Managing Dairy Profit Risk Using Weather Derivatives

Gang Chen, Matthew C. Roberts, Cameron Thraen*

Paper presented at the NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management
St. Louis, Missouri, April 21-22, 2003

---

*The authors are, respectively, Ph.D. student, assistant professor, and associate professor, Department of Agricultural, Environmental, and Development Economics, The Ohio State University, Columbus, OH 43210. Corresponding author is Gang Chen at chen.796@osu.edu. We thank Normand R. St-Pierre for his comments and suggestions.

Managing Dairy Profit Risk Using Weather Derivatives

Practitioner's Abstract

Weather conditions are the primary dairy production risk. Hot and humid weather induces heat stress, which reduces both the quantity and quality of milk production. Traditional heat abatement technologies control the environment through ventilation, misting or evaporative cooling. Usually, they can increase the producers' expected profit, but cannot cover all the profit losses from heat stress. Weather derivatives could reduce weather-induced profit risk and thus act as a substitute for traditional abatement technologies in the aspect of risk management. We test the risk management value of weather derivatives in a utility maximization framework. The result is that weather derivatives offer an opportunity to improve the efficient portfolio frontier, and simultaneously using weather derivatives and abatement equipment is more favorable than using each of them alone.

Keywords: abatement technology, mean-variance efficiency, profit risk, weather derivatives

Introduction

Weather conditions are the primary dairy production risk. Hot and humid weather induces heat stress, which reduces both the quantity and quality of dairy production (Barth; Thompson) . Traditional heat abatement technologies control the environment through ventilation, misting or evaporative cooling (Turner et al.; Lin et al.) . Adoption of abatement equipment, however, is hindered by its high initial cost and possibly long payback period, especially for small- and medium-scale firms. Moreover, the abatement equipment is only seasonally useful. Weather-based derivatives are a relatively new financial product that pay during undesirable weather conditions. These products cannot reduce production risk but can offset profit losses. They can be purchased to cover only certain time periods, and may be substitutes for abatement equipment at the margin. The objective of this study is to test the risk management value of weather derivatives to reduce weather-induced profit risk and to act as a substitute to traditional abatement technologies.

The analysis is conducted by first constructing two profit models. One is for a representative producer's profit without using weather derivatives or abatement technologies; the other is for his profit with using both of these two instruments. Then the producer's optimal portfolio choice is derived in a utility maximization framework. From the utility framework, the benefit of using weather derivatives for managing risk is measured. The assumptions implicit in this paper are that (1) the producer has Pratt's absolute risk aversion and choose mean-variance efficient portfolios with a one-period horizon; (2) weather conditions are the only common risk factor to all producers in summer; and (3) the market is fully efficient in that there are no transaction costs, indivisibilities, taxes, or basis risk.

The 35-year weather data of Summit County, Ohio are used for the empirical illustration. Corresponding to the weather data, the representative producer's milk loss from heat stress and reduced loss from using abatement are derived by employing the results of St-Pierre, Cobanov, and Schnitkey (SCS) . Our result is that although abatement technologies are effective at reducing economic losses from heat stress, using weather derivatives can significantly increase the producer's utility compared to only using traditional abatement technologies.

Background Knowledge

Weather derivatives have several unique properties. The payoff is calculated based on a weather index. So weather derivatives have an advantage over traditional insurance for hedging against weather-related losses. Because there is no need to prove damages to receive payoffs, there is little moral hazard. Moreover, since weather information is perfectly symmetric, adverse selection is eliminated.

Weather derivatives have been the focus of much research. Dischel argues that due to the non-tradable nature of weather, weather derivatives cannot be valued by the Black-Scholes option pricing model, and instead a stochastic Monte Carlo simulation with a weather forecast model may be more effective. Turvey examines the weather effects on crop yields and states that weather derivatives might be used as a form of agricultural insurance. Cao and Wei propose a model for daily temperature, which can incorporate several key properties such as seasonal cycles and uneven variations throughout the year and develop a pricing model based on Lucas' equilibrium asset pricing model. Diebold and Campbell propose a non-structural time series model of daily average temperature, which incorporates seasonal changes of temperature levels and variations throughout the year. Most previous research only examines temperature and/or rainfall derivatives to manage weather risk for energy and field crop markets. To our knowledge, there has been no research on the potential of using weather derivatives to hedge against livestock profit risk.

Economic losses are induced in the dairy industry when effective temperature conditions are out of dairy cows' thermal comfort zone. According to SCS, heat stress in dairy cattle is a function of the Temperature-Humidity Index (THI, also known informally as the 'heat index'). Johnson reports that a THI higher than 72 degrees is likely to have adverse effects on per-cow yield. In SCS, it is suggested that the threshold of THI to trigger heat stress should be lowered to 70 degrees because of the lower heat tolerance of the current selection of dairy cows. So 70 degrees is used as a threshold for risk from heat stress, $THI_{threshold}$. According to NOAA , the standard formula of THI is: THI = T – (0.55 – 0.55 RH) (T – 58), where T is temperature in degrees Fahrenheit and RH is relative humidity in percent. Since RH is is expressed as a percentage, it is easy to see that THI is positively correlated with temperature.

Theoretical Analysis

Consider a dairy producer who produces without using abatement equipment or weather derivatives. His profit is $\widetilde{y} = P \cdot \widetilde{Q} - TC$, where P is milk price, $\widetilde{Q}$ is the stochastic yield, and TC denotes a total cost. For analytical simplicity, it is assumed there is no price risk; therefore price is normalized to unity. The tilde ($\widetilde{\ }$) denotes a random variable.

Suppose expected profit of a producer is his historical average, μ, so the difference between $\widetilde{y}$ and μ is his profit risk. The profit risk is orthogonally decomposed into two parts. One is systematic risk which comes from weather conditions; the other is nonsystematic risk which reflects the individual's production variability not arising from weather and is assumed uncorrelated with weather conditions.

(1) $$\widetilde{y} = \mu + \theta \cdot f(\widetilde{x}) + \widetilde{\varepsilon},$$

where

(2) $$\widetilde{x} = E(\widetilde{z}) - \widetilde{z}$$

(3) $$\widetilde{z} = \max(\widetilde{THI} - THI_{threshold},\ 0)$$

(4) $$\theta = \mathrm{cov}(\widetilde{y}, f(\widetilde{x}))/\mathrm{var}(f(\widetilde{x}))$$

(5) $$E(\widetilde{y}) = \mu,\quad E(\widetilde{\varepsilon}) = 0,\quad \mathrm{var}(\widetilde{\varepsilon}) = \sigma^2_{\widetilde{\varepsilon}},\quad \mathrm{cov}(\widetilde{z}, \widetilde{\varepsilon}) = 0,\quad \mathrm{cov}(\widetilde{x}, \widetilde{\varepsilon}) = 0.$$

The coefficient θ quantifies the sensitivity of the producer's individual profit to systematic risk. The factor $\widetilde{z}$, which is common to all producers in a region, measures the degree of heat stress, and the factor $\widetilde{x}$ denotes the weather condition compared to its expectation. If $\widetilde{z}$ is lower than $E(\widetilde{z})$, that means the heat stress is milder than its expectation. In this case, $\widetilde{x}$ is positive. And $f(\widetilde{x})$ captures systematic risk and increases with $\widetilde{x}$. The functional form of $f(\widetilde{x})$ is assumed to be linear, i.e. $f(\widetilde{x}) = \alpha \cdot \widetilde{x}$, where α is a positive parameter of the linear relationship. The final term $\widetilde{\varepsilon}$ is a nonsystematic risk component.

Then equation (1) becomes,

(6) $$\widetilde{y} = \mu + \theta \cdot \alpha \cdot \widetilde{x} + \widetilde{\varepsilon} = \mu + \beta \cdot \widetilde{x} + \widetilde{\varepsilon}$$

where

(7) $$\beta = \mathrm{cov}(\widetilde{y}, \widetilde{x})/\ \mathrm{var}(\widetilde{x}).$$

Suppose that weather derivatives are available for purchase. Since here the risk is from excessively high THI, weather derivatives that will be used are focused on weather call options. The underlying index is $\widetilde{THI}$, and the strike price is $THI_{threshold}$. The payoff from a weather call option is:

$$\widetilde{n} = \max(\widetilde{THI} - THI_{threshold},\ 0) = \widetilde{z}. \tag{8}$$

The hypothetical[1] option premium is calculated on the basis of actuarial fairness. So purchasing weather options cannot change the producer's expected profit. The option premium equals the expected payoff:

$$\pi = E(\widetilde{n}) = E(\widetilde{z}). \tag{9}$$

Also suppose that the producer is free to choose his abatement equipment investment η ($\eta \geq 0$; where $\eta = 0$ means he does not install abatement equipment). By using abatement equipment, the production loss from heat stress can be reduced. The biological functional form of the effectiveness of abatement equipment is formulated as:

$$\Delta loss = g(\eta, \widetilde{THI}) = (a + b \cdot \widetilde{THI}) \cdot \sqrt{\eta} \tag{10}$$

where $\Delta loss$ is the reduced profit loss, i.e. the profit gain from using abatement, η is abatement investment,[2] and a and b are parameters.

It is easy to see that $\Delta loss$ is increasing with η and $\widetilde{THI}$. When $\eta = 0$, $\Delta loss$ is also equal to 0. And with fixed η, $\Delta loss$ is increasing with $\widetilde{THI}$. That is because although the profit is low when $\widetilde{THI}$ is high, the reduced profit loss will be high with abatement equipment; on the other hand, when $\widetilde{THI}$ is low (i.e. weather is good), the abatement equipment is not of much use, so the reduced loss is low. Since the net profit from using abatement technology is $(a + b \cdot \widetilde{THI}) \cdot \sqrt{\eta} - \eta$, that is to say if $\widetilde{THI}$ is high enough, the net profit from investing abatement equipment will be positive; otherwise, the net profit is negative.

With weather options and abatement equipment, the producer's net profit equals:

$$\widetilde{y}^{net} = \widetilde{y} + \phi \cdot (\widetilde{n} - \pi) + \Delta loss - \eta \tag{11}$$

where ϕ is weather option purchase amount. Therefore, there are two elements that the producer is free to choose: spending on weather options, ϕ, and spending on abatement, η. It is assumed these two choices are determined simultaneously in a portfolio taking the remaining parameters as given.

The producer's optimal portfolio choice of weather option purchase and abatement investment is derived using a utility maximization model.[3] The producer is assumed to

[1] There has not been a weather derivative on THI in the security market yet.

[2] Since abatement equipment is useful for many years once installed, the installation cost is annualized at a certain rate (say 15%) for yearly analysis. When "burn-rate" method is used to forecast weather, the expected THI will vary little over years. So, the producer's yearly optimal decision on weather option purchase amount and abatement investment will not change much over years once determined based on current information. This is a simple one-period, one-agent model.

[3] This framework is equivalent to expected utility maximization if (net) profit is distributed normally

have a mean-variance utility function of

$$U = E(\bullet) - \frac{1}{2}A \cdot \mathrm{var}(\bullet) \tag{12}$$

where A is an index of agents' aversion to taking on risk. Then the representative producer's objective is to choose his optimal option purchase ϕ and abatement spending η to maximize his utility from using weather options and abatement equipment:

$$\max_{\phi,\ \eta}\ U^{net} = E(\widetilde{y}^{net}) - \frac{1}{2}A \cdot \mathrm{var}(\widetilde{y}^{net})\,. \tag{13}$$

Specifically,

$$\begin{aligned} \max_{\phi,\ \eta}\ U^{net} \ &= \ E(\widetilde{y}) + E(\Delta loss - \eta) - \frac{1}{2}A \cdot [\mathrm{var}(\widetilde{y}) + \phi^2 \mathrm{var}(\widetilde{n}) + \mathrm{var}(\Delta loss) \\ &\quad +2\phi \mathrm{cov}(\widetilde{y}, \widetilde{n}) + 2\mathrm{cov}(\widetilde{y}, \Delta loss) + 2\phi \mathrm{cov}(\widetilde{n}, \Delta loss)]\,, \end{aligned} \tag{14}$$

$$\begin{aligned} U^{net} \ &= \ \mu + (a + b\mu_{\widetilde{THI}})\sqrt{\eta} - \eta - \frac{1}{2}A \cdot [\beta^2 \sigma^2_{\widetilde{z}} + \sigma^2_{\widetilde{\varepsilon}} + \phi^2 \sigma^2_{\widetilde{z}} + b^2 \eta \sigma^2_{\widetilde{THI}} \\ &\quad -2\beta\phi\sigma^2_{\widetilde{z}} - 2\beta b \sqrt{\eta} \mathrm{cov}(\widetilde{THI}, \widetilde{z}) + 2\phi b \sqrt{\eta} \mathrm{cov}(\widetilde{THI}, \widetilde{z})]. \end{aligned} \tag{15}$$

Take first order condition with respect to ϕ and η respectively,

$$\phi\sigma^2_{\widetilde{z}} - \beta\sigma^2_{\widetilde{z}} + b\sqrt{\eta}\mathrm{cov}(\widetilde{THI}, \widetilde{z}) = 0\,, \tag{16}$$

$$(a+b\mu_{\widetilde{THI}}) \cdot \frac{1}{2}\eta^{-\frac{1}{2}} - 1 - \frac{1}{2}A[b^2\sigma^2_{\widetilde{THI}} - \beta b \eta^{-\frac{1}{2}} \mathrm{cov}(\widetilde{THI}, \widetilde{z}) + \phi b \eta^{-\frac{1}{2}} \mathrm{cov}(\widetilde{THI}, \widetilde{z})] = 0\,. \tag{17}$$

Then equation system of (16) and (17) can be solved simultaneously.

It follows from (16) that

$$\phi^* = \beta - b\frac{\mathrm{cov}(\widetilde{THI}, \widetilde{z})}{\sigma^2_{\widetilde{z}}}\sqrt{\eta}\,. \tag{18}$$

Substituting (18) into (17) and rearranging, it follows

$$\sqrt{\eta}^* = \frac{a + b\mu_{\widetilde{THI}}}{2 + Ab^2[\sigma^2_{\widetilde{THI}} - \frac{\mathrm{cov}^2(\widetilde{THI}, \widetilde{z})}{\sigma^2_{\widetilde{z}}}]}\,. \tag{19}$$

It follows from (18) that:

Proposition 1 . *The optimal weather option purchase amount is decreasing with abatement equipment investment. Thus it indicates that weather options can act as*

and producers' utility function is exponential. See Pratt (1964) and Meyer (1987).

a substitute for abatement equipment.

In (19), it is not difficult to see that the denominator is positive, because $b^2[\sigma^2_{\widetilde{THI}} - \frac{\text{cov}^2(\widetilde{THI},\tilde{z})}{\sigma^2_{\tilde{z}}}] = b^2\sigma^2_{\widetilde{THI}}[1 - \frac{\rho^2_{\widetilde{THI},\tilde{z}} \cdot \sigma^2_{\widetilde{THI}} \cdot \sigma^2_{\tilde{z}}}{\sigma^2_{\widetilde{THI}} \cdot \sigma^2_{\tilde{z}}}] = b^2\sigma^2_{\widetilde{THI}}(1 - \rho^2_{\widetilde{THI},\tilde{z}}) > 0$, since the correlation coefficient $\rho_{\widetilde{THI},\tilde{z}} \in (0,1)$.

Since $\Delta loss = (a+b\widetilde{THI}) \cdot \sqrt{\eta}$, this inequality $(a+b\mu_{\widetilde{THI}}) > 0$ implies that abatement investment can reduce profit loss from heat stress. Then the numerator of (19) is also positive. So it follows that:

Proposition 2 . *The optimal abatement investment is positive.*

And Proposition 3 also follows from (19):

Proposition 3 . *The optimal abatement investment is negatively related to the producer's risk aversion degree (i.e. A). That is, the more risk-averse the producer, the less he would invest in abatement equipment.*

By substituting (19) back into (18), it follows:

$$(20) \qquad \begin{aligned} \phi^* &= \beta - b\frac{\text{cov}(\widetilde{THI},\tilde{z})}{\sigma^2_{\tilde{z}}} \cdot \frac{a + b\mu_{\widetilde{THI}}}{2 + Ab^2[\sigma^2_{\widetilde{THI}} - \frac{\text{cov}^2(\widetilde{THI},\tilde{z})}{\sigma^2_{\tilde{z}}}]} \\ &= \beta - b(a + b\mu_{\widetilde{THI}}) \cdot \frac{\rho_{\widetilde{THI},\tilde{z}}\sigma_{\widetilde{THI}}}{[2 + Ab^2\sigma^2_{\widetilde{THI}}(1 - \rho^2_{\widetilde{THI},\tilde{z}})] \cdot \sigma_{\tilde{z}}}. \end{aligned}$$

From (20), it follows that:

Proposition 4 . *The optimal option purchase amount is increasing with β. That means that the more the producer's profit is sensitive to the systematic risk, the more options he should purchase, ceteris paribus.*

It also follows from (20):

Proposition 5 . *The optimal option purchase amount is increasing with producer's risk aversion degree, A.*

Proposition 6 . *The optimal option purchase is decreasing with a and b.*

By substituting (20) and (19) back into (15), the maximized increased utility from using weather options and abatement can be derived from:

$$(21)\qquad \Delta U = U^{net}(\phi^*,\eta^*) - U(0,0)$$
$$= (a + b\mu_{\widetilde{THI}})\cdot\sqrt{\eta} - \eta - \frac{1}{2}A[\phi^2\sigma^2_{\widetilde{z}} + b^2\eta\sigma^2_{\widetilde{THI}}$$
$$-2\beta\phi\sigma^2_{\widetilde{z}} - 2\beta b\sqrt{\eta}\mathrm{cov}(\widetilde{THI},\widetilde{z}) + 2\phi b\sqrt{\eta}\mathrm{cov}(\widetilde{THI},\widetilde{z})]\,.$$

It is also viable to compare it with the cases in which the producer only uses one of these two instruments. The simultaneous usage of weather options and abatement equipment will be more favorable. So, weather derivatives can act as a substitute for traditional abatement technologies.

Data and Empirical Results

Data

For the empirical part of the study, we need to estimate equations (6) and (10). Three types of data are needed: weather data, profit data and abatement investment data. The 35-year (1949 to 1964 and 1984 to 2002) weather data of Summit County, Ohio are used.[4] The weather data include daily maximum and minimum temperature and daily maximum and minimum relative humidity. Daily temperature and dew point[5] both follow routinely seasonal patterns each year. So the " burn-rate" method works well with them for pricing weather options. Daily maximum temperature-humidity index (THI) can be derived from daily maximum temperature and minimum relative humidity.[6] Note in the models, $\widetilde{THI}$ corresponds to maximum THI. When maximum THI is lower than 70 degrees in a day, there is no heat stress for dairy cows.

Corresponding to the weather data, a representative producer's milk loss from heat stress and reduced loss from using abatement equipment are generated by employing the results in SCS.[7] Abatement investment cannot change in a relatively long period once fixed. Also weather options are assumed to be written on summer basis, i.e. the payoff is cumulative $\widetilde{n}$ of a summer and premium is the expected payoff. Thus, equations (6) and (10) are estimated based on cumulative summer data. Summer period is set from May 1st to Oct. 31st every year, because 97% of heat stress occurs in this period.

---

[4]It is a quite common phenomenon that daily relative humidity data are missing across weather stations in NOAA database.

[5]Dew point measures how much water vapor is in the air. In many places, the air's total vapor content varies only slightly during an entire day, and so it is the changing air temperature that primarily regulates the full variation in relative humidity. Related information can be found at: http://www.usatoday.com/weather .

[6]In a day, the maximum THI is in the afternoon, when the temperature is highest and relative humidity is lowest; and the minimum THI is at night, when the temperature is lowest and relative humidity is highest.

[7]See the Appendix for detail.

Table 1 shows the descriptive statistics of the cumulative summer weather data. And figure 1 is the histogram of cumulative $\widetilde{z}$.

Estimate β in Equation (6)

Following SCS, $THI_{threshold}$ is set as 70 degrees. From the weather data and the SCS milk loss model, we calculate the daily milk loss during summers of the 35 years and the corresponding daily $\widetilde{THI}$. Then by accumulating the milk loss and $\widetilde{z} = \max(\widetilde{THI} - THI_{threshold},\ 0)$ during each summer in the 35 years, we have 35 observations of accumulated profit loss and $\widetilde{x} = E(\widetilde{z}) - \widetilde{z}$. From a least squares regression, β is estimated, which is 0.5635 kg milk per cow. That is to say each degree of $\widetilde{z}$ beyond its mean will induce 0.5635 kg milk loss. The milk price is set as \$ 0.287/kg as in SCS, so the milk loss is \$ 0.1617 per degree of $\widetilde{x}$.

Estimate a and b in Equation (10)

We put the daily summer weather data into the SCS abatement effect model[8] to calculate the daily reduced THI corresponding to seven abatement levels. Multiplying the estimated β and milk price, we calculate the reduced profit loss (in dollars) due to abatement investment (in dollars). The daily reduced profit loss and THI are accumulated for each summer. Thus there are 35 observations of accumulated reduced profit loss and accumulated THI for each of the seven abatement investment levels. By a least squares regression, a and b are estimated as -57.4080 and 0.005107 respectively.

Results

In the following three scenarios, the producer's risk aversion level, which is represented by Pratt's Absolute Risk Aversion (PARA), is set as 0.20.[9]

Scenario 1: With the estimates of a, b and β, we can calculate the optimal portfolio choice and the corresponding increased utility.

[8] In SCS there are three abatement effect models corresponding to three abatement intensity levels. The first model is for only using fans or sprinklers; the second model is for a combination of fans and sprinklers; and the third model is for a specific system, the Korral Cool system, which is used in the Southwest and other dry and hot areas. In the research, we use the second model, and based on this model, we linearly simulate six abatement effect functions corresponding to six different fixed cost levels. See Appendix B.

[9] See, for example, Pratt (1964). Note that in this paper, we make no assumption about whether the risk aversion parameter is constant, decreasing, or increasing with initial wealth levels. We are studying a representative farmer faces an opportunity to buy weather options which will not change his expected wealth level and needs to decide how much money to invest on weather options. So we have an implicit assumption that changes of expectation and variance of profit due to using abatement equipment and weather options will not affect his risk aversion degree.

By equations (19) and (20), for a yearly management decision, the optimal abatement investment for a cow is 16.4275 dollars, and the optimal option purchase amount is 0.13535 shares (i.e. $ 75.2821) per cow. The maximized increased utility is 52.3362 dollars in certainty equivalent.

Scenario 2: If the producer only uses abatement system to manage production risk, the optimal investment level is 26.7004 dollars per cow. The increased utility is 28.7252 dollars.

Scenario 3: If the producer only uses weather options to manage profit risk, the optimal purchase amount is 0.16171 shares (i.e. $ 89.9429) per cow. The increased utility is 35.6091 dollars.

The annual net revenue from a dairy cow typically is around $ 330.[10] And according to our data, the mean and variance of the annual revenue loss due to heat stress are $ 49.6856 and 411.1898 respectively. Thus by the mean-variance model, the utility loss of a farmer with PARA of 0.20 is $(-49.6856 - \frac{1}{2} \cdot 0.2 \cdot 411.1898) = -90.8046$ dollars in certainty equivalent. In scenario 1, with using both weather options and abatement equipment, the increased utility is 52.3362 dollars in certainty equivalent. So the optimal use of these two instruments can reduce utility loss by 57.64%.

Figure 3 shows the increased utility corresponding to different PARAs in the three scenarios. The PARAs range from 0 to 0.30. The optimal portfolio choices bring more utility than only using abatement equipment or weather options. If the producer's PARA is less than 0.14, using abatement equipment alone will bring more utility than using weather options alone; if his PARA is higher than 0.14, using weather options alone will be more favorable than using abatement equipment alone. An extreme case is that the producer is risk neutral, i.e., his PARA is zero. Then using weather options will bring no benefit to him because weather options are actuarially-fairly priced. Hence the increased utility from optimal portfolio is equal to that from solely using abatement equipment, which is 17.0322 dollars in certainty equivalent.

Table 4 shows that the representative producer's portfolio decisions vary with his risk aversion level. With the increase of Pratt's absolute risk aversion, the producer's optimal weather option purchase is increasing and his optimal abatement investment is decreasing; also the increased utility is increasing.

Suppose the producer can choose the strike prices of weather options, in scenario 3, the optimal strike price is 70 degrees no matter what level his PARA is. But for scenario 1, no theoretical results of the optimal strike price can be derived. In table 4 and figure 4, we see that the increased utility is first increasing and then decreasing with the strike prices. The maximum increased utility corresponds to 71 degrees. That is because the effectiveness of abatement equipment is also increasing with heat stress and thus abatement equipment to some extent is also an insurance tool.

[10] It is calculated based on Gayle S. Willett's report "How Much Debt can a Dairy Cow Carry?" at http://cru.cahe.wsu.edu/CEPublications/eb1762/eb1762.html.

Cross Validation Analysis

We can test the robustness of our results by Cross-Validation. Specifically, we use every 34-year data out of the 35-year data to estimate the β in milk loss model [equation (6)] and a and b in abatement effect model [equation (10)]. From the parameter estimates, we derive the optimal portfolio choice, i.e. ϕ^* and η^* . And we do the out-of-sample evaluation of the net profit from using weather options and abatement equipment by applying the optimal portfolio to the year left. We do this 35 times by successively omitting one of the 35 observations each time.

Table 5 shows that the estimates of β , a , and b are quite robust. For instance, the mean of the 35 estimates of β is 0.5636 kg/cow, and the standard deviation of these estimates is 0.03421. And the corresponding optimal portfolio choices are robust as well. We compare the profit loss without using these two instruments and the net profit loss with using them. The risk aversion degree is still set as 0.20 here. We find that the mean of 35 out-of-sample profit loss is -49.6856 dollars, and the standard deviation is 20.2778; the mean of 35 out-of-sample net profit loss is -32.6066 dollars, and the standard deviation is 7.9889. Thus, we see that using weather options and abatement equipment can significantly reduce both the mean of profit loss from heat stress in summer and their variance.

From validation analysis, we also observe that there are 25 out of the 35 years where the net profit from using the optimal portfolio is positive. The maximum is 71.8474 dollars and the minimum is -11.9750 dollars. That means in most cases, optimally using weather options and abatement equipment can increase net profits. Moreover, negative net profit from using these instruments only happens when weather conditions favor milk production, namely the milk losses are relatively low. Therefore, using weather options together with abatement equipment can smooth the producer's yearly net revenue. That is a desirable result for a risk averse producer.

Conclusion

This study is the first paper to investigate the potential of weather derivatives in hedging against livestock profit risk, which mainly concerns the profit risk from heat stress in hot and humid summers. A representative dairy producer's profit risk is decomposed into systematic risk from weather conditions and idiosyncratic risk which is uncorrelated with weather condition. With the access to hypothetical weather derivatives and abatement equipment, the producer's optimal portfolio choice of these two instruments is derived in a mean-variance utility maximization framework. The results suggest that weather derivatives can act as a substitute for abatement technologies and the simultaneous usage of them is more favorable than using each of them alone.

This paper provides a link of the burgeoning weather derivatives literature in agricultural economics to a real-world application in which an easily-quantifiable weather metric (daily THI in excess of a biological threshold) is the primary source of production risk for a major agricultural commodity. Further, unlike other possible applications of weather derivatives, dairy is unique in that weather derivatives are likely substitutable for capital investment in heat abatement equipment, such as fans or water misters.

This research also raises many questions of relevance to the economic community, such as the optimal contract design, basis risk from location difference between weather derivatives and actual production area, whether the existence of these contracts reinforces economies of scale in dairy production, what level of sophistication is required to effectively utilize these tools, and finally, what size of a dairy is required to use weather derivatives. These questions may be of interest for further research.

Appendices

A. Milk Loss Function

The milk loss model in SCS (2003) is: $MILK_{loss} = 0.0695 * (THI_{max} - THI_{threshold})^2 * Duration$, where $MILK_{loss}$ is in kilogram, and $Duration$ is the proportion of a day where heat stress occurs (i.e. $THI_{max} > THI_{threshold}$).[11]

With the assumption that daily THI follows a perfect sine function with a period of 24 hour,[12] the process to calculate the $Duration$ of heat stress:

$THI_{mean} = (THI_{max} + THI_{min})/2$
if $THI_{max} < THI_{threshold}$
    $Duration = 0$
elseif $THI_{min} >= THI_{threshold}$
    $Duration = 24$
elseif $THI_{mean} > THI_{threshold}$
    $Duration = (PI - 2 * \arcsin(\frac{THI_{threshold} - THI_{mean}}{THI_{max} - THI_{mean}}))/PI * 12$
else $Duration = (PI + 2 * \arcsin(\frac{THI_{mean} - THI_{threshold}}{THI_{max} - THI_{mean}}))/PI * 12$
end

where $PI = 3.1415...$

B. Abatement Effect Function

In SCS, for a 50 m^2 cow pen, which can hold 7.1759 dairy cows, when the annualized fixed costs are \$310, the corresponding operating costs are \$0.0685/hour of operation.

[11] This equation is applicable to dairy cows maintained in a system of minimal cooling.

[12] This assumption is set for accounting for the extent and cumulative severity of heat stress within days. It is stated that this assumption underestimates duration of heat stress at higher latitudes in summers, but gains in accuracy from using more complex models are overall small.

And the abatement effect is: $\Delta THI = -17.6 + (0.36 * T) + (0.04 * H)$, where ΔTHI is the change in apparent THI, T is ambient temperature (°C), and H is ambient relative humidity in percent.

Based on the above specifications, we linearly simulate six abatement effect functions corresponding to six fixed cost levels. The six fixed cost levels are 130, 190, 250, 370, 430, 490 dollars respectively. That is, all the parameters in a simulated model are proportional to those in the SCS model, with the proportion equal to the ratio of fixed cost levels.

We define the reduced profit loss by:
$\Delta loss = \max(\min(THI_{max} - THI_{threshold},\ \Delta THI),\ 0) * \beta * MILKprice.$

References

Barth, C. L. "State-of-the-art for Summer Cooling for Dairy Cows." 52-61, in *Livestock Environment II, Proceedings of the Second International Livestock Environment Symposium*, Scheman Center, Iowa State Univeristy, April 20-23, 1982.

Black, F. and M. Scholes. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* 81(1973): 1429-45.

Cao, M. and J. Wei. "Equilibrium Valuation of Weather Derivatives." Working Paper, Rotman School of Management, University of Toronto, Oct. 2001.

Diebold, F. and S. Campbell. "Weather Forecasting for Weather Derivatives." Working Paper, University of Pennsylvania, September 2002.

Dischel, B. "Black-Scholes Won't Do." *Weather Risk Special Report, Energy and Power Risk Management/Risk*, Oct. 1998.

Johnson, H. D. "Bioclimate Effects on Growth, Reproduction and Milk Production." p. 35, in Johnson, H. D. (ed.) *Bioclimatology and the Adaptation of Livestock*. Elsevier, Amsterdam, The Netherlands, 1980.

Lin, J. C., B. R. Moss, J. L. Koon, C. A. Flood, R. C. Smith III, K. A. Cummins and D. A. Coleman. "Comparison of Various Fan, Sprinkler, and Mister Systems in Reducing Heat Stress in Dairy Cows." *Applied Engineering in Agriculture* 14(1998): 177-182.

Lucas, R. "Asset Prices in an Exchange Economy." *Econometrica* 46(1978): 1429-45.

Miranda, M. "Area-Yield Crop Insurance Reconsidered." *American Journal of Agricultural Economics* 73(1991): 233-242.

National Oceanic and Atomospheric Adiministration, "Livestock Hot Weather Stress." Reg. Operations Letter C-31-76. US Department of Commerce, National Weather Service, Central Region, Kansas City, Missouri, 1976.

Pratt, J. "Risk Aversion in the Small and in the Large." *Econometrica* 32(1964): 122-136.

St-Pierre, N. R., B. Cobanov and G. Schnitkey. "Economic Cost of Heat Stress: Economic Losses from Heat Stress by U.S. Livestock Industries." *Journal of Dairy Science* 86(2003): 52-77.

Thompson, G. E. "Review of the Progress of Dairy Science Climatic Physiology of Cattle." *Journal of Dairy Research* 49(1973): 441-473.

Turner, L.W., J. P. Chastain, R. W. Hemken, R. S. Gates and W. L. Crist. "Reducing Heat Stress in Dairy Cows through Sprinkler and Fan Cooling." *Applied Engineering in Agriculture* 8(1992): 251-256.

Turvey, C. "Weather Derivatives for Specific Event Risks in Agriculture." *Review of Agricultural Economics* 33(2001): 335-351.

Table 1: Descriptive Statistics of Accumulated Weather Data

Category	Symbol	
Expected $\widetilde{z}$	$\mu_{\widetilde{z}}$	556.185
Expected $\widetilde{THI}$	$\mu_{\widetilde{THI}}$	12856.434
Standard Deviation $\widetilde{z}$	$\sigma_{\widetilde{z}}$	118.393
Standard Deviation $\widetilde{THI}$	$\sigma_{\widetilde{THI}}$	172.991
Corr Coeff b/t $\widetilde{THI}$ and $\widetilde{z}$	$\rho_{\widetilde{THI},\widetilde{z}}$	0.871

Table 2: Estimate Profit Sensitivity β In Equation (6)

Parameter	Coefficient	Std. Error	T-Statistic	Prob.	R^2
β	0.5635	0.0337	16.7118	0.0000	0.89

Table 3: Estimate Abatement Effectiveness in Equation (10)

Parameter	Coefficient	Std. Error	T-Statistic	Prob.	R^2
a	-57.4080	3.4461	-16.6587	0.0000	0.91
b	0.005107	0.0002679	19.0627	0.0000	

Table 4: Optimal Decisions Corresponding to Different Strike Prices

Panel A: PARA = 0.10

Strike	Optimal Portfolio				Only Abatement		Only Options		
Price	η^*	ϕ^*	π^*	ΔU	η^*	ΔU	ϕ^*	π^*	ΔU
65	16.5754	0.10632	124.2863	33.8037	21.7727	22.5983	0.12941	151.281	17.1457
66	16.5862	0.1108	114.4016	34.0441	21.7727	22.5983	0.13434	138.7007	17.3618
67	16.6105	0.11565	104.2994	34.2622	21.7727	22.5983	0.13968	125.9643	17.5408
68	16.642	0.12088	94.1039	34.4489	21.7727	22.5983	0.14547	113.2519	17.6814
69	16.6825	0.12705	84.283	34.5958	21.7727	22.5983	0.15242	101.1141	17.7732
70	16.7258	0.13512	75.1496	34.6828	21.7727	22.5983	0.16171	89.9429	17.8045
71	16.7777	0.14592	66.8074	34.7083	21.7727	22.5983	0.17434	79.8152	17.7663
72	16.8574	0.15953	58.9627	34.664	21.7727	22.5983	0.19028	70.325	17.6275
73	16.9545	0.17707	51.5593	34.5347	21.7727	22.5983	0.21096	61.4273	17.386
74	17.0882	0.2002	44.6208	34.2971	21.7727	22.5983	0.23833	53.1186	16.996
75	17.2855	0.23222	38.3168	33.9456	21.7727	22.5983	0.27609	45.5557	16.4194
76	17.5448	0.27785	32.6604	33.4388	21.7727	22.5983	0.32991	38.7809	15.6178
77	17.848	0.34373	27.301	32.7755	21.7727	22.5983	0.40784	32.3923	14.6118
78	18.2372	0.44274	22.018	31.795	21.7727	22.5983	0.52569	26.1434	13.1949
79	18.7304	0.59961	17.3896	30.7043	21.7727	22.5983	0.71038	20.6022	11.5444
80	19.2525	0.82954	13.1802	29.3786	21.7727	22.5983	0.98282	15.6155	9.6304

Panel B: PARA = 0.15

Strike	Optimal Portfolio				Only Abatement		Only Options		
Price	η^*	ϕ^*	π^*	ΔU	η^*	ΔU	ϕ^*	π^*	ΔU
65	16.3511	0.10648	124.4696	42.1918	24.218	25.5954	0.12941	151.281	25.7186
66	16.3674	0.11096	114.5624	42.5523	24.218	25.5954	0.13434	138.7007	26.0428
67	16.4038	0.1158	104.4347	42.8792	24.218	25.5954	0.13968	125.9643	26.3112
68	16.4508	0.12102	94.2142	43.1589	24.218	25.5954	0.14547	113.2519	26.522
69	16.5112	0.12718	84.3696	43.3789	24.218	25.5954	0.15242	101.1141	26.6598

70	16.5756	0.13524	75.2162	43.5092	24.218	25.5954	0.16171	89.9429	26.7068
71	16.653	0.14603	66.8558	43.547	24.218	25.5954	0.17434	79.8152	26.6495
72	16.7717	0.15961	58.9916	43.4802	24.218	25.5954	0.19028	70.325	26.4413
73	16.9164	0.1771	51.5704	43.286	24.218	25.5954	0.21096	61.4273	26.079
74	17.1158	0.20017	44.6139	42.9297	24.218	25.5954	0.23833	53.1186	25.494
75	17.4102	0.23206	38.2907	42.403	24.218	25.5954	0.27609	45.5557	24.6291
76	17.7979	0.27747	32.6164	41.6449	24.218	25.5954	0.32991	38.7809	23.4267
77	18.2522	0.34301	27.2436	40.6542	24.218	25.5954	0.40784	32.3923	21.9176
78	18.837	0.44139	21.9507	39.1917	24.218	25.5954	0.52569	26.1434	19.7923
79	19.5802	0.59712	17.3175	37.5703	24.218	25.5954	0.71038	20.6022	17.3166
80	20.3699	0.82515	13.1105	35.6022	24.218	25.5954	0.98282	15.6155	14.4456

Panel C: PARA = 0.20

Strike	Optimal Portfolio				Only Abatement		Only Options		
Price	η^*	ϕ^*	π^*	ΔU	η^*	ΔU	ϕ^*	π^*	ΔU
65	16.1293	0.10663	124.6521	50.5814	26.7004	28.7252	0.12941	151.281	34.2915
66	16.1512	0.11111	114.7224	51.0619	26.7004	28.7252	0.13434	138.7007	34.7237
67	16.1996	0.11595	104.5691	51.4975	26.7004	28.7252	0.13968	125.9643	35.0815
68	16.262	0.12116	94.3237	51.8701	26.7004	28.7252	0.14547	113.2519	35.3627
69	16.3422	0.12731	84.4556	52.163	26.7004	28.7252	0.15242	101.1141	35.5464
70	16.4275	0.13535	75.2821	52.3362	26.7004	28.7252	0.16171	89.9429	35.6091
71	16.53	0.14613	66.9038	52.3862	26.7004	28.7252	0.17434	79.8152	35.5326
72	16.6872	0.15969	59.0202	52.2966	26.7004	28.7252	0.19028	70.325	35.2551
73	16.8787	0.17714	51.5813	52.0373	26.7004	28.7252	0.21096	61.4273	34.772
74	17.143	0.20014	44.6072	51.5622	26.7004	28.7252	0.23833	53.1186	33.992
75	17.5337	0.2319	38.265	50.8609	26.7004	28.7252	0.27609	45.5557	32.8388
76	18.049	0.2771	32.5731	49.8528	26.7004	28.7252	0.32991	38.7809	31.2356
77	18.654	0.3423	27.1873	48.5375	26.7004	28.7252	0.40784	32.3923	29.2235
78	19.4349	0.44006	21.8847	46.5982	26.7004	28.7252	0.52569	26.1434	26.3898
79	20.4297	0.59469	17.247	44.4553	26.7004	28.7252	0.71038	20.6022	23.0888

80	21.4902	0.82088	13.0425	41.8577	26.7004	28.7252	0.98282	15.6155	19.2608

Panel D: PARA = 0.25

Strike	Optimal Portfolio				Only Abatement		Only Options		
Price	η^*	ϕ^*	π^*	ΔU	η^*	ΔU	ϕ^*	π^*	ΔU
65	15.9102	0.10679	124.8336	58.9725	29.2118	31.9807	0.12941	151.281	42.8643
66	15.9377	0.11127	114.8814	59.5729	29.2118	31.9807	0.13434	138.7007	43.4046
67	15.9981	0.1161	104.7026	60.1171	29.2118	31.9807	0.13968	125.9643	43.8519
68	16.0758	0.1213	94.4324	60.5823	29.2118	31.9807	0.14547	113.2519	44.2034
69	16.1754	0.12744	84.5408	60.9479	29.2118	31.9807	0.15242	101.1141	44.4329
70	16.2814	0.13547	75.3474	61.1639	29.2118	31.9807	0.16171	89.9429	44.5114
71	16.4085	0.14624	66.9513	61.2259	29.2118	31.9807	0.17434	79.8152	44.4158
72	16.6037	0.15977	59.0485	61.1133	29.2118	31.9807	0.19028	70.325	44.0689
73	16.8415	0.17718	51.5922	60.7887	29.2118	31.9807	0.21096	61.4273	43.465
74	17.1699	0.20011	44.6005	60.1948	29.2118	31.9807	0.23833	53.1186	42.49
75	17.6559	0.23175	38.2396	59.3192	29.2118	31.9807	0.27609	45.5557	41.0484
76	18.2979	0.27674	32.5304	58.0626	29.2118	31.9807	0.32991	38.7809	39.0445
77	19.0532	0.3416	27.1319	56.4253	29.2118	31.9807	0.40784	32.3923	36.5294
78	20.0304	0.43876	21.8199	54.0142	29.2118	31.9807	0.52569	26.1434	32.9872
79	21.2781	0.59231	17.1781	51.3588	29.2118	31.9807	0.71038	20.6022	28.861
80	22.6124	0.8167	12.9762	48.144	29.2118	31.9807	0.98282	15.6155	24.076

Note: η^* is optimal abatement investment (\$); ϕ^* is optimal weather option purchase (share); π^* is optimal weather option purchase (\$); and ΔU is the certainty equivalent of increased utility (\$).

Table 5: Cross Validation Parameter Estimates

	$\hat{\beta}$	$\hat{a}$	$\hat{b}$
Mean	0.5636	-57.3957	0.005107
Standard Deviation	0.03421	3.5007	0.0002722

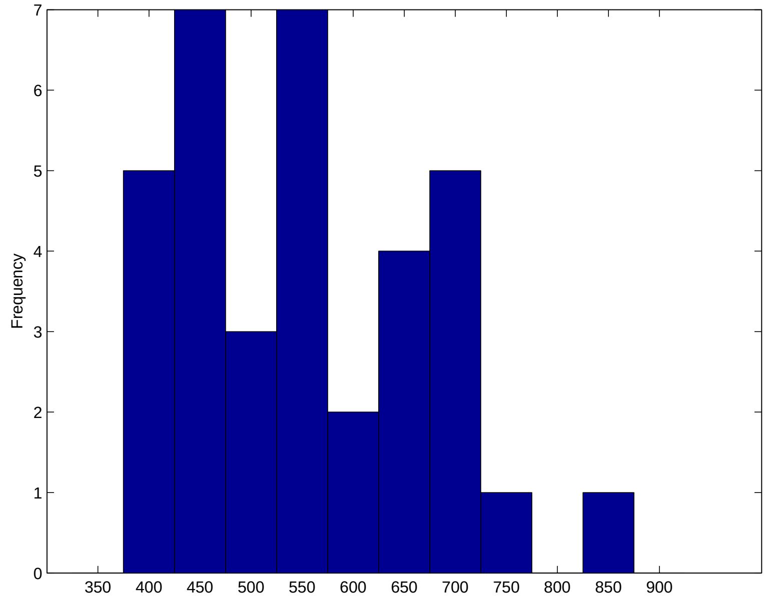

Figure 1: Histogram of Cumulative $\tilde{\tilde{z}}$

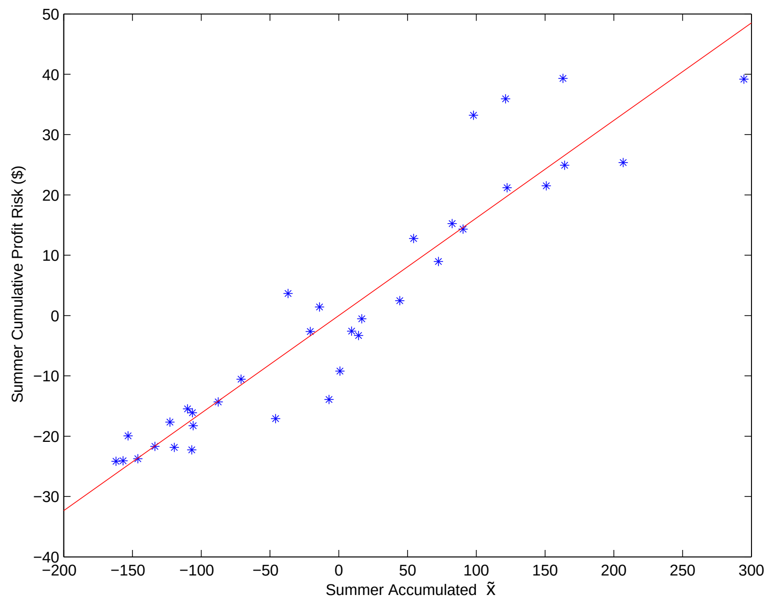

Figure 2: Estimate Profit Sensitivity β in Equation 6

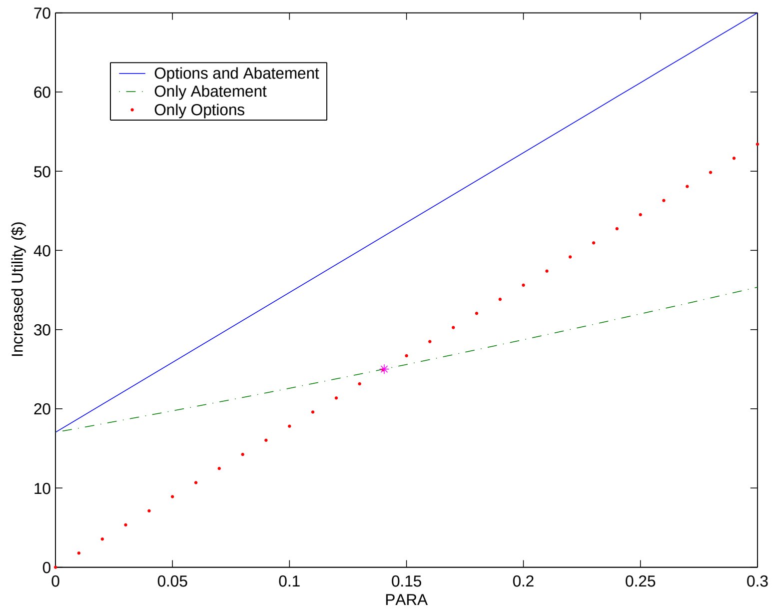

Figure 3: Increased Utility with Different PARAs

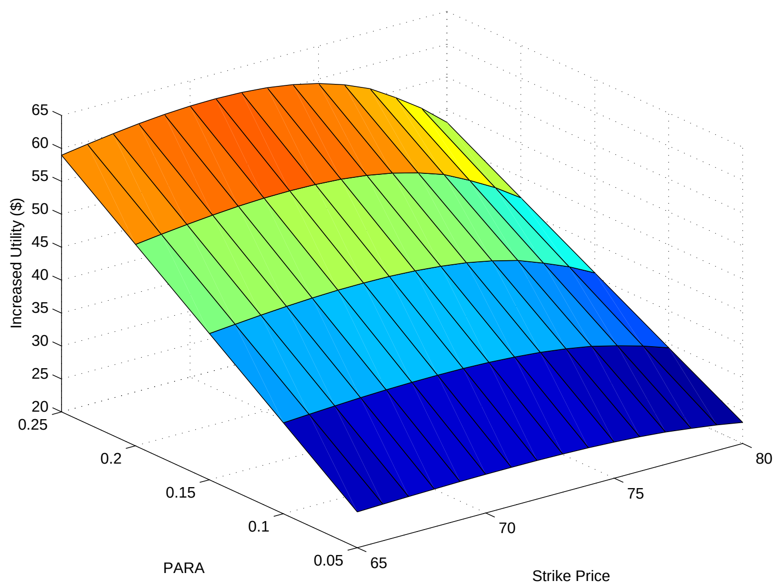

Figure 4: Increased Utility with Different PARAs and Strike Prices