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Social Consistency and Individual Rationality

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Abstract

This paper aims at proving that social interactions can easily be rationalized by individual preferences as defined in standard microeconomic theory. For that purpose, we show *individual choice rationality* to be logically equivalent to *social consistency*, when individual rationality means that individual preferences are completely ordered and social consistency that there is a one-to-one mapping between a given family of social communities and the existence of a particular (unique, reflexive and symmetric) interaction relation between individuals. Moreover, continuity and monotonicity of individual preferences are shown to fit the modeling of *group loyalty* when group loyalty is defined as the ability to freely accept a personal loss for the global gain of a particular population.

Cet article vise à montrer que les interactions sociales peuvent être déduites des relations individuelles de préférence (et vice versa). Nous montrons ainsi que le postulat de *rationalité individuelle* est logiquement équivalent à celui de *cohérence sociale* dès lors que le postulat de rationalité individuelle signifie que les préférences des agents sont des préordres complets et celui de cohérence sociale, qu'il existe une bijection entre chaque système de coalitions et la relation d'interaction (unique, réflexive et symétrique) qui le soutend. De plus, on montre que la continuité et la monotonie des préférences individuelles permettent de modéliser la notion de *loyauté communautaire* dès lors que la loyauté communautaire est définie en tant que la capacité des agents à accepter une perte personnelle d'utilité au profit d'un gain collectif pour une coalition particulière.

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1 Introduction

1.1. Social Interaction within Economic Theory. Standard economic theory is generally considered unable to incorporate social phenomena because

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it rests on the methodological assumption of individualism. Hence, to build on maximizing behavior, most economic models require that individuals' utility must depend on a quite limited set of arguments: while workers are supposed to be concerned with wages, relationships with colleagues or supervisors are assumed to be indifferent; while consumers are supposed to be concerned with prices, neighbors' consumptions are assumed to be irrelevant; while voters are supposed to be concerned with candidates, friends' political preferences are assumed to be without influence. In other words, people are supposed to choose jobs, goods or representatives independently of their social environment. Conformism or mimetism, fashion or tradition are then hardly seen as economic phenomena. As noticed by Postlewaite (1998): '*At the same time that single-minded materialistic focus has made economics successful as a discipline, it has led to a belief that economics methodology is inadequate to understand important aspects of human behavior, particularly in settings in which individuals are concerned about the opinion of others.*' Even though most economists are sympathetic to the idea that social relations are important, they are generally reluctant to explicitly incorporate them within economic theory. Two reasons could explain this reluctance: first, increasing the number of variables which affect individuals' utility necessarily decreases the predictive power of the models, especially if coordination problems arise; second, when interactions produce externalities which can be directly captured within groups (or social networks), interactions can be considered negligible (see Becker, 1971).

1.2. An Emerging Theory of Social Interaction. In the last decade, several authors have nevertheless proposed new themes and new analyses which try to explicitly link sociological considerations and economic ones: for instance, Akerlof (1997) with the concept of social distance, Becker (1991) with that of social influence, Benabou (1996) for the notion of community structure, Manski (1993) for the so-called 'reflection problem'... and so forth. Empirical researchers have also investigated the importance of peer effects and neighborhood effects in crime (Sacerdote and Scheinkman, 1996), unemployment (Topa, 2001) or group loyalty (Luttmer, 2001). The key point of all these contributions, say the *emerging theory of social interaction* (Akerlof 1997, p. 1005), is the fact that individuals are assumed to be deeply influenced in choices and economic behaviors by the social environment, that is they are no longer considered as Robinsons but rather as the members of particular groups, class, city... However, the link between individuals and social groups is typically symmetric: individuals' choices are determined by the social environment as well as the environment is determined by individuals' choices (see e.g. Stauffer and Sahini, 2006). While basic economic theory is just concerned with individual preferences and their consequences on choices, the economic theory of social interaction is also concerned with *identities* and the way these identities influence choices (see Kasher and Rubinstein, 1997; Samet and Schmeidler, 2002): while the former theory only wonders 'what do individuals prefer?', the latter also asks 'who are they?', this last question meaning first 'with whom do they interact?' and second 'how does social interaction influence their choices?'.

Methodologically, the great majority of the theoretical papers in this new field have chosen to use a common modeling framework stemmed from the analysis of interacting particle systems in physics: they assumed that any individual's optimal choice varies positively with the average choice of his group. However, several recent contributions (see e.g. Morris, 1997; 2000) try to emphasize social interactions outside of this current methodology in using another formal analogy: the one existing between *populations* - or communities - within a *social structure* and *events* - or beliefs - within an *information structure*. Actually, we can observe first that a social group is a subset of individuals linked by a social relation while an event is a subset of possible worlds linked by an accessibility relation (Kripke, 1963; Aumann, 1976; Billot and Walliser, 1999). Second, we can also observe that beliefs and communities are conceptually twins since they are both based on elements - worlds on one side, individuals on the other - which are sufficiently joined by a relation - of accessibility on one side, of interaction on the other - to be finally 'isolated' from the others elements. Then, the basic intuition of this paper is the following: while Kripke (1963) describes these isolated elements as 'indistinguishable' when defined as abstract worlds, we describe the individuals as 'interdependent' when preferences are mutually sensitive. This way, we suggest that social interactions between individuals are depending on their preferences as well as the individual preferences are depending on the social interactions. In other words, individual preferences and social interactions are two possible representations of the same evidence. In order to prove it, we establish that the standard assumption in microeconomics of complete ordered individual preferences is logically equivalent to the consistency of the social system, that is the existence of a unique reflexive and symmetric binary relation between individuals which perfectly describes all the social interactions.

1.3. Community and Loyalty. In this paper, we use three main tools: *interaction*, *interdependence* and *group loyalty*. The two firsts are not original. They already exist in the literature (Akerlof, 1997) and the third one is used as an illustration of the intrinsic relevance of microeconomics to model some social evidences.

• An *interaction* is a social situation where two individuals are sufficiently connected to consider themselves as social neighbors and we define a *social community* as a subset of individuals whose social neighborhood is fully included in the same particular population. When assuming the social influence to be encapsulated within social neighbors, a social community is therefore an exclusive concept in the sense where it forbids to be influenced by anyone belonging to another population (Lickel *et al.*, 2006). In fact, belonging to what we call a 'native population' - to be gay or hispanic, protestant or catholic - is just a given individual data which does not necessarily imply to belong to the associated social community.[1] Belonging to a social community is much more demanding than belonging to a particular population: it requires to never significantly interact with people of other native populations. An individual may be French and catholic and may have jewish friends, work with muslim people and be married with an Italian woman, i.e. may deeply interact outside his native population. Then, because of his various social interactions, he may be sensitive to various populations' lot, jewish, muslim, European... In this case, it is reasonable to consider that he does not belong to the French catholic community even if he belongs to the French catholic native population. On contrary, a Chinese immigrant who only interacts within his native population (because, for instance, of

[1] We use the term 'native' as a synonymous for 'exogeneous'. A native population gathers people who have not previously decided to be gathered but whose idiosyncratic identity, according to a particular criterion, is very similar.

the language), does not regularly see and meet individuals outside the Chinese population and then he is a natural candidate to be a member of the Chinese community.

• *Interdependence* is used to describe an individual preference focused on a particular population in such a way that this individual feels himself only concerned with what economically happens to this particular population. We then define an *economic community* as a subset of interdependent individuals, that is a community of economic interests. A Chinese immigrant whose preferences translate that he only cares about the Chinese population issues (and then neglects the other populations' interest) obviously belongs to the Chinese economic community. On the contrary, even if he belongs to the French catholic population, an individual may neglect the very interests of this population and may be particularly sensitive to what happens to the jewish or muslim populations; this way, it is reasonable to consider that he has no real true common interest with French catholics and consequently that he does not belong to the economic community they form.

A priori, the two kinds of communities are conceptually independent since they do not involve the same kind of relationships. Nevertheless, our purpose is precisely to show that this conceptual independence is no longer valid when individual preferences are constrained to satisfy standard properties. More precisely, when individuals are assumed to be rational in the standard sense, that is when their preferences are completely ordered as in an Arrow-Debreu framework, we prove that the economic communities form a social consistent system. This result also proves that the language of preferences can be used to describe some social basic evidences. In order to illustrate this last thesis, we later show how to translate the concept of group loyalty as defined by Luttmer (2001) in terms of preferences properties, that is continuity and monotonicity.

1.4. The Results. The organization of the paper is the following: in Section 2, the formal analogy between information structure and social structure is used to define social communities in terms of neighborhoods based on interactions between individuals, that is a reflexive and symmetric binary relation. Then, we prove (Theorem 1) that a social system, i.e. the set of all social communities within a great set of individuals called society, is consistent w.r.t the individual interactions if and only if (i) the society is a social community itself and (ii) the social system is monotone. In Section 3, while defining an economic community as a subset of individuals who are interdependent in terms of preferences (over alternatives), we prove (Theorem 2) that an economic system, i.e. the set of all economic communities, is consistent w.r.t the individual interactions if and only if individual preferences are completely ordered. This way, we show the conditions under which social interactions can be rationalized by individual preferences. In Section 4, we introduce the notion of social loyalty and prove (i) (Theorem 3) that any neighborhood is made of loyal individuals if preferences are continuous and monotone and (ii) (Theorem 4) that any social community is made of loyal individuals if preferences are continuous and monotone and if individuals are empathic.

2 Interactions and Social Communities

Consider a fixed finite set S of n *individuals*. Call *society* the set S and *population* any nonempty element of 2^S. Take as a simple representation of *social interaction* a binary relation between individuals denoted N: write $j \in N(i)$ if individual j socially interacts with (i.e. is a *social neighbor* of) individual i. This way, the subset $N(i)$ defines the (social) *neighborhood* of i. We assume that the interaction N is (1) reflexive: $\forall\ i \in S$, $i \in N(i)$ and (2) symmetric: $\forall i, j \in S$, $i \in N(j) \Rightarrow j \in N(i)$.

One natural way to interpret the social relation N is to consider that two individuals i and j are said to be neighbors if the intensity of their relationship is 'significant': for instance if they spend together a significative part of their time (greater than a given amount under which they consider that there is no genuine relation) but also if they share a significative number of cultural values meaning that they both belong to the same circumscribed population of individuals within S even if they seldom meet. The first kind of interaction is rather spatial while the second is rather cultural but in both cases, the concept of interaction translates a certain sort of 'proximity'. Then, what we called 'interaction' throughout these pages is conceptually and even formally more general than Akerlof's since not necessarily related to a distance (even social). Actually, this definition leads us to distinguish people who meet at random in the bus or in the street for a short while (who then do not really interact) from people who are truly in interaction such like those who work in the same office, those who share regular cultural activities and so forth...

We simply propose to define a *social community* (based on a *native* population A) as the larger 1-cohesive[2] nonempty subset of A.

Formally, we denote by $soc(A)$ the *social community based on native population* A:

$$soc(A) = \{i \in S : N(i) \subset A\}. \tag{1}$$

Indeed, each social community $soc(A)$ forms a more cohesive population than its complement in S. A social community is then a population of individuals locally closed by the social interaction. Each social community is a population of S but each population S does not necessarily form a social community.

Empirically, except the set of all human beings, there is nowadays very few 'native populations' which can be said 1-cohesive (except maybe some fully isolated social organizations such like primitive tribes who do not even know that they are not alone on Earth ...); generally, in each potential native population, there exist individuals who socially interact with 'outside' people, that is individuals who then do not belong to the community associated with their native population. In our words, this means that communities are w.l.o.g. smaller than

[2] According to Ellison (1993) and Morris (2000), a population A of individuals is said to be *p-cohesive* if every member $i \in A$ is such that there exists at least a proportion p of his social neighbors who belong to A. Hence, a population A is 1-cohesive whenever the subset of all individuals $i \in S$ such that $N(i) \subset A$ is equal to A. However, for every nonempty population $A \subset S$, it is possible (at least for a modeler) to point out individuals $i \in S$ such that $N(i) \subset A$ and to isolate them from individuals $j \in S$ for which there exist individuals $k \notin A$ such that $k \in N(j)$. Note that no one can be sure that the latter individuals are sufficiently cohesive to form a true community (since $p \in [0, 1]$) while the former ones can always revendicate a strong specific link ($p = 1$).

populations since more demanding (exclusive) in terms of social interactions.[3] This criterion - allowing to distinguish a native population from its community - is not purely formal. In most big western cities, Paris, London or New York City..., one can easily observe the different local communities (jewish, Chinese, Irish, gay...) living in some specific areas ('Le Marais' for Parisian gays or 'East London' for Indian people) where the density of the corresponding native populations is so high that the possibility to socially interact with someone belonging to another population is low. Nevertheless, in the same time, one can find gays in each Parisian borough even far from Le Marais as well as Indian people in the West London, which illustrates the empirical difference between a population and a community. Note also that nothing prevents to understand the definition of a social community in a broader sense where the native population is not racially or sexually but spatially (e.g., a village) or culturally (e.g., a linguistic minority) specified.

Formally, a *social system*[4] is a mapping *soc* from 2^S towards 2^S. In short, we say that the social system *soc represents* the social interaction N if (1) holds for any $A \subset S$.

Definition 1 : *A social system soc is* consistent *if there exists an interaction N such that soc represents the relation N.*

Theorem 1 : *A social system soc is consistent if and only if*
*(*A1*) the society S forms a social community: $soc(S) = S$,*
*(*A2*) for two populations $A, B \subset S$ such that $A \subset B$: $soc(A) \subset soc(B)$.*

(All proofs are in Appendix.)

A1 is called *inclusivity* and means that the society S is assumed to be 1-cohesive whatever happens to smaller communities. **A1** requires that any individual in S knows that S exists and is the greatest social community he belongs to. **A1** has also a flavor of a 'civil peace' requirement as explicitly shown by Lemma 1 below.

A2 corresponds to *monotonicity* of the social communities with respect to the size of the associated sequence of nested native populations. This means that a community based on a big native population is bigger than a community based on a smaller nested one. For instance, the American population is obviously bigger than the Afro-American one and the two associated communities, i.e. the individuals who revendicate and essentially define themselves as Americans or Afro-Americans are ranked the same way. This axiom then introduces some minimal regularity in the sociological phenomena we study which seems to be empirically relevant (see e.g. Wellman and Berkowitz, 1988, ch. 5).

Theorem 1 states that if a social system consistently represents an interaction, it satisfies the two conditions.[5] Conversely, if a given social system satisfies the two conditions, then there exists an interaction which generates the social system. Actually, the proof illustrates the *uniqueness* of the interaction for a given system.

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[3] This is consistent with Lévi-Strauss's analysis (e.g., 1961) and overall the recent Guérin's works (2001).

[4] This concept is close to that of a *local interaction system* as defined by Morris (2000).

[5] Note that inclusivity and monotonicity can be seen as the set-theoretic translation of the two conditions that define, in decision theory, a set function as a Choquet capacity.

Remark 1 : *When characterized by* $N(i) = \{j \in S : i \notin N(S_{-j})\}$ *(for all individuals* i *of* S*), the interaction* N *means that an individual* j *is a social neighbor of individual* i *iff the complement population* S_{-j} *does not form a social community.*

3 Preferences and Economic Communities

Consider a fixed finite set X of *options* $a, b, c...$ Call an *issue* any vector x of X^n such that the ith coordinate x_i corresponds to a particular option of X that is allocated to (or chosen by) individual i. Basically, any option can be allocated to (or chosen by) any individual. The set X^n is the n-size set of all possible individual issues.[6] For any population $A \subset S$, x_A denotes the tuple $\{x_i\}_{i \in A}$. Furthermore, if $z = (x_A, y_{-A})$, it means that $z_i = x_i \in X$ for all individuals $i \in A$ and $z_j = y_j \in X$ for all the other individuals $j \notin A$. Each individual $i \in S$ is assumed to rank the issues according to a *preference relation* $\succcurlyeq_i$, that is a complete binary relation defined over $X^n \times X^n$: $\forall (x, y) \in (X^n)^2$: $x \succcurlyeq_i y$ or $y \succcurlyeq_i x$. This way, $x \succcurlyeq_i y$ means that for i, issue x is at least as good as issue y. Two issues x, y are considered indifferent, that is denoted $x \sim_i y$ when, simultaneously, $x \succcurlyeq_i y$ and $y \succcurlyeq_i x$. A complete relation $\succcurlyeq$ is ordered if it satisfies transitivity: for all issues $x, y, z \in X^n$, $x \succcurlyeq y$ and $y \succcurlyeq z$ implies $x \succcurlyeq z$. Note that any complete binary relation which is ordered is also reflexive: for all issues $x \in X^n$, $x \succcurlyeq x$.

Let an individual $i \in S$ be such that

$$(x_A, y_{-A}) \sim_i (x_A, z_{-A}) \text{ for all } x, y, z \in X^n. \tag{2}$$

Such an individual i is said to be *interdependent* with the population A and the subset of all individuals interdependent with A is called the *economic community* (based on A) and denoted $eco(A)$.

An economic community is a set of individuals who share a common interest for a particular population (without necessarily being members of it). More precisely, the individuals who belong to the same economic community reveal that they are always sensitive to what happens to a particular population while never sensitive to the rest of the society; they expand (or export) the selfish device beyond themselves but untill the border of a given population. For instance, parents are generally sensitive to the economic condition in life of their children and little children. If they restrict their interest to this small subset of individuals, they form an economic community who can behave in a way which makes the whole family happy (in sharing by advance the capital with equity, in helping the youngest...). If a family (even spatially and then socially scattered) is a natural candidate for being an economic community, note that an individual can form his own economic community when he is selfish (this situation corresponds to the Senian liberal one, see Billot, 2003). By the way, (2) also can be seen as an equation for the first Marxists-socialists who considered the industrial proletariat, that is here the native A, as the only population whose economic

[6] Hence, if $S = \{1, 2\}$ and $X = \{a, b\}$, then the set X^2 of issues is:

$$\{(a; 1, a; 2), (a; 1, b; 2), (b; 1, b; 2), (b; 1, a; 2)\}$$

where $x = (a; 1, b; 2)$ means that $x_1 = a$ while $x_2 = b$, that is 1 chooses a while 2 chooses b.

situation, that is x_A, fully determines the historical and political evolution of a society. In short, there are numerous potential economic communities: friends circle, families, firms, classes, villages, minorities...

An *economic system* is a mapping *eco* from 2^S towards 2^S.

In what follows, we investigate the relationship between the properties of the social system and the properties of the economic system, that is the theoretical link between the interaction N via the social communities with the individual preferences via the economic communities. The intuition is that standard microeconomics requirements about individual rationality implicitly correspond to a particular organization of the social system:

Theorem 2 : *The economic system eco forms a consistent social system if and only if preference relations are completely ordered.*

It is also possible to prove a converse result: for any consistent social system, there exist complete ordered preference relations such that each social community is also an economic community.

From Theorem 1, we know that a social system is consistent iff **A1** and **A2** are satisfied. Hence, one way to prove Theorem 2 is to prove that, when the agents are interdependent, complete ordered relations lead to set up a system of economic communities which satisfy **A1** and **A2**. For that purpose, we establish the two following lemmas:

Lemma 1 : *Preference relations are reflexive if and only if* A1 *holds.*

By (2), preference reflexivity must be understood as an interdependence constraint for all individuals in S. This way, reflexivity ensures that the society S is formed with people who define a community from both sides, social and economic.

Lemma 2 : *Preference relations are transitive if and only if* A2 *holds.*

Preference transitivity implies a kind of consistency between a first pair of issues, a second, a third, etc... and a last one which collapses all the intermediate steps. The property of monotonicity corresponds somehow to the same kind of consistency: inclusion of a sequence of native populations can be collapsed in saying that the biggest native population (which is actually the society itself by **A1**) includes all other communities with respect to the same nesting rule. The intuition is simple. Consider the French last presidential elections:[7] the global set of individuals who 'choose among several candidates and vote for Jospin' is included in the set of individuals who 'choose among several parties and vote for the Socialist one' which in turn is included in the set of individuals who 'choose between Right and Left and vote for the Left'. In terms of native populations, being in the first population naturally implies to be in the second one which in turn implies to be in the third one. Then, by (2), the community based on the first population, say the Jospinist trend within the Socialist Party, cannot be greater than that based on the second population, say the voters of the Socialist Party itself, nor that based on the third. Hence, preferences transitivity and communities monotonicity translate the same kind of consistency in their respective context.

[7] In 2002.

Theorem 2 exactly states that the interaction N is fairly represented by the economic system *eco* when individual preferences are completely ordered. Furthermore, if preference relations are completely ordered, each economic community exactly defines a social one, i.e. the two mappings, *soc* and *eco*, are identical:

$$[(x_A, y_{-A}) \sim_i (x_A, z_{-A}), \forall x, y, z \in X^n] \Leftrightarrow [N(i) \subset A]. \tag{3}$$

In this case, when $soc(A) = eco(A)$, we simply call it a 'community'. Hence, the right interpretation of Theorem 2 seems to be the following: standard basic assumptions about the individual behavior (that is, complete ordered preferences as a signal of rationality) are logically equivalent to an assumption about the social organization of the society (that is, consistency of the social system). *In other words, individual rationality and social consistency can be considered as the two faces of the same coin.*

$$\left\{ \begin{array}{ccc} \text{reflexivity of preferences} & \Longleftrightarrow & \text{inclusivity of communities} \\ \text{transitivity of preferences} & \Longleftrightarrow & \text{monotonicity of communities} \end{array} \right\}$$

i.e.:

$$\text{individual rationality} \quad \Longleftrightarrow \quad \text{social consistency.}$$

Beyond this formal equivalence, Theorem 2 means that the two following statements are equivalent:

- When rational, individuals are interdependent with individuals with whom they interact;
- When rational, individuals choose to interact with individuals with whom they are interdependent.

In short, to be sensitive for instance in France to the so-called *second generation*'s situation in life requires to be in social contact with the immigrant population. Hence, the existence of suburban ghettos (where immigrants often represent the 2/3 of the inhabitants) prevent any real global sensitivity (of the rest of the society) to the problems of this population. In the same time, being a priori interdependent with such a population prevents to interact with people outside and then can induce some 'communitarist' behaviors such as religious fundamentalism, political extremism... Individual rationality, that is social consistency, seems then to lead to a segregated (at least partitioned) society where the different communities are economically and socially disjoint, except at the level of the society itself, that is for some public events such as elections or wars for which votes and lifes are totally aggregated within the national community (**A1**).

Since individual rationality (in terms of preferences) is logically equivalent to social consistency of the interaction system (in terms of communities), we propose to go on the analysis in pointing out other kinds of equivalence between well-known properties of the preferences (e.g. continuity and monotonicity) and specific properties of the social system, e.g. group loyalty.

4 Group Loyalty

When assuming preference relations to be completely ordered, we know by (3) that if an individual i socially interacts with an individual j and belongs to a community $eco\,(A)$, then j belongs to the same community $eco\,(A)$. This relationship is clearly strong. Similarly, economic interdependence is modeled by means of the individual's indifference between all situations where the native population, say A, gets the same issue, i.e. x_A, *whatever happens to the rest of the society, i.e.* y_{-A} *or* z_{-A}*...* To weaken the possible relationship between two individuals, we propose to introduce the concept of *group loyalty.* Actually, in a recent paper, Luttmer (2001) shows that individual preferences for redistribution are not only determined by self-interest but also affected by the characteristics of the social environment (district, family...). One of the main explanations for this empirical feature is the *racial* group loyalty: individuals increase their support for welfare spending as the share of local recipients from their own racial group rises. In our (more abstract terms), this phenomenon means that when individuals are loyal with a given native population, they can accept to loose something within a particular issue if it is beneficial to the rest of this native population (eventually a subset of it). Let us consider an individual who is the born-in-France son of a Maghrebian immigrant, that is he belongs to the 'second generation'. Assume he becomes a famous and rich actor or a soccer player (like Zinedine Zidane for instance). If he accepts (in supporting or voting for a leftist party) to pay more taxes (i.e., to decrease his own utility) in order to improve the standards of life of the French Muslims by means of the redistribution system (i.e., to increase their utilities), he can be said loyal with his native population. In short, group loyalty may translate a link which is weaker than interdependence but nevertheless significant.

Hence, in what follows, we seek for the conditions under which the previous social entities (neighborhood and community) can constitute a population of individuals who are 'loyal' .

Write $x \geq y$ if $x_i \geq y_i$ for all $i \in S$, $x > y$ if $x \geq y$ and $x_i > y_i$ for some $i \in S$ and $x \gg y$ if $x_i > y_i$ for all $i \in S$.

Definition 2 : *Let an individual* $i \in S$. *If for all issues* $x \gg y$, *there exist issues* $z \ll y$ *such that the individual* $j \neq i$ *is better off with* (x_j, z_i, z_{-ij}) *rather than with* y $((x_j, z_i, z_{-ij}) \succ_i y$ *with* $x_j \gg y_j$, $z_i \ll y_i$ *and* $z_{-ij} \ll y_{-ij})$, *then such an individual* i *is said to be* loyal *with the individual* $j \in S$.

4.1 Neighborhood Loyalty

In the same spirit than previously, we search for proving that the concept of group loyalty can be encapsulated within preferences properties.

A preference relation $\succcurlyeq$ is *continuous* if the set of issues $\{x \in X^n : x \succcurlyeq y\}$ is closed for every $y \in X^n$ and *monotone* if $(x_i, z_{-i}) \succcurlyeq (y_i, z_{-i})$ for some issues $x, y, z \in X^n$ implies that $(x_i, z_{-i}) \succ (y_i, z_{-i})$ for all $x, y, z \in X^n$ such that $x_i > y_i$.

Theorem 3 : *Let a consistent social system soc. If preference relations are continuous and monotone, then any neighborhood is a subset of mutually loyal individuals.*

Remark 2 : *Note that, thanks to Theorem 2, Theorem 3 can be rewritten as follows: if preference relations are continuous and monotone, then any neighborhood is a subset of mutually loyal individuals. This result means that, in order to be loyal with someone, an individual needs to socially interact with him: loyalty is not altruism, it is rather the economic counterpart of social sensitiveness.*

4.2 Community Loyalty

Are continuity and monotonicity of preference relations sufficient to ensure that the result established in Theorem 3 can be naturally extended from neighborhoods to communities? Intuitively, since a community requires more structure than a neighborhood, the answer should be negative. Hence, we propose the following characterization of preference relations in order to extend the result. Given preference relations and a particular neighborhood $N(i)$, write $x \succcurlyeq_{N(i)} y$ when $x \succcurlyeq_j y$ for all individuals $j \in N(i)$ and $x \succ_{N(i)} y$ when $x \succcurlyeq_{N(i)} y$ and $x \succ_j y$ for some $j \in N(i)$.

Definition 3 : *Let an individual $i \in S$ such that if $x \succcurlyeq_i y$, then $x \succcurlyeq_{N(i)} y$, for all $x, y \in X^n$. Such an individual i is said to be* empathic *with his neighborhood $N(i)$.*

Empathy requires that the preferences of a given individual are sensitive (in the most minimal way) to what other individuals' preferences are when these individuals belong to the social neighborhood of the first individual. It can be also written as: if $x \succ_{N(i)} y$, then $x \succ_i y$, for all $x, y \in X^n$.

Theorem 4 : *If preference relations are continuous and monotone and if individuals are empathic with their neighborhoods, then each smallest community is a subset of mutually loyal individuals.*

The following lemma is central for proving the theorem.

Lemma 3 : *If all individuals are socially empathic with their neighborhood, then $i \in N(j)$ implies $N(i) = N(j)$, for all $i, j \in S$.*

References

Akerlof, G.A. (1997): 'Social Distance and Social Decisions,' *Econometrica* **65**, 1005-1027.

Arrow, K.J. (1951): *Social Choice and Individual Values.* Wiley: New York.

Aumann, R.J. (1976): 'Agreeing to Disagree,' *Annals of Statistics* **4**, 1236-1239.

Becker, G.S. (1971): *The Economics of Discrimination.* Chicago University Press: Chicago.

Becker, G.S. (1971): 'A Note on Restaurant Pricing and Other Examples of Social Influence on Prices,' *Journal of Political Economy* **99**, 1109-1116.

Benabou, R. (1996): 'Workings of a City: Location, Education, and Production,' *American Economic Review* **86**, 584-609.

Billot, A. (2003): 'How Liberalism Kills Democracy: Sen's Theorem Revisited,' *Public Choice* **116**, 247-270.

Billot, A. and B. Walliser (1999): 'A Mixed Knowledge Hierarchy,' *Journal of Mathematical Economics* **32**, 185-206.

Elisson, G. (1993): 'Learning, Local Interaction, and Coordination,' *Econometrica* **61**, 1047-1071.

Guerin, F. (2001): *Histoire et épistémologie du concept de communauté dans la sociologie française aux XIXe - XXe siècles.* Ph.D.dissertation, EHESS, Paris.

Kasher, A. and A. Rubinstein (1997): 'On the Question "Who is a J?": A Social Choice Approach,' *Logique et Analyse*, **160**, 385-395.

Kripke, S. (1963) : 'Semantical Analysis of Modal Logic,' *Zeitschrift fur Mathematische Logik und Grundlagen der Mathematik* **9**, 62-67.

Levy Strauss, C. (1961): *Race et Histoire.* Gonthier: Paris.

Lickel, B., Rutchik, A., Hamilton, D.L. and S.J. Sherman (2006): 'Intuitive Theories of Group Types and Relational Principles,'*Journal of Experimental Social Psychology* **42**, 28-39.

Luttmer, E.F.P. (2001): 'Group Loyalty and the Taste for Redistribution,' *Journal of Political Economy* **109**, 500-528.

Manski, C.F. (1993): 'Identification of Endogenous Social Effects: The Reflection Problem,' *Review of Economic Studies* **60**, 531-542.

Mongin, P. (2000): 'Does Optimization imply Rationality,' *Synthese* **124**, 73-111.

Morris, S. (1997): 'Interaction Games: A Unified Analysis of Incomplete Information, Local Interaction and Random Matching,' *CARESS Working Paper* **97-02**, University of Pennsylvania.

Morris, S. (2000): 'Contagion,' *Review of Economic Studies* **67**, 57-78.

Postlewaite, A. (1998): 'The Social Basis of Interdependent Preferences,' *European Economic Review* **42**, 779-800.

Sacerdote, B. and J.A. Scheinkman (1996): 'Crime and Social Interactions,' *Quarterly Journal of Economics* **111**, 507-548.

Samet, D. and D. Schmeidler (2003): 'Between Liberalism and Democracy,' *Journal of Economic Theory* **110**, 213-233.

Stauffer, D. and M. Sahimi (2006): 'Discrete Simulation of the Dynamics of Spread of Extreme Opinions in a Society,' *Physica A: Statistical Mechanics and Applications* **364**, 537-543.

Topa, G. (2001): 'Social Interaction, Local Spillovers, and Unemployment,' *Review of Economic Studies* **68**, 261-295.

Wellman, B. and S.D. Berkowitz (1988): *Social systems: A Network Approach.* Cambridge University Press: Cambridge.

5 Appendix

Proof of Theorem 1. Suppose that a social system *soc* represents an interaction N. Then, $soc(A \cap B) = \{i \in S : N(i) \subset A \cap B\}$ which is obviously equal to $\{i \in S : N(i) \subset A\} \cap \{i \in S : N(i) \subset B\}$, that is $soc(A) \cap soc(B)$. Now, suppose that *sc* satisfies **A1-A2**. By **A2**, $A \subset B \Rightarrow soc(A) \subset soc(B)$ for all populations $A, B \subset S$. Write S_{-j} for $S \backslash \{j\}$. Define the interaction N as follows: $N(i) = \{j \in S : i \notin N(S_{-j})\}$. Show that the system *soc* represents the interaction as defined above. (i) Suppose $i \in soc(A)$. Then, $A \subset S_{-j}$ implies by **A2** $i \in soc(S_{-j})$ which implies $j \notin N(i)$. Thus, $j \in N(i)$ implies $A \nsubseteq S_{-j}$

which implies $j \in A$. Hence, $N(i) \subset A$. (ii) Conversely, suppose $N(i) \subset A$. If $A = S$, $i \in soc(S)$ by **A1**. Suppose then that we consider a population $A \neq S$. Then, $A = \cap_{j \notin A} S_{-j}$. Since $i \notin soc(S_{-j})$ implies $j \in A$, $j \notin A$ implies $i \in soc(S_{-j})$. Hence, $i \in soc(S_{-j})$ for all $j \notin A$ implies, by the above definition of A, $i \in soc(A)$.■

Proof of Lemma 1. Write $y_{\emptyset}$ for y_{-S}. We have by definition of an economic community:

$$\begin{aligned} eco(S) &= \{i \in S : (x_S, y_{-S}) \succcurlyeq_i (x_S, z_{-S}) \text{ for all } x, y, z \in X^n\} \\ &= \{i \in S : (x_S, y_{\emptyset}) \succcurlyeq_i (x_S, z_{\emptyset}) \text{ for all } x, y, z \in X^n\} \\ &= \{i \in S : x \succcurlyeq_i x \text{ for all } x \in X^n\} \end{aligned}$$

since $x_S = x$. Then, it is obvious that $eco(S) = S$ iff each preference relation is reflexive.■

Proof of Lemma 2. (i) By construction, $A \subset B \Rightarrow eco(A) \subset eco(B)$ is equivalent to $eco(A) \cap eco(B) = eco(A \cap B)$. Hence, take any individual $i \in S$ and two populations A and B such that $i \in eco(A) \cap eco(B)$. By definition of an economic community, for all profiles $x, y, z \in X^n$: $i \in eco(A) \Rightarrow \left(x_{A\cap B}, y_{-(A\cap B)}\right) \succcurlyeq_i (x_{A\cap B}, y_{A\cap -B}, z_{-A})$ and $i \in eco(B) \Rightarrow (x_{A\cap B}, y_{A\cap -B}, z_{-A}) \succcurlyeq_i \left(x_{A\cap B}, z_{-(A\cap B)}\right)$. Thus by transitivity: $\left(x_{A\cap B}, y_{-(A\cap B)}\right) \succcurlyeq_i \left(x_{A\cap B}, z_{-(A\cap B)}\right)$ for all $x, y, z \in X^n$. So, $i \in eco(A \cap B)$. (ii) Suppose now an agent $i \in S$ such that $i \in eco(A \cap B)$. Then, for all $x, y, z \in X^n$: $\left(x_{A\cap B}, y_{-(A\cap B)}\right) \succcurlyeq_i \left(x_{A\cap B}, z_{-(A\cap B)}\right)$.Thus, $(x_A, y_{-A}) \succcurlyeq_i (x_A, z_{-A})$ and $(x_B, y_{-B}) \succcurlyeq_i (x_B, z_{-B})$ which implies: $i \in eco(A) \cap eco(B)$.■

Proof of Theorem 2: By Lemmas 1 and 2, we know that preference relations are ordered if and only if **A1** and **A2** hold. By Theorem 1, we know a social system is consistent if and only if **A1** and **A2** hold. Then, the economic system *eco* is a consistent social system.■

Proof of Theorem 3. (i) Let an individual $j \in N(i)$. By monotonicity of i's preference relation $\succcurlyeq_i$, we have: $(x_j, y_{-j}) \succ_i y$ for all issues $x, y \in X^n$ where $x_j > y_j$. Now, by continuity, there always exists an issue $z \ll y$ such that $(x_j, z_{-j}) \succ_i y$. Hence, i is loyal with j. (ii) Suppose now that i is loyal with j. Then, by definition, $(x_j, z_{-j}) \succ_i y$ for some $x \gg y \gg z$. Thus, by monotonicity of i's preference relation $\succcurlyeq_i$, we have: $(x_j, z_{-j}) \succcurlyeq_i (y_j, z_{-j}) \succ_i y$. Hence, $i \in eco(j)$ by (2), that is $j \in N(i)$ since the interaction N is reflexive and symmetric. Hence, any neighborhood is a subset of mutually loyal individuals.■

Proof of Lemma 3. Empathy can also be written: if $x \succ_{N(i)} y$, then $x \succ_i y$, for all $x, y \in X^n$. With symmetry of the relation N, if $i \in N(j)$ then $j \in N(i)$. Hence, $N(i) = N(j)$, for all $i, j \in S$, requires that if $i \in N(j)$, then $N(i) \subset N(j)$, namely the transitivity of the social interaction N. Prove the contrapositive of the required property. Then, suppose that there exists three individuals $1, 2, 3 \in S$ such that $1 \in N(2)$, $2 \in N(3)$ but $1 \notin N(3)$. Now, let $x_1 = 1$ while $x_i = 0$ for all individual $i \neq 1$. By continuity of preference relations, we have $x \succ_2 0^n$ where 0^n is the null vector of X^n. Then, $x \succcurlyeq_i 0^n$ for all $i \in S$. Thus, it is true that $x \succ_{N(i)} 0^n$. But, in the same time, since $1 \notin N(3)$, then

$0^n \sim_3 x$. Now, $x \succ_{N(i)} 0^n$ for all $i \in S$ and $0^n \sim_3 x$ is a contradiction. So, if $i \in N(j)$, then $N(i) \subset N(j)$ and, by symmetry, $N(i) = N(j)$.■

Proof of Theorem 4. The result is immediate by (1) and Lemma 3 since $eco(N(i)) = N(i)$. This way, smallest communities exactly correspond to neighborhoods as implied by the social interaction and then, Theorem 3 holds.■