium turns significantly positive across the maturity spectrum. These results support Fama's explanation of the FPP as an omitted-variable bias.

To explore the nature of the latent common factor, we extract a time-series approximation of the latent factor associated with each futures contract in our sample. We find that it is significantly correlated with measures and determinants of currency risk that have been used elsewhere in the literature. These include sovereign credit ratings, the general risk attitude of global investors, and some monetary variables, which is in line with recent results by Burnside, Eichenbaum, Kleshchelski and Rebelo (2006). Together, these results are consistent with the notion that at least part of the the latent factor is a risk premium.

The remainder of this paper proceeds as follows. Section 2 restates the FPP and describes the data. Section 3 presents the traditional tests of the EH. Sections 4 introduces the CCE estimator and gives the estimates of the premium coefficient in the presence of the common factor, and the nature of the unobserved factor is analyzed in section 5. The section 6 concludes.

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rates (which are driven by Central Bank rates), and hence may easily reflect motives for e.g. carry trade.

2 The Expectations Hypothesis and Futures Data

Let s_t denote the log of the spot exchange rate at time t and f_{t-m}^t the log of the futures exchange rate at time $t - m$ with delivery at time t and maturity m. Under the EH, f_{t-m}^t should be an efficient predictor of the spot exchange rate s_t. This could be tested empirically by regressing s_t on f_{t-m}^t. However, exchange rates are known to be nonstationary, and one cannot test the EH in levels. Instead, a lagged spot exchange rate is subtracted from both sides to obtain stationary data:

$$s_t - s_{t-m} = \alpha + \beta(f_{t-m}^t - s_{t-m}) + \varepsilon_t. \tag{1}$$

Under the EH, $\alpha = 0$ and $\beta = 1$.

Our empirical work uses daily closing spot exchange rates and daily closing prices for three-months futures contracts for US\$/DM, US\$/GBP, and US\$/Yen exchange rates. After the introduction of the Euro, we use US\$/EUR rates instead of US\$/DM rates. The delivery dates for the futures contracts are the third Wednesdays of March, June, September, and December. For all three currencies our data set contains 59 futures contracts with settlement days between 11 June 1993 and 26 June 2007. For each contract we consider the futures prices with maturities running from two days to six months or 126 business days.[4] Let $k = 1, ..., 59$ be the index for the delivery dates of the individual futures contracts and $m \in \{2, ..., 126\}$ the length of time to maturity. Each day in our sample is defined by a tuple (m, k) which denotes m days before the delivery date k. Define the log spot return $y_{m,k} = s_{0,k} - s_{m,k}$ and the futures premium $x_{m,k} = f_{m,k}^k - s_{m,k}$. With these definitions, we rewrite regression equation (1) as follows:

$$y_{m,k} = \alpha_m + \beta_m x_{m,k} + \varepsilon_{m,k}. \tag{2}$$

Under the EH, $\alpha_m = 0$ and $\beta_m = 1$ for all $m \in \{2, ..., 126\}$. We call equation (2) the *Fama Regression.*

One way to estimate equation (2) is to apply an OLS estimator separately for every $m \in \{2, \ldots, 126\}$. In this case we would end up with 125 individual regressions each based on 59 observations. However, since futures and spot prices are correlated across maturities, these regressions would not be independent of each other and the error vectors $\varepsilon_{m,k}$ would be correlated

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[4]Dickey-Fuller tests show that the futures premium and the differenced exchange rates are stationary.

across m. Taking this dependence into account leads to more accurate estimates of the standard errors. To do that, we treat equation (2) as a panel with maturity length m as the cross-sectional and the maturity dates of the individual contracts as the time dimension and estimate it applying the Beck and Katz (1995) OLS estimator with panel-corrected standard errors. The estimator corrects for heteroskedasticity, correlation across maturities and, if necessary, serial correlation. Figures 1 - 3 in the appendix plot the estimates for the slope coefficients together with their 95 percent confidence intervals against the maturity length, m.

For all three currencies we observe that the slope coefficients decrease with the length of the maturity m. For maturities shorter than one month (22 working days) the estimated slope coefficients are generally positive. Of these, four are significantly different from zero for US$/DM or US$/Yen futures contracts, respectively, and two for US$/Pound futures contracts. For maturities ranging between one and slightly under two months, the slope coefficients are close to zero and ambiguous in sign. However, for maturities exceeding 60 days, the slope coefficients for US$/DM and US$/Yen futures rates are with two exceptions all negative. The same result holds for US$/Pound futures rates for maturities exceeding 70 days. In 37 regressions based on US$/DM contracts, 25 regressions based on US$/Yen contracts and ten regressions based on US$/Pound contracts, the coefficients are statistically significantly negative at the 5% level.

Table 1: Relationship between 'Fama coefficients' and maturity length

β_m	US$/DM	US$/Yen	US$/Pound
intercept	1.359	1.494	1.065
	(0.00)	(0.00)	(0.00)
maturity	-0.062	-0.053	-0.052
	(0.00)	(0.00)	(0.00)
maturity^2	0.0003	0.0002	0.0003
	(0.00)	(0.03)	(0.00)
R^2	0.68	0.62	0.49
N	125	125	125
H_0: cons=1 [a]	0.05	0.11	0.76
Link test [a]	0.51	0.79	0.06

Notes: p-values in parentheses.
[a] Test results represented in p-values.

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To summarize the relationship between the estimated slope coefficients and the maturity length of the futures contracts, we regress the estimated β_m's on an intercept and the maturity length m in levels and squared values.[5] Table 1 has the results. The R^2 values suggest that maturity length explains roughly two thirds of the variation in the estimated slope coefficients. The estimated intercepts are positive and highly significant, and, as shown by a t-test, not significantly different from one. The linear maturity terms have negative and significant coefficients, while the squared maturity terms have significantly positive coefficients that are much smaller in absolute value. Thus, with increasing maturity, m, the slope coefficients of the 'Fama regression' first decline and become negative, but eventually increase and become positive again. For example, the results for the DM/Euro futures contracts indicate that the slope coefficient of the 'Fama regression' turns negative for maturity horizons longer than 26 days, reaches its minimum value of -2.00 at a maturity length of 116 days, and thereafter slowly increases again. Although this is not observed in our sample, the results suggest that the slope coefficient would eventually become positive again with maturities larger than 194 days.[6]

One might suggest that the result that the slope coefficients in the 'Fama regression' depend on the number of days left to maturity of a contract is an artefact due to a winding down of trading activity as the expiration day of a contract comes closer. To assure that this is not the case, we analyse the trading volume, the average daily return and the annualized volatility of the futures prices with respect to maturity. The results are shown in Figures 4, 5 and 6 in the appendix. We find that the liquidity of futures markets seems to be high also for futures contracts close to expiry. For very short maturities of less than five days, the trading volume is somewhat lower, but it increases steadily in maturity and reaches its maximum seven days prior

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[5]We test for the validity of this functional form by performing a link test. This test takes the fitted values of the residuals from the original regression and squares them, then reinserts these into the model as an additional regressor. The latter is never significant at the five percent level, indicating that the quadratic form is adequate.

[6]This is in line with the results of Alexius (2001), Chinn and Meredith (2004) and Chinn (2006), who focus on UIP at the multi-year level and find that the rejection of the UIP becomes less decisive if the maturity horizon increases. As an alternative to the regressions in Table 1, we also estimate a panel of equations covering each maturity $m = 2, 3...$ imposing the linear quadratic relationship as a cross-equation restriction on the coefficients β_m. The results are similar to those reported in Table 1, confirming the negative linear and positive quadratic term.

to expiry. For maturities up to 77 days the liquidity stays high. However, thereafter the trading volume decreases rapidly, which confirms the usual finding that investors tend to invest mostly in the nearest-maturity futures contract. However, the pattern of the trading volume seems to have no significant effect on the daily returns nor on the volatility of futures prices, see Figures 5 and 6.[7]

To summarize, there exists a significantly negative relationship between the slope coefficients and the maturity horizon of the futures contracts. For small maturity horizons, the rejection of the EH is less decisive. Thus, the FPP emerges only emerges gradually with increasing maturity.

3 A latent factor model of foreign exchange returns

One common interpretation of the FPP is that foreign exchange returns contain an unobserved variable such as a risk premium in addition to the rational expectation of the exchange rate and the associated expectation error. Thus, let $\Pi_{m,k}$ be an unobserved variable such that:[8]

$$y_{m,k} = \beta'_m x_{m,k} + \Pi_{m,k} + \varepsilon'_{m,k}. \tag{3}$$

The relationship between the foreign exchange return and the futures premium is now clouded by the unobservable variable. It is well known that the OLS estimate of the slope coefficient β'_m is downwards biased, if the unobservable variable $\Pi_{m,k}$ and the futures premium are negatively correlated. From this perspective, the observation that the slope coefficient is a decreasing function of the maturity length suggests that the negative correlation between the unobservable variable and the futures premium grows stronger as the time to maturity becomes longer.

This suggestion is supported by the results of Hodrick and Srivastava (1987), who show that the dynamic relationship between futures premiums with different maturities and the same delivery date can be approximated by the following autoregressive structure:

$$(f^t_{t-m+n} - s_{t-m}) = \tau_n(f^t_{t-m} - s_{t-m}) + \psi_{t-m+n}, \tag{4}$$

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[7]We have also repeated all our estimations dropping all observations for maturities longer than 77 days and all our results were unaffected.

[8]Note, that for simplicity we drop the intercept term from our notation. However, in our regressions we control for the possible presence of an intercept.

where $\tau_n \simeq (\tau_1)^n < 1$. Hodrick and Srivastava conclude that daily risk premiums must be highly positively autocorrelated. Expressing equation (4) in expectations based on the information available at time $t - m$, we obtain:

$$E_{t-m}(f_{t-m+n}^t - s_{t-m}) = \tau_n(f_{t-m}^t - s_{t-m}), \tag{5}$$

Setting $m = n$ and using the boundary condition $f_t^t - s_t = 0$, it follows:

$$E_{t-m}(s_t - s_{t-m}) = \tau_m(f_{t-m}^t - s_{t-m}). \tag{6}$$

Taking expectations of the latent factor model (3) and subtracting (6) gives the following expression for the expected unobserved factor $\Pi_{m,k}$:

$$E_{t-m}(\Pi_{m,k}) = -\left(\beta_m' - \tau_m\right) x_{mk}. \tag{7}$$

Thus, if $\beta_m' > \tau_m$, which would be true if $\beta_m' = 1$, the latent variable $\Pi_{m,k}$ associated with an m-period foreign exchange return is indeed negatively correlated with the m-period futures premium.

Furthermore, we observe that the futures premiums of a given contract with different maturities are highly correlated. On average we estimate a correlation coefficient of 0.90 between the futures premiums x_{mk} and $x_{m-1,k}$. Combining the correlation structure between futures premia of a given contract k with equation (7), we get:

$$E_{t-M+n}(\Pi_{M-n,k}) = -\rho^n \left(\beta_{M-n}' - \tau_{M-n}\right) x_{Mk}, \tag{8}$$

where M denotes the maturity of the first trading day of a futures contract k, and where ρ measures the correlation between two consecutive forward premia of a given contract. Thus, the unobserved variables associated with the same contract k and different maturities are all linked in expectation to the same unexpected variable on the first trading day of the contract.

This, together with our empirical results from section 2, suggests the following factor structure of the unobserved variable:

$$\Pi_{m,k} = \gamma_m g_k + \psi_{m,k}, \tag{9}$$

such that the first part of the first term, γ_m, depends only on maturity, m, while the second part, g_k, is specific to each futures contract but independent of maturity, and $\psi_{m,k}$ is the stochastic part of the latent variable $\Pi_{m,k}$. We can now rewrite equation (3) as:

$$y_{m,k} = \beta_m' x_{m,k} + \gamma_m g_k + \eta_{m,k}, \tag{10}$$

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where $\eta_{m,k} = \varepsilon_{m,k} + \psi_{m,k}$. In Appendix A we show how the factor structure described by equation (9) can be derived from expected utility maximization and a pricing kernel.

To estimate this model, we regard equation (10) again as a panel data model with maturity m as the cross-section and maturity date k as the time-series dimension. We apply Pesaran's (2006) common correlated effects (CCE) estimator, which allows the unobserved factor g_k to be correlated with the regressor $x_{m,k}$ and the random variable $\eta_{m,k}$ to be serially correlated and heteroscedastic. Furthermore, the CCE estimator does not require the maturity-specific regressors to be identically and/or independently distributed over the cross-section units, which is particularly relevant for the analysis of our data set, since the futures premium and foreign exchange returns for different maturities are not independent of each other.

Since, for a given contract, g_k is fixed, we can average foreign exchange returns across maturities:

$$\overline{y_k} = \overline{\beta'_m x_k} + \overline{\gamma} g_k + \overline{\eta_k}, \tag{11}$$

where the bars indicate averages. Assuming that the errors are i.i.d, the law of large numbers implies that $\overline{\eta_k} \simeq 0$ for sufficiently large M. This allows us to approximate the unobserved factor for each contract k by:[9]

$$g_k \simeq \frac{1}{\overline{\gamma}} \overline{y_k} - \frac{\overline{\beta'_m}}{\overline{\gamma}} \overline{x_k}. \tag{12}$$

Substituting g_k into the equation (12) gives:

$$y_{mk} = \beta'_m x_{mk} + a_m \overline{y_k} + d_m \overline{x_k} + \eta'_{mk} \tag{13}$$

with $a_m = \frac{\gamma_m}{\overline{\gamma}}$, $d_m = -\frac{\gamma_m \overline{\beta_m}}{\overline{\gamma}}$ and $\eta'_{mk} = \varepsilon_{mk} - a_m \overline{\eta_k}$. Pesaran (2006) shows that by running a multiple regression of y_{mk} on $(x_{mk}, \overline{y_k}, \overline{x_k})$ for a given m, one obtains a consistent and unbiased estimate of β'_m. A simplified proof is provided in Appendix B.[10]

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[9]The CCE estimator is based on the assumption that the slope coefficient β'_m follows a random coefficient model, thus $\beta'_m = \beta' + v_m$, where the random deviations v_m are distributed independently from the model coefficients and other error terms. Therefore, it holds that $\overline{\beta'_m x_k} = \overline{\beta'_m} \overline{x_k}$.

[10]In appendix C we show that the approximation of the unobserved factor by a linear combination of the cross-maturity averages of the futures premium and the forex returns is closely related to the model for the term premium proposed by cochrane and Piazzesi (2005).

Estimating equation (13) also yields consistent estimates of the coefficients a_m and d_m. Thus, given that $a_m = \gamma_m / \overline{\gamma}$, we are able to identify the individual factor loadings γ_m up to a scaling factor $\overline{\gamma}$. Further, using the consistent estimates of β'_m for $m = 2, \ldots, M$, we can recover $\overline{\gamma} g_k$ from equation (12) for every contract $k = 1, \ldots, T$. Therefore, we are also able to identify the unobserved factor g_k up to the scaling factor $\overline{\gamma}$.

We implement the CCE estimator in two ways. First, we allow the intercept and the coefficient β'_m to vary with maturities. The resulting estimates of β'_m are plotted in Figures 7 to 9 together with their 95% confidence interval. The figures show that the downward trend in the slope coefficient has has vanished for all three currencies. The slope coefficients are now generally positive and very often significantly larger than zero. Regressions of the slope coefficient on the maturity and the squared maturity of the same type as in table 1 confirm the visual impression, i.e., that neither regressor has a significant coefficient.

Table 2 reports the average intercept and β'_m obtained from these estimates together with the standard deviations of the mean. Pesaran (2006) calls this the *CCE mean group estimator*. For all three currencies, the average intercept is not significantly different from zero. The slope coefficients are all significantly positive, but smaller in magnitude than one. However, only for the US\$/Yen contracts, the Null Hypothesis that the mean group estimator equals one cannot be rejected at standard significance levels, although even here the p-value is 0.09.

Next, we estimate the *CCE pooled estimator*, which restricts the slope coefficients β'_m to be the same for all maturities m (see Pesaran (2006)). The lower panel of Table 2 reports these estimates. Again, we find that the slope coefficient is significantly positive, but it is also still significantly different from one. We investigate the robustness of this latter conclusion further at the end of this section.

The relevance of adding $(\overline{y_k}, \overline{x_k})$ to the standard 'Fama specification' (2) can be tested by means of the Likelihood ratio test. Table 2 reports the p-values of testing (13) against (2) on basis of the mean group estimator and the pooled estimator. It appears that in none of the cases one can omit the regressors $(\overline{y_k}, \overline{x_k})$ on basis of their statistical significance. Moreover, as already noted, adding these regressors to the estimation equation substantially increases the value of the futures premium coefficient, indicating that the coefficient estimates from the standard specification (2) are severely downward biased.

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Table 2: Slope Coefficients from CCE Estimator

	US$/DM	US$/Yen	US$/Pound
Mean Group Estimator			
average intercept	0.002	0.019	0.003
	(0.90)	(0.76)	(0.92)
average β'	0.688	0.749	0.822
	(0.00)	(0.00)	(0.00)
LR test$^{a)}$:	0.00	0.00	0.00
$H_0 : \beta' = 1^{a)}$	0.00	0.09	0.03
Pooled Estimator			
average intercept	0.00	0.00	0.00
	(0.99)	(1.00)	(1.00)
β'	0.664	0.656	0.780
	(0.00)	(0.00)	(0.00)
LR test$^{a)}$:	0.00	0.00	0.00
$H_0 : \beta' = 1^{a)}$	0.00	0.00	0.01

Notes: p-values shown in paranthesis.

$^{a)}$) Test results represented in p-values.

Figures 10 to 12 in the appendix plot the factor loadings γ_m against the maturity m resulting from the mean group CCE estimator.[11] The figures show that the factor loadings increase with increasing maturity. This confirms our suggestion from above: OLS estimates of the slope coefficients decline with increasing maturity, because the importance of the latent factor for the foreign exchange return increases. The figures also suggest that we can approximate the factor loadings by a linear-quadratic function of maturity,

$$\gamma_m = \delta_0 + \delta_1 m + \delta_2 m^2 + u_m, \qquad (14)$$

with $\delta_2 < 0 < \delta_1$.

Table 3 reports the results of OLS estimates of equation (14) for all three currencies. It shows that the linear-quadratic function indeed fits the factor loadings extremely well. The coefficients on the linear and the quadratic maturity are almost identical for all three currencies. Note that the linear quadratic functions imply that the factor loadings eventually decrease in

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[11]The factors and factor loadings estimated by the mean group CCE, pooled CCE and the error-in-variables CCE regressions look almost identical.

maturity. Based on our estimates, they would reach zero again after approximately 248 working days, or one year. Although this is not in our sample, it is consistent with the results from the previous section.

Table 3: Factor Loadings

	US\$/DM	US\$/Yen	US\$/Pound
δ_0	0.048	0.012	0.106
	(0.03)	(0.00)	(0.00)
δ_1	0.024	0.023	0.023
	(0.00)	(0.00)	(0.00)
δ_2	-0.0001	-0.0001	-0.0001
	(0.00)	(0.00)	(0.00)
R^2	0.98	0.99	0.92

Notes: p-values represented in paranthesis.

To summarize, the main results from this section are three-fold. First, we have shown that we can characterize the term structure of daily foreign exchange returns by the corresponding futures premium and a latent, futures-contract specific factor. Second, the factor loadings can be characterized by a simple, linear-quadratic function, a result which is reminiscent of Cochrane and Piazzesi's (2005) findings for the term structure of interest rates. Third, taking into account the latent factor and the bias it causes in OLS estimates, the slope coefficients on the futures premium are positive and close to but significantly below one.

4 Robustness

We investigate the robustness of our estimates against two possible problems. First, as noted above, the trading volume for very short and for longer maturities decreases, see Figure 4 in the Appendix. Although Figures 5 and 6 suggest that the trading volume has no significant impact on the daily returns nor on the volatility of futures prices, we re-estimate the CCE regressions by dropping all observations for maturities shorter than 15 and larger than 77 days. The estimates are reported in Table 5 in the appendix. In case of the US\$/DM futures contracts, the CEE mean group estimator and the CCE pooled estimator hardly change in comparison with the estimates

that are based on the complete maturity spectrum. For the US\$/Yen and the US\$/Pound futures contracts the slope coefficients are lower. However, for all three currencies, the slope coefficients remain significantly positive.

A second potential problem is the timing mismatch between when the futures exchange rate f_{t-m}^t and the spot exchange rate s_{t-m} are recorded.[12] If this mismatch has an impact at all, it has more impact at short maturities than at long maturities, since the variance of the futures premium σ_{x_m} increases with the maturity m.[13] Therefore, we re-estimate our estimations by deleting this time only the futures prices with maturities shorter than 15 days from the sample. The estimates are shown in Table 6 in the appendix. For both, the CCE mean group estimator and the CCE pooled estimator, dropping the 15 maturities at the short end hardly changes the estimates in comparison with the results that are based on the complete maturity spectrum.

5   The nature of the unobserved factor

Figures 13 to 15 show the latent factors $\hat{g}_k$ estimated for the three currencies using the CCE estimator. Remember that these latent factors are identified up to a scaling factor. However, since we are interested in analysing the driving forces of the unobserved factors and less so in their levels, this identification problem can be ignored. There are some large movements in the factors in the mid-1990s, around the turn of the millenium and between the third quarter of 2001 and the second quarter of 2002.

The correlation between the US\$/Yen factor and the US\$/Pound factor is 0.34, the correlation between the US\$/Yen factor and the US\$/DM factor is 0.44, while the correlation between the US\$/Pound and the US\$/DM factors is 0.73. Market fluctuations seem to be more interdependent in European markets than between European and Asian markets.

To explore the nature of these factors further, we analyze the correlation between the contract-specific factors and several conventional measures of

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[12]The spot exchange rates are recorded by JP Morgan at 4 p.m. London time, while the closing prices of the futures exchange rates are recorded at 2 p.m. at the Chicago Mercantile Exchange. Thus, there is a timing mismatch of four hours between the futures and the spot prices.

[13]The increase in σ_{x_m} reduces the attenuation bias due to the measurement error as the timing mismatch does not increase with the maturity m. Estimates of σ_{x_m} are available from the authors on request.

financial market risk. The first is the spread between BBB US corporate bonds and US Treasury bonds, a commonly used indicator of risk aversion in international financial markets. In addition, we use historical (20 days rolling window) volatilities of stock indices, 10-year government bonds, and currency indices in the US, Germany, Japan, and the UK. This is motivated by dynamic international asset pricing model that suggests that the risk premium in forward prices is closely related to the realized volatility of exchange rates. Further, we add some macro economic variables that are commonly regarded as determinants of currency risk and country risk; see e.g. Burnside et al. (2006). Specifically, we use the three countries' debt service to export ratios, their current account balances relative to GDP, and the growth rates of broad money (M2).[14] Burnside et al (2006) show that primarily monetary variables have a significant impact on risk factors in currency markets. Additionally, we use the growth rates of real GDP to capture business cycle effects and economic dynamics and a variable measuring the size of central bank foreign exchange interventions. Finally, we include a measure for the recession probability in the USA [15] to measure external shocks and changes in investors' risk attitudes.

To condense the information contained in these time series, we use factor analysis. While the scree plot suggest that the optimal number of factors to describe our set of variables is four, the Kaiser-Guttman rule[16] suggest that the optimal number is three. We decided to retain three factors, since they explain around 78% of the variation of our explanatory variables, and the fourth factors does not add a lot. We extract the factors loadings by applying the principle factor method and redefine these further by oblique rotation. The estimates for the factor loadings are listed in the lower panel of Table 4.

The first and third factor focus entirely on US variables. The first factor captures to a large extent information on US M2 growth, the US debt service ratio and the historical volatility of US stock indeces. The third factor is related to the variables capturing the real economic position of the USA, i.e. the US recession probability measure and US GDP growth and also two

---

[14]We also added the inflation rate, the debt to GDP ratio and the reserves to import ratio, but these variables had not much additional explanatory power so that we dropped these from the model.

[15]Source: `http://www.uoregon.edu/~jpiger/us_recession_probs.htm`

[16]The Kaiser-Guttman rule says that the optimal choice of factors is determined by the number of factors with eigenvalue greater than 1.0.

Table 4: Determinants of Unobserved Factor

	US$/DM	US$/Yen	US$/Pound
Factor 1	-1.46	-1.39	-1.07
	(0.00)	(0.08)	(0.00)
Factor 2	1.65	3.06	0.87
	(0.10)	(0.00)	(0.33)
Factor 3	1.08	-0.35	0.57
	(0.03)	(0.55)	(0.09)
Constant	0.29	-3.20	0.83
	(0.58)	(0.00)	(0.25)
DW	1.95	1.72	1.81
R2	0.29	0.16	0.19
N	59	59	59

Factor loadings

Variable	Factor1	Factor2	Factor3
Prob.of US Recession	0.241	-0.091	0.631
M2 Growth	0.025	-0.562	-0.009
M2 Growth US	0.636	0.007	0.577
CB intervention	-0.106	0.366	0.091
Fed intervention	-0.355	0.133	-0.166
Debt service	-0.333	0.717	-0.080
Debt service US	-0.904	0.042	0.043
CA Balance	-0.121	0.519	0.074
CA Balance US	-0.108	0.119	-0.384
GDP growth	0.113	-0.450	-0.286
GDP growth US	0.264	0.161	-0.711
Spread	0.212	0.089	0.882
HV stock index	0.373	0.377	0.500
HV stock index US	0.663	0.123	0.512
HV interest	0.165	0.715	0.169
HV interest US	-0.021	0.186	0.702
HV forex	0.287	0.687	-0.018
HV forex US	-0.025	0.221	0.216

DW= Durbin-Watson statistic, Prob. of US recession=Recession probability in the USA; see `http://www.uoregon.edu/~jpiger/us_recession_probs.htm`, CB and Fed interventions=purchases of Million USD, CA=current account; Spread=interest rate spread between low graded US corp.bonds and govern.bonds; HV=historical volatility (20 day window)

variables measuring financial market risk, i.e. the interest spread between low graded corporate bonds and US government bonds and the historical volatility of US interest rate index. The second factor is closely related to domestic determinants, namely the domestic debt service ratio, domestic monetary variables, i.e. M2 growth and current account balance, and the historical volatilities of the 10-government bond and forex index.

We then regress the latent variables from our CCE model on these three factors and a constant. The Durbin Watson statistic gives no indication for autocorrelation in the error terms.[17] The results are shown in the upper panel of Table 4. In the case of US\$/DM futures, the fit of the regression is quite good, the R^2 being 0.29. For US\$/Pound and US\$/Yen futures, the explanatory power is somewhat less.

For all three currencies, the coefficient on the first factor, which reflects to a large extent US monetary variables and the US debt service ratio, turns out to be significant. The second factor, which is closely related to domestic monetary variables and domestic financial risk indicators, has significant coefficients in the regressions for US\$/DM and US\$/Yen futures. Finally, the third factor, which reflects the condition of the US economy, has significant coefficients in the US\$/DM and the US\$/Pound regressions.

These results are consistent with the notion that the latent variables extracted from the 'Fama regressions' contain a risk premium as suggested by Fama (1984). They are also in line with the results of Burnside et al. (2008).

6   Conclusion

The forward premium puzzle is the empirical observation that foreign exchange forward premiums and the realized foreign exchange returns tend to be negatively correlated. This is in stark contrast to the Expectations Hypothesis, which holds that forward rates should be efficient predictors of future spot rates and which is a cornerstone of conventional models of international macro economics and finance.

In this paper we have reconsidered the puzzle using futures instead of forward rates for the US dollar exchange rates against the Yen, the British

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[17]As a robustness check we have repeated the regression with an additional lagged dependent variable to control for possible autocorrelation, but this variable turned out to be highly insignificant and the regression results did not differ much.

Pound, and the German DM (the Euro after 1999). Futures rates allow us to analyze the relationship between realized returns and futures premiums at different maturity horizons, ranging from the first until the last trading day of a futures contract. This creates a panel structure of our data with time to maturity as the cross sectional dimension and the maturity dates of individual contracts as the time series dimension.

Our first result is that the slope coefficient of the conventional 'Fama regression' (1984) is a linear-quadratic function of the time to maturity. It is positive for short maturities, declines to negative values for maturities longer than 60 days, and eventually increases again for maturities exceeding 116 days.

Our second result is that the residual of the conventional 'Fama regression' contains a latent factor. Thanks to the panel structure of our data, we can extract this latentn factor by means of Pesaran's (2006) common correlated effects estimator. The latent factor is contract specific with factor loadings that have a linear-quadratic structure and which increase with declining increments in the time to maturity. Once we account for this factor, the slope coefficients between the realized foreign exchange returns and the futures premiums are consistently positive and no longer depend on the time to maturity. This result is reminiscent of Cochrane's and Piazzesi's (2005) recent findings concerning the Expectations Hypothesis for the term structure of interest rates.

Our third result is that the latent variable extracted from the data is correlated with a number of observables that are conventionally regarded as variables which are linked to currency risk. This suggests that the latent factor at least partially has the nature of a risk premium. Exploring its nature in more detail and modeling it more appropriately remains a challenge for future research.

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A   Risk premium structure - A Pricing Kernel Approach

In this section we show that formally obtain the somewhat intuitive derivation of the factor specification described in equation (9) from a pricing kernel cum utility analysis. Consider the following consumer problem at time $t - m + n$ for consumption at $t - m + n$, $n \leq m$, and future consumption at time t with discount factor δ:[18]

$$\max_{C_{t-m+n}, C_t} U(C_{t-m+n}) + \delta^{m-n} E_{t-m+n}[U(C_t)].$$

The following two budget constraints apply

$$W_{t-m+n} = C_{t-m+n} + D$$

and

$$0 = C_t - RD - F^t_{t-m+n} A + S_t A.$$

Here W_{t-m+n} is wealth at time $t - m + n$ and D is the amount invested in a riskless domestic bond that pays gross interest R at time t. At time t the consumer can also buy currency futures contracts A that mature at time t and promise delivery of foreign currency at price F^t_{t-m+n}. The spot price of foreign currency at time t is denoted by S_t. For simplicity, we assume that there are no margin payments to be made up front. All costs for the futures contract are in the agreed contract price. This explains why the currency transactions only appear in the second budget constraint. Per contrast, in case of spot speculation one buys foreign currency (bonds) in the first period and sells the proceeds the second period.

Substituting the budget constraints into the utility function, we get:

$$\max_{D,A} \delta^{m-n} E_{t-m+n}[U(RD + F^t_{t-m+n} A - S_t A)] + [U(W_{t-m+n} - D)].$$

From the first order conditions we have a.o.

$$\delta^{m-n} E_{t-m+n}\left[\frac{\partial U(C_t)}{\partial C_t}(F^t_{t-m+n} - S_t)\right] = 0.$$

---

[18]Note that the additive expected utility structure easily permits an extension to incorporation of a multi-period decision problem.

Since the forward rate F^t_{t-m+n} is known at time t, we can write:

$$1 = \frac{E_{t-m+n}\left[\frac{\partial U(C_t)}{\partial C_t}\frac{S_t}{F^t_{t-m+n}}\right]}{E_{t-m+n}\left[\frac{\partial U(C_t)}{\partial C_t}\right]}.$$

Furthermore we can re-express this as

$$
\begin{aligned}
1 &= E_{t-m+n}\left[\frac{\partial U(C_t)}{\partial C_t}\frac{S_t}{S_{t-m+n}}\frac{S_{t-m+n}}{F^t_{t-m+n}}\right]/E_{t-m+n}\left[\frac{\partial U(C_t)}{\partial C_t}\right] \\
&= E_{t-m+n}\left[\frac{\partial U(C_t)}{\partial C_t}e^{y_{m-n,k}-x_{m-n,k}}\right]/E_{t-m+n}\left[\frac{\partial U(C_t)}{\partial C_t}\right]. \quad (15)
\end{aligned}
$$

Suppose that as a simplification of (3) and (10), omitting superfluous indices and assuming $\beta'_m = 1$,

$$y_{m-n} = x_{m-n} + \gamma g + \varepsilon.$$

Hence, (15) can be written as

$$1 = E_{t-m+n}\left[\frac{\partial U(C_t)}{\partial C_t}e^{\gamma g+\varepsilon}\right]/E_{t-m+n}\left[\frac{\partial U(C_t)}{\partial C_t}\right]. \quad (16)$$

Postulate the following specification for the marginal utility

$$\frac{\partial U(C_t)}{\partial C_t} = \exp\left(-\frac{1}{2}\lambda^2\sigma^2 - \lambda\varepsilon\right)$$

and where ε follows a mean zero normal distribution with variance σ^2. Thus $\partial U(C_t)/\partial C_t$ follows a lognormal distribution and has expected value

$$E_{t-m}\left[\frac{\partial U(C_t)}{\partial C_t}\right] = \exp\left(-\frac{1}{2}\lambda^2\sigma^2 + \frac{1}{2}\lambda^2\sigma^2\right) = 1.$$

Combining expressions and using (16) gives

$$
\begin{aligned}
\frac{\partial U(C_t)}{\partial C_t}e^{y_{m-n}-x_{m-n}} &= \exp\left(-\frac{1}{2}\lambda^2\sigma^2 - \lambda\varepsilon + \gamma g + \varepsilon\right) \\
&= \exp\left(-\frac{1}{2}\lambda^2\sigma^2 + (1-\lambda)\varepsilon + \gamma g\right).
\end{aligned}
$$

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Hence,

$$E_{t-m+n}[\frac{\partial U(C_t)}{\partial C_t}e^{y_{m-n}-x_{m-n}}] = E_{t-m}[\exp\left(-\frac{1}{2}\lambda^2\sigma^2 + (1-\lambda)\varepsilon + \gamma g\right)]$$

$$= \exp\left(-\frac{1}{2}\lambda^2\sigma^2 + \gamma g + \frac{1}{2}(1-\lambda)^2\sigma^2\right)$$

$$= \exp\left(\gamma g - \lambda\sigma^2 + \frac{1}{2}\sigma^2\right).$$

Substitute all this into the first order condition (16) to get

$$1 = e^{\gamma g - \lambda\sigma^2 + \frac{1}{2}\sigma^2}.$$

Take logarithms on both sides to solve for λ

$$\lambda = \gamma g/\sigma^2 + \frac{1}{2}.$$

To conclude, we have given a form of the utility function and pricing kernel consistent with the linear specification (10) that is adopted for the regression analysis.

B   Consistency and unbiasedness of the common correlated effects estimator

Consider (10) from the main text

$$y_{m,k} = \beta'_m x_{m,k} + \gamma_m g_k + \eta_{m,k}, \tag{17}$$

Take averages across maturities m to get the equivalent of (11)

$$\overline{y_k} = \overline{\beta'_m x_k} + \overline{\gamma} g_k + \overline{\eta_k}.$$

Hence, one can express the unobserved factor as

$$g_k = \frac{1}{\overline{\gamma}}\overline{y_k} - \frac{1}{\overline{\gamma}}\overline{\beta'_m x_k} - \frac{1}{\overline{\gamma}}\overline{\eta_k}. \tag{18}$$

Substitute this expression for g_k into (17), to get

$$y_{m,k} = \beta'_m x_{m,k} + \frac{\gamma_m}{\overline{\gamma}}\overline{y_k} - \frac{\gamma_m}{\overline{\gamma}}\overline{\beta'_m x_k} - \frac{\gamma_m}{\overline{\gamma}}\overline{\eta_k} + \eta_{m,k}. \tag{19}$$

Under the assumption that β'_m follows a random coefficient model, i.e. $\beta'_m = \beta' + v_m$, we can write

$$\overline{\beta'_m x_k} = \beta' \overline{x_k} + \overline{v_m x_k},$$

where v_m is an IID error term. Subsequently, we can restate (19) as follows

$$y_{m,k} = \beta'_m x_{m,k} + \frac{\gamma_m}{\overline{\gamma}} \overline{y_k} - \beta' \frac{\gamma_m}{\overline{\gamma}} \overline{x_k} - \frac{\gamma_m}{\overline{\gamma}} \overline{v_m x_k} - \frac{\gamma_m}{\overline{\gamma}} \overline{\eta_k} + \eta_{m,k}.$$

Using matrix notation, this can further be written as

$$y_m = X_1 \beta'_m + X_2 c + \Upsilon_m \qquad (20)$$

where $c^T = (\gamma_m/\overline{\gamma}, -\beta'\gamma_m/\overline{\gamma})$, $X_1 = x_m$ and $X_2 = (\overline{y_k}, \overline{x_k})$ and

$$\Upsilon_m = -\frac{\gamma_m}{\overline{\gamma}} \overline{v_m x_k} - \frac{\gamma_m}{\overline{\gamma}} \overline{\eta_k} + \eta_{m,k}.$$

Note that X_2 is obseravble, i.e. can be contsructed from the data.

An OLS regression across k contracts and maturities m using only X_1 as regressor, as is the standard procedure in the premium regressions, gives

$$
\begin{aligned}
\widehat{\beta'_m} &= \left(X_1^T X_1\right)^{-1} X_1^T y_m \\
&= \left(X_1^T X_1\right)^{-1} X_1^T [X_1 \beta'_m + X_2 c + \Upsilon_m] \\
&= \beta'_m + \left(X_1^T X_1\right)^{-1} X_1^T X_2 c + \left(X_1^T X_1\right)^{-1} X_1^T \Upsilon_m.
\end{aligned}
$$

and where

$$\left(X_1^T X_1\right)^{-1} X_1^T \Upsilon_m$$

is assumed to be small (see below). We see that if X_1 and X_2 are correlated, then the second term $\left(X_1^T X_1\right)^{-1} X_1^T X_2 c$ will be non-zero. In this case, ignoring the presence of the unobserved factor in equation (17), which corresponds to the 'Fama regression', gives a biased estimate of $\widehat{\beta'_m}$.

With prior adjustment, see Maddala (1977, p. 462), one uses the idempotent matrix

$$Q = I - X_2 \left(X_2^T X_2\right)^{-1} X_2^T$$

to transform the dependent variable and regressor by respectively $Q y_m$ and $Q X_1$ prior to running the regression. With prior adjustment the OLS esti-

mator becomes

$$
\begin{aligned}
\widehat{\widehat{\beta'_m}} &= \left(X_1^T Q^2 X_1\right)^{-1} X_1^T Q^2 y_m \\
&= \left(X_1^T Q X_1\right)^{-1} X_1^T Q [X_1 \beta'_m + X_2 c + \Upsilon_m] \\
&= \beta'_m + \left(X_1^T Q X_1\right)^{-1} X_1^T Q X_2 c + \left(X_1^T Q X_1\right)^{-1} X_1^T Q \Upsilon_m \\
&= \beta'_m + \left(X_1^T Q X_1\right)^{-1} X_1^T Q \Upsilon_m,
\end{aligned}
$$

where we used the two features of an idempotent matrix that $Q^2 = Q$ and

$$
\begin{aligned}
QX_2 &= X_2 - X_2 \left(X_2^T X_2\right)^{-1} X_2^T X_2 \\
&= X_2 - X_2 = 0.
\end{aligned}
$$

Thus beta is now consistently estimated by $\widehat{\widehat{\beta'_m}}$ if $\left(X_1^T Q X_1\right)^{-1} X_1^T Q \Upsilon_m$ is small. Pesaran (2006) gives sufficient conditions for this latter requirement to hold.[19]

C Combining Pesaran's CCE estimator (2006) with Cochrane and Piazzesi (2005)

Cochrane and Piazzesi (2005) show in their seminal paper that a single factor, described by a linear combination of forward rates, has a high predictive power for the excess returns on one- to five-year maturity bonds. In this section we transfer this idea to our set-up and show that Cochrane and Piazzesi's finding is closely related to Pesaran's CCE estimator applied in our empirical section.

In the related literature on the forward premium puzzle, the difference between the realized spot exchange rate innovation, $y_{m,k}$, and the forward premium, $x_{m,k}$, is very often denoted as the 'excess return'. Similar to Cochrane and Piazzesi (2005) we relate this 'excess return', which is expressed in our model by the unoberved factor $\gamma_m g_k$, to a set of the forward premia. Suppose that g_k is linear in all the forward premia concerning contract k.

$$
g_k = \sum_{i=1}^{M} \lambda_i x_{i,k}. \tag{21}
$$

---

[19]E.g. either by assumptions on the Euclidean norm, if beta is non-stochastic, or if $\beta'_m - \beta = v_m$ are mean zero iid across maturities.

Thus, conditional on the information available at maturity $t - m$, the excess return contained in futures contracts at time of contract k is described by:

$$
\begin{aligned}
y_{m,k} - x_{m,k} &= \gamma_m \{ \lambda_M x_{M,k} + ... + \lambda_m x_{m,k} + \\
&\quad E_m \left[\lambda_{m-1} x_{m-1,k} + ... + \lambda_1 x_{1k} \right] \} + \eta_{m,k}.
\end{aligned}
$$

Suppose that
$$
E_m \left[\lambda_{m-1} x_{m-1,k} + ... + \lambda_1 x_{1k} \right]
$$
can be estimated by linear projection on the realized ex-post premia
$$
\lambda_{m-1} x_{m-1,k} + ... + \lambda_1 x_{1k}.
$$

Then the regression model becomes

$$
y_{m,k} = x_{m,k} + \gamma_m \sum_{i=1}^{M} \lambda_i x_{i,k} + \eta_{m,k},
$$

which is very much related to Piazzesi and Cochrane (2005).

Averaging across maturities gives

$$
\frac{1}{M} \sum_{m=1}^{M} (y_{m,k} - x_{m,k}) = \left(\sum_{i=1}^{M} \lambda_i x_{i,k} \right) \left(\frac{1}{M} \sum_{m=1}^{M} \gamma_m \right)
$$

or in shorthand notation

$$
\frac{1}{\overline{\gamma}} \left(\overline{y_k} - \overline{x_k} \right) = \sum_{i=1}^{m} \lambda_i x_{i,k} = g_k.
$$

The left hand side of the latter expression conforms to the way in which Pesaran's estimator constructs the factor to correct for the omitted bias. The expression in the middle for g_k corresponds to the linear factor structure suggested by the Cochrane and Piazzesi paper. This offers the link between Pesaran's CCE estimator and Cochrane and Piazzesi's result that the excess return of a financial asset (in there case a bond) can be explained by a linear combination of forward premia.

D Figures
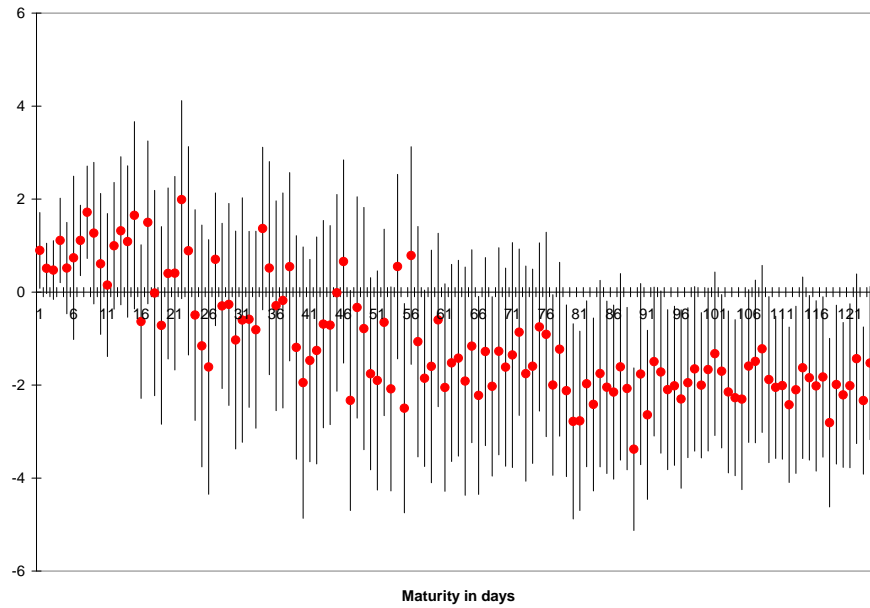
Figure 1: β_m, DM(Euro)/US\$ futures contracts - OLS
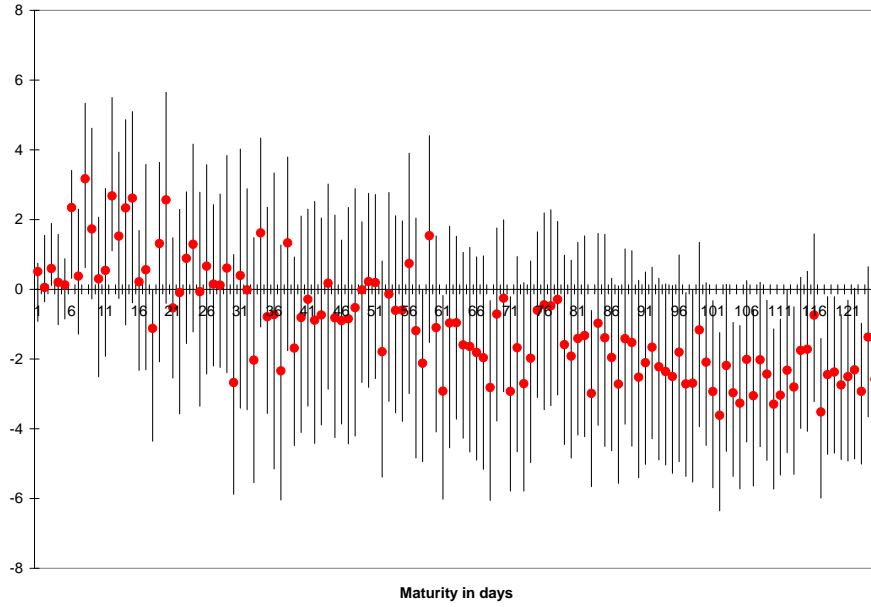
Figure 2: β_m, US\$/Yen futures contracts - OLS

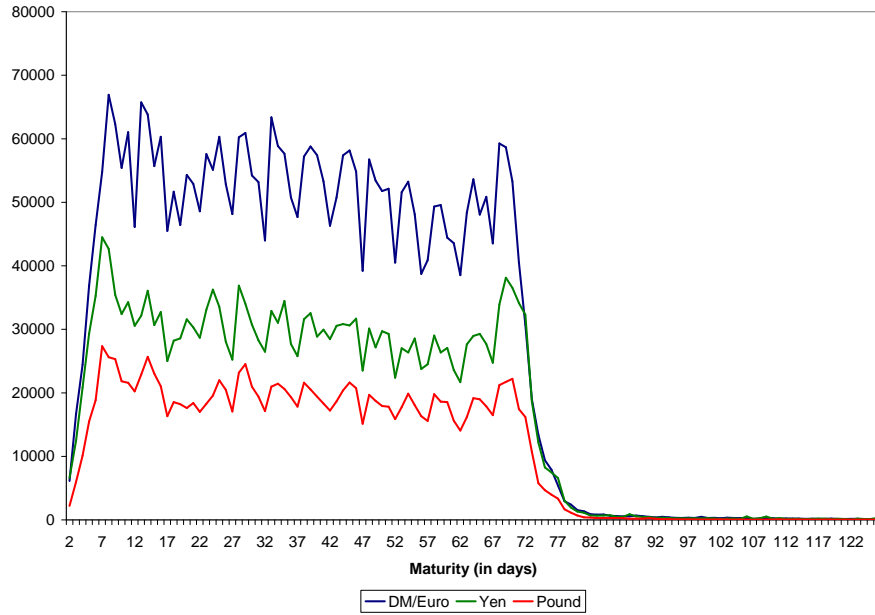Figure 3: β_m, US\$/Pound futures contracts - OLS

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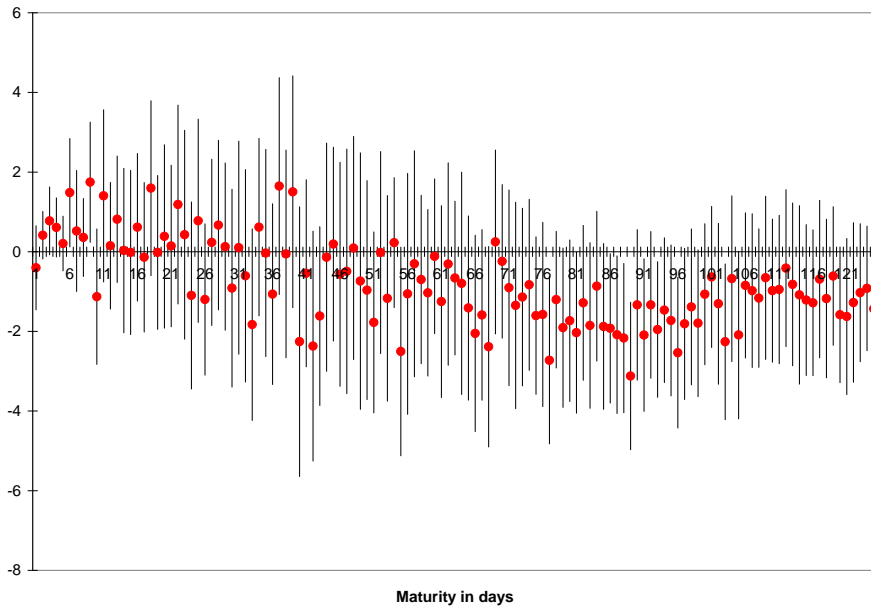
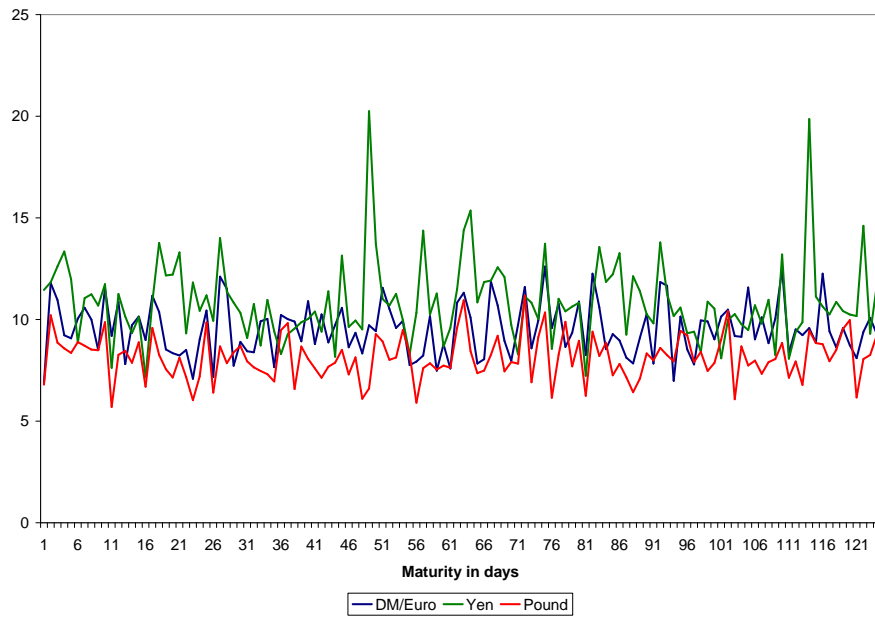Figure 4: Trading volume of futures contracts
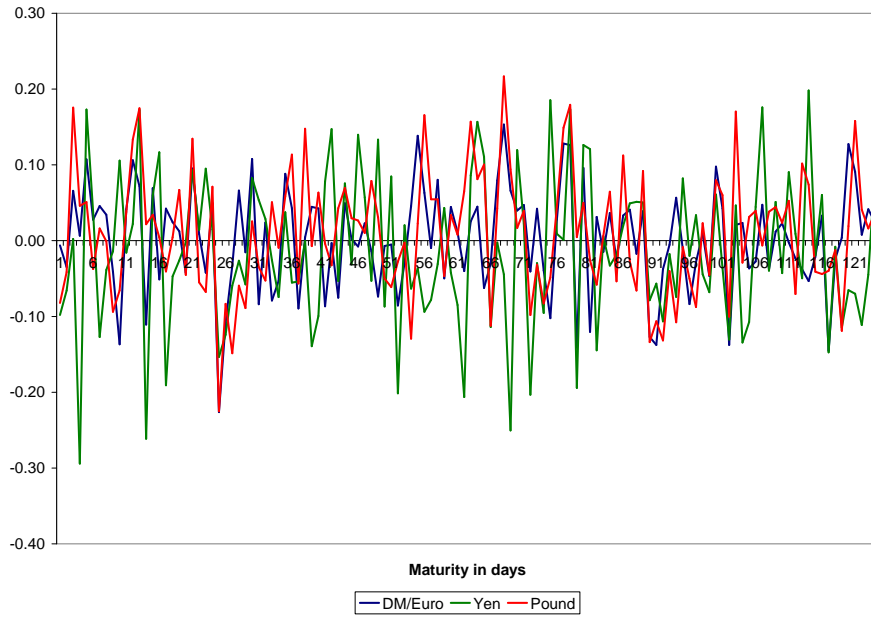

Figure 5: Annulized volatility of daily futures returns
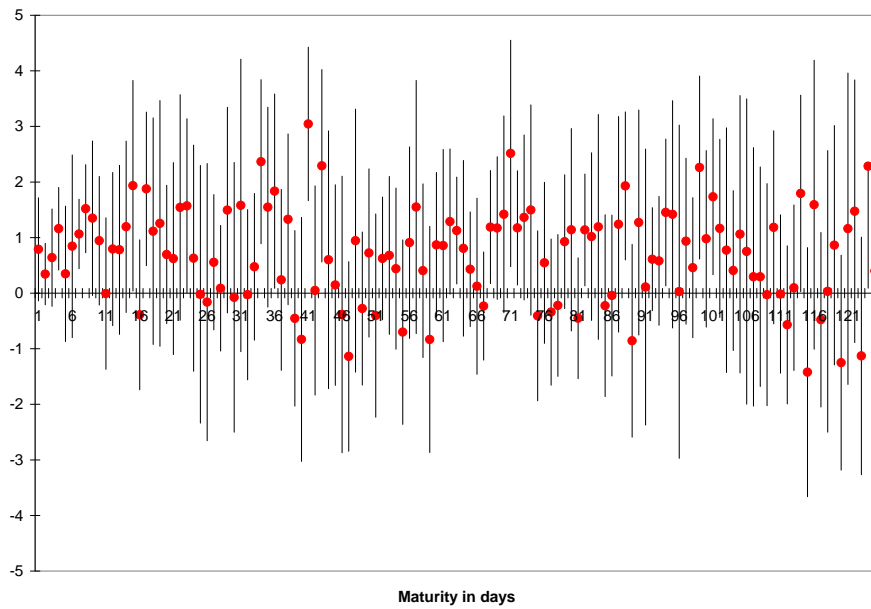
Figure 6: Average daily return of futures prices

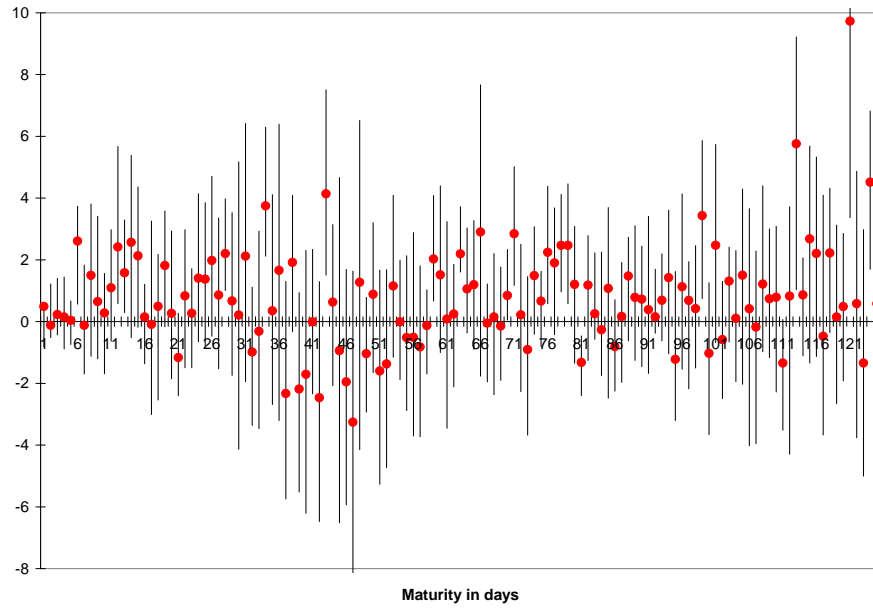Figure 7: β_m, DM(Euro)/US\$ futures contracts - CCE estimations

Figure 8: β_m, US\$/Yen futures contracts - CCE estimations

Figure 9: β_m, US\$/Pound futures contracts - CCE estimations

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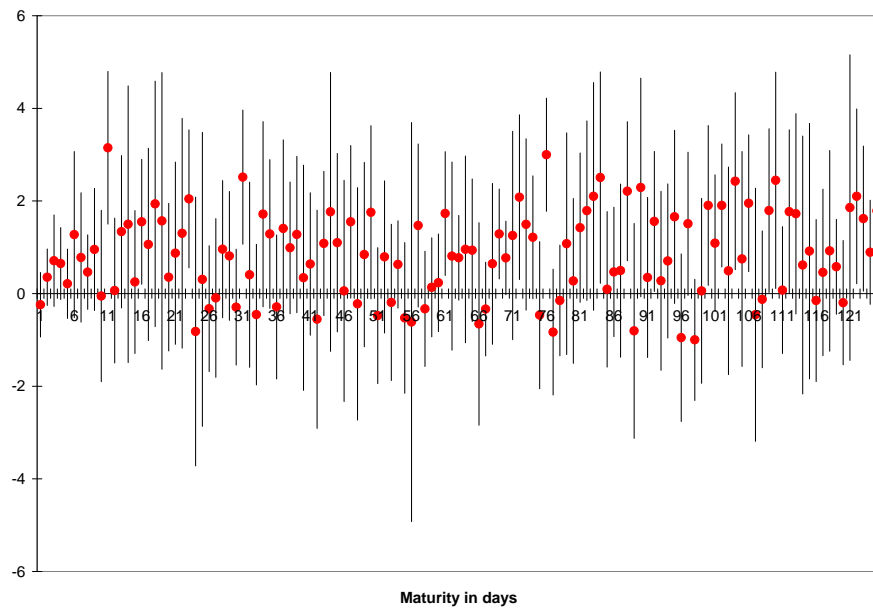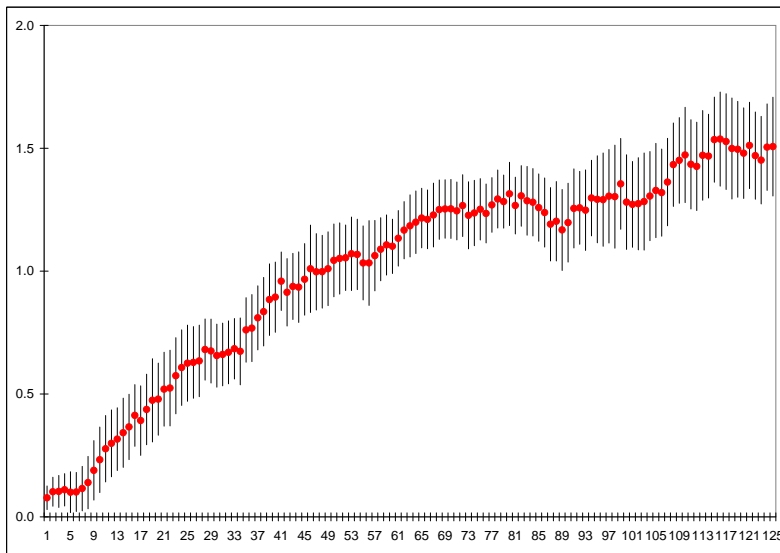
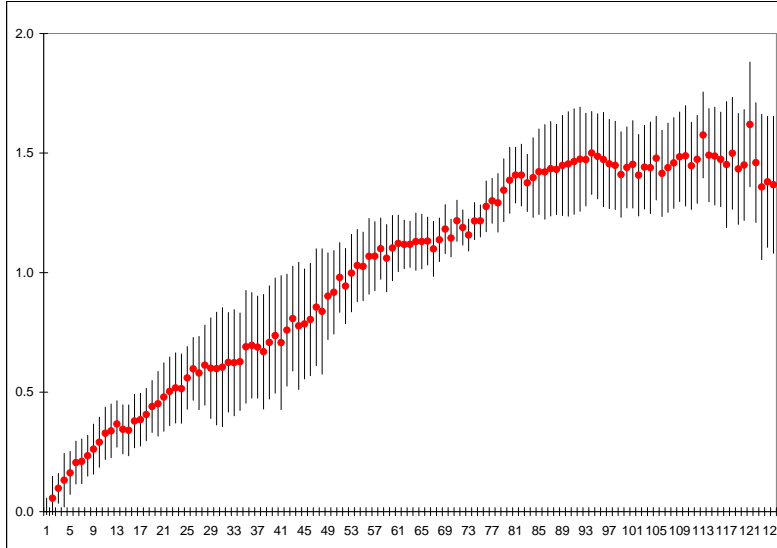Figure 10: Scaled factor loadings across maturity - DM futures contracts

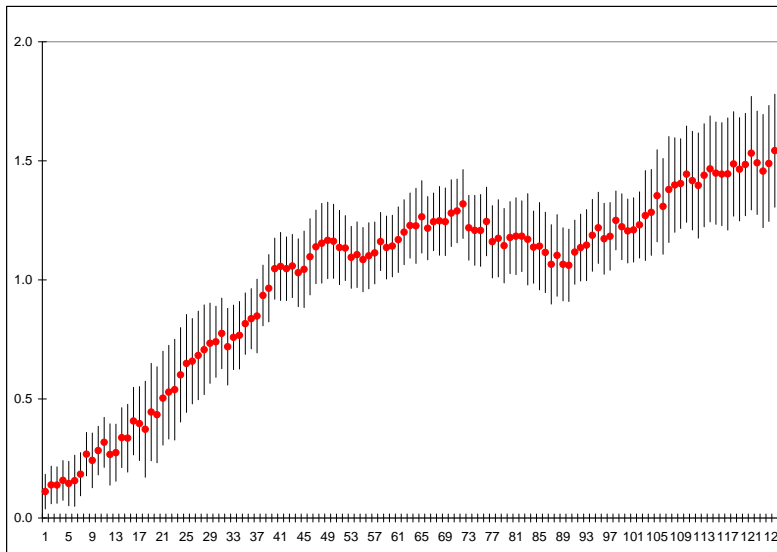Figure 11: Scaled factor loadings across maturity - Yen futures contracts

Figure 12: Scaled factor loadings across maturity - Pound futures contracts
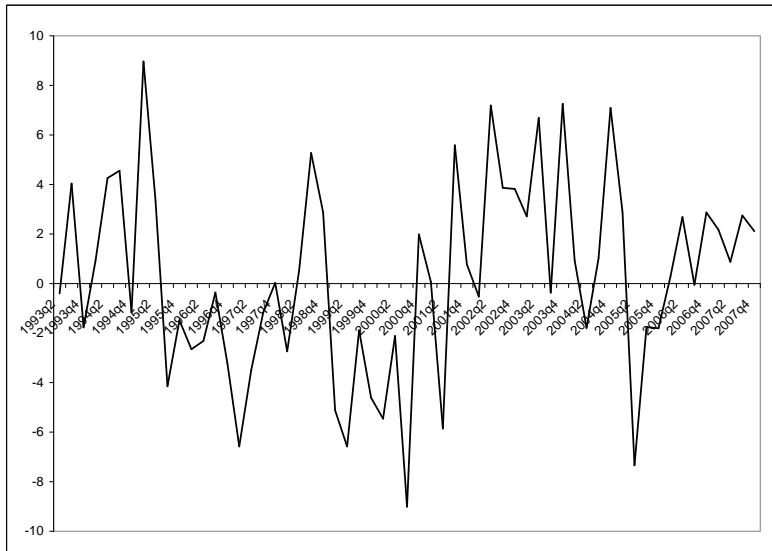
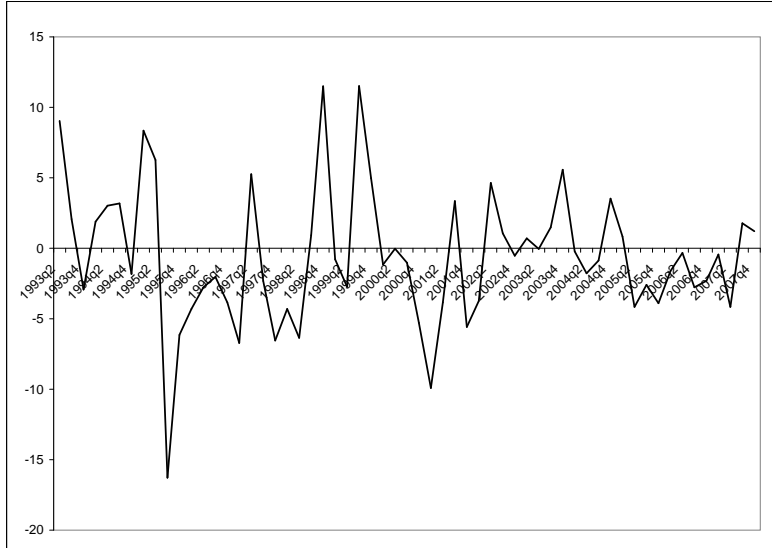Figure 13: Scaled factor across time - DM futures contracts

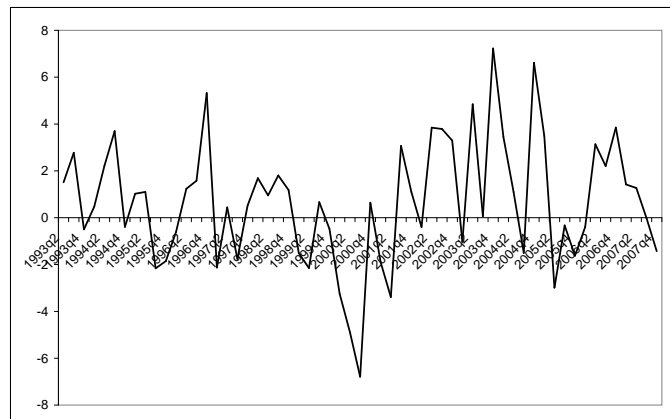Figure 14: Scaled factor across time - Yen futures contracts

Figure 15: Scaled factor across time - Pound futures contracts

E Tables

Table 5: Slope Coefficients from CCE Estimator dropping maturities $m < 15$ & $m > 77$

	US\$/DM	US\$/Yen	US\$/Pound
Mean Group Estimator			
average intercept	0.004	0.015	0.000
	(0.84)	(0.66)	(0.99)
average β'	0.616	0.328	0.540
	(0.00)	(0.00)	(0.00)
$H_0 : \beta' = 1^a)$	0.00	0.00	0.00
Pooled Estimator			
average intercept	0.00	0.00	0.00
	(1.00)	(1.00)	(1.00)
β'	0.608	0.309	0.540
	(0.00)	(0.04)	(0.00)
$H_0 : \beta' = 1^a)$	0.00	0.00	0.00

Notes: p-values shown in paranthesis.
[a]) Test results represented in p-values.

Table 6: Slope Coefficients from CCE Estimator dropping maturities $m < 15$

	US$/DM	US$/Yen	US$/Pound
Mean Group Estimator			
average intercept	0.00	0.02	0.00
	(0.94)	(0.74)	(0.92)
average β'	0.669	0.701	0.803
	(0.00)	(0.00)	(0.00)
$H_0 : \beta' = 1^a)$	0.00	0.05	0.02
Pooled Estimator			
average intercept	0.00	0.00	0.00
	(1.00)	(1.00)	(1.00)
β'	0.641	0.620	0.777
	(0.00)	(0.01)	(0.01)
$H_0 : \beta' = 1^a)$	0.00	0.01	0.01

Notes: p-values shown in paranthesis.

^{a}) Test results represented in p-values.