# Estimating Parameters of <br> a Renewable Resource Model Without Population Data 

Robert G. Chambers<br>Ivar E. Strand, Jr.<br>Department of Agricultural and Resource Economics University of Maryland College Park, Maryland


#### Abstract

A general approach to determining parameters of a traditional bioeconomic model is offered for the situation in which knowledge of resource abundance is unknown. Production parameters (such as catchability coefficients) and biological factors (such as natural mortality and recruitment) are included in the model. The general model is articulated for a typical fishery and further specified to obtain estimates of parameters for the St. John's River shad fishery. The results, considering the illustrative nature of the analysis, are promising and suggest avenues of additional research.


## Introduction

Economists are often faced with the problem of determining parameters associated with resource stock dynamics. Catchability coefficients, natural mortalities, and recruitment relationships are among the desired information. Economists have used steady-state models (e.g. Bell, 1972; and Hall, 1977) and short-run production models (e.g., Griffith et al., 1976) but have not attempted to incorporate both the intertemporal recruitment relationships with the more

[^0]sophisticated intratemporal production relationships. Taylor and Prochaska (1984) have recently attempted to remove resource stock effects by use of spectral analysis. Their approach assumes an exogenously determined structure associated with the intertemporal resource stock dynamics.

In this article, we draw from well-established biological (Beverton and Holt, 1957) and economic (Koyck, 1954) literature to present a model which is estimable and incorporates both realistic intraseasonal production and endogenous intertemporal resource stock dynamics. It is particularly useful when independent estimates of the resource stock are not available and relies on a maximum likelihood estimation procedure. A parametric representation of the resource stock variable based on biological theory is first specified and then used as the basic for estimating biological production parameters. The approach is presented in general terms and then specialized for particular assumptions concerning the bioeconomic interactions. The presentation is in terms of the fishing industry, a choice which makes the argument less abstract and permits direct application of the approach to a selected fishery. An anadromous species, shad (Alosa sapidissima), with biological characteristics that simplify the problem greatly, is chosen for an empirical illustration of the approach.

## A General Production Model

In their classic work on fishery population dynamics, Beverton and Holt (1957) derived the following mathematical relationship relating the harvest level at time $t\left(h_{t}\right)$ to the size of the resource stock at time $t\left(X_{t}\right)$ and the vector of effort levels $\left(\mathbf{E}_{t}\right)$ from different gear expended in harvesting the resource:

$$
\begin{equation*}
h_{t}=\frac{\mathbf{C} \cdot \mathbf{E}_{t}}{\mathbf{C} \cdot \mathbf{E}_{t}+M}\left(1-\gamma_{t}\right) X_{t}=\eta_{t} X_{t} \tag{1}
\end{equation*}
$$

where $\mathbf{C}$ is a vector of instantaneous rates of mortality due to a unit of effort for a given gear type, $M$ is the instantaneous natural mortality, $\gamma_{t}$ is the percentage of current population transmitted
from time period $t$ to time period $t+1$ and is equal to $e^{-M-C E} t$, $\eta_{t}$ is the percent of the population caught, and ( $\cdot$ ) denotes inner product.

Often, a fundamental problem with using Equation (1) to estimate the biological parameters $(\mathbf{C}, M)$ is the absence of independent estimates of the stock level. If they were available, Equation (1) could serve as the basis for parameter estimation. Alternative procedures must be sought because independent stock estimates are not usually known. Beddington and Cooke (1982) offer an excellent review of a variety of ways of handling the problem. Ours falls into the category most similar to the one proposed by Schnute (1977) and Deriso (1980).

Below, a general approach to obtaining estimates of the parameters is presented. The general solution, however, simplifies for certain circumstances and several of these cases are presented. The key to the general approach is the construction of a parametric representation of the stock variable which can be used to specify a priori the structure of the error term over time. The specification of the error structure is then used to generate maximum likelihood estimates.

In general, current resource stock depends on past stocks, environmental factors (both random, $U_{t}$, and nonrandom, $Z_{t}$ ), and past levels of effort. This is formalized as

$$
\begin{equation*}
B(L) X_{t}=C(L) g\left(\mathbf{E}_{t}, Z_{t}, U_{t}\right) \tag{2}
\end{equation*}
$$

where $B(L)$ and $C(L)$ are generalized lag operators and $g(\cdot)$ translates effort and environmental factors into effects. If $B(L)$ is invertible, Equation (2) can be rewritten as

$$
\begin{equation*}
X_{t}=B^{-1}(L) C(L) g\left(\mathbf{E}_{t}, Z_{t}, U_{t}\right) \tag{3}
\end{equation*}
$$

The importance of (3) is that the current stock variable can be represented solely as a function of past effort, environmental factors, and stochastic disturbances. Data are normally available on past effort levels and environmental factors, and thus expression (3) is a potentially estimable representation of the current stock variable.

Equation (3) substituted into Equation (1) yields

$$
\begin{equation*}
h_{t}=\eta_{t} B^{-1}(L) C(L) g\left(\mathbf{E}_{t}, Z_{t}, U_{t}\right) \tag{4}
\end{equation*}
$$

which is a function of observable variables, and a random error component. Estimates of the parameters of $\eta_{t}, B^{-1}(L), C(L)$, and $g(\cdot)$ are possible to obtain using nonlinear least squares or a maximum likelihood procedure. In practice, estimation will be greatly facilitated by imposing certain types of separability on $g(\cdot)$.

## A Specific Production Model

Equation (4) is written in a general manner that allows the researcher to adapt it to the specific case being addressed. Circumstances in fisheries vary widely and each researcher would likely have to tailor the expression to the requirements of the particular circumstance. Below, we present one derivation based on particular assumptions about the nature of the lag structure and random component. It is not meant to be applicable to all fisheries but rather illustrative of a possible specification procedure.

It is useful to return to Equation (3). This equation, relating current stock to previous events, is the fundamental link in the system. Fortunately, there is a wealth of biological literature on the specification of such processes (e.g., Hilborn, 1979). We choose a stochastic representation of the transition

$$
\begin{equation*}
X_{t}=\gamma_{t-1} X_{t-1}+r\left(Z_{t-1}\right) \gamma_{t-1} X_{t-1}+U_{t} \tag{5}
\end{equation*}
$$

where $r\left(Z_{t-1}\right)$ is a recruitment function. ${ }^{1}$ Simply stated, current stocks are the sum of carryover ( $\gamma_{t-1} X_{t-1}$ ), new recruitment $\left(r\left(Z_{t-1}\right) \gamma_{t-1} X_{t-1}\right)$, and a random error component $U_{t}$. For the purpose of estimation, it is assumed that $U_{t}$ is i.i.d normal with mean zero and constant variance $\sigma_{u}^{2}$. In this formulation, spawning occurs after harvest in $t-1$.

It must be remembered that a model is being developed to estimate certain key parameters after events have taken place, i.e., ex post. Thus, weather elements, which are random ex ante to harvest, can be introduced, in principle, into the recruitment func-
tion. Equation (5) is a stochastic difference equation with a solution

$$
\begin{align*}
X_{t}= & \sum_{i=0}^{m} U_{t-i} \prod_{j=0}^{i-1} \gamma_{t-j-1}\left(1+r\left(Z_{t-j-1}\right)\right) \\
& +X_{t-m} \prod_{j=0}^{m}\left(1+r\left(Z_{t-j-1}\right)\right) \gamma_{t-j-1} \tag{6}
\end{align*}
$$

where $X_{t-m}$ is assumed finite, $\prod_{j=0}^{-1}(\cdot)=0$, and $\prod_{j=0}^{0}(\cdot)=1$. The second term may not be observationally important, however, because in many instances it tends to vanish. To see this, note that this term is the contribution to current stock coming from carryover in period $t-m\left(\gamma_{t-1} \gamma_{t-2} \cdots \gamma_{t-m} X_{t-m}\right)$ and the recruitment generated by it $\left(1+r\left(Z_{t-1}\right)\right)\left(1+r\left(Z_{t-2}\right)\right) \cdots\left(1+r\left(Z_{t-m-1}\right)\right)$. In some species it is plausible that the products of these items go to zero since carryover in many populations is quite small, say $<10 \%$. These are also the species for which management is the most critical. In the following we presume it is sufficiently close to zero to be ignored in the empirical analysis.

Setting $m$ equal to $\bar{m}$, the remainder of Equation (6) is approximated by

$$
\begin{equation*}
X_{t}=\sum_{j=0}^{\bar{m}} U_{t-1} \prod_{j=0}^{i-1} \gamma_{t-j-1}\left(1+r\left(Z_{t-j-1}\right)\right) \tag{7}
\end{equation*}
$$

which can be substituted into (1) to get

$$
\begin{equation*}
h_{t}=\eta_{t}\left[\sum_{i=0}^{\bar{m}} U_{t-1} \prod_{j=0}^{i-1} \gamma_{t-j-1}\left(1+r\left(Z_{t-j-1}\right)\right)\right] \tag{8}
\end{equation*}
$$

Although this process has eliminated the stock variable and has only limited data requirements, it still remains to estimate the parameters of $\eta_{t}, \gamma_{t-j-1}$, and $r\left(Z_{t-j-1}\right)$.

One might be tempted to transform into (8) logarithms to get

$$
\begin{equation*}
\log h_{t}=\log \eta_{t}+v_{t} \tag{9}
\end{equation*}
$$

where $v_{t}$ is the $\log$ of the bracketed term in (8). Clearly, $v_{t}$ is autocorrelated and methods of maximum likelihood with prespecified
variance-covariance structure could yield estimates for the parameters. These estimates would likely be consistent and efficient, although proof of that is beyond our paper.

An approximation to the maximum likelihood estimator can be obtained using the Koyck transformation. Returning to (8), we have, for $\bar{M}=3$,

$$
\begin{align*}
h_{t}= & \eta_{t}\left(U_{t}+U_{t-1} \gamma_{t-1}\left(1+r_{t-1}\right)\right) \\
& +U_{t-2} \gamma_{t-2} \gamma_{t-1}\left(1+r_{t-1}\right)\left(1+r_{t-2}\right) \\
& +X_{t-3} \gamma_{t-3} \gamma_{t-2} \gamma_{t-1}\left(1+r_{t-3}\right)\left(1+r_{t-2}\right)\left(1+r_{t-1}\right) \tag{10}
\end{align*}
$$

If we assume that the recruitment function does not vary over time (i.e., $r=r_{t}=r_{t-1}=r_{t-2}$ ) and there is very small carryover (high mortality) between one period and the next, Equation (10) can be rewritten as

$$
\begin{equation*}
h_{t} / \eta_{t}=U_{t}+U_{t-1} \gamma_{t-1}(1+r)+U_{t-2} \gamma_{t-1} \gamma_{t-2}(1+r)^{2} \tag{11}
\end{equation*}
$$

We know that Equation (11) also holds in $t-1$
$h_{t-1} / \eta_{t-1}=U_{t-1}+U_{t-2} \gamma_{t-2}(1+r)+U_{t-3} \gamma_{t-2} \gamma_{t-3}(1+r)^{2}$
Multiplying Equation (12) by $\gamma_{t-1}(1+r)$ and subtracting that product from (12) yields

$$
\begin{equation*}
h_{t} / \eta_{t}-\gamma_{t-1}(1+r)\left(h_{t-1} / \eta_{t-1}\right)=U_{t}-U_{t-3} \gamma_{t-1} \gamma_{t-2} \gamma_{t-3}(1+r)^{3} \tag{13}
\end{equation*}
$$

If the $\gamma_{t-i}$ representing carryover are very small, the final term on the right-hand side of Equation (13) also tends to zero. Thus, the left-hand side approximately represents a normally distributed expression and is potentially useful in estimating parameters.

## An Illustration

Because the argument thus far has been extremely abstract, we present a rather straightforward illustration of the general meth-
odology. An attractive special case is one in which exploitation of a single cohort occurs. This eliminates the accounting of cohorts as they pass through the exploitation and spawning phases of their life cycle. Although some populations of salmon and shrimp may have the requisite characteristic, a more narrow and manageable example is Florida's St. John's River shad (Alosa sapidissima) population. American shad is an anadromous fish, spawning in estuaries, migrating as juveniles into the ocean and returning as adults approximately 5 years later to spawn in the same estuary. American shad populations farther north enter the estuaries to spawn more than once and therefore are subject to more than one year's exploitation. St. John's River shad, however, are believed to spawn only once and die after the arduous trip up the river and intense spawning activity. There appears to be heavier mortality on the females because they carry valuable roe. It is a fishery in which stocks were believed to be declining in the early 1970s (Williams and Bruger, 1972).

Shad are commercially harvested near the mouth of the St. John's River with three gear types: drift gill nets, anchored gill net, and haul seines. In the late 1960s, the fishery had around 100 drift gill nets, 20 anchored gill nets, and 20 haul seines. Landings have reached $\frac{3}{4}$ of a million pounds (round weight) several times between 1950 and 1976. However, since 1971, landings have only exceeded $\frac{1}{4}$ of a million pounds once and have been as low as $\frac{1}{40}$ of a million pounds.

To make the general model more particular to the St. John's shad experience, several modifications are required in Equation (5). First, the information content each year during the month-long "shad run" is believed to be too short to make reasonable estimates of natural mortality $(M)$. We thus assume that $M=0$ during the shad run. Second, the illustrative nature of our estimation made collecting environmental data less relevant and we assumed a constant recruitment parameter, $r$, over the entire period of analyses. Finally, the absence of carryover reduced the term $\left(1+r_{t}\right)$ to $r$. We were then left with a resource transition equation of the form

$$
\begin{equation*}
X_{t}=r \gamma_{t-5} X_{t-5}+U_{t} \tag{14}
\end{equation*}
$$

where $\gamma_{t-5}$ was defined as $\exp \left(-c_{1} E_{1 t}-C_{2} E_{2 t}-C_{3} E_{3 t}\right)$, and $E_{1 t}$ as the number of drift gill nets in period $t, E_{2 t}$ as the number of anchor gill nets, and $E_{3 t}$ as the number of haul seines.

Following the derivation of Equation (13), we obtain

$$
\begin{equation*}
\frac{h_{t}}{\eta_{t}}-\frac{r \gamma_{t-5} h_{t-5}}{\eta_{t-5}}=U_{t} \tag{15}
\end{equation*}
$$

Assuming normality and intertemporal independence in $U_{t}$, the log-likelihood function for the parameters can be written as

$$
\begin{equation*}
L\left(r, c_{1}, c_{2}, c_{3}, \sigma_{u}\right)=-T \ln \sigma-\frac{T}{2} \ln 2 \pi-\sum_{t=1}^{T} \frac{U_{t}^{2}}{2 \sigma_{u}^{2}} \tag{16}
\end{equation*}
$$

where $T$ is the number of observations and $\sigma_{u}^{2}$ is the variance of $U_{t}$.
Maximum likelihood estimates are obtained for Equation (15) with respect to the parameter vector $(r, C, \sigma)$ by numerical optimization using the Berndt-Hall-Hall-Hausman algorithm. We could, however, have used some sound a priori information on the elements of the parameter vector. Obviously, it is unreasonable to expect any element of the parameter vector to take a negative value. Although we did not impose these conditions, there are several possible ways to incorporate constraints into our model prior to estimation. ${ }^{2}$

Data for the period (1960-1976) are used to estimate parameters and standard errors for Equation (16). The results are shown in Table 1. The data from 1955 through 1959 provide the lagged landings and effort requirements of the first five periods. The catch-

Table 1
Estimates of Production and Recruitment Parameters

| Parameter | Estimate | Standard Error |
| :--- | :---: | :---: |
| $C_{1}$ (drift gill nets) | 0.104 | 0.118 |
| $C_{2}$ (anchored gill nets) | 0.007 | 0.065 |
| $C_{3}$ (haul seine) | 0.093 | 0.099 |
| $r$ (recruitment) | 13.986 | 0.812 |
| $\sigma_{u}$ | 349.49 | 58.71 |

ability coefficients $\left(C_{i}\right)$ all have the expected sign and reasonable magnitude relative to biological estimates for other species (see Beddington and Cook, 1982). Introduction of environmental information might improve the estimates as well as reduce the estimated error of $U_{t}$. The estimated recruitment parameter ( $r=13.6$ ) suggests that, on average, each thousand pounds of shad not commercially harvested returns 13.6 thousand pounds of shad to the mouth of the St. John's River 5 years hence.

Given this apparently high level of recruitment over the period, one might ask why the resource was seen by Williams and Bruger (1972) to be declining. Although we do not have information on the environmental changes, we can provide some information on whether our evidence suggests a declining resource base and whether overfishing could be responsible for it. Table 2 presents our estimates of the returning stock $\left(\hat{X}_{t}=h_{t} / \hat{\eta}_{t}\right)$, the estimated parent population of the returnees, and the fishing mortality of the parent population. Clearly, the estimates of fishing mortality on parent populations are quite high for the 1955-1971 period, ranging from a low of 0.883 to a high of 0.999 . These high mortalities (column 3) may have contributed to the declining stocks shown in the second column, but the other evidence in Table 2 raises questions as to whether commercial fishing mortality is the only factor in the resource's decline.

Considering the returnees' parent stock estimate (based on lagged returning stock minus lagged commercial landings), one finds that rather low estimated parent stocks produced substantial returning through 1969. Since then, relatively high estimated parent stocks have produced lower returning stock, suggesting a shift in the system. This may be the result of increased sportfishing in the upper estuary, greater mortality at sea, or environmental changes such as canals or water projects.

## Summary

Above, an argument was advanced that certain circumstances allow estimation of bioeconomic parameters of a natural resource system with catch and effort data but without stock information. The procedure requires parameterization of the resource stock and use of

Table 2
Estimates of Returning Stocks, Parent Stocks, and Commercial Fishing Mortality Associated with the Parent Stock, 1960-1976

|  | Estimated <br> Returning Stock <br> $(1000 \mathrm{lb})$ | Estimated Parent <br> Stock <br> $(1000 \mathrm{lb})$ | Estimated Fishing <br> Mortality Associated <br> with Parent Stock |
| :---: | :---: | :---: | :---: |
| 1955 | 485.61 | 35.41 | 0.927 |
| 1956 | 387.52 | 12.91 | 0.966 |
| 1957 | 411.58 | 51.18 | 0.875 |
| 1958 | 586.40 | 0.01 | 0.999 |
| 1959 | 529.80 | 0.01 | 0.999 |
| 1960 | 470.52 | 2.32 | 0.995 |
| 1961 | 425.81 | 0.52 | 0.998 |
| 1962 | 760.85 | 1.06 | 0.998 |
| 1963 | 632.16 | 42.27 | 0.933 |
| 1964 | 646.99 | 33.80 | 0.947 |
| 1965 | 768.12 | 10.43 | 0.986 |
| 1966 | 534.61 | 5.11 | 0.990 |
| 1967 | 320.74 | 2.05 | 0.994 |
| 1968 | 551.83 | 20.62 | 0.963 |
| 1969 | 411.37 | 22.57 | 0.945 |
| 1970 | 237.27 | 18.97 | 0.920 |
| 1971 | 295.35 | 42.85 | 0.855 |
| 1972 | 176.99 | 56.99 | 0.678 |
| 1973 | 136.80 | 37.81 | 0.724 |
| 1974 | 420.46 | 336.36 | 0.200 |
| 1975 | 271.235 | 239.15 | 0.118 |
| 1976 | 233.23 | 205.63 | 0.118 |

maximum likelihood estimation techniques. Intuitively, the argument is simple: recruitment will change stocks and these stocks, in turn, will influence future stocks and catch. By structuring the interand intratemporal relationships among catch, stock, and effort, estimates of useful bioeconomic parameters can be obtained.

An illustration was provided for the simple case in which the stock died after one period of exploitation. Some discussion of the effects of multiperiod exploitation seems appropriate. The extent of a resource's period of exploitation depends on its age at entry and
maximum age. A species such as striped bass has been susceptible to exploitation as a 2 -year-old and has a maximum age greater than 10 years. In this instance, Equation (10) would be more complex than the shad example and the lagging procedure would result in an autocorrelated error structure. Although complicated lag structures can be taken into consideration, more complexity and data requirements are introduced in the estimation procedure. It may not be too critical in many instances because the high level of exploitation may substantially reduce the carryover between periods and effectively reduce the order of autocorrelations. More research is required before the ultimate effectiveness of the suggested approach is established.

## Acknowledgments

This research was funded by Maryland Agricultural Experiment Station project MD-A-26-DN (Scientific Article No. A2821). Computer funds were provided by the Computer Science Center, University of Maryland.

## Notes

1. We choose to illustrate with a one-period lagged recruitment. Greater lags are possible although accompanied by greater difficulty in deriving closed-form solutions to implied difference equations. We also choose $U_{t}$ to be separable from stocks, an assumption that may not always be useful (see Cushing, 1971). We believe, however, the assumption is reasonable given nonextreme values of stocks.
2. One possible approach is Bayesian in nature and involves specifying prior densities for the parameters that either totally preclude negative parameter estimates or make the probability of obtaining, say, a negative $c_{i}$, very, very small. Under a quadratic loss criterion, one would then calculate the posterior means of the marginal densities to obtain a point estimate of the appropriate parameter. This approach seems quite attractive, but the complicated nonlinear relationship between effort and landings and the form of (15) suggests that finding an appropriate conjugate prior and actual numerical computation of the posterior means will be quite complex. A second possible approach to incorporating the prior information of nonnegative parameters is to reparameterize the model in terms of transformations that guarantee estimates which are nonnegative. The
most obvious approach here is the "method of squaring" suggested by Lau (1978). Put simply, if the parameter is to be nonnegative, this approach transforms $c$ into the parameter $c^{* 2}$ and the estimation problem becomes that of choosing $c^{*}$ and not $c$.

## References

Beddington, J. R., and J. G. Cooke. 1982. Estimating the response of population to exploitation from catch and effort data. Lecture Notes in Biomathematics, Vol. 54, ed. S. Levin and T. Hallam. New York: Springer-Verlag, pp. 247-261.
Bell, F. W. 1972. Technological externalities and common property resources: an empirical study of the U.S. lobster industry. J. Polit. Econ. 80: 148-158.
Beverton, J. H., and S. V. Holt. 1957. On the dynamics of exploited fish populations. Fishery Investigations II, Vol. 19. London: HMSO.
Cushing, D. H. 1971. The dependence of recruitment on parent stock in different groups of fishes. J. Conseil Permanent Int. Explor. Mer 33(May): 340-350.
Deriso, R. B. 1980. Harvesting strategies and parameters estimation for an agestructured model. Can. J. Fish. Aquat. Sci. 37: 268-282.
Griffin, W. L., R. D. Lacewell, and J. P. Nichols. 1976. Optimum effort and rent distribution in the Gulf of Mexico shrimp fishery, Am. J. Agric. Econ. 58: 644-652.
Hall, D. C. 1977. A note on natural production functions. J. Environ. Econ. Manag. 4: 258-264.
Hilborn, R. 1979. Comparison of fisheries control systems that utilize catch and effort data. J. Fish. Res. Board Can. 36: 1477-1489.
Koyck, L. M. 1954. Distributed lags and investment analysis. Amsterdam: NorthHolland.
Lau, L. J. 1978. Testing and imposing monotonicity, convexity and quasi-convexity constraints. In Production economics: a dual approach to theory and applications, Vol. 1, ed. M. Fass and D. McFadden. Amsterdam: North-Holland, pp. 409-454.
Taylor, T. B., and F. J. Prochaska 1984. Incorporating unobserved cyclical stock movements in fishery catch equations: an application to the Florida blue crab fishery. N. Am. J. Fish. Manag. 4: 67-74.
Schnute, J. 1977. Improved estimates from the Schafer production model: theoretical considerations. J. Fish. Res. Board Can. 34: 583-603.
Williams, R. O., and G. E. Bruger. 1972. Investigations on American shad in the St. John's River. Tech. Ser. 66. St. Petersburg, FL: Florida Department of Natural Resources.

Copyright of Marine Resource Economics is the property of Marine Resources Foundation. The copyright in an individual article may be maintained by the author in certain cases. Content may not be copied or emailed to multiple sites or posted to a listserv without the copyright holder's express written permission. However, users may print, download, or email articles for individual use.


[^0]:    Marine Resource Economics, Volume 2, Number 3
    0738-1360/86/010263-00\$02.00/0
    Copyright (C) 1986 Crane, Russak \& Company, Inc.

