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Technical progress in North and welfare gains in South under nonhomothetic preferences

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Abstract

The paper proposes a theoretical model investigating the welfare consequences of technological shocks in a Ricardian framework (*a la* Dornbush, Fisher and Samuelson, 1977). Contrary to existing literature, the model incorporates a nonhomothetic demand function whose price and income elasticities are endogenously determined by technology. The model is applied to the case of trade between two economies with different development levels. It is shown in particular that the developing country can experience a fall in utility as a result of technical progress in the developed country. This result depends on the type of technological shock assumed (biased vs. uniform technical progress), as well as on the size of the development gap.

Keywords : Dornbush-Fisher-Samuelson Ricardian model ; Technology and Trade ; North-South Trade ; Nonhomothetic preferences ; Hierarchic needs ; Hierarchic purchases

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1 Introduction

The role of nonhomothetic preferences in the analysis of trade has experienced a sharp rise in importance in the recent literature. This assumption about consumption behavior has been included in a bunch of papers dealing with different questions such as the predicted factor content of trade (Dinopoulos, Fujiwara and Shimomura (2006), Chung (2003)), the effects of trade liberalization or preferential trade agreements on welfare (Stibora and de Vaal (2007), Stibora and de Vaal (2006)), the relation between income inequality and trade flows (Mitra and Trinidade (2003), Kugler and Zweimüller (2002)) and the influence of technological progress on the distribution of gains from trade (Matsuyama (2000)).

The reason for including nonhomothetic preferences in a trade model, despite the lower tractability of this kind of function, is that there is much evidence to show that deviation from homothetic preferences is not marginal. One illustration of this is provided by a major finding in the empirical literature devoted to this issue: Hunter (1991) proposes a counter-factual analysis and shows that nonhomothetic preferences account for 25% of world trade. In the same line, several empirical contributions prove that income elasticities in trade are actually non unitary (see, for instance, Magee and Houthakker (1969), Atesoglu (1993, 1994); Bairam, Dempster (1991); Perraton (2003) and Muscatelli et al. (1994, 1995)). However, most contributions in the existing literature are based on the assumption of homothetic preferences. This raises the question of whether certain important theoretical results in trade literature are robust to changes in the specification of preferences.

This paper is devoted to the old question of the international distribution of welfare gains induced by technical progress. More specifically, I look at the relative ability of a developing country to benefit (in terms of welfare) from the external technical progress through trade. Among the large literature devoted to this issue, the standard approach is the Ricardian framework : according to Dornbush, Fisher and Samuelson (hereafter, DFS (1977)), technical progress in one country always benefits the trading partner, through a terms-of-trade deterioration for the fast-growing country. This theoretical result relies on the use of a Cobb Douglas utility function yielding unitary income and price elasticities in trade. It prevails in major inter-sectorial trade models such those of endogenous growth (see for instance, Grossman and Helpman, chapter 7, 1991).

I propose here a ricardian model where non unitary elasticities in trade are microfunded on the basis of non homothetic preferences[1]. This model is related to the Ricardian inter-sectorial trade models of Matsuyama (2000)[2]. Actually, Matsuyama (2000) proposes a nonhomothetic model which is considered as an alternative to the standard homothetic model of DFS (1977). Matsuyama models nonhomothetic preferences through a function of hierarchic desires, where agents extend the range of consumed goods when real income increases. As the quantities of each good consumed are exogenously fixed to one, the evolution of consumption can only be extensive. The common point shared by the Matsuyama and DFS models is that they fix, a priori, the evolution of consumption patterns in the event of technical progress. In the DFS model, demand shifts towards those goods which display the greatest decrease in relative price (so as to maintain the initial distribution of spending on each good). In Matsuyama's approach, the change in demand favours the lower priority goods produced by North.

By allowing agents to choose both the quantities and the range of goods consumed, I

[1] The theoretical interpretation of non-unitary income elasticities in trade is not straightforward. According to McGregor and Swales (1986, 1991) and Krugman (1989), this result is a statistical artefact resulting from a supply composition effect. On the other hand, nonhomothetic preferences occupy a central position in the post-Keynesian analysis of income elasticities of trade (Thirlwall (1979), Kaldor (1970)). This debate lies outside the purposes of this paper. The choice of a nonhomothetic preference-based approach is found in some empirical studies reported in section 2.1. More details can also be found in studies which control for the supply side composition effect in their estimations of export and import functions: Amable (1992), Fagerberg (1988), Feenstra (1994), Gagnon (2003), Funke and Runwedel (2001), Bayoumi (1999).

[2] We can also refer to the nonhomothetic intra-sectorial models of Flam-Helpman (1987) and Stockey (1991), which are discussed in detail in Matsuyama (2000).

propose a new framework to account for non homothetic preferences alternative to Matsuyama (2000). The main specificity of my approach is the link established between the price and income elasticities of external demand, and the nature of the goods in which each country is specialized. This link forwards by two transmission channels. Firstly, I consider hierarchic consumption behavior. Goods are then distinguished according to their relative degree of priority, which also determines their respective income and price elasticity values. Secondly, the technological dimension is linked to the consumption behavior through a function of hierarchic purchases. In particular, I allow technical progress to change the relative degree of priority of each good. The intuition behind this is that technical progress, by the creation of new goods, leads to the creation of new needs, and then modifies the perception of agents regarding their pre-existing needs. This effect is formally expressed by the fact that elasticities are partially determined by technical coefficients of production. Hence, technical progress has a direct impact on the distribution of spending devoted to each good and on the range and quantities of goods consumed.

The aim of the paper is then to analyze the impact of technical progress on the international distribution of welfare, in a framework where hierarchic consumption, non-uniform substitution and income effects are simultaneously included. Our main result is that South can be harmed by technical progress in the North. This happens if technical progress is biased toward lower priority goods and if the size of the technological gap between the two countries is large. The decrease in South's welfare occurs despite the fact that South's agent consumes a wider range of goods produced by North. This result derives directly from the link established between preferences and technology. It is totally at odds with results from main inter-sectorial Ricardian models, where an increase in the existing pattern of comparative advantages leads to increased gains from trade (DFS (1977), Krugman (1990), Matsuyama (2000)). On the contrary, our result suggests that, for less advanced countries to avoid being harmed by North's technical progress, there exists an optimal level of development gap with their trading partners. Due to the complexity of the model, I analytically study the case of a very large development gap between North and South. I also provide simulations for the more general case.

The rest of the paper is organized as follows. In the next section, a closed-economy model and the main properties of the demand functions are presented. In the third section, these functions are incorporated into a Ricardian model of the type developed by DFS. A model of North-South trade is then obtained where the price and income elasticities of external demand are non-unitary and micro-founded. The final section is devoted to the analysis of the impact of North's technical progress on South's welfare.

2 A nonhomothetic closed-economy model

2.1 Characteristics of consumption behavior

A series of macro- and microeconomic empirical studies on demand behavior provides direct validation for the hypothesis of nonhomothetic preferences. We can notably consider the following evidences. Firstly, Falkinger, Zweimüller (1996) and Jackson (1984) show that a rise in income is accompanied by an extension in the range of goods consumed by agents. Secondly, Hunter (1991), Hunter-Markusen (1988), Jackson (1984) and Fillat-Francois (2004), test the empirical relevance of some specific nonhomothetic demand functions (respectively, Stone Geary, Almost Ideal Demand System and hierarchic consumption). They verify that when their income rises, agents do not distribute this increase uniformly over all the goods they buy[3]. Finally, a corollary from the empirical validity of the previous nonhomothetic demand functions is that sector income elasticities depend on the agent's level of income.

[3] It should be noted that, as the Hunter (1991), Hunter-Markusen (1988) and Francois-Fillat (2004) studies rely on cross section data, they indirectly control for the impact of supply change composition on demand income elasticities. One can also refer to the paper by Bills and Klenow (2000), who calculate Engel's curve by controlling for quality.

We require a demand function which is consistent with these three previous stylized facts: hierarchic consumption, non unitary income elasticities at the sectorial level, and a link between demand income elasticities and agents' levels of income. From a theoretical point of view, our approach to preferences is then related to that of Jackson (1984) and Hunter-Markusen (1988). They use a utility function of the following type:

$$U = \sum \beta_i \log(q_i + \gamma_i)di \tag{1}$$

where q_i corresponds to the quantity of good i consumed.

This quasi-homothetic utility function can produce a linear expenditure system (LES) or a hierarchic linear expenditure system (HLES), depending on the sign of the constant terms γ_i. For instance, Hunter and Markusen (1988) consider negative constant terms. In this way, they account for the need to satisfy a minimum consumption of an exogenous number of sectors[4]. Subsequently, the function produces (non unitary) income and substitution effects. On the contrary, Jackson (1984) assumes a positive sign for the constant terms: the utility function relies more on an endowment effect, which produces hierarchic consumption of an endogenous range of consumed goods. For some goods, the non negativity constraint may effectively become binding. The main interest of this approach is that it allows both for extensive and intensive demand behavior[5].

We follow Jackson's approach. In this way, we can simultaneously account for non unitary elasticities of demand at the sectorial level and hierarchic consumption. Nevertheless, we modify the previous utility function in three ways. Firstly, it is expressed in continuum so that we can obtain an expression for the marginal good. Secondly, the standard Cobb Douglas coefficient β_i is assumed to be equal to one. All goods in the utility function are then symmetric, except as far as the endowment effect is concerned. Thirdly, we make a linear transformation by dividing the term in brackets in equation 1 by γ_i. The purpose of this last transformation is to obtain a function where only consumed goods enter into the agent's final utility[6]. The individual utility function at the heart of our model can then be written:

$$U = \int_0^\infty \log(\frac{q_i}{\gamma_i} + 1)di \tag{2}$$

The maximization utility program of the representative consumer can be written:

$$\begin{aligned} Max \quad U \quad s.t. \quad \int_0^\infty p_i q_i di &= y \\ and \qquad\qquad q_i &\geq 0 \end{aligned}$$

where p_i and y correspond respectively to the price of good i and to the agent's income. U is given by equation 2.

We can write the Khun-Tucker conditions:

$$\text{for } i \in K, \quad \frac{1}{p_i\gamma_i} > \lambda \quad => \quad q_i = \frac{1}{p_i\lambda} - \gamma_i \tag{3}$$

$$\text{for } i \notin K, \quad \frac{1}{p_i\gamma_i} < \lambda \quad => \quad q_i = 0 \tag{4}$$

$$\text{for } i = J, \quad \frac{1}{p_J\gamma_J} = \lambda \quad => \quad q_i = 0 \tag{5}$$

[4] One could also refer to the approaches of Markusen (1986), Matsuyama (1992) and Puga-Venables (1999), who apply the minimum consumption framework in a model with two sectors.

[5] Foellmi and Zweimüller (2006) propose a closed-economy model where the utility function presents the same characteristics.

[6] Otherwise, according to equation 1, for all goods consumed, utility is determined by $\beta_i \log(q_i + \gamma_i)$, but non-consumed goods also enter into the agent's utility through $\beta_i \log(\gamma_i)$. Our transformation removes this effect.

According to these conditions, a good will or will not be consumed depending on its value for the term $p_i\gamma_i$. This term determines hierarchic consumption and can then be considered as an entry criterion. *Ceteris paribus*, the lower the value of this term, the more likely it is that the good will be consumed, and the higher the priority attached to it. Agents will consume lower priority goods only if what are perceived as more basic needs have been already satisfied in terms of quantity (following the equation 3)[7].

To order the consumption process explicitly and determine the marginal good J, we have to assume that the entry criterion follows an increasing monotonic function. This assumption enables us to divide the continuum into two segments: that of consumed goods $i \in K = [0, J[$ and that of non-consumed goods $i \notin K = [J, \infty[$ (figure 1).

We can then write the Lagrange multiplier:

$$\lambda = \frac{\int_{i\in K} di}{y + \int_0^J p_i\gamma_i di} \tag{6}$$

Using equations 5 and 6, we can determine an implicit equation in J for the extensive demand function. Similarly, on the basis of equations 3 and 6 and simplifying by equation 7, we obtain an intensive demand function (q_i):

$$\begin{aligned} J &= \frac{y + \int_0^J p_i\gamma_i di}{p_J\gamma_J} && (7) \\ p_iq_i &= p_J\gamma_J - p_i\gamma_i \quad \text{if} \quad p_i\gamma_i < p_J\gamma_J && (8) \\ p_iq_i &= 0 \quad \text{if} \quad p_i\gamma_i > p_J\gamma_J \end{aligned}$$

In addition, using equations 3 and 6 and deriving the induced demand function, we obtain an expression for demand income elasticity. In the same way, we can also deduce two useful expressions for own-price and cross-price elasticities.

$$\begin{aligned} \eta_y^i &= \frac{1}{J}.\frac{y}{p_iq_i} && (9) \\ \eta_p^i &= -1 - \frac{\gamma_i}{q_i}.\left(1 - \frac{1}{J}\right) < -1 \\ \eta_p^i &= -1 - \left(\frac{p_J\gamma_J}{p_iq_i} - 1\right).\left(1 - \frac{1}{J}\right) && (10) \\ \eta_{p_k}^i &= \frac{1}{J}.\frac{p_k\gamma_k}{p_iq_i} > 0 && (11) \end{aligned}$$

We now present changes in consumption behavior according to changes in income (y) and in the ranking of the good in the continuum (i). The main properties of our demand functions are summarized by seven theorems reported in table 1. We briefly present the proofs for these theorems in annex 1.

From these theorems, we can highlight that in our model, the heterogeneity of agents' consumption baskets depends partly on their levels of income, and partly on the characteristics of the goods supplied (if it has higher or lower priority). Changes in consumption

[7] On this point, we should explain why we have chosen to maintain sectorial differences for the γ term. By applying the same value for γ_i, the entry criterion would only depend on relative prices (i.e. on unit values). We wanted to avoid this effect, as the model refers to an inter-sectorial approach. Effectively, in this case, units are likely to vary from one sector to another, and there is no justification for choosing unit values as an entry criterion. The weighting with γ_i allows us to get around this problem. The term γ_i then refers to a pure preference effect, which determines the willingness of the agent to consume a good. In addition, multiplication by p_i accounts for the possibility of consuming it according to the price structure (given the level of income). Because of this influence of price on the composition of consumption, preferences reflect hierachic purchases rather than hierachic desires.

behavior resulting from an increase in income take the form of access to what had initially been non-priority (T1), together with a non uniform variation in the spending on each good (T2, T3). Based on equation 7, figure 3 represents graphically the evolution of intensive and extensive consumption with income[8]. Our extensive demand function presents also specific characteristics: income elasticities are in accordance with the stylized facts we aim to account for, i.e. non uniform evolution of expenditures between sectors and agents. More specifically, the last good to enter the consumption basket has the highest income elasticity of demand (T5). Thus, every good, except the first one always consumed, can be considered as a "luxury good" the first time it enters the agent's consumption basket. With the entry of new goods, the previously-consumed goods tend to behave more and more like necessity goods, as and when their weight in the consumption increases. Furthermore, the agents' perceptions of the degree of priority of each good vary with their respective income levels. This is formally reflected by the fact that the lower the agent's relative level of income, the more sensitive demand for good i will be to variation in income (T4).

Our extensive demand function presents two further main characteristics concerning price elasticities. As with income elasticities, a good's level of sensitivity to price variations depends on its novelty. Agents tend to adapt their consumption in favour of the lower priority goods when there is an uniform fall in prices, and to the detriment of these same goods when there is a rise in prices (T6). These increasing values for price elasticities along the continuum seem reasonable: the higher the good's priority, the less it is substitutable. Finally, according to the cross-price elasticity expression, the substitution effect is higher than the income effect. In addition, the lower the good's priority, the more its demand is sensitive to the price variation of another good (T7).

The ranking of goods in the continuum according to their degree of priority and the previous properties of our model both hold under the assumption of an increasing monotonic function for the entry criterion $p_i\gamma_i$. The supply side of the model is defined such as to verify this relation. In this way, we establish a link between preferences and technology : agents' perceptions of the priority of each good appear to be dependant on the technological dimension.

2.2 The link between technology and preferences

The supply side of the closed economy is modeled in a very simple way. We assume a single-input (labor) production function with constant returns. There are no profits in the economy. Each agent's income is then given by the wage level. The national income corresponds to total wages uniformly distributed over the population. We denote a_i, the quantity of labor required to produce one unit of good i. We assume perfect competition, so that the prices of goods are given by wage (w) and the inverse of labor productivity (a_i):

$$p_i = a_i w \tag{12}$$

Without losing generality, we can assume that goods are ranked in the continuum in increasing order, so that the parameters of the model satisfy the following relation:

$$a_i\gamma_i \text{ increasing function of } i \tag{13}$$

Thus, using equation 12, $p_i\gamma_i$ is also an increasing function of i. All the properties of the consumption function described above are therefore maintained.

We should highlight the fact that the way technical progress will influence equilibrium consumption derives straightforwardly from our way of modeling the hierarchic consumption behavior. By applying equation 12 and proposition 13 to our HLES, we effectively

[8] It should be noted that the choice of a convex curve for $p_i\gamma_i$ is arbitrary. Theorems are valid for the general case.

establish a link between technical progress and preferences. Technical progress, by modifying the values of $a_i\gamma_i$ and the entry criterion $p_i\gamma_i$, will impact on the range of consumed goods, the share of spending devoted to each good, and their respective income and price elasticities of demand (equations 7-11). Actually, our goods ranking, which combines a technological criterion[9] (a_i) and a criterion based on the utility function (γ_i), expresses the fact that a technological shock (via a_i) or a preference shock (via γ_i) have the same impact on the composition and evolution of consumption. This equivalence is supported by two arguments. Firstly, technical progress and increases in real income are two sides of the same phenomenon. Accordingly, every increase in income (or technical advance) involves changes in the distribution of expenditures between goods, because of Engel's law [Pasinetti, 1983, p. 69]. Secondly, the equivalence between a technological shock and a preference shock is based on the following intuition. When a technical advance results in the creation of new goods, it also entails the creation of new needs and modifies agents' perceptions of pre-existing goods (in terms of needs)[10]. We could say that the boundary line between "essential" needs and "psychological" needs is modified[11].

The mechanism of an endogenous change in consumption behavior with technical progress is at the heart of our theoretical results. Technical progress, by changing real income, is likely to modify the range of consumed goods (theorem 1). But at the same time, in our model, technical progress also induces a change in the threshold level of the quantities consumed necessary to extend the range of consumption (equation 8). Variation in utility therefore depends on the evolution of these two components, quantities and range of goods consumed, which may not change in the same way. This kind of endogenous change in consumption is graphically represented in figure 4.

At this point, it may be useful to compare our approach to preferences with that of DFS and Matsuyama. The differences in demand behavior in the three models are presented in table 2. Our model is clearly distinct from DFS, where there is an homothetic demand function of the Cobb Douglas type. This exogenously constraints the range of consumed goods and the distribution of spending between goods. Nevertheless, the two models can considered close insofar as they both allow for endogenous determination of quantities consumed. Our approach also appears to be closely related to that of Matsuyama (2000), based on an utility function of hierarchic desires[12]. In this case, agents consume one unit of each good according to an order of priority that is fixed *a priori*. As in our model, the range of consumed goods, and subsequently the share of spending devoted to each good, are endogenous. Nevertheless, his approach differs from our own in that he only allows for an extensive change in consumption behavior: income and price elasticities are null for each good.

We now embed our demand function in a Ricardian trade framework. The differences described above concerning the utility function of the three approaches will be reflected by differences in the mechanisms involved in the trade balance equilibrium.

[9] Fixed capital is not explicitly introduced into the model. However, we consider that the input of labor represents effective labor, i.e. a composite (comprising labor, capital, and human capital).

[10] Like Young (1991), we have not modeled the innovation of goods in our model: an infinite number of goods are theoretically available for consumption, but some of them are too expensive to be produced. Technical progress makes it possible to produce these goods by reducing production costs.

[11] We should stress the fact that in our model, no goods, except the first one, can be considered essential in physiological terms. It is only the perception of the agents (according to income and price structure) which makes each of them relatively essential.

[12] The function of hierarchic desires is often used to link consumption expenditures with income distribution (Murphy, Shleifer and Vishny (1989) or Zweimüller (2000)). This is also the case in Matsuyama (2000). To be able to compare his approach with our own, we only consider the characteristics of his model under the assumption of a representative agent (i.e. with an uniform income distribution).

3 The nonhomothetic Ricardian model

3.1 International specializations and consumption

We consider two economies with different levels of development. The more developed economy is the foreign one, and it is denoted by an asterisk. The foreign wage is the numeraire ($w^* = 1$) and the domestic wage (w) then reflects the relative wage. The general structure of the open-economy model is established following the standard hypotheses of DFS (1977). Firstly, it is assumed that competition in the two countries is perfect. There is therefore an international price p_i^m, which corresponds to the trade equilibrium at $p_i^m = Min\{p_i, p_i^*\}$. Secondly, the foreign economy is more developed in the sense that it is more productive, in absolute terms, in every sector $a_i^* < a_i$. Thirdly, the productivity advantage of the developed country increases along its goods continuum, i.e. $\frac{a_i}{a_i^*}$ is an increasing function of i.

We can demonstrate that under the hypotheses of a technological gap increasing along the whole goods continuum of the developed country, and of an identical utility function in the two countries, the ranking of goods (in terms of priority level) in the developed country entails an identical ranking in the developing country:

$$\text{if} \quad \frac{\partial a_i^* \gamma_i}{\partial i} > 0 \quad \text{and} \quad \frac{\partial \frac{a_i}{a_i^*}}{\partial i} > 0 \quad \text{then} \quad \frac{\partial a_i \gamma_i}{\partial i} > 0$$

In other words, the ranking of goods established for the developed economy is an "international" ranking. We can therefore identify the comparative advantage of each country in terms of the nature of goods respectively traded. Like Matsuyama (2000), we thus establish a relation between the characteristics of a country (its level of development) and the characteristics of the goods in which it specializes (relative degree of priority). Formally, the specialization equation which determines the segment of South's specializations $\left[0, \bar{i}\right]$ and, which holds also in the models of Matsuyama and DFS, can be written:

$$w = \frac{a_{\bar{i}}^*}{a_{\bar{i}}} \tag{14}$$

Given the hypothesis of the technology gap, trade can only take place if, for all $i \in \left[0, \bar{i}\right]$, $p_i^* > p_i$. For the developing country to be competitive in this goods segment, the minimum condition is that $w < 1$. The developing economy will then be specialized in the beginning of the goods continuum, i.e. in goods with higher priorities. These goods present a relatively lower technological gap and lower income elasticity of demand (see theorem 5)[13].

Concerning consumption, the marginal goods consumed in the two countries can be determined using equation 7:

$$J = \frac{w + \int_0^J p_i^m \gamma_i di}{\gamma_J p_J^m} \tag{15}$$

$$J^* = \frac{1 + \int_0^{J^*} p_i^m \gamma_i di}{\gamma_{J^*} p_{J^*}^m} \tag{16}$$

[13] It can be noted that this characteristic allows us to interpret the degree of priority of each good in terms of the degree of sophistication. Empirical studies provide evidence in favor of a positive relation between income level and the share of expenditures devoted to the importated technological, differentiated, and higher quality goods (see respectively, Meliciani (2002), Francois and Kaplan (1996), Hallak (2003)).

Because agents in the two countries face identical prices for goods, the difference in relative wage expresses a difference in real income. We can therefore apply the main theorems defined in the closed economy.

Corollary 1: *The developed country consumes more products and these products are lower priority.*

Effectively, using theorem 1 and relation 13, and given that the relative wage is less than one, we can write $J^* > J$ and therefore $p^m_{J^*}\gamma_{J^*} > p^m_J\gamma_J$.

According to this corollary and the previous assumptions about the technological dimension, the structure of consumption and specializations induced by the model can be summarized as in table 3[14].

3.2 Trade equilibrium

We make the standard assumption of a balance of trade equilibrium constraint where X and M represent respectively the exports of the developing country (imports of the developed country) and the imports of the developing country (exports of the developed country). On the basis of equations 8 and 12, we can write:

$$\begin{aligned} X &= M \\ \int_0^{\bar{i}} p^m_i q^*_i L^* di &= \int_{\bar{i}}^{J} p^m_i q_i L di \\ \bar{i} L^* a^*_{J^*}\gamma_{J^*} - wL^* \int_0^{\bar{i}} a_i\gamma_i di &= (J-\bar{i}) L a^*_J \gamma_J - L\int_{\bar{i}}^{J} a^*_i \gamma_i di \end{aligned}$$

Re-writing the last term (of the right hand side) using equation 15, we obtain, after simplification:

$$w = \frac{\bar{i}\left[L^* a^*_{J^*}\gamma_{J^*} + La^*_J\gamma_J\right]}{L + (L+L^*)\int_0^{\bar{i}} a_i\gamma_i di} \tag{17}$$

We have finally obtained a system of four equations (14-17) with four unknowns, which must satisfy the following conditions: $0 < \bar{i} < J < J^*$.

Concerning the last expression, it should be noted that it is no easy task analyzing this trade balance equation, as it includes many endogenous variables $(w, \bar{i}, J, J^*)$. Nevertheless, we can make some comments on the partial effects of the entry criterion terms $(a_i\gamma_i)$, that which will highlight the potential impact of technical progress on trade equilibrium. Firstly, in the numerator, $a^*_{J^*}\gamma_{J^*}$ and $a^*_J\gamma_J$, refer to the threshold value of the marginal consumed good in each country. According to equation 8, the spending on each good is positively related to these terms. At the same time, according to equations 15-16, the range of consumed goods in each country decreases with these terms. Thus, a decrease in these boundary terms (through North's technical progress) is likely to hurt South's relative wage by lowering the spending devoted on its goods. On the contrary, a decrease in the integral value in the denominator reflects an higher gap between the threshold value of the marginal consumed good $(a^*_{J^*}\gamma_{J^*}, a^*_J\gamma_J)$, and the threshold value of South's products $(a_i\gamma_i)$. This is likely to result in higher spending on each of South's products (equation 8), and then in an increase in the relative wage.

[14] Concerning the structure of trade, we can also note that North is specialized in a wider range of products. This characteristic comes directly from theorem 2 and 3 applied to the trade balance condition. It is in accordance with some recent empirical studies which highlight the fact that high income per capita countries export a wider range of products. See, in particular, Hummels and Klenow's paper (2005).

At this point, it is again worthy to compare the mechanisms involved in the adjustment of our trade balance equilibrium with those of Matsuyama and DFS. Looking at table 4, the differences between the three models are straightforward[15]. In DFS, the trade balance equilibrium does not depend on technical coefficients. This is due to the Cobb Douglas function, according to which the share of spending on each good is fixed by the parameter β_i. Every technical progress will have the effect of increasing the quantity demanded in proportion to the fall in price. In Matsuyama, the trade balance equilibrium no longer depends on the relative wage, but only on South's technical coefficients. This comes from the use of the function of hierarchic desires, where the quantities demanded are constrained to one. As North is specialized in lower priority goods, this entails that agents in the two countries only use their increase in real income to demand goods produced by North. It follows that demand for South's products is insensitive to price variation, and hence to wage variation. The only way to modify external demand for South is through its own technical progress, enabling it to extend its range of specializations. In other words, under a function of hierarchic desires and an uniform distribution of income within each country, differences in income elasticities in trade are "apparent". They are determined by the relative ability of each country to extend its range of traded goods[16]. North is in this case more favored, as every change in real income is used to demand its (lower priority) goods. Contrary to our approach, non unitary income elasticities in trade, in Matsuyama's model, are then not founded on the characteristics of the good itself, but on the assumption of an unconditional preference for North's goods. This distinction between the two models is at the heart of the differences in the trade balance equations.

Finally, one should insist on the fact that the simultaneous inclusion of extensive and intensive consumption differentiates strongly our model from these two approaches. Relative wage is still a determinant of our trade balance equilibrium, because of demand sensitivity to price variation. Moreover, the inclusion of both countries' technical coefficients in the trade balance equilibrium accounts for our endogenous elasticities. This reflects the impact of every technical progress on the share of spending devoted to each country.

We now turn to the implication of our approach for the analysis of the international diffusion of welfare gains through trade. According to the above comparisons, we have decided to devote a particular intention to the impact (on South's welfare) of a North's technical progress biased toward the less priority goods in the case of a large technological gap. Effectively, we expect specific results in that configuration at least for two reasons. Firstly, in our model, the *size of the technological gap* between the two countries will have a direct influence on the value of trade elasticities, as the latter depends on the level of income of the agents. Secondly, the *nature of the technical progress* is also expected to be crucial because a good's demand elasticity depends directly on its technical coefficient of production. The analytical study of this case is conduced under specific functional forms and completed by simulations.

4 The effect of a North's technical progress on South's welfare

4.1 The critical influence of the size of the technological gap and the nature of the technical progress

The choice of a functional form for a Ricardian inter-sectorial trade framework is not a trivial issue. In the standard interpretation of Dornbush, Fisher and Samuelson (1977), the

[15] The main equations of the models of Matsuyama and DFS are presented in annex 3a.

[16] Following Krugman (1989), these aggregate elasticities of external demand can be viewed as "apparent" because they do not refer to the evolution of demand for a given consumption basket, but to the changes in the composition of traded goods.

continuum of goods involves different sectors, the characteristics of which are not defined. The only precision made concerning sector characteristics is a ranking of goods along the continuum as a function of the relative productivity gap between the two countries. Krugman (1990) proposed an extension of the theoretical interpretation that can be made of the DFS model: he argued that the ranking of goods according to the productivity gap corresponds to the ranking of goods according to their technological content. However, choosing a functional form which can take into account the link between technological content and sector productivity is no easy task, insofar as that if the continuum involves different sectors, the respective units of each good are likely to be different. In other words, it is impossible directly to establish the relation between a_i and i. In our model, we get around this problem by not directly fixing the functional form of a_i, but only that of $a_i\gamma_i$, which we have defined as an increasing function of i (relation 13).In other words, we do not fix the primitives of the expressions for $a_i\gamma_i$ and $a_i^*\gamma_i$. We propose to use the following functional forms:

$$a_i\gamma_i = \alpha + \beta.i \qquad (18)$$
$$a_i^*\gamma_i = \alpha^* + \beta^*.i \qquad (19)$$

In addition, our hypotheses about the technological gap will enable us to constrain parameter values. For an absolute productivity advantage increasing along the continuum, the sufficient condition to allow South to be competitive in the first part of the continuum is:

$$\frac{\beta}{\beta^*} > \frac{\alpha}{\alpha^*} > 1$$

In this case, the ratio of production costs ($\frac{a_i^*}{a_i}$) decreases along the whole continuum with an asymptote tending to $\frac{\beta^*}{\beta}$.

We rewrite the general equilibrium model by applying the equations 18-19 to the system 14-17 and verify that an analytical solution exists[17]. We then analyze the effect on South's utility of a biased technical progress (toward the less priority goods) in North when the technological gap is large This configuration is formally equivalent to a decrease in the value of β^* when this parameter is very small (relatively to β). To simplify the analysis, we study trade equilibrium around $\beta^* = 0$.

To do this, we denote $\varepsilon = \sqrt{\beta^*}$ and proceed to marginal developments of our equilibrium equations around $\varepsilon = 0$. Results are the following ones :

$$\bar{i} = \widetilde{i}(1 - a.\varepsilon) \qquad (20)$$
$$w = \widetilde{w}.(1 + b.\varepsilon) \qquad (21)$$
$$J = \frac{\widetilde{j}}{\varepsilon}.(1 + c.\varepsilon) \qquad (22)$$
$$J^* = \frac{\widetilde{j}^*}{\varepsilon}.(1 - d.\varepsilon) \qquad (23)$$

$\widetilde{i}, \widetilde{w}, \widetilde{j}, \widetilde{j}^*, a, b, c$ and d are all positive parameters obtained after marginal development of equilibrium equations (their expressions are reported in annex 2c).

By including (20-23) in equation 2 we obtain an expression for South's utility. With the same methodology (marginal developments), we can demonstrate that South's utility can be represented, around $\varepsilon = 0$, by an equation of the following form :

$$U = A + B.\varepsilon \qquad (24)$$

where :

[17] See annexes 2a and 2b.

$$
\begin{aligned}
A &= \frac{\alpha}{\beta}.\log\frac{\alpha}{(\alpha+\beta\widetilde{i})}+\widetilde{i}+\frac{l}{(1+l)}.\frac{1}{(\alpha+\beta\widetilde{i})} \\
B &= \frac{1}{(\alpha+\beta\widetilde{i}).\alpha^{*}.(1+l)}.\left(\frac{\widetilde{j}^{*}-\widetilde{j}}{1+l}+\frac{\widetilde{j}}{3}\right)
\end{aligned}
$$

In figure 5, we have represented for specific value of parameters the equation 24 as well as the general utility equation (2). This confirms that equation (24) is good approximation of our utility function around $\varepsilon = 0$.

The two terms A and B are positive ($\widetilde{j}^{*} > \widetilde{j}$). This implies that the utility of the developing country is an increasing function of the term $\varepsilon = \sqrt{\beta^{*}}$. Hence, the result of this analysis is that when the size of the technological gap increases (when ε tends toward 0), the utility of the developing country decreases.

As a first comment, it is worthy to compare this result with those obtained by Matsuyama (2000) and DFS (1977) under the same configuration. In their model, the following Ricardian property is always verified: as trade gains are derived from the existence of differences between countries, when these differences grow larger, i.e. when technological shocks augment the existing terms of comparative advantage, the transaction gains increase[18]. In other words, a biased technical progress in North improves South's welfare. Actually, the difference between these both approaches and our own model can be explained on the basis of equation 20. Effectively, looking at equations 20-23, one can note that around $\varepsilon = 0$, if the impact of β^{*} on w and on J, J^{*} is standard (respectively positive and negative), this is no more the case for $\bar{i}$. The equation 20 implies that, the range of South's specialization is negatively related with the cost of production of North. Before discussing in more details the mechanisms at the heart of this "non standard" relationship, we aim to complete our analysis by looking for what happens outside the previous configuration. We proceed by simulations and relax first, the assumption of a very large technological gap, and second that of a biased technical progress.

For our simulations, we consider three different equilibriums, in order to measure the extent to which the qualitative results produced by the simulations are sensitive to the size of technological gap (simul 1, 2 and 3). These configurations differ in the value given to the parameter β^{*}. Table 5 presents the values taken by the different variables for these three initial equilibriums. We adopt a comparative static procedure and then study South's utility in the case of a foreign technical progress biased towards the most sophisticated goods (FNUTS, variation of β^{*}), and in the case of a foreign uniform technical shock (FUTS, identical variation in α^{*}, β^{*}). These technological shocks are formally reflected by a reduction in the parameters concerned by one half.

Simulations results confirm that the size and the nature of the shock are two conditions for the appearance of a configuration where South's utility decreases as a result of a technical progress in North. Effectively, by comparing tables 5 and 6, we can see that even in the case of a very large technological gap, the South always benefits from North's technical progress when the latter is uniform along the continuum. Also, in the case of a technical progress in the developed country growing over $\left[\bar{i}, J^{*}\right]$, the utility of the developing country fall only under a large technological gap. This configuration appears for simul 2 and 3 (compare tables 5 and 7). The existence of a threshold effect after which the developing country may present a decrease of its utility appears clearly in figure 6.

We now turn to the description of the main mechanisms embedded in our model which are at the heart of the process of immeserizing in South after a technical progress in North.

[18] Result of Matsuyama and DFS can directly be derived from their conditions of specialization (equation 14) and their trade balance equilibrium (table 3). We have also reported graphically the impact of technical shock in figures of annex 3b.

4.2 Mechanisms at the heart of an immeserizing effect of external growth

It is worthy to begin by an explanation of our results in the case of an uniform technical progress in the North. According to our simulations, in this configuration, the classical effect of welfare gain diffusion holds, whatever the size of the technological gap. This can be explained in the following way. Given uniform technical progress in the developed country, the productivity gap between the two countries increases uniformly. According to the specialization equation (14), this entails a fall in the relative wage. But this fall stimulates demand for goods produced by the developing country, limiting the deterioration in its terms of trade. At the new equilibrium, the fall in the relative wage does not completely offset the initial productivity gains (figures 7). Welfare increases in both countries. It can be noted that this result tallies with that of DFS, but differs from that of Matsuyama, where the developing country is totally impervious to the developed country's technical progress. The comparison between our result and that of Matsuyama is particularly interesting. This highlights the fact that, despite unfavorable elasticities in trade, trade remains a good way to benefit from external technical progress, once we have allowed South to benefit from price competitiveness.

Concerning now the differences implied by a biased technical progress, the decrease in South's utility can be explained in the following way. Our modeling of preferences entails that the development gap (the difference in per capita income) determines the differences in income and price elasticities between the two countries. We can make two observations about this effect. Firstly, in a similar fashion to the uniform shock described above, the developing country is relatively penalized by its elasticities in trade. Secondly, according to theorem 6, we have seen that in our model, price elasticities increase along the continuum. Under a shock biased towards the lower priority goods, the goods which benefit from the highest price elasticities are then those which benefit from the largest falls in price. The price elasticity and income elasticity effects contribute jointly to a pronounced change in the distribution of expenditures in favor of the goods produced by the developed country. The balance of trade equilibrium condition therefore requires a fall in the relative wage, such that the developing country may see a fall in its aggregate real income. As can be seen in figures 8, the real costs of some consumed goods for the developing country have risen. More precisely, the adverse impact of North's biased technical progress appears to be conduced by two effects. Firstly, some goods which were previously produced by North are now produced by South, and consumed with higher real prices. Secondly, the fall in the relative wage reduces South's purchasing power for some of North's goods. Actually, South does not benefit from an reinforcement of the existing terms of comparative advantage, because the strong change in the spending patterns forces South to extend its specializations, despite the reduction in its comparative advantages (see tables 5 and 7; equation 22). The wider the development gap between the two countries, the more this extension, necessary to return to the trade balance equilibrium, will penalize the developing country in terms of changes in purchasing power.

One should also insist on another characteristic of our model : technical progress hurts the South despite the fact that South's agents ultimately consume more sophisticated goods at lower prices (J increases). This result differs strongly from Matsuyama's approach, where J can be viewed as a direct measure of utility. In our model, changes in agents' utilities have to be apprehended through changes in the quantities and range of goods consumed. What happens here is that the increase in utility induced by a wider range of consumed goods is offset by lower quantities of each good consumed. With this kind of shock, there is actually a stronger change in the slope of the curve $\frac{p_i \gamma_i}{w}$, compared with the changes entailed by an uniform shock (see figures 7 and 8). This change reflects the strong modification in the consumption behavior of the agents, especially in their perception of the relative priority of each good. With a flatter curve, agents view goods that had been previously considered as lower priority goods, as now having relatively more priority. They extend the range of consumed goods accordingly. At the same time, the spending on relatively higher priority goods decreases (equation 8). This means that the fall in real income results in a reduction in the level of satisfaction provided by each good. In other words, in our model, changes in price structure indirectly account for changes in what we can call the "standard of consumption": South's agents import a wider range of sophisticated goods (for instance, televisions or cell phones), to the detriment of the quantities of each good consumed (housing, clothes, etc.)[19].

Finally, it is worthy to briefly explain the reasons for which this configuration doesn't appear in the approaches of Matsuyama (2000) and DFS (1977). The difference with our own model comes from the fact that, according to their assumption on preferences, in both other models the range of specialization does not increase as a result of this kind of shock. Notably, as it has been already noted, in Matsuyama's model the trade balance condition is not modified by biased foreign technical progress. Consequently, the range of South's specializations remains constant. The fall in the relative wage is directly determined in equation 14 by the decrease in productivity of the marginal good $\bar{i}$. As the productivity gains are increasing along $\left[\bar{i}, J^*\right]$, this implies that South's consumers benefit from an increase in their purchasing power for all goods after $\bar{i}$. In the DFS model, North's technical progress is accompanied by a reduction in South's specializations. South's consumers benefit from an increase in welfare through the increasing purchasing power gains along the goods continuum.

To conclude, the endogenous change in consumption and specialization in our model can be interpreted from the perspective of the product cycle theory[20]. The biased technical progress in North has two main effects on specializations. On the one hand, it stimulates North's production of sophisticated products, and on the other hand, it induces North to give up the production of less sophisticated goods (which are being produced by South). South's access to more sophisticated goods increases, because of the standardization of North's production. At the same time, South becomes more competitive in some products, because of the increase in its relative poverty.

5 CONCLUSION

This paper proposes a new framework for analyzing the changes in South's welfare induced by domestic and external technical progress. Our main contribution concerns the way we model consumption behavior. We have considered a hierarchic linear expenditure system. Goods can then be distinguished according to their order of entry into the consumption basket. This determines their relative degree of priority, and their respective income and price elasticity values. Moreover, we have established a link between these goods' characteristics and the technological dimension. In this way, agents' perceptions of the relative priority of goods are dependent on the type of technical progress. We have compared our results with those of Matsuyama (2000) and DFS (1977). As in our model, Matsuyama establishes a connection between technical progress, its income effect, and the change in spending patterns due to nonhomothetic preferences. This also highlights the differential impact of technical progress on welfare, depending on specialization patterns.

[19] Some vertically differentiated models also produce a configuration where North's technical progress may be immiserizing for South. Nevertheless, the mechanisms involved are different. For instance, Stockey (1991) assumes a model where vertically differentiated goods are imperfect substitutes. North's agents then consume a wider range of quality goods. She shows that external progress is immiserizing for South, if it is biased toward non-traded goods (i.e. high quality goods consumed only by North). In our model, this configuration appears even if the technical progress benefits traded goods. Flam and Helpam (1987), consider a model with heterogenous population in each country. South (North) produces a low (high) quality good consumed by poor agents in the two countries. In this case, the global distribution of spending is dependant on the distribution of income. If the distribution of income shifts toward the agents who consume high quality goods, this entails a change of spending patterns in favor of North. The subsequent improvement in its terms of trade may be immiserizing for South. In our model, South can lose even under our assumption of a representative agent.

[20] See for instance, the three stages of product developement in Vernon (1966, 1979), the models of Grossman and Helpman (1991), and the empirical studies of Schoot (2004), Feenstra-Rose (2000), and Hummels-Klenow (2005).

The main contribution of our model, compared with that of Matsuyama, involves our endogenous demand for quantities (and the subsequent endogenous income and price elasticities). By giving up the constraint of the (0,1) consumption framework, we bring new insights concerning the mechanisms at the heart of evolutions in welfare. More specifically, technical progress, by changing price structure, also modifies the relative priority of goods. This effect is notably reflected by a change in the level of spending (devoted to each good), necessary to extend the range of consumption. We have called this effect the endogenous change in the "standard of consumption". We have shown that our model can present results that are *a priori* paradoxical in a Ricardian context: when there is technical progress in North biased towards the most technological goods, the greater the difference between the two countries, the less the developing country gains from trade. Technical progress in North may then be immeserizing for South. This implies that there exists a level of development gap which maximize the welfare gains from trade. This result can be viewed as the reflection of an adverse impact of a product cycle mechanism, which is driven by consumption behavior.

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TABLES

Table 1: Main characteristics of our demand function

	Income variation (y)	Rank of good variation (i)
Range of products	$\frac{\partial J}{\partial y} > 0$ T1	
Share of spending	$\frac{\partial p_i q_i}{\partial y} > 0$ T2	$\frac{\partial p_i q_i}{\partial i} < 0$ T3
Income elasticities	$\frac{\partial \eta_y^i}{\partial y} < 0$ T4	$\frac{\partial \eta_y^i}{\partial i} > 0$ T5
Price elasticities		$\frac{\partial \lvert \eta_p^i \rvert}{\partial i} > 0$ T6
Cross-price elasticities		$\frac{\partial \eta_{pk}^i}{\partial i} > 0$ T7

Table 2: Main properties of the demand function of the three models

	DFS (1977) Cobb Douglas	Mats (2000) Hierarchic desires	Current model Hierarchic purchases
$\eta_y^i =$	1	0	$\eta_y^i(y,i)$
$\eta_p^i =$	1	0	$\eta_p^i(y,i)$
$J =$	$J^* = cst$	$J(y,p)$	$J(y,p)$
$q_i =$	$q_i(y,p)$	1	$q_i(y,p)$
$\frac{p_i q_i}{y} =$	$\beta_i = cst$	$\frac{p_i q_i}{y}(J,p)$	$\frac{p_i q_i}{y}(J,q_i,p)$

Table 3: Structure of consumption and specializations in the two countries

Specializations	Price	Country of production	Country of consumption
$[0,\bar{i}]$	wa_i	Developing country	Both countries
$[\bar{i},J]$	a_i^*	Developed country	Both countries
$[J,J^*]$	a_i^*	Developed country	Developed country

Table 4: Trade balance equilibrium conditions

DFS (1977)	Matsuyama (2000)
$wL\int_{\bar{i}}^{1}\beta_i = L^*\int_0^{\bar{i}}\beta_i$	$L = (L+L^*)\int_0^{\bar{i}} a_i di$

Table 5: Initial equilibriums according the technological gap

Initial equilibriums (for each simul: $\alpha/\alpha^* = 2$)	U	U^*	w	$\bar{i}$	J	J^*
Simul 1 ($\beta/\beta^* = 6,7$)	6,7	15,0	0,31	2,6	14,2	25,7
Simul 2 ($\beta/\beta^* = 66,7$)	7,2	27,4	0,19	4,0	32,7	80,5
Simul 3 ($\beta/\beta^* = 1010,1$)	6.8	38.9	0.14	5.8	97.0	310.2

[$\alpha = 0.045$; $\alpha^* = 0.0225$; $\beta = 0.020$; $L = L^* = 1$]

Table 6: Equilibriums after foreign uniform technical shock (FUTS)

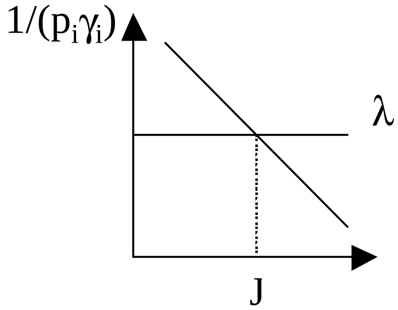
Equilibriums after FUTS (for each simul: $\alpha/\alpha^* = 4$)	U	U^*	w	$\bar{i}$	J	J^*
Simul 1 ($\beta/\beta^* = 13,3$)	6.9	23.6	0.16	2.1	14.6	36.4
Simul 2 ($\beta/\beta^* = 133,3$)	7.5	46.5	0.10	3.5	34.6	114.8
Simul 3 ($\beta/\beta^* = 2020,2$)	7.0	72,0	0,07	5,5	101,7	444,5

Table 7: Equilibriums after foreign non uniform technical shock (FNUTS)

Equilibriums after FNUTS (for each simul: $\alpha/\alpha^* = 2$)	U	U^*	w	$\bar{i}$	J	J^*
Simul 1 ($\beta/\beta^* = 13,3$)	7.0	18.6	0.26	2.9	18.0	36.2
Simul 2 ($\beta/\beta^* = 133,3$)	7.1	31.0	0.17	4.5	42.7	113.5
Simul 3 ($\beta/\beta^* = 2020,2$)	6,77	40,6	0,13	6,1	130,6	438,0

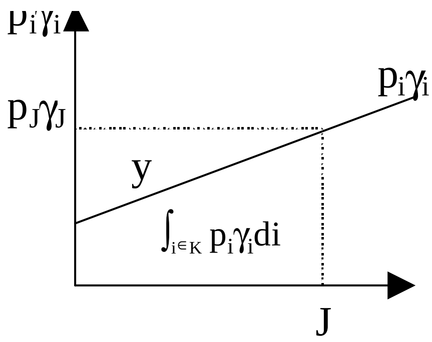
FIGURES

Figure 1: Range of consumed goods according to the entry criterion ($\frac{1}{p_i \gamma_i}$)



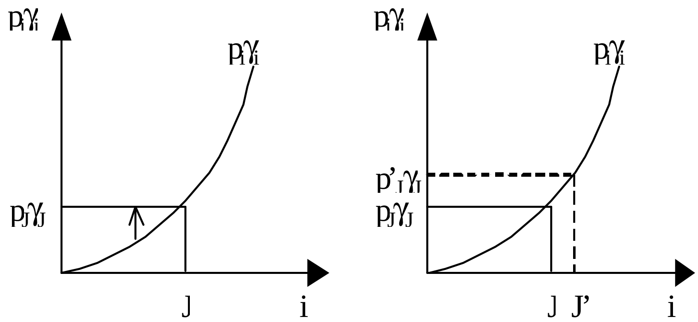
$(0, J)$ represent the segment of consumed goods under the assumption of a decreasing monotonic function for $\frac{1}{p_i \gamma_i}$

Figure 2: Proof of theorem 1: *The number of consumed goods J is an increasing function of y*



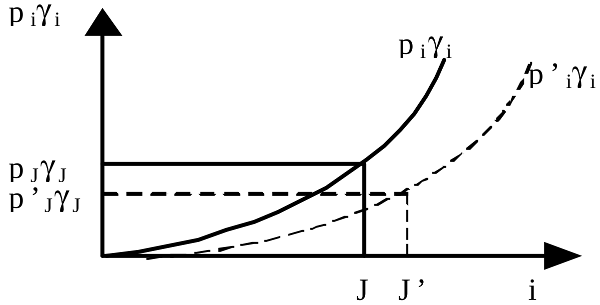
As y increases, the area of the triangle above the curve $p_i \gamma_i$ increases, which is expressed by an increase of consumed goods.

Figure 3: The change of the quantities and varieties consumed with income variation

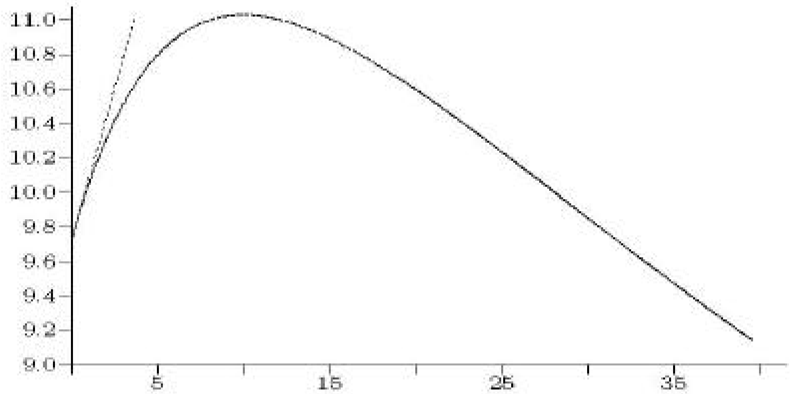


Along the continuum, the distance between $p_J \gamma_J$ and $p_i \gamma_i$ indicates the spending devoted to each good.

Figure 4: The impact of prices change on the composition of consumption

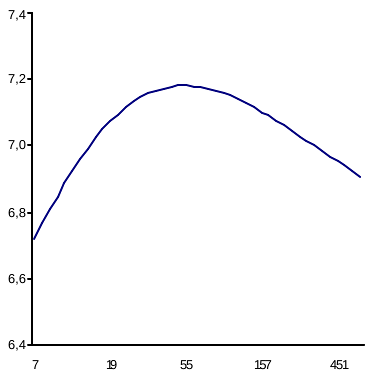


Figures 5: South's utility



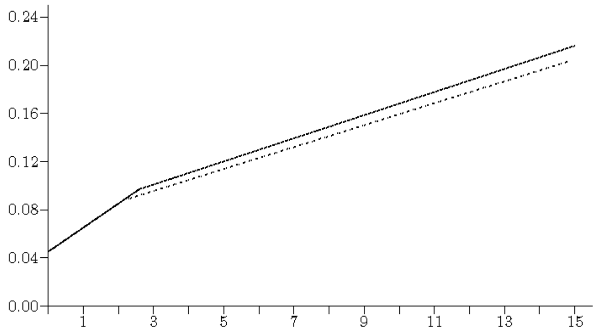
The general function (equation 2) is represented by the normal line and the border case (around $\varepsilon = 0$) by the doted line (equation 24).

Figure 6: Utility in developing country and technological gap

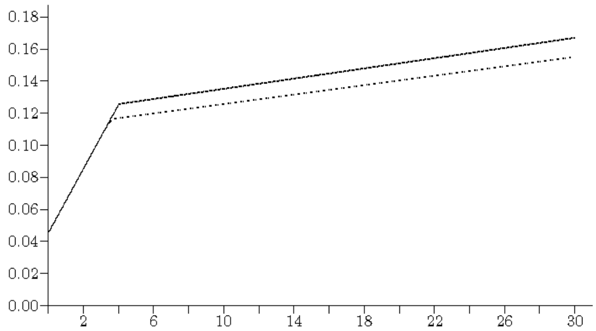


The vertical axis represents the utility of developing country and the horizontal axis reports the value taken by β/β^* (under the asumptions $\alpha/\alpha^* = 2$ and constant)

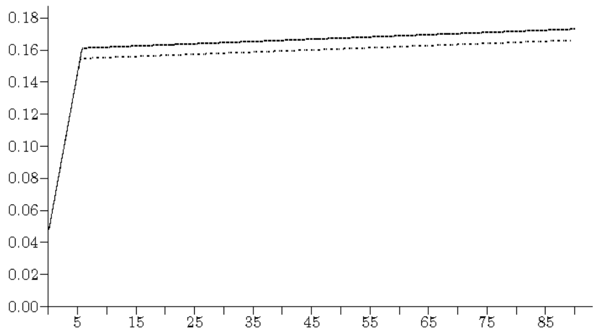
Figures 7: Effect of FUTS on real prices of goods ($\frac{p_i^m}{w}$) consumed by developing country[21]



FUTS / Simul 1



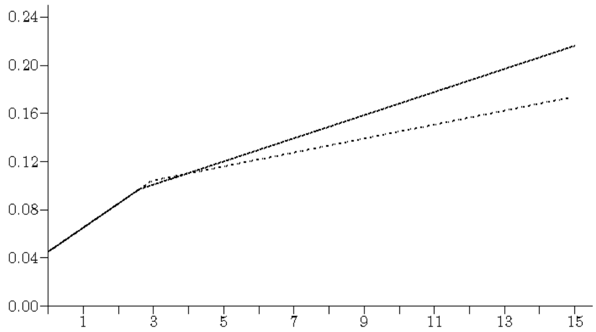
FUTS / Simul 2



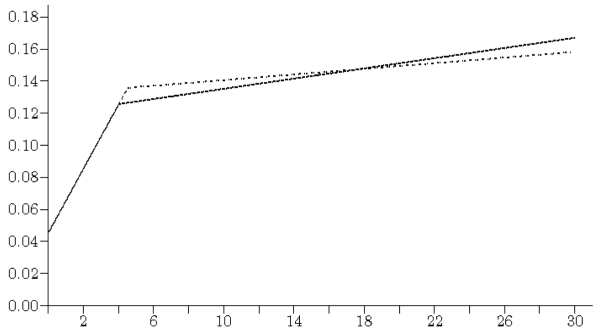
FUTS / Simul 3

[21] The horizontal axis corresponds to the continuum of goods (i). The normal line represents the real price of consumed goods in South before the shock. The doted line represents the real price of these goods after the shock. The break in the lines refers to $\bar{i}$.

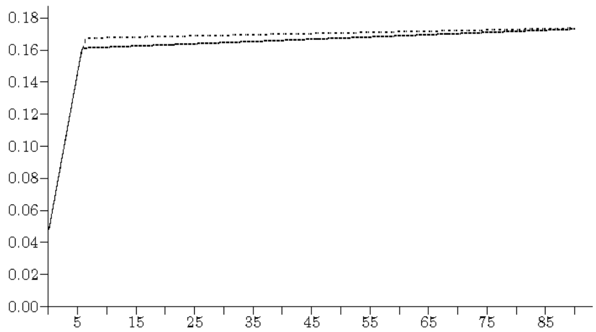
Figures 8: Effect of FNUTS on real prices of goods ($\frac{p_i^m}{w}$) consumed by developing country



FNUTS / Simul 1



FNUTS / Simul 2



FNUTS / Simul 3

ANNEXES

ANNEX 1: Demonstrations of seven theorems on consumption

Theorem 1: The number of goods consumed J is an increasing function of y

This can be demonstrated graphically using equation 7 (figure 2). When y increases, the number of goods consumed J increases, as does its threshold value $p_J\gamma_J$. This is expressed by the increase in the area of the triangle above the curve $p_i\gamma_i$. For given prices of goods, the values of variables $p_J\gamma_J$ and J therefore represent a wealth effect.

Theorem 2: The amount spent on each good p_iq_i is an increasing function of y

This is proved simply by applying theorem 1 in equation 8: as J and therefore $p_J\gamma_J$ are an increasing function of y, it follows that the amount spent on each good p_iq_i is also an increasing function of y.

Theorem 3: The amount spent on each good p_iq_i is a decreasing function of i

This can also be demonstrated on the basis of equation 8, according to which, the amount spent on a good p_iq_i is given by the gap between the threshold effect value of the marginal good and the threshold effect value of the good i. The amount spent on each good is therefore a decreasing function of that good's position in the continuum.

Theorem 4: Income elasticity η_y^i is a decreasing function of y

This is proved by using equation 9 to calculate the expression $p_k\gamma_k$ of good k which satisfies the condition $\eta_y^k = 1$. To prove this theorem, it is then sufficient to verify that when income increases, this good corresponds to a higher index value in the continuum, $\partial p_k\gamma_k/\partial y > 0$. Given that income elasticities increase along the continuum (theorem 5), we denote k ($\eta_y^k = 1$) the good which marks the limit between the "luxury goods" segment ($\eta_y^i > 1$) and the "necessity goods" segment ($\eta_y^j < 1$). Using equations 8 and 9, and after transformation (using equation 7), we can write for this good:

$$p_k\gamma_k = \frac{\int_0^J p_i\gamma_i di}{J} \tag{25}$$

To prove theorem 4, we must verify that when income increases, the good k (which satisfies the condition $\eta_y^k = 1$) corresponds to a higher index value in the continuum of goods($\frac{\partial(p_k\gamma_k)}{\partial y} > 0$). Using partial derivative, one can first deduce from theorem 1 that $\frac{\partial J}{\partial y} > 0$. Second, it is straightforward to show that when we calculate the partial derivative according to J and simplify the result (using equation 7), we obtain:

$$\frac{\partial(p_k\gamma_k)}{\partial J} = \frac{y}{J^2} > 0$$

Theorem 5: *Income elasticity η_y^i is an increasing function of i*

This theorem is directly proved by applying theorem 3 to equation 9.

Theorem 6: The absolute value of price elasticity $\left|\eta_p^i\right|$ is an increasing function of i

This can be directly derived from the application of theorem 3 to equation 10.

Theorem 7: The substitution effect is an increasing function of i (for a given change in the price of a good k)

This is proved directly by applying theorem 3 to equation 11.

ANNEX 2a: New equilibrium conditions

On the basis of equations (14-17) and (18-19), we can write :

$$w = \frac{\alpha^* + \beta^*.\bar{i}}{\alpha + \beta.\bar{i}} \tag{26}$$

$$J^2 = \bar{i}^2 + \frac{w}{\beta^*}(2 - \beta\bar{i}^2) \tag{27}$$

$$J^{*2} = \bar{i}^2 + \frac{(2 - w\beta\bar{i}^2)}{\beta^*} \tag{28}$$

Consequently, it is indeed verified that $J^* > J$ when $w < 1$.

On the basis of these three equations, we can determine a polynomial for $\bar{i}$:

$$\begin{aligned} P(\bar{i}) &= \\ & \alpha^* \left(l - (1+l)\frac{\beta\bar{i}^2}{2} \right) \\ & +\beta^*\bar{i} \left(l + (1+l)\bar{i}(\alpha + \frac{\beta\bar{i}}{2}) - \left(\alpha + \beta\bar{i}\right)(J^* + lJ) \right) \\ &= 0 \end{aligned} \tag{29}$$

With $l = \frac{L}{L^*}$

ANNEX 2b: Verification of the existence of an analytical solution

We shall now demonstrate that the polynomial which determines the expression of $\bar{i}$ presents at least one economically possible solution ($\bar{i} > 0$). To do so, we simply need to verify that the solution can be flanked by two values (one positive and one negative). We therefore flank $P(\bar{i})$ by two extreme values of $\bar{i}$:

- The developing country does not specialize in any good ($\bar{i} = 0$). If we define $\bar{i} = \bar{i}_0 = 0$, then equation 29 can be written:

$$P(\bar{i}_0) = \alpha^* l > 0 \tag{30}$$

- The developed country satisfies all its needs through its domestic production ($\bar{i} = J$). According to the expression of the marginal good consumed by the domestic economy (equation 27), the value of $\bar{i}$ corresponding to this configuration is $\bar{i} = \bar{i}_1 = \sqrt{\frac{2}{\beta}}$. With this value, we obtain:

$$P(\bar{i}_1) = -\alpha^* + \beta^*\bar{i}_1 l + \beta^*\frac{2}{\beta}(1+l)(\alpha + \frac{\beta\bar{i}_1}{2}) - \beta^*\bar{i}_1\left(\alpha + \beta\bar{i}_1\right)(J^* + lJ) \tag{31}$$

To determine the sign of this polynomial, we proceed as follows:

With $\bar{i}_1 = \sqrt{\frac{2}{\beta}}$, we know that $\bar{i} = J$ and $J^* > J = \bar{i}_1$.

Consequently, we can write the following relation for the last term of the polynomial:

$$\begin{aligned} -\beta^*\bar{i}_1\left(\alpha + \beta\bar{i}_1\right)(J^* + lJ) &< -\beta^*\bar{i}_1\left(\alpha + \beta\bar{i}_1\right)(\bar{i}_1 + \bar{i}_1 l) \\ -\beta^*\bar{i}_1\left(\alpha + \beta\bar{i}_1\right)(J^* + lJ) &< -\beta^*\left(\bar{i}_1\right)^2\left(\alpha + \beta\bar{i}_1\right)(1+l) \\ -\beta^*\bar{i}_1\left(\alpha + \beta\bar{i}_1\right)(J^* + lJ) &< -\beta^*\frac{2}{\beta}(1+l)\alpha - 2\beta^*\bar{i}_1(1+l) \end{aligned}$$

By substituting the right-hand expression for the last term of equation 31, we verify:

$$\begin{aligned} P(\bar{i}_1) &< -\alpha^* + \beta^*\bar{i}_1(1+2l) + \beta^*\frac{2}{\beta}(1+l)\alpha - \beta^*\frac{2}{\beta}(1+l)\alpha - 2\beta^*\bar{i}_1(1+l) \\ P(\bar{i}_1) &< -\alpha^* - \beta^*\bar{i}_1 < 0 \end{aligned}$$

So, by flanking $\bar{i}$ in this way, we have demonstrated that our polynomial has at least one root and that this root, with a value between $\bar{i}_0$ and $\bar{i}_1$, is economically possible.

ANNEX 2c : Equilibrium around $\varepsilon = 0$

By proceeding to marginal development of equations 26-29, we have obtained equations 20-23. Their respective parameters can be written in the following way:

$$\widetilde{i} = \left(\frac{2.l}{\beta(1+l)}\right)^{1/2} \quad (32)$$

$$\widetilde{w} = \frac{\alpha^*}{\alpha + \beta.\widetilde{i}} \quad (33)$$

$$\widetilde{j} = \left(\frac{2.\widetilde{w}}{1+l}\right)^{1/2} \quad (34)$$

$$\widetilde{j}^* = \left(2 - 2\frac{\widetilde{w}.l}{1+l}\right)^{1/2} \quad (35)$$

and :

$$a = \frac{\widetilde{j}^* + l.\widetilde{j}}{\widetilde{w}(1+l)\beta}.\frac{1}{\widetilde{i}} \quad (36)$$

$$b = \frac{a.\beta.\widetilde{w}}{\alpha^*}.\widetilde{i} \quad (37)$$

$$c = \frac{a.\beta}{2}.\widetilde{i}.\left(\widetilde{i}.(1+l) + \frac{\widetilde{w}}{\alpha^*}\right) \quad (38)$$

$$d = \frac{a.\widetilde{i}^2.\beta.\widetilde{w}}{\widetilde{j}^{*2}}.\left(1 + \frac{\widetilde{i}\beta.\widetilde{w}}{2.\alpha^*}\right) \quad (39)$$

ANNEX 3a: The equations of the DFS and Matsuyama models

The supply side hypotheses are similar to our own model. Consequently, the condition of specialization, i.e. equation 14, is identical in all three models. The differences in the choice of utility function, on the other hand, induce different equations for the balance of trade equilibrium. To make it easier to compare the three models, the equilibrium equations are presented under the hypothesis $\beta_i = 1$.

DFS (1977)	**Matsuyama** (2000)	
$Max \quad U = \int_0^J \beta_i \ln q_i di$	$Max \quad U = \int_0^\infty \beta_i q_i di$	$(A1)$
$sc \qquad w = \int_0^J p_i q_i di$	$sc \qquad w = \int_0^\infty p_i q_i di$	$(A2)$
$X_i = p_i^m q_i^* .L^* = \frac{1}{J}.L^*$	$X_i = p_i^m q_i^* L^* = w a_i L^*$	$(A3)$
$M_i = p_i^m q_i .L = \frac{w}{J}.L$	$M_i = p_i^m q_i L = a_i^* L$	$(A4)$
$J = J^* = cste$	$wL = Lw \int_0^{\bar{i}} a_i di + L \int_{\bar{i}}^J a_i^* di$	$(A5)$
	$L^* = L^* w \int_0^{\bar{i}} a_i di + L^* \int_{\bar{i}}^{J^*} a_i^* di$	$(A6)$
$X=M \Rightarrow$ $wL \int_0^{\bar{i}} \beta_i = L^* \int_{\bar{i}}^1 \beta_i$	$X=M \Rightarrow$ $L = (L + L^*) \int_0^{\bar{i}} a_i di$	$(A7)$

- (A1) and (A2) represent the agent's maximization programme.

- (A2) and (A3) represent the demand for imports per product addressed to the developing country and the demand for exports that this country addresses to the developed country.

- The balance of trade equilibrium condition (A7) is obtained through (A2) and (A3). For Matsuyama, this expression is simplified with the help of equation (A5).

- (A5) and (A6) represent the exhaustion of the budget constraint in the two countries and makes it possible to determine the number of varieties consumed in Matsuyama's model.

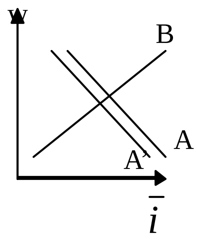
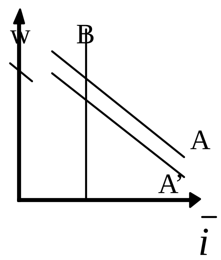
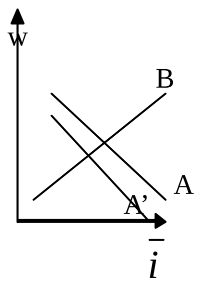
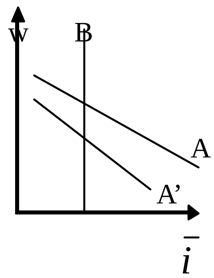
ANNEX 3b: Comparison of the effect of technological shocks

The line A represents the condition of specialization (equation 14) and the line B represents the balance of trade equilibrium condition (equations A7). B' and A' represent the shifting of these lines with the occurrence of technical progress (TP). It should be noted that the graphic representation proposed here is deliberately simplified[22]. It should be read in the following manner:

- Along the y axis, the shift from one equilibrium to another gives the size of the variation in relative wage

- The distance between A and A', at any given point on the x axis, tells us the size of the productivity gains for a good i.

From the comparison of these two distances, we can deduce the variation in the purchasing power of an agent for every good consumed.

	Dornbush, Fisher, Samuleson (1977)	Matsuyama (2000)
Uniform technical progress in North (FUTS)		
Biased technical progress In North (FUTS)		

[22] This presentation is intuitive. For more rigour, the reasoning in terms of variations would require modification of the scale in logarithm or the representation of shifts in a non-linear form. This would complicate the presentation without modifying the results.