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SOE$^{PL\ 2009}$ – An Estimated Dynamic Stochastic General Equilibrium Model for Policy Analysis And Forecasting

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Introduction

The paper documents the effects of work on the dynamic stochastic general equilibrium (DSGE) $\mathbb{SOE}^{\mathrm{PL}}$ model that has been carried out in the recent years at the National Bank of Poland, initially at the Bureau of Macroeconomic Research and lately at the Bureau of Applied Research of the Economic Institute. In 2009, a team consisting of the authors of this paper developed a new version of the model, called $\mathbb{SOE}^{\mathrm{PL}-2009}$ which in 2010 is to be used to obtain routine mid-term forecasts of the inflation processes and the economic trends, supporting and supplementing the traditional structural macroeconometric model and experts' forecasts applied so far.

In the recent years many researchers have engaged in the work over a class of estimated macroeconomic models (of the business cycle) integrating the effects of at least three important lines of economic and econometric research:

- methods of macroeconomic modelling (gradual departure from the traditional structural models towards models resistant to Lucas' and Sims' critique, strongly motivated with microeconomics);
- micro- and macroeconomic theories (monetary policy issues, with emphasis on the consequence of imperfect competition, the role of nominal and real rigidities, as well as anticipating and optimising behaviours of agents in an uncertain environment, with a strong shift of point of view towards general equilibrium);
- estimation techniques (reduction in parameters calibration, shift from classical techniques to Bayesian techniques with Bayesian-specific risk quantification as well as systematic and controlled introduction of experts' knowledge, improvement of projections accuracy).

Merger of the three trends has brought about a class of models — DSGE models — with high analytical and developmental potential. The very potential of the models of this class seems to be the most important reason for the interest of central banks in that area[1], research that may be directly translated into the practice of monetary policy.

[1] Attention is being drawn to the fact that in the general case the DSGE models do not have to be based on the new Keynesian perspective of economy and do not have to be estimated with the use of Bayesian techniques.

Along with the development of numerical, econometric methods and the theory of economics, a number of central banks supplement or even replace the traditional structural macroeconometric models, whose forecasting applications are enhanced with experts' knowledge, with estimated DSGE models, namely models which attempt to translate the economic processes in a more explicit and systematic manner, whereby experts' knowledge is introduced through Bayesian methods[2]. It happens although no formal reasons exist for which the *ex post* verified accuracy of forecasts within the DSGE models should be higher than that of classical models[3]. DSGE models give, however, a chance of structural (internally consistent and microfounded) explanations of the reasons for the recently observed phenomena and their consequences for the future. DSGE models present a different image of economic processes than classical macroeconometric models — they capture the world from the perspective of structural disturbances. These disturbances set the economy in motion and economic agents respond to them in an optimal way, which eliminates the consequences of the disturbances, i.e. restores the economy to equilibrium. The analytical knowledge and experience gathered in contact with the traditional structural models rather interferes with than helps interpret the results of DSGE models. In econometric categories, the results of DSGE models are, nevertheless, at least partially compliant with that which may be achieved with VAR and SVAR models, thus, it is hard to speak about revolution here.

Following the events of 2008–2009 (global financial crisis), while searching for the reasons for the problems' occurrence, the usefulness of formalised tools constructed on a uniform, internally coherent (but also restrictive) paradigm for macroeconomic policy tends to be questioned. The reasons for the global economy problems are searched for in models oversimplifying perception of the world and burdening the decisions regarding economic policy. We have noticed that the critique refers to a larger extent to the models as such (i.e. tools) and less to the practice of applying them (i.e. the user). Therefore, we consider that conclusions from a deeper analysis of the sources of 2008–2009 crisis, verification of the directions of economic research and methods of the research, which is likely to be held, as well as the analysis of the current policy less influenced by its rationalisation shall confirm the legitimacy of building and applying models, particularly DSGE class models. The issue of applications using the strong sides of the models remains, however, open. In our opinion, the best we can do is to try to use our model, gather and exchange experience, develop new procedures and thoroughly verify the results.

The model whose details we shall present further herein derives from the structure developed at Riksbank — DSGE model for the euro area[4] see Adolfson et al. (2005b). The euro area DSGE model, know-how, methods of estimation and applications received within the technical support of Riksbank enabled us to start several experiments, build different versions of DSGE model (a family of $\mathbb{SOE}^{PL}$ models) and develop our own procedures of the model application. Some of the experiments have been described in separate papers, e.g. Grabek et al. (2007), Grabek

[2]For complete image, we wish to add that there are also arguments against the engagement of central banks in constructing DSGE models, see e.g. Orphanides (2007).

[3]The issue of correct measuring of accuracy of forecasts, in which the contribution of experts' knowledge is considerable but non-formalised or systematic, has been omitted here. In such a situation it is hard to assess whether it is the model that failed or the expert. Generally, it may be argued whether the forecasting applications of DSGE models expose the strongest sides of this class of models.

[4]The euro zone model by Riksbank develops the ideas mentioned among others by Christiano et al. (2001, 2003, 2005), Altig et al. (2004a), as well as Smets and Wouters (2004).

and Kłos (2009), Grabek and Utzig-Lenarczyk (2009). The alternative we present in this paper summarizes some of the gathered experience.

We pass the DSGE $\mathbb{SOE}^{PL-2009}$ model for use, with a view to considering and analysing other interpretation and understanding of economic processes than that proposed by the traditional models. Additionally, systematic work with the model (preparing forecasts and analyses of their accuracy, simulation experiments and analytical works) may reveal issues and problems that will have to be solved. Resulting knowledge shall enable the preparation of a more thorough future modification of the model, taking into account the effects of the parallel research and the conclusions arrived at during use.

This paper consists of three basic parts. In the first part — relatively independent of the other parts — we have made an attempt to outline the development of the methods of macroeconomic (macroeconometric) modelling and the economic thought related to monetary policy, which brought about the creation of dynamic stochastic general equilibrium models, pushing aside other classes of models — at least in the academic world. The considerations are illustrated with simple models of real business cycles (RBC) and DSGE model based on new Keynesian paradigm. The second chapter of the first part focuses on the technical aspects of construction, estimation and application of DSGE models, drawing attention to mathematical, statistical and numerical instruments. Although it presents only the keynotes, outlines and ideas, the formalisation and precision of presentation required in that case makes the fragment of the paper slightly hermetic — a reader less interested in the techniques may omit that chapter.

The further parts of the paper refer to specification, results of estimations and properties of the DSGE $\mathbb{SOE}^{PL-2009}$ model. We present, therefore, a general non-technical outline of the basic features of the model, illustrating at the same time the correlations with other DSGE models (Chapter 3). The next chapter defines decision-making problems of the optimising agents, their equilibrium conditions as well as characteristics of behaviours of the non- optimising agents. The description of the model specification is completed with balance conditions on a macro scale. The $\mathbb{SOE}^{PL-2009}$ model has been estimated with the use of Bayesian techniques. Identically as in all estimated DSGE models we are aware of, the Bayesian estimation refers solely to some of the parameters (the rest of the parameters have been calibrated). Although due to the application of the Bayesian techniques, the number of calibrated parameters has been clearly reduced, being aware of the consequences of faulty calibration we conducted a sort of sensitivity analysis (examination of the influence of changes in the calibration of parameters on the characteristics of the model). The presented $\mathbb{SOE}^{PL-2009}$ version takes into account the conclusions we arrived at based on the analysis. For the purposes of this paper and the first forecast experiments we use only point estimates of the parameters reflecting the modal value of posterior distribution, in other words our reasoning omits — hopefully temporarily — the issue of uncertainty of the parameters. The results of the estimation of parameters and assumptions made at the subsequent stages of the work (calibrated values, characteristics of prior distributions) have been presented in Chapter 6.

A synthetic image of the model characteristics has been presented in Chapters 7–8, which describes the responses of observable variables to structural disturbances taken into account in the model (i.e. impulse response functions), variances decompositions (formally — forecast error decomposition), thanks to which the structure (relative role) of the impact of shocks on the observable variables may be assessed, estimation (identification) of structural disturbances in the sample, examples of historical decompositions (counterfactual experiments) and information about the ex post accuracy of forecasts — this is, thus, a typical set of information allowing understanding the consequences of the assumptions made at the stage of constructing decision-making problems (model specification) and choice of parameters.

The Appendix presents structural form equations, equations used to determine value at a steady state and a list of variables of the $\mathbb{SOE}^{\mathrm{PL}-2009}$ model.

Part I

Genesis and anatomy of dynamic stochastic general equilibrium models

1
Genesis and anatomy of dynamic stochastic general equilibrium models

The development of economic models for the purpose of monetary policy analysis has been one of the most exploited research programs within macroeconomics in the last two decades. A lot of effort has been paid to make attempts to understand the correlations between monetary policy, inflation and business cycle. The research is deemed to produce a sort of consensus as to the specification of the key elements of a model of economy, within which modern macroeconomic analysis is being carried out, mainly in the aspects important for the monetary policy applied by central banks. The specification has been named a new Keynesian model. The new Keynesian model is a dynamic, stochastic model of general equilibrium and, thus, a model deriving from neo-classical trend. The basis for its architecture is a model of real business cycle, on which Keynesian elements in the form of real and/or nominal frictions[1] are imposed. Such originating trend of macroeconomic analysis is called a new neoclassical synthesis, see Goodfriend and King (1997).

1.1 Methods of the Cowles Commission

In the 1940s and 1950s government institutions of the most important economies started to collect in a systematic way national statistics regarding economic activity. The economists gained material which helped them specify quantitative models of national economy and analyze them in empirical tests. Early works over the econometric models of national economies complied with

[1] E.g.: monopolistic competition, frictions in the process of adjusting prices, wages, frictions in the financial market.

the paradigms developed during the works carried out by the Cowles Commission. Empirical macroeconomic analysis was at that time carried out with often large[2], dynamic, most often linear, multi-equation econometric models. Their specification was based mainly on statistical tests[3], while the role of economic theory was limited to preparing a list of regressors to be taken into account in the particular equations. The choice of variables was based mainly on Keynesian IS-LM type models, i.e. on theories which ignored both the supply side of economy and changes in relative prices. The models were called ”structural”, as they enabled the consideration of, among other things, feedback nature relationships between variables. Such relations and the resulting problem of simultaneity or interrelation, important for model estimation techniques, were the focus of interest of macroeconometrics and the theory of estimation at that time. Therefore, it is considered that econometrics practiced in the spirit of the Cowles Commission emphasised the structural aspects. Nevertheless, according to today's understanding of structurality in macroeconomics, it may be concluded that structurality, or practically its absence primarily accounted for the failure of this trend of modelling. Although model equations were aimed at presenting the dynamics, which reflects the decisions made by economic agents, forms of equations assumed *ad hoc* failed to comply with any mechanism of individual choice. Dynamics of each of the variables would be modelled with a single equation. Groups of variables formed the blocks of a model and each block was researched by a separate group of experts. The resultant equations were later on combined in a complete model of economy, additionally taking into account the interactions occurring among the variables within the different blocks. The constructed models allowed, seemingly, correct quantification of the consequences of controlling variables that depended on the persons making decisions with regard to economic policy. They were, however, too large to arrive at a general image of the mechanism according to which the propagation of shocks in an economic system took place. It was also hard to research the mechanism of system response to a change in control of economic policy in a longer time perspective. According to the philosophy of the Keynesian trend, the emphasis was put on short-term analysis of economic aggregates dynamics. The economy was out of (partial) equilibrium for a short period and model simulations answered the question of how to effectively bring the economy to equilibrium, i.e. how to stabilise it. Thus, macroeconometrics dealt mainly with the analysis of variables dynamics in a short time with the use of partial equilibrium model. An example of a model within the discussed class is provided in Klein and Goldberger (1955).

1.1.1 Problems with identification

In the terminology derived from the paper of Spanos (1990), the reasons for failure of the originators of models maintained in the tradition of the Cowles Commission as regards the application of the models to economic policy analysis may be divided into two groups. These are problems with structural identification and problems with statistical identification.

As mentioned by Hendry (1976) or Qin (1993), in the period of works of the Cowles Commission and the later development of multi-equation models maintained in this tradition, economet-

[2]Consisting of several hundred or, sometimes, even several thousand equations.
[3]A statistical identification of model is often being mentioned.

rics focused to a large extent on the theory of estimation[4], and a smaller emphasis was put on the assessment of the quality of models by virtue of diagnosis of statistical specification errors. Structural identification was a priority and as a result, often proved *ex post*, the models maintained in this convention were not able to sufficiently accurately replicate the statistical properties of the processes the dynamics of which they were supposed to represent. Although the Cowles Commission was putting emphasis on structural identification of the model, the developed methods are considered, from the perspective of the present day, to be unsatisfactory.

Problems with structural identification were more fundamental. They are presented, among others, in two papers from the 1970s and 1980s: Lucas (1976) and Sims (1980). Lucas (1976) criticises the status of exogeneity of variables — the controls of economic policy. He points out that the model of structural identification proposed by the Cowles Commission does not explicitly take into account the expectations of economic agents. Therefore, the parameters of the mode[5], which were deemed to be structural[6], are actually a mixture of structural parameters and parameters related to the expectations of economic agents and, thus, may not be considered to be fixed for various economic policy regimes. The estimations of parameters within a model estimated on data originating from a specific economic policy regime shall no longer be valid if the policy regime changes. Therefore, a model estimated within one regime may not be extrapolated outside of the regime and, in consequence, may not be applied to analyse the consequences of a change in the regime. Due to instability of parameters, the traditional structural macro-models, according to Lucas, are worthless for simulation of the effects of changes in economic policy, which is exactly the purpose for which they were created. Sims (1980) only supports the comments by Lucas claiming that no variable may be deemed exogenous in the world of economic agents that anticipate future events (forward-looking agents) and whose behaviour is based on intertemporal optimisation. As a result of endogenous economic policy macroeconomic variables correlate with variables — the controls of policy. By assuming erroneously that the policy is exogenous, the endogeneity may be falsely interpreted as a causal relation, and may seem to identify the channel of policy's impact on economy.

Finally, the stagflation of the 1970s and the related failures of economic policy based on the traditional macroeconometric models disqualified, in the academic opinion, the approach of the Cowles Commission. Pesaran and Smith (1995) conclude that the models represented neither the data, nor the theory, and therefore were ineffective for practical purposes of forecasting and policy.

1.2 LSE and VAR models

Problems with statistical and structural identification of traditional multi-equation econometric models brought about the development of several trends out of which two had the largest impact

[4]Important in the works of the Commission and later works were mainly the issues regarding simultaneity, i.e. interrelation of the modelled phenomena.

[5]This specifically refers to the so called reduced form of the model.

[6]Parameters are deemed to be structural or deep if their value does not change under the impact of a change in the economic policy regime.

on the practice of macroeconometrics: the so-called LSE method (London School of Economics), see Hendry (1995), and SCVAR method (Structural Cointegrated Vector Autoregression), i.e. structural vector autoregression for cointegrated variables, see Lütkepohl (2008), which currently — next to the DSGE model — is the basic tool of macroeconomic analysis.

1.2.1 LSE methodology

There may be numerous possible sources of errors in statistical identification of a model. The errors include, but are not limited to, omitting important variables, erroneous dynamic structure or illegitimate restrictions regarding exogeneity imposed on the variables. The LSE approach is an attempt to overcome the problems with statistical identification. It is based on the so-called reduction principle. An econometric model is understood as a simplified representation of an unknown and unobservable stochastic process, which generated the researched economic observations. It originates from a dynamic model of possibly general specification in a given class of models, so as to cover possibly many various processes. Further on, the model is reduced sequentially to the final form. The reduction step entails the elimination of a variable or a group of variables from the model and is made with the use of statistical tests. For the resulting representation of a process that generates data to be complete, loss of information by virtue of reduction must be insignificant from the point of view of the modelled process. A completeness of the model is confirmed by the statistical properties of the residuals vector. Any deviations from the Gaussian white noise certify that specification is faulty. Thus, the LSE approach emphasises the correct statistical identification of a model but is not an attempt to solve problems related to structural identification.

1.2.2 VAR methodology

Similar problems are reflected in the approach based on the analysis of time series with the use of non-structural models of vector autoregression (VAR). Non-structural VAR models, or VAR models in reduced form, are in fact the generalisation of the LSE approach to vector time series. They are models expressing endogenous variables by their lagged values. A VAR model of order $K \geq 1$ has the following form:

$$y_t = \sum_{k=1}^{K} A_k y_{t-k} + e_t, \qquad \text{or} \qquad y_t = A y_{t-1} + e_t, \tag{1.1}$$

where y_t is a vector of endogenous variables in t period, A_k matrices for $k = 1, 2, ..., K$ are autoregressive matrices. Another representation is called a cumulative representation and A matrix is an autoregressive companion matrix. The e_t process covers shocks controlling the dynamics of endogenous variables. The e_t shocks control the dynamics of endogenous variables such that the variables may be presented as a function of the history of shocks. This is the so-called moving average representation of the process (y_t):

$$y_t = A^t y_0 + \sum_{k=0}^{t-1} A^k e_{t-k}, \tag{1.2}$$

where y_0 is the initial value of the process (y_t), or — assuming infinite history of the process of endogenous variables:

$$y_t = \sum_{k=0}^{\infty} A^k e_{t-k}. \tag{1.3}$$

The basic purpose of the macroeconomic analysis carried out in accordance with the concept of a shock impacting the system of endogenous variables as a source of their dynamics is to identify shocks of structural nature, i.e. independent shocks with explicit and cohesive economic interpretation. The e_t shocks may not, however, be structurally interpreted as they do not need to be independent, and generally are not, which should be expected from shocks of structural nature. The e_t shocks have the nature of errors or regression residuals (forecast errors) and may form linear combinations of the actual structural shocks which determine the dynamics of endogenous variables. It may, then, generally happen that $e_t = B\epsilon_t$, where ϵ_t are structural shocks, i.e. independent ones. In particular it is assumed that the covariance matrix of ϵ_t shocks is the identity matrix, i.e. $\mathbb{D}(\epsilon_t) = \mathrm{I}$. This leads to the approach giving structural interpretation to VAR models. The approach is called structural vector autoregression, shortly SVAR (from structural VAR). Next to the advantages of the LSE approach in the form of representation of a process generating the dynamics of endogenous variables through a model of rich dynamic structure, namely VAR, the SVAR methodology makes it possible to provide shocks with structural interpretation.

The SVAR methodology has been developed not only for stationary processes on which the approach based on the Cowles Commission paradigms must rely, but may be easily generalised to non-stationary cointegrated processes. Such approach is called structural cointegrated vector autoregression method, shortly SCVAR, or more often — structural vector error correction model, shortly SVECM.

SVAR and SVECM models attempt to solve problems regarding statistical identification and some of the problems regarding structural identification, which were identified in the classical multi-equation models. These advantages explain the popularity of VAR class models in empirical macroeconomics. In order to emphasise that there have been proposed at least partial solutions not only for statistical identification problems but also for structural identification, modern macroeconomic analysis is often referred to as structural macroeconometrics. Despite the aforementioned advantages, VAR methods are not free of drawbacks. Mainly, VAR models specification lacks any theoretical basis. The relations between the variables are of clearly statistical nature and even in the case of structural models they may not be referred to any economic mechanism on which the modelled process is founded. Thus, a VAR model lacks a theoretical framework, and upon assessment it may bring about, and in practice often brings, non-intuitive implications in the form of responses to shocks that may hardly be rationalised and non-cohesive forecasts. Due to the foregoing disadvantages, macroeconomic analysis, particularly at central banks, more and more often refers to models having theoretical foundations, such as DSGE models.

DSGE models - through economic theory underlying their specification — constitute another step forward to solve the problems with structural identification. The step is taken at the cost of statistical quality of the model, however not so significant because the DSGE models, and more specifically their approximate solutions, are directly related to VAR class models. Specifically, the so-called reduced form of DSGE model, namely the form which sets dynamics of endogenous variables around the equilibrium, is a VAR model. It may be considered as a VAR model on the parameters of which, i.e. on elements of matrices A and B, a set of restrictions has been imposed. The restrictions originate in the theory of economics underlying the specification of a DSGE model. They limit the dynamic structure of the model, yet ensure its internal cohesion and guarantee structural nature of the identified shocks. The DSGE model specification founded on the theory of economics ensures that restrictions imposed on the dynamic structure of its reduced form derive from the optimal decision-making rules of rational economic agents operating in an internally cohesive economic reality. The methodologies of DSGE models are, therefore, at least theoretically, an answer to the problems related to structural identification. They also, to a major extent, reply to the problems related to statistical identification, as the solution of a DSGE model is a VAR class model. In that scope a DSGE model provides, nevertheless, only partial solutions, as the structural nature of its specification imposes strong restrictions on the dynamic structure of the reduced form. Thus, a DSGE model expresses a trade-off between the proper statistical and structural identification of macroeconomic processes.

1.3 Methodology of modern macroeconomics

The views with regard to the method of analysing the dynamics of economic aggregates have significantly changed over the last 30 years. The evolution followed from models specified in the tradition of the Cowles Commission, through VAR class models, to dynamic stochastic general equilibrium models based on the theory of economics.

Modern macroeconomics attempts to explain the dynamics of economic aggregates with the use of models based on the so-called microfoundations. Hence, as opposed to traditional Keynesian models or multi-equation macroeconometric models where the forms of interrelations between economic variables were assumed *ad hoc*, the mechanisms shaping the decision-making rules of economic agents are explicitly modelled. Most often four types of agents are distinguished. They are households, firms as well as government and central bank. Whereas the decision-making rules of households reflect the process of optimisation of welfare, i.e. a discounted stream of the expected utility, those of firms result from maximisation of the expected profit. Optimisation takes place in a stochastic economic environment and in the case of households the structure of their preferences — the so-called utility function — is also set. The decisions of agents are, thus, always optimal a priori, i.e. the best of possible taking into account the available information. Usually, they refer to three types of economic categories: products and services, work or assets, both physical such as capital and financial such as bonds or money. Households decide about consumption, i.e. demand for a product, supply of labour and change in the structure of the portfolio of financial assets, the level of investments, the level of use of the available capital, etc. Firms decide about product supply and demand for labour[7]. The government determines the value of public spending, collects taxes, expends transfers and incurs public debt. The central bank controls the nominal interest rate and/or the supply of money. The decision rules of the government and the central bank are usually made *ad hoc*. Beside of the optimisation criterion, the process of making decisions by economic agents takes into account several categories of limiting conditions. These are most often budget constraints, initial conditions, equilibrium conditions and constraints regarding the available technology and information structure in the economy[8].

A central issue in the theory of dynamic general equilibrium is the intertemporal dimension of the decision-making process. The decisions of agents consist of intertemporal allocation of the available resources. Today's income may be allocated to future consumption and future income may finance today's consumption expenditures. Intertemporal substitution of resources is possible by virtue of participation in the market of financial assets, e.g. purchase or sale of bonds. The individual decisions are coordinated by the market, which brings about decentralised allocation of resources.

Formally, economy is described as a dynamic system. It reflects a short-term equilibrium in at least two meanings. Firstly, in each point of time economy reflects a general equilibrium as understood by Walras. It is, thus, assumed that prices always clear the markets. Secondly, economic agents make optimal decisions, i.e. *ex post* they do not make mistakes in a systematic manner as to the actions undertaken. In that meaning their decisions are called rational. Should it appear *ex post* that the decisions of the agents are not the best possible to be taken, this is only due to information gap, i.e. due to the fact that after the moment of making the decision an event occurred that could not have been foreseen by the agent, e.g. an unexpected exogenous growth in productivity took place. The agents build expectations as to the future values of economic variables through the operator of conditional expected value. In that sense the mechanism of building expectations by the economic agents is rational. It is, therefore, assumed that agents know the complete model of economy, namely that they know (the real) principles governing the world in which they live and the values of all of its parameters. They are also able, based on the model, to set out optimal decision-making rules for all agents and apply them, which would require in practice the possibility of making a perfect filtration, i.e. a perfect measurement of the value of all of the variables and shocks impacting the economy in any period. That omnipresent transparency and rationality draws critique on the part of alternative paradigms of economic modelling such as multi-agent modelling (agent-based computational economics), see Fagiolo and Roventini (2008).

Even though the economy is — in the above meaning — assumed to remain at all times in short-term equilibrium, in a short period it may be out of long-term equilibrium. Long-term equilibrium is also called stationary state or steady state. The names are founded on mathematics.

[7]The mentioned division into the decisions of households and firms is conventional, however, characteristic to the literature of the subject matter.

[8]*Ex post* there are also imposed the so called transversality conditions of no-Ponzi game type.

Long-term equilibrium is a mathematical concept and refers to the model of economy instead of the economy in itself. There is no direct equivalent in the real world. Economy reflects long-term equilibrium if all of the variables grow from one period to another according to fixed growth rates[9]. Economy in a steady state may be knocked out from the state. It happens because stochastic disturbance, i.e. structural shocks or structural innovations impacts the economy. Examples of structural shocks are technological shocks (increase or drop in total factor productivity), shock of preferences or contractionary monetary policy. If the effects of shocks abate, the economy comes back to long-term equilibrium but not necessarily to its pre-shock state. This shall happen if the effect of shock is permanent. If economy is knocked out of long-term equilibrium by a shock of temporary effect, it shall asymptotically come back to the same equilibrium. As in reality structural shocks happen at any moment, the real economy shall never come to the same steady state. It shall, however, fluctuate in its environment. The steady state path may be interpreted as the path taken by the economy would it not be for the shocks. A model implying the comeback of economy to the steady state upon structural shock occurrence is called a steady-state or stationary model.

Modern macroeconomic analysis decomposes the time series of economic aggregates into two basic components, namely long-term trend and short-term cyclic fluctuations around the trend, i.e. the so-called business cycle. It shall be emphasised that both the trend and the cycle are statistical fiction and have no equivalent in the real world. Both categories are a product of data filtration and their form depends on the chosen method of filtration. DSGE models, as ones deriving from the RBC models family, are used in cycle analysis and due to their construction are not able to answer any question regarding the forming of the trend. This is the subject of the theory of economic growth, which is introduced to DSGE models in an exogenous manner.

1.3.1 Real Business Cycle theory

Since the beginning of 1980s, when the first papers of the type Kydland and Prescott (1982) appeared, the theory of Real Business Cycle (shortly RBC) has gained the status of a leading macroeconomic theory with regard to economic fluctuations analysis — a model that may be called a standard RBC model from today's perspective, the basic tool of business cycle analysis. The influence of the RBC revolution on understanding short- and medium-term economic fluctuations has two perspectives — both methodical and conceptual.

From the methodological point of view, the theory of Real Business Cycle made the dynamic stochastic general equilibrium model a basic tool for macroeconomic analysis. *Ad hoc* assumed behavioural equations describing economic aggregates gave way to equations of motion derived from intertemporal solutions of optimisation problems of the economic agents operating in perfectly competitive and friction-less markets. *Ad hoc* assumptions made with regard to the mechanisms of forming expectations by economic agents gave way to rational expectations. The creators of the RBC trend emphasised as well the importance of quantitative aspects of macroeconomic analysis, which was reflected in the use of the methods of calibration, simulation and validation of RBC models.

[9] The definition covers the case in which some or all variables are unchanged from period to period.

Equally fundamental proved to be the conceptual implications of the RBC theory. Firstly, it appears from the RBC theory that a business cycle presents an effective, i.e. optimal path of economic aggregates. In other words, both expansion and contraction result from optimal reaction of economic agents to exogenous shocks impacting the real sphere of economy, mainly technological shocks. Thus, cyclic fluctuations — including recessions — do not bring about ineffective allocation of resources — they are fully optimal and do not result from market imperfections. The RBC theory implies, therefore, that the stabilisation policy of a government may change resources' allocation but only to less efficient one. This clearly contradicted the standard Keynesian interpretation of recession as a period in which economic resources are used ineffectively and the end of which may be advanced by stimulation of aggregated demand.

The second implication of the RBC theory is the conclusion that the main reason for economic fluctuations are technological shocks, which temporarily improve or deteriorate the total factor productivity in the economy. RBC models try to replicate the fluctuations of products and other economic aggregates, even if the only shock impacting economy is the shock of productivity. Such interpretation of economic fluctuations contradicted the traditional view that changes in productivity contributed to economic growth, which has nothing to do with a business cycle.

The third fundamental conceptual implication of the RBC theory is the limited, or practically non-existent, role of monetary factors in economy. A standard RBC model implies the neutrality of money even in a short period. Hence, the dynamics of real variables in economy, such as production, consumption or employment, do not depend on monetary policy. Monetary policy may only affect the nominal values such as the nominal interest rate or nominal supply of money, namely inflation, whose values do not affect the real sphere of economy. The results contradicted the general opinion that monetary policy impacts the real economy in a short run, see Friedman and Schwartz (1963) and Christiano et al. (1998). Even though the RBC theory had a considerable influence on the method of understanding business cycles, particularly in the academia, RBC models — due to the results regarding the neutrality of money — were not accepted by central banks. The central banks continued to focus on classical multi-equation models and more and more often on LSE type approach, particularly on vector autoregression methods.

Due to the growing evidence of contradictions between the Real Business Cycle theory and empirical research, as well as rift between its implications and the practice of economic policy, RBC methods could not be considered satisfactory. On the other hand, RBC methods proposed solutions to problems that discredited the traditional approach based on paradigms developed by the Cowles Commission. RBC models are dynamic stochastic general equilibrium models instead of partial equilibrium models. They have an internally cohesive theoretical structure. Therefore, even though they are calibrated (today, estimated) based on the data that may be generated by non-intuitive statistical artefacts, the results of simulation experiments (e.g. forecasts, shock response analyses) reflect internal cohesion. This is not the case with VAR class models, even the structural ones. The decision-making rules of economic agents in an RBC model are structural, i.e. derived from microfoundations instead o f being assumed *ad hoc*. The expectations of agents are rational, while anticipation of future events influences their today's behaviour. RBC models reflect, therefore, the achievements of the revolution of rational expectations. RBC models parameters, or at least some of them, are deep parameters. Thus, the models are at least partially resistant to Lucas' and Sims' critique. In other words, the models allow us to correctly analyse the effects of changes in economic policy regime. All those features indicate that RBC methods well manage the problems of structural identification. Additionally, the reduced form of an RBC model is a VAR type model[10], namely a model covering a wide class of stochastic processes. This feature shows that problems related to statistical identification are not ignored in the RBC methods.

It is clear that RBC methods could not be simply rejected. Thus, attempts of such modification of the Real Business Cycle theory have been made so that the internally cohesive structure of an RBC model be preserved, their dynamics be related to VAR class models, while the role of money be increased in a short time. The introduction of the elements of new Keynesian economics to the classical RBC model proved to be a solution — such derived model is called a new Keynesian model. We will start with the frictionless monetary model of the economy and then introduce some frictions which render money non-neutral.

1.3.2 Frictionless economy

A standard frictionless monetary model is presented below[11]. Despite a simple structure, it presents the most important components of a DSGE model. In the following paragraph the model is extended with Keynesian elements, which leads to the basic form of a new Keynesian DSGE model. A standard DSGE economy consists of a representative household, a representative firm and a central bank. The presentation of the standard DSGE model in this paragraph has been founded on the monograph by Gali (2008).

1.3.3 Representative household

In exchange for nominal wage W_t,a household provides a representative firm in each of the periods $t = 0, 1, 2, ...$ with a homogenous labour supply[12] H_t^s. The labour market is perfectly competitive. In exchange, a household receives wage of the worth of $H_t W_t$. The wage, together with the savings B_{t-1} in period $t-1$, is the income of a household $D_t = H_t W_t + B_{t-1}$, which is divided between consumption C_t and savings B_t. A household consumes homogenous goods (products) manufactured by the firm, while the unit price of the product is P_t. Savings are located in bonds with no risk. A unit price of bonds in period t amounts to $Q_t = \frac{1}{1+i_t}$, where i_t is the nominal interest rate determined by the central bank. One bond purchased in period t at price Q_t is worth in period $t+1$ one monetary unit. The expenditures of a household in period t are, thus, $P_t C_t + Q_t B_t$. Household's welfare is measured with a utility function $U(C_t, H_t)$, while consumption increases and labour decreases welfare.

[10]The reduced form of a DSGE model is a process of VARMA class, perhaps of infinite order, however, in practice the process is approximated with a finite order VAR model.

[11]The presented version of the model abstracts from investments and government sector. Beside those simplifications, the model corresponds to the standard specification of an monetary model which builds on the RBC origins. The assumed simplifications are non-significant from the point of view of the presented conclusions.

[12]Subscript s stands for the supply of labour offered by a household. Subscript d shall stand for demand for labour reported by a firm. In general equilibrium $H_t^s = h_t^d$, therefore, subscripts s or d shall be omitted when it must be emphasised that we mean the labour in general equilibrium.

In period t, a representative household must, then, make a decision as to variables $\{C_t, H_t^s, B_t\}$, such as to maximise utility and by virtue of spending $P_tC_t + Q_tB_t$ not to exceed the budget of the value of $W_tH_t^s + B_{t-1}$. Decisions of a household are not, however, one-period problem, i.e. their consequences are not important only "here and now". The decision-making process has a dynamic, multiperiod nature, as today's decisions affect the value of resources available tomorrow. A household must, then, decide not only about variables $\{C_t, H_t^s, B_t\}$ for the determined t but also for all of the $t = 0, 1, 2, ...$ at the same time. In order to know how to do that, in $t = 0$ period a household solves the problem of maximisation of the discounted stream of the expected utility:

$$\max_{\{C_t, H_t^s, t=0,1,2,...\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, H_t^s), \tag{1.4}$$

subject to a sequence of budget constraints:

$$P_tC_t + Q_tB_t = W_tH_t^s + B_{t-1}, \tag{1.5}$$

where $\mathbb{E}_t$ is the operator of the conditional expected value. Problem (1.4–1.5) may be solved by using Lagrange's functional in the form:

$$\mathscr{L} = \mathbb{E}\left\{\sum_{t=0}^{\infty} \beta^t [U(C_t, H_t^s) - \omega_t(P_tC_t + Q_tB_t - B_{t-1} - W_tH_t^s)]\right\}. \tag{1.6}$$

First order conditions are as follows:

$$\omega_t = \frac{\partial U(C_t, H_t^s)}{\partial C_t}\frac{1}{P_t}, \quad \omega_t = -\frac{\partial U(C_t, H_t^s)}{\partial H_t^s}\frac{1}{W_t} \quad \text{oraz} \quad \omega_t = \frac{\beta}{Q_t}\mathbb{E}_t\omega_{t+1}, \tag{1.7}$$

and for CRRA[13]type utility function, the decision rules of a household are the following:

$$C_t = \left[\frac{Q_t}{\beta}\mathbb{E}_t\{(C_{t+1})^{\sigma_c}\Pi_{t+1}\}\right]^{\frac{1}{\sigma_c}}, \quad \frac{W_t}{P_t} = (H_t^s)^{\sigma_h}C_t^{\sigma_c}. \tag{1.8}$$

System (1.8) is recursive. The first equation, i.e. consumption's law of motion, determines the value of consumption C_t depending on the expected consumption C_{t+1} and inflation Π_{t+1}. The second equation — labour supply equation — determines, for the assumed real wage value, the supply of labour that needs to be provided by a household in order to generate income necessary to cover the costs of consumption. The choice of $\{C_t, H_t^s, t = 0, 1, 2, ...\}$ shall maximise the expected welfare of the household.

[13]The *constant relative risk aversion* utility function is defined as:

$$U(C_t, H_t) = \frac{(C_t)^{1-\sigma_c} - 1}{1-\sigma_c} - \frac{(H_t)^{1+\sigma_h} - 1}{1-\sigma_h}.$$

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1.3.4 Representative firm

A representative firm hires labour supplied by a household H_t^d in exchange for nominal wage W_t. It manufactures a homogenous real product Y_t with the use of production function $Y_t = A_t N_t$, where A_t is an exogenous process of total factor productivity. The product market is perfectly competitive. A decision-making variable in the case of a firm is the demand for labour H_t^d, which — for the assumed technology level — determines the value of the product. In order to determine the H_t^d value, a firm solves in each period a static problem of profit maximisation:

$$\max_{H_t^d} \left\{ P_t Y_t - W_t H_t^d \right\},$$

subject to the technological constraint:

$$Y_t = A_t H_t^d,$$

the solution of which implies equating real wage with marginal productivity of labour:

$$A_t = \frac{W_t}{P_t},$$

which is exogenous.

1.3.5 General equilibrium

Upon determination of optimal decision rules of a representative household and a representative firm, market clearing conditions are imposed on all markets, i.e. the economy is assumed to remain in general equilibrium state in every period.

The model in its existing form is non-linear. Generally, no satisfactory methods exist to solve non-linear DSGE models. Therefore, the equilibrium conditions are superimposed on the simplified, log-linear form of the model, which for the presented frictionless model reads as follows[14]:

$$\begin{aligned} c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma_c}(i_t - \mathbb{E}_t \pi_{t+1} - \rho), \\ w_t - p_t = \sigma_c c_t + \sigma_h h_t^s, \\ w_t - p_t = a_t, \\ y_t = a_t + h_t^d, \end{aligned} \tag{1.9}$$

where $\rho = -\ln\beta$.

In the basic model there are three markets — product market (consumption balancing), labour market and savings (bonds) market. The product market's clearing condition states that the

[14]Small letters denote in this chapter percentage (logarithmic) deviations from the steady state. Exceptions: real and nominal interest rates, which are expressed in absolute values, as well as real marginal costs and monopolistic markup which are expressed in logs. Timeless variables denote steady state values.

whole product supply y_t is subject to consumption:

$$y_t = c_t$$

the labour market clears when:

$$h_t^d = h_t^s = h_t.$$

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Whereas households trade bonds only between one another, the savings market clears automatically:

$$b_t = 0.$$

The condition equating real wage with marginal productivity of labour, after log-linearisation $w_t - p_t = a_t$, may be read out as the determination of real marginal cost (denoted by mc_t) or — equally — monopolistic markup $\lambda_t = -mc_t$, equal to one (zero for the logarithm):

$$mc_t = -\lambda_t = w_t - p_t - mpn_t = 0,$$

where $mpn_t = a_t$ stands for marginal labour productivity.

1.3.6 Consequences for monetary policy

Based on the conditions (1.9) and the market clearing conditions, there may be determined the dynamics of real categories of product y_t, employment h_t, real wage $w_t - p_t$ and real interest rate $r_t = i_t - \mathbb{E}_t \pi_{t+1}$:

$$\begin{aligned} c_t = y_t &= \frac{\sigma_h + 1}{\sigma_h + \sigma_c} a_t, \\ h_t &= \frac{\sigma_h}{\sigma_c} \frac{1 - \sigma_c}{\sigma_h + \sigma_c} a_t, \\ w_t - p_t &= a_t, \\ r_t &= \rho + \frac{\sigma_c(\sigma_h + 1)}{\sigma_h + \sigma_c}(\rho - 1) a_t, \end{aligned} \tag{1.10}$$

where technology[15] is controlled with an exogenous stationary process in the form of:

$$a_t = \rho a_{t-1} + \epsilon_t,$$

where $\epsilon_t \sim N(0, \sigma)$ is a structural shock of labour productivity. In a standard RBC model this is the only structural shock. The last of the equations above, i.e. the real interest rate equation $r_t = i_t - \mathbb{E}_t \pi_{t+1}$, results from Euler household equation.

It appears then that the dynamics of real variables in a standard frictionless economy model depends only on the level of technology a_t[16]. Therefore, from Fisher equation: $r_t = i_t - \mathbb{E}_t \pi_{t+1}$, it results that a change in the nominal interest rate is one to one translated into a change in

[15] More precisely, the process of total factor productivity (TFP), here — labour productivity.

[16] And in the case of real interest rate — on the expected change in technology $\mathbb{E}_t \Delta a_{t+1} = (1 - \rho) a_t$.

inflation expectations. The equilibrium dynamics of real categories does not depend on the nominal interest rate i_t, as the interest rate is translated only to nominal category — the expected inflation. Monetary policy does not affect the decision rules of firms and is neutral for the welfare of households. Consequently, a standard RBC model implies that implementation of monetary policy, requiring high monetary and institutional expenditures, is non-productive.

1.4 New neoclassical synthesis

The conclusion regarding the neutrality of money in a standard frictionless monetary model of the economy was a decisive factor that the models of that class could not raise serious interest at institutions such as central banks, whose practice and understanding of economic categories fluctuations contradicted the implications of Real Business Cycle theory. On the other hand, microfounded models gained the interest of academic communities because — compared to previous macroeconometric models — they made a significant step ahead from the point of view of the method of economic modelling. They appeared particularly attractive due to explicit modelling of the decision-making process of economic agents and specification of the model based on structural parameters — unchanging in the analysis of alternative scenarios of economic policy, including monetary policy applications. Therefore, the normal course of events seems to be the beginning of construction of a new theoretical trend based on a possibly simple case, which would, however, reflect the central elements of the trend. In the case of the trend that originated as an effect of merger of the RBC theory with the elements of new Keynesian economics — the trend that is currently called new neoclassical synthesis — the starting point was the model of general equilibrium of frictionless economy (or distortion-free economy). Neutrality of money and the primacy of technological shocks in the course of the cycle contradicted economic practice and the concept that demand factors and monetary shocks should play more than marginal role in forming the cycle. This leads to many attempts of developing the originally frictionless monetary models towards specifications abiding by the methodical achievements of that trend, however, implying non-neutrality of money in a short run.

In the 1980s and 1990s the analysis covered various microeconomic mechanisms, mainly the so-called distortions or non-effectiveness, the purpose of which was to bring the general equilibrium models closer to economic reality. A number of papers proposed methods to consider nominal rigidities in the dynamic model of general equilibrium. Initially — analogically to new Keynesian literature — emphasis was being put on rigidities in the process of adjusting prices. The proposed models in fact extended the standard frictionless framework to the case of firms operating in the market of monopolistic competition which are not always able to determine markups or, equivalently, prices at the optimal level. The choice of distortions in the process of price or markup determination offered the simplest solution guaranteeing that monetary shocks shall have real effects and shall reflect adequate persistence. In the following paragraph, we present a standard new Keynesian model based on a standard frictionless engine, emphasising the reasons for non-neutrality of money.

1.4.1 Standard new-Keynesian model

Economy consists of a representative household, firms — no longer a representative firm — and central bank. The economic environment in which a representative household is operating is identical as in the standard frictionless monetary model. Hence, that decision-making problem of a representative household, as well as its decision-making functions are the same as in the standard frictionless monetary model. The central bank implements monetary policy applying the monetary policy rule, e.g. Taylor rule. The main difference appears in the sector of firms that possess monopolistic power, so they are no longer price takers. It is assumed that there is a continuum of firms and each of them is represented by a point on $[0,1]$[17] interval. They operate in the market of imperfect competition, usually monopolistic competition. Thus, firms have monopolistic power and may determine the prices of their products. Each firm manufactures a product different from the products manufactured by other firms (a variety), while the elasticity of substitution between products manufactured by various firms is finite. The products of the particular firms are distinguishable, e.g. based on brand naming.

1.4.2 Representative household

Whilst consumption of a representative household consists of a continuum of products, it faces the same decision-making problem as in the frictionless model. In other words, households face the same decision rules as in the frictionless model, i.e. after log-linearisation they take the following form:

$$\begin{aligned} c_t &= \mathbb{E}_t c_{t+1} - \frac{1}{\sigma_c}(i_t - \mathbb{E}_t \pi_{t+1} - \rho), \\ w_t - p_t &= \sigma_c c_t + \sigma_h h_t^s. \end{aligned} \tag{1.11}$$

In period t, a representative household consumes $C_t(i)$ of product i, while total consumption in economy originates as a result of averaging the consumption of the particular products with the use of CES integral aggregator:

$$C_t = \left(\int_{[0,1]} C_t(i)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}, \tag{1.12}$$

where η is the elasticity of substitution between any two products in economy. Budget constraints, then, take the form of:

$$\int_{[0,1]} P_t(i) C_t(i) di + Q_t B_t = B_{t-1} + W_t H_t, \tag{1.13}$$

where $P_t(i)$ stands for the price of product i determined by firm i in period t. If we assume that

[17] There is no obstacle for the number of products to be finite. Integral aggregators are, then, replaced with sum-based aggregators. Yet, in literature the application of integral aggregators is common.

prices $P_t(i)$ aggregate to the average price P_t in period t according to:

$$P_t = \left(\int_{[0,1]} P_t(i)^{1-\eta} di\right)^{\frac{1}{1-\eta}} \tag{1.14}$$

the budget constraint aggregates to:

$$P_t C_t + Q_t B_t = B_{t-1} + W_t H_t \tag{1.15}$$

i.e. to the constraint identical as in the frictionless model. A household decides the share in the total consumption C_t of particular products by virtue of maximisation of consumption

$$C_t = (\int_{[0,1]} C_t(i)^{\frac{\eta-1}{\eta}} di)^{\frac{\eta}{\eta-1}}$$

for the assumed value of consumption expenditures $\int_{[0,1]} P_t(i)C_t(i)di$, i.e. for each $i \in [0,1]$, solving the problem:

$$\begin{aligned} &\max_{C_t(i)} \quad C_t = \left(\int_{[0,1]} C_t(i)^{\frac{\eta-1}{\eta}} di\right)^{\frac{\eta}{\eta-1}}, \\ &\text{subject to} \quad \int_{[0,1]} P_t(i)C_t(i)di = Z_t, \end{aligned} \tag{1.16}$$

where Z_t is an auxiliary variable determining the value of consumption expenditures. The solution to this problem leads to the function of demand in the form of:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\eta} C_t. \tag{1.17}$$

1.4.3 Firms

A firm i in period t is assumed to manufacture $Y_t(i)$ of product i with the use of production function such as that applied by a representative firm in the frictionless model:

$$Y_t(i) = A_t H_t^d(i) \tag{1.18}$$

The level of stationary technology A_t is common to all firms.

As firms have monopolistic power, they can set the prices of their products. In period t a firm i determines the price of a product i at the level of $P_t^\star(i)$, which maximises the expected discounted flow of its profits. The process of determining prices is subject to frictions. Price rigidities of the Calvo (1983) type are most frequently applied. Hence, a firm i in period t may determine an optimal price of its product $P_t^\star(i)$, with probability equal to $1-\xi$. With probability equal to ξ the firm is forced to leave the price of its product at the previously determined level. The probability ξ does not depend on time that elapsed from the last period in which the firm had the possibility to re-optimise the price of its product. In other words, all the firms which in period t may re-optimise prices, solve the same optimisation problem, namely determine the same price $P_t^\star$. The price $P_t^\star$ solves the following problem:

$$\max_{P_t^\star} \quad \sum_{k=0}^{\infty} (\beta\xi)^k \mathbb{E}_t\{Z_{t,t+k}(P_t^\star Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}))\}, \tag{1.19}$$

where $Z_{t,t+k}$ stands for the stochastic discount factor, $Y_{t+k|t} = (\frac{P_t^\star}{P_{t+k}})^{-\eta} C_{t+k}$ stands for the households' demand in period $t+k$ for the output of a firm that had the latest opportunity to re-optimise prices in period t, and $\Psi_{t+k}(Y_{t+k|t}) = W_{t+k} H_{t+k}(Y_{t+k|t})$ stands for the nominal cost of such firm. Thus, as opposed to the frictionless model, in the new Keynesian model the mechanism of inflation is modelled explicitly. Firms generate inflation by determining an optimal price at the level different from the average price level in the previous period.

1.4.4 General equilibrium

The condition of clearing the product markets makes demand equal to supply in each market:

$$C_t(i) = Y_t(i)$$

for every $i \in [0, 1]$ and entails the clearing of an aggregated product market:

$$Y_t = C_t$$

providing that product aggregation is carried out with the use of aggregator in the following form:

$$Y_t = \left(\int_{[0,1]} Y_t(i)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}.$$

After log-linearisation, the condition takes the form of:

$$y_t = c_t.$$

Demand for labour is aggregated with the use of $H_t^d = \int_{[0,1]} H^d(i) di$. aggregator. It may be shown that such aggregation leads to a relation between product, productivity and employment in the form of[18]:

$$y_t = a_t + h_t^d.$$

The condition of labour market clearing requires that the aggregated demand for labour be equal to supply of labour by a representative household:

$$h_t^d = h_t^s = h_t$$

[18] This dependence requires the elimination of the effects of wage dispersion in economy.

The bonds market clears automatically:

$$b_t = 0.$$

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1.4.5 Consequences for monetary policy

The decision rules of households and aggregated production function of a standard new-Keynesian model corresponds to that in the presented frictionless monetary economy:

$$\begin{aligned} c_t &= \mathbb{E}_t c_{t+1} - \frac{1}{\sigma_c}(i_t - \mathbb{E}_t \pi_{t+1} - \rho), \\ w_t - p_t &= \sigma_c c_t + \sigma_h h_t^s, \\ y_t &= a_t + h_t^d, \end{aligned} \tag{1.20}$$

where $\rho = -\ln \beta$.

In the frictionless monetary model the decision rules of firms — making the real wage equal to marginal productivity of labour or, equivalently, making the nominal marginal cost of production equal to market price — made the real wage dependent on employment and productivity. Thus, they force the real marginal cost to be equal to unity[19]:

$$mc_t = w_t - p_t - mph_t = w_t - p_t - a_t = 0. \tag{1.21}$$

Equation which made the real wage dependent on productivity only, along with the conditions of market clearing, closed the frictionless model in the sense that it was possible to determine the dynamics of real variables with no need to refer to the rules of monetary policy. Euler equation determined only the dynamics of real interest rate $r_t = i_t - \mathbb{E}_t \pi_{t+1}$. When firms operate in monopolistic competition conditions, the average real marginal cost in economy mc_t does not have to be equal to unity but it is equal to the inverse of the average monopolistic markup in economy λ_t:

$$mc_t = w_t - p_t - mph_t = w_t - p_t - a_t = -\lambda_t \neq 0 \tag{1.22}$$

as firms that may re-optimise the price determine it above the marginal nominal cost. Equation 1.22 as firms that may re-optimise the price determine it above the marginal nominal cost. Equation 1.21 which — making the real wage dependent only on labour productivity — closed the frictionless model and allowed to determine dynamics of real categories with no need to define the method of determining the nominal interest rate i_t[20]. Equation 1.22 relates the real wage with productivity and average monopolistic markup. Assuming the exogeneity of the process generating average markups λ_t, the real variables of the new Keynesian model may be

[19]Which means that the real cost logarithm mc_t shall be zero.

[20]Actually,

rate and the expected inflation was included by virtue of the Fisher rule.

expressed by RBC model variables and by average markups:

$$\begin{aligned}
y_t &= y_t^\star - \frac{1}{\sigma_c + \sigma_h}\lambda_t, \\
h_t &= h_t^\star - \frac{1}{\sigma_c + \sigma_h}\lambda_t, \\
w_t - p_t &= (w_t - p_t)^\star - \frac{\sigma_h}{\sigma_c + \sigma_h}\lambda_t, \\
r_t &= r_t^\star - \frac{1}{\sigma_c + \sigma_h}\mathbb{E}_t(\lambda_{t+1} - \lambda_t),
\end{aligned} \tag{1.23}$$

where the values of $y_t^\star$, $h_t^\star$, $w_t^\star - p_t^\star$ and $r_t^\star$ variables are that resulting from the frictionless model, namely in the case of absence of price rigidities and in perfect competition. Thus, it may be seen that non-neutrality of monetary policy in the presented standard model takes place by virtue of the process of determining optimal margins or, equivalently, by the process of determining optimal prices. More precisely, the process of determining optimal prices must be disturbed because if we assume perfect flexibility of prices, i.e. exclude price rigidities, the sector of firms shall be reduced to a representative monopolist determining the optimal price in each period at the level of:

$$P_t^\star = \frac{\eta}{\eta - 1}\psi_t$$

where ψ_t stands for the nominal marginal cost of a representative monopolist. The markup shall, then, be fixed and equal to $\lambda_t = \ln \frac{\eta}{\eta-1}$, which would close the considered model so that the dynamics of real categories would be independent of monetary policy. In the case of price rigidities of Calvo type, the solution of the decision-making problem of firms leads to a stochastic difference equation for the optimal price, which upon linearisation has the form of:

$$p_t^\star = p_{t-1} + \beta\xi\mathbb{E}_t\{p_{t+1}^\star - p_t\} + (1 - \beta\xi)\hat{mc}_t + \pi_t, \tag{1.24}$$

where $\hat{mc}_t = mc_t - mc = -(\lambda_t - \lambda)$ stands for the deviation from the average real marginal cost mc_t in economy from its value in steady state mc or, equivalently, the minus deviation of the average markup λ_t in economy from its value in steady state $\lambda = -mc$. Based on that equation, the average markup may be determined as the function of the expected optimal price in the next period:

$$\lambda_t = \lambda + \frac{\beta\xi}{(1 - \beta\xi)}\mathbb{E}_t(p_{t+1}^\star - p_t^\star).$$

Consequently, it may be seen that the model with price rigidities shall not be closed such as the frictionless model or model with a representative monopolist did. Average markups depend on the expected increment of the optimal level of prices, which within the model are determined endogenously. Due to disturbances in the prices formation process changes of the nominal interest rate are not one to one translated into the expected inflation but affect the real interest rate. In reply to the monetary policy shock not all firms shall be able to determine markup on the level of the optimal response. Therefore, as long as the monetary shock has not expired, the monetary policy shall influence real categories.

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Optimal price policy of firms implies the inflation equation in the following form:

$$\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa \tilde{y}_t,$$

where $\kappa = \frac{(1-\xi)(1-\beta\xi)(\sigma_c+\sigma_h)}{\xi}$, and $\tilde{y}_t = y_t - y_t^n$ stands for the output gap, i.e. the difference between product y_t and the so-called natural product y_t^n, namely the product that would be materialised in the case of perfectly flexible prices, meaning one that may be expressed by the level of technology a_t. The equation is known as the new-Keynesian Philips curve. Euler equation brings about the dependence of output gap on the path of differences between the real interest rate and its natural level, i.e. to the so-called dynamic IS equation:

$$\tilde{y}_t = \mathbb{E}_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma_c}(i_t - \mathbb{E}_t\{\pi_{t+1}\} - r_t^n),$$

where:

$$r_t^n = \rho + \sigma_c \mathbb{E}_t \frac{1+\sigma_h}{\sigma_c(1-\alpha)+\sigma_c+\alpha}(1-\rho)a_t$$

is the natural interest rate. The last three equations and the process defining the technology $a_t = \rho_a a_{t-1} + \eta_t$ define the form of the new Keynesian model with regard to π_t, $\tilde{y}_t$, r_t^n and a_t. Nevertheless, as may be expected in the light of the presented motivation, equations includes the i_t variable — the nominal interest rate, the level of which is determined by the central bank. In order to be able to solve the model, an equation for i_t,shall be introduced, i.e. it shall be decided how the central bank is to control the nominal interest rate. Most commonly, an equation for i_t is assumed in the form of the so-called monetary policy rules. They are *ad hoc* assumed equations defining the response of interest rate to the changes in macroeconomic aggregates. If the interest rate reacts to inflation changes and output gap, the form of the rule is following:

$$i_t = \rho + \phi_y \tilde{y}_t + \phi_\pi \pi_t.$$

Such type of rule is known as Taylor rule.

2
DSGE model – anatomy

Formally, a DSGE model is a system of first-order conditions for economic agents, conditions constraining their decisions and equilibrium conditions. Should all the variables of the model, except structural shocks, be gathered in y_t, vector and structural shocks in vector ϵ_t, the DSGE model shall be a stochastic difference equation in the following form[1]:

$$\mathbb{E}_t \left\{ f(y_{t+1}, y_t, y_{t-1}, \epsilon_t) \right\} = 0, \tag{2.1}$$

where $\mathbb{E}\{\epsilon_t | I_{t-k}\} = 0$ for $k \geq 1$ and I_t represents the information about the state of the world possessed by the economic agents in t period, i.e. the so called information set. The form (2.1) is called a structural form of a DSGE model, i.e. it represents the economic mechanism determining the dynamic properties of economy. The y_t vector is called a vector of endogenous variables of the model, state vector or simply state. The mechanism represented by the structural form of the model includes the encoded decision rules of economic agents. In order to solve the model, it is necessary to decode the rules and represent them in operational form, i.e. form enabling their direct implementation. The operationality of the model solution from the perspective of t period is understood to be such property that today's y_t state may be determined based on the knowledge of its previous value y_{t-1} and the value of ϵ_t, shocks that have materialised today. Both former states and today's shocks come within the information set I_t, possessed by the economic agents in t period. Knowing the solution of the model, economic agents are able to implement their decision-making rules. Formally, the solution of model (2.1) is any g function in the form of:

$$y_t = g(y_{t-1}, \epsilon_t), \tag{2.2}$$

[1] Vector y_t includes all of the variables of the model except structural shocks. In particular, it may include autoregressive variables in the form of: $\theta_t = \rho\, \theta_{t-1} + \varepsilon_t$, where ε_t is a structural shock. Variables in the form of $\theta_t = \rho\, \theta_{t-1} + \varepsilon_t$ are called model disturbances. From the point of view of model representation in the so called state space, disturbances are endogenous variables, i.e. states.

which solves the structural model, i.e. shall satisfy the equation (2.1):

$$\mathbb{E}_t \left\{ f \left(g \left(g(y_{t-1}, \epsilon_t \right), \epsilon_{t+1}), g(y_{t-1}, \epsilon_t), y_{t-1}, \epsilon_t \right) \right\} = 0. \tag{2.3}$$

The condition implies that the decision rules represented by the g function are actually possible and optimal. The g function is called a reduced form of a DSGE model, its solution, a decision rule, the law of motion for the state or policy function. The latter name originates from the optimal control theory. The g function represents the decision rules of economic agents decoded from the structural form of the model, describes the laws of motion for all of the variables of the model, and thus, determines the dynamics of the model. Equations of motion determine the values of all variables of the model in t period, i.e. the state vector y_t, based on the previous state of economy y_{t-1} and structural shocks ϵ_t that occurred in the current period. Determining the g function is equivalent to solving the DSGE model.

Solving a DSGE model is a difficult task in general, due to the non-linear form of the f function and absence of adequately general analytical or numerical methods. Therefore, instead of considering the non-linear model (2.1), its linear approximation is being considered. This is the so-called expansion into the first-order Taylor series, (eg. Birkholc, 2002). This considerably simplifies the procedure of model solving, however, does not lead to determination of the g function but a linear approximation. Moreover, the model is most often expressed in logarithms of variables such that after linearisation, the values of their increments[2] dy_t have the interpretation of percentage deviations from the point in the neighbourhood of which the model is approximated. Such approximation is called a logarithmic-linear (log-linear) approximation. If the model is log-linearised in the neighbourhood of a steady state, the values of the increments of variables dy_t are interpreted as the percentage deviations from the value in the steady state, i.e. from the values consistent with the long-term equilibrium.

The expansion of model (2.1) into the Taylor series takes place in the neighbourhood of a chosen point. It has become customary that the point is the point of long-term economy equilibrium, i.e. model (2.1) is expanded into first-order Taylor series in the neighbourhood of a deterministic steady state, in this chapter identified as $\bar{y}$. The deterministic steady state is defined as each $\bar{y}$ point satisfying:

$$f(\bar{y}, \bar{y}, \bar{y}, 0) = 0. \tag{2.4}$$

In other words, steady state is a state for which self-repeating shall be optimal, if no structural shock occur — economic agents in the steady state have no motivation to change their decisions. Thus, steady state is also a fixed point of g representation, assuming zero values of structural shocks $\epsilon_t = 0$:

$$\bar{y} = g(\bar{y}, 0). \tag{2.5}$$

Since $y_{t+1} = g(g(y_{t-1}, \epsilon_t), \epsilon_{t+1})$ and $f(\bar{y}, \bar{y}, \bar{y}, 0) = 0$, expansion of (2.1) into first-order Taylor

[2]Formally — the values of the differentials.

series in the neighbourhood of steady state $\bar{y}$ has the form of:

$$\begin{aligned}\mathbb{E}_t\Bigg\{ &\left(\frac{\partial f}{\partial y_{t+1}}(\bar{y})[\frac{\partial g}{\partial y}(\bar{y})]^2 + \frac{\partial f}{\partial y_t}(\bar{y})\frac{\partial g}{\partial y}(\bar{y}) + \frac{\partial f}{\partial y_{t-1}}(\bar{y})\right) dy_{t-1} + \\ &\left(\frac{\partial f}{\partial y_{t+1}}(\bar{y})\frac{\partial g}{\partial y}(\bar{y})\frac{\partial g}{\partial \epsilon}(\bar{y}) + \frac{\partial f}{\partial y_t}(\bar{y})\frac{\partial g}{\partial \epsilon}(\bar{y}) + \frac{\partial f}{\partial \epsilon}(\bar{y})\right) d\epsilon_t\Bigg\} = 0.\end{aligned} \tag{2.6}$$

Denoting matrices $\frac{\partial f}{\partial y_{t+1}}(\bar{y})$, $\frac{\partial g}{\partial y}(\bar{y})$, $\frac{\partial f}{\partial y_t}(\bar{y})$, $\frac{\partial f}{\partial y_{t-1}}(\bar{y})$, $\frac{\partial g}{\partial \epsilon}(\bar{y})$ and $\frac{\partial f}{\partial \epsilon}(\bar{y})$ by A_0, A, A_1, A_2, B and S respectively, the linear approximation of the model takes the form of:

$$\mathbb{E}_t\left\{(A_0A^2 + A_1A + A_2)dy_{t-1} + (A_0AB + A_1B + S)d\epsilon_t\right\} = 0. \tag{2.7}$$

System 2.7 is a stochastic matrix difference equation with unknown matrices A and B. Determination of A and B matrices satisfying equation (2.7), i.e. solution of equation (2.7), enables the determination of linear approximation of policy function g, because up to the first-order Taylor series expansion around $\bar{y}$ deterministic steady state reads $y_t = g(y_{t-1}, \epsilon_t) \approx \bar{y} + Ady_{t-1} + Bd\epsilon_t$ or, equivalently:

$$dy_t \approx Ady_{t-1} + Bd\epsilon_t.$$

And the differential dy_t represents the deviation of state y_t from the steady state and, if the variables of a model are expressed in the form of logarithms, the deviation is interpreted as percentage deviation. The last equation indicates that the reduced form of a DSGE model has the representation of a VAR model, with accuracy to linear approximation.

As equation (2.7) must be satisfied for every dy_t and $d\epsilon_t$, both addends of a sum under the sign of the expected value must be equal to zero, i.e. it is required that:

$$\mathbb{E}_t\left\{(A_0A^2 + A_1A + A_2)dy_{t-1}\right\} = 0 \tag{2.8}$$

for every t and:

$$\mathbb{E}_t\left\{(A_0AB + A_1B + S)du_t\right\} = 0 \tag{2.9}$$

for every t. The former equation is a stochastic matrix quadratic equation with respect to A, and the latter — if only A has been determined — shall enable to calculate B from the condition:

$$B = -(A_0A + A_0)^{-1}S. \tag{2.10}$$

Thus, the solution of the model boils down to the determination of matrix A from equation 2.9. Special numerical algorithms are applied for that purpose, e.g the basic Blanchard-Kahn method or the Anderson-Moore algorithm.

The solution of linear approximation of a DSGE model or, equivalently, the linear approximation of its reduced form[3]:

$$y_t = Ay_{t-1} + B\epsilon_t$$

is represented in the so-called state-space:

$$\begin{aligned} Y_t &= Hy_t + u_t, \\ y_t &= Ay_{t-1} + B\epsilon_t, \end{aligned} \tag{2.11}$$

and:

$$u_t \sim N(0, R) \quad \text{oraz} \quad \epsilon_t \sim N(0, \mathrm{I}).$$

The former equation is called the measurement equation and presents the relation between observations, namely observable variables Y_t, and endogenous variables y_t of the model, which in the context of model representation in the state-space are called state variables or simply states. Observations may be functions of endogenous variables y_t, functions of exogenous variables, and in particular they may include trends. They may also depend on the values of states in steady state[4]. Random variable u_t is called a measurement error or disturbance. The latter equation is called transition equation and describes the dynamics of state variables y_t, i.e. the already known linear approximation of the model solution. This equation is a linear approximation of the reduced form, i.e. solution, of the model. Hereinafter we shall call it a solution or reduced form. The elements of matrices A, B and H are non-linear functions of model parameters, which shall be identified with θ. Formally, $\theta \in \Theta \subset \mathrm{R}^n$ for some n. Representation of the reduced form of a DSGE model in the state space (2.11) together with the assumptions of ϵ_t shocks distribution and u_t observation errors shall be abbreviated with the symbol $\mathscr{M}$ and called the $\mathscr{M}$ model.

From the statistical — classical — point of view, the $\mathscr{M}$ model may be considered a family of conditional distributions of observables with regard to non-observable data, i.e. with regard to states, shocks, measurement errors and parameters:

$$\left\{p(Y|y, \epsilon, u, \theta), \, y \in \Upsilon, \, \epsilon \in \Xi, \, u \in \Omega, \, \theta \in \Theta\right\}, \tag{2.12}$$

where Ξ is the space of shocks ϵ, Υ is the state space y, Ω is the space of measurement errors u, and Θ is the parameters space. Alternatively, but also from a classical point of view, the model $\mathscr{M}$ represents the family of conditional probability distributions of its variables conditional upon the θ parameters:

$$\left\{p(Y, y, \epsilon, u|\theta), \theta \in \Theta\right\}. \tag{2.13}$$

The form, upon integration of states, shocks and disturbances from the distribution $p(Y, y, \epsilon, u|\theta)$

[3]From now on, to the end of this chapter, we assume that the variables of the model are expressed in the form of percentage deviations from the steady state.

[4]For the simplicity of notation, the representation of model (2.11) makes the observations dependent only on states.

leads to the likelihood function $L(\theta|Y)$:

$$L(\theta|Y) = p(Y|\theta) = \int_{\Omega}\int_{\Xi}\int_{\Upsilon} p(Y, y, \epsilon, u|\theta) dy d\epsilon du, \tag{2.14}$$

which is the central object for the estimation of the model parameters.

From the statistical — Bayesian — point of view the $\mathscr{M}$ model may be considered joint distribution of observations, states, shocks, disturbances and parameters, which gives rise to factorisation into the conditional distribution of observations, states, shocks and disturbances with regard to parameters, and unconditional distribution — the so-called prior distribution — of parameters:

$$p(Y, y, \epsilon, u, \theta) = p(Y, y, \epsilon, u|\theta)p(\theta). \tag{2.15}$$

Upon integration of states, shocks and disturbances from the joint distribution, we receive the so-called kernel of posterior distribution (posterior kernel) $\mathscr{K}(\theta|Y)$:

$$\mathscr{K}(\theta|Y) = \int_{\Omega}\int_{\Xi}\int_{\Upsilon} p(Y, y, \epsilon, u|\theta)p(\theta) dy d\epsilon du = p(Y|\theta)p(\theta) \tag{2.16}$$

being the product of the likelihood function of parameters and density of the probability of their prior distribution. The kernel of posterior distribution of parameters is the central object of Bayesian estimation of parameters.

The process of solving a DSGE model requires calculations made on matrices A_0, A_1, A_2 i S, which define the linear approximation of its structural form. In order to solve the model, the elements of the matrices — parameters of the model — shall be assigned numerical values. A solved DSGE model, i.e. its reduced form, may next serve various simulation experiments, both positive and counterfactual. It is also possible to quantify the response of economy to structural shocks, estimate the values of shocks in a sample, forecast endogenous variables and their functions — the so-called observable variables. Nevertheless, all the experiments require the solution of the model, i.e. prior assignment of numerical values to its parameters. This may be done in many ways.

2.1 Values of parameters

In order to be able to solve a model and then carry out analyses based thereon, the parameters of the model shall be assigned numerical values. This may be done at least in two ways — the values of parameters may be calibrated or estimated. In practice, calibration and estimation are applied in a complementary manner, i.e. some of the parameters of the model are calibrated and some of them are estimated. Among the methods of estimation, the most often used is the maximum likelihood method, method of moments and Bayesian estimation. Currently, the Bayesian approach becomes more and more popular. This initially resulted from the fact that the estimation of DSGE models with the maximum likelihood method with the use of deterministic optimisation algorithms results in numerical problems that prevent the satisfactory use of the method. Currently, the Bayesian approach to DSGE models estimation becomes more common than only the methods of solving technical problems. This results mainly from the fact that the approach enables the consideration of knowledge a priori, in an explicit and formal manner.

2.1.1 Calibration

Calibration may entail the assignment of values to the model parameters in an arbitrary manner or can be based on other research. As an example, there are empirical studies dedicated to the estimation of elasticity of demand for domestic and foreign products in various sectors of many economies. The elasticities specified therein may be assumed to be the one chosen for the DSGE model. On the other hand, in the calibration process, such values of DSGE model parameters may be assumed as to make its dynamics (response of economy to structural shocks) and long-term equilibrium correspond with economic intuition, the results of other studies or indication of models of more empirical purpose, such as VAR class models. In practice, calibration is a mixture of both techniques. From the point of view of Bayesian estimation, calibration boils down to the assignment to the calibrated parameters a possibly informative prior distribution. Thus, such method of reasoning enables the formal reflection of that what is common in the classical approach but happens *ad hoc* and in an implicit manner.

2.1.2 Maximum likelihood estimation

Among the classical methods of estimation the most often used is the maximum likelihood method. It entails the determination of the values of θ, parameters, for which the likelihood function $\mathscr{L}(\theta|Y)$ obtains the highest value. The method requires the determination of the analytical form of $\mathscr{L}(\theta|Y)$ function or development of a numerical method of its approximation. Formally, the likelihood function $\mathscr{L}(\theta|Y)$ is a density function of conditional distribution of observables Y conditional with respect to θ parameters::

$$\mathscr{L}(\theta|Y) = p(Y|\theta).$$

The likelihood function may be considered a function measuring the likelihood of θ parameters in the sense that it takes higher value for such configuration of parameters for which the density of probability — determined by $p(Y|\theta)$ that data Y was generated by the $\mathscr{M}$model whose parameters have θ value — is larger. The likelihood function $\mathscr{L}(\theta, Y) = p(Y|\theta)$ originates as a result of integration of y states, ϵ shocks and u disturbances from the conditional density $p(Y, y, \epsilon, u|\theta)$. Technically, marginalisation of states, shocks and disturbances takes place in the process of their filtration. Filtration is conditioned with regard to the $\mathscr{M}$ model. In order to emphasise this fact, the likelihood function is denoted by $\mathscr{L}(\theta|Y, \mathscr{F}(\mathscr{M}))$, where $\mathscr{F}(\mathscr{M})$ stands for filtration made with the use of the $\mathscr{M}$ model.

From the operational point of view, likelihood $\mathscr{L}(\theta|Y, \mathscr{F}(\mathscr{M}))$ is measured by using an iterative formula:

$$\mathscr{L}(\theta|Y, \mathscr{F}(\mathscr{M})) = p(Y|\theta, \mathscr{F}(\mathscr{M})) = p(Y_0)\prod_{t=1}^{T} p(Y_t|I_{t-1}),$$

where $I_t = \{Y_0, Y_1, ..., Y_t\}$ denotes the information set in t period. The use of such formula requires the determination of probability density $p(Y_t|I_{t-1})$ for $t = 1, 2, ..., T$. Assuming the normality of shocks and disturbances, the density is the density of normal distribution:

$$\begin{aligned} p(Y_t|I_{t-1}) =& (2\pi)^{-\frac{Tk}{2}} |(\mathbb{D}(Y_t|I_{t-1})|^{-\frac{1}{2}} \times \\ & \times \exp\left\{-\frac{1}{2}\left(Y_t - \mathbb{E}(Y_t|I_{t-1})\right)' (\mathbb{D}(Y_t|I_{t-1}))^{-1} \left(Y_t - \mathbb{E}(Y_t|I_{t-1})\right)\right\}. \end{aligned}$$

In order to calculate the likelihood of parameters, it is vital to know expected values $\mathbb{E}(Y_t|I_{t-1})$ and variances $\mathbb{D}(Y_t|I_{t-1})$ for $t = 1, 2, ..., T$. For that purpose Kalman filter is used, the equations of which are — in the case of normal distribution — identical as the analytically derived formulas for the values.

2

In order to determine values of the θ parameters maximising the likelihood function $\mathscr{L}(\theta|Y, \mathscr{F}(\mathscr{M}))$ iterative methods based on Newton-Raphson method are most often used (see Żak and Chong, 2008). The general sequence of estimation of DSGE model parameters with the maximum likelihood method is the following:

1. Parameters assume the value θ_k, $k = 0, 1, 2, ..., K$. For $k = 0$ the initial value of parameters must be assumed.
2. Solution of the $\mathscr{M}$ model for $\theta = \theta_k$, i.e. determination of matrices $A = \frac{\partial g}{\partial y}|_{\theta=\theta_k}(\bar{y})$ and $B = \frac{\partial g}{\partial \epsilon}|_{\theta=\theta_k}(\bar{y})$.
3. Determination of the value of the likelihood function for $\theta = \theta_k$, i.e. determination of $\mathscr{L}(\theta_k|Y, \mathscr{F}(\mathscr{M}))$ in one run of the Kalman filter.
4. Numerical determination of the first two differentials of function $\mathscr{L}(\theta|Y, \mathscr{F}(\mathscr{M}))$ for $\theta = \theta_k$, i.e. the gradient $\bigtriangledown_{\mathscr{L}(\theta_k|Y,\mathscr{F}(\mathscr{M}))}$ and hessian $\mathrm{H}_{\mathscr{L}(\theta_k|Y,\mathscr{F}(\mathscr{M}))}$.
5. Making an iterative step of the Newton-Raphson method, i.e. assumption that $\theta_{k+1} = \theta_k - \bigtriangledown_{\mathscr{L}(\theta_k|Y,\mathscr{F}(\mathscr{M}))}(\mathrm{H}_{\mathscr{L}(\theta_k|Y,\mathscr{F}(\mathscr{M}))})^{-1}$.
6. Verification of algorithm convergence criteria.

DSGE model estimation with the maximum likelihood method, with the use of Newton-Raphson method is time consuming. This is because Newton-Raphson method is iterative and each iteration requires the solution of a model (point 2) and running the Kalman filter in order to determine the value of the likelihood function $\mathscr{L}(\theta|Y, \mathscr{F}(\mathscr{M}))$ (point 3). Additionally, it requires numerical determination of a gradient and Hessian of the likelihood function, which is a numerically instable task. Finally, Newton-Raphson method requires the specification of initial values of θ_0 parameters and is a local method — i.e. it does not guarantee that the obtained estimation θ_K maximises globally the likelihood function. Hence, stochastic optimisation methods have recently become more popular, particularly simulated annealing. Yet, the largest problem related to the application of the maximum likelihood method to estimate DSGE model parameters is insufficient curvature of the likelihood function, namely the presence of the so-called plateau. Deterministic iterative optimisation methods are then ineffective and usually the obtained θ_K estimation is not much different from the specified initial values of θ_0.Therefore, mainly for that reason, DSGE models are often estimated with Bayesian methods, which allow to increase the likelihood function curvature by virtue of a change of its shape with the so-called prior distribution. Maximum likelihood method poses also other difficulties.

2.1.3 Bayesian estimation

Although the initial reasons for applying Bayesian estimation to DSGE models were strictly technical, the Bayesian approach makes the nature of statistical analysis of DSGE models other than classical. Bayesian inference fundamentally differs from the classical one. Unlike as in the classical case, in the Bayesian interference the probability of the formulated hypotheses — and the statement that θ parameters of the $\mathscr{M}$ DSGE model take a specific value illustrates such hypothesis[5] — is assessed not only on the basis of how strongly the Y data confirm the hypothesis, but also based on subjective assessment of the probability that is not related to the Y data. Such subjective assessment is called a priori assessment. A priori assessment merged with classical likelihood of parameters, i.e. with the likelihood function $\mathscr{L}(\theta|Y,\mathscr{F}(\mathscr{M}))$, results in the a posteriori assessment of probability of the truth of the postulated hypothesis. The application of the Bayesian approach to estimate parameters equals — from the numerical point of view — the use of the maximum likelihood method with full information. Hence, it is also possible to formally include a priori knowledge, consider uncertainty with parameters in the form of posterior distribution and make formal small-sample inference that does not refer to asymptotic properties.

Formally, the Bayesian estimation task entails the determination of the posterior distribution:

$$p(\theta|Y) = \frac{p(\theta)p(Y|\theta)}{p(Y)}$$

of parameters θ. The analytical determination of posterior distribution is usually impossible, so simulation approach, e.g. Metropolis-Hastings algorithm, is applied. For point estimates of θ parameters one assumes, for example, the maximum of posterior density function in order to approximate its modal value. Density $p(Y|\theta)$ is obtained based on the $\mathscr{L}(\theta|Y,\mathscr{F}(\mathscr{M}))$ likelihood, which is determined in the process of filtration conditionally with regard to the model $\mathscr{M}$. Such procedure leads to posterior distribution:

$$p(\theta|Y,\mathscr{F}(\mathscr{M})) = \frac{p(\theta)p(Y|\theta,\mathscr{F}(\mathscr{M}))}{p(Y|\mathscr{F}(\mathscr{M}))}, \tag{2.17}$$

where $p(Y|\mathscr{F}(\mathscr{M}))$ is a normalising factor — a density function of data subject to filtration $\mathscr{F}(\mathscr{M})$. Analytical determination of marginal distribution $p(Y|\mathscr{F}(\mathscr{M}))$ by calculation is usually. Therefore, while determining the maximum of posterior distribution, the normalising constant tends to be ignored and maximisation is applied only to the product of posterior distribution and likelihood function:

$$\mathscr{K}(\theta|Y,\mathscr{F}(\mathscr{M})) = p(\theta)p(Y|\theta,\mathscr{F}(\mathscr{M}))$$

[5] The hypothesis states that the $\mathscr{M}$,model, whose parameters have the θ value, generated the data Y.

i.e. the so-called posterior kernel. Rationalisation of such conduct results from the fact that posterior distribution is proportional to its kernel:

$$p(\theta|Y,\mathscr{F}(\mathscr{M})) \propto \mathscr{K}(\theta|Y,\mathscr{F}(\mathscr{M})).$$

Prior distribution is assumed arbitrarily. This may be both, informative distribution and uninformative distribution. The informative nature of prior distribution increases along with the decrease of its variance, i.e. subjective assessment draws a relatively growing attention along with a drop in variance. If prior distribution is highly informative, it is considered to have a "strong prior", in contrast to uninformative distribution.

Determination of posterior kernel requires — as in the case of the maximum likelihood method — the determination of the likelihood function $\mathscr{L}(\theta|Y,\mathscr{F}(\mathscr{M}))$, and strictly speaking — its logarithm, as instead of maximisation of the kernel $\mathscr{K}(\theta|Y,\mathscr{F}(\mathscr{M}))$, it is possible to equivalently maximise its logarithm:

$$\ln \mathscr{K}(\theta|Y,\mathscr{F}(\mathscr{M})) = \ln p(\theta) + \ln p(Y|\theta,\mathscr{F}(\mathscr{M})),$$

which is a sum of prior distribution logarithm $\ln p(\theta)$ and a log-likelihood function $\ln p(Y|\theta,\mathscr{F}(\mathscr{M}))$. The value of likelihood function may be determined with the use of Kalman filter equations.

2.2 Kalman filter

A DSGE model may be perceived from many perspectives. It may be perceived as a system of conditions defining the optimal behaviour of economic agents or as a stochastic matrix difference equation. It may also be viewed as the so-called filter. Let us assume that we have observable data, $Y = \{Y_T, Y_{T-1}, ..., Y_0\}$, namely observations from periods $t = 0, 1, 2, ..., T$. The filter is a mechanism processing Y data in order to determine the expected values and variances of $y_t$endogenous variables of the model, Y_t observations, ϵ_t shocks and u_t disturbances in t period, under the condition of an informative set I_t, then we speak of a filter, or under the condition of an information set I_T, then then we speak about a smoother. The task of determination of the expected values and variances under the condition of set I_{t+k} for $k = 0, 1, 2, ..., T-t$ is called a filtration, while the task of determining the expected values and variances under the condition of set I_{t-k} for $k = 1, 2, ..., t$ is called a prediction. Since we operate on a linear approximation of a DSGE model, the approximation is a linear filter. There are many methods of filtration and prediction. In the context of DSGE models, most often used is the so-called Kalman filter. The equations of the Kalman filter may be derived from the optimisation calculus by minimisation of one-period forecast-errors of observable variables of the model. Assuming the normality of shocks and measurement errors of the model, filtration and prediction may also be derived analytically from the probabilistic viewpoint. Obtained derivations coincide with Kalman filter equations. Hence the filter has statistical interpretation and the likelihood function based thereon is not the approximation which would occur in the case of more general assumptions with regard to the distribution of shocks and measurement errors, but constitutes an exact result.

The effect of the filter application has several dimensions. Firstly — the determination of the expected values and variances:

$$\mathbb{E}(y_t|I_t) \quad \text{and} \quad \mathbb{D}(y_t|I_t) \tag{2.18}$$

or:

$$\mathbb{E}(y_t|I_T) \quad \text{and} \quad \mathbb{D}(y_t|I_T) \tag{2.19}$$

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of endogenous variables, which enables one to know the estimated values of those among them which are not perfectly observable, i.e. do not come within set Y. Secondly — determination of the expected values and variances:

$$\mathbb{E}(\epsilon_t|I_t) \quad \text{and} \quad \mathbb{D}(\epsilon_t|I_t) \tag{2.20}$$

or:

$$\mathbb{E}(\epsilon_t|I_T) \quad \text{and} \quad \mathbb{D}(\epsilon_t|I_T) \tag{2.21}$$

of structural shocks, which is equivalent to their identification. Thirdly — determination of the expected values and variances:

$$\mathbb{E}(Y_t|I_{t-1}) \quad \text{and} \quad \mathbb{D}(Y_t|I_{t-1}) \tag{2.22}$$

enables one to determine the value of the likelihood function $\mathscr{L}(\theta|Y,\mathscr{M})$ for θ. Determination of the value of $\mathscr{L}(\theta|Y,\mathscr{M})$ is an element of estimation both according to the maximum likelihood method and Bayesian estimation.

Deriving the Kalman filter that has statistical interpretation requires an assumption that structural shocks of the model have normal distribution. This assumption, provided that it is actually fulfilled, has some advantages. It appears that Kalman filter — if shocks reflect normal distribution - is an optimal filter with regard to minimisation of variances of the estimations of states, shocks and observations. If the assumption of normality is not fulfilled, the Kalman filter is optimal in the class of linear filters, however, there are more effective non-linear filters in the aforementioned sense. In consequence of the assumptions regarding the normality of structural shocks, the density function of Y_t — conditional upon I_{t-1}, is a density function of a multidimensional normal distribution, so its analytical form is known. Kalman filter enables, therefore, the determination of the distribution of Y_t observations in each t period, conditional with regard to information set of $t-1$ period. Since it is a multidimensional normal distribution, an analytical form of the iterative likelihood function of Y observations is known.

For the initial values of $\mathbb{E}(y_0)$ and $\mathbb{D}(y_0)$, the Kalman filter includes the following equations. The expected values $\mathbb{E}(y_t|I_t)$ and variances $\mathbb{D}(y_t|I_t)$ of states for $t=1,2,...,T$ are determined recurrently based on the following relations:

$$\begin{aligned} \mathbb{E}(y_t|I_t) &= A\mathbb{E}(y_{t-1}|I_t) + B\,\mathbb{E}(\epsilon_t|I_t), \\ \mathbb{D}(y_t|I_t) &= A\mathbb{D}(y_{t-1}|I_t)A' + B\mathbb{D}(\epsilon_t|I_t)B', \end{aligned} \tag{2.23}$$

while assuming the normality of structural shocks, the following applies:

$$\begin{aligned}\mathbb{E}(y_{t-1}|I_t) &= \mathbb{E}(y_{t-1}|I_{t-1}) + \mathrm{cov}(y_{t-1}, Y_t|I_{t-1})\,\mathbb{D}(Y_t|I_{t-1})^{-1}(Y_t - \mathbb{E}(Y_t|I_{t-1})),\\ \mathbb{D}(y_{t-1}|I_t) &= \mathbb{D}(y_{t-1}|I_{t-1}) - \mathrm{cov}(y_{t-1}, Y_t|I_{t-1})\,\mathbb{D}(Y_t|I_{t-1})^{-1}\mathrm{cov}(y_{t-1}, Y_t|I_{t-1}).\end{aligned} \tag{2.24}$$

The expected values $\mathbb{E}(\epsilon_t|I_t)$ and variances $\mathbb{D}(\epsilon_t|I_t)$ of structural shocks for $t = 1, 2, ..., T$ are determined recursively, assuming the normality of structural shocks, based on the following relationships:

$$\begin{aligned}\mathbb{E}(\epsilon_t|I_t) &= \mathrm{cov}(\epsilon_{t-1}, Y_t|I_{t-1})\,\mathbb{D}(Y_t|I_{t-1})^{-1}(Y_t - \mathbb{E}(Y_t|I_{t-1})),\\ \mathbb{D}(\epsilon_t|I_t) &= \Psi - \mathrm{cov}(\epsilon_t, Y_t|I_{t-1})\,\mathbb{D}(Y_t|I_{t-1})^{-1}\mathrm{cov}(\epsilon_t, Y_t|I_{t-1}),\end{aligned} \tag{2.25}$$

The expected values $\mathbb{E}(Y_t|I_{t-1})$ and variances $\mathbb{D}(Y_t|I_{t-1})$ of observations for $t = 1, 2, ..., T$ are determined recursively based on the following relationships:

$$\begin{aligned}\mathbb{E}(Y_t|I_{t-1}) &= HA\,\mathbb{E}(y_{t-1}|I_{t-1}) + A_x\, x_t,\\ \mathbb{D}(Y_t|I_{t-1}) &= HA\,\mathbb{D}(y_{t-1}|I_{t-1})A'H' + R.\end{aligned} \tag{2.26}$$

2.3 Metropolis-Hastings algorithm

Bayesian estimation entails the determination of posterior distribution $p(\theta|Y, \mathscr{F}(\mathscr{M}))$ of parameters. The distribution is not normal with regard to θ parameters (it is normal with regard to non-linear functions of θ parameters). Thus, its analytical form is not known in the general case. The shape of the density function $p(\theta|Y, \mathscr{F}(\mathscr{M}))$, or, equivalently, the shape of the $\mathscr{K}(\theta|Y, \mathscr{F}(\mathscr{M}))$ function is approximated with simulation methods. For that purpose sampling methods are used, most often the Metropolis-Hastings algorithm. The result of the operation of the algorithm is a sequence of parameters $\Theta = \{\theta_0, \theta_1, ..., \theta_M\}$, the subsequence of which $\theta_m, \theta_{m+1}, ..., \theta_M$ originates — for an adequately large m — from unknown posterior distribution $p(\theta|Y, \mathscr{F}(\mathscr{M}))$. Formally, the Θ sequence is a Markov chain, whose stationary distribution is the $p(\theta|Y, \mathscr{F}(\mathscr{M}))$. distribution. The approximation of posterior distribution $p(\theta|Y, \mathscr{F}(\mathscr{M}))$ involves the determination of a histogram of the elements of the Θ set. Based on the members of the Θ set, also the moments of parameter estimates may be determined.

Let θ_0 be an initial value of parameters, e.g. the modal value of $\mathscr{K}(\theta|Y, \mathscr{F}(\mathscr{M}))$ kernel of $p(\theta|Y, \mathscr{F}(\mathscr{M}))$ posterior distribution, let Σ be a numerically determined second differential of $\mathscr{K}(\theta|Y, \mathscr{F}(\mathscr{M}))$ in θ_0, and let γ be a positive constant, and let us assume that $\Theta = \emptyset$. The k-th, $k = 1, 2, ..., M$, step of Metropolis-Hastings algorithm runs as follows:

1. Draw θ from the normal distribution $N(\theta_{k-1}, \gamma\Sigma)$ with the expected value θ_{k-1} and variance $\gamma\Sigma$.
2. Assume $\theta_k = \begin{cases} \theta, & \text{with probability } \min(1, r);\\ \theta_{k-1}, & \text{otherwise.}\end{cases}$
   where acceptance threshold r is specified by: $r = \frac{\mathscr{K}(\theta|Y, \mathscr{F}(\mathscr{M}))}{\mathscr{K}(\theta_{k-1}|Y, \mathscr{F}(\mathscr{M}))} = \frac{p(\theta|Y, \mathscr{F}(\mathscr{M}))}{p(\theta_{k-1}|Y, \mathscr{F}(\mathscr{M}))}$.
3. Assume: $\Theta = \Theta \cup \{\theta_k\}$.

Calculation of the value of $\mathscr{K}(\theta|Y,\mathscr{F}(\mathscr{M}))$ (point 2) requires making one run of the Kalman filter, thus, the filter is run in each iteration of the Metropolis-Hastings algorithm. The Metropolis-Hastings method is highly time-consuming, as it requires that chain Θ be convergent to stationary distribution, which happens for very large M values from the practical point of view. There are analytical tools, i.e. convergence diagnostics, which enable the determination of Θ chain convergence. Without convergence verification, approximation of posterior distribution $p(\theta|Y,\mathscr{F}(\mathscr{M}))$ is not credible.

To verify convergence of Θ we may apply Gelman and Rubin diagnostic or the Geweke diagnostic. The Gelman and Rubin diagnostic suggests that for, each parameters, the shrink or scale reduction factor:

$$r = \sqrt{\frac{(1-\frac{1}{n})W + \frac{1}{n}B}{W}}$$

should not be too hign (perhaps lower than $1.1 - 1.2$), where W and B are estimates of within and between chain variances of this parameter when length of each chain equals n. The Geweke test simply applies the difference of means test to two overlapping parts of the chain to check if the two parts of the chain come from the distribution with the same mean.

2.4 Model selection

In the process of model building one usually works with alternative specifications or alternative calibrations of the same model. At the end of the day a single specification is used[6]. To select a single model $\mathscr{M}$ from a family of models $\mathscr{W}$ we may want to compare them in a pairwise manner by ratios of their posterior distributions (the so called posterior odds ratio):

$$\frac{p(\mathscr{M}_1)|Y}{\mathscr{M}_2|Y} = \frac{p(\mathscr{M}_1)p(Y|\mathscr{F}(\mathscr{M}_1))}{p(\mathscr{M}_2)p(Y|\mathscr{F}(\mathscr{M}_2))} \tag{2.27}$$

for $\mathscr{M}_1, \mathscr{M}_2 \in \mathscr{W}$ where:

$$p(Y|\mathscr{F}(\mathscr{M})) = \int_{\theta\in\Theta} p(\theta, Y|\mathscr{F}(\mathscr{M}))d\theta = \int_{\theta\in\Theta} p(\theta)p(Y|\theta,\mathscr{F}(\mathscr{M}))d\theta \tag{2.28}$$

is a marginal density of data Y provided that model $\mathscr{M}$ is used. This integral in most cases is a difficult one and has to approximated. One way to approximate (2.28) is to assume a functional form of the posterior kernel. In case of normal distribution this method is known as Laplace approximation and the estimate of $p(Y|\mathscr{F}(\mathscr{M}))$ is:

$$p(Y|\mathscr{F}(\mathscr{M})) = (2\pi)^{\frac{n}{2}}\sqrt{(|\Sigma_{\hat{\theta}_{\mathscr{M}}}|)}p(\hat{\theta}|Y,\mathscr{F}(\mathscr{M}))p(\hat{\theta}_{\mathscr{M}}|\mathscr{F}(\mathscr{M})) \tag{2.29}$$

where $\hat{\theta}_{\mathscr{M}}$ and $\Sigma_{\hat{\theta}_{\mathscr{M}}}$ denote estimate of θ using model $\mathscr{M}$ (e.g. the posterior mode) and its covariance.

[6]However, in order to reduce the modele selection risk, one could in principle work with many models using methods of bayesian averaging for policy experiments like stochastic simulations or forecasting.

Comparison of posterior odds ratios is a technical procedure. It should be supplemented by verification of models' in sample and, if possible, out of sample forecasting power as well as by qualitative inspection of models' theoretical structure.

2.5 Applications

With a reduced form of a DSGE model, it is possible to do several standard exercises: to identify structural shocks, analyse responses of endogenous (observable) variables to structural shocks, perform variance decomposition of endogenous variables, carry out historical decompositions and forecasts.

All the exercises are based on the property that the reduced form of a DSGE model, i.e. its solution, has a structure of a VAR model in reduced form:

$$y_t = Ay_{t-1} + B\epsilon_t$$

Based on Y data all the simulation exercises that have been developed for VAR class models may be done for a DSGE model. Let us shortly discuss them in the following paragraphs.

2.5.1 Structural shocks identification

The basic exercise that may be done on basis of the reduced form DSGE model is identification of structural disturbances ϵ_t in a sample, i.e. for periods $0, 1, 2, ..., T$. Shocks identification consists in derivation of expected values $\mathbb{E}(\epsilon_t|I_t)$ and variances $\mathbb{D}(\epsilon_t|I_t)$. As it has been said, this may be done in the course of one run of the Kalman filter. In this context, solution or the reduced form of a DSGE model may be considered not only as a VAR model but also as a linear filter.

2.5.2 Impulse response analysis

Analysis of response functions (or impulse response analysis) replies to the question of what happens in the economy after the occurrence of a structural shock. More precisely, the analysis aims to quantify the responses of endogenous variables in periods t, $t+1$,..., i.e. values of $y_t, y_{t+1}, ...$, to an impulse from a structural shock ϵ_t in period t. The dynamic reaction of the economy may be determined on the basis of the reduced form of a DSGE model, with the use of the following relationship:

$$\frac{\partial y_{t+k}}{\partial \epsilon_t} = A^k B\epsilon_t,$$

where:

$$A = \frac{\partial g}{\partial y}(\bar{y}) \text{ oraz } B = \frac{\partial g}{\partial \epsilon}(\bar{y})$$

Presentation of the response of endogenous variables to a shock which takes place in period t depending on index $k \geq 1$ is called an impulse response function (shortly IRF). It is assumed, at the same time, that before t period the economy stays in the long-term equilibrium $\bar{y}$, in t period a structural shock ϵ_t, occurs, while in the periods $t+1, t+2, ..., t+k$ no structural shocks occur, i.e. $\epsilon_{t+1} = \epsilon_{t+2} = ... = \epsilon_{t+k} = 0$.

Based on the above relationship, it may be seen that the necessary and sufficient condition for stability of the long-term equilibrium is convergence of matrix A^k along with k to a zero matrix:

$$\lim_{k\to\infty} A^k = 0.$$

The condition is satisfied if and only if all the eigenvalues of matrix A are smaller in modulus than 1. If this is the case, all the shocks of the model are of transitory or stationary nature. Economy being in a steady state $\bar{y}$, should it be subjected to shock ϵ_t, it shall asymptotically return to the same steady state $\bar{y}$.

Nevertheless, some of the structural shocks do not need to be transitory — they may have permanent, or long-term, non-stationary effects. If a DSGE model specification includes such shocks, the condition of stability is not fulfilled. It is replaced with a more general condition. Should the economy in a steady state $\bar{y}$ be subjected to a shock ϵ_t with permanent effects, it returns to a steady state $\tilde{y}$, different than the one in which it was before the occurrence of the shock, so it is permitted that $\tilde{y} \neq \bar{y}$. Therefore, it is required that upon the occurrence of a shock with long-term effect, the model variables move away from the initial steady state $\bar{y}$ by a finite value at the most, i.e. that $|\bar{y} - \tilde{y}| < \infty$. The condition is satisfied if and only if:

$$\lim_{k\to\infty} A^k = \tilde{A}$$

for a some matrix $\tilde{A}$ with finite elements, i.e. if and only if the all the eigenvalues of matrix A are not larger than 1 in modulus. This condition ensures that the response of the model to any type of structural shock — with stationary or non-stationary effects — shall not be exploding. Shocks with exploding effects are excluded for lack of economic interpretation.

2.5.3 Variance decomposition

Variance decomposition of endogenous variables answers the question which structural shocks have the largest importance for the dynamics of given variables. Thus, it is possible to determine which shocks and to what extent determine the dynamics of the particular variables.

As the reduced form of a DSGE model has the form of an autoregressive equation:

$$y_t = Ay_{t-1} + B\epsilon_t$$

the iterative or mechanical point forecasts of the values of endogenous variables y_{t+h} for $h = 1, 2, ..., H$, read:

$$\hat{y}_{t+h} = A\hat{y}_{t+h-1}$$

and in period t, since dy_t is known (estimated), one assumes $d\hat{y}_t = dy_t$. Thus, the recursive forecast error may be calculated in period $t+h$ from:

$$\Delta_h = y_{t+h} - \hat{y}_{t+h} = \sum_{k=0}^{h-1} A^k B\epsilon_{t+h}$$

as well as the covariance matrix:

$$\mathbb{D}(\Delta_h) = \sum_{k=0}^{h-1} A^k BB'(A^k)'.$$

We shall identify the elements of matrix $\mathbb{D}(\Delta_h)$ by d_{ij}^h, i.e. we shall assume that $\mathbb{D}(\Delta_h) = [d_{ij}^h]$. The diagonal elements d_{ii}^h of matrix $\mathbb{D}(\Delta_h)$ are variances of forecast errors of endogenous variables in the forecast horizon h. By identifying the ij element of matrix $A^k B(B)'(A^k)'$ as p_{ij}^k, we get that the contribution of the j-th structural shock to the variance of an i-th endogenous variable of the model in the horizon of the forecast h amounts to:

$$\sigma_i^h(j) = \frac{\sum_{k=0}^{h-1}(p_{ij}^k)^2}{d_{ii}^h}.$$

2.5.4 Unconditional forecasts

The forecasting process may be of at least dual nature. Firstly, we may be interested in determination of the so-called central forecast path, which usually corresponds to the expected value of forecast variables or the modal value of their predictive distribution. Additionally, we may be interested in quantification of uncertainty related to the determined central path, namely the determination of the distribution of forecast variables in the forecast horizon. The distribution is called a predictive distribution.

Let us assume that the forecast horizon is $h \geq 1$ periods, i.e. we are interested in calculation of forecasts for periods $T+1, T+2, ..., T+h$. The forecasting process may refer both to observable variables, i.e. $Y_{T+1}, Y_{T+2}, ..., Y_{T+h}$ observations, and endogenous variables, i.e. $y_{T+1}, y_{T+2}, ..., y_{T+h}$ states. If the forecast variables are observable variables, their predictive distribution is given by:

$$p_T(Y_{T+1}, Y_{T+2}, ..., Y_{T+h}) = \int_{\Theta} p(Y_{T+1}, Y_{T+2}, ..., Y_{T+h}|\theta) p_T(\theta) d\theta, \tag{2.30}$$

where: $p_T(Y_{T+1}, Y_{T+2}, ..., Y_{T+h}) = p(Y_{T+1}, Y_{T+2}, ..., Y_{T+h}|I_T, \mathscr{M})$, and $p_T(\theta) = p(\theta|I_T)$ is a posterior distribution of θ, where $I_T = Y$. Determination of the central path of observable variables involves, usually, the determination of the modal value of predictive posterior distribution, while the determination of uncertainty related to the forecast based on the central path entails the determination of the distribution of observable variables in the forecast horizon, i.e. distribution with the density $p_T(Y_{T+1}, Y_{T+2}, ..., Y_{T+h})$. The density is a multidimensional integral with a large support and may not be calculated analytically. Therefore, simulation methods are applied. Below we present an algorithm simulating the predictive distribution of observable variables and endogenous variables of the $\mathscr{M}$. Let us assume that $y_+ = \emptyset$ and $Y_+ = \emptyset$.

1. Draw parameters θ parameters from the posterior distribution of the model $\mathscr{M}$, ie. $\theta \in p(\theta|Y)$.

2. Draw states y_T from normal distribution with mean $\mathbb{E}(y_T|I_T)$ and variance $\mathbb{D}(y_T|I_T)$, ie. $y_T \sim \mathrm{N}\left(\mathbb{E}(y_T|I_T), \mathbb{D}(y_T|I_T)\right)$.

3. Draw a sequence of structural shocks $(\epsilon) = (\epsilon_{T+1}, \epsilon_{T+2}, ..., \epsilon_{T+h})$, in which every element has a normal distribution with expected value equal to zero and Ψ, ie. $\epsilon_{T+i} \sim \mathrm{N}(0, \Psi)$, $i = 1, 2, ..., h$. Using the sequence (ϵ) and equation of motion of states generate a respective sequence of state variables $(y) = (y_{T+1}, y_{T+2}, ..., y_{T+h})$.

4. Draw a sequence of observation errors $(u) = (u_{T+1}, u_{T+2}, ..., u_{T+h})$, in which every element has a normal distribution with expected value equal to zero and variance R, ie. $u_{T+i} \sim \mathrm{N}(0, R)$, $i = 1, 2, ..., h$. Using (u) and measurement equation generate a respective sequence of observable variable $(Y) = (Y_{T+1}, Y_{T+2}, ..., Y_{T+h})$.

5. Assume $y_+ = y_+ \cup (y)$ and $Y_+ = Y_+ \cup (Y)$.

2

Distribution $p(\theta|Y)$ in point 1 has been determined with the use of Metropolis-Hastings algorithm. The expected value $\mathbb{E}(y_T|I_T)$ and variance $\mathbb{D}(y_T|I_T)$ have been determined with the use of the Kalman filter.

Repeating steps 1-5, for a large number of times leads to sets y_+ and Y_+. The set y_+ includes realizations of forecast paths of unknown values of endogenous variables in periods $t+1$, $t+2$,...,$t+h$. These paths come from the predictive distribution of endogenous variables. The set Y_+ includes the realizations of forecast paths of unknown values of observable variables in periods $t+1$, $t+2$,...,$t+h$. The paths come from the predictive distribution of observable variables. By inspecting the statistical values of sets y_+ and Y_+ we may learn the central paths of forecasts (e.g. by determining the modal values of sets y_+ and Y_+) and quantify the uncertainties related to them (e.g. by calculating the variance of forecasts). The approximation of predictive distributions of states and observations entails the generation of histograms from the obtained samples of y_+ and Y_+ respectively. Uncertainty of forecast can therefore be modelled by virtue of consideration of the four sources of uncertainty, namely:

1. structural shocks variance,

2. measurement errors variance,

3. variance of the estimator of the current state y_T, and:

4. variance of the estimator of parameters θ[7].

The uncertainty resulting from the variance of measurement errors may be eliminated by assuming $(u) = (0, 0, ..., 0)$ in step three. The uncertainty resulting from the estimation of the current state y_T may be eliminated by assuming $y_T = \mathbb{E}(y_T|I_T)$ in step two. The uncertainty resulting from the estimation of parameters θ may be eliminated by assuming in step one that θ is always the mode of posterior distribution $p(\theta|Y)$.

[7]The uncertainty shall not be present, if parameters have been calibrated.

Part II

Specification of DSGE $\mathbb{SOE}^{\mathrm{PL}-2009}$ model

3
SOE$^{PL–2009}$ – general outline

3

3.1 $\mathbb{SOE}^{\text{Euro}}$ model — prototype of $\mathbb{SOE}^{\text{PL}}$ family models

The family of $\mathbb{SOE}^{\text{PL}}$ models originates directly from the estimated DSGE model of the euro area developed by the analysts of the Central Bank of Sweden (Sveriges Riksbank), see (por. Adolfson et al., 2005b, 2007a) — hereinafter the model shall be referred to as $\mathbb{SOE}^{\text{Euro}}$. The Riksbank's DSGE model for the euro area uses the pattern of a small open economy, which enables its application to the description of the Polish economy. The Swedish analysts acted similarly. They used the $\mathbb{SOE}^{\text{Euro}}$ elements to construct a model describing the economy of Sweden — this is how the RAMSES DSGE model of the Riksbank (see Adolfson et al., 2007b) was created. The $\mathbb{SOE}^{\text{Euro}}$ model was based on the ideas included in the model by L. Christiano, M. Eichenbaum and Ch. Evans (Christiano et al., 2001, 2003, 2005). Also the influence of the model by Smets and Wouters (2002, 2004) may be noticed. The three models define the line or school of constructing DSGE models, from which the family of DSGE $\mathbb{SOE}^{\text{PL}}$ models derives. An important source of ideas we used when modifying the initial specification of the models were the subsequent works by L. Christiano, and in particular Altig et al. (2004a), Christiano et al. (2007c,d) and Christiano et al. (2007a,b). More complete references are provided at the end of the paper.

The family of $\mathbb{SOE}$ models refer to the economic ideas of the $\mathbb{SOE}^{\text{Euro}}$ model by Riksbank. They also use the broader understood methods of constructing and applying DSGE models, as well as Bayesian estimation[1]. The software we use is a modification (usually very far-reaching) of the scripts prepared at Riksbank for the purposes of $\mathbb{SOE}^{\text{Euro}}$. During the several years' work

[1] The authors of the $\mathbb{SOE}^{\text{Euro}}$ model used the work and experience of other researchers, e.g. Schorfheide (2000), the aforementioned F. Smets and R. Wouters, and also (partially) the authors of the Dynare package (M. Juillard, S. Adjemian et al.). On the other hand, the experience of Riksbank has been used by the analysts of the European Central Bank building the NAWM model for the purposes of the ECB (see. Christoffel et al., 2007a). The essence of the methods applied by the ECB may be reconstructed by virtue of analysis of the construction of the YADA package (a collection of scripts of the Matlab software), see Warne (2009).

and repeated reconstruction of the model and of the computation procedures, the logic of work with a DSGE model has remained unchanged — upon log-linearisation of equations, the $\mathbb{SOE}^{PL}$ model is solved numerically (brought to the reduced form) with the Anderson-Moore algorithm (Anderson and Moore, 1985) and, then, expressed in the state space representation, which enables — by the application of the Kalman filter — to determine the value of the likelihood function and further on to apply the formalised (classical or Bayesian) techniques of parameters estimation. It must be emphasised in that context that it is vital to construct a block of measurement equations in the state space model (equation approximating the relations of the variables of a theoretical model with observable variables). In our opinion, the emphasis put on that aspect distinguishes the methods applied by Riksbank (and further by the ECB and by us) from the techniques applied e.g. by a large number of Dynare package users.

3

As we have mentioned before, the starting point for the formation of the $\mathbb{SOE}^{Euro}$ model by Riksbank was the DSGE model of a closed economy by Christiano et al. (to which we shall hereinafter refer as CEE model) — a model representing the new-Keynesian point of view with regard to the economic processes (see Gali, 2008; Woodford, 2003). J. Lindé and his team have supplemented the CEE model specification with issues related to international exchange, following the hints included in the literature of the so-called new open economy macroeconomics, see e.g. Lane (1999). As a result, a model was created in which the optimising (rational) households maximise the utility originating (among others) from the consumption of products manufactured from domestic and imported components. An infinite number of specialised agents manufacture domestic products and import consumption and investment goods. The specialisation of manufacturers enables them to set prices in a manner characteristic of imperfect competition. The price-setting mechanism is related to the appearance of nominal rigidities (delays in adjustment of prices to market conditions) — a phenomenon with which the new Keynesian school explains the effectiveness of macroeconomic (monetary) policy in a short run. The rigidities of the prices of imported and exported products (in the $\mathbb{SOE}^{Euro}$ approximated with Calvo model (Calvo, 1983)) cause also that the exchange rate pass-through is incomplete. Another characteristic of the new Keynesian school are real rigidities (an idea derived from the Real Business Cycle school (RBC)), which along with the stochastic nature of the technical progress explain the business cycle. In the $\mathbb{SOE}^{Euro}$ model it has been assumed that there exists consumption habit persistence, variable capital utilisation, and capital adjustment costs.

The characteristic feature of the $\mathbb{SOE}^{Euro}$ model was also the use of a single, non-stationary disturbance (a stochastic trend) interpreted as the trend of technical progress (see also Altig et al., 2004b, 2005). By including non-stationary disturbance in the specification of a general equilibrium model, we enable the growth of all of the variables indicating the trend (here e.g. investments, consumption, GDP, foreign trade turnover, real wages), while the characteristics of the growth (i.e. characteristics of non-stationary disturbance) are elements of the model specification. Hence, it is possible, at least partially, to exceed the short-term analysis horizon characteristic of business cycle models — the $\mathbb{SOE}^{Euro}$ model had also the potential to explain mid-term trends[2].

Another distinguishing feature of the $\mathbb{SOE}^{\text{Euro}}$ model was a large number of shocks (disturbances) — larger than in other then constructed models — including structural shocks: several technological shocks (stationary, non-stationary, stationary investment-specific), markup shocks (domestic products, imported investment goods, imported consumption goods, exported products), preference shocks (consumption, labour supply, cash demand), as well as observable disturbances (fiscal and originating from the world's economy — derived from separately estimated SVAR models). The authors of the model have emphasised that thanks to the above, the assessment of a relative role of the disturbance in shaping business cycles is possible, and the assessment is more reliable as the parameters of the model are (largely) estimated.

3

In the group of solutions to which the authors of the $\mathbb{SOE}^{\text{Euro}}$ model attached greater significance was the so-called working capital channel, the solution determining the demand of manufacturing firms for money (partial payment of wages in advance, financed from a loan, what translates into direct, positive influence of the interest rate (the cost of working capital loan) on the marginal cost of production of intermediate products and inflation); the demand for money by households results — traditionally — from the utility provided by cash to the households. However, the implemented alternative of working capital (in stochastic version, when the share of payments made in advance is subject to stochastic disturbance) proved to be hardly useful in most of the applications. The reason was also a relatively small role of monetary aggregates in the policy of central banks, therefore, the solution was reduced to deterministic version or even marginalised. It seems to us now, that the addition of the financial sector to DSGE models would restore importance of this solution.

3.2 Family of $\mathbb{SOE}^{\text{PL}}$ models, $\mathbb{SOE}^{\text{PL}-2009}$ version

In the recent years, the original version of the $\mathbb{SOE}^{\text{Euro}}$ model was subject to a series of experiments conducted at the National Bank of Poland (NBP). There were attempts to estimate the model (with Bayesian techniques) on the Polish data. We made experiments with various collections of observable variables, the fiscal rule, the interest rate rule, the construction of premium for foreign exchange risk and the construction of the tax system (more precisely the role of taxes and national insurance contributions in the process of generation of the manufacturing costs). Structural changes were taken into account, as well as stochastic wage markup. Whereas some of the experiments implemented ideas present in other models (e.g. extended risk premium — RAMSES), other attempted to better adjust the model to institutional framework of the Polish economy (taxes, contributions). Experiments with stochastic wage markup were the consequence of problems with interpretation of the labour supply disturbance identified in the sample, etc. In 2008 wider studies were conducted of the problems related to Poland's accession to the euro area. A special version of a DSGE $\mathbb{SOE}^{\text{PL}}_{€}$ created for the purposes of that study (see Grabek and Kłos, 2009) allowed (among others) to compare the method of absorption of disturbances in a

[2] We shall remember, however, that technical progress has an exogenous nature, so the conclusions regarding a several-year's horizon shall be very cautious.

small open economy functioning within a monetary union and outside of it. In that version of the model, the world economy consisted of two areas: the monetary union and the rest of the world.

The above experience, as well as additional ideas resulting from separate research fed into the construction of the newest version of the DSGE model named $\mathbb{SOE}^{PL-2009}$. An important role in designing the changes in the specification was also played by the analyses of accuracy of the forecasts received from the earlier, experimental versions. As an example, the analyses showed that forecasts of the dynamics of investment expenditures and investment deflator were highly imprecise. Although it is very hard to model (and forecast) investment expenditures, it did not justify such enormous errors. On the other hand, due to limited resources (time, computational capacity of computers, etc.) we could not implement in the discussed version the solutions that were the object of our former works (e.g. extension of the financial sector). In the current economic situation these could increase the chances for the model to better explain the ongoing processes and, therefore, probably improve the accuracy of forecasts. In the hereinafter presented version of the model there are also many problems that have not been solved — they have only been outlined. In some cases the applied simplifications are disputable. An example is the labour market in which (as in $\mathbb{SOE}^{PL}$, and $\mathbb{SOE}^{PLEuro}$) unemployment may not occur, the issue of catching up and the changes in the share of foreign trade in the GDP, mid-term trends (appreciation) of foreign exchange rates, or the issue of indebtedness of households and governments. At least some of the problems shall be the subject of our work in future but it is worth mentioning in advance that the growing size of the model is a natural barrier and it may not be expected that one model shall answer all the questions.

Although the $\mathbb{SOE}^{PL-2009}$ version was created in consideration of forecasting applications, the criterion of quality (accuracy) of forecasts has not dominated our choices. In any case, when the quality of forecasts would force the resignation from logic or coherence — the economic contents of the model — priority was given to the economic contents. The $\mathbb{SOE}^{PL-2009}$ model is, therefore, a dynamic stochastic general equilibrium model to be used for forecasting purposes and not a ”forecasting model”. The preference for future applications is, however, reflected in omission of interesting threads and deeper research of problems that are not directly related to forecasting — here an example may be the hypotheses related to structural changes that have probably occurred at the end of the 20th century and beginning of the 21st century in the Polish economy.

3.3 Basic features of $\mathbb{SOE}^{PL-2009}$ model

The specification of $\mathbb{SOE}^{PL-2009}$ is based on a framework typical in the class of DSGE models derived from CEE model: a representative, forward-looking and optimising consumer; imperfect competition in intermediate products and labour markets; perfect competition in the markets of final products and capital services; nominal and real rigidities. Final products are assembled from domestic and imported intermediate products. Merging of domestic and imported components into final products is made with the use of CES function, according to the logic of Dixit-Stiglitz aggregator (Dixit and Stiglitz, 1977). Finally two types of goods for domestic use (consumption and investment goods) are created, as well as products to be exported. A departure from the aforesaid principle refers to the goods consumed by the government — the goods consist only of the domestic component[3]. The prices of intermediate products (and also wages) are set under imperfect competition with rigidities in the adjustment processes. The mechanism of prices rigidity is based on a slightly modified Calvo model. Upon the solution of the respective decision-making problems, the dynamics of prices (and wages) is described by Phillips curves, in which inflation expectations appear explicitly (the anticipated rate of inflation).

It is assumed that there are infinitely many rational (optimising and forward-looking) households in the economy. Households maximise utility attained from the consumption (with some habit persistence), leisure and cash holdings under budget constraint. Maximisation of utility takes place in the perspective of infinite horizon with classical time discount. Households are the sole administrator of labour force, while the unique qualifications of each of the households give them a monopolistic position in the processes of wage negotiations (see Erceg et al., 2000). Households are also the owners of fixed capital assets and they receive income on account of lease of the assets. The possibility of generating income from leasing the capital services causes that households are interested in increasing this resource — investments. Another source of income is interest from domestic deposits and deposits in foreign currencies. In the $\mathbb{SOE}^{PL-2009}$ model we have assumed that households may deposit their savings in domestic currency and in euro or dollar. Interest on each type of currency deposit is calculated taking into account risk premium, different for each of the currencies. Additional sources of income are profits of intermediate goods producers as well as transfers from the budget. Generally, all the households reflect the same consumption pattern, which is ensured by a special type of insurance levelling the income. The technique enables merging of the pattern of a representative consumer with differentiation of qualification of the labour force supply by households (households are different but their consumption standards, or more generally — the structure of expenditures, are the same). All of the income and also consumption expenditures are burdened with a set of taxes.

Next to households, an infinite number of domestic firms produce heterogeneous intermediate goods by using the Cobb-Douglas technology with homogenous capital and labour inputs. The labour and capital services are purchased in competitive market and set to minimise the cost of production. Part of the capital lease rent (use of capital services) and wage bill must be paid in advance and is financed with a working capital loan. The marginal costs of domestic production of intermediate products depend on the costs of capital and labour, the costs of working credit and national insurance contributions paid be the employers (an additional charge on the labour costs). Yet, given the specific nature of manufactured goods, manufacturers may set prices of their products in a manner characteristic of monopolies, while the applied markup has a stochastic nature. In order to set the prices of intermediate products, the manufacturers solve a dynamic (intertemporal) decision-making problem, in which the Calvo type rigidity mechanism is assumed.

[3] $\mathbb{SOE}^{Euro}$ only consumer and investment goods cover the imported component. Pragmatic reasons (avoidance of the further extension of the model and complication of equations) caused that the goods consumed by the government do not include imported components. A side effect of such simplification is the understating of the steady state share of foreign trade in GDP.

A special solution applied in $\mathbb{SOE}^{PL-2009}$ is the inclusion in the model of a disturbance representing the effects of fluctuations in the prices of energy (e.g. oil). The disturbance has a structural nature[4] and affects the economic processes through two channels. The first one works through the costs related to capital utilisation rate, such as it is proposed e.g. by Christiano et al. (2007a), see also Leduc and Sill (2001). The second one is based on the direct influence of that disturbance on the marginal costs of domestically manufactured intermediate products and, thus, also on the prices of all of the final products.

Final products (consumption, investment and export goods) consist of domestic intermediate products and imported components. Importing firms purchase homogenous goods in the world's market (the euro area and the USA), transforming them into heterogeneous products (e.g. by branding), set their prices and charge a markup (it is assumed that markups have a stochastic nature). This is, thus, another segment of economy with imperfect competition. The marginal cost of imports is the function of the price of goods in the world's markets weighted with the geographic structure.

Exporting firms purchase domestic and imported components and produce heterogeneous export goods, the prices of which are, hence, set by maximising profits under monopolistic competition. The market in which the exporters sell their products is fully competitive, consumers in both parts of the world pay identical prices. In other words, the geographical structure of exports has no importance for the relations described in the model.

We define in the model two specific agents who do not have clear objective functions and their behaviour is described with *ad hoc* rules. These are the government dealing with the collection of taxes and expending income and the central bank that controls the interest rate. The government plays only a passive role, distributes the income from taxes without creating the budget deficit. No public debt category appears here. The government assigns its expenditures for public (collective) consumption and lump-sum transfers to households. Theoretically, the transfers may be considered negative, which would mean the appearance of budget deficit in periods and repayment of the debt in others. The budget deficit in such situation immediately reduces the disposable income but the rational, optimising and forward-looking households may, in spite of that, maintain the level of expenditures, if they decide to avail of foreign deposits. The whole reflects the Ricardian behaviour. The interest rate of the central bank follows the interest rate rule. The rule is an effect of the manner in which the rational and forward-looking agents — households and firms — perceive the behaviour of the central bank. In other words, the model characterises the point of view of economic agents and their perception of monetary policy, instead of the actual decision-making process observed from inside of the bank.

The equilibrium at a micro scale results from first order conditions for each group of optimising agents. Decisions regarding consumption, investments, savings (and their currency structure), the level of capital utilisation (etc.) bring households to the maximum of their expected utility. Decisions regarding the production level and proportion of production factors bring firms to minimise the costs, while the decisions regarding the prices (wages) lead to maximisation of

[4] We have also carried out experiments with a version assuming the observable nature of such shock.

expected profits. Equilibrium conditions on every market depend, among others on the type of competition and the specific values of elasticity of substitution — generally, it is also required that all markets are clear. Macro-scale balances are satisfied, which guarantees that demand for services of production factors in each period are equal to the total supply of these factors, the total value of expenditures is equal to the total income and the state budget revenues is equal to expenditures.

In the $\mathbb{SOE}^{PL-2009}$ model there are several groups of stochastic shocks (disturbances). The first, basic group, are structural unanticipated shocks. Economic growth is described with a stochastic trend — non-stationary disturbance characterising technical progress, thanks to which the time series reflecting the trend related to technical progress are modelled in consideration of that process. More precisely, we assume that the trend is a resultant of two processes (non-stationary disturbances). The first directly impacts labour[5] such as in the original version of $\mathbb{SOE}^{Euro}$ by Riksbank. The second process impacts the prices of investment expenditures and fixed assets as proposed by Christiano et al. (2007c,d), see also Altig et al. (2004a, 2005); Burriel et al. (2009). The total impact of both types of technical progress gives, however, identical dynamics to all of the variables growing in steady state. Identical for all of the growing variables is the dynamics of values and the division of the dynamics into the growth of volume and prices. An exception are investment expenditures, whose dynamics of value is the same as other variables but the division into the growth of volume and price is different — accordingly to the characteristics of the second of non-stationary disturbances and the applied technology. Thus, in a long run real investments do not have to grow at the same rate as consumption or GDP. Other structural disturbances (technology, preferences, markups, risk premium, etc.) have stationary nature — it has been assumed that they shall have the nature of first-order autoregressive process (AR(1)).

Another group of disturbances are observable disturbances (also of unanticipated nature). As we have mentioned before, the world's economy is heterogeneous in $\mathbb{SOE}^{PL-2009}$ and consists of two areas: the euro area and the dollar area (the rest of the world identified with the USA). The key characteristics of the world's economy and their relations are approximated with a structural vector autoregression (SVAR) model. The SVAR model is estimated separately and is used to describe the mechanisms generating the shocks (here observable disturbances) from the foreign environment. In similar manner the fragments of the fiscal block have been treated — here also the SVAR model has been used for approximation of the interdependencies between budget expenditures and (a part of) budget revenues.

Additionally, beside of the unanticipated structural and observable disturbances, which are sort of a standard, in $\mathbb{SOE}^{PL-2009}$ the possibility of occurrence of disturbances anticipated by the agents has been permitted. Formally, the structure of anticipated disturbances coincides with the supplementation of the disturbance structure with MA class component — we have used here the convention proposed by Schmitt-Grohé and Uribe (2008), a similar version is also proposed by Christiano et al. (2007a,b). The anticipated disturbances are, however, an option of which we have not availed in the current version (and, therefore, omit it in the further description).

[5] It is also proposed here to extend the interpretation of the disturbance for demographic effects.

Nevertheless, the analytical potential of this solution seems to be considerable, so we shall return to the issue of the role of anticipated disturbances at further stages of the research.

Looking from another angle, the $\mathbb{SOE}^{\mathrm{PL-2009}}$ specification brings the model to neoclassical responses in the long run, with explicitly and clearly defined steady state[6], while the short-term effects — thanks to inclusion in the model of real and nominal rigidities — shall have a more Keynesian nature. For example, fiscal stimulation of the economy is effective only in a short run. Also the stimulation of economic activity with monetary policy instruments may be effective in a short run however, in the long run the only growth determinant is the (exogenous) technical progress, and the possible deviations from the long-term trends are absorbed already in medium term, while the costs of such adjustments may exceed the previous gains. The characteristics of adjustment processes, i.e. model dynamics, is a consequence of optimising (forward-looking) behaviours of agents (the decision-making problems of agents formally derived from intertemporal optimisation problems). The function of institutions managing the macroeconomic policy has been adjusted to the above logic (i.e. the rules of behaviours derived by the rational agents based on the observation of the activities of institutions). Their policy excludes any forms of game with the agents and is time-consisted. As a result, the model may be used solely for making analyses in which the condition shall be satisfied at least approximately.

The existence and functions of money result from the households' objective function (holding some cash resources is useful for households). Additionally, demand for money is also reported by firms that need to pay in advance for some share of labour services and of capital[7]. Stochastic monetary effects (disturbances in cash demand among households and demand among firms) — as a result of several experiments made before the year 2009 — proved to be unsuccessful as the disturbances of this class have not increased the potential of the model and, therefore, accordingly as in $\mathbb{SOE}^{\mathrm{Euro}}$ they have been excluded. In consequence, the monetary variables do not appear in the set of observable variables.

[6]We use interchangeably the terms of steady state and long-term equilibrium.

[7]A competitive method of deriving cash models is the definition of the costs of transaction. In effect of optimisation of such costs, agents decide to possess cash resources, see e.g. Coenen et al. (2006).

4
Decision-making problems, equilibrium conditions, macroeconomic balance of the model

4

4.1 Growth

The $\mathbb{SOE}^{\mathrm{PL-2009}}$ is a model of exogenous, stochastic growth which is driven by changes in the level of technology (z_t). The growth rate of technology, $\mu_{z,t} \equiv \frac{z_t}{z_{t-1}}$, is governed by the stochastic process:

$$\mu_{z,t} = \left(1 - \rho_{\mu_z}\right)\mu_z + \rho_{\mu_z}\mu_{z,t-1} + \varepsilon_{\mu_z,t}, \qquad \varepsilon_{\mu_z,t} \sim N\left(0, \mu_z \sigma_{\mu_z}\right), \qquad \mathbb{E}\,\mu_{z,t} = \mu_z,$$

where ρ_{μ_z} is the persistence coefficient, and μ_z is a long-term growth rate of technology.

The technological trend has a neutral nature — it refers to all of the macroeconomic categories characterised by growth. Beside of that, we assume the existence of a technological trend specific for capital/investment goods $\left(\Psi_t\right)$, whose changes, $\mu_{\Psi,t} \equiv \frac{\Psi_t}{\Psi_{t-1}}$, are governed by the process:

$$\mu_{\Psi,t} = \left(1 - \rho_{\mu_\Psi}\right)\mu_\Psi + \rho_{\mu_\Psi}\mu_{\Psi,t-1} + \varepsilon_{\mu_\Psi,t} \qquad \varepsilon_{\mu_\Psi,t} \sim N\left(0, \mu_\Psi \sigma_{\mu_\Psi}\right), \qquad \mathbb{E}\,\mu_{\Psi,t} = \mu_\Psi.$$

The presence of an additional technological trend specific for capital goods, by use of capital as a factor of production (see Chapter 4.3.2), is translated into other macroeconomic categories and extends the neutral technological trend. The common technological trend (z_t^+) for all growing variables, except capital goods, may be presented as:

$$z_t^+ = z_t \Psi_t^{\frac{\varpi}{1-\varpi}}, \qquad \mu_{z^+,t} = \mu_{z,t} \mu_{\Psi,t}^{\frac{\varpi}{1-\varpi}},$$

where ϖ is the share of capital in production. The level of technology for capital goods $\left(z_t^+ \Psi_t\right)$ and all the other categories $\left(z_t^+\right)$ allow to express the growing variables in a stationary form (usually denoted with small letters), i.e.:

$$y_t \equiv \frac{Y_t}{z_t^+}, \qquad i_t \equiv \frac{I_t}{z_t^+ \Psi_t}, \qquad \text{etc.} \tag{4.1}$$

Additionally, nominal variables are stationarized with the use of the level of prices $\left(P_t^d\right)$, e.g. nominal wages are translated into stationary real wages:

$$w_t \equiv \frac{W_t}{P_t^d z_t^+}. \tag{4.2}$$

Therefore, at the final stage the whole model may be presented with the use of stationary variables and explicitly determined steady state.

4

4.2 Foreign economy

Domestic economy functions in the environment of two foreign economies: the euro area and the rest of the world. Interactions with those economies entail exchange of goods and financial flows. We assume that the currency of the rest of the world is the dollar and in the euro area — the euro. We use, therefore, three nominal exchange rates: dollar/zloty, euro/zloty and dollar/euro, denoted respectively by: S_t^u, S_t^e, S_t^x. The exchange rates satisfy:

$$S_t^u = S_t^x S_t^e. \tag{4.3}$$

Additionally, we define real exchange rates:

$$x_t^x \equiv \frac{S_t^x P_t^u}{P_t^e}, \qquad x_t^e \equiv \frac{S_t^e P_t^e}{P_t^c}, \qquad x_t^u \equiv \frac{S_t^e S_t^x P_t^u}{P_t^c}, \tag{4.4}$$

where P_t^u is the level of prices in the rest of the world, P_t^e is the level of prices in the euro area, and P_t^c is the level of domestic consumer prices. To real exchange rates, analogically as for the nominal rates, the following applies:

$$x_t^u = x_t^x x_t^e. \tag{4.5}$$

4.3 Producers

There are five markets of intermediate goods: domestic goods, imported consumption, investment and export goods, as well as export goods. In each of the markets there are infinitely many agents (continuum determined in the [0,1] interval) manufacturing heterogeneous intermediate products of a given type that are aggregated to a homogenous final product representing the production of the given market.

4.3.1 Aggregators

Heterogeneous intermediate products must be aggregated[1]. For each market we assume the existence of infinitely many firms (the agents do not consume resources or generate added value), which operate under perfect competition and use the same production function. They purchase heterogeneous intermediate products and transform them into a homogenous final product (taking the prices of intermediate products and the price of the final product as given).

The production function of the final good in each of the markets O $(O \in \{Y, C^m, I^m, X^m, X\})$ takes the form of the CES function:

$$O_t = \left[\int_0^1 O_{i,t}^{\frac{1}{\lambda_t^o}} \, di \right]^{\lambda_t^o}, \qquad 1 \leq \lambda_t^o < \infty, \qquad o \in \{d, mc, mi, mx, x\}, \tag{4.6}$$

where O_t is the production of the final good, $O_{i,t}$ is the production by the i-th intermediate goods producer, λ_t^o is the markup in the market o, and o identifies market: domestic products (d), imported consumption goods (mc), imported investment goods (mi), imported goods intended for export (mx), export products (x). Markups specific for each of the markets are described with stochastic processes:

$$\lambda_t^o = \left(1 - \rho_{\lambda^o}\right) \lambda^o + \rho_{\lambda^o} \lambda_{t-1}^o + \varepsilon_{\lambda^o,t}, \qquad \varepsilon_{\lambda^o,t} \sim N\left(0, \lambda^o \sigma_{\lambda^o}\right), \qquad \mathbb{E}\,\lambda_t^o = \lambda^o, \tag{4.7}$$

where λ^o is the value of markup in steady state.

Profit maximisation by the aggregator leads to the demand function for intermediate products of the i-th producer:

$$O_{i,t} = \left[\frac{P_{i,t}^o}{P_t^o} \right]^{-\frac{\lambda_t^o}{1-\lambda_t^o}} O_t, \tag{4.8}$$

where P_t^o is the price of the homogenous final product in market o, $P_{i,t}^o$ is the price of the intermediate product of the i-th producer.

Using equations (4.6) and (4.8) we obtain the equation for the price of the homogenous final product in the given market:

$$P_t^o = \left[\int_0^1 \left(P_{i,t}^o\right)^{\frac{1}{1-\lambda_t^o}} \, di \right]^{1-\lambda_t^o}, \qquad o \in \{d, mc, mi, mx, x\}. \tag{4.9}$$

[1] Aggregation of heterogeneous products into a homogenous product is a technical operation necessary from the point of view of the model operability, however, we may apply economic interpretation thereto.

4.3.2 Domestic intermediate goods firms

The producers of domestic intermediate goods are the only actual generators of the GDP. Using the Cobb-Douglas production function, with production technology identical for all of the producers, they use individually determined labour and capital inputs to produce:

$$Y_{i,t} = \epsilon_t \left(z_t H_{i,t}\right)^{1-\varpi} \left(K_{i,t}\right)^{\varpi}, \tag{4.10}$$

where $H_{i,t}$ and $K_{i,t}$ are the inputs of labour (hours) and capital services determined by the i^{th} producer. The total factor productivity (ϵ_t) is described with a stochastic process:

$$\epsilon_t = 1 - \rho_\epsilon + \rho_\epsilon \epsilon_{t-1} + \varepsilon_{\epsilon,t}, \qquad \varepsilon_{\epsilon,t} \sim N\left(0, \sigma_{\varepsilon_\epsilon}\right), \qquad \mathbb{E}\epsilon_t = 1. \tag{4.11}$$

The optimal values for inputs of capital and labour are determined based on the problem of costs minimisation:

$$\min_{K_{i,t},H_{i,t}} R_t^{fw} F_t^\tau W_t H_{i,t}\left(1+\tau_t^s\right) + R_t^{fk} F_t^\tau R_t^k K_{i,t} - \lambda_t P_{i,t}\left[Y_{i,t} - z_t^{1-\varpi}\epsilon_t K_{i,t}^{\varpi} H_{i,t}^{1-\varpi}\right], \tag{4.12}$$

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where W_t is the nominal wage, τ_t^s is the rate of national insurance contribution paid by the employer, R_t^k is the gross nominal rental rate per unit of capital services, λ_t is the Lagrange multiplier. We assume that in each period a fraction of the wage and capital fund, ν^w and ν^k, must be financed with a working capital loan hence the presence of effective gross nominal interest rates, R_t^{fw} and R_t^{fk}, in the cost function, given by:

$$R_t^{fw} \equiv \nu^w R_{t-1} + 1 - \nu^w, \qquad R_t^{fk} \equiv \nu^k R_{t-1} + 1 - \nu^k, \tag{4.13}$$

where R_{t-1} is the gross nominal interest rate. We assume also that the use of labour and capital services involves the use of "energy", whose costs are represented by F_t^τ function. The $F_t^\tau(\cdot,\cdot)$ function, specified explicitly only at the level of log-linearised form, is a linear function of structural shock representing the dynamics of the prices of energy (e.g. oil):

$$\pi_t^{\text{oil}} = \left(1-\rho_{\pi^{\text{oil}}}\right)\pi^{\text{oil}} + \rho_{\pi^{\text{oil}}}\pi_{t-1}^{\text{oil}} + \varepsilon_{\pi^{\text{oil}},t}, \qquad \varepsilon_{\pi^{\text{oil}},t} \sim N\left(0, \sigma_{\varepsilon_{\pi^{\text{oil}}}}\right). \tag{4.14}$$

First order conditions of the decision-making problem (4.12) with respect to $H_{i,t}$, $K_{i,t}$ and λ_t are:

$$R_t^{fw} F_t^\tau W_t\left(1+\tau_t^s\right) = (1-\varpi)\lambda_t P_{i,t} z_t^{1-\varpi}\epsilon_t K_{i,t}^{\varpi} H_{i,t}^{-\varpi},$$

$$R_t^{fk} F_t^\tau R_t^k = \varpi\lambda_t P_{i,t} z_t^{1-\varpi}\epsilon_t K_{i,t}^{\varpi-1} H_{i,t}^{1-\varpi},$$

$$Y_{i,t} = z_t^{1-\varpi}\epsilon_t K_{i,t}^{\varpi} H_{i,t}^{1-\varpi}.$$

Based on the first order conditions of the problem of costs minimisation, we arrive at the equation of real marginal cost of the domestic intermediate goods producers:

$$mc_t^d \equiv \lambda_t = \frac{1}{\epsilon_t}\left(\frac{1}{\varpi}\right)^{\varpi}\left(\frac{1}{1-\varpi}\right)^{1-\varpi}\left(\overline{r}_t^k R_t^{fk}\right)^{\varpi}\left(w_t R_t^{fw}\right)^{1-\varpi} F_t^{\tau}. \tag{4.15}$$

The market of domestic intermediate products is characterised with monopolistic competition, which means that manufacturers produce heterogeneous products and may set their prices. At the same time, there are some limitations in the spirit of Calvo price setting (Calvo (1983)). In every period any of the manufacturers, with probability $1-\xi_d$, may set the optimal price of its output $\left(P_t^{d,new}\right)$. With probability ξ_d the price cannot be set in the optimal way and it is then indexed to previous inflation (with weight κ_d) and the current inflation target[2](with weight $1-\kappa_d$):

$$P_{t+1}^d = \left(\pi_t^d\right)^{\kappa_d}\left(\overline{\pi}_{t+1}^c\right)^{1-\kappa_d} P_t^d. \tag{4.17}$$

If a producer is allowed to reoptimise its price, it sets its price to maximise the flow of future profits, assuming that it will not be allowed to reoptimise the price in the future. Thus, the decision-making problem takes the form:

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$$\max_{P_t^{d,new}} \mathbb{E}_t \sum_{s=0}^{\infty} \upsilon_{t+s}\left(\beta\xi_d\right)^s\left[\left(\pi_t^d \ldots \pi_{t+s-1}^d\right)^{\kappa_d}\left(\overline{\pi}_{t+1}^c \ldots \overline{\pi}_{t+s}^c\right)^{1-\kappa_d} P_t^{d,new} Y_{i,t+s} - MC_{i,t+s}^d Y_{i,t+s}\right],$$

where υ_{t+s} is the marginal utility of the households' nominal income[3], and β is a discount factor. When solving the profit maximisation problem above, the producer takes into account the demand for their ouput given by equation (4.8). The solution of the problem takes the form of the Phillips curve for domestic intermediate goods, in which the main inflation determinants become the real marginal costs $\left(mc_t^d\right)$, given by (4.15), and markup $\left(\lambda_t^d\right)$, described with the exogenous process (4.7)[4].

The last problem the manufacturers have to cope with is the determination of the optimal level of employment (number of full time employees), based on the number of hours worked determined in the process of costs minimisation. The process of adjusting employment involves Calvo-type

[2]The inflation target $\left(\overline{\pi}_t^c\right)$ has a stochastic nature and is given by exogenous process:

$$\overline{\pi}_t^c = \left(1-\rho_{\overline{\pi}^c}\right)\overline{\pi}^c + \rho_{\overline{\pi}^c}\overline{\pi}_{t-1}^c + \varepsilon_{\overline{\pi}^c,t}, \qquad \varepsilon_{\overline{\pi}^c,t} \sim N\left(0, \overline{\pi}^c \sigma_{\varepsilon_{\overline{\pi}^c}}\right). \tag{4.16}$$

In steady state inflation target is equal to the steady state level of inflation $\left(\overline{\pi}^c \equiv \pi^d\right)$.

[3]Due to the fact that in each period the profit generated by the firm is transferred to households, the profit in the particular period is weighted with the marginal utility of the households' nominal income.

[4]Below we present an example of a Phillips curve (binding for each of the markets of intermediate products), already in a log-linearised but relatively legible form:

$$\begin{aligned}\widehat{\pi}_t^o = &\frac{\beta\mu}{1+\kappa_o\beta\mu}\left(\widehat{\pi}_{t+1}^o - \widehat{\overline{\pi}}_{t+1}^c\right) + \frac{\kappa_o}{1+\kappa_o\beta\mu}\left(\widehat{\pi}_{t-1}^o - \widehat{\overline{\pi}}_t^c\right)\\ &+ \frac{1+\kappa_o\beta\mu\rho_\pi}{1+\kappa_o\beta\mu}\widehat{\overline{\pi}}_t^c + \frac{\left(1-\xi_o\beta\mu\right)\left(1-\xi_o\right)}{\xi_o\left(1+\kappa_o\beta\mu\right)}\left(\widehat{\lambda}_t^o + \widehat{mc}_t^o\right).\end{aligned} \tag{4.18}$$

Current inflation depends on the difference between the inflation target and past/expected inflation, directly on the very inflation target and the current standing of producers (their markup and marginal costs). Also the impact of price rigidities can be seen — the smaller the price rigidity (smaller ξ_o), the larger is the importance of marginal costs and markup in the market for the current inflation.

rigidities — with probability $1-\xi_e$ the producer is allowed to set the level of employment in an optimal way, while with the probability ξ_e the producer cannot change the level of employment. When producer is allowed to re-optimise the level of employment, the decision-making problem takes the form:

$$\min_{E_{i,t}^{new}} \sum_{s=0}^{\infty} \left(\beta\xi_e\right)^s \left(n_i E_{i,t}^{new} - H_{i,t+s}\right)^2, \tag{4.19}$$

where n the number of hours per employee. The solution of the decision-making problem describes the level of employment in the economy.

4.3.3 Importers

The imported consumption, investment and export goods make three separate markets of imported products. In each of the markets the importers purchase foreign goods (from the euro area and the rest of the world — we assume the stability of the geographic structure of the import), and differentiate them. Heterogeneous products are, then, purchased by the aggregators and transformed into homogenous final products. The monopolistic competition implies that importers may set prices of their products, while the process runs similarly to the case of domestic intermediate goods producers (with specific ξ_o, κ_o and λ^o, $(o \in \{mc, mi, mx\})$ parameters for each market of imported products). Solving the problem of maximisation of the importers' profit, we arrive at three Phillips curves in the form compliant with the formula presented in the footnote — full versions are provided in the Appendix.

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The marginal costs, with fixed geographic structure of import, take the form:

$$\begin{aligned} mc_t^{mc} &\equiv \frac{S_t^u P_t^u}{P_t^{mc}} \omega^{mc,u} + \frac{S_t^e P_t^e}{P_t^{mc}} \left(1-\omega^{mc,u}\right), \\ mc_t^{mi} &\equiv \frac{S_t^u P_t^u}{P_t^{mi}} \omega^{mi,u} + \frac{S_t^e P_t^e}{P_t^{mi}} \left(1-\omega^{mi,u}\right), \\ mc_t^{mx} &\equiv \frac{S_t^u P_t^u}{P_t^{mx}} \omega^{mx,u} + \frac{S_t^e P_t^e}{P_t^{mx}} \left(1-\omega^{mx,u}\right), \end{aligned} \tag{4.20}$$

where $\omega^{mc,u}$, $\omega^{mi,u}$ and $\omega^{mx,u}$ determine the share of the rest of the world in the basket of imported consumption, investment and export goods.

The real marginal costs and markups in the markets, described with exogenous processes (4.7), determine inflation for imported consumption goods, investment goods and goods intended for export. The total demand for imported consumption and investment goods depends on the decisions of households (see Chapter 4.4), while the demand for import of goods intended for export is determined by the exporters.

4.3.4 Exporters

Similarly to the domestic intermediate goods producers and importers, exporters produce, under monopolistic competition, heterogeneous export goods $\left(X_{i,t}\right)$, for which they may set prices $\left(P_{i,t}^x\right)$. Due to the fact that production is intended for the world market, the prices set by exporters are expressed in dollars. As we assume free flow of products between the euro area and the rest of the world, the price for the euro area market is the same price converted into euro based on the USD/EUR exchange rate $\left(P^x_{i,t} S^x_t\right)$. The process of setting the price runs similarly as in the case of domestic goods producers and importers with parameters ξ_x, κ_x i λ^x specific for the export market. After solving the problem of profit maximisation we obtain the Phillips curve for the export market.

Export good of the i^{th} exporter is produced with the use of domestic products $\left(X^d_{i,t}\right)$ and imported products $\left(X^m_{i,t}\right)$. The production function takes the form:

$$X_{i,t} = \left[\omega_x^{\frac{1}{\eta_{xx}}} \left(X^m_{i,t}\right)^{\frac{\eta_{xx}-1}{\eta_{xx}}} + \left(1-\omega_x\right)^{\frac{1}{\eta_{xx}}} \left(X^d_{i,t}\right)^{\frac{\eta_{xx}-1}{\eta_{xx}}} \right]^{\frac{\eta_{xx}}{\eta_{xx}-1}}, \tag{4.21}$$

where η_{xx} is the elasticity of substitution between domestic and imported products, and ω_x is the share of the imported component. Each exporter must solve the problem of costs minimisation:

$$\min_{X^m_{i,t}, X^d_{i,t}} P^{mx}_t X^m_{i,t} + P^d_t X^d_{i,t}, \tag{4.22}$$

where P^{mx}_t is the price of imported component, P^d_t is the price of the domestic component, subject to (4.21).
The solution of the problem leads to the marginal costs of exporters:

$$mc^x_t = \frac{1}{S^e_t S^x P^x_t} \left[\omega_x \left(P^{mx}_t\right)^{1-\eta_{xx}} + \left(1-\omega_x\right) \left(P^d_t\right)^{1-\eta_{xx}} \right]^{\frac{1}{1-\eta_{xx}}}.$$

The marginal cost together with the markup described with the exogenous process (4.7) is reflected in the Phillips curve for the export market.

Assuming that consumption and investments in the euro area and in the rest of the world are determined based on CES functions with domestic export being one of the inputs, the demand for domestic export on the part of both economies and both types of products is expressed with the following equations:

$$C^{x,u}_t = \left[\frac{P^x_t}{P^u_t}\right]^{-\eta_{f,u}} C^u_t, \qquad I^{x,u}_t = \left[\frac{P^x_t}{P^u_t}\right]^{-\eta_{f,u}} I^u_t,$$

$$C^{x,e}_t = \left[\frac{S^x_t P^x_t}{P^e_t}\right]^{-\eta_{f,e}} C^e_t, \qquad I^{x,e}_t = \left[\frac{S^x_t P^x_t}{P^e_t}\right]^{-\eta_{f,e}} I^e_t,$$

where C^e_t $\left(C^u_t\right)$ and I^e_t $\left(I^u_t\right)$ are consumption and investments in the euro area (rest of the world), while $\eta_{f,e}$ $\left(\eta_{f,u}\right)$ is the elasticity of substitution between domestic export products and the euro area (the rest of the world) products. Assuming that the income of foreign economies is entirely divided between consumption and investments, we may express the demand for domestic export as a function of foreign income:

$$X_t \equiv C^{x,e}_t + I^{x,e}_t + C^{x,u}_t + I^{x,u}_t = \left[\frac{P^x_t}{P^u_t}\right]^{-\eta_{f,u}} Y^u_t + \left[\frac{S^x_t P^x_t}{P^e_t}\right]^{-\eta_{f,e}} Y^e_t. \tag{4.23}$$

Thus, the demand for domestic export depends on the relation of export prices and world prices and income (output) abroad.

4.4 Households

The households maximise utility consisting of consumption, leisure and cash. Households provide labour and capital services to the producers of domestic intermediate products. In each period households divide their income between domestic and foreign deposits, consumption, investments and purchase/sale of new, installed capital, as well as cover the cost of maintenance of capital that has not been lent to producers. The income of households consists of domestic and foreign deposits plus interest, remuneration for the labour and capital services, as well as profits transferred in the form of dividend. All of the income is adequately taxed, and additionally direct transfers from the state budget are allowed. Moreover, we assume that financial markets are complete. This enables households to acquire state contingent securities making them homogenous with regard to the possessed resources and incurred expenditures, thanks to which the model may be made operational.

Households are characterised with the so called internal habit persistence, which means that utility is derived not so much from the absolute current level of consumption as from a change in the level of consumption in reference to the previous period. The utility function of a j-th household takes the form:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} \left[\zeta_{t+s}^{c} \ln\left(C_{j,t+s} - bC_{j,t+s-1}\right) - \zeta_{t+s}^{h} A_L \frac{h_{j,t}^{1+\sigma_L}}{1+\sigma_L} + A_q \, \zeta_{t+s}^{h} \frac{\left(\frac{Q_{j,t}}{z_t P_t}\right)^{1-\sigma_q}}{1-\sigma_q} \right], \tag{4.24}$$

where $C_{j,t}$ is the consumption in period t, $h_{j,t}$ is the supply of labour (hours), $Q_{j,t}$ is cash holdings, σ_L is the inverse of the elasticity of labour supply with respect to wage, σ_q is the elasticity of demand for cash with respect to interest rate. The preferences regarding consumption, leisure and cash holdings, ζ_t^c, ζ_t^h and ζ_t^q, are given by exogenous processes:

$$\zeta_t^l = 1 - \rho_{\zeta^l} + \rho_{\zeta^l}\zeta_{t-1}^l + \varepsilon_{\zeta^l}, \qquad \varepsilon_{\zeta^l} \sim N\left(0, \sigma_{\zeta^l}\right), \qquad \mathbb{E}\,\zeta_t^l = 1, \quad l \in \{c, h, q\}. \tag{4.25}$$

Consumption and investment goods purchased by households consist of domestic and imported products:

$$\begin{aligned} C_t &= \left[(1-\omega_c)^{\frac{1}{\eta_c}} \left(C_t^d\right)^{\frac{\eta_c-1}{\eta_c}} + (\omega_c)^{\frac{1}{\eta_c}} \left(C_t^m\right)^{\frac{\eta_c-1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c-1}}, \\ I_t &= \left[(1-\omega_i)^{\frac{1}{\eta_i}} \left(I_t^d\right)^{\frac{\eta_i-1}{\eta_i}} + (\omega_i)^{\frac{1}{\eta_i}} \left(I_t^m\right)^{\frac{\eta_i-1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i-1}}, \end{aligned} \tag{4.26}$$

where C_t^d $\left(I_t^d\right)$ and C_t^m $\left(I_t^m\right)$ are domestic and import components of consumption (investments), ω_c $\left(\omega_i\right)$ is the share of import in consumption (investments) and η_c $\left(\eta_i\right)$ is the elasticity of substitution between domestic consumption (investment) goods and imported goods.

We assume that aggregation is made in such a way as to maximise the values C_t and I_t subject to budget constraints:

$$P_t^d C_t^d + P_t^{mc} C_t^m = P_t^c C_t, \qquad P^t \frac{I_t^d}{\Psi_t} + P_t^{mi} I_t^m = P_t^i I_t, \tag{4.27}$$

where P_t^d, P_t^{mc} and P_t^{mi} are the prices of the domestic components and imported consumption and investment components. Solving the problems of consumption and investment maximisation, we arrive at the equation of demand for domestic and imported components of consumption and investment:

$$C_t^d = (1-\omega_c)\left[\frac{P_t^c}{P_t^d}\right]^{\eta_c} C_t, \qquad C_t^m = \omega_c \left[\frac{P_t^c}{P_t^{mc}}\right]^{\eta_c} C_t,$$

$$I_t^d = (1-\omega_i)\left[\frac{P_t^i \Psi_t}{P_t^d}\right]^{\eta_i} I_t, \qquad I_t^m = \omega_i \left[\frac{P_t^i \Psi_t}{P_t^{mi}}\right]^{\eta_i} \frac{I_t}{\Psi_t}.$$

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The prices of final consumption goods $\left(P_t^c\right)$ and investment goods $\left(P_t^i\right)$ are then expressed with the following equations:

$$\begin{aligned} P_t^c &= \left[(1-\omega_c)\left(P_t^d\right)^{1-\eta_c} + \omega_c \left(P^{mc}\right)^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}}, \\ P_t^i \Psi_t &= \left[(1-\omega_i)\left(P_t^d\right)^{1-\eta_i} + \omega_i \left(P_t^{mi}\right)^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}}. \end{aligned} \tag{4.28}$$

The households' physical capital stock $\left(\overline{K}_{j,t+1}\right)$ evolves according to:

$$\overline{K}_{j,t+1} = (1-\delta)\overline{K}_{k,t} + \Upsilon_t F\left(I_{j,t}, I_{j,t-1}\right) + \Delta_{j,t}, \tag{4.29}$$

where δ is the capital depreciation rate, $\Delta_{j,t}$ is the purchase/sale of new, installed capital. Function F is the function of transformation of investment expenditures into physical capital:

$$F\left(I_{j,t}, I_{j,t-1}\right) = \left(1 - \tilde{S}\left(\frac{I_{j,t}}{I_{j,t-1}}\right)\right) I_{j,t}. \tag{4.30}$$

Function $\tilde{S}$ is not explicitly specified, we assume that:

$$\tilde{S}(x) = \tilde{S}'(x) = 0 \qquad \text{and} \qquad \tilde{S}''(x) \equiv \tilde{S}'' > 0, \qquad x = \mu_z^+ \mu_\Psi. \tag{4.31}$$

This means that full transformation of investments into physical capital takes place when investment expenditures grow at the steady state level. In other words, fluctuations in investment expenditures generate costs, which creates the mechanism of smoothening of investment expenditures. An additional factor affecting the effectiveness of transformation of investments into capital goods is the exogenous process Υ_t, called the investment-specific technology shock or effectiveness of transformation of investment into capital:

$$\Upsilon_t = 1 - \rho_\Upsilon + \rho_\Upsilon \Upsilon_{t-1} + \varepsilon_{\Upsilon,t}, \qquad \varepsilon_{\Upsilon,t} \sim N\left(0, \sigma_\Upsilon\right), \qquad \mathbb{E}\,\Upsilon_t = 1. \tag{4.32}$$

The physical capital stock is fully or partially leased to the intermediate goods producers in the form of capital services $K_{j,t}$. With $u_{j,t}$, $u_{j,t} \equiv \frac{K_{j,t}}{\overline{K}_{j,t}}$, we denote utilization rate of capital (in steady state $u = 1$). We assume that incomplete use of the capital resource generates cost for households, depending on the utilitzation rate — $F^\tau_{a,t} a(u_{j,t}) \frac{\overline{K}_{j,t}}{\Psi_t}$. Function $F^\tau_{a,t}$ represents a part of the cost depending on the changes in the prices of energy and — similarly to the function F^τ_t — is a function of energy price shock $\left(\pi^{\text{oil}}_t\right)$ (the solution is based on the work by Christiano et al. (2007a)). Function $a(u_{j,t})$ is not explicitly specified, we assume only that $a(1) = 0$ and $a'' \geq 0$.

The budget constraint of households takes the form of:

$$\begin{aligned}
& M_{j,t+1} + S^e_t B^e_{j,t+1} + S^e_t S^x_t B^u_{j,t+1} + P^c_t C_{j,t}\left(1 + \tau^c_t\right) + P^i_t I_t \\
& + P^d_t \left(F^\tau_{a,t} \frac{a(u_{j,t})}{\Psi_t} \overline{K}_{j,t} + P_{k',t} \Delta_t \right) = R_{t-1}\left(M_{j,t} - Q_{j,t}\right) + Q_{j,t} + \left(1 - \tau^k_t\right) \Pi_t \\
& + \left(1 - \tau^p_t\right) R^k_t u_{j,t} \overline{K}_{j,t} + \left(1 - \tau^y_t\right)\left(1 - \tau^w_t\right) W_{j,t} h_{j,t} \\
& + R^e_{t-1} \Phi \left(\frac{A^e_{t-1}}{z^+_{t-1}}, \mathbb{E}_t s^e_t s^e_{t-1}, \tilde{\phi}^e_{t-1} \right) S^e_t B^e_{j,t} \\
& + R^u_{t-1} \Phi \left(\frac{A^u_{t-1}}{z^+_{t-1}}, \mathbb{E}_t s^u_t s^u_{t-1}, \tilde{\phi}^u_{t-1} \right) S^e_t S^x_t B^u_{j,t} + TR_t + D_{j,t} \\
& - \tau^k_t \Bigg[\left(R^b_{t-1} - 1\right)\left(M_{j,t} - Q_{j,t}\right) + \left(R^e_{t-1} \Phi \left(\frac{A^e_{t-1}}{z^+_{t-1}}, \mathbb{E}_t s^e_t s^e_{t-1}, \tilde{\phi}^e_{t-1} \right) - 1 \right) S^e_t B^e_{j,t} \\
& \qquad + \left(R^u_{t-1} \Phi \left(\frac{A^u_{t-1}}{z^+_{t-1}}, \mathbb{E}_t s^u_t s^u_{t-1}, \tilde{\phi}^u_{t-1} \right) - 1 \right) S^e_t S^x_t B^u_{j,t} \\
& \qquad + B^e_{j,t}\left(S^e_t - S^e_{t-1}\right) + B^u_{j,t}\left(S^e_t S^x_t - S^e_{t-1} S^x_{t-1}\right) \Bigg] \\
& + \tau^p_t P^d_t F^\tau_t \frac{a(u_{j,t})}{\Psi_t} \overline{K}_{j,t} + \tau^p_t P_t P_{k',t} \delta \overline{K}_{j,t},
\end{aligned} \tag{4.33}$$

where $M_{j,t}$ are domestic financial assets, $B^e_{j,t}$ and $B^u_{j,t}$ are assets denominated in euro and dollar, $P_{k',t}$ is the relative price of capital goods, τ^c_t is the consumption tax rate, τ^k_t is the capital tax rate (on interest from deposits and dividends), τ^p_t is the corporate income tax rate, Π_t are the profits of intermediate goods producers (domestic, exporters and importers), TR_t are lump-sum transfers from state budget, $D_{j,t}$ is the income from state contingent securities, τ^y_t and τ^w_t are the personal income tax rate and the rate of national insurance contribution paid by an employee.

Foreign assets, $B^e_{j,t}$ and $B^u_{j,t}$, bear interest according to the interest rates for the euro area, R^e_t, and the rest of the world, R^u_t, adjusted for risk premium, see e.g. (Adolfson et al., 2007a, page 8) and (Schmitt-Grohé and Uribe, 2003; Engel, 1996):

$$\begin{array}{ll} \Phi\left(\frac{A_{t-1}^{e}}{z_{t-1}^{+}}, \mathbb{E}_t s_t^e s_{t-1}^e, \tilde{\phi}_{t-1}^e\right) & \text{for assets denominated in euro,} \\ \Phi\left(\frac{A_{t-1}^{u}}{z_{t-1}^{+}}, \mathbb{E}_t s_t^u s_{t-1}^u, \tilde{\phi}_{t-1}^u\right) & \text{for assets denominated in dollar,} \end{array} \tag{4.34}$$

where:

$$A_t^e \equiv \frac{S_t^e B_{t+1}^e}{P_t^d}, \qquad A_t^u \equiv \frac{S_t^u B_{t+1}^u}{P_t}, \qquad s_t^e \equiv \frac{S_t^e}{S_{t-1}^e}, \qquad s_t^u \equiv \frac{S_t^u}{S_{t-1}^u},$$

while $\tilde{\phi}_t^e$ and $\tilde{\phi}_t^u$ are the risk premium shocks described with stochastic processes:

$$\begin{array}{lll} \tilde{\phi}_t^e = \rho_{\tilde{\phi}^e}\tilde{\phi}_{t-1}^e + \varepsilon_{\tilde{\phi}^e,t}, & \varepsilon_{\tilde{\phi}^e,t} \sim N\left(0, \sigma_{\tilde{\phi}^e}\right), & \mathbb{E}\,\tilde{\phi}_t^e = 0, \\ \tilde{\phi}_t^u = \rho_{\tilde{\phi}^u}\tilde{\phi}_{t-1}^u + \varepsilon_{\tilde{\phi}^u,t}, & \varepsilon_{\tilde{\phi}^u,t} \sim N\left(0, \sigma_{\tilde{\phi}^u}\right), & \mathbb{E}\,\tilde{\phi}_t^u = 0. \end{array} \tag{4.35}$$

Risk premium for assets in the given currency depends on the position in those asses at the scale of the whole economy, while function Φ is strictly decreasing in A_t^e $\left(A_t^u\right)$. For total foreign assets, $a_t \equiv \frac{A_t}{z_t^+} = \frac{A_t^e}{z_t^+} + \frac{A_t^u}{z_t^+}$, we assume that in steady state they are equal to 0, while foreign assets denominated in euro are positive (then $a^u = -a^e$).

Based on the utility function (4.24), budget constraint (4.33) and the law of motion of capital (4.29) we may formulate an optimisation problem and the Lagrange functional:

$$\max_{c_{j,t}, i_{j,t}, u_{j,t}, \overline{k}_{j,t+1}, m_{j,t+1}, q_{j,t}, \tilde{\Delta}_{j,t}, b_{j,t+1}^e, b_{j,t+1}^u} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathscr{L}_t,$$

$$\begin{aligned}
\mathscr{L}_t = {} & \zeta_{t+s}^c \ln\left(C_{j,t+s} - bC_{j,t+s-1}\right) - \zeta_{t+s}^h A_L \frac{h_{j,t}^{1+\sigma_L}}{1+\sigma_L} + A_q \zeta_{t+s}^q \frac{\left(\frac{Q_{j,t}}{z_t P_t}\right)^{1-\sigma_q}}{1-\sigma_q} \\
& + \upsilon_t \Bigg[R_{t-1}\left(M_{j,t} - Q_{j,t}\right) + Q_{j,t} + TR_t + D_{j,t} + \left(1-\tau_t^k\right)\Pi_t + \left(1-\tau_t^p\right) R_t^k u_{j,t} \overline{K}_{j,t} \\
& \quad + \left(1-\tau_t^y\right)\left(1-\tau_t^w\right) W_{j,t} h_{j,t} + R_{t-1}^e \Phi\left(\frac{A_{t-1}^e}{z_{t-1}^+}, \mathbb{E}_t s_t^e s_{t-1}^e, \tilde{\phi}_{t-1}^e\right) S_t^e B_{j,t}^e \\
& \quad + R_{t-1}^u \Phi\left(\frac{A_{t-1}^u}{z_{t-1}^+}, \mathbb{E}_t s_t^u s_{t-1}^u, \tilde{\phi}_{t-1}^u\right) S_t^e S_t^x B_{j,t}^u + \tau_t^p P_t^d F_{a,t}^\tau \frac{a(u_{j,t})}{\Psi_t} \overline{K}_{j,t} + \tau_t^p P_t P_{k',t} \delta \overline{K}_{j,t} \\
& \quad - \tau_t^k \Bigg[\left(R_{t-1} - 1\right)\left(M_{j,t} - Q_{j,t}\right) + \left(R_{t-1}^e \Phi\left(\frac{A_{t-1}^e}{z_{t-1}^+}, \mathbb{E}_t s_t^e s_{t-1}^e, \tilde{\phi}_{t-1}^e\right) - 1\right) S_t^e B_{j,t}^e \\
& \quad \quad + \left(R_{t-1}^u \Phi\left(\frac{A_{t-1}^u}{z_{t-1}^+}, \mathbb{E}_t s_t^u s_{t-1}^u, \tilde{\phi}_{t-1}^u\right) - 1\right) S_t^e S_t^x B_{j,t}^u + B_{j,t}^e\left(S_t^e - S_{t-1}^e\right) \\
& \quad \quad + B_{j,t}^u\left(S_t^e S_t^x - S_{t-1}^e S_{t-1}^x\right) \Bigg] - \Bigg(M_{j,t+1} + S_t^e B_{j,t+1}^e + S_t^e S_t^x B_{j,t+1}^u \\
& \quad + P_t^c C_{j,t}\left(1+\tau_t^c\right) + P_t^i I_t + P_t^d \left(F_{a,t}^\tau \frac{a(u_{j,t})}{\Psi_t} \overline{K}_{j,t} + P_{k',t} \Delta_t\right) \Bigg) \Bigg] \\
& \quad + \omega_t \left[(1-\delta)\overline{K}_{k,t} + \Upsilon_t F\left(I_{j,t}, I_{j,t-1}\right) + \Delta_{j,t} - \overline{K}_{j,t+1}\right]
\end{aligned}$$

where υ_t and ω_t are Lagrange multipliers. First order conditions of the above decision-making problem create a system of equations determining consumption, investments, physical capital stock and its utilization rate, cash holdings, foreign assets denominated in euro, marginal utility of nominal income and nominal exchange rate (USD/PLN). After stationarization, for which we additionally define:

$$\upsilon_t z_t^+ P_t^d \equiv \psi_{z+,t} \tag{4.36}$$

and upon the consideration of the solution symmetry, the conditions may be presented as follows:

$$\frac{\zeta_t^c}{c_t - bc_{t-1}\frac{1}{\mu_{z^+,t}}} - \beta b \mathop{\mathbb{E}}_t \frac{\zeta_{t+1}^c}{c_{t+1}\mu_{z^+,t+1} - bc_t} - \psi_{z^+,t}\gamma_t^{cd}\left(1+\tau_t^c\right) = 0,$$

$$\frac{\upsilon_t P_t^d p_{k',t}}{\Psi_t} = \omega_t,$$

$$\begin{aligned}-\psi_{z^+,t}\gamma_t^{id} + \psi_{z^+,t}p_{k',t}\Upsilon_t F_1\left(i_t, i_{t-1}, \mu_{z^+,t}\mu_{\Psi,t}\right)\\ + \beta \mathop{\mathbb{E}}_t \psi_{z^+,t+1}p_{k',t+1}\Upsilon_{t+1}F_2\left(i_{t+1}, i_t, \mu_{z^+,t+1}\mu_{\Psi,t+1}\right) = 0,\end{aligned}$$

$$-\psi_{z^+,t} + \beta \mathop{\mathbb{E}}_t \frac{\psi_{z^+,t+1}}{\pi_{t+1}\mu_{z^+,t+1}}\left[\left(1-\tau_{t+1}^k\right)\left(R_t - 1\right) + 1\right] = 0,$$

$$\begin{aligned}-\psi_{z^+,t}p_{k',t} + \beta \mathop{\mathbb{E}}_t \frac{\psi_{z^+,t+1}}{\mu_{z^+,t+1}\mu_{\Psi,t+1}}\Big[\left(1-\tau_{t+1}^p\right)\left(\bar{r}_{t+1}^k u_{t+1} - F_{t+1}^\tau a(u_{t+1})\right)\\ + \tau_{t+1}^p p_{k',t+1}\delta + p_{k',t+1}\left(1-\delta\right)\Big] = 0,\end{aligned}$$

$$\bar{r}_t^k = F_{a,t}^\tau a'(u_{j,t}),$$

$$\zeta_t^q A_q q_t^{-\sigma_q} - \psi_{z^+,t}\left(1-\tau_t^k\right)\left(R_{t-1}^b - 1\right) = 0,$$

$$-\psi_{z^+,t} + \beta \mathop{\mathbb{E}}_t \frac{\psi_{z^+,t+1}}{\mu_{z^+,t+1}\pi_{t+1}}\left[s_{t+1}^e R_t^e \Phi\left(a_t^e, s_{t+1}^e s_t^e, \tilde{\phi}_t^e\right)\left(1-\tau_{t+1}^k\right) + \tau_{t+1}^k\right] = 0,$$

$$-\psi_{z^+,t} + \beta \mathop{\mathbb{E}}_t \frac{\psi_{z^+,t+1}}{\mu_{z^+,t+1}\pi_{t+1}}\left[s_{t+1}^e s_{t+1}^x R_t^u \Phi\left(a_t^u, \mathop{\mathbb{E}}_t s_{t+1}^u s_t^u, \tilde{\phi}_t^u\right)\left(1-\tau_{t+1}^k\right) + \tau_{t+1}^k\right] = 0.$$

The labour market is characterised by monopolistic competition — households provide heterogeneous labour services $\left(h_{j,t}\right)$ and set wages $\left(W_{j,t}\right)$. In a similar manner to creating homogenous final products by the aggregators, also the heterogeneous labour services are aggregated into a homogenous labour services $\left(H_t\right)$, which may be then used by the domestic intermediate goods producers:

$$H_t = \left[\int_0^1 \left(h_{j,t}\right)^{\frac{1}{\lambda_t^w}} dj\right]^{\lambda_t^w}, \qquad 1 \leq \lambda_t^w < \infty,$$

where wage markup is described with an exogenous process:

$$\lambda_t^w = \left(1-\rho_{\lambda^w}\right)\lambda^w + \rho_{\lambda^w}\lambda_{t-1}^w + \varepsilon_{\lambda^w,t}, \qquad \varepsilon_{\lambda^w,t} \sim N\left(0, \lambda^w \sigma_{\lambda^w}\right), \qquad \mathbb{E}\,\lambda_t^w = \lambda^w. \tag{4.37}$$

The process of wage setting runs similarly to the process of price setting by the producers (Calvo model) — in each period, with probability $1-\xi_w$, a household may set optimal wage; with probability ξ_w wage cannot be re-optimised, it may only be indexed to previous inflation of consumer prices (with weight κ_w), the current value of the inflation target (with weight $1-\kappa_w$) and the current technology growth:

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$$W_{j,t+1} = \left(\pi_t^c\right)^{\kappa_w}\left(\overline{\pi}_{t+1}^c\right)^{1-\kappa_w}\mu_{z^+,t+1}W_{j,t}.$$

When a household is allowed to set the wage in an optimal way, it maximises the difference between the utility of income on account of wage and disutility of leisure reduction:

$$\begin{aligned}\max_{W_t^{new}} \mathbb{E}_t \sum_{s=0}^{\infty}\left(\beta\xi_w\right)^s \Bigg[& -\zeta_{t+s}^h A_L \frac{\left(h_{j,t+s}\right)^{1+\sigma_L}}{1+\sigma_L} + \upsilon_{t+s}\left(1-\tau_{t+s}^y\right)\left(1-\tau_{t+s}^w\right)\times \\ & \times\left(\left(\pi_t^c \ldots \pi_{t+s-1}^c\right)^{\kappa_w}\left(\overline{\pi}_{t+1}^c \ldots \overline{\pi}_{t+s}^c\right)^{1-\kappa_w}\times \right.\\ & \left.\times\left(\mu_{z^+,t+1}\ldots\mu_{z^+,t+s}\right)W_t^{new}\right)h_{j,t+s}\Bigg].\end{aligned}$$

The first order condition of the above decision-making problem leads to the equation of real wage in economy.

4.5 Behaviour of other agents

Apart from optimising agents (households, firms), the $\mathbb{SOE}^{\mathrm{PL}}$ model explicitly considers the existence of two additional agents — the central bank and the government. The agents have not been assigned any formal, autonomous object functions. It is only assumed that the purpose of the central bank is to control price dynamics, and the only instrument the bank has at its disposal is the interest rate. The other agent — the government — fulfils a passive function of managing budget funds, i.e. charges taxes from which expenditures are financed.

4.5.1 Central bank

The reactions of the central bank[5] are characterised from an external point of view, i.e. rational and anticipating active participants of the economic processes — firms and households. From the actually applied monetary policy, the agents derive characteristics of the rules of behaviour of the central bank: interest rate persistence, sensitivity of the interest rate to inflation deviations from the reference point (inflation target[6]), sensitivity of the interest rate to GDP deviations from its level in a steady state, etc. The interpretation of activities of the central bank and independent conclusions regarding the perspectives of the inflation processes, as well as further policy of the central bank must be reliable. In microeconomic decision-making problems of households and firms there appear values related to the interest rate policy, therefore, optimal decisions of setting prices and wages depend on the values. On the other hand, it is silently assumed that the central bank is not trying to carry out any form of game intended for exploitation of the perception error created by the policy. In other words, we assume that monetary policy is reliable. Possibly, temporary incoherence or inconsistency is absorbed by the fluctuations of the inflation target.

4

As a consequence of the above mentioned assumptions, the interest rate rule describing the activities of the central bank is defined directly in a log-linearised form, without prior reference to the decision-making problem. The form of the rule is sufficiently general for covering a possibly broad spectrum of interest rate policies. During estimation we allow the possibility of a structural change occurrence, i.e. change in the value of some of the parameters of the rule.

We assume that the real exchange rate present in the rule is the effective rate defined as:

$$\widehat{x}_t^{ue} = \widehat{x}_t^{u} + \left(1 - \gamma^{xux}\right)\widehat{x}_t^{x}, \tag{4.38}$$

where γ^{xux} determines the currency structure of international settlements. The standard version of the interest rate rule has then the form:

$$\begin{aligned}\widehat{R}_t = \rho_R \widehat{R}_{t-1} + \left(1-\rho_R\right)\Big[r_{\overline{\pi}^c}\widehat{\overline{\pi}}_t^c + r_\pi\left(\widehat{\pi}_{t-1}^c - \widehat{\overline{\pi}}_t^c\right) + r_y \widehat{y}_{t-1} + r_x \widehat{x}_{t-1}^{eu}\Big] \\ + r_{\Delta\pi}\Delta\widehat{\pi}_t^c + r_{\Delta y}\Delta\widehat{y}_t + \epsilon_{R,t}.\end{aligned} \tag{4.39}$$

In the current version of the model we assume that $r_{\overline{\pi}^c} \equiv 1$, therefore, $\widehat{\overline{\pi}}_t^c$ shall be interpreted as the perception of the policy of the central bank (inflation target) by the agents. The disturbance of the interest rate (monetary policy, monetary disturbance) $\epsilon_{R,t}$, — contrary to other shocks appearing in the model — is defined as innovation. The interpretation of the component

[5] Depending on the type of interest rate to which the model refers, an extending interpretation of the agent or group of agents responsible for interest rate change is possible. In a typical situation, when interest rate is derived from the interbank market, there are grounds for claiming that interest rate fluctuations result at the same time from the activities of the central bank and the responses to the current events in the interbank market. Such interpretation eliminates the automatic assignment of any interest rate changes to the decisions of the central bank.

[6] We differentiate the inflation target declared by the central bank from the inflation target being the result of perception of the inflation processes and monetary policy by the optimising agents. The second concept allows for a stochastic nature of the target oscillating around the stationary value (steady state). The stationary value of the target (in the second meaning) may but does not have to be compliant with the target declared by the institution responsible for the monetary policy. In each case when we speak about the inflation target, we mean the second concept. This model does not refer to the officially declared targets of the monetary policy.

suggesting itself — discretionary component of the monetary policy — is, however, disputable. Traditionally, it has been assumed that monetary disturbance is not correlated with variables present in the information set (see e.g. Christiano et al., 1998). Therefore, it should be assumed that the information set of agents (model) and the information set of the central bank are different.

4.5.2 Government

In the class of models deriving from the research of L. Christiano, such as in the known models built for the purposes of central banks, the function of government is reduced to administration of the state income redistribution. Taxes are not an instrument of fiscal policy — their level is relatively constant, there are only observed temporary deviations from the long-term level (steady state). The deviations have the nature of disturbances. In the $\mathbb{SOE}^{\mathrm{PL}-2009}$ model the disturbances are approximated with the AR(1) process or a SVAR model, such as it is made in DSGE models with exogenous variables. Although the fluctuations of tax rates have been treated in a slightly simplified manner, the specification of decision-making problems of firms and households shows the role of national insurance contributions and taxes in determining the budget of the agents and, therefore, their behaviour. In the $\mathbb{SOE}^{\mathrm{PL}-2009}$ model there are explicitly present national insurance contributions paid by the employers (τ^s) and employees (τ^w), the capital tax (τ^k), the tax on consumption (τ^c), the income tax (τ^y) and the corporate income tax (τ^p). This is, thus, a set of the basic fiscal charges imposed on agents. Therefore, the fiscal policy impact is represented on the micro level.

Macroeconomic consequences of fiscal policy are treated in a more simplified manner. The model is based on the pattern of a representative consumer; one of the functions included in the fiscal policy is, thus, such redistribution of the state revenues that the differentiated households reflect a uniform pattern of consumption behaviour. Formally, in the households budget constraint the category of special state contingent insurance (D) appears, however, the system is not characterised in detail, so it must be financed by the budget. An element of the system is also the pension and disability-pension block. We do not define separate institutions dealing with management of such class of funds — in the applied pattern of a representative agent there is no place for households living only of the pension payments, therefore, national insurance contributions are treated as the revenues of the government (budget) that are spent on a current basis.

Within the above outlined redistribution system, the existence of budget deficit and public debt has not been explicitly provided. The Ricardian logic of optimising and forward-looking households rationalises this solution (today's budget deficit is balanced with the growth of taxes tomorrow, namely reduction of disposable income, therefore, it requires a reduction in spending already today). Technically, transfers (TR) received by households may be negative, so households shall immediately and directly reduce disposable income. If households are not inclined to reduce their expenditures, they may finance them with negative foreign deposits. Thus, the reasoning here is cohesive. Emphasis should be placed on the fact that the aforesaid approximation may be considered to be sufficient for an economy that only sporadically experiences a deficit or surplus in the budget (public finance). If the budget deficit has a structural nature and the public debt reflects no trend of stabilisation, the role of the government sector may be larger than it appears from the proposed approximation.

The amount of government expenditures (G_t) has, similarly as the tax rates, a stochastic nature. The fluctuations of government expenditures (government consumption, collective consumption) around a steady state are approximated with a (separately estimated) SVAR model. More precisely, a separate model has been built trying to explain jointly the fluctuations of government expenditures and the rates of income tax, corporate income tax and tax on consumption. The model has the following form:

$$\Gamma_0 \,\widehat{\tau}_t = \Gamma_0 \,\Gamma(L)\, \widehat{\tau}_{t-1} + \Gamma_0 \,\epsilon_{\tau,t}, \qquad \epsilon_{\tau,t} \sim \mathbb{N}\left(0, \Sigma_\tau\right), \tag{4.40}$$

$$\widehat{\tau}_t \equiv \left[\widehat{\tau}^p_t, \widehat{\tau}^y_t, \widehat{\tau}^c_t, \widehat{\widehat{G}}_t\right]', \qquad \epsilon_{\tau,t} \equiv \left[\epsilon_{\tau^p,t}, \epsilon_{\tau^y,t}, \epsilon_{\tau^c,t}, \epsilon_{\tau^{\widehat{G}},t}\right]',$$

Structural decomposition of disturbances is given by:

$$\Gamma_0 \,\epsilon_{\tau,t} = B_{0,\tau}\, u_t, \qquad \mathbb{E}\, u \cdot u' = I. \tag{4.41}$$

The estimated matrices of parameters (Γ_0) i ($B_{0,\tau}$) enable the determination of the matrix of disturbance covariance (Σ_τ). The fluctuations (deviations from the steady state) of the tax on capital and national insurance rates are treated as structural disturbances and are approximated in the DSGE model with the form of AR(1) process.

Joint reflection of budget expenditures and income leads to the following specification:

$$\begin{aligned} P_t\, G_t + \left(TR_t + D_t\right) = {} & R_{t-1} \left(M_{t+1} - M_t\right) + \tau^c_t\, P^c_t\, C_t + \left(\tau^w_t + \tau^s_t + \tau^y_t\, \left(1 - \tau^w_t\right)\right) W_t\, H_t \\ & + \tau^k_t \Big[\Pi_t + \left(R_{t-1} - 1\right) \left(M_t - Q_t\right) \\ & \quad + \left(R^e_{t-1} \Phi^e(a^e_{t-1}, ..., \tilde{\phi}^e_{t-1}) - 1\right) S^e_t\, B^e_t + B^e_t \left(S^e_t - S^e_{t-1}\right) \\ & \quad + \left(R^u_{t-1} \Phi^u(a^u_{t-1}, ..., \tilde{\phi}^u_{t-1}) - 1\right) S^u_t\, B^u_t + B^u_t \left(S^u_t - S^u_{t-1}\right) \Big] \\ & + \tau^p_t \left[R^k_t\, u_t\, \overline{K}_t - \frac{1}{\Psi_t} P_t\, F^\tau_{a,t}\, a\left(u_t\right) \overline{K}_t - P_t\, P_{k',t}\, \delta\, \overline{K}_t \right]. \end{aligned} \tag{4.42}$$

A new solution introduced to the discussed version of the $\mathbb{SOE}^{\text{PL}}$ model is the use of the above specification in building a consolidated (integrated) balance of total expenditures and income, which means that the changes e.g. in government expenditures impact e.g. net foreign assets. The issue is discussed in more detail in the following paragraphs.

4.6 Macroeconomic balance conditions

At macro scale, equilibrium is identified with simultaneous balancing of income and expenditures of households and the government, the aggregated supply and demand (factor resources), as well as the balance of the banking sector and international transactions (payment balance). As a result, we obtain a system guaranteeing that in each period the total of expenditures is equal to the total of income, the stream of services of productivity factors is sufficient for manufacturing domestic intermediate products, which in combination with the carried out import and export ensures the levelling of demand and supply at macro scale. Net export is balanced with foreign net assets.

The system of macroeconomic balance is considerably much more extended in comparison with the previous versions of $\mathbb{SOE}^{\text{PL}}$ and $\mathbb{SOE}^{\text{Euro}}$. In particular, for the first time macroeconomic balance of income and expenditures of households has been introduced, calculated at current prices, which requires, among others, the derivation of firms' profit accounts (being the income of households). A new solution also consists in the explicit derivation of the balance of expenditures and income of the government (equivalent to state budget), which upon integration with the balance of income and expenditures of households covers all of the expenditures at the scale of the whole economy and the methods of their financing (sources of income). The balance enables the determination of net foreign assets, so it replaces a classical balance of payments.

4

4.6.1 Profits in economy

We take into account firms manufacturing intermediate products, importers and exporters, who transfer their profits to households where the profits are taxed with the tax on dividend (tax on capital). We assume that total profit in economy is the sum of the profits generated in particular sectors of economy (domestic production, export and import), i.e.:

$$\Pi_t = \Pi_t^d + \Pi_t^x + \Pi_t^m.$$

According to the suggestion of Christiano et al. (2007b, page 26–28), profits generated by firms may be estimated at macro scale with the use of marginal costs assessment, taking into account the ineffectiveness of allocation resulting from price setting with the use of the Calvo model. More precisely, profits result from a difference between the marginal cost and the actual price. Contrary to the previous versions of the model, we assume that fixed costs are absent in domestic production, instead profits appear in steady state.

Domestic products manufacturers

In the case of intermediate products manufacturers we have:

$$\Pi_t^d = \int_0^1 P_{jt}^d\, Y_{jt}\, \mathrm{d}\, j - P_t^d\, mc_t^d \left(\int_0^1 Y_{jt}\, \mathrm{d}\, j \right) = P_t^d\, Y_t - P_t^d\, mc_t^d \left(Y_t \left(\frac{\bar{P}_t^d}{P_t^d} \right)^{\frac{\lambda_t^d}{1-\lambda_t^d}} \right).$$

The value $\left(\frac{\bar{P}_t^d}{P_t^d} \right)^{\frac{\lambda_t^d}{1-\lambda_t^d}}$ is the allocation inefficiency. Such expression may be approximated with the function of markup $f(\lambda_t^d, \ldots) \equiv f_t^d$, however, in the case of log-linearisation it is also justified to treat the price relations as equal one. Taking into account the above, profits at macro scale may be estimated as:

$$\Pi_t^d = P_t^d\, Y_t - P_t^d\, mc_t^d \left(Y_t\, f_t^d \right). \tag{4.43}$$

Making it stationary and then bringing down to fixed prices, we obtain:

$$\varpi_t^d \equiv \frac{\Pi_t^d}{P_t^d\, z_t^+} = y_t - mc_t^d\, y_t\, f_t^d = y_t\left(1 - mc_t^d\, f_t^d\right), \tag{4.44}$$

where the marginal cost mc^d is expressed with equation (4.15).

Profits in export

Assuming, further, that profits in export (calculated in domestic currency) are subject to domestic tax, omitting the existence of fixed costs, we have:

$$\Pi_t^x = P_t^x\, S_t^e\, S_t^x\, \left(C_t^x + I_t^x\right)\left(1 - mc_t^x\, f_t^x\right), \tag{4.45}$$

where f_t^x is defined as f_t^d. After making it stationary, we obtain:

$$\varpi_t^x = \frac{P_t^x\, S_t^e\, S_t^x}{P_t^d}\left(1 - mc_t^x\, f_t^x\right)\left(c_t^x + i_t^x\right) = \gamma_t^{xd}\left(1 - mc_t^x\, f_t^x\right)\left(c_t^x + i_t^x\right). \tag{4.46}$$

Profits in import

Out of the two possible methods of defining profits (on the micro level with further aggregation or with direct reference to the macro scale), we have used the macroeconomic convention:

$$\Pi_t^m = P_t^{mc}\, C_t^m + P_t^{mi}\, I_t^m + P_t^{mx}\, X_t^m - S_t^e\, S_t^x\, P_t^\star\left(C_t^m + I_t^m + X_t^m\right).$$

Bringing it to the stationary version, we arrive at:

$$\begin{aligned}\varpi_t^m = \frac{\Pi_t^m}{z_t^+\, P^d{}_t} &= \gamma_t^{mcd}\, c_t^m + \gamma_t^{mid}\, i_t^m + \gamma_t^{mxd}\, x_t^m - \gamma_t^{\star,d}\left(c_t^m + i_t^m + x_t^m\right) = \\ &= \left(\gamma_t^{mcd} - \gamma_t^{\star,d}\right) c_t^m + \left(\gamma_t^{mid} - \gamma_t^{\star,d}\right) i_t^m + \left(\gamma_t^{mxd} - \gamma_t^{\star,d}\right) x_t^m.\end{aligned} \tag{4.47}$$

4.6.2 Income and expenditures of households

We assume that the total of net income and expenditures of households is balanced. Merging the income side with the side of expenditures, we arrive at the aggregated version of the budget condition of households:

$$
\begin{aligned}
&\left(1-\tau_t^k\right)\left(R_{t-1}-1\right)\left(M_t-Q_t\right)\\
&+\left(1-\tau_t^k\right)R_{t-1}^e\Phi\left(a_{t-1}^e,s_t^e,s_{t-1}^e,\tilde{\phi}_{t-1}^e\right)S_t^eB_t^e+\tau_t^kS_{t-1}^eB_t^e\\
&+\left(1-\tau_t^k\right)R_{t-1}^u\Phi\left(a_{t-1}^u,s_t^x,s_t^e,s_{t-1}^x,s_{t-1}^e,\tilde{\phi}_{t-1}^u\right)S_t^eS_t^xB_t^u+\tau_t^kS_{t-1}^eS_{t-1}^xB_t^u\\
&+\left(1-\tau_t^k\right)\Pi_t+\left(1-\tau_t^p\right)\left(R_t^ku_t\overline{K}_t-\frac{1}{\Psi_t}P_tF_{a,t}^\tau a\left(u_t\right)\overline{K}_t\right)+\tau_t^pP_tP_{k',t}\delta\overline{K}_t\\
&+\left(1-\tau_t^y\right)\left(1-\tau_t^w\right)W_tH_t+M_t+TR_t+D_t\\
&-M_{t+1}-S_t^eB_{t+1}^e-S_t^eS_t^xB_{t+1}^u-\left(1+\tau_t^c\right)P_t^cC_t-P_t^iI_t-P_tP_{k',t}\Delta_t=0.
\end{aligned}
\tag{4.48}
$$

4.6.3 State budget

The term state budget or ”government budget” means here, approximately, the sector of public finance and a fragment of the financial sector specialising in pension insurance.

The standard form of the balance of revenues and expenditures of a government stems from the original version of the $\mathbb{SOE}^{\text{Euro}}$. Upon adaptation to the domestic tax system and the possibility of depositing savings in two currency markets (next to domestic deposits), we arrive at (see equation (4.42)):

$$
\begin{aligned}
P_tG_t+\left(TR_t+D_t\right)=R_{t-1}\left(M_{t+1}-M_t\right)+\tau_t^cP_t^cC_t+\left(\tau_t^w+\tau_t^s+\tau_t^y\ \left(1-\tau_t^w\right)\right)W_tH_t\\
+\tau_t^k\Big[\Pi_t+\left(R_{t-1}-1\right)\left(M_t-Q_t\right)\\
+\left(R_{t-1}^e\Phi^e(a_{t-1}^e,\ldots,\tilde{\phi}_{t-1}^e)-1\right)S_t^eB_t^e+B_t^e\left(S_t^e-S_{t-1}^e\right)\\
+\left(R_{t-1}^u\Phi^u(a_{t-1}^u,\ldots,\tilde{\phi}_{t-1}^u)-1\right)S_t^uB_t^u+B_t^u\left(S_t^u-S_{t-1}^u\right)\Big]\\
+\tau_t^p\left[R_t^ku_t\overline{K}_t-\frac{1}{\Psi_t}P_tF_{a,t}^\tau a\left(u_t\right)\ \overline{K}_t-P_tP_{k',t}\delta\overline{K}_t\right].
\end{aligned}
\tag{4.49}
$$

In the above formula we assume that national insurance contributions paid by the employees and the employer represent the revenues of the government budget.

4.6.4 Monetary balance

As in the original version of the SOE model, we define the broad money dynamics as:

$$
\mu_t\equiv\frac{M_{t+1}}{M_t}=\frac{m_{t+1}}{m_t}\mu_{z^+,t}\,\pi_t^d.
\tag{4.50}
$$

The equation (after log-linearisation) shall be used for explaining the resource of money. We notice that in the steady state the following applies:

$$
\pi^d=\frac{\mu}{\mu_{z^+}}.
$$

The banking system must provide financing of firms with working capital loans and must provide cash for households. Therefore, we have the dependence:

$$\nu_t^k \, \mathrm{F}_t^\tau \, R_t^k K_t + \nu_t^w \, \mathrm{F}_t^\tau \, \left(1+\tau_r^s\right) W_t H_t = M_{t+1} - Q_t, \tag{4.51}$$

where $M_{t+1} = \mu_t M_t$.

Making it stationary and expressing in fixed prices, we obtain the following form:

$$\nu_t^k \, \mathrm{F}_t^\tau \, \frac{\overline{r}_t^k k_t}{\mu_{\Psi,t}\, \mu_{z^+,t}} + \nu_t^w \, \mathrm{F}_t^\tau \, \left(1+\tau_r^s\right) \overline{w}_t H_t = \frac{\mu_t m_t}{\pi_t^d \mu_{z^+,t}} - q_t. \tag{4.52}$$

4.6.5 Balance of payment

The basic function of the payment balance in the model is the determination of the value of net foreign assets compliant with the level of activity in foreign trade, expenditures and income in economy. Foreign assets may be determined at least in two ways: from the classical version of balance, when we consider net export, and from the integrated balance of expenditures and incomes. In $\mathbb{SOE}^{\mathrm{PL}-2009}$ both solutions were implemented. The selection was to be done by virtue of experiments, estimation of parameters, analyses of the model characteristics (impulse response function) and analyses of forecasting accuracy. Yet, the experiments showed that the alternative of the income balance increases the potential possibility of the model, therefore, it was the one finally chosen. Below we present both alternatives.

4

Classical version

In the payment balance, as in the $\mathbb{SOE}_{€}^{\mathrm{PL}}$ model, the geographic structure of net foreign assets is presented explicitly (see Kłos, 2008). However, we add import to the specification, and it becomes a component of export. Doubts appear at the moment of choosing the price index to be used for expressing import at current prices. Here we assume that in the balance of payment import is calculated at global prices, i.e. prices at which the products were purchased. Such approach complies with the approach assumed in the previous versions of the $\mathbb{SOE}$ model.

$$\begin{aligned} S_t^e S_t^x B_{t+1}^u + S_t^e B_{t+1}^e = {} & S_t^e S_t^x P_t^x \left(C_t^x + I_t^x\right) - S_t^e S_t^x P_t^\star \left[C_t^m + I_t^m + X_t^m\right] \\ & + S_t^e S_t^x B_t^u R_{t-1}^u \Phi^u\left(a_{t-1}^u, \tilde{\phi}_{t-1}^u, \ldots\right) \\ & + S_t^e B_t^e R_{t-1}^e \Phi^e\left(a_{t-1}^e, \tilde{\phi}_{t-1}^e, \ldots\right). \end{aligned} \tag{4.53}$$

The classical form of the payment balance (after bringing it to stationary state) has the form:

$$\begin{aligned} \left(a_t^u + a_t^e\right) = {} & \gamma_t^{x,d} x_t - \gamma_t^{\star,d} \left(c_t^m + i_t^m + x_t^m\right) \\ & + s_t^e s_t^x \frac{a_{t-1}^u}{\pi_t^d \mu_{z^+,t}} R_{t-1}^u \Phi^u\left(a_{t-1}^u, \tilde{\phi}_{t-1}^u, s_t^e, s_t^x, s_{t-1}^e, s_{t-1}^x\right) \\ & + s_t^e \frac{a_{t-1}^e}{\pi_t^d \mu_{z^+,t}} R_{t-1}^e \Phi^e\left(a_{t-1}^e, \tilde{\phi}_{t-1}^e, s_t^e, s_{t-1}^e\right), \end{aligned} \tag{4.54}$$

4

where:

$$\Phi^i(a_t^i, s_{t+1}^i, s_t^i, \tilde{\phi}_t^i) = \exp\left\{-\tilde{\phi}_a^i \left(a_t^i - a^i\right) - \tilde{\phi}_s^i \left(\mathop{\mathbb{E}}_t s_{t+1}^i s_t^i - 1\right) + \tilde{\phi}_t^i\right\}, \qquad i \in \{u, e\} \quad (4.55)$$

and

$$a_t^u = \frac{S_t^e S_t^x B_{t+1}^u}{P_t^d z_t^+}, \quad a_t^e = \frac{S_t^e B_{t+1}^e}{P_t^d z_t^+}, \quad \gamma_t^{x,d} = \frac{S_t^e S_t^x P_t^x}{P_t^d},$$

$$\gamma_t^{\star,d} = \frac{S_t^e S_t^x P_t^\star}{P_t^d}, \quad s_t^i = \frac{S_t^i}{S_{t-1}^i} \quad i \in \{u, e, x\}.$$

Version with the income balance

Merging the income and expenditure balance of households (4.48) and the state budget (4.42), we arrive at the formula, which upon simplification takes the following form:

$$\begin{aligned}
P_t^g G_t + P_t^i I_t + P_t^c C_t + S_t^e B_{t+1}^e + S_t^e S_t^x B_{t+1}^u = {} & \Pi_t + \left(1 + \tau_t^s\right) W_t H_t \\
& + \left(R_t^k u_t - \frac{1}{\Psi_t} P_t F_a^\tau a\left(u_t\right)\right) \overline{K}_t \\
& + \left(R_{t-1} - 1\right)\left(M_{t+1} - Q_t\right) \\
& + R_{t-1}^e \Phi\left(a_{t-1}^e, \ldots, \tilde{\phi}_{t-1}^e\right) S_t^e B_t^e \\
& + R_{t-1}^u \Phi\left(a_{t-1}^u, \ldots, \tilde{\phi}_{t-1}^u\right) S_t^e S_t^x B_t^u.
\end{aligned}$$

Taking into account balance of the banking sector, we obtain:

$$\begin{aligned}
P_t^g G_t + P_t^i I_t + P_t^c C_t + S_t^e B_{t+1}^e + S_t^e S_t^x B_{t+1}^u = {} & \Pi_t + \left(1 + \tau_t^s\right)\left[\mathrm{F}_t^\tau \left(R_t^{fw} - 1\right) + 1\right] W_t H_t \\
& + \left[\left[\mathrm{F}_t^\tau \left(R_t^{fk} - 1\right) + 1\right] R_t^k u_t - \frac{1}{\Psi_t} P_t \mathrm{F}_{a,t}^\tau a\left(u_t\right)\right] \overline{K}_t \\
& + R_{t-1}^e \Phi\left(a_{t-1}^e, \ldots, \tilde{\phi}_{t-1}^e\right) S_t^e B_t^e + R_{t-1}^u \Phi\left(a_{t-1}^u, \ldots, \tilde{\phi}_{t-1}^u\right) S_t^e S_t^x B_t^u.
\end{aligned} \quad (4.56)$$

Finally, the payment balance obtained by the integration of income and expenditures of households, the income and expenditures of the budget and the balance of the banking sector (after making it stationary and bringing down to real categories) takes the form:

$$\begin{aligned}
& a_t^e + a_t^u + g_t + \gamma_t^{id} i_t + \gamma_t^{cd} c_t + \frac{\mathrm{F}_{a,t}^\tau a\left(u_t\right) \overline{k}_t}{\mu_{\Psi,t} \mu_{z^+,t}} = \\
& = \overline{\varpi}_t^d + \overline{\varpi}_t^m + \overline{\varpi}_t^x + \left(1 + \tau_t^s\right)\left[\mathrm{F}_t^\tau \left(R_t^{fw} - 1\right) + 1\right] \overline{w}_t H_t \\
& + \left[\mathrm{F}_t^\tau \left(R_t^{fk} - 1\right) + 1\right] \frac{\overline{r}_t^k k_t}{\mu_{\Psi,t} \mu_{z^+,t}} \\
& + R_{t-1}^e \Phi\left(a_{t-1}^e, \ldots, \tilde{\phi}_{t-1}^e\right) \frac{s_t^e a_{t-1}^e}{\pi_t^d \mu_{z^+,t}} + R_{t-1}^u \Phi\left(a_{t-1}^u, \ldots, \tilde{\phi}_{t-1}^u\right) \frac{s_t^e s_t^x a_{t-1}^u}{\pi_t^d \mu_{z^+,t}}.
\end{aligned} \quad (4.57)$$

The above equation shows that expenditures calculated at the macro level of economy (consumer expenditures, investment expenditures, government expenditures, new foreign deposits (net foreign assets), and capital adjustment are financed from the profits, income from labour, income from capital and revenues from (mature) foreign deposits. Net foreign assets amount in total to $a_t = a_t^e + a_t^u$.

4.6.6 The aggregate resource constraint

The starting point for further considerations is the formula in which all of the components are expressed at fixed prices. In resources constraint we omit the factor characterising the ineffectiveness of allocation (the effect of Calvo price settings). The obtained inequality has the form:

$$G_t + C_t^d + \frac{1}{\Psi_t}\left[I_t^d + F_{a,t}^{\tau}\, a\left(u_t\right) \overline{K}_t\right] + X_t^d \leq \varepsilon_t\, z_t^{1-\varpi}\, K_t^{\varpi}\, H_t^{1-\varpi} \tag{4.58}$$

or assuming equality:

$$\frac{G_t}{z_t^+} + \frac{C_t^d}{z_t^+} + \frac{1}{\Psi_t\, z_t^+}\left[I_t^d + F_{a,t}^{\tau}\, a\left(u_t\right) \overline{K}_t\right] + \frac{X_t^d}{z_t^+} = y_t \tag{4.59}$$

and using the solution of the decision problem of exporters (4.22), we may determine:

$$x_t^d = \frac{X_t^d}{z_t^+} = \left(1-\omega_x\right) f_t^x \left[\omega_x \left(\gamma_t^{mx,d}\right)^{1-\eta_{xx}} + \left(1-\omega_x\right)\right]^{\frac{\eta_{xx}}{1-\eta_{xx}}} \frac{X_t}{z_t^+}.$$

Merging the above and assuming equality of resources and demand for the resources, plus bringing it to the stationary form, we obtain:

$$\begin{aligned} & g_t + c_t^d + i_t^d + \left(1-\omega_x\right) f_t^x \left[\omega_x \left(\gamma_t^{mx,d}\right)^{1-\eta_{xx}} + \left(1-\omega_x\right)\right]^{\frac{\eta_{xx}}{1-\eta_{xx}}} x_t \\ = & \frac{\left(\Psi_{t-1}\, z_{t-1}^+\right)^{\varpi} z_t^{1-\varpi}}{z_t^+} \varepsilon_t\, H_t^{1-\varpi}\, k_t^{\varpi} - \frac{F_a^{\tau}}{\mu_{z^+,t}\, \mu_{\Psi,t}} a\left(u_t\right) \overline{k}_t - \phi. \end{aligned}$$

More generally, starting with the identity of GDP calculated in real terms:

$$g_t + c_t^d + i_t^d + x_t^d = \tilde{y}_t \tag{4.60}$$

we arrive at an equation that upon log-linearisation will explain the real income:

$$\begin{aligned} y_t = & \, g_t + c_t^d + i_t^d + \left(1-\omega_x\right) f_t^x \left[\omega_x \left(\gamma_t^{mx,d}\right)^{1-\eta_{xx}} + \left(1-\omega_x\right)\right]^{\frac{\eta_{xx}}{1-\eta_{xx}}} x_t \\ & + \frac{F_a^{\tau}}{\mu_{z^+,t}\, \mu_{\Psi,t}} a\left(u_t\right) \overline{k}_t. \end{aligned} \tag{4.61}$$

4

Part III

Results of estimation and characteristic features of the DSGE $\mathbb{SOE}^{\mathrm{PL}-2009}$ model

5
Forms of model, data, SVAR models

5.1 Forms of model

Upon the determination of the first order conditions from the decision-making problems presented earlier and their log-linearisation, we arrive at the linear structural form of the model which may be presented in matrix form in the following manner:

$$\begin{cases} \mathbb{E}_t \left\{ \alpha_0 \tilde{z}_{t+1} + \alpha_1 \tilde{z}_t + \alpha_2 \tilde{z}_{t-1} + \beta_0 \theta_{t+1} + \beta_1 \theta_t \right\} = 0, \\ \theta_t = \rho \; \theta_{t-1} + \varepsilon_t \end{cases} \tag{5.1}$$

where the vector of exogenous variables (θ_t disturbances) consists of structural disturbances θ_t^s and observable fiscal disturbances θ_t^τ and observable foreign disturbances $\theta_t^\star$. The approximation of the processes controlling observable disturbances is received by separate estimation of two SVAR models: fiscal model and world's economy model. Fragments of matrix ρ and the matrix of observable disturbances covariance are equivalent to the respective matrices of the SVAR model, i.e. they remain fixed during (Bayesian) estimation of the whole DSGE model. In the Appendix we shall present a list of variables of the structural form and a list of log-linearised equations which constitute the structural form of the model.

Transition to the reduced form of the model — which eliminates the forward-looking variables — is made with the use of Anderson-Moore algorithm. Thus, we obtain:

$$\begin{cases} \tilde{z}_{t+1} = A \tilde{z}_t + B \, \theta_{t+1}, \\ \theta_{t+1} = \rho \, \theta_t + \epsilon_{t+1}. \end{cases}$$

The system may be transformed in the state space form of the model. Then the transition equation shall be:

$$\underbrace{\begin{bmatrix} \tilde{z}_{t+1} \\ \theta_{t+1} \end{bmatrix}}_{\xi_{t+1}} = \underbrace{\begin{bmatrix} A & B\rho \\ 0 & \rho \end{bmatrix}}_{F_\xi} \underbrace{\begin{bmatrix} \tilde{z}_t \\ \theta_t \end{bmatrix}}_{\xi_t} + \underbrace{\begin{bmatrix} B \\ I \end{bmatrix} \epsilon_{t+1}}_{\upsilon_{t+1}}$$

and the state space model takes the form of:

$$\begin{cases} \xi_{t+1} = F_\xi\, \xi_t + \upsilon_{t+1}, & \mathbb{E}(\upsilon_{t+1}\, \upsilon'_{t+1}) = Q, \\ \underline{Y}_t = A'_x\, x_t + H'\, \xi_t + u_t, & \mathbb{E}(u_t\, u'_t) = R, \end{cases} \tag{5.2}$$

where $\underline{Y}_t$ is a vector of observed variables and u_t is a vector of measurement errors.

During the construction of the state space model, the state vector is supplemented with lagged variables. Variables that are not necessary in the measurement equations may be eliminated. In order to avoid introduction of further symbols, we shall omit the fact that the $\tilde{z}_t$ vector present in the structural form and the $\tilde{z}_t$ vector in the general case are different. As results from the notation of equations, on a standard basis we assume the appearance of measurement errors in the model (state space representation). The elements of the variance-covariance matrix of measurement errors R increase the pool of the model parameters. For simplicity, we shall assume that R is diagonal. The appearance of measurement errors in the equation is explained by inaccuracies in statistics, approximation errors appearing in measurement equations (log-linearisation of non-linear dependencies) and the possibility of errors in model specification[1], therefore, arrival at (additional) statistical identification is a side effect[2].

Formally, in the presented version of the model there are in total 30 shocks, including 11 observable disturbances (processes estimated with SVAR models). A list of the model variables and a list of equations of the structural form presented in the Appendix characterise a more general version of the $\mathbb{SOE}^{\mathrm{PL-2009}}$ model. The version discussed further on originated as an effect of elimination of the disturbance of national insurance contributions paid by the employer and the assumption that there is only one interest rate risk premium for the tested economy, common for both currency markets ($\tilde{\phi}^u_t = \tilde{\phi}^e_t = \tilde{\phi}_t$). In both cases we deal with a simplification of the model motivated by the problems with shocks identification.

5.2 Observable variables, data

The data used in a DSGE model estimation and SVAR models come from the official publications of the National Bank of Poland (NBP), the Central Statistical Office (GUS), the ECB and

[1] The subject of the selection of observable variables and the role of measurement errors specified in DSGE models were analysed by Guerron-Quintara (2009). According to his conclusions, consideration of measurement errors during a DSGE model estimation allows to increase the model resistance to specification errors and incorrect selection of observable variables; see also Boivin and Giannoni (2005).

[2] The issue of statistical identification was discussed in more detail by e.g. Ireland (2004), Alvarez-Lois et al. (2005), Canova and Sala (2005).

OECD available till November 2009. The observable variables in the $\underline{Y}_t$ vector in equation (5.2) are: GDP deflator, investment deflator, consumer price index (CPI), GDP, consumption (calculated together with changes in inventories), investment expenditures, export, import, total employment, real wages, domestic short-term interest rate (Wibor 3M), real USD/PLN exchange rate, GDP deflator in the euro area, GDP deflator in the USA, short-term interest rate for dollar (OECD assessments of three-month's dollar rate), short-term interest rate for euro (Euribor 3M), GDP in the USA, GDP in the euro area, the nominal cross rate (dollar/euro)[3] — 19 series in total. The whole sample for the DSGE $\mathbb{SOE}^{\mathrm{PL}-2009}$ model covers the period of 1996:2–2009:3. For the estimation of the SVAR model of the world's economy, we have used the longest available series.

The data referring to domestic national accounts stem from the system of measurement at fixed prices of the year 2000. At the initial stages of preparing the data, the consistency of the calculations (volume and deflators) was verified, and then the series were subjected to transformations that referred to the elimination of their seasonality with the X12 method (only variables that may reflect quarterly seasonality), elimination of means (exchange rate), finding the logarithm, transformation of variables into quarterly growth rates. Exceptions are the dynamics of prices, which have an annualised form, interest rate (remaining in the natural form) and employment. In the case of the last variable, the data regarding the level have been filtrated (with Hodrick-Prescott filter) and deviations from the HP trend act as an observable variable.

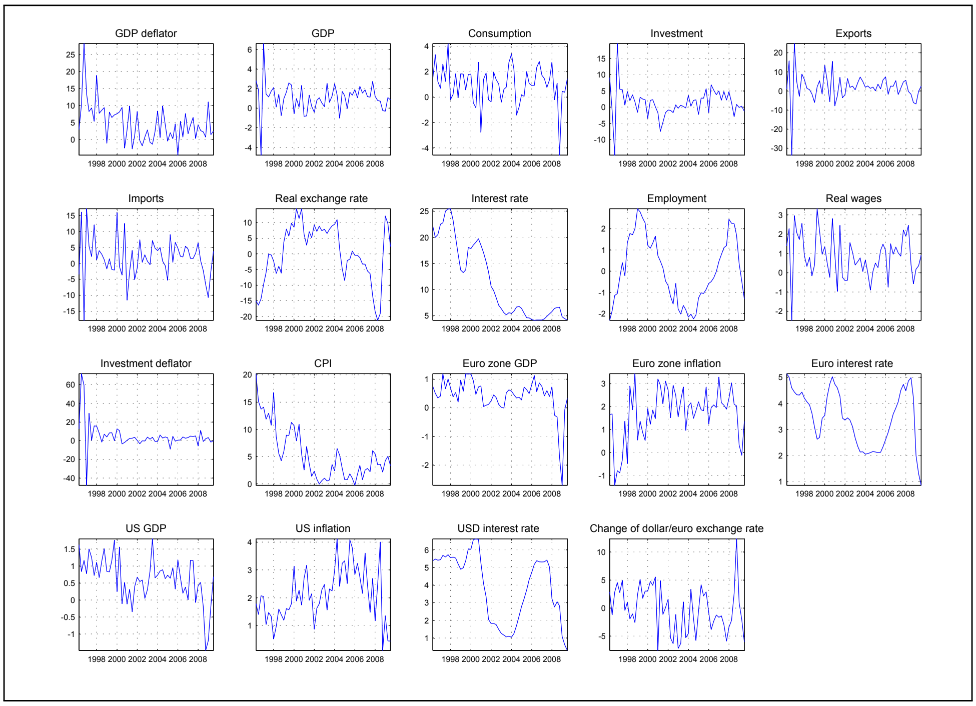
Bearing in mind the solutions assumed by the authors of the $\mathbb{SOE}^{\mathrm{Euro}}$ model, an additional adjustment of the export and import series was made. Whereas the model does not attempt to explain why the share of foreign trade in the GDP changes (grows), an adjustment ensuring constant share of import and export was made. Another arbitrary correction refers to the trend appearing in the observable exchange rate (real exchange rate of dollar/zloty). The phenomenon of nominal exchange rate appreciation (in consequence also the real exchange rate) is specific for economies catching-up with the development gap and similarly as in the case of trade share in the GDP, the $\mathbb{SOE}^{\mathrm{PL}-2009}$ model does not undertake to explain the process, therefore, at the final phase of transformation (linear) trend is eliminated from the exchange rate. Although the practice of elimination of series of various arbitrarily defined components (generally — the filtration practice) used in estimation of DSGE or (S)VAR models is widespread, it is worth to remember that the informative content of the series and their possible interrelations are distorted. As an example, by eliminating a trend from employment or exchange rate, we leave the consequences of the existence of the trends in other series (e.g. prices, GDP) and such set of data is further used for searching relations between series.

We wish to point out that foreign variables are present in the set of observable variables, i.e. the variables fulfil a double function in the model: they are disturbances and observable variables. Thus, we receive a better identification of the other disturbances referring to the world (export

[3]We present here a set of variables that was used in the estimation of the discussed version of the model. The version is an effect of many experiments, out of which a part referred to the selection of observable variables. We have experimented (among others) with monetary aggregates, export deflators, import deflators, consumption deflators (including changes in inventories), fiscal series (government expenditures, revenues from VAT tax), competitive versions of employment series and change in the real exchange rate of dollar.

and import markups, asymmetry in the level of technology, risk premium, etc.) and, additionally, one of the fundamental assumptions of the model of a small open economy is verified: the common steady state (identical inflation, interest rate and economic growth in domestic and world economy). Any possible departures from that principle are compensated with constants in the block of measurement equations. Figure 5.1 illustrates the graphs of observable variables of the model after all of the transformations.

Figure 5.1. Data



5.3 SVAR models

SVAR models are a separate component of the model. The purpose of fiscal SVAR model and world's economy SVAR model is to receive a description of a stochastic process approximating the disturbances represented by the variables of the SVAR models.

5.3.1 Fiscal SVAR

Fiscal SVAR, formally represented by equation (4.40), consists of four variables: effective personal income tax rate (calculated as the revenues of state budget from personal income tax by the estimates of households income, increased for the health insurance contribution), effective corporate income tax rate (the revenues of state budget from corporate income tax by the GDP decreased for households income), consumption tax (state budget revenues from intermediate taxes referred to individual consumption) and government expenditures (public consumption). The above described raw time series are subject to transformations. Tax rates are seasonally adjusted with X12 method and then converted into percentage deviations from the mean in the sample (according to the logic of log-linearised variables of the model). A logarithm was found for the series of government expenditures at fixed prices and then the series was seasonally adjusted with X12 method and converted into percentage deviations from the trend determined with the use of Hodrick-Prescott filter. Such constructed SVAR model has been estimated on a sample 1996:1–2009:3.

5.3.2 World's economy SVAR

The task of the VAR model is to determine the dynamics of foreign variables and of dollar/euro nominal exchange rate. Foreign variables of the model are exogenous and consist of two areas: the euro area and the dollar area. The dollar area is approximated by the USA economy. Variables of the euro area shall be identified with superscript e, while the variables of the dollar area with subscript u. For modelling each of the areas we have used three variables: real product (y_t^e and y_t^u), inflation (π_t^e and π_t^u) and nominal interest rate (R_t^e and R_t^u). Beside of the variables characteristic of each of the areas, the VAR model includes the dynamics of the nominal exchange rate of USD/EUR (Δx_t). The model consists, thus, of three variables for the euro area and three variables for the dollar area, as well as the cross rate USD/EUR. Vector autoregression system in reduced form is given by:

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + d + \omega t + e_t$$

where: $y_t = (y_t^e, \pi_t^e, R_t^e, y_t^u, \pi_t^u, R_t^u, \Delta x_t)$.

Matrices A_1 and A_2 are autoregressive matrices, vector d consists of constants and vector ω consists of the trend parameters, while the vector of error terms e_t depends on the vector of structural shocks through matrix B, i.e. $e_t = B u_t$. The lag order of the system is 2, the model includes a deterministic trend and a constant, i.e. the reduced form covers 144 parameters.

The sample based on which the model parameters have been estimated covers 166 observations for each of the variables, starting from the first quarter of 1980. The data for the dollar area come from the OECD and Eurostat databases. The data for the euro area starting from the first quarter 1995 come from the OECD and Eurostat databases, while the data from before 1995 have been reconstructed based on the database of a Area Wide Model (AWM) of the European Central Bank. A series of the nominal USD/EUR exchange rate comes from the Eurostat database. The variables have been seasonally adjusted with the ARIMA X12 method. Inflation and the nominal exchange rate are included in the model as quarterly logarithmic growth rates of the series of GDP deflators and nominal exchange rate of USD/EUR. Output is included in the model as a cyclic component of a quarterly HP filter applied to the logarithm of an output expressed at market prices deflated with the GDP deflator. Interest rate is expressed in percentage points at quarterly values.

For the purpose of estimation of the structural form of the VAR model within each of the areas, Cholesky type structuralisation has been applied in the following sequence: production, inflation and interest rate. For the propagation of structural disturbances between the euro area and the dollar area one quarter delay has been assumed. An exchange rate shock enters the structuralisation orthogonally to the other shocks. The parameters of the reduced form, i.e. the elements of matrices A_1 and A_2, as well as the elements of vectors d and ω, have been estimated on quarterly data with estimated general least squares method (EGLS). On the parameters of the reduced form dynamic restrictions have been imposed (the matrix structure, the imposed restrictions and the results of the estimation have been presented in the Annexe). Matrix B making the forecasts errors e_t dependent on structural shocks u_t has been estimated with the maximum likelihood method.

Figure 5.2 presents the effect of positive structural shock to the interest rate (R^e for the euro area and R^u for the dollar area) of the value of 1 p.p. on the product (y^e, y^u), inflation (π^e and π^u) and the interest rate (R^e and R^u). After monetary policy contraction in the euro area, output declines by about 1%, while the maximum drop occurs 9 quarters after the shock and the effect becomes statistically insignificant after about 3 years. Inflation drops by about 0.25%, while the effect is significant between the 5th and 15th quarter and the maximum effect materializes in the 12th quarter. Prices, thus, display larger inertia than output. An effect of contractionary monetary policy of the value of 1 percentage point in the dollar area is similar in terms of quality, however, the response of inflation is much weaker — drop by about 0.05%, insignificant along the whole length of the impulse response function. Output responds significantly between the 2nd and 5th quarter and drops by about 0.65% in the 6th quarter. Impulse responses seem to be sensible.

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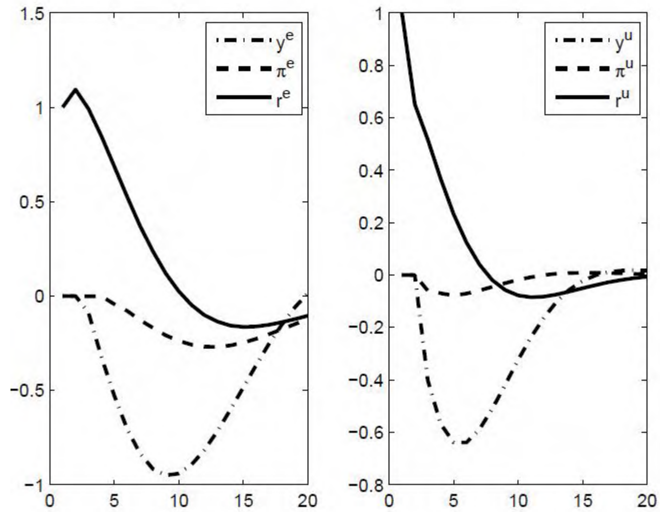
Figure 5.2. Contractionary monetary policy shock in a SVAR model



6
Assessment of parameters – calibration, optimisation, steady state

The procedure of estimation of large DSGE models, motivated with Bayesian ideas[1], consists of two basic stages. At the first stage we determine with numerical optimisation methods the approximation of modal value of posterior distribution, i.e. point estimates of the parameters. At the second stage we estimate the shape of parameters distributions in the neighbourhood of the posterior mode, by applying MCMC techniques (e.g. Metropolis algorithm versions). Thus we generate a sample from model parameters posterior distribution[2]. Practice has shown that already at the stage of optimisation, problems with parameters identification appear, while convergence of numerical procedures is achieved only if prior distributions shall explicitly determine at least a subset of parameters. In such a situation most of the authors of the DSGE models decide to divide parameters into two groups: calibrated and estimated parameters. The estimation procedure (e.g. with Bayesian techniques) is then applied only to some of the parameters. In principle, one would have to be absolutely certain about the values of the calibrated parameters and only those parameters to which our knowledge (which we represent by prior distribution) is uncertain shall be (Bayesian) estimated. Unfortunately, in our case (or probably not only in our case, see Adolfson et al. (2007a, page 12)), this was not the motivation for applying calibration to at least some of the parameters — we calibrated parameters which cannot be estimated because of technical constraints.

[1]We refer here to practice, the formulated generalisation refers to the users of the DYNARE package (academic communities), YADA package (ECB) and Riksbank. Generally, DSGE models do not have to be estimated with Bayesian techniques and the manner of the Bayesian techniques application may be different than we suggest.

[2]We omit here (and further on in the presentation) the fact that some of the parameters of the model are estimated separately with the use of SVAR models.

6.1 Calibration of parameters

In the $\mathbb{SOE}^{\mathrm{PL-2009}}$ model, which as for the standards of estimated DSGE models may be deemed to be large, about 80-90 parameters from a set of over 150 parameters are calibrated. The calibration refers, among others, to 19 variances of the measurement errors of observable data, 27 parameters directly or indirectly characterising the steady state, 22 parameters characterising the stochastic disturbance processes, 8 parameters centring the variables included in the SVAR model, as well as 12 other parameters which from economic point of view may be treated as important. Above enumeration omits zero covariance of measurement errors and a collection of parameters related to the stochastic processes of observable disturbances (SVAR), yet, it includes the values of parameters that are subject to structural changes (11) and those that are determined by the specification (economic content) of the model. The percentage of calibrated parameters is relatively higher, in reference to the estimated DSGE models developed in other countries, but our model is larger and data sample remains short and non-homogenous. Thus we face greater challenges than the authors of models in e.g. the euro area countries or the USA.

Due to the diversity of the parameters excluded from the estimation, no uniform procedure has been applied to determine their values. In case of characteristics of steady state variables that have their equivalents among observable data, the average values of the sample were the main indicator. In a number of cases, the first approximations were the values assumed by other authors of DSGE models, and in particular the assumptions made for the models of $\mathbb{SOE}^{\mathrm{Euro}}$, MEDEA (Burriel et al., 2009), NAWM and RAMSES (Adolfson et al., 2009), however, we always made attempts to verify such assumptions by experimenting with competitive values, i.e. we tested the consequences of changes in the values of such parameters, making a sort of sensitivity analysis, such as e.g. the authors of NAWM (Christoffel et al., 2007a, pages 44–45). At the initial stages of the works over the model, we used Laplace approximation of marginal likelihood as the criterion of assessment of the model, later we additionally applied the measures of accuracy of *ex post* forecasts for the last 4-5 years of the sample, all the time controlling the dynamic features of the model (impulse response functions).

The experiments proved that restrictions imposed on a DSGE model by virtue of calibration of some of the parameters were very strong but in many cases failed confrontation with empirical material. Seemingly small adjustments of calibrated parameters significantly influenced the accuracy of forecasts, yet, they are not recognisable for other criteria (e.g. marginal likelihood, impulse response functions). Small departure from the “typical” calibrated values of parameters enabled us to improve the quality of the $\mathbb{SOE}^{\mathrm{PL-2009}}$ model, or at least its forecasting features, without a major breach of its economic content. Despite the efforts, we are aware that there is large spare capacity in that area and potential consequences of assuming wrong values are serious. Specification of the most important calibrated parameters of the model has been provided in Table 6.1. The specification omits the parameters obtained from estimation of SVAR models.

Table 6.1. Values of the most important calibrated parameters in the $\mathbb{SOE}^{\mathrm{PL}-2009}$ model

Parameter	Value	Comments	Source
ϖ	0.250		based on literature, verified by experiment
δ	0.025		based on literature and sample means of the great ratios
β	0.9995		based on literature, verified by experiment
A_L	7.500		based on literature
A_q	0.135		based on data
σ_L	1.000		based on literature
σ_q	10.620		based on literature
ν^{fk}	0.400		based on literature, verified by experiment
ν^{fw}	0.600		based on literature, verified by experiment
μ	1.0165	steady state	determined by experiment
g_r	0.190	steady state	mean from the sample of share of consumption in GDP
τ^k	0.0125	steady state	empirically verified expert assessments
τ^w	0.180	steady state	average national insurance contributions
τ^s	0.180	steady state	average national insurance contributions
τ^c	0.1772	steady state	mean from the sample of effective VAT rate
τ^p	0.0125	steady state	mean relation of revenues from CIT to GDP
τ^y	0.1301	steady state	mean from the sample of effective PIT rate
ω_c	0.450		determined by experiment
ω_i	0.650		determined by experiment
ω_x	0.330		determined by experiment
ω_{mcu}	0.460		determined by experiment
ω_{miu}	0.370		determined by experiment
ω_{mxu}	0.333		determined by experiment
η_{fu}	4.500		determined by experiment
η_{fe}	3.200		determined by experiment
$u_{y^{e,\star}}$	0.6810		determined by experiment
$u_{a_m^e}$	-0.0095		determined by experiment
b	0.6401		determined by experiment
$\tilde{S}''$	6.9682		determined by experiment
σ_a	1.1276		determined by experiment
λ^x	1.0000		conclusion from model assumptions
λ^{mx}	1.0000		conclusion from model assumptions
ρ_ϵ	0.700		determined by experiment
ρ_{λ_d}	0.100		based on literature, verified by experiment
$\rho_{\lambda_{mx}}$	0.500		based on literature
$\rho_{\epsilon,R}$	0.000		based on literature
$\rho_{\bar{\pi}^c}$	0.900		based on literature, verified by experiment
ρ_{τ_k}	0.900		determined by experiment
ρ_{τ_s}	0.900		determined by experiment
$\rho_{\pi^{oil}}$	0.900		determined by experiment
cR^d	-1.3500	observable variable adjustment	determined by experiment
R_{kraj}	0.0400	variance of measurement error	expert assessments
$R_{otoczenie}$	0.0100	variance of measurement error	expert assessments
$\rho_{\lambda_w,1}$	0.6774	change of regime	determined by experiment
$\rho_{\tilde{\Phi},1}$	0.6431	change of regime	determined by experiment
$\sigma_{\lambda_w,1}$	0.7959	change of regime	determined by experiment
$\sigma_{\tilde{\Phi},1}$	1.7921	change of regime	determined by experiment
$\sigma_{\epsilon_{R,1}}$	0.1727	change of regime	determined by experiment
$\sigma_{\bar{\pi}^c,1}$	0.3482	change of regime	determined by experiment
$\lambda_{w,1}$	1.1366	change of regime	determined by experiment
$\rho_{R,1}$	0.7359	change of regime	determined by experiment
$r_{\pi,1}$	1.4497	change of regime	determined by experiment
$r_{x,1}$	-0.0100	change of regime	determined by experiment
$r_{y,1}$	0.1783	change of regime	determined by experiment

Source: Prepared by the authors.

6.2 Prior distributions and results of estimation

As it has been mentioned before, the DSGE model estimation is a two-stage process. At the first stage we search for the values of parameters that maximise the value of the function, which is given by the sum of the logarithms of the likelihood function and prior distribution. These parameters represent the mode of posterior distribution and the stage is called optimisation. At the second stage an attempt is made to generate a sample from (posterior) distribution of

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parameters by using e.g. an iterative algorithm in which the starting point is the distribution modal value, and consecutive values depend on the Hessian obtained during optimisation. Generation of such sample allows — among others — to better understand the scale of uncertainty related to the estimated parameters. That piece of information may be used to asses forecasts uncertainty. The formal aspects of Bayesian estimation, MCMC techniques and specifics of Bayesian estimation of DSGE models have been shortly outlined in the first part of the paper. Other recommended sources include i.e.: Osiewalski (2001), Koop (2003), Geweke (1999), Geweke (2005), Hastings (1970), Chib and Greenberg (1995), Brooks (1998), Brooks and Gelman (1996) and Schorfheide (2000), An and Schorfheide (2005), Canova (2007), Warne (2009), Fernández-Villaverde (2009), providing the details of the use of Bayesian techniques in DSGE models estimation.

Table 6.2. Results of estimation — characteristics of prior and posterior distributions

Para-	Prior distribution			Optimisation results		Sample form posterior distribution				
meters	Type	CVal	Std/DF	Mode	InvH	Mean	Mode	Median	5% perc.	95% perc.
Price rigidities										
ξ_w	Beta	0.600	0.100	0.558	0.079	0.558	0.547	0.557	0.427	0.690
ξ_d	Beta	0.600	0.100	0.790	0.026	0.794	0.790	0.795	0.749	0.837
ξ_{mc}	Beta	0.600	0.100	0.677	0.057	0.675	0.672	0.675	0.573	0.774
ξ_{mi}	Beta	0.600	0.100	0.687	0.062	0.683	0.690	0.685	0.576	0.783
ξ_{mx}	Beta	0.600	0.100	0.554	0.081	0.552	0.562	0.553	0.416	0.687
ξ_x	Beta	0.600	0.100	0.527	0.060	0.530	0.530	0.530	0.431	0.631
ξ_e	Beta	0.600	0.100	0.698	0.034	0.701	0.704	0.704	0.641	0.755
κ_w	Beta	0.500	0.140	0.417	0.124	0.423	0.385	0.419	0.231	0.628
κ_d	Beta	0.500	0.140	0.164	0.062	0.171	0.146	0.164	0.079	0.286
κ_{mc}	Beta	0.500	0.140	0.441	0.144	0.446	0.402	0.440	0.228	0.682
κ_{mi}	Beta	0.500	0.140	0.291	0.113	0.310	0.278	0.297	0.138	0.529
κ_{mx}	Beta	0.500	0.140	0.350	0.123	0.357	0.333	0.347	0.172	0.577
κ_x	Beta	0.500	0.140	0.202	0.081	0.215	0.175	0.204	0.095	0.375
Foreign exchange rates and energy prices effects										
ν^{τ}	TNor	0.125	0.075	0.150	0.068	0.125	0.122	0.121	0.021	0.240
$\tilde{\phi}_a^u$	InvG	0.125	2	0.382	0.206	0.610	0.295	0.429	0.184	1.483
$\tilde{\phi}_a^e$	InvG	0.125	2	0.190	0.063	0.196	0.164	0.186	0.109	0.318
$\tilde{\phi}_s^u$	Beta	0.250	0.150	0.217	0.114	0.208	0.133	0.193	0.051	0.421
$\tilde{\phi}_s^e$	Beta	0.250	0.150	0.196	0.082	0.182	0.202	0.180	0.053	0.316
Interest rate rule										
$\rho_{R,2}$	Beta	0.800	0.085	0.820	0.035	0.825	0.829	0.827	0.768	0.877
$r_{\pi,2}$	TNor	1.700	0.150	1.800	0.124	1.798	1.782	1.795	1.596	2.010
$r_{\Delta\pi}$	TNor	0.300	0.065	0.237	0.031	0.234	0.231	0.234	0.184	0.286
$r_{x,2}$	Norm	0.000	0.065	-0.022	0.017	-0.020	-0.015	-0.020	-0.048	0.005
$r_{y,2}$	Norm	0.125	0.065	0.128	0.057	0.130	0.120	0.128	0.039	0.225
$r_{\Delta y}$	TNor	0.075	0.065	0.128	0.033	0.128	0.124	0.127	0.072	0.187
Markets organisation, technological progress										
λ_2^w	TNor	1.100	0.075	1.150	0.060	1.142	1.121	1.137	1.055	1.241
λ^d	TNor	1.200	0.075	1.213	0.063	1.204	1.209	1.203	1.098	1.311
λ^{mc}	TNor	1.200	0.075	1.299	0.052	1.290	1.286	1.290	1.199	1.383
λ^{mi}	TNor	1.200	0.075	1.160	0.071	1.146	1.119	1.139	1.033	1.283
η_c	InvG	5.000	2	3.740	0.454	3.861	3.723	3.792	3.146	4.813
η_i	InvG	5.000	2	3.964	0.607	4.454	3.777	4.097	3.271	5.880
η_{xx}	InvG	5.000	2	4.544	0.861	4.800	4.362	4.622	3.540	6.637
μ_z	TNor	1.008	0.002	1.008	0.001	1.008	1.008	1.008	1.007	1.009
μ_Ψ	TNor	1.008	0.002	1.007	0.001	1.007	1.007	1.007	1.005	1.009
Characteristics of disturbances										
ρ_Υ	Beta	0.700	0.100	0.773	0.052	0.774	0.785	0.777	0.683	0.855
$\rho_{\tilde{z}^\star}$	Beta	0.900	0.100	0.957	0.044	0.953	0.999	0.971	0.848	0.999
ρ_{μ_z}	Beta	0.600	0.100	0.592	0.097	0.593	0.607	0.596	0.430	0.748
ρ_{μ_Ψ}	Beta	0.600	0.100	0.586	0.099	0.583	0.573	0.585	0.412	0.741
ρ_{ζ^c}	Beta	0.750	0.100	0.778	0.053	0.778	0.774	0.782	0.683	0.862
ρ_{ζ^h}	Beta	0.750	0.100	0.625	0.102	0.612	0.629	0.614	0.436	0.782
$\rho_{\lambda^{mc}}$	Beta	0.750	0.100	0.685	0.101	0.678	0.698	0.685	0.498	0.833

Continued on next page

Para-meters	Prior distribution			Optimisation results		Sample form posterior distribution				
	Type	CVal	Std/DF	Mode	InvH	Mean	Mode	Median	5% perc.	95% perc.
$\rho_{\lambda^{mi}}$	Beta	0.700	0.100	0.544	0.114	0.557	0.555	0.558	0.370	0.739
$\rho_{\lambda^{x}}$	Beta	0.500	0.100	0.633	0.092	0.617	0.631	0.623	0.453	0.761
$\rho_{\lambda^{w},2}$	Beta	0.666	0.100	0.584	0.102	0.561	0.565	0.563	0.396	0.714
$\rho_{\tilde{\phi}^{e},2}$	Beta	0.666	0.100	0.571	0.081	0.582	0.602	0.583	0.449	0.711
σ_{ϵ}	InvG	0.750	2	2.043	0.382	2.107	1.971	2.060	1.524	2.843
σ_{Υ}	InvG	0.750	2	0.663	0.095	0.675	0.639	0.664	0.532	0.856
$\sigma_{\tilde{z}^{\star}}$	InvG	0.750	2	0.359	0.055	0.372	0.361	0.365	0.285	0.483
σ_{μ_z}	InvG	0.500	2	0.277	0.047	0.287	0.264	0.282	0.214	0.379
$\sigma_{\mu_{\Psi}}$	InvG	0.500	2	0.425	0.121	0.489	0.396	0.445	0.289	0.794
$\sigma_{\zeta^{c}}$	InvG	0.500	2	0.687	0.079	0.707	0.697	0.700	0.582	0.855
$\sigma_{\zeta^{h}}$	InvG	0.500	2	0.295	0.058	0.306	0.289	0.298	0.220	0.418
$\sigma_{\lambda^{d}}$	InvG	0.750	2	0.844	0.102	0.869	0.846	0.860	0.707	1.062
$\sigma_{\lambda^{mc}}$	InvG	1.000	2	0.773	0.205	0.856	0.757	0.810	0.537	1.322
$\sigma_{\lambda^{mi}}$	InvG	1.000	2	1.484	0.261	1.547	1.417	1.513	1.104	2.101
$\sigma_{\lambda^{mx}}$	InvG	1.000	2	5.961	1.531	6.304	5.425	6.055	3.954	9.553
$\sigma_{\lambda^{x}}$	InvG	1.000	2	2.546	0.436	2.682	2.423	2.610	2.025	3.570
$\sigma_{\lambda^{w},2}$	InvG	0.750	2	0.380	0.064	0.392	0.384	0.385	0.296	0.515
$\sigma^{e}_{\tilde{\phi},2}$	InvG	1.000	2	2.129	0.504	2.180	2.088	2.115	1.443	3.139
$\sigma_{\tau^{k}}$	InvG	0.010	2	0.012	0.006	0.021	0.010	0.015	0.007	0.051
$\sigma_{\tau^{s}}$	InvG	0.025	2	0.031	0.015	0.053	0.025	0.037	0.018	0.134
$\sigma_{\gamma^{oil}}$	InvG	2.000	2	2.688	1.274	2.994	2.220	2.791	1.440	5.208
$\sigma_{R,2}$	InvG	0.250	2	0.195	0.031	0.205	0.192	0.202	0.155	0.263
$\sigma_{\bar{\pi}^{c},2}$	InvG	0.250	2	0.466	0.205	0.467	0.371	0.442	0.216	0.802

Source: Prepared by the authors.

Note: Nrm — normal distribution, InvG — inverse Gamma distribution, TNor — truncated normal distribution; for the inverse Gamma distribution the modal value and the number of the degrees of freedom have been specified. HInv — approximation of standard deviation from the assessment of parameter based on Hessian inverse. CVal — central value.

Calibration of parameters does not solve all the problems which appear during the optimisation, e.g. selection of an effective algorithm for optimisation[3]. We also continue to stick to conditional estimation — optimisation takes place in the presence of the expert knowledge introduced formally with prior distribution. The type of prior distribution assumed for particular parameters results from knowledge about the allowed interval of values of the parameters, therefore, on a standard basis there is assumed Beta distribution for e.g. Calvo probabilities or correlation coefficient (AR processes of the shocks), an inverse Gamma distribution for variances of disturbance innovation, normal distribution for parameters that may assume any values, etc. Variances (degrees of freedom) of prior distributions have been defined such as to prefer information included in the sample (unless we had a stronger belief regarding the value of the parameters and the sample proved to be sufficiently informative)[4]. The central values of the distributions, however, proved to be debatable. As in the case of calibrated parameters, extensive literature describes the experience of the authors of Bayesian estimated DSGE models[5], of which we have, of course, availed while verifying our own analyses and beliefs. Also in this case it appears that relatively small changes in the central values of distributions, completely undistinguishable for an

[3]Often applied by researchers, implemented in Dynare and YADA packages, *csminwel* algorithm by Ch. Sims (provided by the author on the Internet site in the form of Matlab scripts, also available in the form of C++ function) proved to be ineffective in our case. After several experiments it appeared that the unconditional optimisation algorithm implemented in the optimisation toolbox of Matlab deals much more effectively with the extreme's determination, however, at the cost of considerable longer time of computation.

[4]It is worth to remember that reaching for Bayesian techniques is not always the consequence of the Bayesian point of view or Bayesian beliefs of the authors. Sometimes, as in this case, the Bayesian procedures are applied instead of the classical procedures, due to the fact that the available sample does not include a sufficient amount of information to carry out the traditional estimation (flat likelihood function). Instead of additionally extending the set of calibrated parameters, we avail of techniques that slightly reduce the arbitrariness of the study. Therefore, pragmatic considerations are decisive.

[5]Grabek et al. (2007, pages 69–70) present several examples drawn from the DSGE models with structure similar to $\mathbb{SOE}^{\mathrm{PL-2009}}$. In the specification provided there, prior distributions of models built before 2006 are available.

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expert, are important — they impact the characteristics of the model and its forecasting features. The final result of our research — a complete set of assumptions regarding prior distributions, as well as assessments of parameters and approximations of standard errors arrived at during optimisation have been provided in Table 6.2.

Table 6.2 provides also selected characteristics of the distribution received as a result of the application of Metropolis algorithm (in the random walk version), which generates a sample of distribution convergent toward posterior distribution. During the second stage of the estimation procedure, Markov chain of the length of 750,000 elements has been generated, while the data presented in the Table have been taken from a sample of 500,000 elements remaining upon the rejection of the first 250,000 elements. Already a superficial analysis of the values of parameters for the modal value received from optimisation and determined from the Markov chain reflects a considerable divergence between the corresponding values, suggesting lack of convergence. More thorough convergence analyses (logarithms of posterior probability for the elements of the chain, Geweke tests for the particular parameters, test statistics based on cumulated sums, etc.) provided additional doubts as to whether the convergence is actually present. Due to limited resources at our disposal we could not further generate the chain[6]. Therefore, during further works we used only the point estimates of parameters originating from the first stage of estimation — optimisation. Generally, a significant issue of the uncertainty of parameters at the current stage of the works has not been completed — we shall come back to the issue after the existing technical limitations has been reduced.

6.3 Assessment of parameters — conclusions

6.3.1 Steady state

When all the disturbances (shocks) described in the model expire, the economy reaches a steady state. Then, all the nominal categories grow at an identical rate determined by the calibrated parameter μ. The annualised value of the parameter implies the growth rate at the level of about 6.6%. The division of the value dynamics into the dynamics of volume and prices is identical for all the growing categories, except investments. The division depends on the estimated parameters characterising the growth rate of technical progress μ_z and μ_Ψ, as well as the calibrated share of capital in a product (ϖ — parameter of Cobb-Douglas production

[6]As an example, the authors of the $\mathbb{SOE}^{\text{Euro}}$ model generated a chain of the length of 700,000-800,000 elements using 500,000 end elements for inference see (see Adolfson et al., 2005b, page 28). On the other hand, S. Schmitt-Grohé and M. Uribe used the end 4,000,000 elements from a chain of 10,000,000 elements for a significantly smaller DSGE model (see Schmitt-Grohé and Uribe, 2008, page 16). The characteristics of posterior distribution of the NAWM model (ECB) have been determined based on a chain of the length of 500,000 elements (with first 50,000 elements rejected), however, convergence was tested with the use of four parallel chains of the length of 500,000 elements each (see Christoffel et al., 2007a, page 42). In the DSGE model built for the Central Bank of Hungary, the Metropolis procedure was used for generation of two Markov chains of the lengths of 300,000 elements, and for reasoning a sample of 500,000 elements was used, originating from the merger of the last 250,000 elements of each of the chains (see Jakab and Világi, 2008, page 19). Reviewing the descriptions of other Bayesian estimated models, it is hard to resist the feeling that some of the authors of the DSGE models treat convergence in a sort of ritual manner — this is a qualitative difference with regard to the conduct of researchers specialising in the Bayesian techniques.

function) and amounts to about 2.5% of inflation and about 4.1% of volume dynamics[7]. The annual interest rate of about 7.1% corresponds to those values.

In all the DSGE models we are aware of, the steady state level of interest rate has very high values. Formally, this is so because its level derives from the value dynamics (μ), discount and the rate of tax on capital in steady state. Within that logic, whilst expecting volume growth dynamics at the level of 3-4% and inflation at about 2%, it is hard to arrive at interest rate lower than 5-6%, even if the discount implies the minimal weight applied to the future and the tax on capital is quite arbitrarily assumed at low values (or even zero, see Adolfson et al. (2009, page 29)). There is only one interest rate present in the model, no difference exists for the rate of the central bank, deposits, loans, short- or long-term rate[8], while observable equivalents are three-month rates in the interbank market (Wibor, Euribor). In order to at least slightly compensate for this inconsistency, in the block of measurement equations of the model adjusting constants have been introduced. It has been determined by experiment (taking into account the quality of forecasts and marginal likelihood) that adjustment compliant with data for the domestic interest rate amounts to 1.35%, 3.4% for the euro area and 2.5% for the USA. Therefore, by and large, interest rates in Poland shall be by about 2 p.p. higher than the rates in the euro area in steady state. Corresponding adjustments have also been introduced to the GDP growth rate and inflation in the euro area and in the USA[9].

Also the share of trade in GDP may raise doubts. In steady state net exports amount to zero and the share of import in GDP has the level of about 36%, which is below the values that we have observed (we expected the value of about 40%). In reference to the previous version of the $\mathbb{SOE}^{\mathrm{PL}}$ model this is, however, a considerable progress (we earlier had less than 29%) achieved thanks to the explicit consideration of import intensity of exports. Nevertheless, the model continues to omit import intensity of government consumption, so there are reasons for underestimated share of trade in GDP.

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The values of the most important variables and proportions in the steady state are presented in Table 6.3. In the Appendix we present an example forecast with a very long horizon, which (among others) reflects the rate of convergence toward the steady state.

Table 6.3. Values of selected variables and the great ratios in steady state

Parameter	Value	Comments
Annual growth rate of technological progress	0.041181	annualised μ_{z^+}
Annual growth rate of total investments	0.050825	
Annual inflation rate	0.025424	annualised π
Annual interest rate	0.070696	R
Share of consumption in GDP	0.626330	
Share of investments in GDP	0.183670	
Share of government consumption in GDP	0.190000	g_r
Share of import (export) in GDP	0.359007	

Source: Prepared by the authors.

[7]The carried out experiments, e.g. with higher value dynamics μ, brought about higher inflation (instead of growth), generating at the same time larger *ex post* forecast errors. Also the received approximation of the marginal likelihood did not provide arguments for such solution.

[8]Based on the structure of the model, all of the transactions have the horizon of one quarter.

[9]For inflation this was 0.25 p.p. in the euro area and 0.25 in the USA; for the annualised GDP dynamics it was 2.00 p.p. for the euro area and about 1.332 p.p. for the USA.

6.3.2 Structural changes

The $\mathbb{SOE}^{\mathrm{PL}-2009}$ model allows for structural changes regarding monetary policy, exchange rate policy and labour market characteristics. We have assumed that due to official introduction of direct inflation targeting in Poland in 1999, interest rate rule parameter values may have changed. This concerns also characteristics of inflation target disturbance. Already before introduction of targeting, exchange rate regime had gradually been shifting towards free float, which — in our opinion — affected the exchange rate risk premium shock (characteristics of a stochastic process). Additionally, we assume that introduction of a new pension system and health insurance system in 1999, with stricter control of the flow of contributions and pension entitlements, affected the level of wage markup in steady state and the characteristics of the stochastic process governing this markup. In both cases, we assumed for simplicity the simultaneous occurrence of such regime changes as from the second quarter 1999. The experiments showed that more refined defining of moments of change, better reflecting the historical events, practically does not affect the marginal likelihood and the forecasting features of the model, however, leads to a considerable elongation of the time of computation. Therefore, we have assumed the simplification.

Searching for further structural changes, the year of Poland's accession to the European Union was also tested, as the possible moment of further structural changes (covering i.e. the labour market, budget expenditures, effects related to exchange rate). The results of experiments have not confirmed such assumptions[10]. As a result there are two regimes present in the model, while change appears at the beginning of the sample, which has an advantage of decreasing the heterogeneity of the sample, enabling at the same time slightly better identification of parameters in the second regime. A more radical solution — estimation starting from the year 2000 — seemed to us to be too costly. A drawback of the assumed solution is, however, a dozen or so additional parameters to be estimated or calibrated. Finally, we decided to calibrate the parameters for the first regime, based on the results of experiments made earlier. In the above-presented tables, the changing parameters have been identified with subscripts, respectively 1 and 2 for the given regimes. In Table 6.4 we present a specification of parameters whose values change in effect of structural changes.

From a formal point of view, the method of representation in the model of the aforesaid structural changes gives the changes deterministic nature, however, unforeseeable by the agents. This simplification results from the applied technique[11]. We think that the simplification may be, however, rationalised by reference also to subject-matter arguments. The introduction of the social security system reform (affecting the labour market) and changes in the strategy of monetary policy was announced earlier and in the world of rational agents everybody was aware of that. The information had both a general and qualitative nature. The actual consequences of such activities were not known even to the authors of the reforms and in that sense the

[10]The scope of the experiments made by us was, obviously, limited and several hypotheses remain open. As an example, the authors of the Hungarian DSGE model, when estimating their model for two regimes of monetary policy, also arrived at large differences of Calvo probabilities and indexation parameters (see Jakab and Világi, 2008, page 21). Yet, we have not verified that issue.

[11]Other authors model structural changes with the use of stochastic switching Markov chain, see e.g. (Justiniano and Primiceri, 2006; Lou et al., 2007; Sims and Zha, 2005). It is also proposed to estimate two models in separate sub-samples (Smets and Wouters, 2007).

resultant of all of the changes was unforeseeable, particularly, while speaking about the values of deep parameters. However, the phenomenon of the growth of general uncertainty of the agent actions in face of the expected complex structural changes has been omitted. The effect may be compensated by defining a set of anticipated disturbances — yet, we have left that issue for separate study.

Table 6.4. Parameters changing values as a result of structural changes

Parameter	Regime 1 (calibrated value)	Regime 2 (optimised value)
Wage markup		
λ_w	1.1366	1.1504
ρ_{λ_w}	0.6774	0.5839
σ_{λ_w}	0.7959	0.3802
Risk premium		
$\rho_{\tilde{\Phi}}$	0.6431	0.5712
$\sigma_{\tilde{\Phi}}$	1.7921	2.1293
Interest rate rule and inflation target		
ρ_R	0.7359	0.8204
r_π	1.4497	1.7995
r_x	-0.0100	-0.0219
r_y	0.1783	0.1284
σ_{ϵ_R}	0.1727	0.1949
$\sigma_{\bar{\pi}^c}$	0.3482	0.4658

Source: Prepared by the authors.

From the data provided in the Table 6.4 it appears that the changes in monetary policy discussed here have been interpreted by agents as the increase in interest rate sensitivity to inflation fluctuations (also in reference to other determinants of the interest rate defined in the rule). With increase in the variance of interest rate shock, also the persistence of the interest rate increased. On the other hand, after the year 1999 the inflation target (innovations) has also featured larger variance. At the same time the variance of exchange risk premium innovation increased with simultaneous decrease in the disturbance's persistence. More transparent seem to be the changes in the labour market, where tightening of the insurance system meant the increase in the level of markup with decrease in the innovation variance and persistence of the disturbance. The effects of such changes for the impact of the disturbances on observable variables are illustrated by variances decompositions and reaction functions presented further herein.

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6.3.3 Nominal rigidities

A distinguishing feature of a DSGE model based on the paradigm of a new Keynesian school is the presence of nominal rigidities — delays in the adjustment of prices. While modelling the rigidities of prices (wages) in the $\mathbb{SOE}^{\text{PL}-2009}$ model, we have applied the Calvo model supplemented with the mechanism of dynamic indexation, discussed earlier in more detail (see e.g. equation (4.17)). In Table 6.5 there are specified the assessments of parameters characterising Calvo rigidities. For comparison, there have been provided assessments of analogical parameters (modal value) received from the earlier version of the $\mathbb{SOE}^{\text{PL}}$ model (see Grabek et al., 2007), the RAMSES model (case with UIP condition analogical as in the $\mathbb{SOE}^{\text{PL}-2009}$, (see Adolfson et al., 2007a, page 25), $\mathbb{SOE}^{\text{Euro}}$ model of the euro area (case with variable utilisation (see Adolfson et al., 2005b, page 58), and NAWM ECB (see Christoffel et al., 2007a, page 82), as well as the values characterising rigidities for the Hungarian economy (see Jakab and Világi, 2008, page 23) for the case of carrying out the strategy of direct inflation target. The cited values comply with the modal value of posterior distribution or median (RAMSES).

From the Table 6.5 it results, generally, that the $\mathbb{SOE}^{\mathrm{PL-2009}}$ model describes economy with a more flexible wages and prices than the models of the euro area, Sweden or Hungary — at least prices are less susceptible to marginal costs' changes[12]. The difference is particularly visible in the labour market, however, more persistent indexation mechanism lowers the effects of shorter delay in re-optimisation of wages. Interesting are also changes in the assessments of parameters in the subsequent versions of the $\mathbb{SOE}^{\mathrm{PL}}$, model: the version estimated in 2006 reflected relatively stronger adaptative indexation of the prices of domestically manufactured products; now the κ_d parameter characterising the effect of inertia is minimal, so with slightly longer periods between the subsequent re-optimisations indexation takes place mainly based on the expected (instead of former) prices. Obviously, the change results also from a change in the specification of the model, so we have no grounds to claim that the prices setting mechanism has changed e.g. after the accession to the European Union.

Table 6.5. Rigidities of prices and wages in the chosen DSGE models

Parameter	$\mathbb{SOE}^{\mathrm{PL-2009}}$	$\mathbb{SOE}^{\mathrm{PL-2006}}$	RAMSES	$\mathbb{SOE}^{\mathrm{Euro}}$	NAWM (EBC)	MNB model
Calvo probability						
ξ_w	0.558	0.586	0.752	0.716	0.765	0.711
ξ_d	0.794	0.680	0.838	0.895	0.920	-
ξ_{mc}	0.675	0.585	0.901	0.523	-	-
ξ_{mi}	0.683	0.617	0.944	0.743	-	-
ξ_{mx}	0.552	-	-	-	-	-
ξ_x	0.530	0.600	0.883	0.630	0.770	0.827
Dynamic indexation parameter						
κ_w	0.417	0.350	0.313	0.453	0.635	
κ_d	0.164	0.434	-	0.173	0.417	-
κ_{mc}	0.441	0.439	-	0.128	-	-
κ_{mi}	0.291	0.440	-	0.192	-	-
κ_{mx}	0.350	-	-	-	-	-
κ_x	0.202	0.508	-	0.148	0.489	

Source: Prepared by the authors; detailed information in the text of the paper.

[12] Christoffel et al. (2007a, page 43) mention that Calvo probability ξ and the average time between price re-optimisation $\frac{1}{1-\xi}$ implied by the value does not necessarily inform about the scale of nominal rigidities because in the Phillips curve built on the basis of the Calvo model, real and nominal rigidities may not be identified (discriminated) separately.

7
Variance decompositions, impulse response functions and estimation of disturbances

7.1 Variance decompositions

In the following Tables (7.1–7.4) we have presented variances decompositions (formally — forecast errors), separately for regime I covering the period up to the first quarter of 1999 (Tables 7.1–7.2) and regime II characterising the relationships observed in the other part of the sample (Tables 7.3–7.2). As we have mentioned before, all of the calculations were made for parameter values corresponding to the modal value of posterior distribution determined at the stage of optimisation (we omit the uncertainty of parameters). The horizon has been limited to short-term and medium-term period (maximum 20 quarters), which from the point of view of the current applications of the model seems to be sufficient. For longer horizons, the results depend on model specification. Real growing variables are determined with non-stationary disturbances, while the rest of them — with disturbance of the largest persistence. In the earlier versions of the $\mathbb{SOE}^{\text{PL}}$ model, as well as in the $\mathbb{SOE}^{\text{Euro}}$ model, this was the disturbance of the inflation target (the calibrated autocorrelation coefficient amounted then to 0.975–0.985). In the current version, the largest persistence is reflected by foreign variables which constitute the SVAR[1]. Certainly, this does not seriously affect the steady state or short- and mid-term model

[1] The aforementioned experiments (sensitivity analyses) showed that autocorrelation of inflation target disturbance at the level of 0.975–0.985 deteriorates the quality of forecasts, therefore, a slightly lower value of the parameter (0.900) has been assumed. Higher persistence has been achieved by estimation of the characteristics of AR(1) process for an asymmetric shock and by constructing relations between world's economy SVAR model and the basic part of the model (real dollar/euro rate). It is the real dollar/euro rate that currently determines the joint persistence of the observable external disturbances.

dynamics we are the most interested in.

The data gathered in the Tables present the role of 17 single structural disturbances and two blocks of shocks. The block of external/foreign shocks covers all of the (observable) disturbances characterising the euro area and the USA, present in the SVAR model. In the block of fiscal disturbances all of the disturbances modelled with the use of SVAR model and additionally structural disturbances of national insurance contributions and capital tax have been jointly reflected.

The first general observation refers to the scale of impact (share) of fiscal variables and foreign variables on the observable variables. Comparing variances decompositions received in the previous versions of the $\mathbb{SOE}^{PL}$ model (see Grabek et al., 2007, pages 100–101) with the current version of the model, the growth of importance of the aforesaid shocks is clearly visible. The marginal role of impulses originating from the world's economy was one of the most important deficiencies of previous versions — the effects observed earlier (at the level of 1–3%) were inconsistent with the concept of a small open economy. Therefore, the currently observed results are quite interesting, although we think that this time the effects have been overestimated. The growth of importance of fiscal disturbances is also visible, however, it still seems to be underestimated. Yet, this is — at least partially — caused by the very structure of the DSGE model (a Ricardian household, no resource effects related to public debt, etc.).

7.1.1 Regime I

In the short run (1–4 quarters), the main inflation determinant before 1999:2 was the situation in the labour market (wage markups), markups in the market of domestic products (GDP deflator, CPI) and markups on the imported investment goods (investment deflator). Additionally, the role of fiscal shocks with regard to CPI is visible. Along with the extension of the observation horizon (8–20 quarters), the impact of disturbances originating from the world's economy, the inflation target disturbances and technological disturbances (stationary in investments) is growing. In the same regime, GDP depends (on short-term basis) on external factors (which quickly abate) and on domestic shocks related to foreign trade (exchange rate risk premium, export markups, markups on the import of export components). Further, fiscal factors, markups on domestic intermediate products and wage markups manifest themselves. Along with the longer horizon of analysis, the impact of technological disturbances (stationary and non-stationary) has been growing, with the maintained role of shocks originating in the labour market.

According to the data provided in Table 7.1, short-term consumption is determined by disturbances in consumption preferences, the role of which abates in the subsequent quarters, as well as by disturbances in the wage markup (here we observe an inverse trend). Also a moderate influence of fiscal disturbances may be perceived in mid-term horizon (8–12 quarters). Along with the extension of lag, the impact of stationary technological disturbance and non-stationary disturbances in investments has been growing. In the case of investments, a dominating role is played by stationary technological disturbances in investments (non-stationary disturbance in a longer horizon). Additionally, with a lag of 4–12 quarters, the impact of shocks on wage markup and consumption preferences may be observed.

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The disturbance of markups in the labour market — in regime I — is the most important determinant of employment and real wages, particularly in shorter horizons. Employment fluctuations are also triggered by the shocks of consumption preferences and labour supply (2–12 quarters), fiscal shocks (8–20 quarters) and technological shocks (stationary in investments, in the long-term horizon). On short-term basis (1 quarter) important are also: stationary disturbance in technology, export markups, exchange rate risk premium and internal disturbances. Real wages fluctuations result (beside of wage markup shocks) in a short-term period from fluctuations of labour supply preferences and domestic product markups. In a longer period, technology is a dominating factor.

The real exchange rate of US dollar and the interest rate seem to be dominated by external disturbances along the whole horizon. Interest rate reacts also to fluctuations in labour market markups and inflation target, and in the short period the interest rate shock, exchange rate risk premium and export markups are important as well. The real exchange rate in the short run (1–4 quarters) reacts to exchange rate risk premium, export markups and markups in consumption import. “Fundamental” domestic factors affecting the real exchange rate of the dollar on a mid-term basis are clearly missing.

7.1.2 Regime II

In regime II, in the short run inflation depends on the inflation target, markup on products manufactured domestically (markup of imported investment goods in the case of investment deflator), fiscal disturbances (in the case of CPI) and foreign variables (whose impact grows in time). In the horizon of 1–4 quarters, also wage markup is important, however, generally the role of these disturbances (compared to regime I) clearly decreased, particularly in a mid-term horizon. Drop of importance of wage markup is also observable for GDP. Similarly as in regime I, in a short period of time GDP variance results from disturbances in the markup of products manufactured domestically and disturbances related to foreign trade (external disturbances, export markup, risk premium). Impulses coming from the labour market are slightly more visible with a lag of 4-12 quarters. Yet, this time technological disturbances (stationary in investments, non-stationary) and disturbance in labour supply preferences are more important.

The lowering importance of wage markup disturbance is also confirmed in the case of consumption and investments. In regime II, in the short-term horizon (1 quarter), consumption is clearly impacted only by the shock of consumption preferences and domestic products markups. Roughly 8–9 shocks (e.g. external shocks, stationary shocks in technology, fiscal shocks, inflation target shocks) have the remaining 50% share in the variance (of forecasts errors), while none of them brings about a stronger impact. In a longer horizon — as in regime I — consumption clearly depends on fiscal and technological shocks. The role of wage markup disturbance has been taken over, to some extent, by the disturbance in labour supply preferences. For investments, stationary investment disturbance remains the main determinant on short- and mid-term basis. The importance of non-stationary shocks is growing along the horizon.

Conclusions formulated when discussing factors affecting employment and real wages in regime I remain valid for regime II. We only observe — as in the previous cases — a lower role of wage markup disturbance, with only a partial take-over of the impact by the disturbance in labour supply preferences.

Although one of the phenomena motivating separate treatment of the two regimes were changes in the currency market, it is hard to perceive larger changes in the group of the most important shocks affecting the real exchange rate of the dollar. As before, external disturbances are dominating. In a very short horizon (1–4 quarters), we may talk about the role of markups in export, consumption goods import and import of export components as well as risk premium. The influence of interest rate disturbance, even in the short-term horizon, is small (3.6% for the first quarters and dropping) — it is smaller than the influence of fiscal disturbances (about 5.9–5.0% for 1–12 quarters). On the other hand, the consequences of the adoption of the strategy of direct inflation target are bit clearer. The importance of inflation target disturbance grew and the impact of monetary policy shock is stronger (but only in the first quarter, in further quarters a drop may be observed). Clearly, the impact of wage markup disturbance is lower, while higher is the share of external disturbances.

Thus, investigating the structure of the impact of disturbances in both regimes in a general way, we see that from the point of view of variance decompositions the essence of changes that occurred in 1999 was a sort of breakdown in the institutional structure responsible for setting the price of labour. This phenomenon seems to be the most important, despite hardly spectacular manifestation (see the respective values of parameters in Table 6.4). In some cases the function of wage markup disturbance was taken over by the disturbance in the labour supply preferences, which may suggest that institutional phenomena have been supported by phenomena related to preferences. Nevertheless, wage markup disturbance is non-identifiable from the point of view of specification of the structural form of wage equation — discrimination requires relatively extensive expert knowledge (introduced by the specification of prior distribution at the stage of estimation), so it is hard to talk about tangible evidence.

Table 7.1. Variance decompositions. Regime 1, part 1

Disturbance	GDP deflator					GDP					Consumption					Investment				
	1q	4q	8q	12q	20q	1q	4q	8q	12q	20q	1q	4q	8q	12q	20q	1q	4q	8q	12q	20q
Stationary technology	9.6	5.0	0.8	0.5	1.2	0.7	6.5	3.7	1.1	0.6	3.5	5.0	5.0	3.7	1.9	1.1	3.2	3.5	3.3	1.8
Investment specific technology	2.2	4.8	8.8	12.2	13.3	2.7	8.0	12.9	16.9	12.2	5.8	5.1	1.3	7.4	20.7	51.3	39.8	27.5	17.3	5.7
Asymmetric technology	0.0	0.2	0.3	0.5	0.7	0.6	0.0	0.0	0.1	0.3	0.1	0.1	0.2	0.3	0.4	0.3	0.3	0.5	0.6	1.0
Non-stationary technology	0.8	1.3	1.6	1.8	1.7	2.8	6.9	11.3	19.9	39.1	5.6	7.2	11.3	17.1	23.7	3.0	3.4	4.4	6.4	15.5
Investment non-stationary technology	1.4	2.3	3.2	3.8	3.6	1.1	2.2	4.0	8.5	19.3	1.5	1.4	1.7	2.6	5.9	0.3	2.4	5.2	8.6	21.6
Consumption preference	1.1	1.8	2.9	3.9	4.4	6.5	8.4	5.8	3.4	1.3	38.9	29.3	16.7	5.8	4.1	7.8	8.9	9.6	10.0	7.5
Labour supply	8.3	10.1	5.9	2.9	2.1	1.4	7.7	7.9	4.9	1.5	2.8	5.8	7.9	7.3	3.6	1.8	3.7	4.8	5.4	4.2
Domestic goods markup	27.4	4.5	1.7	0.4	0.9	12.2	5.3	1.2	1.4	0.4	7.9	4.8	2.5	1.3	1.1	6.5	3.7	2.6	1.9	0.3
Imported consumption goods markup	1.3	2.4	1.4	0.1	1.3	0.7	1.2	1.7	2.5	1.4	0.6	0.9	0.4	2.0	2.1	0.4	2.2	3.7	4.1	2.6
Imported investment goods markup	0.3	0.1	0.1	0.8	1.4	1.9	0.5	3.6	4.1	2.0	1.7	2.2	2.5	1.1	2.2	5.0	7.4	5.7	3.0	1.5
Imported export component markup	0.3	2.4	3.1	2.6	2.1	10.7	3.5	4.0	0.7	1.4	2.5	4.5	5.3	4.3	3.0	4.7	5.3	5.5	4.9	2.0
Export markup	0.8	0.8	2.5	1.9	0.7	14.9	6.0	2.0	1.4	0.5	4.5	3.1	0.9	0.0	0.6	0.1	0.6	1.3	1.3	0.5
Wage markup	27.4	35.4	23.7	12.7	7.8	4.7	25.9	28.5	19.3	6.2	9.1	19.5	27.8	26.4	13.2	5.8	12.2	16.7	19.1	16.1
Risk premium	1.8	1.2	1.8	2.2	1.4	10.2	5.6	3.2	2.8	0.2	1.7	0.0	3.1	3.9	1.8	1.2	0.4	1.8	3.3	3.7
Interest rate	0.7	0.9	0.3	0.2	0.3	2.4	2.2	0.1	0.1	0.2	2.5	1.5	0.8	0.5	0.4	2.1	1.3	0.9	0.6	0.1
Inflation target	7.2	10.6	13.2	13.5	9.4	0.8	1.2	3.0	3.0	2.2	2.7	0.9	1.2	2.5	2.2	2.0	0.7	0.6	1.7	4.1
Energy prices	1.5	0.1	0.1	0.2	0.2	2.9	0.0	0.2	0.1	0.1	0.6	0.8	0.6	0.4	0.3	2.3	0.7	0.6	0.5	0.2
Fiscal	2.7	5.1	7.9	9.1	7.0	6.8	4.2	5.3	7.1	7.6	4.0	6.0	8.2	8.9	8.9	1.0	0.8	1.0	1.6	3.0
World economy (SVAR)	5.0	10.9	20.6	30.7	40.4	15.9	4.6	1.6	2.8	3.4	4.0	1.7	2.5	4.5	4.0	3.3	3.1	4.1	6.2	8.6

Disturbance	Exports					Imports					Real exchange rate					Interest rate				
	1q	4q	8q	12q	20q	1q	4q	8q	12q	20q	1q	4q	8q	12q	20q	1q	4q	8q	12q	20q
Stationary technology	0.3	1.3	0.2	2.1	0.2	1.9	1.8	2.7	4.1	1.7	0.0	1.0	0.2	0.4	0.0	4.2	5.7	4.4	2.8	1.2
Investment specific technology	0.3	2.5	10.6	17.6	12.6	3.0	6.2	10.5	2.6	3.3	0.1	1.6	2.8	3.0	1.5	1.2	1.2	1.5	0.7	11.3
Asymmetric technology	1.2	0.9	2.0	3.2	4.2	0.8	1.3	3.6	3.3	4.0	0.5	0.4	0.4	0.4	0.3	0.4	0.3	0.3	0.4	0.5
Non-stationary technology	1.2	2.6	8.2	16.0	29.9	1.3	3.5	12.4	13.2	23.3	0.1	0.3	0.2	0.1	0.0	0.2	0.7	1.2	1.8	2.6
Investment non-stationary technology	0.5	0.7	2.7	6.7	15.2	0.3	1.4	6.3	6.9	11.6	0.0	0.5	0.5	0.3	0.0	0.3	1.2	2.2	3.5	5.5
Consumption preference	1.5	1.8	0.9	3.5	0.6	3.2	4.0	1.7	6.8	5.6	2.3	1.2	0.4	0.7	0.0	6.0	7.5	9.3	8.8	5.8
Labour supply	0.1	1.9	2.1	1.2	0.8	1.1	1.9	1.0	4.8	3.8	0.7	1.5	0.4	0.3	0.1	3.1	7.6	7.9	6.3	2.9
Domestic goods markup	0.8	0.1	3.2	2.5	0.5	8.5	0.7	6.4	3.6	0.3	1.3	0.5	0.9	0.3	0.1	2.2	3.9	1.8	0.7	0.5
Imported consumption goods markup	5.5	8.7	5.5	1.9	0.2	6.0	8.8	5.5	1.1	1.7	11.1	12.1	3.5	0.9	0.0	5.6	8.0	0.6	1.7	1.5
Imported investment goods markup	2.1	3.8	6.0	6.0	2.7	4.9	5.7	2.8	1.0	1.7	2.9	2.5	1.4	0.9	0.3	1.3	0.7	2.1	1.7	0.8
Imported export component markup	19.4	30.3	18.0	5.7	0.5	34.0	27.9	0.9	6.0	1.8	8.8	4.4	1.3	0.6	0.1	7.9	4.3	5.7	3.4	1.9
Export markup	30.4	25.8	12.0	2.6	0.6	17.2	23.0	21.3	5.5	0.4	11.2	6.9	1.8	0.5	0.0	11.8	3.3	0.3	0.3	0.4
Wage markup	0.2	6.6	8.5	2.6	3.0	3.6	6.2	2.4	15.7	14.1	2.6	5.1	1.6	0.9	0.3	10.3	25.8	28.3	23.6	11.2
Risk premium	9.0	3.1	6.2	6.9	1.5	2.0	2.5	7.1	7.1	3.4	18.6	0.0	1.9	1.1	0.1	11.4	7.4	2.8	4.1	1.7
Interest rate	1.4	0.7	0.5	0.5	0.1	0.1	0.1	1.5	0.9	0.1	2.8	0.2	0.2	0.1	0.0	10.7	1.2	0.5	0.3	0.2
Inflation target	0.3	1.3	3.3	2.3	0.7	0.7	0.6	0.2	2.2	4.0	1.8	0.7	0.5	0.1	0.1	1.6	6.4	8.5	9.5	8.5
Energy prices	0.5	0.5	0.6	0.5	0.1	0.2	0.4	1.4	0.8	0.2	0.9	0.3	0.2	0.1	0.0	1.7	0.9	0.6	0.3	0.2
Fiscal	0.3	0.6	1.7	1.3	0.5	0.6	0.9	0.7	1.5	3.4	5.9	5.3	5.0	3.9	3.2	5.9	3.0	5.6	7.2	6.7
World economy (SVAR)	24.9	6.7	7.7	17.1	26.2	10.6	3.1	11.7	13.0	15.4	28.2	55.6	76.7	85.6	93.7	14.3	10.8	16.3	23.0	36.6

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Table 7.2. Variance decompositions. Regime 1, part 2

Disturbance	Employment					Real wages					Investment deflator					CPI				
	1q	4q	8q	12q	20q	1q	4q	8q	12q	20q	1q	4q	8q	12q	20q	1q	4q	8q	12q	20q
Stationary technology	12.6	5.9	2.3	2.1	3.8	3.2	4.5	6.2	5.3	1.9	4.3	5.7	2.8	1.1	0.7	6.6	5.7	1.8	0.6	1.0
Investment specific technology	4.1	6.3	8.1	6.1	18.6	0.4	2.0	8.7	18.8	19.0	1.1	2.7	5.2	11.1	16.9	1.6	4.1	7.0	11.6	15.1
Asymmetric technology	0.3	0.1	0.0	0.0	0.1	0.1	0.1	0.1	0.3	0.3	0.3	0.3	0.2	0.2	0.4	0.2	0.3	0.2	0.4	0.6
Non-stationary technology	0.1	0.2	0.7	1.7	6.2	5.7	9.4	18.3	29.9	31.4	0.4	1.0	1.3	1.8	1.9	0.6	1.2	1.4	1.8	1.8
Investment non-stationary technology	0.5	0.7	1.6	3.6	12.9	2.2	3.1	5.3	9.0	11.5	4.1	0.3	2.1	3.7	4.1	1.0	2.2	2.8	3.7	3.9
Consumption preference	8.0	10.0	10.8	10.5	13.0	1.4	2.9	5.5	7.6	5.6	0.2	2.1	3.3	3.5	3.5	0.4	2.1	3.2	3.6	4.1
Labour supply	5.7	9.6	11.4	8.4	0.3	15.0	13.9	6.9	0.2	2.6	4.5	8.6	6.5	4.7	1.7	6.1	10.0	6.3	3.6	1.9
Domestic goods markup	7.9	5.0	0.0	2.3	2.8	19.7	8.5	5.3	2.9	1.1	9.4	2.5	0.3	1.0	0.6	17.7	1.4	1.2	0.4	0.8
Imported consumption goods markup	0.9	0.9	0.9	1.8	0.8	1.3	1.4	1.8	2.7	2.1	2.8	2.0	5.2	2.7	0.8	8.6	0.2	4.5	2.7	1.5
Imported investment goods markup	0.8	0.6	3.0	3.5	0.3	0.5	0.2	0.7	2.0	2.3	30.5	16.9	14.3	3.8	2.2	0.5	0.2	0.8	1.4	1.9
Imported export component markup	4.5	1.8	5.4	4.4	4.5	1.6	0.9	1.5	2.7	2.6	4.0	1.5	0.7	0.2	1.6	2.0	2.3	0.9	1.3	2.0
Export markup	11.3	7.1	0.6	1.5	1.2	0.9	0.2	0.9	0.6	0.5	4.6	0.5	3.2	2.1	0.3	1.7	0.8	0.8	0.0	0.6
Wage markup	18.8	32.6	40.6	32.2	2.0	42.4	44.1	26.1	2.2	9.0	15.1	29.6	24.0	18.5	6.7	20.0	34.8	24.1	14.9	7.3
Risk premium	8.6	5.5	1.3	3.7	3.8	0.5	1.8	1.2	1.3	1.2	5.6	3.5	2.6	0.4	0.2	3.5	1.0	2.4	1.2	0.9
Interest rate	2.3	2.0	0.7	0.1	0.7	0.8	1.3	1.1	0.8	0.4	1.1	0.1	0.0	0.1	0.2	0.8	0.6	0.1	0.0	0.3
Inflation target	0.1	1.5	3.5	4.2	4.4	0.8	1.0	3.1	4.8	2.9	5.3	9.4	9.4	11.7	9.2	5.8	10.7	11.3	12.4	9.5
Energy prices	0.5	0.4	0.4	0.4	0.5	0.9	0.6	0.5	0.4	0.3	0.9	0.4	0.0	0.1	0.2	1.2	0.2	0.0	0.1	0.2
Fiscal	3.7	4.2	6.8	10.3	20.5	1.2	2.5	3.7	3.9	1.7	1.9	4.4	5.6	8.1	7.3	16.7	9.7	13.4	12.6	6.2
World economy (SVAR)	9.3	5.5	2.0	3.2	3.5	1.4	1.6	2.9	4.5	3.4	3.9	8.4	13.4	25.4	41.5	5.2	12.6	17.7	27.7	40.5

Table 7.3. Variance decompositions. Regime 2, part 1

Disturbance	GDP deflator					GDP					Consumption					Investment				
	1q	4q	8q	12q	20q	1q	4q	8q	12q	20q	1q	4q	8q	12q	20q	1q	4q	8q	12q	20q
Stationary technology	10.9	4.6	2.2	1.7	0.7	0.8	8.0	6.1	2.6	0.9	4.2	6.1	7.0	5.2	2.4	1.8	4.1	4.6	4.4	2.2
Investment specific technology	0.7	2.0	3.9	5.4	7.2	2.8	9.1	15.6	19.1	13.7	4.7	4.6	0.9	9.3	22.7	50.2	41.1	30.2	19.8	6.0
Asymmetric technology	0.0	0.1	0.3	0.4	0.5	0.6	0.0	0.0	0.1	0.3	0.1	0.2	0.3	0.3	0.4	0.3	0.4	0.5	0.7	1.1
Non-stationary technology	0.8	1.2	1.1	1.0	1.1	2.7	7.8	13.4	21.1	38.0	5.4	7.5	13.4	20.2	25.4	2.7	3.3	4.6	6.9	17.4
Investment non-stationary technology	1.2	2.1	2.2	2.2	2.3	1.0	2.5	4.7	8.7	18.4	1.3	1.3	1.8	3.0	6.2	0.6	2.2	5.3	9.3	24.3
Consumption preference	1.7	3.0	3.3	2.9	2.5	6.7	10.6	7.9	4.1	1.4	39.1	31.6	20.7	7.3	4.2	6.9	8.4	9.8	10.7	8.5
Labour supply	8.4	9.7	1.7	1.4	0.6	1.5	8.8	10.8	7.2	2.3	3.6	6.8	10.2	9.2	4.1	2.6	4.6	6.1	6.7	4.9
Domestic goods markup	33.8	8.5	3.7	0.0	0.9	12.2	7.7	0.0	1.2	0.5	8.7	6.2	4.0	2.1	1.5	7.3	4.7	3.7	2.8	0.3
Imported consumption goods markup	1.1	2.4	1.6	1.0	0.3	0.8	2.2	0.4	2.2	1.6	1.1	1.6	0.3	1.9	2.1	0.1	1.5	3.3	4.1	3.0
Imported investment goods markup	0.1	0.5	1.1	0.5	0.6	1.9	0.4	4.3	4.7	2.3	1.6	2.3	2.9	1.2	2.4	4.9	7.6	6.2	3.5	1.7
Imported export component markup	0.2	3.2	3.7	2.2	1.3	10.5	3.7	5.2	0.9	1.6	2.7	4.9	6.3	5.1	3.2	4.7	5.4	5.9	5.5	2.4
Export markup	1.5	0.6	3.4	2.6	0.6	15.0	8.0	2.1	1.9	0.5	3.8	2.6	0.7	0.1	0.8	0.6	1.2	1.8	1.7	0.6
Wage markup	9.5	10.3	1.1	1.8	0.7	1.7	9.7	11.4	7.3	2.3	4.1	7.5	11.0	9.7	4.3	2.9	5.1	6.6	7.2	5.0
Risk premium	2.4	1.4	2.6	2.1	0.8	11.1	6.1	4.5	2.8	0.4	0.7	0.9	3.7	4.0	1.7	0.2	0.4	2.3	3.5	3.3
Interest rate	1.9	3.0	1.6	0.2	0.4	3.3	4.4	1.0	0.1	0.3	3.8	2.9	1.9	1.1	0.8	3.3	2.4	1.8	1.3	0.1
Inflation target	13.1	21.7	23.7	20.3	12.9	2.1	0.7	3.6	4.2	3.0	5.1	2.8	0.7	3.3	2.7	4.0	2.2	0.1	1.9	6.1
Energy prices	1.8	0.1	0.1	0.1	0.2	2.8	0.1	0.1	0.1	0.2	0.7	0.9	0.8	0.6	0.4	2.3	0.8	0.7	0.6	0.2
Fiscal	2.2	4.4	5.5	5.2	3.3	6.8	4.6	6.6	8.5	9.0	4.4	6.7	10.4	11.0	10.0	0.6	0.7	1.3	2.2	4.0
World economy (SVAR)	8.7	21.1	37.2	49.0	63.1	15.7	5.6	2.4	3.3	3.3	4.9	2.6	3.0	5.4	4.7	3.9	3.8	5.2	7.3	9.0

Disturbance	Exports					Imports					Real exchange rate					Interest rate				
	1q	4q	8q	12q	20q	1q	4q	8q	12q	20q	1q	4q	8q	12q	20q	1q	4q	8q	12q	20q
Stationary technology	0.3	1.5	0.9	1.0	0.2	1.8	1.5	3.1	4.8	2.0	0.0	1.1	0.1	0.2	0.0	3.7	5.2	4.2	2.7	0.8
Investment specific technology	0.4	2.3	10.7	18.9	14.6	3.4	7.2	11.1	3.3	4.2	0.1	1.5	2.8	3.2	1.7	2.5	5.2	7.0	5.6	6.1
Asymmetric technology	1.2	0.9	2.1	3.3	4.2	0.8	1.4	3.4	3.6	4.5	0.5	0.5	0.4	0.4	0.3	0.5	0.3	0.3	0.3	0.3
Non-stationary technology	1.2	2.7	8.3	16.2	29.5	1.3	3.6	11.4	14.3	26.1	0.0	0.2	0.2	0.1	0.0	0.0	0.4	0.9	1.5	2.4
Investment non-stationary technology	0.5	0.9	2.7	6.5	14.7	0.2	1.3	5.6	7.5	13.1	0.0	0.5	0.5	0.3	0.0	0.0	0.6	1.4	2.7	4.8
Consumption preference	1.3	1.5	1.6	3.8	0.3	3.4	4.3	1.4	7.1	6.1	2.1	0.9	0.6	0.7	0.0	6.8	9.9	10.3	8.9	4.6
Labour supply	0.1	1.8	3.2	0.9	0.2	0.8	1.4	1.8	5.3	4.0	0.6	1.4	0.7	0.0	0.0	2.2	6.0	6.2	5.0	1.9
Domestic goods markup	0.9	0.5	2.5	2.5	0.5	8.5	0.4	6.5	4.5	0.5	1.5	0.0	0.8	0.4	0.1	1.3	4.8	2.6	1.0	0.3
Imported consumption goods markup	5.4	9.5	6.7	2.1	0.4	6.3	9.6	5.2	0.9	1.8	10.9	12.9	3.8	0.9	0.0	5.5	9.7	1.7	0.9	0.8
Imported investment goods markup	2.0	3.9	6.1	6.4	3.0	5.0	6.2	2.8	1.1	2.0	2.8	2.5	1.4	1.0	0.3	1.3	1.5	3.4	3.0	0.1
Imported export component markup	18.9	31.8	18.7	5.9	0.7	34.8	29.6	1.1	6.6	2.0	8.4	4.6	1.4	0.6	0.1	8.6	6.2	6.5	3.2	1.2
Export markup	30.0	27.5	12.2	2.2	0.6	17.7	24.4	20.2	6.4	0.5	10.7	6.9	1.9	0.6	0.1	13.2	3.8	1.0	0.5	0.2
Wage markup	0.1	2.0	3.3	0.6	0.2	1.0	1.6	2.2	5.9	4.1	0.6	1.5	0.7	0.0	0.0	2.5	6.6	6.7	5.2	2.0
Risk premium	9.6	2.6	6.9	6.2	0.7	2.0	2.1	7.3	7.1	2.9	20.4	0.7	2.0	1.0	0.1	13.4	7.0	4.3	4.0	1.0
Interest rate	1.9	1.4	0.3	0.6	0.3	0.3	0.3	2.2	1.7	0.2	3.6	0.6	0.2	0.1	0.0	11.7	2.0	0.9	0.4	0.2
Inflation target	1.0	0.9	4.1	3.5	1.1	1.1	1.0	0.5	2.5	5.7	3.2	0.3	0.7	0.2	0.1	2.9	10.3	12.5	13.9	11.9
Energy prices	0.5	0.5	0.6	0.5	0.1	0.2	0.5	1.3	0.9	0.2	0.9	0.2	0.2	0.1	0.0	1.9	1.1	0.6	0.3	0.1
Fiscal	0.3	0.4	1.7	2.2	2.2	0.5	0.6	0.7	1.9	3.5	5.7	5.4	5.1	4.1	3.5	6.3	1.8	3.7	4.6	3.8
World economy (SVAR)	24.6	7.5	7.5	16.9	26.4	10.9	3.1	12.1	14.8	16.6	28.0	58.2	76.6	86.2	93.6	15.7	17.5	25.6	36.2	57.4

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Table 7.4. Variance decompositions. Regime 1, part 2

Disturbance	Employment					Real wages					Investment deflator					CPI				
	1q	4q	8q	12q	20q	1q	4q	8q	12q	20q	1q	4q	8q	12q	20q	1q	4q	8q	12q	20q
Stationary technology	13.0	6.3	1.0	0.6	3.4	4.1	6.7	9.4	6.4	2.4	4.1	5.2	1.5	0.3	0.2	6.9	5.0	0.4	1.2	0.5
Investment specific technology	4.5	7.7	10.7	8.7	15.4	0.3	2.7	11.2	18.1	20.0	0.1	0.4	1.3	5.7	11.2	0.3	0.9	2.6	5.7	8.9
Asymmetric technology	0.3	0.1	0.0	0.0	0.1	0.1	0.1	0.2	0.2	0.3	0.3	0.3	0.1	0.1	0.2	0.2	0.3	0.2	0.3	0.4
Non-stationary technology	0.1	0.2	0.7	1.9	5.9	7.4	13.1	23.3	28.3	32.1	0.3	0.8	0.9	1.3	1.3	0.5	1.0	1.1	1.2	1.2
Investment non-stationary technology	0.5	0.8	1.9	4.0	12.2	2.8	4.3	6.7	8.4	11.6	4.9	1.5	1.3	2.6	2.8	0.8	1.7	2.1	2.5	2.5
Consumption preference	9.2	13.4	15.3	13.7	12.3	1.6	3.2	5.7	6.3	5.4	0.1	3.3	3.9	2.7	1.5	0.8	3.2	3.9	2.8	2.1
Labour supply	6.2	12.0	16.0	12.7	1.2	19.2	18.4	6.8	2.3	3.4	3.8	7.4	4.1	1.6	0.3	5.6	8.9	3.0	0.2	0.4
Domestic goods markup	9.4	7.8	1.8	2.0	3.4	25.9	13.1	8.5	3.6	1.4	9.8	1.5	1.1	1.5	0.4	20.1	3.9	2.7	0.8	0.7
Imported consumption goods markup	1.4	2.1	0.7	1.2	0.1	2.0	1.2	0.8	1.7	1.9	3.5	1.5	6.9	4.6	0.2	9.7	1.5	6.7	2.4	0.5
Imported investment goods markup	1.0	0.6	3.8	4.6	0.3	0.7	0.4	0.8	2.0	2.5	34.1	23.5	20.5	6.3	1.5	0.7	0.9	0.0	0.5	1.1
Imported export component markup	5.0	2.2	7.3	5.7	4.1	2.2	1.4	1.6	2.4	2.6	4.3	1.9	1.4	1.3	0.6	2.2	2.8	0.8	0.6	1.0
Export markup	12.7	9.8	1.4	2.0	1.7	1.6	0.6	0.5	0.4	0.6	4.8	0.1	4.2	2.6	0.1	1.6	0.5	1.0	0.3	0.5
Wage markup	6.9	13.2	17.1	13.0	0.7	23.8	20.9	6.6	3.0	3.6	4.2	8.0	4.2	1.4	0.3	6.3	9.5	2.8	0.5	0.5
Risk premium	9.7	6.6	2.3	4.7	3.2	1.1	3.1	1.5	1.0	1.2	6.6	5.6	3.0	0.3	0.3	4.4	2.0	3.2	0.9	0.4
Interest rate	3.9	4.4	2.3	0.3	1.2	1.7	3.3	3.0	1.6	0.8	2.2	1.6	0.6	0.5	0.2	1.9	2.4	1.0	0.3	0.3
Inflation target	1.5	0.0	4.1	6.4	7.0	0.7	0.1	3.4	5.1	3.7	8.8	18.0	16.6	18.8	12.4	9.9	20.5	20.9	20.2	12.6
Energy prices	0.4	0.3	0.4	0.5	0.5	1.2	0.9	0.8	0.5	0.3	1.0	0.3	0.2	0.2	0.1	1.3	0.1	0.2	0.1	0.1
Fiscal	4.1	5.4	9.6	13.8	24.1	1.7	3.6	4.5	3.0	1.1	1.3	3.4	3.9	4.9	3.5	19.1	12.6	14.6	11.9	4.7
World economy (SVAR)	10.3	7.1	3.5	4.2	3.0	2.0	2.7	4.9	5.7	5.1	5.9	15.6	24.4	43.4	63.0	7.9	22.1	32.8	47.7	61.6

7.2 Impulse response functions

Figures 7.1–7.19 present the responses of selected observable variables to specified structural disturbances of the $\mathbb{SOE}^{\mathrm{PL}-2009}$ model, i.e. impulse response function (IRF). Impulse response functions take into account the existence of all of the simultaneous and lagged interrelations among model variables, persistence of impulses and (in the case of observable disturbances) the possible correlation between the disturbances. It is, thus, an illustration of the dynamic characteristics of the model and the consequences of interrelations present in the model. The tool is used both for diagnosing the very model (enables to better understand the functioning of the model, detect the possible incoherence) and — assuming that the model is an adequate description of a fragment of reality — for the provision of information about the effects of disturbances, key reactions to macroeconomic policy variables (direction, strength, lag distribution, etc.).

As in the case of decomposition of forecast error variances, impulse response functions presented further herein have been determined for the point estimates of parameters corresponding to the posterior mode. Thus, the effects of uncertainty of parameters have not been included. The impulse has the value of one standard deviation and lasts one quarter, taking into account autocorrelation of the disturbance. So, although each time the impulse is different, it complies with the values typical for the sample (see Tables 6.2 and 6.1).

For observable variables expressed in annualised form as a percentage (GDP deflator, investment deflator, CPI, inflation in the euro area, inflation in the USA and interest rates), IRF graphs present deviations from the steady state calculated in percentage points[2]. Impulse response functions of observable real variables (GDP, consumption, investments, consumption, export, import, real wages), as well as exchange rates and employment are characterised with percentage deviations from the steady state. Each of the response functions is presented for both regimes included in the $\mathbb{SOE}^{\mathrm{PL}-2009}$ model, thus, it must be remembered that differences in responses to some of the shocks result also from differences in the sizes of impulses and autocorrelations of impulses.

From the point of view of the purpose for which the $\mathbb{SOE}^{\mathrm{PL}-2009}$ model has been built, interesting are the impulse response functions enabling the analysis of the characteristics of monetary transmission mechanism, i.e. response of the economy to monetary impulse (interest rate), inflation target and risk premium. The impulse response functions of the monetary disturbance (Figure 7.15) have standard characteristics, foreseen by the authors of DSGE models. Increase in domestic short-term interest rate following the impulse is transferred to the nominal and real exchange rate, whose appreciation leads to a drop in exports. The increase of import is, however, limited due to the income effect — rational forward looking agents reduce investments and consumption, which is translated into a drop of GDP, employment and wages (with multiplier effects for consumption and import). Due to the fact that dropping export comprises an import component, which to some extent may substitute the domestic component, import fluctuations

[2]In technical sense, the method of presentation of the response function for growing variables enables their treatment as the so called cumulative interim multipliers. They inform about the cumulative (in time) effect of the studied impulse on growing observable variables.

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occur in the first quarters. Domestic currency appreciation, decrease of aggregated demand and wages result in a drop of inflation[3]. Domestic prices, although rigid, include a currently optimised component, so domestic inflation reacts without a lag[4]. Comparing the response functions for the two regimes we shall notice that a relatively small change in the size of monetary impulse is accompanied by quite a considerable change of the scale of variables responses — in the second regime both prices and real categories react more suddenly to the impulse — in the case of CPI, the maximum response nearly doubled. Characteristic is the fact that this has not significantly affected the time of shock absorption — it is similar in both regimes.

Also responses to exchange rate risk premium impulse seem to be similar to the responses received in the class of models assuming forward-looking behaviour of agents and motivating the exchange rate changes with uncovered interest rate parity (UIP). In such a situation a growth in risk premium results in immediate depreciation of (nominal and real) exchange rate, leading to a growth of prices of imported components of consumption, investment and export goods. This brings about a fast reduction of import and increase in export. Moreover, imported components are being replaced with domestic components, which despite the drop of consumption and investments result in an increase in GDP and employment. With increase in demand for domestic products also the prices of domestically manufactured goods rise, so an interest rate increase is inevitable. Absorption takes place quite quickly — e.g. the fast response of interest rate to premium disturbance eliminates the scale of depreciation. A change in regime that — according to the estimation results — took place in 1999, practically did not change the responses of most of the observable variables to premium shock (including inflation), with the exception of consumption and investments (with weaker responses in the new regime) and employment.

Another impulse response function that is interesting for the central bank — the IRF of the inflation target disturbance (see Figure 7.14) — describes the consequences of unexpected emergence among the manufacturers and consumers of a belief (expectations?) as to the (temporary) higher inflation (inflation target). Due to the fact that the inflation target is a basis for indexation of prices and wages in economy — which is its basic function — inflation rises automatically, while that increase exceeds the scale of the shock. Despite the fact that, in the perception of agents, higher inflation target considerably alleviates interest rate policy (the central bank accepts higher inflation levels), multiplier effects pushing inflation up above the current target force an increase in interest rate. The real exchange rate strengthens upon relatively large but short-lasting depreciation (only slightly affecting export) — thus, the consequences of the expected interest rate increase prevail. Temporary exchange rate depreciation stimulates export and output, but after the increase in interest rate and exchange rate appreciation, output, consumption, investment, export and import fall. The impulse response functions of inflation target may, thus, be treated as an illustration of the costs of inflation suppression, when such phenomenon intensifies. Benefits (increase in production, consumption, etc.) abate quickly,

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[3]In the categories of a theoretical model, whose structural form we have presented in the Appendix, the sequence of events is initiated with a growth of marginal income utility (a consequence of interest rate growth), which leads to immediate reduction of consumption.

[4]On the other hand, the cost of working capital bind directly the marginal cost of domestic manufactured products with interest rate. In this case the interrelation diminishes the anti-inflation effects of monetary disturbance.

while costs (higher inflation, drop in employment, output) are perceptible much longer. The new regime that appeared post 1999 caused much stronger positive reactions of output, consumption and investments in the first quarters. The reactions were strong enough to result in a temporary increase in employment and real wages (which has not been observed in the first regime). In subsequent quarters the responses of variables are similar in both regimes.

In the group of stationary technological disturbances (TFP, investment-specific and asymmetric disturbances), the first of them (see Figure 7.1) represents an increase in total factor productivity leading to a decrease in the marginal costs of domestic production and demand for production factors (per unit of product). The effect of such a situation is a drop in the dynamics of prices and increase in output, consumption, investments and real wages. Lower inflation implies a reduction of interest rate and depreciation of real exchange rate resulting from anticipated expansionary monetary policy, thus, contributing to increase in the share of domestic products in the final product. The whole leads, of course, to an increase in export and decrease in import. Also the proportion of productivity factors changes — employment decreases and investments increase. The fact that the wages are rigid and remuneration for capital services is flexible proves to be an important element.

Figure 7.1. Stationary technology shock

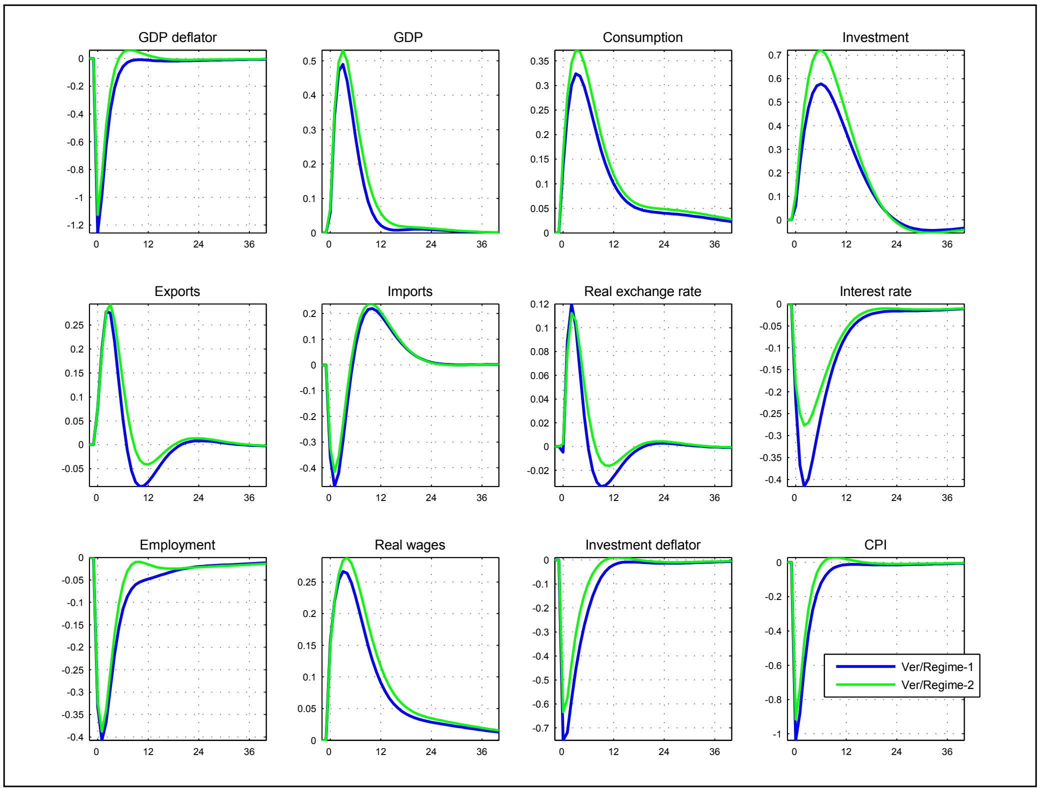

Figure 7.2. Stationary investment shock

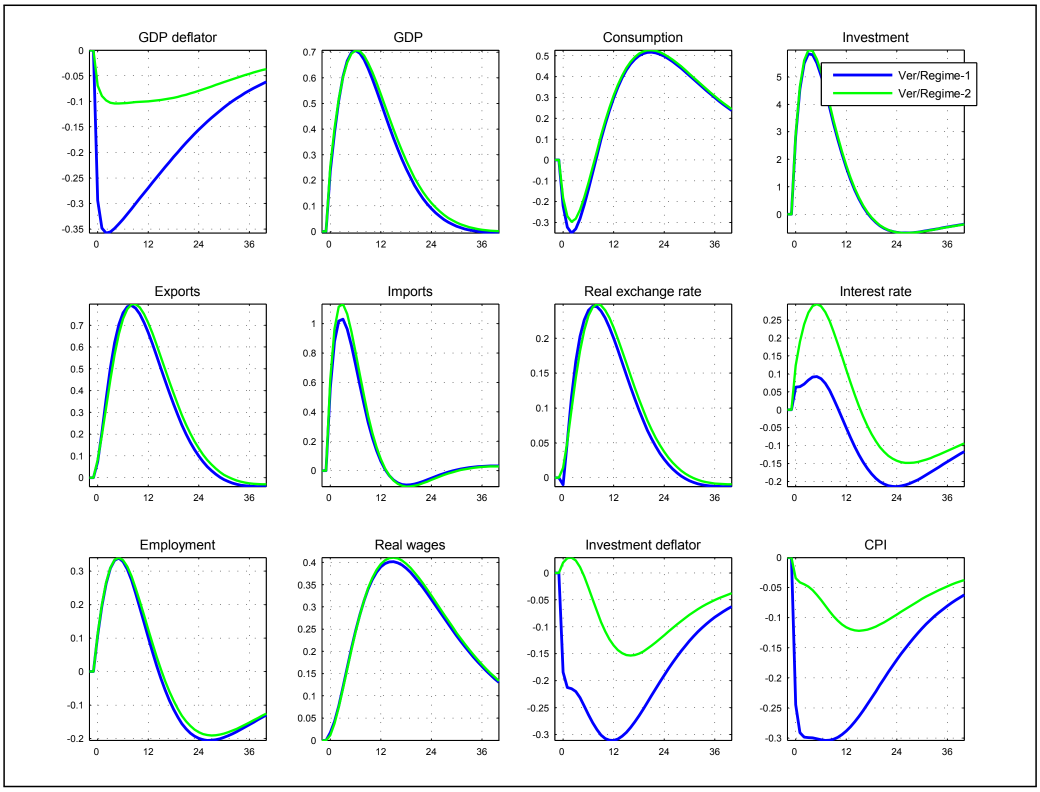

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Figure 7.3. Asymmetric technology shock

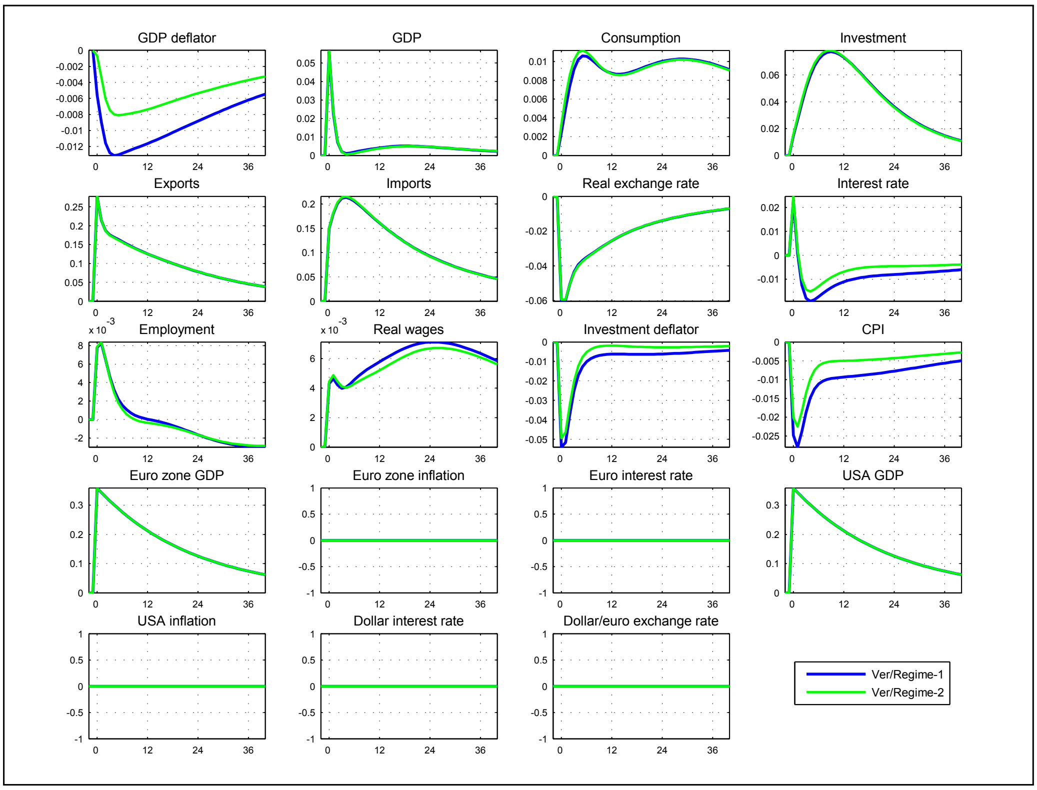

Figure 7.4. Non-stationary technology shock

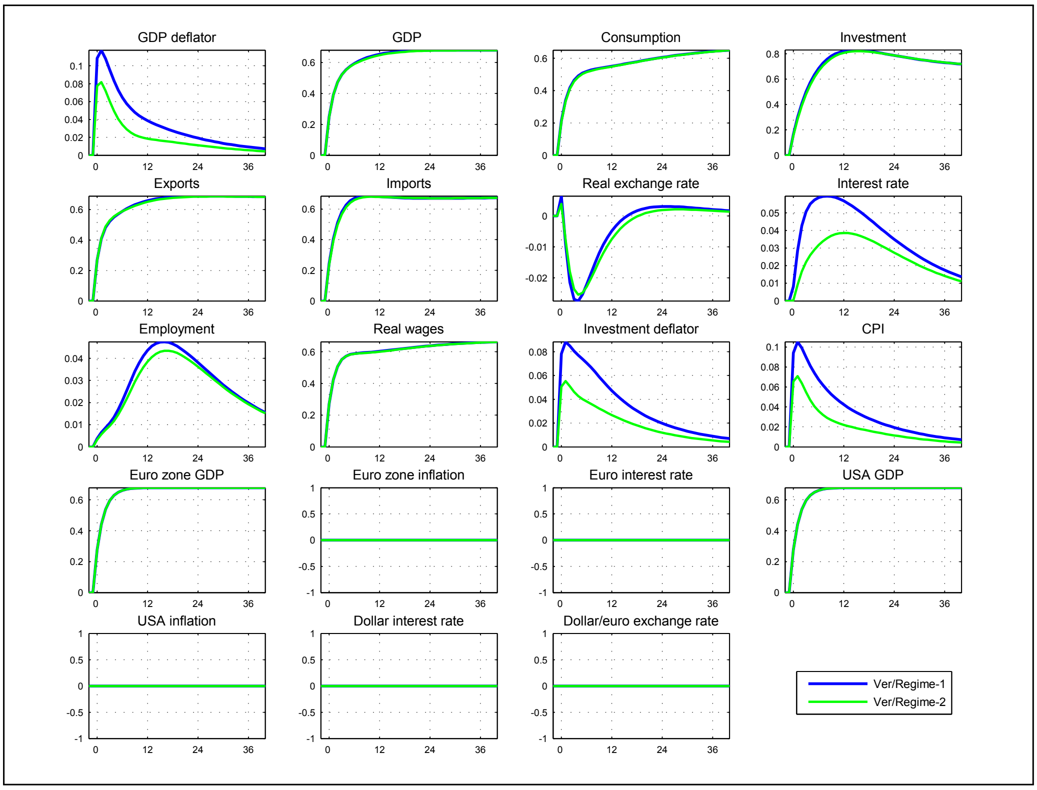

Figure 7.5. Investment non-stationary technology shock

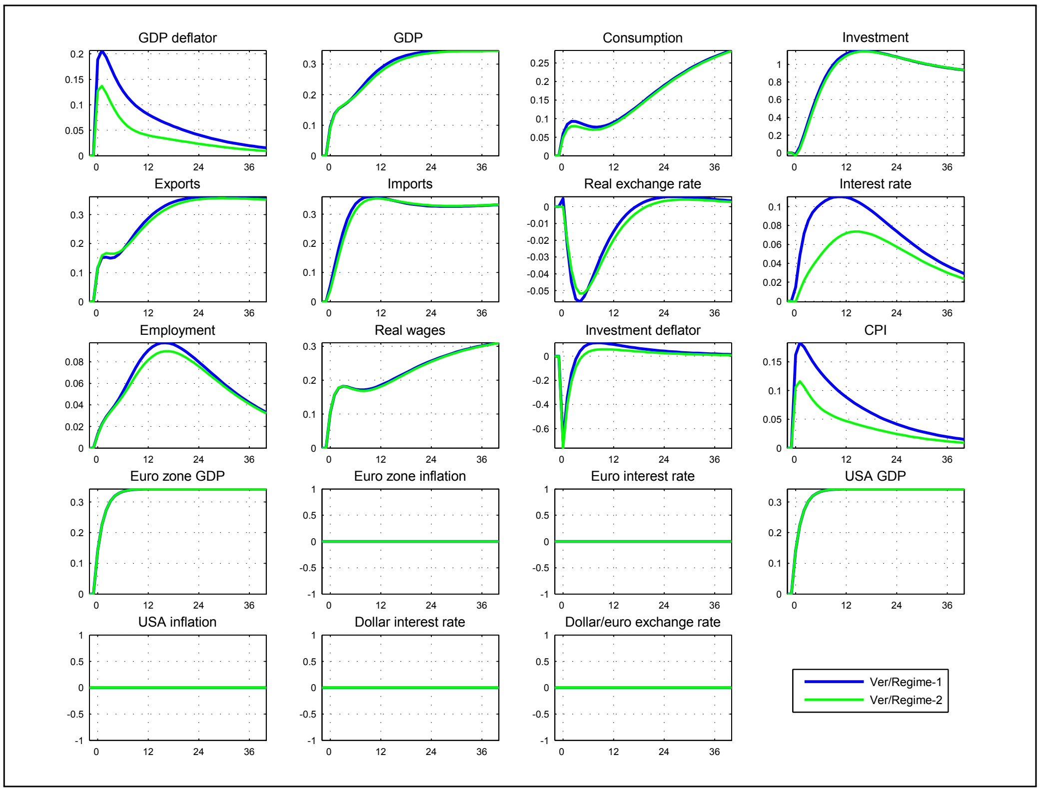

Figure 7.6. Consumption preference shock

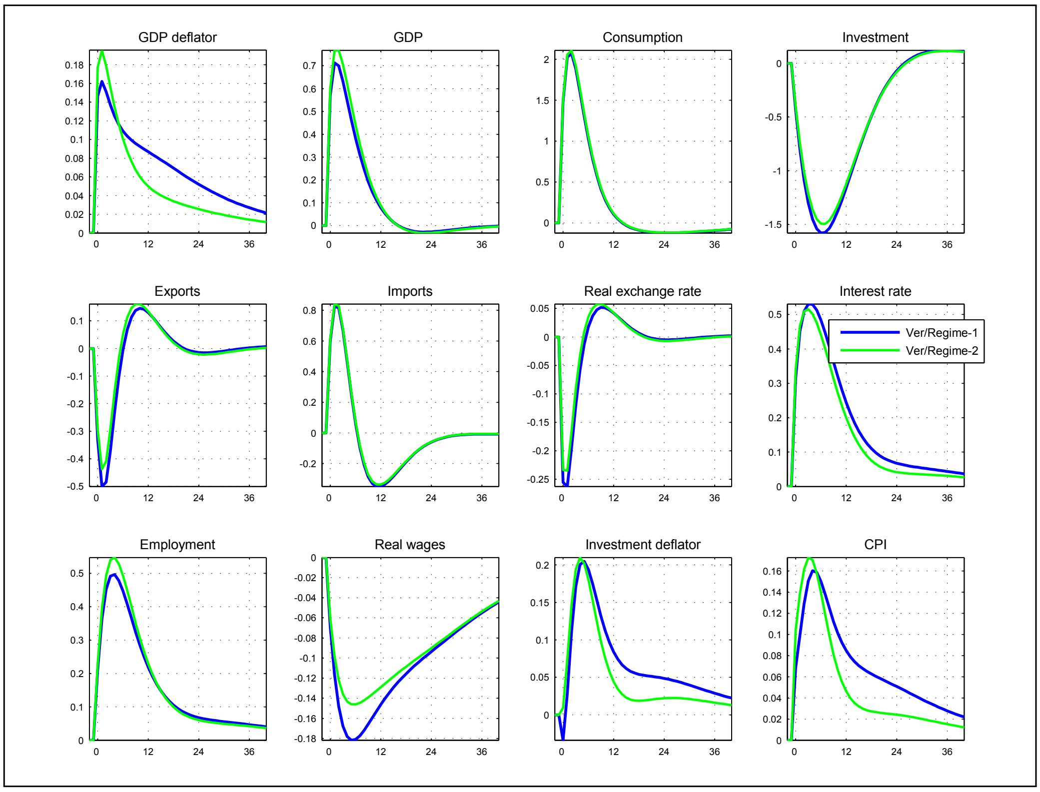

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Figure 7.7. Labour supply shock

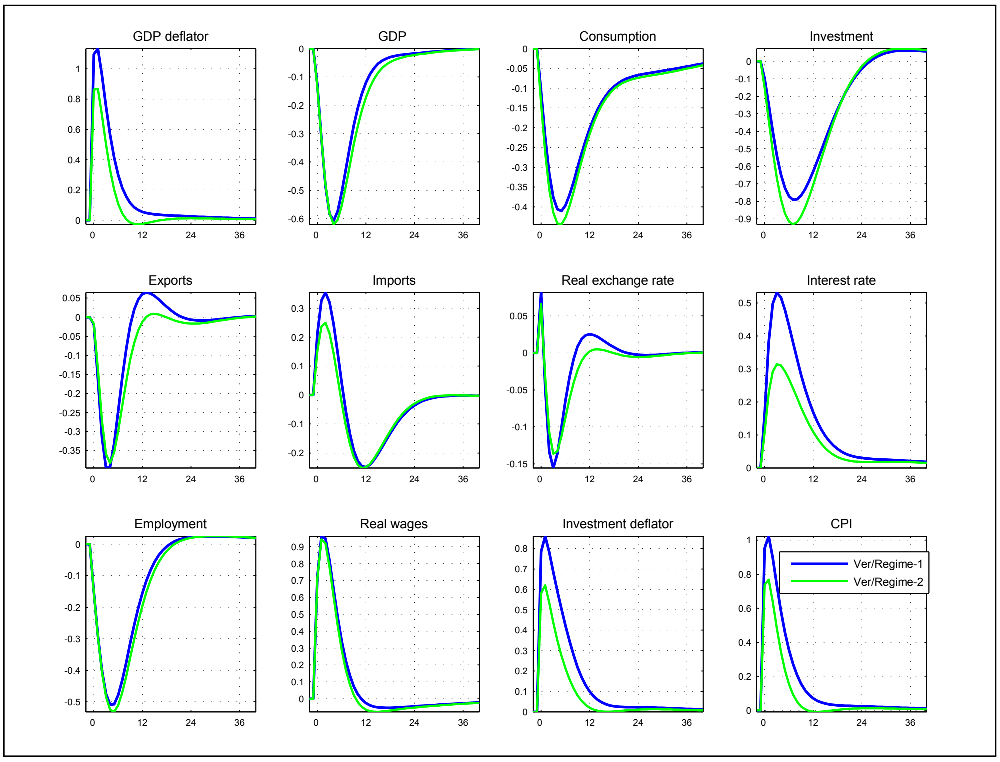

Figure 7.8. Domestic goods markup shock

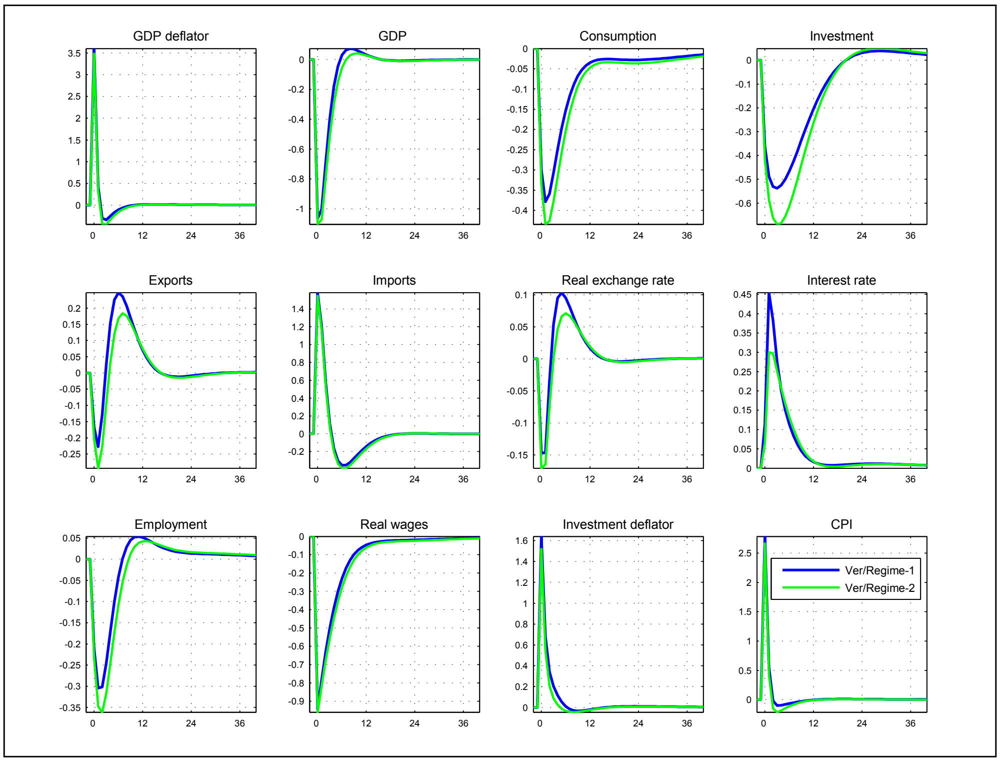

Figure 7.9. Imported consumption goods markup shock

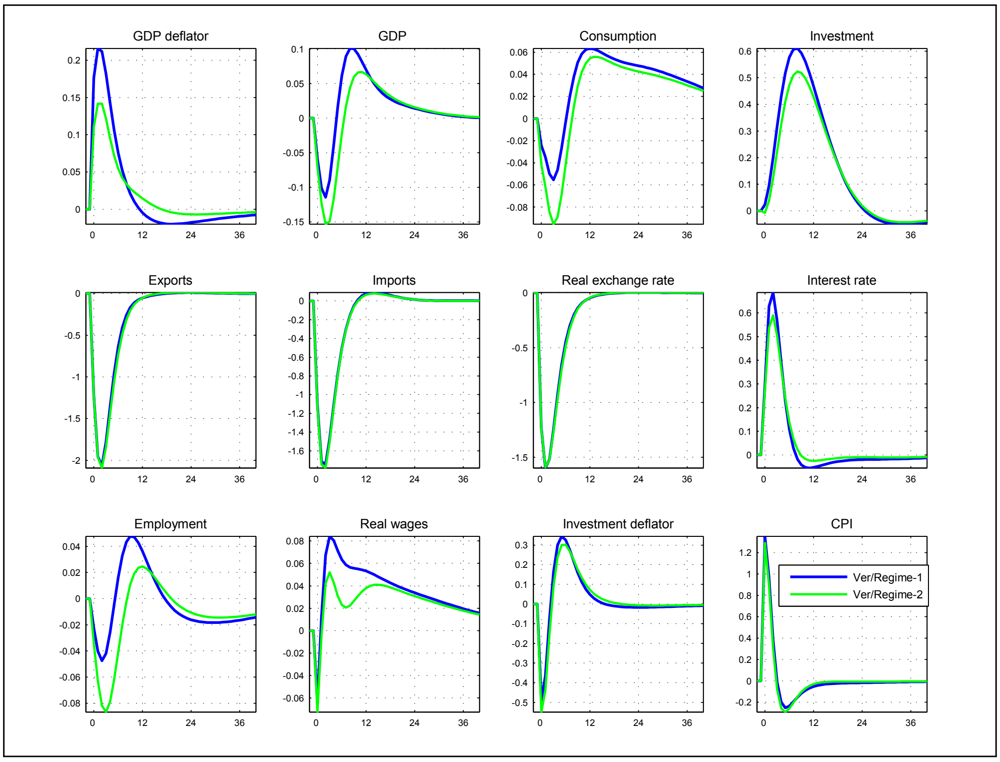

Figure 7.10. Imported investment goods markup shock

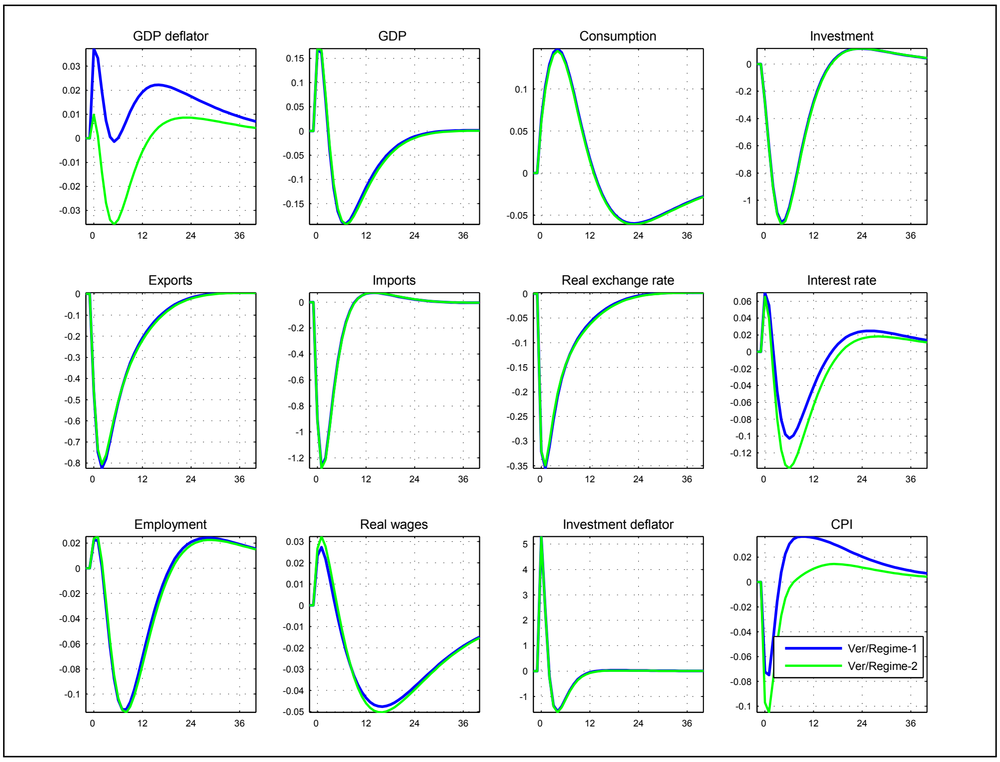

Figure 7.11. Imported export component markup shock

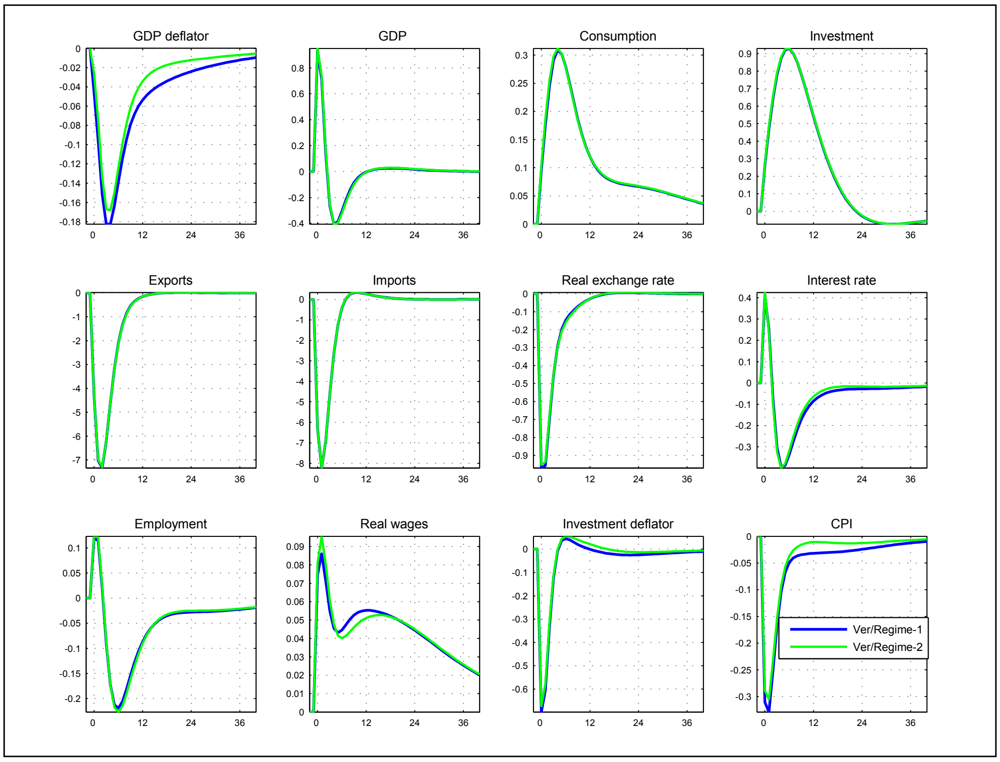

Figure 7.12. Export markup shock

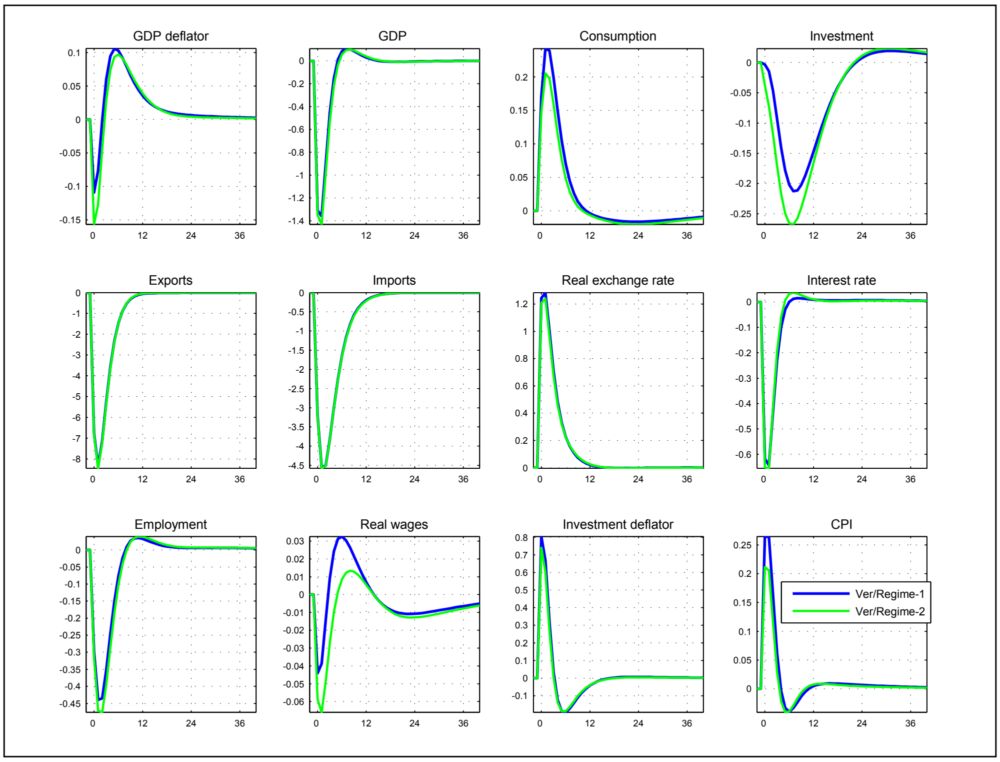

Figure 7.13. Risk premium shock

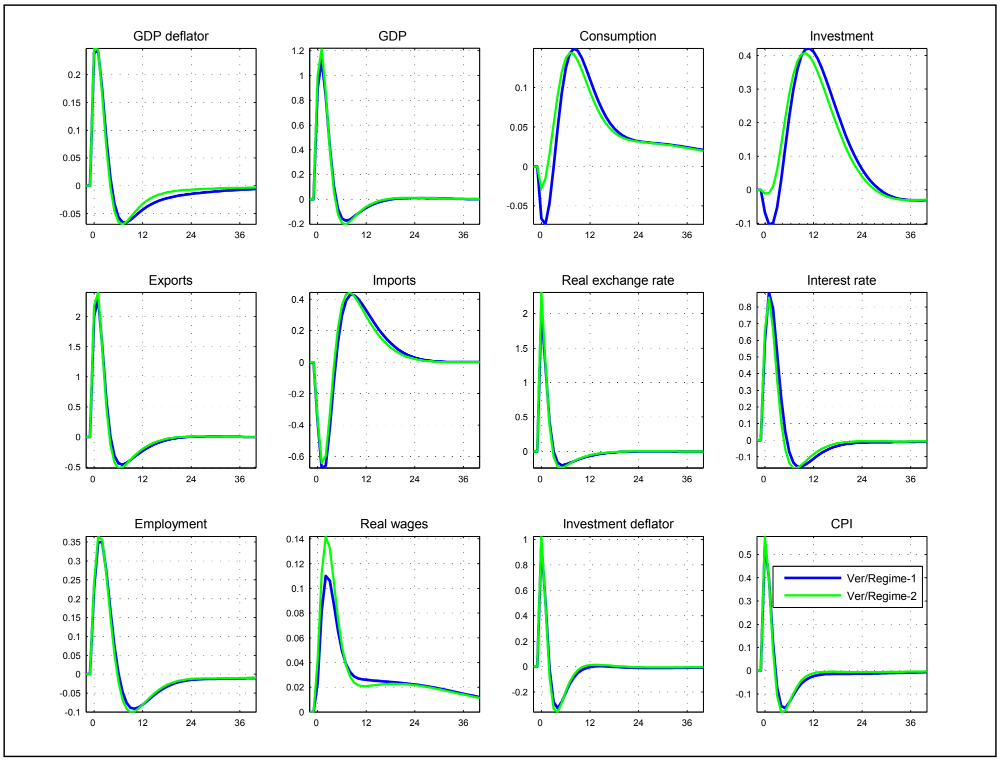

Figure 7.14. Inflation target shock

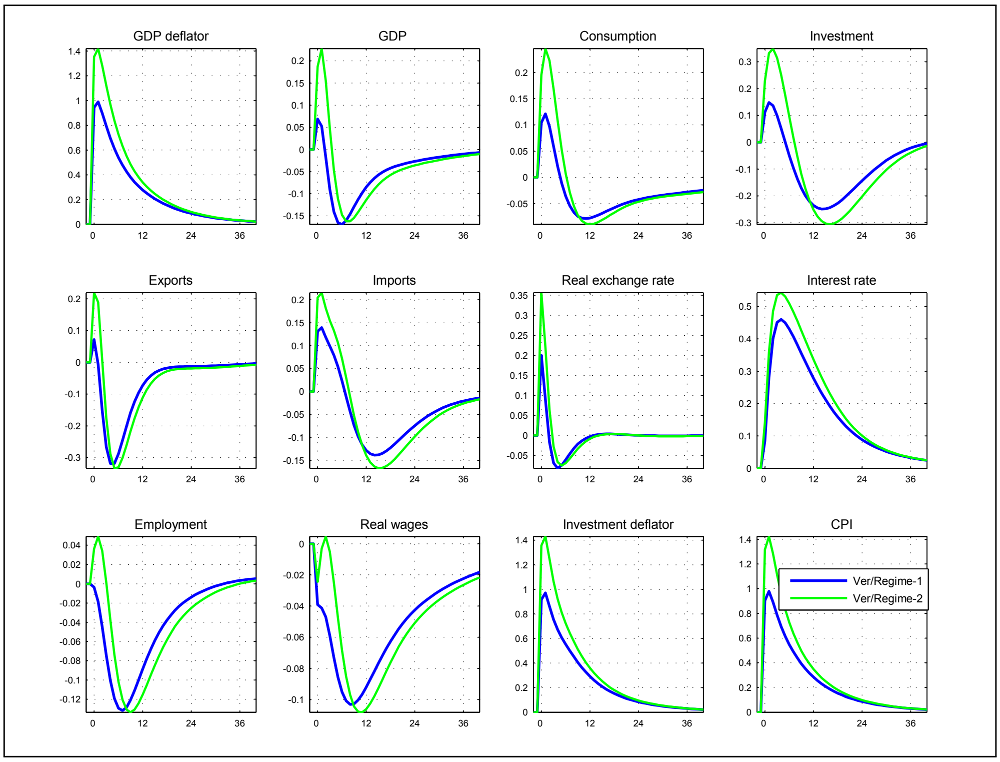

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Figure 7.15. Monetary policy (interest rate) shock

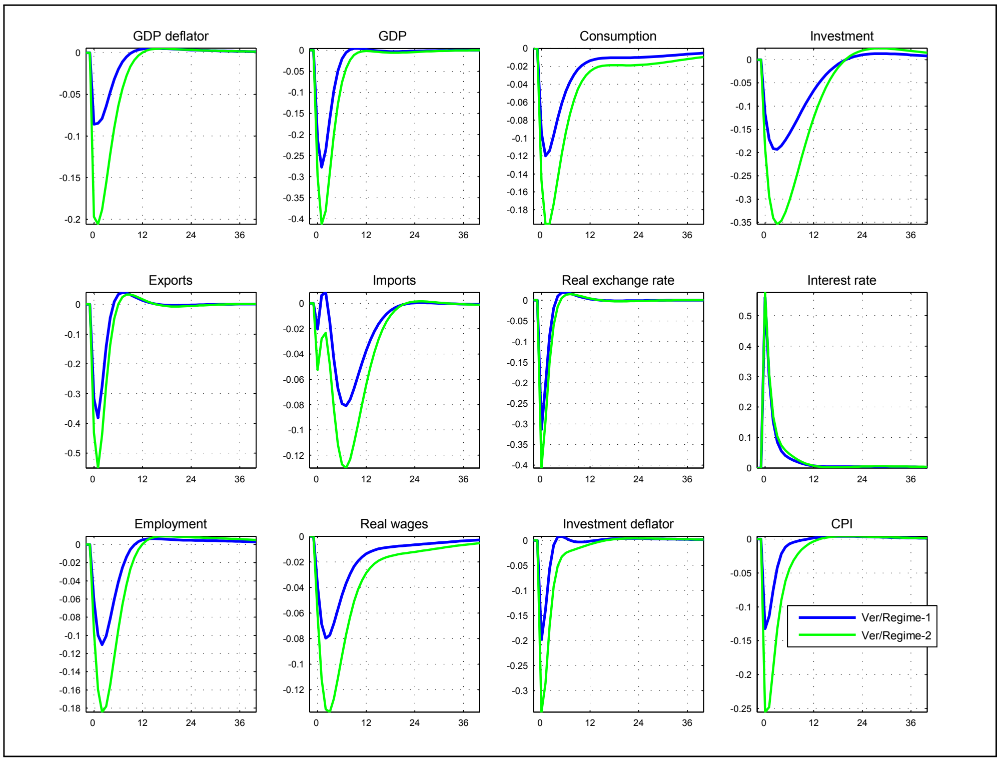

Figure 7.16. Energy prices shock

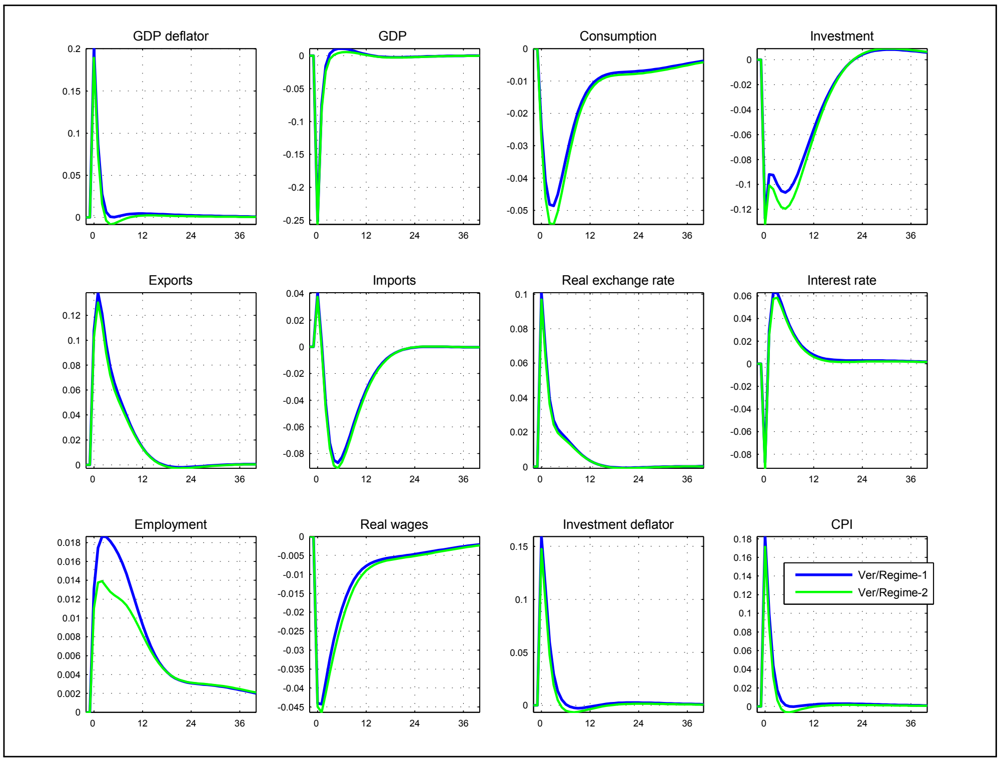

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Figure 7.17. Capital tax shock

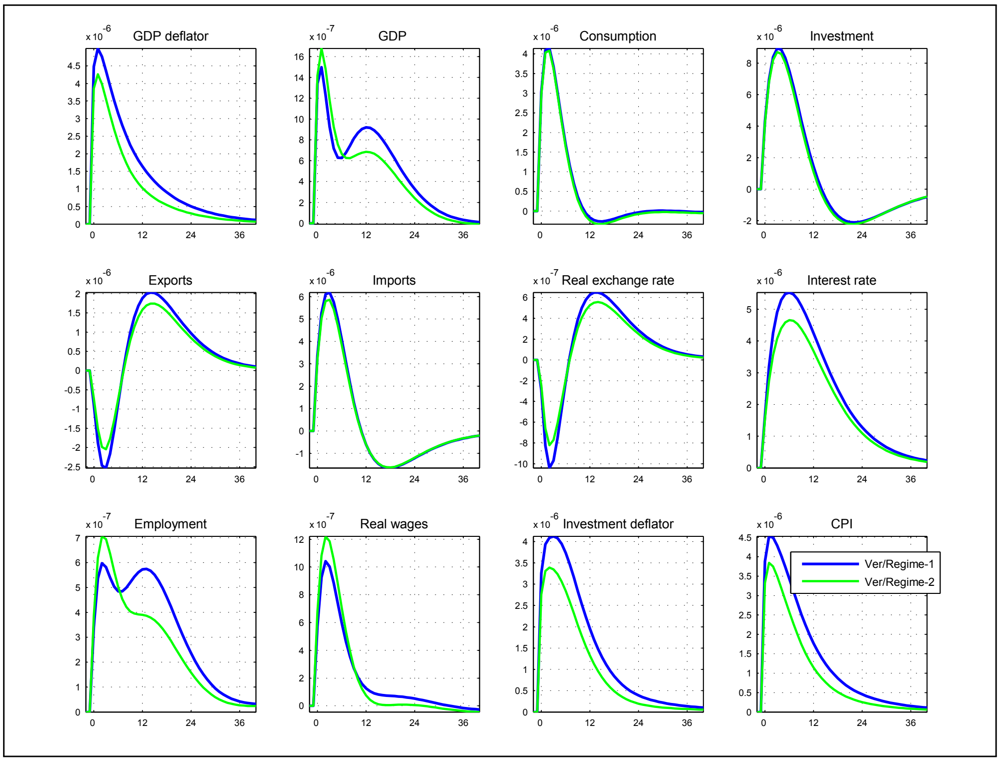

Figure 7.18. Employer social contribution shock

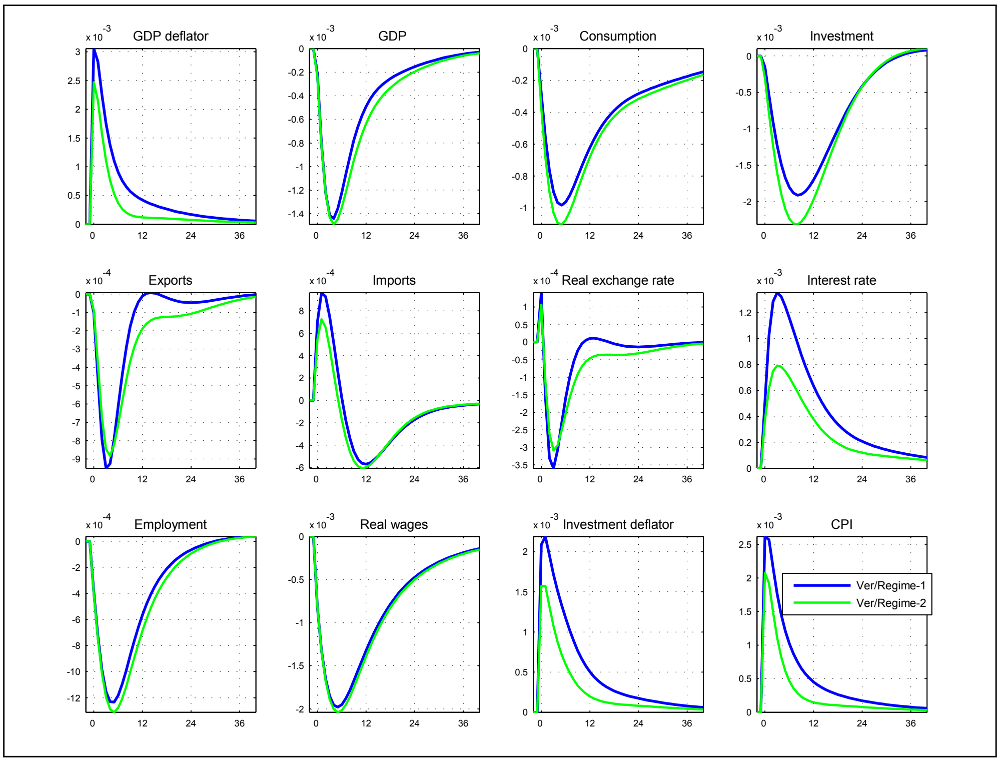

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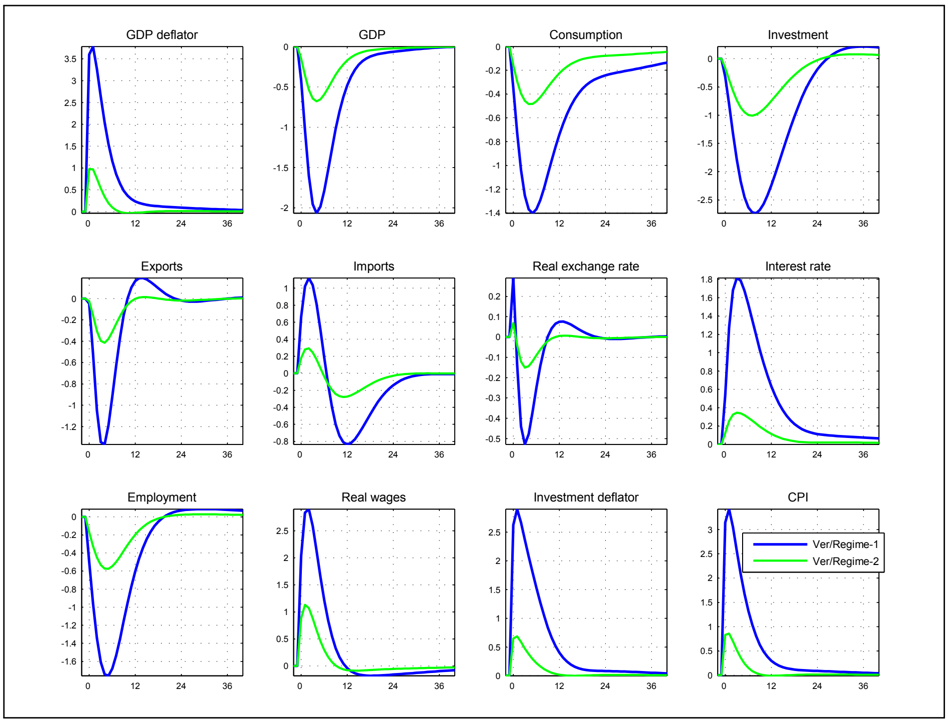
Figure 7.19. Wage markup shock



Investment-specific technology shock results in an increase in efficiency of the transformation of investment expenditures into fixed assets. Therefore, from a unit of expenditures we receive more capital which brings rent to households. Thus, the appearance of such shock (see Figure 7.2) results in a large increase in investment expenditures. The increase during the first period leads even to a fall in consumption in the pool of household expenditures. At the same time, a drop in the price of capital services reduces the marginal cost of domestically produced goods, which — *ceteris paribus* — lowers domestic inflation and increases export competitiveness. The total effect is the increase in export but even bigger increase in import (due to high level of import in investments), and thus appearance of negative net export, which gives an impulse for real exchange rate depreciation. In the face of exchange rate depreciation and fast GDP growth, the interest rate is rising.

In a small open economy, the rate of technological progress is identical to the rate in the rest of the world, there may, however, be present temporary disturbances differentiating growth dynamics. Such function is fulfilled by asymmetric technology disturbance. Appearance of such a shock increases the dynamics of output in the world, increasing demand for goods exported from the domestic economy, while the increase in dynamics in the world's output does not impact the world's prices, interest rates and cross rate. The situation is illustrated by the impulse response functions in Figure 7.3. Faster increase in export than in import leads to (nominal and real) exchange rate appreciation, which translates into a drop in prices of imported products. The domestic interest rate reacts stronger to the appearing supply gap than the minimally decreasing domestic inflation. Generally, asymmetric technology shock weakly affects the economy, as in the previous versions of the $\mathbb{SOE}^{\mathrm{PL}}$ model.

Peculiar reactions are observed for non-stationary disturbances (in technology — see Figure 7.4 and in investments — see Figure 7.5). Non-stationary nature of disturbances means a permanent influence of shocks on the level of growing observable variables (GDP, consumption, investments, export, import, real wages, external GDP). The shocks have no permanent impact on inflation, employment, exchange rate dynamics and interest rate.

Thus, real observable variables move to a new path, while price fluctuations are a consequence of relation between the marginal cost of domestic production and the rate of growth of technical progress $\mu_{z,t}$[5]. The graphs of impulse response functions for both aforementioned disturbances are similar and result from the same mechanism. The only difference refers to investment deflator. In the case of disturbance in technology, responses of the deflator are the same as the responses of other prices. In the case of shock in investments — based on disturbance definition — we observe in its wake a drop in investment prices, as faster growth of volume reduces price dynamics.

The consequences of consumption preference shock and labour supply (leisure preference) shock are illustrated in Figures 7.6 and 7.7. Momentary increase in interest in consumption means simultaneous decrease of the relative importance of the other sources of utility specified in the function maximised by households. An immediate effect of the shock is an increase in consumption and a drop in investments. A relative decrease in the importance of leisure leads to a rise in employment. A combined effect results in an increase in output, employment and domestic prices, and in real wages decrease. Due to the fact that consumption goods include imported components, and domestic components become more expensive, import grows (despite a drop in investments). At the same time — as a result of the increase in prices of domestic products — the competitiveness of export diminishes, which results in drop of export. With the rise in inflation of the domestically manufactured products, the increase of interest rate becomes inevitable and leads to nominal exchange rate appreciation — the scale of appreciation is high enough to translate into the appreciation of the real exchange rate. Changes in exchange rates increase the depth of response of export and import. Viewing all of the responses to shock in preferences as boiling down to increase in production and prices, the shock may be treated as a demand shock, however, the terms “demand/supply shocks” do not fit the logic of the DSGE models.

Labour supply shock means an increase in the importance of leisure (which automatically lowers labour supply), and thus a decrease in the relative importance of consumption in the households’ utility function. The effect of such impulse is the reduction of consumption, investments, GDP and employment. The drop of labour supply is compensated with increased capital utilisation rate, but due to increase in the price of labour services, the marginal costs of domestic production rise and thus results in an increase in inflation. Increase in domestic prices reduces export competitiveness and stirs import, which rises despite a drop in domestic demand (the price effect proves to be stronger than the income effect). Negative net export along with the increase

[5] More specifically, the rate affects the real rent for capital services present in the marginal cost $\overline{r}_t$.

in domestic prices results in real exchange rate depreciation, but in the face of inevitable interest rate increase the exchange rate ultimately appreciates. The total view boils down to increase in inflation and a drop in the level of economic activity.

Taking into account the specification of the structural form of the model, the wage markup shock (markup in the labour market) fulfils an analogical function as the labour supply shock[6], therefore, the response functions presented in Figure 7.19 replicate Figure 7.7, with accuracy to the response scale. Nevertheless, the sense assigned to the disturbances is completely different. In the case of wage markup disturbance, we speak about phenomena of institutional nature, regulations changing (for example) the position of trade unions in wage negotiations. The IRF for wage markup shock in both of the regimes explains why the disturbance was so important in the first regime.

A large group of disturbances are markup shocks (for domestically manufactured products, imported consumer goods, investment goods, as well as re-exported products and export products, see Figures 7.8–7.12). Already a superficial analysis of Phillips curve specification for domestic prices shows that the markup has an identical impact on inflation to marginal cost (see stationary disturbance in technology, however, with a reverse sign). In the case of markup on import products, where an analogical situation as for domestic products markup is present, there appear more extensive effects related to substitution of imported components with domestic production, and the resulting changes in the proportion of the components of aggregated demand. And consequently, a shock in the markup on imported consumer goods (7.9) increases the prices of imported components of consumption and automatically the CPI is growing (causing a response of interest rate). Thus, general consumption is reduced, including consumption import. Consequently, in the basket of households there will be more relatively cheaper investment goods but despite the growth of investment import, the import in total decreases, so there appears a positive net export and exchange rate appreciation. The increase in the share of domestic production in consumption means also a growth of the prices of components manufactured domestically, which stirs the growth of inflation. As regards disturbance in markup on investment import, the reactions of observable variables are analogical (see Figure 7.10) — in this case, however, consumption is growing at the cost of investments.

The shock of markup of imported export component (Figure 7.11) results in replacement of import with domestic products, while the appearance of positive net export results in exchange rate appreciation, which immediately impacts export. Growth of demand for domestic products results in the growth of employment and wages, and the suddenness of this reaction results in instantaneous growth of the interest rate (the rate depends on the level of gap and its dynamics). When the effects of appreciation are translated into domestic prices, a similarly sudden interest rate decrease occurs. With a lower interest rate and higher employment (at least at the beginning), consumption and investments grow. The shock of export markup (Figure 7.12) reduces export, which immediately leads to a drop in GDP and employment, despite import reduction. Momentarily there appears a surplus of domestic products and drop in the

[6] In the formal statistical sense, the disturbances are undistinguishable. In a DSGE model we identify them only thanks to the knowledge provided with prior distribution in Bayesian estimation.

prices of products. However, depreciation resulting from the appearance of negative net export makes import more expensive and therefore CPI grows. In the case of drop of inflation for the domestically manufactured products and a drop in GDP, the response of the interest rate is opposite to the CPI movement — the interest rate decreases. This is caused by a large scale of GDP drop (calculated in percentage points), with a relatively low increase in CPI inflation (a tenth parts of a percentage point).

7.3 Smoothing — estimation of structural disturbances

Figures 7.20–7.21 present historical decompositions of most important structural disturbances (and the innovations respective for the disturbances) present in the $\mathbb{SOE}^{\mathrm{PL}-2009}$ model. According to the earlier suggestions, in DSGE models the disturbances are the most important factor "driving" the fluctuations of variables. An economy in long-term equilibrium is knocked out of the equilibrium by the appearance of subsequent shocks, while the decisions made by the agents (at macro scale) cause the absorption of the disturbances. Was it not for the disturbances, the economy would develop in compliance with the characteristics of the steady state. In the world of DSGE models, the very (structural) disturbances are a type of ultimate cause of the phenomena and events that we may observe in the economy. Therefore, the question about the sources of observable events shall be answered by searching for disturbances (innovations) that have occurred. The structural disturbances are, however, a set of artefacts that we construct or quantify with the use of a specific DSGE model. Two different DSGE models of the same object in the same period, usually differently recreate the disturbances that formally fulfil the same function in the models — different series are received. Therefore, the analysis of disturbances (or broader — historical decompositions) refers to a specific model (see also Canova and Sala, 2005). Thus, the series presented in the subsequent Figures show an image of a fragment of the history of Polish economy seen from the perspective of the $\mathbb{SOE}^{\mathrm{PL}-2009}$ model. This is an image specific for the version of the model[7].

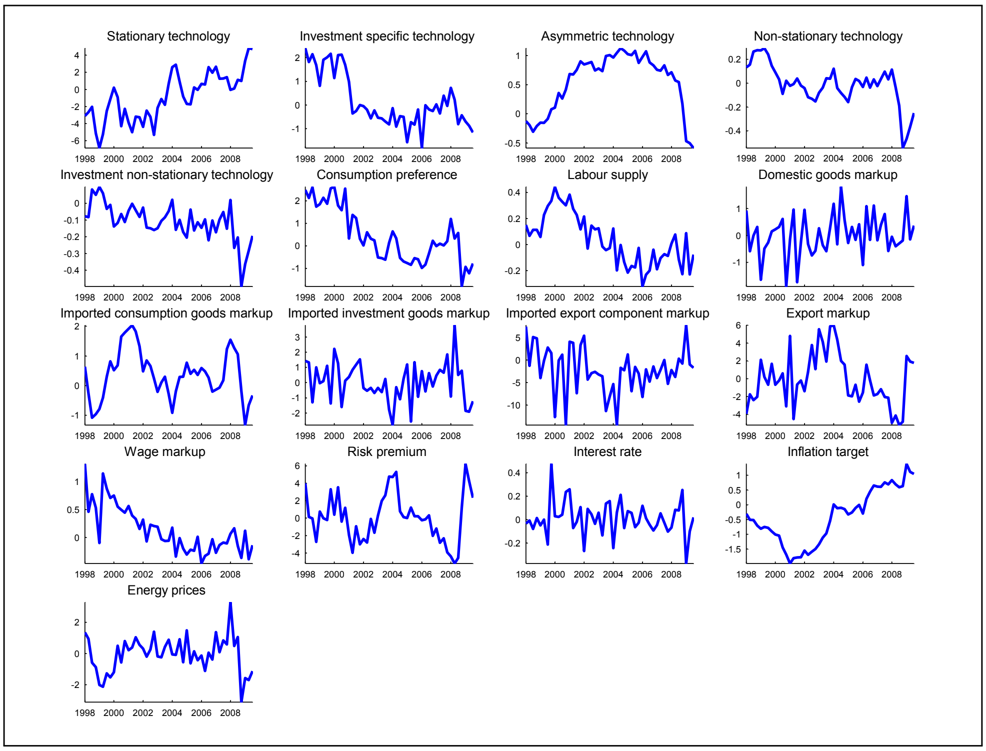
In the $\mathbb{SOE}^{\mathrm{PL}-2009}$ model, it has been assumed that structural disturbances are generated by a stochastic process of AR(1) class and only in a few cases the autocorrelation coefficient takes zero values (e.g. for interest rate disturbance) — then the graphs of disturbances and innovations are identical. Different from zero autocorrelation coefficients make nearly all of the disturbances presented in Figure 7.20 reflect a relatively high inertia[8]. The largest inertia is reflected by the inflation target disturbance and asymmetric disturbance in technology.

Due to the fact that the "natural" value of disturbances is zero, stronger positive/negative deviations will be eliminated in a shorter or longer period of time. Therefore, on the basis of the current estimations of disturbances and the knowledge about the way observable variables responds, speculations may be formulated regarding the paths of observable variables. Such is the logic of the forecasting technique with the use of a DSGE model.

[7]From the technical side, the procedure of determination/estimation of non-observable structural disturbances based on the model having the form of state space model is called smoothing, see Hamilton (1994, Chapter 13).

[8]It is worth remembering that some of the autocorrelation coefficients of disturbances have been calibrated.

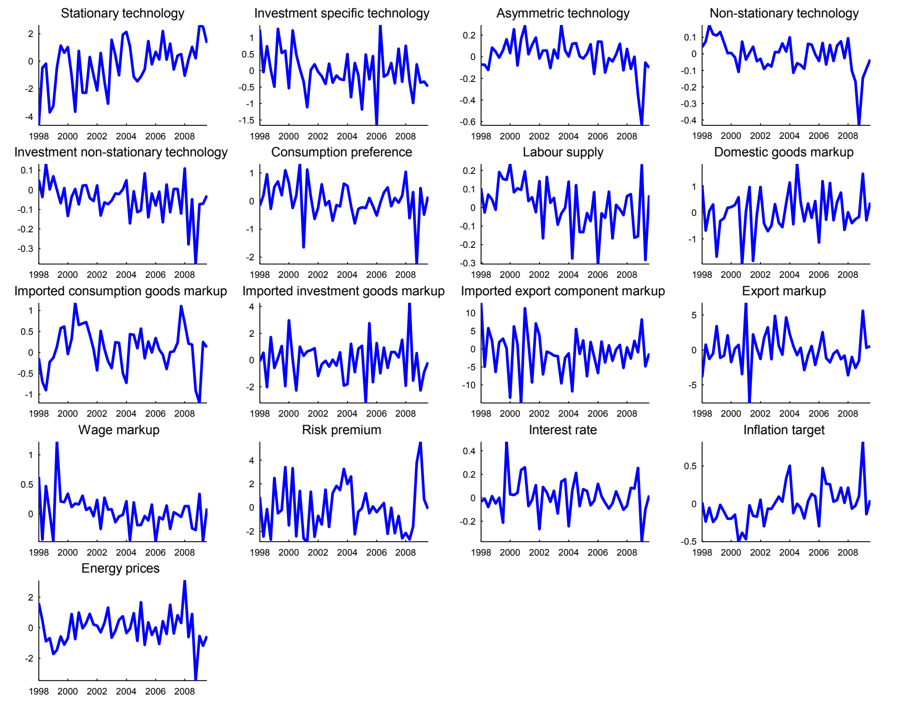
Figure 7.20. Estimates of structural disturbances



Trying to interpret some of the paths of disturbances presented in the graphs, we see a specific role of non-stationary disturbances in technology. In the previous versions of the $\mathbb{SOE}^{PL}$ model there was only one non-stationary disturbance in technology present. Its graph represented approximately the world's business cycle. Currently, the effect is broken down into two disturbances and is no longer so visible, particularly in the view of the fret that deep (negative) innovations appearing at the end of the sample (related to the crisis in financial markets) additionally obscure the proportions (scale). From the beginning of the 21st century, non-stationary disturbance in investments is negative, yet, there appear positive innovations of the stationary disturbance in investments. Thus, the increase of investment dynamics observed in the years 2005–2007 was — accordingly to the logic of the model — specific for the Polish economy and short-lasting. A similar conclusion may be formulated when viewing the stationary disturbance in technology (of the TFP type), which has been larger than zero since 2004 (with short episode of negative innovation around the year 2005), while strong positive innovations compensate the negative shocks resulting from trends common for the whole global economy shaped by non-stationary disturbances.

The graph, or even mid-term trends that may be observed for the disturbance in the inflation target, suggests that after a period of relative stabilisation in the vicinity of steady state in the years 2003–2005, there appeared a trend to increase the target, i.e. the agents index the prices of their products based on increasingly growing rates of (future) inflation and think

Figure 7.21. Estimates of structural innovations



that the central bank also adjusts the interest rate by accepting higher inflation[9]. A relatively high positive value of the disturbance implies, however, a change in the trend. In the graph of innovation, the inflation episode related to the accession to the European Union is clearly visible. The fluctuations from 2000–2001 and 2006 are more difficult to interpret.

Disturbance in foreign exchange risk premium approximately coincides with intuition. Larger innovations in risk premium are related, among others, to the financial crisis of 2008–2009, the period of zloty liberation (1999–2000) and accession to the European Union. In the graph of the premium disturbance a phase of strong (speculative?) appreciation in the years of 2006–2008 is visible, with strong depreciation later on.

Interest rate disturbance (originally identical with innovations) reflects more sudden deviations around the years 1999–2000, which seems to be natural taking into account the introduction of the floating exchange rate regime and implementation of the strategy of direct inflation targeting. Explicit consideration of a structural change to interest rate rule resulted in a more homogenous graph of the disturbance path — earlier versions of the model identified phases of sudden fluctuations in the years 1997–1999 and relative stabilisation (clear decrease in variance) after the year 2001. The fluctuations at the end of the sample suggest, however, the strongest discretionary negative impulse of interest rate in the last 10 years.

[9] Of course, an opposite conclusion may be formulated for the period before the year 2004.

8
Historical decompositions and forecasts

8.1 Historical decompositions

The set of calculation procedures that has been built for models in the state space representation enables to carry out historical decompositions and, thus, to study the impact of shocks (groups of shocks) on the path of observable variables in the whole historical context of events. The procedures allow making counterfactual scenarios to answer the question of how the paths of variables would have been if some of the shocks had not occurred. Therefore, we assess the influence and role of the given disturbances for the actual series of events. We are not referring here a theoretical or potential impact of disturbances on the observable variables of the model, as in the case of variance decompositions or impulse response functions, but about the actual influence on historical events. Variance decompositions and impulse response functions are determined by the assumption that economy is in steady state, while the studied disturbance innovation (shock) appears in isolation and its effects are not distorted by other shocks. Historical decompositions are made in consideration of possible deviations from the steady state of all of the variables, as well as overlapping/neutralisation of the effects of other shocks. Using such tools, we may, thus, make an attempt to identify disturbances that had the largest importance for the given variables or/and historical episodes. As a presentation of the analytical capacity of the tools, we show an exercise regarding the search for key disturbances in the graphs of the particular observable variables (the analysis is limited to the period post 2004).

Figures 8.1–8.2 present the paths of observable variables, their historical graphs (green line) and the graph of their hypothetical development (blue line). In the first case we take into account only the disturbance in consumption preference and in the second case — all of the disturbances are present, except the disturbance in consumption preference. In each case we receive paths of variables resulting from the deviation of economy from the steady state, the considered disturbances and the relationships between the endogenous variables of the model. The aforementioned paths show, therefore, that in the studied period consumption was a category dependent practically on the exogenous shock of preference only, the role of fiscal or technological shocks or work supply preference mentioned as important in variance decompositions (particularly in a longer horizon, see Chapter 7.1) is hard to perceive. Carrying out analogical study we shall notice that the path of investment expenditures is determined by a stationary technological disturbance in investments. It is difficult to observe the role of non-stationary shocks or preferences. In the case of export a single disturbance cannot be found to explain the dominating/larger part of the variance of the variable (enabling the recreation of its historical path). It has been determined by experiment that for the recreation of the graph of export in the studied fragment of the sample, external disturbances included in the SVAR model are necessary (except interest rates), as well as disturbances in export markup and markup on the import of export components. Key factors of import proved to be the disturbances in markup on imported goods (consumer and investment goods, and export components), as well as export markups.

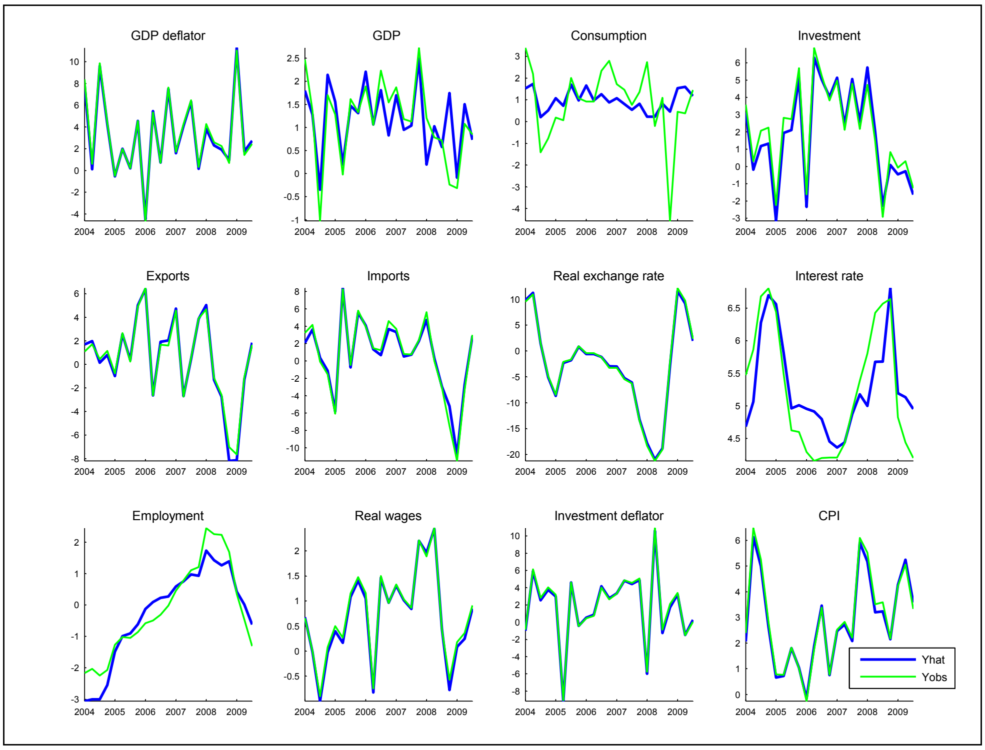
The method proposed above enables to state that GDP deflator fluctuations result in the studied period from the realisation of the disturbance in inflation target and markup on domestically manufactured products. On the other hand, the path of the investment deflator may be recreated with the use of disturbance in the markup on imported investment goods and stationary technological disturbance in investments. Both for the GDP deflator and the investment deflator in the last 2–3 years, the exchange rate risk premium is important as well. For the recreation (arrival at a relatively correct approximation) of the path of real wages in the years 2004–2009, four disturbances are necessary: wage markup, labour supply preferences, inflation target and markup on domestically manufactured products.

As shown in Figures 8.3 and 8.4, the historical graph of the real dollar/zloty exchange rate may be recreated with the use of risk premium disturbance and three components of the SVAR model: change in the nominal dollar/euro rate and inflation in the USA and the euro area (the real cross rate). For the other observable variables (GDP, CPI, domestic interest rate, employment) it is very difficult to determine a narrow set of shocks to allow the recreation of the paths in the tested sample.

8.2 Forecasting technique

The $\mathbb{SOE}^{PL-2009}$ model is an example of a linear DSGE model formulated in a state space representation. For that class of models the initial stage of making a forecast is the identification of the value of the vector of state variables (with the use of Kalman filter) at the starting point of the forecast, i.e. determination in what state, from the perspective of the model, the economy is. Having the values for the vector of state variables at the starting point, we are able to make a conditional or unconditional forecast, point forecast or stochastic forecast. Due to the fact that all of the exogenous variables of the model (disturbances) are described with known stochastic

Figure 8.1. Historical decomposition; excluded consumption preference shock

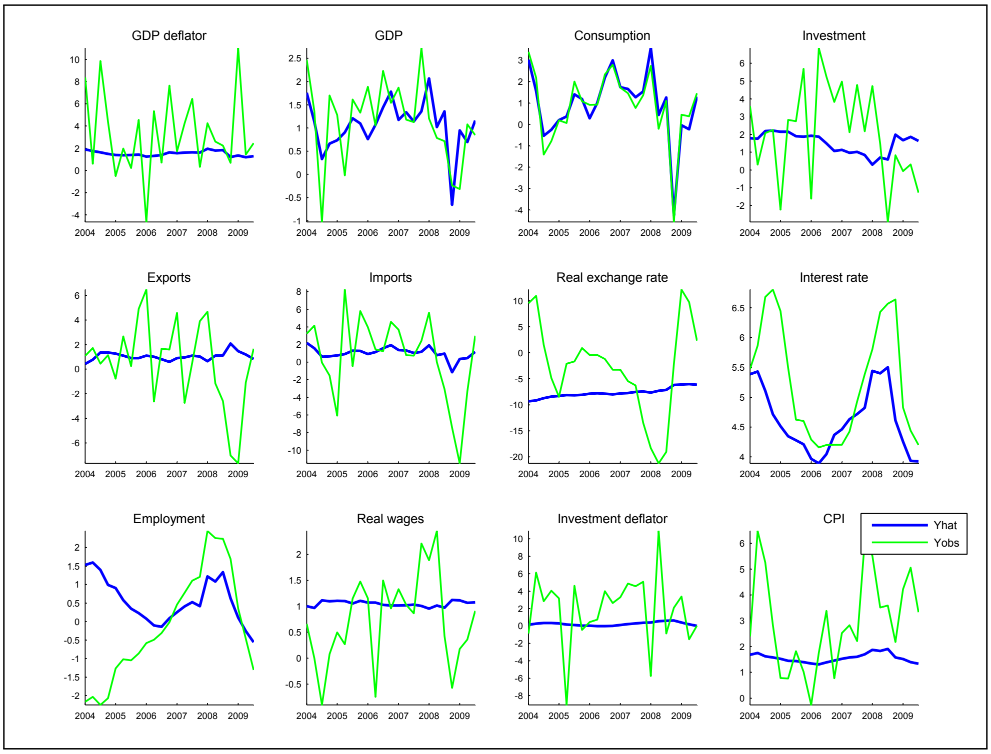


processes, no external assumptions regarding exogenous variables are necessary. Unconditional forecast, assuming that in the future no new shocks shall occur, means that all of the disturbances identified at the starting point shall expire and upon an adequately long period of time — due to the structure of the model — the variables shall come back to steady state. This is illustrated e.g. in Appendix C. In the simplest case, such prepared forecast may be point forecast, yet, a more natural environment for the estimated DSGE models (including $\mathbb{SOE}^{\mathrm{PL}-2009}$) is the world taking into account the existence of uncertainties. In our case it is possible to quantify the risk related to:

- uncertainty of parameters — when (full) Bayesian estimation is made, we have the information about whole distributions, and in the case of classical estimation, the standard errors may be used;
- state uncertainty — considering the uncertainty related to the forecast of state variables;
- uncertainty of future shocks — information about the variances of shocks enables the generation of uncertainty related to the realisation of shocks in the future;
- measurement errors — forecasts of observable variables are additionally burdened with uncertainty related to the measurement (observation) of observable variables.

The above uncertainties are taken into account by conducting stochastic simulations where we draw parameters (from the posterior distribution), state variables at the starting point, a sequence of structural shocks and a sequence of measurement errors. We then end up with the estimation of the predictive distribution which can be depicted in the form of a fan chart or used

Figure 8.2. Historical decomposition; included only consumption preference shock



to calculate probabilities of reaching certain levels by certain variables.

Although the list does not account for all of the potentially important sources of uncertainties (e.g. broader understood uncertainty of the model, data, uncertainty according to F. Knight, see e.g. Kłos (2004)), the very fact of the possibility of (selective) quantification of major risk factors, determination of e.g. confidence intervals, shows potential of such class of DSGE models.

8

With the use of the $\mathbb{SOE}^{\text{PL}-2009}$ model there may also be created conditional forecasts (scenarios), i.e. with the assumed path of one or more observable variables[1]. Due to the fact that the principle that shocks explain the behaviour of variables is valid all the time, the user must indicate what disturbances are to guarantee the fulfilment of the assumed path, which enables explicit determination of the values of shocks[2]. A modest intervention test (modesty metric) by Leeper and Zha (2003), see also Adolfson et al. (2005a), allows to verify whether the values of such shocks are not too high. High values of shocks lower the likelihood of conditional forecast, as in reality they could be treated by economic agents as a change in the prevailing economic regime or might change the agent's behaviour (according to Lucas critique).

[1] Such forecasts are known in literature as conditional forecasts by Waggoner-Zha or Leeper-Zha, see e.g. Waggoner and Zha (1999).

[2] When preparing a scenario, it is possible to start with the shocks, i.e. declare specific values of the shocks as events whose effects we study. Although disturbances have economic interpretation, which may help us in determining the correct value of innovation, the most frequent method of conduct requires monitoring of the response of observable data, as to which we have intuition and knowledge, and only that gives us basis for final determination of the value of the shocks.

Usually, in the above technique of constructing conditional forecasts, unanticipated shocks are used. In certain situations, the conditional paths may be generally known (e.g. expected tax changes) and may not/should not be treated as unexpected events. Then, it is possible to use the anticipated shocks that shall enable agents to react to the very information about a future event and the realisation of a shock in future shall not be a surprise.

A special case of conditional forecasts, interesting from the point of view of the central bank, are forecasts using the assumptions with regard to interest rate. According to the historical tradition, forecasts — or rather projections — are (often) created at central banks with the assumption of exogenous interest rate (constant or implied by market expectations). In the world of rational agents (forward-looking and optimising) it is hard, however, to answer the question what would their responses (decisions) be, when the central bank stopped to use the rule known to the agents for several or a dozen or so quarters, i.e. when it stopped to modify the interest rate accordingly to the occurring events. A break in the functioning of a central bank lasting many quarters does not comply with the standards, so the knowledge that the central bank shall start to act on a "non-standard" basis shall force the agents to "non-standard" behaviours. If — by force of assumption — we exclude the possibility of foreseeing by the agents the "non-standard" approach of the central bank, we arrive at a contradiction with the initial assumption about the rationality of the agents. In the first case we receive "irrational" forecasts of variables (the effect of assumption of "non-standard" policy of the bank — the problem has been discussed by e.g. Laséen and Svensson (2009)). In the second case, the possible projections shall be internally

Figure 8.3. Historical decomposition; excluded risk premium and real dollar/euro rate shocks

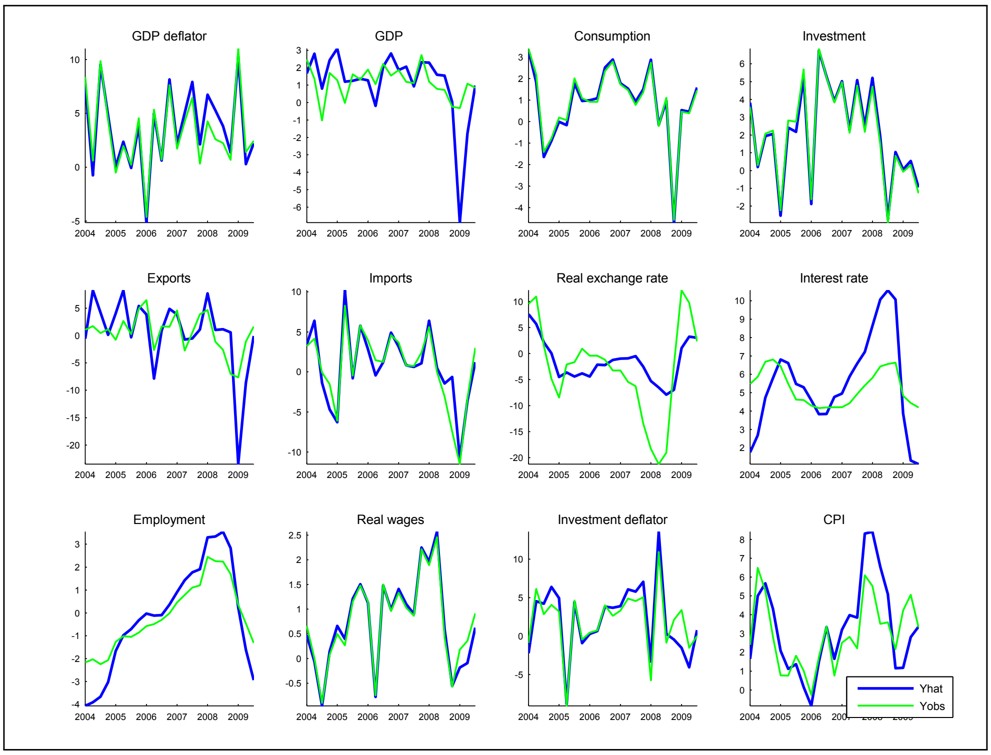
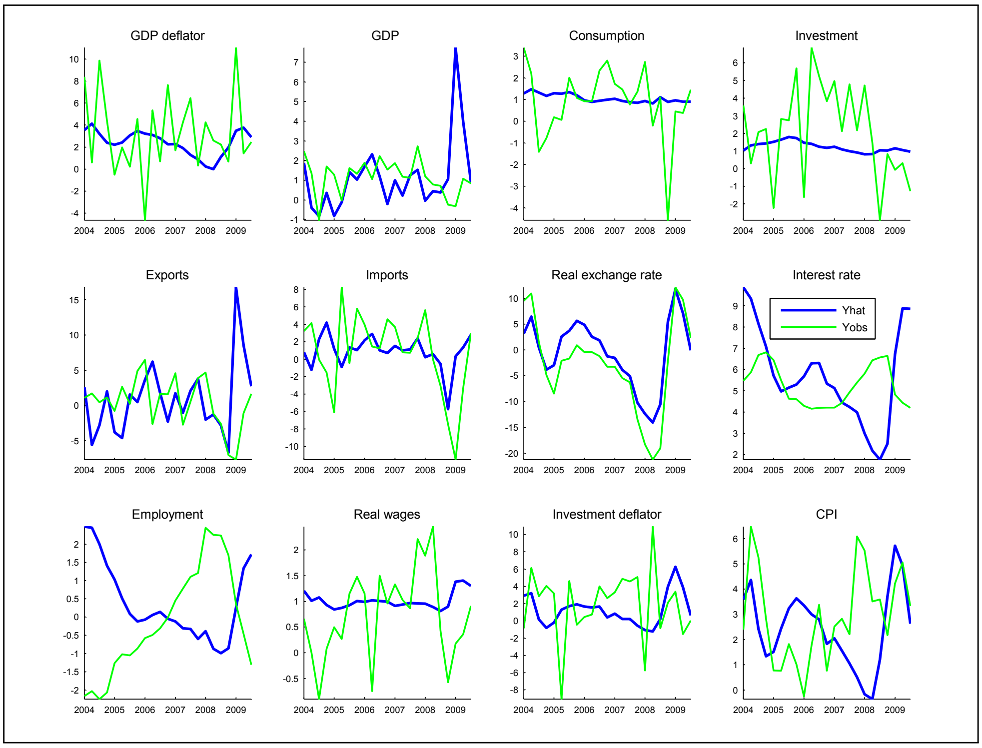


Figure 8.4. Historical decomposition; included only risk premium and real dollar/euro rate shock



inconsistent and shall be subject to Lucas critique. In each case the projection will not provide a reliable, methodologically correct answer to the asked question. A solution of the problem is the preparation of forecasts with the assumption of endogenous interest rate.

8

8.3 Ex-post forecasting accuracy of the $\mathbb{SOE}^{\text{PL}-2009}$ model

Using the observations of observable data Y_t for $t = 0, 1, 2, ..., T$, the quality of forecasts of the model in a sample may be verified by comparison of the theoretical values of the observable data identified as $\hat{Y}_t$, with their real values in the sample Y_t for $t = 1, 2, ..., T-1$. The observable variables are forecasted for $h = 1, 2, ..., H$ periods ahead, where H is the maximum forecast horizon. The forecast of observable variables for period $t+h$ made in period t is denoted as $\hat{Y}_{t+h}$, and the actual value of the observable variables in period $t+h$ as Y_{t+h}. The difference:

$$\gamma_{t+h} = Y_{t+h} - \hat{Y}_{t+h}$$

is called a forecast error in period t for the h forecast horizon. Going through $t = 0, 1, 2, ..., T-h$, we may determine various statistics of forecast errors γ_{t+h} depending on the determined forecast horizon $h = 1, 2, ..., H$. Based thereon, the measures of the forecasting quality of the model are constructed. The analysis will cover one-dimensional and multi-dimensional measures (joint measures).

In the exercises presented below, we assume the maximum forecast horizon of $H = 12$ quarters, and the sample covers the period of 2004:4–2009:3, i.e. the period after Poland's accession to the European Union. Thus, the test covers the most turbulent period of financial crisis (2008:2–2009:3), which may hardly be deemed typical or representative.

8.3.1 One-dimensional measures of forecasting quality

From the group of one-dimensional forecasting quality measures we have used two: root mean square error (RMSE) and mean percentage error (MPE).

The root mean square error for the forecast horizon $h = 1, 2, ..., H$ for the i^{th} observable variable is calculated as:

$$RMSE = \sqrt{\frac{\sum_{t=0}^{T-h}(\gamma_{t+h}(i))^2}{T-h+1}},$$

where $\gamma_{t+h}(i)$ is the i^{th} element of the vector of γ_{t+h}. RMSE tells about how much the $\hat{Y}_{t+h}$ forecast is different on the average from the actual value of the observable variable Y_t. The mean percentage error for the forecast horizon $h = 1, 2, ..., H$ for the i^{th} observable variable is calculated as:

$$MPE = \frac{\sum_{t=0}^{T-h}\frac{\gamma_{t+h}(i)}{Y_{t+h}}}{T-h+1}.$$

The value tells by what percentage the $\hat{Y}_{t+h}$ forecast is different on the average from the real value of the observable variable Y_t. The basic function of this measure is the reflection of the possible bias. If the actual value of the forecast variable takes values close to zero, the relation $\frac{\gamma_{t+h}(i)}{Y_{t+h}}$ becomes very large even in the case of moderate $\gamma_{t+h}(i)$.

The analysis of RMSE shows several systematic features of the forecasting properties of the DSGE $\mathbb{SOE}^{\text{PL}-2009}$ model. Firstly, the inflation forecast errors (domestic inflation, consumption goods, investment goods and CPI) increase to the horizon of about 3–4 quarters and then fall. This means that least susceptible to errors is the forecast of inflation for 1–2 quarters, and then around the horizon of 7–11 quarters, depending on the type of inflation. The inflation in the perspective of 3–5 quarters is associated with the largest error. The forecasting accuracy of the model with respect to forecast horizon is different for other variables where the RMSE increase along the horizon. Secondly, beside single cases, the model's forecasts always win with the naive forecasts. The advantage of a DSGE model over naive forecasts increases along the forecast horizon. Thirdly, the inspection of MPE errors shows that the forecasts of most of the variables are biased, which means that they are systematically underestimated or overestimated, regardless of the forecast horizon. The conclusions have been formulated based on the data that are analysed in details in Appendix D.

8.3.2 Multidimensional measures of forecasting accuracy

Multidimensional analysis of forecast errors of the model[3] is based on the covariance matrix of errors Ω_h, which is determined based on the residuals vector $\gamma_{t+h} = Y_{t+h} = \hat{Y}_{t+h}$ for the forecast horizon $h = 1, 2, ..., H$:

$$\Omega_h = \frac{\sum_{t=1}^{T-H} \gamma_{t+h} \Phi \gamma_{t+h}^T}{T-h},$$

where Ψ is the scaling matrix. In the calculations it has been assumed that Ψ is a diagonal matrix with variances of the subsequent forecasts errors on the main diagonal. For matrix Ω_h its eigenvalues λ_i^h have been determined, as well as the respective v_i^h eigenvectors. The determination of the eigenvectors for matrix Ω_h is equivalent to the determination of the so called principal components of γ_{t+h} forecasts for $t = 0, 1, 2, ..., T - H$. Principal components are orthogonal directions along which the specified value of forecast error variations (variances) materializes. Each eigenvector of matrix Ω_h determines one such direction. The larger the eigenvalue related to the eigenvector, the larger the variability of errors in the corresponding direction. Should we select eigenvectors or directions — principal components to which the largest eigenvalues correspond, and check which variables contribute to variability of these directions to the largest extent, it shall prove which variables generate the largest errors in the forecasting process. Thus, it is easy to quantify contribution of subsequent variables to the variance along any given direction, since these contributions are proportional to squares of their coordinates in the eigenvector defining that direction. Such analysis may be made for every forecast horizon $h = 1, 2, ..., H$, which gives a clear picture of forecast errors depending on the forecast horizon. For the presentation purposes, we provide results for horizons $h = 1, 4, 8, 12$ and for each horizon for the directions related to the four largest eigenvalues.

Tables 8.1–8.4 present how the share of variables generating the largest forecast errors in the total variance of the errors change depending on the forecast horizon. Variables of at least 10% contributions were considered. Table 8.1 shows that the first — most important — of the identified directions, realizes mainly the errors of export, import and real exchange rate of the dollar. The contribution of each of the variables is significant, regardless of the forecast horizon, however, the share of exchange rate diminishes for the horizon of over 9–10 quarters.

Table 8.1. The first direction with the largest variance of forecast errors

Variable	h=1	h=4	h=8	h=12
Export	0.26	0.27	0.20	0.37
Import	0.40	0.33	0.33	0.25
Real USD/PLN exchange rate	0.20	0.30	0.40	0.27

Note: The eigenvalues for the first direction amount to 72, 478, 407 and 583 for the forecast horizons of 1, 4, 8 and 12, respectively.

Table 8.2 shows that in the second of the verified directions the largest share in the forecast errors has the real exchange rate of the dollar, with the share for the horizons over 8 quarters,

[3]An interesting review of the techniques of DSGE models forecast errors analysis is presented by Adolfson et al. (2005c).

Table 8.2. The second direction with the largest variance of forecast errors

Variable	h=1	h=4	h=8	h=12
Investments	0.05	0.07	0.04	0.13
Export	0.04	0.04	0.16	0.07
Import	0.08	0.16	0.20	0.24
Real USD/PLN exchange rate	0.68	0.62	0.41	0.25
Nominal USD/EUR exchange rate	0.04	0.00	0.11	0.16

Note: The eigenvalues for the second direction amount to 26, 110, 104 and 188 for the forecast horizons of 1, 4, 8 and 12, respectively.

amounting even to 80%. For the horizon between 1 and 6 quarters, a major share falls to export and between 7 and 10 quarters — import. For short horizon a contribution that may not be neglected is brought by import and EUR/USD exchange rate, while the share of investments is moderate, yet, realise for most of the horizons.

Table 8.3. The third direction with the largest variance of forecast errors

Variable	h=1	h=4	h=8	h=12
Investments	0.00	0.01	0.26	0.45
Export	0.00	0.00	0.01	0.14
Government expenditers	0.19	0.23	0.13	0.00
Nominal USD/EUR exchange rate	0.62	0.65	0.35	0.23

Note: The eigenvalues for the third direction amount to 11, 25, 32 and 45 for the forecast horizons of 1, 4, 8 and 12, respectively.

As it may be seen from Table 8.3, in the third of the identified directions, in the case of short horizons of 1–4 quarters, the dominating contribution to forecast errors is made by investments, while for horizons longer than 6 quarters — government expenditures and, mainly, the nominal USD/EUR exchange rate.

In the fourth of the identified directions — see Table 8.4 — for horizons up to 5–6 quarters, the largest share in forecast errors falls to investments and nominal USD/EUR exchange rate; for horizons of 6–8 quarters — export and nominal USD/EUR exchange rate, while for the 12 quarters horizon the forecasts are dominated by investments.

Table 8.4. The fourth direction with the largest variance of forecast errors

Variable	h=1	h=4	h=8	h=12
Inflation in investments.	0.10	0.46	0.57	0.1
Investments	0.52	0.46	0.57	0.1
Government expenditures	0.05	0.00	0.10	0.21
Real USD/PLN exchange rate	0.28	0.00	0.02	0.02
Nominal USD/EUR exchange rate	0.13	0.20	0.00	0.00

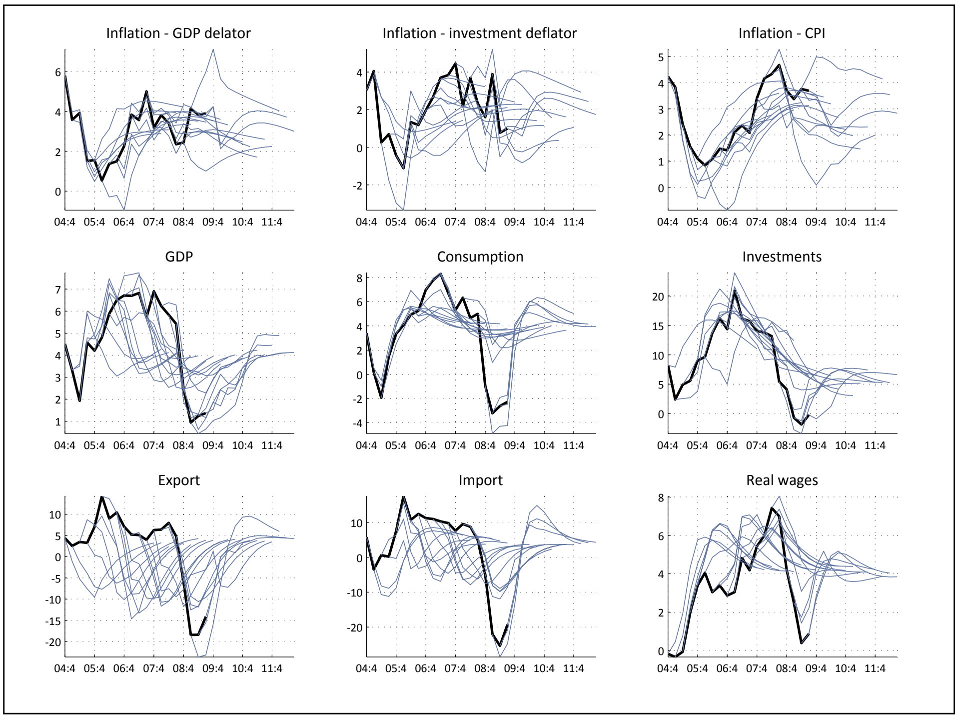
Note: The eigenvalues for the fourth direction amount to 5, 20, 10 and 7 for the forecast horizons of 1, 4, 8 and 12, respectively.

8.3.3 Rolling forecasts

Figures 8.5–8.6 present the historical graphs of observable variables (thick line) and a series of forecasts for 12 quarters horizon (made in compliance with the description provided at the beginning of paragraph 8.3). In the graphs annual dynamics of variables has been presented[4]. Using this illustration, the characteristics of forecasting errors of the $\mathbb{SOE}^{PL2009}$ model, given by formal errors measures, may be supplemented with the illustration of the ability of the model to forecast mid-term trends, turning points or sensitivity of forecasts to new information.

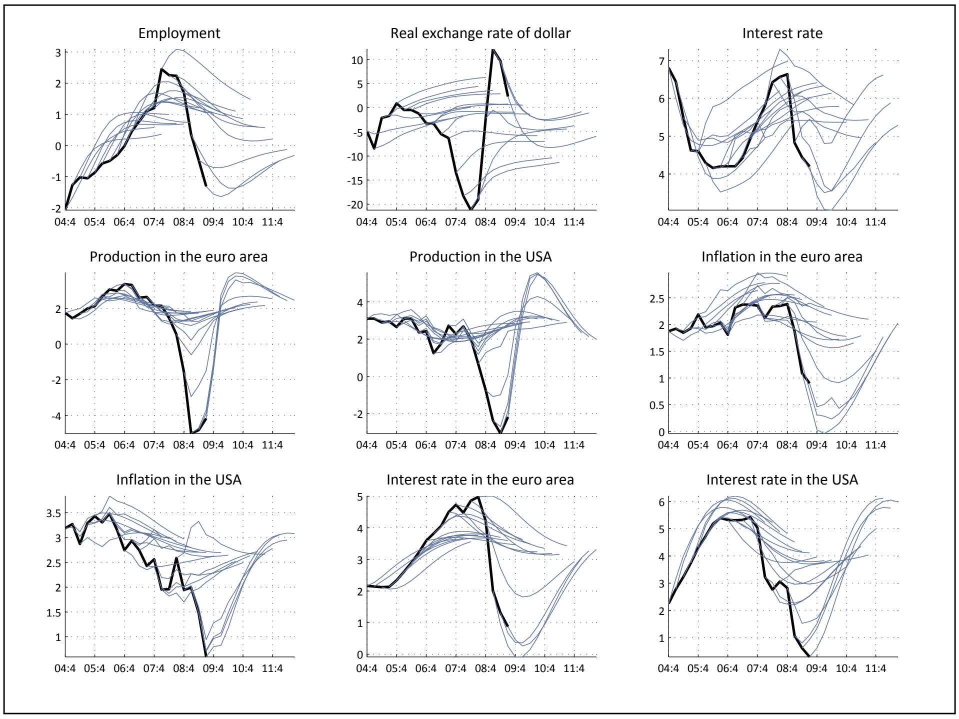
The first observation is that the inaccuracies of forecasts resulting from the earlier presented, one-dimensional measures seem to be slightly exaggerated. In the case of many variables (e.g. consumption, investments), relatively large root mean square errors result from single observations (individual large errors during the financial crisis). Contrary to the above presented measures, consumption and inflation (CPI, GDP deflator) or even investments seem to be relatively well forecasted. An exception is the period when the economy was exposed to the shocks of the global financial crisis. When such sudden shocks occur, subsequent forecasts (for example) become instable (subsequent forecasts differ widely). Tendency of the model to underestimate the dynamics of export and import is confirmed (the effects of which are reflected also in the GDP forecasts), as well as small explanatory power of the model in the case of the exchange rate. It should be emphasized however, that the forecasts of several turning points of investments, GDP and inflation are accurate.

Figure 8.5. *Ex post* forecasts for the selected variables of the model (part 1)



[4]The exception is employment, which has the form of deviations (absolute values) from the HP trend, and real exchange rate of dollar expressed in the form of deviations from the linear trend.

Figure 8.6. *Ex post* forecasts for the selected variables of the model (part 2)



Final comments

The presented material documents the results of works over a version of an estimated dynamic stochastic general equilibrium model $\mathbb{SOE}^{\mathrm{PL-2009}}$, which in 2010 will be used for building mid-term forecasts and projections of inflation processes and business cycle in Poland, i.e. shall support and supplement the forecasting materials prepared based on the traditional macroeconometric model and the opinions of experts at the National Bank of Poland.

DSGE models are one of the most important tools of theoretical analyses of modern macroeconomics. New theoretical concepts are developed based on DSGE models or the environment of economy in dynamic general equilibrium. Being an effective tool of theoretical research, DSGE models are also becoming useful tools for empirical research. More complex estimation of parameters, particularly with the use of the ideas of Bayesian econometrics, enables matching models motivated with economic theory to data — referring to the real existing economies, testing the actual episodes, analysing the reasons of observable events. All of that from the point of view of explicitly declared economic paradigm (model specification). Strict theoretical grounds and explicitly declared paradigm are features that simplify the interpretation of results but — as it seemed initially — lower the forecasting potential of models. When we were interested in the accuracy of forecasts, eclectic models in which more or less random correlations of data enable the reduction of errors dominated the models with clearer (more explicit) economic contents. Therefore forecasting experiments with Bayesian estimated DSGE models carried out by F. Smets and R. Wouters (2004) enjoyed high interest of the institutions managing the macroeconomic policy. The experiments showed that an estimated DSGE model has not only analytical but also forecasting potential. The conclusions were confirmed also for other estimated DSGE models of developed and stable market economies (see e.g. Adolfson et al., 2005c; Christoffel et al., 2007b).

Forecasting of inflation or business cycles for stable developed market economies is different from the forecasting of the responses of agents subjected for a dozen or so years to institutional changes, societies that attempt building market institutions from scratch and shape the macro-and microeconomic rationality of firms, households, social organisations and state institutions. Unchanging interrelationships assumed within the DSGE model (deep parameters defining the structure of the model of economy) are more difficult to identify, while the general uncertainty is much higher. Therefore the task of building a DSGE model for the Polish economy enabling the construction of mid-term forecasts and projections proved to be a harder task than the construction of a analogical model e.g. for the euro area countries. The experiments with the family of $\mathbb{SOE}^{\mathrm{PL}}$ models enable, however, to formulate a cautious conclusion that also for the group of economies in which Poland is included, a DSGE model of significant analytical and forecasting potential may be developed[5]. Using the word "potential" of the model we wish to emphasise that although the progress achieved in reference to the earlier versions of the $\mathbb{SOE}^{\mathrm{PL}}$ family of models is significant, the current features of the $\mathbb{SOE}^{\mathrm{PL-2009}}$ model, including the forecasting characteristics, require further work, while the development potential of DSGE models guarantees the effectiveness of such projects.

[5]This is confirmed by the experience of the analysts dealing with DSGE models at the central banks of Hungary and the Czech Republic (see Benesz et al., 2005; Andrle et al., 2009; Jakab and Világi, 2008).

Part IV

Appendix

Appendix A
List of equations, list of variables

Forms of the model

The structural form of the model can be written as:

$$\begin{cases} \mathbb{E}_t \left\{ \alpha_0 \, \tilde{\mathbf{z}}_{t+1} + \alpha_1 \, \tilde{\mathbf{z}}_t + \alpha_2 \, \tilde{\mathbf{z}}_{t-1} + \beta_0 \, \boldsymbol{\theta}_{t+1} + \beta_1 \, \boldsymbol{\theta}_t \right\} = 0 \\ \boldsymbol{\theta}_t = \rho \, \boldsymbol{\theta}_{t-1} + \boldsymbol{\varepsilon}_t , \end{cases} \tag{A.1}$$

where:

$$\tilde{z}'_{t\,[1x28]} = \begin{bmatrix} & \overset{1}{\widehat{\pi}^d_t} & \overset{2}{\widehat{\pi}^{mc}_t} & \overset{3}{\widehat{\pi}^{mi}_t} & \overset{4}{\widehat{\pi}^{mx}_t} & \overset{5}{\widehat{\pi}^x_t} & \overset{6}{\widehat{\widehat{w}}_t} & \overset{7}{\widehat{H}_t} & \overset{8}{\widehat{c}_t} & \overset{9}{\widehat{i}_t} & \overset{10}{\widehat{\psi}_{z,t}} & \cdots \\ \cdots & \overset{11}{\widehat{p}_{k',t}} & \overset{12}{\Delta\widehat{S}^u_t} & \overset{13}{\widehat{y}_t} & \overset{14}{\widehat{\widehat{k}}_{t+1}} & \overset{15}{\widehat{u}_t} & \overset{16}{\widehat{q}_t} & \overset{17}{\widehat{m}_{t+1}} & \overset{18}{\widehat{\mu}_t} & \overset{19}{\widehat{a}_t} & \overset{20}{\widehat{a}^e_t} & \cdots \\ \cdots & \overset{21}{\widehat{\gamma}^{mcd}_t} & \overset{22}{\widehat{\gamma}^{mid}_t} & \overset{23}{\widehat{\gamma}^{mxd}_t} & \overset{24}{\widehat{\gamma}^{x,u}_t} & \overset{25}{\widehat{x}^u_t} & \overset{26}{\widehat{x}^x_t} & \overset{27}{\widehat{R}_t} & \overset{28}{\widehat{E}_t} & & & \end{bmatrix}, \tag{A.2}$$

$$\theta'_{t\;[1\times 48]} = \left[\theta^{s\prime}_{t\;[1\times 26]},\; \theta^{\tau\prime}_{t\;[1\times 8]},\; \theta^{\star\prime}_{t\;[1\times 14]} \right], \tag{A.3}$$

$$\theta^{s\prime}_t = \begin{bmatrix} & \overset{1}{\widehat{\epsilon}_t} & \overset{2}{\widehat{\Upsilon}_t} & \overset{3}{\widehat{\tilde{z}}^\star_t} & \overset{4}{\widehat{\mu}_{z,t}} & \overset{5}{\widehat{\mu}_{\Psi,t}} & \overset{6}{\widehat{\zeta}^c_t} & \overset{7}{\widehat{\zeta}^h_t} & \overset{8}{\widehat{\zeta}^q_t} & \overset{9}{\widehat{\lambda}^d_t} & \overset{10}{\widehat{\lambda}^{mc}_t} & \cdots \\ \cdots & \overset{11}{\widehat{\lambda}^{mi}_t} & \overset{12}{\widehat{\lambda}^{mx}_t} & \overset{13}{\widehat{\lambda}^x_t} & \overset{14}{\widehat{\lambda}^w_t} & \overset{15}{\widehat{\tilde{\phi}}^e_t} & \overset{16}{\widehat{\tilde{\phi}}^e_{t-1}} & \overset{17}{\widehat{\tilde{\phi}}^u_t} & \overset{18}{\widehat{\tilde{\phi}}^u_{t-1}} & \overset{19}{\widehat{\epsilon}_{R,t}} & \overset{20}{\widehat{\widehat{\pi}}^c_t} & \cdots \\ \cdots & \overset{21}{\widehat{\nu}^w_t} & \overset{22}{\widehat{\nu}^k_t} & \overset{23}{\widehat{\tau}^k_t} & \overset{24}{\widehat{\tau}^w_t} & \overset{25}{\widehat{\tau}^s_t} & \overset{26}{\widehat{\pi}^{oil}_t} & & & & & \end{bmatrix},$$

$$\theta^{\tau\prime}_t = \begin{bmatrix} \overset{27}{\widehat{\tau}^p_t} & \overset{28}{\widehat{\tau}^y_t} & \overset{29}{\widehat{\tau}^c_t} & \overset{30}{\widehat{g}_t} & \overset{31}{\widehat{\tau}^p_{t-1}} & \overset{32}{\widehat{\tau}^y_{t-1}} & \overset{33}{\widehat{\tau}^c_{t-1}} & \overset{34}{\widehat{g}_{t-1}} \end{bmatrix},$$

$$
\theta^{\star\prime}_t = \begin{bmatrix} & \overset{35}{\widehat{y}^e_t} & \overset{36}{\widehat{\pi}^e_t} & \overset{37}{\widehat{R}^e_t} & \overset{38}{\widehat{y}^u_t} & \overset{39}{\widehat{\pi}^u_t} & \overset{40}{\widehat{R}^u_t} & \overset{41}{\Delta\widehat{S}^x_t} & \cdots \\ \cdots & \overset{42}{\widehat{y}^e_{t-1}} & \overset{43}{\widehat{\pi}^e_{t-1}} & \overset{44}{\widehat{R}^e_{t-1}} & \overset{45}{\widehat{y}^u_{t-1}} & \overset{46}{\widehat{\pi}^u_{t-1}} & \overset{47}{\widehat{R}^u_{t-1}} & \overset{48}{\widehat{S}^x_{t-1}} & \end{bmatrix}.
$$

Upon the solution of the model with Anderson-Moore algorithm, anticipated variables are eliminated (among others) from the structural form. Thus, we can write:

$$
\begin{cases} \tilde{z}_{t+1} = A\tilde{z}_t + B\,\theta_{t+1} \\ \theta_{t+1} = \rho\,\theta_t + \epsilon_{t+1} \end{cases}
$$

or

$$
\underbrace{\begin{bmatrix} \tilde{z}_{t+1} \\ \theta_{t+1} \end{bmatrix}}_{\xi_{t+1}} = \underbrace{\begin{bmatrix} A & B\rho \\ 0 & \rho \end{bmatrix}}_{F_\xi} \underbrace{\begin{bmatrix} \tilde{z}_t \\ \theta_t \end{bmatrix}}_{\xi_t} + \underbrace{\begin{bmatrix} B \\ I \end{bmatrix} \epsilon_{t+1}}_{\upsilon_{t+1}}.
$$

The state space model has the form of:

$$
\begin{cases} \xi_{t+1} = F_\xi\,\xi_t + \upsilon_{t+1}, & \mathbb{E}(\upsilon_{t+1}\,\upsilon'_{t+1}) = Q, \\ \underline{Y}_t = A'_x\,x_t + H'\,\xi_t + u_t, & \mathbb{E}(u_t\,u'_t) = R. \end{cases} \tag{A.4}
$$

The elements of the matrices of the above formulas are functions of DSGE model parameters.

List of model variables

The list of endogenous variables of the $\mathbb{SOE}^{\text{PL}-2009}$ model, components of vector $\tilde{z}_t$

1. π_t^d — inflation of domestic intermediate products $[=\frac{P_t^d}{P_{t-1}^d}]$
2. π_t^x — inflation of export products $[=\frac{P_t^x}{P_{t-1}^x}]$
3. π_t^{mc} — inflation of imported consumer goods $[=\frac{P_t^{mc}}{P_{t-1}^{mc}}]$
4. π_t^{mi} — inflation of imported investment goods $[=\frac{P_t^{mi}}{P_{t-1}^{mi}}]$
5. π_t^{mx} — inflation of imported export components $[=\frac{P_t^{mx}}{P_{t-1}^{mx}}]$
6. $\overline{w}_t$ — real wages
7. H_t — hours worked
8. c_t — consumption
9. i_t — investments
10. $\psi_{z,t}$ — marginal utility of income
11. $p_{k',t}$ — relative price of fixed assets
12. ΔS_t^u — growth of nominal PLN/USD exchange rate
13. y_t — gross domestic product
14. $\overline{k}_{t+1}$ — fixed assets
15. u_t — fixed assets' utilisation rate
16. q_t — cash
17. m_{t+1} — money
18. μ_t — money growth rate
19. a_t — total net foreign assets
20. a_t^e — net foreign assets denominated in euro
21. γ_t^{mcd} — ratio of the prices of imported consumption goods to domestic prices $[=\frac{P_t^{mc}}{P_t^d}]$
22. γ_t^{mid} — ratio of the prices of imported investment goods to domestic prices $[=\frac{P_t^{mi}}{P_t^d}]$
23. γ_t^{mxd} — ratio of the prices of imported components of export to domestic prices $[=\frac{P_t^{mx}}{P_t^d}]$
24. γ_t^{xu} — ratio of export prices to the prices in the rest of the world $[=\frac{P_t^x}{P_t^u}]$
25. x_t^u — real USD/PLN exchange rate
26. x_t^x — real USD/EUR exchange rate
27. R_t^d — domestic short-term interest rate (gross)
28. E_t — employment

List of exogenous variables of the $\mathbb{SOE}^{\mathrm{PL}-2009}$ model, components of vector $\tilde{\theta}_t$, structural disturbances

1. ϵ_t — stationary disturbance in technology (TFP)
2. Υ_t — stationary disturbance in investments
3. $\tilde{z}_t^\star$ — stationary asymmetric disturbance
4. $\mu_{z,t}$ — non-stationary disturbance in technology
5. $\mu_{\Psi,t}$ — non-stationary technological disturbance specific to investments
6. ζ_t^c — disturbance in consumption preferences
7. ζ_t^h — disturbance in labour preferences (labour supply)
8. ζ_t^q — disturbance in preferences of cash holdings
9. λ_t^d — disturbance in markups on domestic intermediate products
10. λ_t^{mc} — disturbance in markups on imported consumption goods
11. λ_t^{mi} — disturbance in markups on imported investment goods
12. λ_t^{mx} — disturbance in markups on imported export components
13. λ_t^x — disturbance in markups on exported products
14. λ_t^w — disturbance in wage markup
15. $\tilde{\phi}_t^e$ — disturbance in risk premium in the euro market
16. $\tilde{\phi}_{t-1}^e$ — as above
17. $\tilde{\phi}_t^u$ — disturbance in risk premium in the dollar market
18. $\tilde{\phi}_{t-1}^u$ — as above
19. $\epsilon_{R,t}$ — disturbance in interest rate (monetary policy)
20. $\overline{\pi}_t^c$ — disturbance in inflation target
21. ν_t^w — disturbance in demand for working capital loan for financing labour services
22. ν_t^k — disturbance in demand for working capital loan for financing capital services
23. τ_t^k — disturbance in capital tax
24. τ_t^w — disturbance in national insurance contributions paid by the employees
25. τ_t^s — disturbance in national insurance contributions paid by the employers
26. π_t^{oil} — disturbance in raw materials (energy, oil) prices

List of exogenous variables of the $\mathbb{SOE}^{\mathrm{PL}-2009}$ model, components of vector $\tilde{\theta}_t$, observable disturbances

27. τ_t^p — disturbance in effective rate of corporate income tax
28. τ_t^y — disturbance in effective rate of income tax
29. τ_t^c — disturbance in effective rate of consumption tax
30. g_t — government consumption

List of exogenous variables of the $\mathbb{SOE}^{\mathrm{PL}-2009}$ model, components of vector $\tilde{\theta}_t$, structural disturbances of the global economy

35. y_t^e — disturbance in GDP in the euro area
36. π_t^e — disturbance in inflation in the euro area
37. R_t^e — disturbance in short-term interest rate in the euro area
38. y_t^u — disturbance in GDP in the USA
39. π_t^u — disturbance in inflation in the USA
40. R_t^u — disturbance in short-term interest rate of the dollar
41. ΔS_t^x — disturbance in the change of nominal USD/EUR exchange rate

A3. List of equations of the structural form of the model

Below we present the log-linearised equations of the structural form of the model (as given by A.1). Log-linearised variables are identified with hats.

Inflation of domestic prices

$$\begin{aligned}(1+\kappa_d\,\beta\,\mu)\,\widehat{\pi}^d_t &= (1-\kappa_d)\,(1-\beta\,\mu\,\rho_\pi)\,\widehat{\overline{\pi}}^c_t + \beta\,\mu\,\widehat{\pi}^d_{t+1} + \kappa_d\,\widehat{\pi}^d_{t-1}\\ &+ \frac{(1-\xi_d)}{\xi_d}\,(1-\beta\,\mu\,\xi_d)\left[\widehat{\overline{w}}_t + \varpi\widehat{H}_t - \varpi\widehat{\overline{k}}_t - \varpi\widehat{u}_t + \frac{\nu^{fw}R}{R^{fw}}\widehat{R}_{t-1} + \frac{\nu^{fw}(R-1)}{R^{fw}}\widehat{\nu}^{fw}_t\right.\\ &\left. + \frac{\varpi}{1-\varpi}\widehat{\mu}_{\Psi,t} + \varpi\widehat{\mu}_{z,t} + \frac{\tau^s}{1+\tau^s}\widehat{\tau}^s_t + \nu^\tau\left(\widehat{\pi}^{oil}_t - \widehat{\pi}^u_t\right) - \widehat{\epsilon}_t + \widehat{\lambda}^d_t\right]\end{aligned} \tag{A.5}$$

Inflation of prices of imported consumption goods

$$\begin{aligned}(1+\kappa_{mc}\,\beta\,\mu)\,\widehat{\pi}^{mc}_t &= (1-\kappa_{mc})\,(1-\beta\,\mu\,\rho_\pi)\,\widehat{\overline{\pi}}^c_t + \beta\,\mu\,\widehat{\pi}^{mc}_{t+1} + \kappa_{mc}\,\widehat{\pi}^{mc}_{t-1}\\ &+ \frac{(1-\xi_{mc})}{\xi_{mc}}\,(1-\beta\,\mu\,\xi_{mc})\left(\widehat{x}^u_t + (\omega^{mcu}-1)\,\widehat{x}^x_t - \frac{c^d}{c\,\gamma^{cd}}\widehat{\gamma}^{mcd}_t + \widehat{\lambda}^{mc}_t\right)\end{aligned} \tag{A.6}$$

Inflation of prices of imported investment goods

$$\begin{aligned}(1+\kappa_{mi}\,\beta\,\mu)\,\widehat{\pi}^{mi}_t &= (1-\kappa_{mi})\,(1-\beta\,\mu\,\rho_\pi)\,\widehat{\overline{\pi}}^c_t + \beta\,\mu\,\widehat{\pi}^{mi}_{t+1} + \kappa_{mi}\,\widehat{\pi}^{mi}_{t-1}\\ &+ \frac{(1-\xi_d)}{\xi_{mi}}\,(1-\beta\,\mu\,\xi_{mi})\left(\widehat{x}^u_t + \left(\omega^{miu}-1\right)\widehat{x}^x_t + \frac{c^m}{c\,\gamma^{cmc}}\widehat{\gamma}^{mcd}_t - \widehat{\gamma}^{mid}_t + \widehat{\lambda}^{mi}_t\right)\end{aligned} \tag{A.7}$$

Inflation of prices of imported export components

$$\begin{aligned}(1+\kappa_{mx}\,\beta\,\mu)\,\widehat{\pi}^{mx}_t &= (1-\kappa_{mx})\,(1-\beta\,\mu\,\rho_\pi)\,\widehat{\overline{\pi}}^c_t + \beta\,\mu\,\widehat{\pi}^{mx}_{t+1} + \kappa_{mx}\,\widehat{\pi}^{mx}_{t-1}\\ &+ \frac{(1-\xi_{mx})}{\xi_{mx}}\,(1-\beta\,\mu\,\xi_{mx})\left(\widehat{x}^u_t + (\omega^{mxu}-1)\widehat{x}^x_t + \frac{c^m}{c\,\gamma^{cmc}}\widehat{\gamma}^{mcd}_t - \widehat{\gamma}^{mxd}_t + \widehat{\lambda}^{mx}_t\right)\end{aligned} \tag{A.8}$$

Inflation of prices of export products

$$\begin{aligned}(1+\kappa_x\,\beta\,\mu)\,\widehat{\pi}^x_t &= (1-\kappa_x)\,(1-\beta\,\mu\,\rho_\pi)\,\widehat{\overline{\pi}}^c_t + \beta\,\mu\,\widehat{\pi}^x_{t+1} + \kappa_x\,\widehat{\pi}^x_{t-1}\\ &+ \frac{(1-\xi_x)}{\xi_x}\,(1-\beta\,\mu\,\xi_x)\left(-\frac{c^m}{c\,\gamma^{cmc}}\,\widehat{\gamma}^{mcd}_t - \widehat{\gamma}^{xu}_t - \widehat{x}^u_t + \omega_x\,\widehat{\gamma}^{mxd}_t + \widehat{\lambda}^x_t\right)\end{aligned} \tag{A.9}$$

Real wages

$$\begin{aligned} & b_w\,\beta\,\xi_w\,\widehat{\pi}^d_{t+1} - b_w\,\xi_w\left(1+\kappa_w\,\beta\,\frac{c^d}{c\,\gamma^{cd}}\right)\widehat{\pi}^d_t + b_w\,\xi_w\,\kappa_w\,\frac{c^d}{c\,\gamma^{cd}}\,\widehat{\pi}^d_{t-1}\\ &\quad -b_w\,\kappa_w\,\beta\,\xi_w\,\frac{c^m}{c\,\gamma^{cmc}}\,\widehat{\pi}^{mc}_t + b_w\,\xi_w\,\kappa_w\,\frac{c^m}{c\,\gamma^{cmc}}\,\widehat{\pi}^{mc}_{t-1}\\ &\quad +b_w\,\beta\,\xi_w\,\widehat{\overline{w}}_{t+1} + \left[\lambda^w\,\sigma_L - b_w\left(1+\beta\,\xi_w^2\right)\right]\widehat{\overline{w}}_t + b_w\,\xi_w\,\widehat{\overline{w}}_{t-1}\\ &\quad -(\lambda^w-1)\,\widehat{\psi}_{z^+,t} + (\lambda^w-1)\,\sigma_L\,\widehat{H}_t\\ &\quad +(\lambda^w-1)\,\widehat{\zeta}^h_t - b_w\,\beta\,\xi_w\,(1-\kappa_w)\,\widehat{\overline{\pi}}^c_{t+1} + b_w\,\xi_w\,(1-\kappa_w)\,\widehat{\overline{\pi}}^c_t\\ &\quad +(\lambda^w-1)\,\widehat{\lambda}^w_t + \left[\frac{\tau_w\,(\lambda^w-1)}{1-\tau^w}\right]\widehat{\tau}^w_t + \left[\frac{\tau_y\,(\lambda^w-1)}{1-\tau^y}\right]\widehat{\tau}^y_t \;=\; 0\end{aligned} \tag{A.10}$$

Demand for labour services

$$\widehat{H}_t = \frac{1}{(1-\varpi)}\widehat{y}_t - \frac{1}{(1-\varpi)}\widehat{\epsilon}_t + \frac{1}{(1-\varpi)}\frac{\varpi}{1-\varpi}\widehat{\mu}_{\Psi,t} + \frac{\varpi}{(1-\varpi)}\widehat{\mu}_{z,t} - \frac{\varpi}{(1-\varpi)}\widehat{\bar{k}}_t - \frac{\varpi}{(1-\varpi)}\widehat{u}_t \tag{A.11}$$

Consumption

$$\begin{aligned}
&\left(\mu_\Psi^{\frac{\varpi}{1-\varpi}}\mu_z\right) b\widehat{c}_{t-1} - \left(\left(\mu_\Psi^{\frac{\varpi}{1-\varpi}}\mu_z\right)^2 + \beta\, b^2\right)\widehat{c}_t + \left(\mu_\Psi^{\frac{\varpi}{1-\varpi}}\mu_z\right) b\,\beta\widehat{c}_{t+1} \\
&- \left(\beta\, b - \left(\mu_\Psi^{\frac{\varpi}{1-\varpi}}\mu_z\right)\right)\left(b - \left(\mu_\Psi^{\frac{\varpi}{1-\varpi}}\mu_z\right)\right)\frac{c^m}{c\,\gamma^{cmc}}\widehat{\gamma}_t^{mcd} \\
&- \left(\beta\, b - \left(\mu_\Psi^{\frac{\varpi}{1-\varpi}}\mu_z\right)\right)\left(b - \left(\mu_\Psi^{\frac{\varpi}{1-\varpi}}\mu_z\right)\right)\widehat{\psi}_{z^+,t} \\
&- b\left(\mu_\Psi^{\frac{\varpi}{1-\varpi}}\mu_z\right)\frac{\varpi}{1-\varpi}\widehat{\mu}_{\Psi,t} + \beta\, b\left(\mu_\Psi^{\frac{\varpi}{1-\varpi}}\mu_z\right)\frac{\varpi}{1-\varpi}\widehat{\mu}_{\Psi,t+1} \\
&- b\left(\mu_\Psi^{\frac{\varpi}{1-\varpi}}\mu_z\right)\widehat{\mu}_{z,t} + \beta\, b\left(\mu_\Psi^{\frac{\varpi}{1-\varpi}}\mu_z\right)\widehat{\mu}_{z,t+1} \\
&- \left(\beta\, b - \left(\mu_\Psi^{\frac{\varpi}{1-\varpi}}\mu_z\right)\right)\left(b - \left(\mu_\Psi^{\frac{\varpi}{1-\varpi}}\mu_z\right)\right)\frac{\tau^c}{(1+\tau^c)}\widehat{\tau}_t^c \\
&- \left(\mu_\Psi^{\frac{\varpi}{1-\varpi}}\mu_z\right)\left(b - \left(\mu_\Psi^{\frac{\varpi}{1-\varpi}}\mu_z\right)\right)\widehat{\zeta}_t^c + \beta\, b\left(b - \left(\mu_\Psi^{\frac{\varpi}{1-\varpi}}\mu_z\right)\right)\widehat{\zeta}_{t+1}^c = 0
\end{aligned} \tag{A.12}$$

Investments

$$\begin{aligned}
&\widehat{p}_{k',t} + \widehat{\Upsilon}_t - \omega_i\left(\gamma^{imi}\right)^{\eta_i-1}\widehat{\gamma}_t^{mid} \\
&+ \left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right)^2\tilde{S}''\,\widehat{i}_{t-1} - (1+\beta)\left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right)^2\tilde{S}''\,\widehat{i}_t + \beta\left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right)^2\tilde{S}''\,\widehat{i}_{t+1} \\
&- \left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right)^2\tilde{S}''\,\widehat{\mu}_{z,t} + \left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right)^2\tilde{S}''\,\beta\widehat{\mu}_{z,t+1} \\
&- \left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right)^2\tilde{S}''\left(\frac{1}{1-\varpi}\widehat{\mu}_{\Psi,t}\right) + \left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right)^2\tilde{S}''\,\beta\left(\frac{1}{1-\varpi}\widehat{\mu}_{\Psi,t+1}\right) = 0
\end{aligned} \tag{A.13}$$

Marginal utility of income

$$\begin{aligned}
\widehat{\psi}_{z^+,t} &= \widehat{\psi}_{z^+,t+1} - \left(\frac{\varpi}{1-\varpi}\widehat{\mu}_{\Psi,t+1} + \widehat{\mu}_{z,t+1}\right) - \widehat{\pi}_{t+1} \\
&\quad - \frac{\tau^k}{(1-\tau^k)}\frac{\pi\,\mu_{z^+} - \beta}{\mu_{z^+}\,\pi}\widehat{\tau}_{t+1}^k + \frac{\pi\,\mu_{z^+} - \beta\tau^k}{\mu_{z^+}\,\pi}\widehat{R}_t
\end{aligned} \tag{A.14}$$

Relative prices of fixed assets

$$
\begin{aligned}
\widehat{p}_{k',t} = & -\widehat{\psi}_{z^+,t} + \widehat{\psi}_{z^+,t+1} + \beta \frac{1-\delta+\delta\,\tau^p}{\left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right)} \widehat{p}_{k',t+1} - \frac{\tau^p}{(1-\tau^p)} \frac{\left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right)-\beta}{\left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right)} \widehat{\tau}^p_{t+1} \\
& + \frac{\left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right) - \beta\,(1+\delta\,\tau^p-\delta)}{\left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right)} \Bigg(\left[\frac{\nu^w R}{R^{fw}} - \frac{\nu^k R}{R^{fk}}\right] \widehat{R}_t + \frac{\nu^w\,(R-1)}{R^{fw}} \widehat{\nu}^w_{t+1} \\
& \qquad - \frac{\nu^k\,(R-1)}{R^{fk}} \widehat{\nu}^k_{t+1} + \widehat{\bar{w}}_{t+1} + \widehat{H}_{t+1} - \left(\widehat{u}_{t+1} + \widehat{\bar{k}}_{t+1}\right) + \frac{\tau^s}{1+\tau^s} \widehat{\tau}^s_{t+1} \Bigg) \\
& - \frac{1}{1-\varpi} \left(\frac{\beta\,(1+\delta\,\tau^p-\delta)}{\left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right)} \right) \widehat{\mu}_{\Psi,t+1} - \left(\frac{\beta\,(1+\delta\,\tau^p-\delta)}{\left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right)} \right) \widehat{\mu}_{z,t+1}
\end{aligned}
\tag{A.15}
$$

Dynamics of the nominal dollar/zloty exchange rate

$$
\left(1-\phi^e_s\right) \Delta\widehat{S}^u_{t+1} = \left(1-\phi^e_s\right) \Delta\widehat{S}^x_{t+1} - \phi^e_s \Delta\widehat{S}^x_t + \phi^e_s \Delta\widehat{S}^u_t + \left(\widehat{R}_t - \widehat{R}^e_t\right) + \tilde{\phi}^e_a\, a^e\, \widehat{a}^e_t - \widehat{\tilde{\phi}}^e_t \tag{A.16}
$$

Real income, balance for the real GDP

$$
\begin{aligned}
\widehat{y}_t = & \frac{g}{y}\widehat{g}_t + \frac{c^d}{y}\widehat{c}_t + \frac{c^d}{y}\eta_c \frac{c^m}{c\,\gamma^{cmc}} \widehat{\gamma}^{mcd}_t + \frac{i^d}{y}\widehat{i}_t + \frac{i^d}{y}\eta_i \frac{i^m}{i\,\gamma^{imi}} \widehat{\gamma}^{mid}_t \\
& + \frac{x^d}{x}\frac{x}{y}\left[\frac{y^e}{y^\star}\widehat{y}^e_t + \left(1-\frac{y^e}{y^\star}\right)\widehat{y}^u_t - \left(\eta_{fu} + (\eta_{fe}-\eta_{fu})\frac{y^e}{y^\star}\right)\widehat{\gamma}^{x,u}_t - \eta_{fe}\frac{y^e}{y^\star}\widehat{x}^x_t + \widehat{\tilde{z}}^{\star,+}_t \right] \\
& + \frac{x}{y}\frac{x^d}{x}\eta_{xx}\,\omega_x\,\widehat{\gamma}^{mxd}_t + \frac{k}{y}\frac{\bar{r}^k}{\left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right)}\widehat{u}_t
\end{aligned}
\tag{A.17}
$$

Fixed assets

$$
\begin{aligned}
\widehat{\bar{k}}_{t+1} = & \frac{1-\delta}{\left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right)}\widehat{\bar{k}}_t + \frac{\left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right)+\delta-1}{\left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right)}\widehat{i}_t \\
& + \frac{\left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right)+\delta-1}{\left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right)}\widehat{\Upsilon}_t - \frac{1-\delta}{\left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right)}\widehat{\mu}_{z,t} - \frac{1-\delta}{\left(\mu_z\,\mu_\Psi^{\frac{1}{1-\varpi}}\right)}\frac{1}{1-\varpi}\widehat{\mu}_{\Psi,t}
\end{aligned}
\tag{A.18}
$$

Fixed assets' utilisation rate

$$
\begin{aligned}
\widehat{u}_t = & \frac{1}{1+\sigma_a}\Bigg(\left[\frac{\nu^w R}{R^{fw}} - \frac{\nu^k R}{R^{fk}}\right]\widehat{R}_{t-1} + \frac{\nu^w\,(R-1)}{R^{fw}}\widehat{\nu}^w_t - \frac{\nu^k\,(R-1)}{R^{fk}}\widehat{\nu}^k_t + \widehat{\bar{w}}_t + \widehat{H}_t - \widehat{\bar{k}}_t \\
& + \frac{1}{1-\varpi}\widehat{\mu}_{\Psi,t} + \widehat{\mu}_{z,t} + \frac{\tau^s}{1+\tau^s}\widehat{\tau}^s_t \Bigg) - \frac{1}{1+\sigma_a}\nu^{a\tau}\left(\Delta\widehat{S}^u_t + \widehat{\pi}^{oil}_t\right)
\end{aligned}
\tag{A.19}
$$

Cash

$$
\widehat{q}_t = \frac{1}{\sigma_q}\left[\widehat{\zeta}^q_t - \widehat{\psi}_{z^+,t} + \frac{\tau^k}{(1-\tau^k)}\,\widehat{\tau}^k - \frac{R}{(R-1)}\widehat{R}_{t-1} \right] \tag{A.20}
$$

Money

$$\widehat{m}_{t+1} = \widehat{\mu}_t + \widehat{m}_t - \widehat{\pi}^d_t - \frac{\varpi}{1-\varpi}\widehat{\mu}_{\Psi,t} - \widehat{\mu}_{z,t} \tag{A.21}$$

Money growth rate

$$\begin{aligned}
\widehat{\mu}_t = {} & \frac{q}{m}\widehat{q}_t - \widehat{m}_t + \widehat{\pi}_t + \frac{\nu^k \,\overline{r}^k \, k R}{m\left(\mu_z \, \mu_\Psi^{\frac{1}{1-\varpi}}\right)} \left[\frac{\nu^w}{R^{fw}} - \frac{\nu^k}{R^{fk}}\right] \widehat{R}_{t-1} \\
& + \frac{\nu^k \,\overline{r}^k \, k}{m\left(\mu_z \, \mu_\Psi^{\frac{1}{1-\varpi}}\right)} \left[1 - \frac{\nu^k (R-1)}{R^{fk}}\right] \widehat{\nu}^k_t \\
& + \frac{\nu^w}{m}\left[\frac{\nu^k \,\overline{r}^k \, k}{\left(\mu_z \, \mu_\Psi^{\frac{1}{1-\varpi}}\right)} \frac{(R-1)}{R^{fw}} + (1+\tau^s)\,\overline{w}\,H\right] \widehat{\nu}^w_t \\
& + \left(\frac{\nu^k \,\overline{r}^k \, k}{m\left(\mu_z \, \mu_\Psi^{\frac{1}{1-\varpi}}\right)} + \frac{\nu^w}{m}\,(1+\tau^s)\,\overline{w}\,H\right)\left[\widehat{\overline{w}}_t + \widehat{H}_t + \frac{\tau^s}{1+\tau^s}\widehat{\tau}^s_t\right] \\
& + \frac{\varpi}{1-\varpi}\widehat{\mu}_{\Psi,t} + \widehat{\mu}_{z,t} + \left(1 - \frac{q}{m}\right)\nu^\tau\left(\widehat{\pi}^{oil}_t - \widehat{\pi}^u_t\right)
\end{aligned} \tag{A.22}$$

Total net foreign assets — classic version

$$\begin{aligned}
\widehat{a}_t = {} & -c^m\,\widehat{c}_t - i^m\,\widehat{i}_t + x\left[1 - (1-u_x)\left(\eta_{fu} + (\eta_{fe} - \eta_{fu})\frac{y^e}{y^\star}\right)\right]\widehat{\gamma}^{x,u}_t \\
& - x\left[u_c\,\omega^{mcu} + u_i\,\omega^{miu} + u_x\,\omega^{mxu} - 1 + (1-u_x)\,\eta_{fe}\frac{y^e}{y^\star}\right]\widehat{x}^x_t \\
& + \frac{c^m\,c^d}{c}\frac{\eta_c}{\gamma^{cd}}\widehat{\gamma}^{mcd}_t + \frac{i^m\,i^d}{i}\frac{\eta_i}{\gamma^{id}}\widehat{\gamma}^{mid}_t + x^m\,\eta_{xx}\,(1-\omega_x)\,\widehat{\gamma}^{mxd}_t \\
& + x\,(1-u_x)\left[\frac{y^e}{y^\star}\widehat{y}^e_t + \left(1 - \frac{y^e}{y^\star}\right)\widehat{y}^u_t + \widehat{\tilde{z}}^{\star,+}_t\right] \\
& \quad + \frac{-R\,a^e}{\pi\,\mu_{z^+}}\Bigg[\left(\widehat{R}^u_{t-1} - \widehat{R}^e_{t-1}\right) + a^e\left(\tilde{\phi}^u + \tilde{\phi}^e_a\right)\widehat{a}^e_{t-1} \\
& \qquad - \left(1 + \tilde{\phi}^u_a\,a^e\right)\frac{1}{a^e}\widehat{a}_{t-1} + \left(\tilde{\phi}^e_s - \tilde{\phi}^u_s\right)\Delta\widehat{S}^u_t \\
& \qquad + \left(1 - \tilde{\phi}^e_s\right)\Delta\widehat{S}^x_t + \left(\tilde{\phi}^e_s - \tilde{\phi}^u_s\right)\Delta\widehat{S}^u_{t-1} - \tilde{\phi}^e_s\Delta\widehat{S}^x_{t-1} + \widehat{\tilde{\phi}}^u_{t-1} - \widehat{\tilde{\phi}}^e_{t-1}\Bigg]
\end{aligned} \tag{A.23}$$

Total net foreign assets — balance of total incomes and total expenditures

$$
\begin{aligned}
& \widehat{a}_t + g\,\widehat{g}_t + \left(c^d + c^m\right)\widehat{c}_t + \left(i^d + i^m\right)\widehat{i}_t = \\
& \quad = \frac{-R\,a^e}{\pi\,\mu_{z^+}}\Big[\left(\widehat{R}^u_{t-1} - \widehat{R}^e_{t-1}\right) + a^e\left(\tilde{\phi}^u + \tilde{\phi}^e_a\right)\widehat{a}^e_{t-1} \\
& \qquad - \left(1 + \tilde{\phi}^u_a\,a^e\right)\frac{1}{a^e}\widehat{a}_{t-1} + \left(\tilde{\phi}^e_s - \tilde{\phi}^u_s\right)\Delta\widehat{S}^u_t \\
& \qquad + \left(1 - \tilde{\phi}^e_s\right)\Delta\widehat{S}^x_t + \left(\tilde{\phi}^e_s - \tilde{\phi}^u_s\right)\Delta\widehat{S}^u_{t-1} - \tilde{\phi}^e_s\Delta\widehat{S}^x_{t-1} + \widehat{\tilde{\phi}}^u_{t-1} - \widehat{\tilde{\phi}}^e_{t-1}\Big] \\
& \quad - x\left[\left(u_c\,\omega^{mcu} + u_i\,\omega^{miu} + u_x\,\omega^{mxu}\right) - 1\right]\widehat{x}^x_t + x\,\widehat{\gamma}^{xu}_t \\
& \quad + \frac{c^d\,c^m}{c}\,\frac{\eta_c}{\gamma^{cd}}\left(1 - \gamma^{mcd}\right)\widehat{\gamma}^{mcd}_t + \frac{i^d\,i^m}{i}\,\frac{\eta_i}{\gamma^{id}}\left(1 - \gamma^{mid}\right)\widehat{\gamma}^{mid}_t \\
& \quad + y\left(\frac{\lambda^d - 1}{\lambda^d}\right)\widehat{y}_t + \frac{y\,(1-\varpi)}{\lambda^d}\widehat{H}_t + \frac{y\,\varpi}{\lambda^d}\widehat{k}_t + \frac{y\,\varpi}{\lambda^d}\left(\frac{R^{fk} - 1}{R^{fk}}\right)\widehat{u}_t \\
& \quad + \frac{y}{\lambda^d}\left((1-\varpi)\,\frac{R^{fw} - 1}{R^{fw}} + \varpi\,\frac{R^{fk} - 1}{R^{fk}} - 1\right)\nu^{\tau}\left(\widehat{\pi}^{oil}_t - \widehat{\pi}^u_t\right) \\
& \quad + \frac{y}{\lambda^d}\,\widehat{\epsilon}_t - \frac{y}{\lambda^d}\,\frac{\varpi}{1-\varpi}\,\widehat{\mu}_{\Psi,t} - \frac{y\,\varpi}{\lambda^d}\widehat{\mu}_{z,t}
\end{aligned}
\tag{A.24}
$$

Net foreign assets denominated in euro

$$
\widehat{a}^e_t = \frac{1}{a^e\,\tilde{\phi}^u_a}\left[\left(\widehat{R}_t - \widehat{R}^u_t\right) - \left(1 - \tilde{\phi}^u_s\right)\Delta\widehat{S}^u_{t+1} + \tilde{\phi}^u_a\,\widehat{a}_t + \tilde{\phi}^u_s\,\Delta\widehat{S}^u_t - \widehat{\tilde{\phi}}^u_t\right] \tag{A.25}
$$

Relation of prices of imported consumption goods

$$
\widehat{\gamma}^{mcd}_t = \widehat{\gamma}^{mcd}_{t-1} + \widehat{\pi}^{mc}_t - \widehat{\pi}^d_t \tag{A.26}
$$

Relation of prices of imported investment goods

$$
\widehat{\gamma}^{mid}_t = \widehat{\gamma}^{mid}_{t-1} + \widehat{\pi}^{mi}_t - \widehat{\pi}^d_t \tag{A.27}
$$

Relation of prices of imported export products

$$
\widehat{\gamma}^{mxd}_t = \widehat{\gamma}^{mxd}_{t-1} + \widehat{\pi}^{mx}_t - \widehat{\pi}^d_t \tag{A.28}
$$

Relation of export prices to the prices in the dollar area

$$
\widehat{\gamma}^{xu}_t = \left(\widehat{\gamma}^{xu}_{t-1} + \widehat{\pi}^x_t - \widehat{\pi}^u_t\right) \tag{A.29}
$$

Real dollar/zloty exchange rate

$$
\widehat{x}^u_t = \widehat{x}^u_{t-1} + \Delta\widehat{S}^u_t + \widehat{\pi}^u_t - \left(1 - \omega_c\right)\left(\gamma^{cd}\right)^{\eta_c - 1}\widehat{\pi}^d_t - \omega_c\left(\gamma^{cmc}\right)^{\eta_c - 1}\widehat{\pi}^{mc}_t \tag{A.30}
$$

Real dollar/euro exchange rate

$$
\widehat{x}^x_t = \left[\rho_{xx}\right]\widehat{x}^x_{t-1} + \widehat{\pi}^u_t - \widehat{\pi}^e_t + \Delta\widehat{S}^x_t \tag{A.31}
$$

Nominal interest rate

$$\begin{aligned}\widehat{R}_t &= \rho_R \widehat{R}_{t-1} + (1-\rho_R)\,(1-r_\pi)\,\widehat{\overline{\pi}}^c_t + r_{\Delta\pi} \frac{c^d}{c\,\gamma^{cd}}\,\widehat{\pi}^d_t + r_{\Delta\pi} \frac{c^m}{c\,\gamma^{cmc}}\,\widehat{\pi}^{mc}_t \\ &+ \left((1-\rho_R)\,r_\pi - r_{\Delta\pi}\right) \frac{c^d}{c\,\gamma^{cd}}\,\widehat{\pi}^d_{t-1} + \left((1-\rho_R)\,r_\pi - r_{\Delta\pi}\right) \frac{c^m}{c\,\gamma^{cmc}}\,\widehat{\pi}^{mc}_{t-1} \\ &+ r_{\Delta y}\,\widehat{y}_t + \left[(1-\rho_R)\,r_y - r_{\Delta y}\right]\widehat{y}_{t-1} + (1-\rho_R)\,r_x\,\widehat{x}^{eu}_{t-1} + \epsilon_{R,t}\end{aligned} \tag{A.32}$$

Employment

$$\beta\,\xi_e \widehat{\widetilde{E}}_{t+1} + (1-\beta\,\xi_e)\,(1-\xi_e)\widehat{H}_t - \left(1+\beta\,\xi_e^2\right)\widehat{\widetilde{E}}_t + \xi_e \widehat{\widetilde{E}}_{t-1} = 0 \tag{A.33}$$

Steady state solution

Based on theoretical analyses, we assume that in the steady state the following are satisfied:

$$\lambda^x = 1, \quad \lambda^{mx} = 1, \quad \gamma^{xu} = 1, \quad \gamma^{xd} = 1, \quad \gamma^{mxd} = 1, \quad \aleph^\star_0\,y^\star = x$$

then the values of other variables are determined in compliance with the following equations:

Technology growth

$$\mu_{z^+} = \mu_\Psi^{\frac{\varpi}{1-\varpi}}\,\mu_z \tag{A.34}$$

Inflation, inflation target

$$\pi = \frac{\mu}{\mu_{z^+}} \tag{A.35}$$

Nominal interest rate

$$R = \frac{\mu - \tau^k\,\beta}{\left(1-\tau^k\right)\,\beta} \tag{A.36}$$

Interest rate of working capital loan for labour services

$$R^{fw} = \nu^w R + 1 - \nu^w \tag{A.37}$$

Interest rate of working capital loan for capital services

$$R^{fk} = \nu^k R + 1 - \nu^k \tag{A.38}$$

Relation of prices of imported consumption goods to domestic prices

$$\gamma^{mcd} = \lambda^{mcd}\,\gamma^{xd} \qquad \text{gdy} \quad \gamma^{xd} = 1 \quad \text{to} \quad \gamma^{mcd} = \lambda^{mc} \tag{A.39}$$

Relation of prices of imported investment goods to domestic prices

$$\gamma^{mid} = \lambda^{mid}\gamma^{xd} \qquad \text{gdy} \quad \gamma^{xd} = 1 \quad \text{to} \quad \gamma^{mid} = \lambda^{mi} \tag{A.40}$$

Relation of prices of consumption goods to domestic prices

$$\gamma^{cd} = \left[(1-\omega_c) + \omega_c\,(\lambda^{mc})^{1-\eta_c}\right]^{\frac{1}{1-\eta_c}} \tag{A.41}$$

Relation of prices of investment goods to domestic prices

$$\gamma^{id} = \left[(1-\omega_i) + \omega_i\,\left(\lambda^{mi}\right)^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}} \tag{A.42}$$

Relation of prices of consumption goods to the prices of imported consumption goods

$$\gamma^{cmc} = \left[(1-\omega_c)\,(\lambda^{mc})^{\eta_c-1} + \omega_c\right]^{\frac{1}{1-\eta_c}} \tag{A.43}$$

Relation of prices of investment goods to the prices of imported investment goods

$$\gamma^{imi} = \left[(1-\omega_i)\,\left(\lambda^{mi}\right)^{\eta_i-1} + \omega_i\right]^{\frac{1}{1-\eta_i}} \tag{A.44}$$

Rent for lease of fixed assets services

$$\overline{r}^{k} = \left[\frac{\beta\,(1-\tau^{p})}{\mu_{z^+}\,\mu_{\Psi} - \beta\,[1+\delta\tau^{p}-\delta]}\right]^{-1}\gamma^{id} \tag{A.45}$$

Real wages

$$\overline{w} = \frac{1}{(1+\tau^{s})R^{fw}}\left[\lambda^{d}\left(\frac{1}{\varpi}\right)^{\varpi}\left(\frac{1}{1-\varpi}\right)^{1-\varpi}\left(R^{fk}\,\overline{r}\right)^{\varpi}\right]^{\frac{1}{\varpi-1}} \tag{A.46}$$

Relation of capital to labour

$$\left(\frac{k}{H}\right) = \frac{\varpi}{1-\varpi}\,\frac{R^{fw}}{R^{fk}}\,\frac{1}{\overline{r}^{k}}\,\left[(1+\tau^{s})\,\overline{w}\,\mu_{\Psi}\,\mu_{z^+}\right] \tag{A.47}$$

Auxiliary variables

$$D_1 = \left[(1-\omega_c) \left[\gamma^{cd} \right]^{\eta_c} + \omega_c \left[\gamma^{cmc} \right]^{\eta_c} \right]$$

$$\begin{aligned} D_2 = \Bigg[& (1-g_r) \left(\frac{1}{\mu_\Psi \mu_{z^+}} \right)^{\varpi} \left(\frac{k}{H} \right)^{\varpi} \\ & - \left((1-\omega_i) \left[\gamma^{id} \right]^{\eta_i} + \omega_i \left[\gamma^{imi} \right]^{\eta_i} \right) \left(\frac{\mu_{z^+} \mu_\Psi + \delta - 1}{\mu_{z^+} \mu_\Psi} \right) \frac{k}{H} \Bigg] \end{aligned} \tag{A.48}$$

$$D_3 = \left[\frac{1}{A_L} (1-\tau^y)(1-\tau^w) \frac{\overline{w}}{\lambda^w} \right]^{\frac{1}{\sigma_L}}$$

$$D_4 = \frac{1}{\gamma^{cd} (1+\tau^c)} \left(\frac{\mu_{z^+} - \beta b}{\mu_{z^+} - b} \right)$$

Labour services

$$H = \left[\frac{D_1 D_4}{D_2} D_3^{\sigma_L} \right]^{\frac{1}{1+\sigma_L}} \tag{A.49}$$

Fixed assets services

$$k = H \frac{k}{H} \tag{A.50}$$

Total consumption

$$c = \frac{D_2}{D_1} H \tag{A.51}$$

Domestically manufactured consumer goods

$$c^d = (1-\omega_c) \left[\gamma^{cd} \right]^{\eta_c} c \tag{A.52}$$

Imported consumer goods

$$c^m = \omega_c \left[\gamma^{cmc} \right]^{\eta_c} c \tag{A.53}$$

Investment expenditures

$$i = k \frac{\mu_{z^+} \mu_\Psi + \delta - 1}{\mu_{z^+} \mu_\Psi} \tag{A.54}$$

Domestic component of investment goods

$$i^d = (1-\omega_i) \left[\gamma^{id} \right]^{\eta_i} i \tag{A.55}$$

Imported component of investment goods

$$i^m = \omega_i \left[\gamma^{imi}\right]^{\eta_i} i \tag{A.56}$$

Imported component of export

$$x^m = \omega_x \, x \tag{A.57}$$

Total export

$$x = \frac{c^m + i^m}{1 - \omega_x} \tag{A.58}$$

Production

$$y = \left[\mu_\Psi^{\frac{-\varpi}{1-\varpi}} \mu_z^{-\varpi} \, H \left(\frac{k}{H}\right)^{\varpi}\right] \tag{A.59}$$

Broad money

$$m = \frac{\nu^k \, \frac{\overline{r}^k \, k}{\mu_\Psi \, \mu_{z^+}} + \nu^w \, (1+\tau^s) \, \overline{w} \, H}{1 - A_q} \tag{A.60}$$

Cash

$$q = A_q \, m \tag{A.61}$$

Appendix B
Global economy SVAR model

SVAR model identification

In the estimation of the reduced form of the global economy model, the following set of restrictions has been assumed for the dynamics of a SVAR model[1]:

$$\begin{pmatrix} y_t^e \\ \pi_t^e \\ r_t^e \\ y_t^u \\ \pi_t^u \\ r_t^u \\ x_t \end{pmatrix} = \begin{pmatrix} * & * & 0 & * & * & * & * \\ * & * & 0 & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & 0 & * \\ * & * & * & * & * & 0 & * \\ * & * & * & * & * & * & * \\ * & * & 0 & * & * & * & * \end{pmatrix} \begin{pmatrix} y_{t-1}^e \\ \pi_{t-1}^e \\ r_{t-1}^e \\ y_{t-1}^u \\ \pi_{t-1}^u \\ r_{t-1}^u \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} * & * & * & * & * & * & * \\ * & * & 0 & * & * & * & * \\ * & * & * & * & * & * & 0 \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & 0 \\ * & * & 0 & * & * & * & * \end{pmatrix} \begin{pmatrix} y_{t-2}^e \\ \pi_{t-2}^e \\ r_{t-2}^e \\ y_{t-2}^u \\ \pi_{t-2}^u \\ r_{t-2}^u \\ x_{t-2} \end{pmatrix} + e_t. \tag{B.1}$$

Results of estimation of the SVAR model

Results of estimation of the reduced form of the SVAR model are the following[2]:

$$\begin{aligned} \begin{pmatrix} y_t^e \\ \pi_t^e \\ r_t^e \\ y_t^u \\ \pi_t^u \\ r_t^u \\ x_t \end{pmatrix} &= \begin{pmatrix} 1.15 & 0.15 & 0.00 & 0.12 & 0.11 & -0.04 & -0.03 \\ 0.06 & 0.28 & 0.00 & 0.00 & 0.10 & 0.04 & 0.00 \\ 0.05 & -0.06 & 1.10 & 0.03 & 0.12 & -0.02 & 0.00 \\ 0.39 & 0.11 & -0.35 & 0.96 & 0.36 & 0.00 & 0.00 \\ 0.03 & 0.10 & 0.18 & 0.05 & 0.34 & 0.00 & 0.00 \\ 0.06 & -0.05 & -0.24 & 0.05 & 0.25 & 0.66 & 0.00 \\ -0.13 & -0.51 & 0.00 & 0.19 & 3.21 & -0.38 & 0.22 \end{pmatrix} \begin{pmatrix} y_{t-1}^e \\ \pi_{t-1}^e \\ r_{t-1}^e \\ y_{t-1}^u \\ \pi_{t-1}^u \\ r_{t-1}^u \\ x_{t-1} \end{pmatrix} + \\ &+ \begin{pmatrix} -0.31 & -0.43 & -0.10 & -0.13 & 0.11 & 0.26 & 0.02 \\ 0.00 & 0.28 & 0.00 & -0.03 & 0.19 & 0.04 & 0.00 \\ 0.01 & -0.07 & -0.22 & -0.05 & 0.12 & 0.02 & 0.00 \\ -0.28 & -0.24 & -0.12 & -0.16 & 0.14 & -0.42 & 0.02 \\ -0.02 & 0.10 & -0.26 & -0.02 & 0.37 & -0.08 & 0.00 \\ -0.06 & -0.06 & 0.09 & -0.01 & 0.27 & 0.08 & 0.00 \\ 0.23 & -2.47 & 0.00 & -0.85 & 1.25 & 2.12 & -0.11 \end{pmatrix} \begin{pmatrix} y_{t-2}^e \\ \pi_{t-2}^e \\ r_{t-2}^e \\ y_{t-2}^u \\ \pi_{t-2}^u \\ r_{t-2}^u \\ x_{t-2} \end{pmatrix} + e_t. \end{aligned} \tag{B.2}$$

The structuralisation mechanism assumed in the identification of structural shocks of the SVAR model was the following:

[1] We omit exogenous variables, as no restriction has been imposed thereon.
[2] Exogenous variables have been omitted.

$$B = \begin{pmatrix} * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & * \end{pmatrix}. \tag{B.3}$$

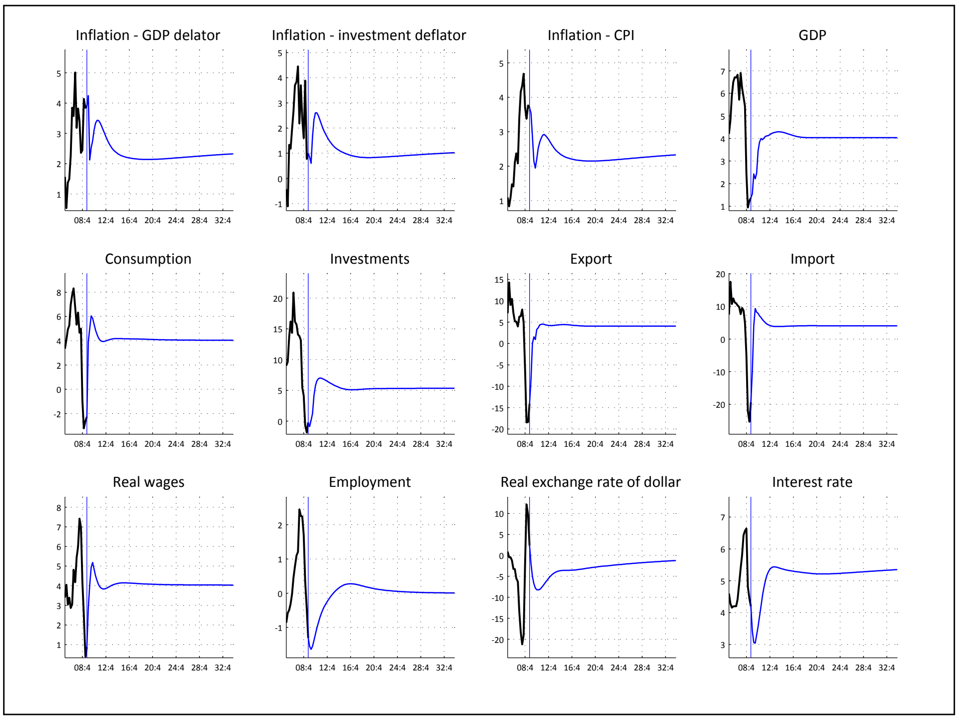
The estimated structural matrix B is the following:

$$B = \begin{pmatrix} 0.43 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.26 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.03 & 0.01 & 0.09 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.52 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.22 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.08 & 0.03 & 0.16 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 4.33 \end{pmatrix}. \tag{B.4}$$

Appendix C
Convergence to the steady state

The figure shows the behaviour of observable variables, if the horizon of the example forecast is extended to 100 quarters. As results from the graph, real categories (GDP, consumption, investments, etc.) converge to the steady state relatively fast. Deviations of inflation (GDP and investments deflator, CPI) from the steady state caused by impulses of the years 2008–2009 abate relatively slowly. Of course, the convergence of observable variables to the steady state is guaranteed by the structure of the model.

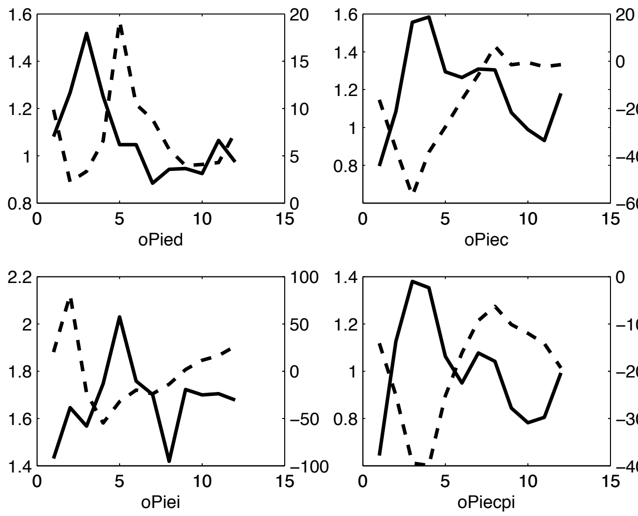
Figure C.1. Example of *ex ante* forecast with long horizon



Appendix D
Analysis of forecasts accuracy

Figures D.1–D.5 present the RMSE and MPE errors for observable variables of the model, depending on the forecast horizon. Tables D.1–D.2 present the mean values and standard deviations of RMSE and MPE errors calculated for the horizons $h = 1, 2, ..., 12$. Table D.3 presents the relations of RMSE errors of a DSGE model and naive forecasts depending on the forecast horizon. The data provided therein shall be analysed further herein in more detail[1].

Figure D.1. RMSE and MPE for deflators: GDP (oPied), consumption (oPiec), investments (oPiei) and CPI (oPiecpi)



RMSE — continuous line and left axis; MPE — dotted line and right axis.

The upper left panel in Figure D.1 shows that the root mean square error of the forecast of domestic inflation is the largest, about 1.5 p.p. for the forecast horizon of 3 quarters, afterwards it drops systematically to about 1 p.p. In relative terms the error fluctuates depending on the forecast horizon from several percent to nearly 20% for the 5 quarters' horizon. Positive

[1]The set of variables for which the accuracy of projections was tested covered — beside of all of the observable variables of the model used in the estimation of parameters — also two additional variables: consumption deflator (identified as $oPiec$) and expenditures on government consumption ($oGov$).

MPE statistics for each horizon means that inflation of domestic products is systematically underestimated — average MPE error amounts to about 7%, and its mean variability is above 4.5 p.p. Average RMSE error amounts to slightly over 1 p.p., while its mean variability is nearly 0.2 p.p. The DSGE model wins with the naive forecast in each forecast horizon, while its advantage grows along the extension of the horizon from about 25% for 1 quarter to 75% for 12 quarters. The mean advantage of the DSGE model over the naive forecast amounts to over 60%.

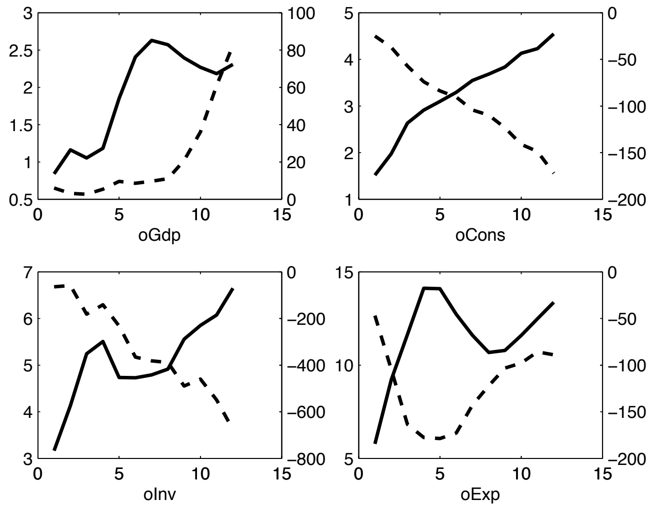
In the case of inflation of consumer goods prices (measured with consumption deflator), the forecast errors are similar, except that the variable is systematically overestimated. The upper right panel in Figure D.1 shows that root mean square error of the forecast of consumer goods inflation is the largest — about 1.6 p.p. — for the forecast horizon of 4 quarters, afterwards it drops systematically to about 1–1.2 p.p. In relative terms the error fluctuates depending on the forecast horizon from several percent to nearly −60% for the 3 quarters horizon. Negative MPE statistics for most of the horizons means that inflation of consumer goods prices is systematically overestimated — the average MPFE error amounts to −16%, however, its mean variability is nearly 20 p.p. An average RMSE error amounts to 1.2 p.p., while its mean variability is nearly 0.25 p.p. The DSGE model wins with the naive forecast in each forecast horizon, except 1 quarter horizon when the naive model proves to be better by slightly over 1%. As in the case of inflation of domestic products prices, the advantage of the DSGE model grows along the forecast horizon — from about 17% for forecasts of 2 quarters to 65% for 12 quarters forecasts. The mean advantage of the DSGE model over the naive forecast amounts to about 45%.

The bottom left panel of Figure D.1 shows that the root mean square error of the forecast of inflation of investment goods prices reaches the highest value — about 2 p.p. — for 5 quarters forecast and drops afterwards to about 1.4 p.p. for 8 quarters forecasts, then it grows again to 1.7 p.p. Before the 5th quarter the error oscillates around 1.4–1.8 p.p. MPE statistics show that inflation of investment goods prices is, thus, underestimated for short horizons of the forecast, overestimated within 3rd–8th quarters and underestimated from the 9th to 12th quarter. The average MPE error amounts to −0.8 p.p., while its mean variability to as much as 35 p.p. The average RMSE error is 1.7 p.p. and its mean variability is nearly 0.15 p.p. The DSGE model wins with the naive forecast in each forecast horizon. Identically as in the case of inflation of the prices of domestic products and consumer goods, the advantage of the DSGE model over the naive forecast amounts to nearly 44%.

Errors of CPI inflation forecasts are similar as in the case of consumption deflator. The bottom right panel of Figure D.1 shows that root mean square error of the forecast of CPI inflation is the highest — nearly 1.4 p.p. — for 3 quarters horizon and afterwards it drops to about 0.8 p.p., then, starting from the 9th quarter, it grows to 1%. In relative terms the error fluctuates depending on the forecast horizon from −40% for 3 quarters horizon to −5% for 8 quarters. MPE for each horizon is negative — CPI inflation is, thus, systematically overestimated, and the average MPE error amounts to −20%, while its mean variability is about 11 p.p. The average RMSE error amounts to 1 p.p. and its mean variability — to 0.22 p.p. The DSGE model wins with the naive forecast in each forecast horizon, beside of the first three periods when the naive model proves to be better by 20–50%. The advantage of the DSGE model grows along the forecast horizon — from 25% for 1 quarter to 65% for 12 quarters. The mean advantage of the DSGE model over the naive forecast amounts to over 56%.

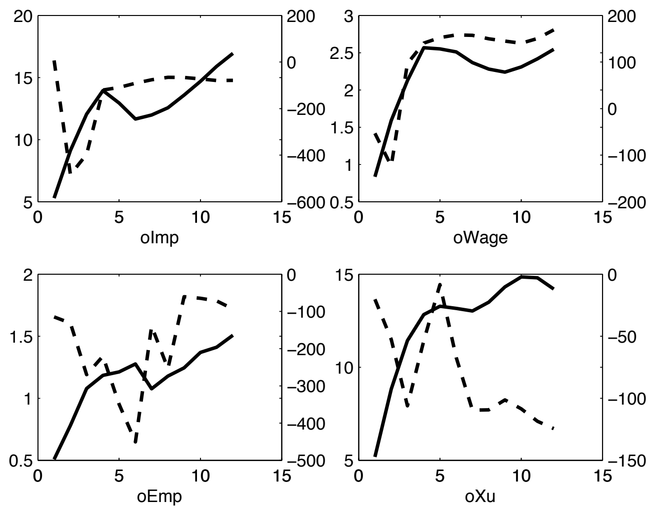
In RMSE errors of the forecasts of inflation of all the four types, a systematic pattern may be perceived, namely the forecasts are the best in the short horizon (1–2 quarters) and longer horizon (over 6–7 quarters), while approximately between the 2nd and the 6th quarter the forecast errors are significantly higher than for short and longer horizons. Additionally, the advantage of the DSGE model over the naive forecast grows systematically along the forecast horizon. In the case of other observable variables, the RMSE error has a tendency to grow with the forecast horizon, which is presented in Figures D.2–D.5.

Figure D.2. RMSE and MPE or the variables of GDP dynamics (oGdp), consumption (oCons), investment expenditures (oInv) and export (oExp)



RMSE — continuous line and left axis; MPE — dotted line and right axis.

Figure D.3. RMSE and MPE for import dynamics (oImp), real wages (oWage), employment (oEmp) and real exchange rate of the dollar (oXu).



RMSE — continuous line and left axis; MPE — dotted line and right axis.

The upper left panel of Figure D.2 shows that root mean square error of the forecast of the annual growth rate of the GDP grows systematically between the 1st and 7th quarter from 0.85 to 2.6 p.p., slightly decreasing after the 7th quarter, but still remains at the level of above 2 p.p. In relative terms, the error grows from several percent in the first quarters to 60–80% in the last quarters. Production is, thus, systematically underestimated. The average MPE error amounts to 21%, while its mean variability is 25 p.p. The average RMSE error amounts to 1.9 p.p. and its mean variability to — 0.65 p.p. The DSGE model wins with the naive forecast in each forecast horizon. The advantage of the DSGE model grows also with extension of the forecast horizon — from about 10% for 1 quarter forecast to nearly 50% for 12 quarters forecasts. The mean advantage of the DSGE model over the naive forecast amounts to 40%.

The upper right panel of Figure D.2 shows that the root mean square error of the forecast of the annual growth rate of consumption systematically grows in the horizon of the forecast from 1.5 p.p. in the 1st quarter to 4.5 p.p. in the 12th quarter. The error grows along the whole forecast horizon also in relative terms — from −24% to −170%. The growth rate of consumption is, thus, systematically overestimated. The average MPE error amounts to −97%, while its mean variability is 45 p.p. The average RMSE error amounts to 3.3 p.p. and its mean variability — to 0.9 p.p. The errors of forecasts of consumption dynamics are, thus, nearly two times higher than the errors of the forecasts of output growth. The DSGE model wins with the naive forecast in each forecast horizon. The advantage of the DSGE model grows with the extension of the forecast horizon — from nearly 20% for 1 quarter to 35% for 12 quarters. The mean advantage of the DSGE model over the naive forecast is 45%.

In absolute terms, even larger forecast errors are generated by the model for annual growth rates of investments, export and import, as well as the exchange rate. The bottom left panel of Figure D.2 shows that the root mean square error of the forecast of annual growth rate of investments grows in the forecast horizon from about 3 p.p in the 1st quarter to nearly 7 p.p. in the 12th quarter. The growing trend is upset only in 5 the quarters' horizon. In relative terms the error grows systematically from −60% to nearly −800%. The growth rate of investments is systematically overestimated. The average MPE error amounts to −100%, while its mean variable — to 45 p.p. The average RMSE error amounts to 5 p.p. and its mean variability equals to 0.9 p.p. The DSGE model wins with the naive forecast in each forecast horizon. The advantage of the DSGE model grows also with the extension of the forecast horizon — from nearly 15% for 1 quarter to 60% for 12 quarters. The mean advantage of the DSGE model over the naive forecast amounts to 45%.

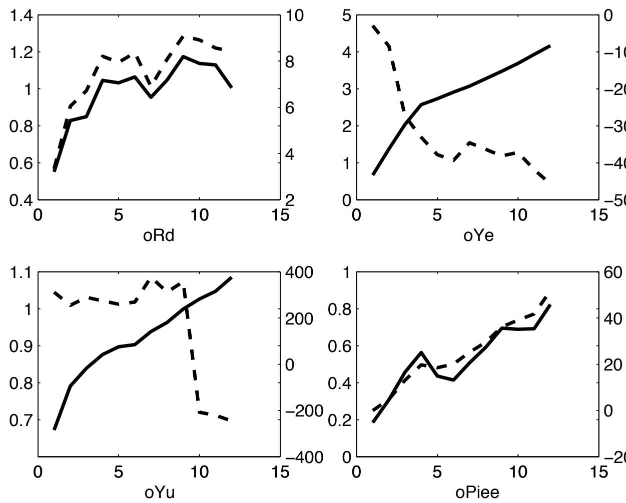
From the accuracy point of view, the errors of import and export forecasts are similar. The bottom right panel of Figure D.2 shows that the root mean square error of the forecast of annual growth rate of export grows in the forecast horizon from about 6 p.p. in the 1st quarter to 14 p.p. in the 5th–6th quarter, and then drops to about 10 p.p. to rise again to about 13 p.p. In relative terms the error grows systematically from minus several or so to −180%, and then drops to about −90%. The growth rate of import is systematically overestimated. The average MPE error amounts to −123%, while its mean variability equals 43 p.p. The average RMSE error amounts to 11.5 p.p. and its mean variability — to 2.3 p.p. The DSGE model wins with the naive forecast in each time horizon, except the horizon of 1 and 2 quarters when the naive forecast is on the average by 36% and 6.5% better, respectively. The advantage of the DSGE model also grows with the forecast horizon, except the 11th quarter — from 5% for 3 quarters to 35% for 12 quarters. The mean advantage of the DSGE model over the naive forecast amounts to 17%.

The upper left panel of Figure D.3 shows that the root mean square error of the forecast of annual growth rate of import grows in the forecast horizon from about 5 p.p. in the 1st quarter to 17 p.p. in the 12th quarter, except quarters 5 and 6. In relative terms the error reaches −480% in the 2nd quarter and then drops systematically to (−60)–(−80)%. The import growth rate is systematically overestimated. The average MPE error amounts to −136% and its mean variability is 146 p.p. The average RMSE error amounts to 12.5 p.p. and its mean variability equals to about 3 p.p. The DSGE model wins with the naive forecast in each forecast horizon. The advantage of the DSGE model grows — except quarters 11 and 12 — with the forecast horizon from 3% for 3 quarters to 37% for 12 quarters. The mean advantage of the DSGE model over the naive forecast amounts to 32%.

The upper right panel of Figure D.3 shows that the root mean square error of the forecast of annual growth rate of real wages grows with the forecast horizon from about 0.8 p.p. in the 1st quarter to 2.5 p.p. in the 12th quarter. In relative terms the error oscillates between −120% in the 2nd quarter and −170% in the 12th quarter. The import growth rate is overestimated for the horizon of 1 and 2 quarters, while it is underestimated for the other horizons. The average MPE error amounts to −106% and its mean variability equals 94 p.p. The average RMSE error amounts to 2.2 p.p. and its mean variability is 0.5 p.p. The DSGE model wins with the naive forecast in each forecast horizon. The advantage of the DSGE model grows with the forecast horizon from 15% for 1 quarter to 38% for 12 quarters. The mean advantage of the DSGE model over the naive forecast amounts to 34%.

The bottom left panel of Figure D.3 shows that the root means square error of the forecast of annual growth rate of employment grows along the forecast horizon from about 0.5 p.p. in the 1^{st} to 1.5 p.p. in the 12^{th} quarter, while the trend is upset only in the horizon of 7 quarters. In relative terms the error reaches the value of −540% in the 6^{th} quarter and falls to (−60)–(−90)% in the 12^{th} quarter. The growth rate of employment is moderately overestimated. The average MPE error amounts to −185% an its mean variability is 125 p.p. The average RMSE error amounts to 1.15 p.p. and its mean variability — to about 0.3 p.p. The DSGE model wins with the naive forecast in each forecast horizon. The advantage of the DSGE model grows with the extension of the forecast horizon — from 13% for 1 quarter to about 43% for 10–12 quarters. The mean advantage of the DSGE model over the naive forecast amounts to 34%.

Figure D.4. RMSE and MPE for the interest rate (oRd), GDP dynamics in the euro area (oYe) and in the USA (oYu), plus inflation in the euro area (oPiee)

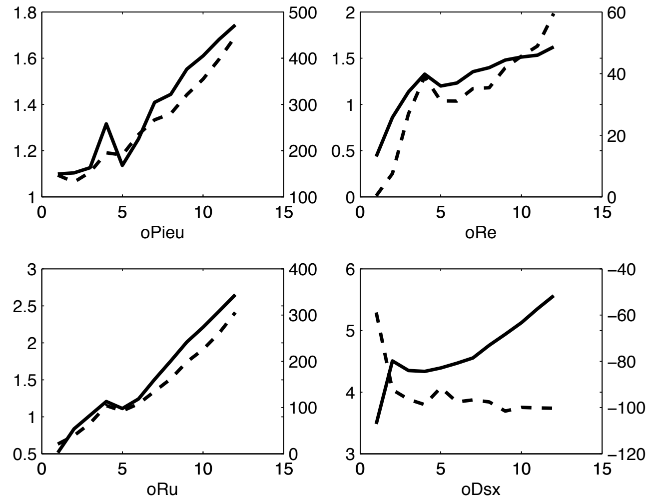


RMSE — continuous line and left axis; MPE — dotted line and right axis.

The upper left panel of Figure D.4 shows that the root mean square error of the forecast of annual nominal interest rate grows together with the forecast horizon, both in absolute and relative terms. The RMSE error grows from 0.5 p.p. in the horizon of 1 quarter to 1–1.5 p.p. for quarters 9–12, while the MPE error grows from 3% to 8.5 − 9%. The nominal interest rate is moderately underestimated — the average MPE error amounts to 7% and its mean variability is about 1 p.p. The average RMSE error amounts to 1 p.p. and its mean variability — to about 0.2 p.p. The DSGE model wins with the naive forecast in each forecast horizon. The advantage of the DSGE model ranges from 5% in the horizon of 1 quarter, through 40% in the horizon of 7 quarters, to 17% for 12 quarters. The mean advantage of the DSGE model over the naive forecast amounts to 28%.

The other three panels of Figure D.4 and the whole Figure D.5 present the errors of forecasts of foreign variables whose dynamics results from the SVAR model which is exogenous in reference to the DSGE model and, therefore, we shall not discuss them in detail. For all of the foreign variables the growing trend of the root mean square error is visible along the forecast horizon. Production in the euro area is moderately overestimated and production in the dollar area is underestimated for the horizons of 1 to 9 quarters and afterwards it is overestimated. Inflation

Figure D.5. RMSE and MPE for inflation in the USA (oPieu), interest rate in the euro area (oRe) and in the USA (oRu), plus the dynamics of the nominal cross rate (oDsx)



RMSE — continuous line and left axis; MPE — dotted line and right axis.

and nominal interest rates in both areas are usually underestimated, while the real exchange rate of EUR/USD is overestimated, on the average. In the case of all of the foreign variables the VAR model generates better forecasts than the naive model for all of the tested forecast horizons, except inflation in the euro area in 11 and 12 quarters forecast horizon and inflation in the dollar area in the horizon of 6 quarters.

Table D.1. Mean and standard deviation of the RMSE statistics

Variable	oPied	oPiec	oPiei	oPiecpi	oGdp	oCons	oInv	oExp	oImp	oWage
Mean	1.08	1.20	1.68	1.00	1.90	3.28	5.11	11.51	12.56	2.19
Standard deviation	0.18	0.24	0.16	0.22	0.66	0.91	0.92	2.31	3.08	0.51
Variable	oEmp	oXu	oRd	oYe	oYu	oPiee	oPieu	oRe	oRu	oDsx
Mean	1.15	12.45	0.98	2.82	0.92	0.53	1.37	1.26	1.54	4.65
Standard deviation	0.28	2.82	0.17	1.04	0.12	0.18	0.24	0.33	0.67	0.55

Table D.2. Mean and standard deviation of the MPE statistics

Variable	oPied	oPiec	oPiei	oPiecpi	oGdp	oCons	oInv	oExp	oImp	oWage
Mean	7.17	-16.48	-0.83	-19.26	21.38	-97.27	-332.39	-123.74	-136.14	106.96
Standard deviation	4.64	19.58	35.39	11.17	25.67	45.04	197.47	42.96	146.55	94.09
Variable	oEmp	oXu	oRd	oYe	oYu	oPiee	oPieu	oRe	oRu	oDsx
Mean	-185.13	-81.69	7.55	-31.92	168.99	24.96	260.11	33.56	143.20	-94.30
Standard deviation	124.96	39.77	1.61	13.09	240.41	15.40	103.26	16.50	89.16	11.58

Table D.3. Comparison of forecasts of the $\mathbb{SOE}^{PL-2009}$ model with the naive forecast

Horizon	oPied	oPiec	oPiei	oPiecpi	oGdp	oCons	oInv	oExp	oImp	oWage
1	0.76	1.01	0.81	1.53	0.91	0.74	0.85	1.37	0.97	0.84
2	0.62	0.84	0.61	1.36	0.69	0.57	0.69	1.06	0.90	0.81
3	0.56	0.81	0.42	1.19	0.44	0.46	0.59	0.95	0.86	0.69
4	0.40	0.68	0.35	0.86	0.38	0.54	0.47	0.92	0.73	0.64
5	0.33	0.53	0.43	0.62	0.54	0.56	0.40	0.85	0.63	0.60
6	0.30	0.50	0.41	0.54	0.64	0.53	0.35	0.75	0.55	0.63
7	0.26	0.50	0.41	0.55	0.66	0.59	0.35	0.76	0.59	0.62
8	0.25	0.50	0.32	0.50	0.73	0.52	0.32	0.69	0.58	0.62
9	0.28	0.41	0.39	0.37	0.63	0.48	0.35	0.65	0.57	0.56
10	0.25	0.33	0.36	0.31	0.53	0.51	0.36	0.67	0.58	0.60
11	0.31	0.30	0.40	0.30	0.51	0.54	0.36	0.75	0.63	0.64
12	0.25	0.35	0.34	0.37	0.52	0.66	0.40	0.66	0.63	0.62
Horizon	oEmp	oXu	oRd	oYe	oYu	oPiee	oPieu	oRe	oRu	oDsx
1	0.87	0.85	0.94	0.83	0.97	0.85	0.82	0.71	0.77	0.79
2	0.78	0.85	0.82	0.78	0.83	0.78	0.82	0.78	0.72	0.72
3	0.79	0.88	0.63	1.05	0.88	0.75	0.94	0.75	0.66	0.60
4	0.73	0.88	0.71	0.98	1.01	0.71	0.79	0.78	0.66	0.67
5	0.69	0.93	0.65	0.88	1.00	0.56	0.98	0.66	0.51	0.81
6	0.68	0.98	0.65	0.81	0.86	0.64	1.09	0.64	0.48	0.93
7	0.56	1.03	0.58	0.84	0.91	0.80	0.83	0.66	0.52	0.90
8	0.56	1.08	0.63	0.81	1.12	0.81	0.94	0.65	0.58	0.83
9	0.55	1.12	0.71	0.80	1.13	0.76	0.90	0.68	0.66	0.76
10	0.57	1.16	0.73	0.78	1.00	0.73	0.81	0.67	0.71	0.73
11	0.56	1.19	0.80	0.78	0.90	1.00	0.97	0.67	0.79	0.88
12	0.58	1.20	0.84	0.80	0.89	1.24	0.85	0.73	0.89	1.25

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