# A Bioeconomic Model for Management of Orange Roughy Stocks 

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#### Abstract

The paper reports the results of a bioeconomic analysis of the exploitation of a recently discovered orange roughy stock located off Tasmania. The parameters of the model are based on the experience derived from the orange roughy fisheries in New Zealand where stocks have been heavily exploited. The model is used to predict the open-access equilibrium stock, and to calculate the stock which maximizes the net present value and the stock level consistent with the $F_{0,1}$ Rule. Assuming a linear approach path, the net present value of the fishery at each of these stocks is calculated. The results are used to estimate the benefit of management and the cost of a conservative stock policy. It is suggested that the results will contribute to the development of a management policy for the Tasmanian stock, and for stocks which are likely to be discovered elsewhere.


Keywords Fishery management, bioeconomic model, orange roughy.

## Introduction

Orange roughy (Hoplostethus atlanticus) occur in northern and southern hemisphere slope waters between 700 and 1500 metres. So far the only commercial exploitation is in the waters around Australia and New Zealand where the orange roughy stocks are the basis of the world's deepest fisheries. The fisheries centre on particular localities, usually near irregularities on the sea bed, which are the focus of spawning and feeding aggregations. Catch rates have been extremely high, with 100 tonne catches being taken in only a few minutes trawling.

The New Zealand fishery began in 1978 (see Robertson (1991)) on the Chatham Rise and expanded to Wairarapa and Challenger in 1981, Kaikoura in 1984, and Ritchie Bank and Westland in 1985. In Australia significant quantities of orange roughy started to be caught after 1986 (see Smith (1991)) off Tasmania and in the Great Australian Bight. A major winter spawning aggregation was discovered in 1989 off the east coast of Tasmania (the St Helens aggregation). Reported catches

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in New Zealand have been 6-11\% by weight of total annual catch of all commercial species. In Tasmania the total orange roughy catch in 1989 was in excess of 25,000 tonnes, with a landed value of $\$ 45$ million. The 1990 catch was in excess of 40,000 tonnes.

Recent research in New Zealand based on samples from the Chatham Rise (see Mace (1990)) suggests that the growth rates of orange roughy are exceptionally slow. Age at maturity is around 20 years and some fish may be as old as 100 years (see Fenton et al. (1991)). The slow growth combined with relatively low fecundity, low natural mortality and aggregating behaviour make the orange roughy stocks susceptible to over-fishing. Robertson (1991) quotes estimates that the Chatham Rise and Challenger stocks have been fished down to around $20 \%$ of virgin biomass in less than 10 years. He suggests that the lessons learned in New Zealand might be helpful in avoiding over-exploitation of Australian orange roughy stocks.

The purpose of the present paper is to use the information available on the Chatham Rise and Challenger fisheries to model the exploitation of the St Helens aggregation which is believed to be similar in size to the initial Challenger stock. A bioeconomic model is developed and used to estimate the equilibrium stock level which is predicted under open access, the stock level which maximizes the expected net present value of the fishery, and the minimum stock level which can confidently be regarded as sustainable in the face of stochastic environmental and harvesting conditions. These stock levels are calculated on the basis of point estimates of economic and biological parameters and should be regarded as indicative only. The net present values at each stock level are computed and compared.

The three stock levels of interest are illustrated in Figure 1. The stock level, $x^{*}$, which maximizes expected net present value is determined by the intersection of the curves depicting the marginal benefit and marginal cost of investing in, or conserving, the stock; the marginal cost is the opportunity cost of forgoing an extra unit of harvest, and the marginal benefit is the present value of the additional sustainable harvest which is obtained as a result of the forgone present harvest. The open-access equilibrium, $\mathrm{x}_{\infty}$, is where the marginal cost curve has a zero value. The diagram illustrates a positive value for both these stock levels but one or both could be zero depending upon the biological and economic characteristics of the fishery. The minimum safely sustainable stock level, $\hat{\mathrm{x}}$, is depicted as being higher than the expected net present value maximizing level, although this is not necessarily the case.

Figure 1 can be used to illustrate the answers to two questions of interest to policy makers. First, what is the benefit of managing the fishery? This benefit, which is also the cost of not managing the fishery, is measured by the shaded area A. Second, what is the cost of a conservative policy which guarantees the preservation of the stock? This cost is measured by the shaded area B. The purpose of this paper is to determine the economic and biological characteristics of the fishery, to use this information to calculate the three stock levels of interest, and to compute the benefit of management and the cost of a conservative policy.

## The Management Objective

It is assumed that the management objective is to maximize the present value of the net revenues from exploiting the fishery, subject to a constraint representing


Figure 1. Marginal benefit and marginal cost of investing in the fish stock.
changes in the biomass over time. This objective corresponds to the welfare maximizing objective when the price of harvest and the unit cost of effort are constant as assumed later in the paper. The changes in the biomass can be represented by a delay-difference equation of the form:

$$
\begin{equation*}
x_{t+1}=\left(x_{t}-h_{t}\right) e^{\delta(x)}+G\left(x_{t-\gamma}\right) \tag{1}
\end{equation*}
$$

where $\quad \mathrm{x}_{\mathrm{t}}=$ biomass at time t
$\mathrm{h}_{\mathrm{t}}=$ harvest at time t
$\delta(\mathrm{x})=$ instantaneous net survival rate (growth less natural mortality) as a function of stock
$\mathrm{G}\left(\mathrm{x}_{\mathrm{t}-\gamma}\right)=$ recruitment at time t as a result of the spawning stock at time $t-\gamma$.

The Lagrangian for the maximization problem is:

$$
\begin{equation*}
L=\Sigma \alpha^{t} \Pi\left(h_{t}, x_{t}\right)-\lambda_{t}\left[x_{t+1}-\left(x_{t}-h_{t}\right) e^{\delta(x)}-G\left(x_{t-\gamma}\right)\right] \tag{2}
\end{equation*}
$$

where $\quad \alpha^{t}=$ discount factor $(1+r)^{-t}$, where $r$ is the real rate of interest $\Pi\left(\mathrm{h}_{\mathrm{t}}, \mathrm{x}_{\mathrm{t}}\right)=$ net revenue at time t at base year prices
$\lambda=$ present value shadow price of the stock.

It is shown in Bjorndal (1988) that the optimal fish stock derived from the maximization problem is given by the solution to:

$$
\begin{equation*}
e^{\delta(x)}\left[\frac{\prod_{x}}{\prod_{h}}+1\right]+\delta_{x}[x-G(x)]+\alpha^{\gamma} G_{x}=1+r \tag{3}
\end{equation*}
$$

where $\Pi_{\mathrm{x}}, \Pi_{\mathrm{h}}, \delta_{\mathrm{x}}$ and $\mathrm{G}_{\mathrm{x}}$ are first derivatives.
To understand the economic logic of equation (3), assume for the moment that $\Pi_{\mathrm{x}}$ and $\delta_{\mathrm{x}}$ are zero. Suppose that there is a marginal investment in the stock of one unit of biomass in period $t$. In period $t+1$ a proportion $\mathrm{e}^{\delta(\mathrm{x})}$ of that unit survives and is added to the spawning stock. In period $t+1+\gamma$ recruitment to the stock rises because of the investment which was made in period $t$, and this recruitment has a present value of $\alpha^{\gamma} G_{x}$ in period $t+1$. Thus the l.h.s. of equation (3) is the value at time $t+1$ of the initial investment in the stock plus the associated net increment. The r.h.s. is the value at time $t+1$ of an alternative investment earning the opportunity cost rate of interest, $r$.

Now consider the effect of the terms $\Pi_{\mathrm{x}}$ and $\delta_{\mathrm{x}}$. Because unit harvesting cost declines as stock increases, $\Pi_{\mathrm{x}}>0$. One benefit of a marginal investment in the stock is the extra profit which will result from the lower harvesting costs. The value of the surviving proportion of the extra unit of the biomass must be adjusted upwards to reflect this extra profit. The adjustment consists of the additional profit, $\Pi_{\mathrm{x}}$, converted to the numeraire good, units of biomass, by means of $\Pi_{\mathrm{h}}$. This value is added to the original investment of one unit of biomass. The biomass survival rate, $\delta(\mathrm{x})$, declines as a result of an investment in the stock because this involves a reduction in harvest which raises the average age of the population. Older fish experience a lower rate of growth than younger fish, whereas natural mortality is assumed to be independent of age. Hence $\delta_{\mathrm{x}}<0$, and an adjustment is required to the marginal benefit of investment in the stock. The term ( $\mathrm{x}-\mathrm{G}(\mathrm{x})$ ) $=(\mathrm{x}-\mathrm{h}) \mathrm{e}^{\delta(\mathrm{x})}$ is the surviving biomass at the original survival rate $\delta(\mathrm{x})$. This must now be adjusted downwards to reflect the decline in the survival rate of all units of biomass resulting from a marginal increase in stock. The marginal cost in terms of biomass survival is $\delta_{\mathrm{x}}(\mathrm{x}-\mathrm{h}) \mathrm{e}^{\delta(\mathrm{x})}$ in period $\mathrm{t}+1$, and this must be subtracted from the surviving proportion of the original investment in biomass.

In order to determine the optimal stock the following sets of biological and economic information need to be obtained: the net instantaneous survival rate, $\delta(\mathrm{x})<0$; the lag between spawning and recruitment, $\gamma ;$ the stock-recruitment function, $\mathrm{G}(\mathrm{x})$; the real rate of interest, r ; and the net revenue function, $\Pi(\mathrm{h}, \mathrm{x})$. The next Section of the paper discusses the biological parameters, and the following Section the economic parameters.

## The Biological Model

The biological model is summarized by equation (1) which is a delay-difference equation for the change in biomass from one period to the next. Full specification of the model requires explicit equations for recruitment, $\mathrm{G}\left(\mathrm{x}_{\mathrm{t}-\gamma}\right)$, and instantaneous survival $\delta(\mathrm{x})$. Specifying the latter involves specifying an equation for the growth in weight of individual fish.

## Recruitment

The stock-recruitment relationship is represented by a Beverton-Holt stockrecruitment function:

$$
\begin{equation*}
\mathrm{G}\left(\mathrm{x}_{\mathrm{t}-\gamma}\right)=\mathrm{ax}_{\mathrm{t}-\gamma}\left(1+\left(\mathrm{b} / \mathrm{x}_{\mathrm{o}}\right) \mathrm{x}_{\mathrm{t}-\gamma}\right)^{-1} \tag{4}
\end{equation*}
$$

where $\mathrm{a}, \mathrm{b}$, and $\gamma$ are parameters, and $\mathrm{x}_{\mathrm{o}}$ is the virgin biomass of the stock.
The parameter $\gamma$, which is the mean age at recruitment, is thought to be around 23 years (Clark and Francis (1990)). The parameter " $a$ " helps to determine both the productivity of the stock and the stock size at which productivity is maximized ( $\mathrm{x}_{\mathrm{msy}}$ ). Since orange roughy stocks have been fished for approximately 10 years only and the age at recruitment is around 23 years, there are no data available to estimate this parameter. Clark and Francis (1990) use a value of around 2.8 which is based on the assumption that the ratio of recruitment at $20 \%$ of virgin biomass to recruitment at virgin biomass-the "steepness parameter"-is 0.95 . This would imply that $\mathrm{x}_{\text {(msy) }}$ is around $10 \%$ of virgin biomass. This is an abnormally low value for $\mathrm{X}_{\text {(msy) }}$ and it may be an unduly optimistic assessment of productivity at low stock levels. In this paper a value of $\mathrm{a}=0.5$ is used which implies a value of $\mathrm{x}_{(\mathrm{msy})}$ equal to around $20 \%$ of virgin biomass. The virgin biomass $\mathrm{x}_{\mathrm{o}}$ is set at 110,000 tonnes based on preliminary biomass estimates from the St. Helens spawning aggregation, and the value of the parameter $b$ is derived from its relationship with other parameters, as shown in the Residual Parameter Section below.

## Growth

Growth of individual fish is modelled using the von Bertalanffy growth function for length at age, together with a power relationship for weight at length. The equations are:

$$
\begin{equation*}
\mathrm{L}(\mathrm{t})=\mathrm{L}_{\infty}\left(1-\mathrm{e}^{-\mathrm{K}\left(\mathrm{t}-\mathrm{t}_{0}\right)}\right) \tag{5}
\end{equation*}
$$

and

$$
\begin{equation*}
W(t)=u L(t)^{v} \tag{6}
\end{equation*}
$$

where $\mathrm{W}(\mathrm{t})=$ weight in kilograms, $\mathrm{t}=$ age in years, $\mathrm{L}_{\infty}=$ maximum length, K $=$ the Brody growth coefficient, $\mathrm{t}_{\mathrm{o}}=$ the intercept at $\mathrm{L}(\mathrm{t})=0$, and u and v are parameters. Mace et al. (1990) give the following estimates of these parameters: $\mathrm{L}=42.5 \mathrm{~cm}, \mathrm{~K}=0.059, \mathrm{t}_{\mathrm{o}}=-0.346, \mathrm{u}=0.0963$, and $\mathrm{v}=2.68$.

## Survival

The term $\mathrm{e}^{\delta(x)}$ in equation (1) is the proportion of a unit of biomass which survives from one time period to the next. Survival has two components: survival of individual fish, and increase in weight of survivors due to growth. The survival rate can be written as: $\delta(x)=g(x)-m(x)$, where $g(x)$ is instantaneous growth rate in weight and $m(x)$ is instantaneous mortality rate due to factors other than fishing.

For orange roughy a constant value of $m=0.05$ has been assumed (see Mace et al. (1990)). Schnute (1987) shows that the growth component in the survival term is equal to $\mathrm{W}^{\prime} / \mathrm{W}$ where W is the average weight of an individual fish in the population at each time step, and $\mathrm{W}^{\prime}$ is the weight to which such a fish would grow over one time step. The effect of fishing is not only to reduce the size of the stock, but also to reduce the mean age and hence the average weight of fish in the stock. However smaller fish grow faster, so the net survival rate of population biomass actually increases as the population decreases. An approximation for $\delta(x)=g(x)$ $-m$ is derived in the Appendix. The resulting equation is of the form:

$$
\begin{equation*}
\delta(\mathrm{x})=\phi\left(\mathrm{x} / \mathrm{x}_{0}\right)^{1 / 2} \tag{7}
\end{equation*}
$$

where $\phi=-0.032$.

## Residual Parameter

In the absence of fishing the stock size will equal the virgin biomass $x_{0}$. From equation (1), and using equations (4) and (7), this implies the following relationship among the parameters:

$$
\begin{equation*}
\left(1-e^{\phi}\right)=a /(1+b) \tag{8}
\end{equation*}
$$

where $b$ is the parameter of the recruitment function which is still to be estimated. The solution value of $b$ from equation (8) is 14.876 .

## The Economic Model

The general form of the profit function is given by:

$$
\begin{equation*}
\Pi(h, x)=p(h) h-c(E) \cdot E \tag{9}
\end{equation*}
$$

where $p(h)$ is the inverse demand curve for the harvest, $c(E)$ is the unit cost of fishing effort, and $\mathrm{E}=\mathrm{E}(\mathrm{h}, \mathrm{x})$ is the amount of effort required to take the harvest, h . The solution to equation (3) can be treated as the solution to a linear autonomous optimal control problem if the profit function is linear in harvest and if the parameters of the economic and biological models are constant over time. Because of the difficulty of solving nonlinear nonautonomous optimal control problems (Clark and Munro (1975)), it will be necessary to proceed on this basis.

Linearity requires the price of harvest, $\mathrm{p}(\mathrm{h})$, and the unit cost of effort, $\mathrm{c}(\mathrm{E})$, to be constant, and the function $\mathrm{E}=\mathrm{E}(\mathrm{h}, \mathrm{x})$, which is the implicit form of the economic production function, to be linear in h . The information available about the New Zealand and Australian orange roughy fisheries can be used to examine whether linearity can reasonably be assumed. It is likely that price and cost will change over time. However it is suggested later in the paper that these changes can be incorporated in a constrained form in an autonomous model.

Since Tasmanian orange roughy is a small proportion of the world supply, and since there are close substitutes for orange roughy it might be reasonable to assume that the Tasmanian industry is a price-taker in world markets. An indirect
way of testing this assumption is to estimate the elasticity of demand for orange roughy in the domestic market. If the industry is a price-taker in the domestic market then it will be a price taker in foreign markets in which there is a greater availability of substitutes. Observations on price and quantity of orange roughy and on the price of a close substitute, smooth dory, were obtained for the Melbourne fish market for the period from September 1989 to July 1990. There were 72 days out of the total 138 in which both species traded. Prices were converted to September 1990 using the quarterly consumer price index. It was assumed that over the short period of the observations income, population and tastes remained unchanged. The inverse demand equation represented by Regression 5, Table 2, was fitted by OLS: the dependent variable is orange roughy price and the independent variables are the quantity of roughy traded, QOR, and the price of a close substitute, smooth dory, SDP. The coefficient on QOR measures the responsiveness of price to a change in quantity supplied. The low value of this coefficient in regression 5 gives an estimate of own price elasticity of demand of 4.6 which suggests that the orange roughy price can be regarded as relatively unresponsive to the quantity of roughy supplied in this market over the range of observed quantities. The mean Melbourne price was $\$ 2.81$ per kilogram but Tasmanian landed prices are lower and more appropriate for the analysis, and a figure of $\$ 1.13$ will be used, which is the weighted mean Tasmanian landed price for 1990.

An estimate of the unit cost of effort is derived from an economic survey of Australia's south-east trawl fishery by Geen, Brown and Pascoe (1989). They report average observations on costs, including the imputed cost of owneroperator labour, depreciation, and capital value, for a sample of 14 of the 40 trawlers operating in the south-west sector of the fishery, which includes the St Helens aggregation, in 1987-88. Using an average cost of capital of $16 \%$ (see McIlgorm (1989)) the average total annual cost for the vessels was $\$ 0.606$ million per year. An estimate of the total number of trawls per vessel per year of 607 was provided by the Australian Fisheries Service. This gives a unit cost of effort of approximately $\$ 1000$. Recent research by Geen, Brown and Pascoe (1990) suggests the presence of considerable over-capacity in the south-east trawl fishery. On this basis it is assumed that effort can be drawn into the orange roughy fishery at constant unit cost.

The relationship between harvest, stock and fishing effort can be expressed as an economic production function, $h=h(E, x)$, where $E$ is fishing effort. A specific form of the Cobb-Douglas production function is often used to represent this relationship:

$$
\begin{equation*}
\mathrm{h}=\mathrm{AEx} \tag{10}
\end{equation*}
$$

where A is a parameter (often called the catchability coefficient). Equation (10) implies that catch per unit effort is always a constant proportion of stock, irrespective of the level of effort. In the case of fish stocks which are harvested in schools it is unlikely that catch per unit effort will decline in proportion to the decline in total stock. If schools are readily located it may be possible to fish the same local density of fish stock down to very low levels of the overall stock. In an extreme case catch per unit effort may remain constant as fish stock declines. Bjorndal (1988) represents the economic production function by:

$$
\begin{equation*}
\mathrm{h}=\mathrm{Ax}^{-\mathrm{z}} \mathrm{Ex} \tag{11}
\end{equation*}
$$

where $\mathrm{z} \leqslant 1$ is a parameter, and $\mathrm{A}(\mathrm{x}) \equiv \mathrm{Ax}^{-\mathrm{z}}$ is the catchability coefficient. When $\mathrm{z}=0$ catch per unit effort is in direct proportion to the stock; when $\mathrm{z}=1$ the catch per unit effort is a constant irrespective of the stock level. The more general form of equation (11) used in this paper allows the coefficient on effort to take a value different from unity:

$$
\begin{equation*}
h=A E^{y} x^{1-z} \tag{12}
\end{equation*}
$$

There are insufficient data on the St Helens fishery to estimate the parameters A, $y$ and $z$ of the economic production function. However data from the New Zealand Challenger Plateau fishery, which has been in operation since 1983, are available. The Challenger Plateau virgin biomass is estimated to have been similar to that of the St Helens stock, but there are important differences between the fisheries. The Challenger fishery is based on spawning aggregations in the winter months and feeding aggregations in the remaining months, whereas the St Helens fishery is restricted to spawning aggregations during a winter season of around three months. The Challenger fishery uses both small ice vessels and large factory trawlers, whereas the St Helens fishery uses only the smaller vessels.

To estimate the economic production function for the St Helens fishery, observations on the harvests and effort of ice vessels on the Challenger Plateau

Table 1
Monthly Harvest, Effort (Trawls) and Biomass Data for the Challenger Plateau Fishery

| Year | June |  |  | July |  |  | August |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | h | E | x | h | E | x | h | E | x |
| 1983 |  |  |  | 420 | 30 | 78252 |  |  |  |
| 1984 |  |  |  | 284 | 21 | 57969 | 160 | 9 | 57425 |
| 1985 | 131 | 7 | 53184 | 478 | 26 | 52245 | 211 | 13 | 51770 |
| 1986 | 1474 | 110 | 48310 | 1290 | 100 | 46381 | 706 | 42 | 45216 |
| 1987 | 1737 | 146 | 36922 | 1279 | 92 | 33665 | 409 | 28 | 32102 |
| 1988 | 957 | 208 | 24575 | 805 | 97 | 19182 | 690 | 138 | 16549 |
| 1989 | 342 | 163 | 13884 | 1560 | 400 | 6781 | 204 | 60 | 6108 |
| 1990 | 179 | 69 | 7073 | 963 | 125 | 4181 | 216 | 49 | 3697 |

$\mathrm{h}=$ harvest (tonnes).
$\mathrm{E}=$ fishing effort (trawls).
$\mathrm{x}=$ biomass (tonnes).
Source: Fisheries Statistics Unit (NZ) for the harvest and effort data, as reported by Clark and Francis (1990) and Clark (1991). Monthly biomass estimates derived, using the population dynamics equation (equation (1)), from the virgin biomass estimate of 91,000 tonnes for the Challenger Plateau, reported in Clark (1991), and from total annual harvest data from Clark and Francis (1990) and Clark (1991), apportioned to months on the basis of monthly harvests reported in Clark and Francis (1990) and Clark (1991).
Note: Harvest and effort data for factory trawlers has been excluded because such vessels are not used on the St. Helens fishery, but total harvest is used in the biomass computations.

Table 2
Regression Results

| No. | Type | Dependent Variable | Independent Variables |  |  | $\begin{gathered} \text { Adjusted } \\ \mathbf{R}^{2} \\ \hline \end{gathered}$ | DurbinWatson | Log of Likelihood Function |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | OLS | $\ln \mathrm{h}$ | $\begin{gathered} \ln \mathrm{A} \\ -1.9226 \\ (-1.4085) \end{gathered}$ | $\ln \mathrm{E}$ <br> 0.8103 <br> (8.1611) | $\begin{gathered} \ln x \\ 0.4827 \\ (4.3777) \end{gathered}$ | 0.7637 | 1.0889 | -9.3044 |
| 2 | AUTO | $\ln \mathrm{h}$ | $\begin{gathered} \ln \mathrm{A} \\ -1.4483 \\ (-1.1173) \end{gathered}$ | $\ln \mathrm{E}$ <br> 0.8205 <br> (9.0097) | $\begin{gathered} \ln x \\ 0.4320 \\ (4.0175) \end{gathered}$ | 0.8157 | 1.7387 | -6.8337 |
| 3 | AUTO $\alpha_{1}=1$ | $\ln \mathrm{h}$ | $\begin{gathered} \ln \mathrm{A} \\ -3.1684 \\ (-3.0649) \end{gathered}$ | $\begin{gathered} \ln \mathrm{E} \\ 1.000 \end{gathered}$ | $\begin{gathered} \ln x \\ 0.5325 \\ (5.2252) \end{gathered}$ | 0.7876 | 1.8100 | -8.88174 |
| 4 | AUTO $\alpha_{1}=1$ $\alpha_{2}=1$ | $\ln \mathrm{h}$ | $\begin{gathered} \ln \mathrm{A} \\ -7.8723 \\ (-42.244) \end{gathered}$ | $\begin{gathered} \ln \mathrm{E} \\ 1.0000 \end{gathered}$ | $\begin{gathered} \ln x \\ 1.0000 \end{gathered}$ | 0.5781 | 1.8709 | $-16.5788$ |
| 5 | OLS | ORP | $\begin{aligned} & \text { Constant } \\ & 2.65 \\ & (6.2716) \\ & \hline \end{aligned}$ | $\begin{gathered} \text { QOR } \\ -0.355 \cdot 10^{-4} \\ (-4.0696) \\ \hline \end{gathered}$ | $\begin{aligned} & \text { SDP } \\ & 0.56838 \\ & (1.9192) \end{aligned}$ | 0.1898 | 2.0577 | -107.576 |

Notes: t values in parentheses.
For Regressions $1-4 \operatorname{Pr}(\mathrm{t}>1.725)=.05$.
For Regressions $5-6 \operatorname{Pr}(\mathrm{t}>1.671)=.05$.
fishery in the winter months were selected. These are reported in Table 1 for the months of June, July, and August from 1983-1990. Table 1 also reports biomass estimates for these months. These estimates were obtained using the population dynamics equation (1) together with total monthly harvest data and an estimate of virgin biomass, $\mathrm{x}_{\mathrm{o}}$. The biomass estimates were used in preference to an alternative procedure suggested by Chambers and Strand (1986), which includes past harvests, modified by the appropriate decay factor, as a proxy for stock in the production function estimation. This procedure is not suitable for fisheries such as orange roughy where the current stock level is strongly influenced by the harvests in several preceding periods.

Clark (1991, Table 5) reports estimates of the virgin biomass of the Challenger stock. An estimate of 91,000 tonnes is derived from trawl survey data together with assumptions about the area swept and the vulnerability of the stock to the gear. An alternative estimate of 122,000 tonnes is derived from catch per unit effort data from the commercial fishery. The estimate of 91,000 tonnes was preferred because it is independent of the commercial catch and effort data which are used to estimate the production function. An analysis of the sensitivity of the results to the choice of the estimate of initial biomass was conducted. Once the estimate of $x_{o}$ is chosen, subsequent estimates of stock are derived from observed harvests and from the parameters of equation (1). It is obvious that the stock estimation process may pose problems for the estimation of the economic production function, but not obvious how these problems can be avoided.

There are two principal features of the economic production function to be established: the nature of the decline (if any) of the catchability term $\mathrm{A}(\mathrm{x})$ as stock declines (the value of $z$ in the coefficient on stock), and the value of the coefficient $y$ on effort. The approach used is to fit the production function with no restriction

Table 3
Optimal Stock and Sustainable Harvest

| Rate of <br> Interest r | $\mathrm{x}^{*}$ <br> (Tonnes) | $\mathrm{h}^{*}$ <br> (Tonnes) |
| :---: | :---: | :---: |
| 0.00 | 28522 | 2561 |
| 0.01 | 24667 | 2511 |
| 0.02 | 21528 | 2484 |
| 0.03 | 18958 | 2442 |
| 0.04 | 16840 | 2389 |
| 0.05 | 15082 | 2331 |
| 0.06 | 13611 | 2269 |
| 0.07 | 12368 | 2205 |
| 0.08 | 11309 | 2142 |
| 0.09 | 10399 | 2079 |
| 0.10 | 9610 | 2018 |
| $\rightarrow \infty$ | 308 |  |
| $\mathrm{x}_{\text {msy }}=28,730 ; \mathrm{h}_{\text {msy }}=2890$. |  |  |

on the value of the coefficients, and then to test hypotheses that the effort coefficient is unity, which is required for linearity of the profit function, and that the stock coefficient is unity indicating a constant catchability coefficient. The regression results are reported in Table 2.

The economic production function, equation (12), is first estimated by OLS with no constraint on the values of the coefficients on E or $x$. A single estimating equation corresponding to equation (12) is used on the assumption that the disturbance terms, reflecting acts of nature, are uncorrelated with the errors made by vessels in the process of attempting to choose effort to maximize expected profit (see Zellner, Kmenta, and Dreze (1966)). The Durbin-Watson statistic suggests the presence of autocorrelation. This is not unexpected since as noted above the stock in a given month depends on the total harvest (harvest of ice boats and factory trawlers) in previous months. A Cochrane-Orcutt procedure was used to correct for first order autocorrelation and Regression 2 in Table 2 was obtained. The value of the Durbin-Watson statistic in Regression 2 indicates the absence of residual autocorrelation. A restricted version of Regression 2, with the coefficient on effort restricted to unity, was fitted and the results reported as Regression 3. Regression 4 was run with the additional restriction that the coefficient on stock is unity.

A comparison of Regressions 2 and 3, by means of a log likelihood ratio test, is used to test the hypothesis that the effort coefficient is unity. The test statistic is $-2(\mathrm{Lr}-\mathrm{Lu})$ which is distributed as $\chi^{2}$ with degrees of freedom equal to the number of restrictions. The test statistic is 4.09618 , indicating that the hypothesis can be rejected at the $5 \%$ confidence level but not at the $2.5 \%$ level. Since the result of this test is inconclusive and since rejecting the hypothesis would involve adopting a nonlinear optimal control approach, it was decided not to reject the hypothesis and to set the coefficient on effort equal to unity. A comparison of Regressions 3 and 4 is used to test the additional restriction that the coefficient on stock is unity. The test statistic is 15.39412 indicating that the hypothesis can be

Table 4
Harvest and Stock (Tonnes) on the Approach Path to Equilibrium Stock

| Year | x* |  | $\mathrm{X}_{\infty}$ |  | $\mathrm{x}_{0.1}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Harvest | Stock | Harvest | Stock | Harvest | Stock |
| 1 | 15,000 | 110,000 | 15,000 | 110,000 | 15,000 | 110,000 |
| 2 | 15,000 | 95,472 | 15,000 | 95,472 | 15,000 | 95,472 |
| 3 | 15,000 | 81,573 | 15,000 | 81,573 | 15,000 | 81,573 |
| 4 | 15,000 | 68,228 | 15,000 | 68,228 | 15,000 | 68,228 |
| 5 | 15,000 | 55,368 | 15,000 | 55,368 | 15,000 | 55,368 |
| 6 | 15,000 | 42,926 | 15,000 | 42,926 | 11,473 | 42,926 |
| 7 | 15,000 | 30,838 | 15,000 | 30,838 | 2,909 | 34,295 |
| 8 | 3,335 | 19,036 | 15,000 | 19,036 | 2,909 | 34,295 |
| 9 | 3,257 | 18,958 | 10,630 | 7,447 | 2,909 | 34,295 |
| 10 | 3,257 | 18,958 | 3,470 | 308 | 2,909 | 34,295 |
| 11 | 3,257 | 18,958 | 3,470 | 308 | 2,909 | 34,295 |
| 12 | 3,257 | 18,958 | 3,470 | 308 | 2,909 | 34,295 |
| 13 | 3,257 | 18,958 | 3,470 | 308 | 2,909 | 34,295 |
| 14 | 3,257 | 18,958 | 3,470 | 308 | 2,909 | 34,295 |
| 15 | 3,257 | 18,958 | 3,470 | 308 | 2,909 | 34,295 |
| 16 | 3,257 | 18,958 | 3,470 | 308 | 2,909 | 34,295 |
| 17 | 3,257 | 18,958 | 3,470 | 308 | 2,909 | 34,295 |
| 18 | 3,257 | 18,958 | 3,470 | 308 | 2,909 | 34,295 |
| 19 | 3,257 | 18,958 | 3,470 | 308 | 2,909 | 34,295 |
| 20 | 3,257 | 18,958 | 3,470 | 308 | 2,909 | 34,295 |
| 21 | 3,257 | 18,958 | 3,470 | 308 | 2,909 | 34,295 |
| 22 | 3,257 | 18,958 | 3,470 | 308 | 2,909 | 34,295 |
| 23 | 3,257 | 18,958 | 3,470 | 308 | 2,909 | 34,295 |
| 24 | 3,224 | 18,958 | 3,437 | 308 | 2,895 | 34,295 |
| 25 | 3,182 | 18,958 | 3,395 | 308 | 2,833 | 34,295 |
| 26 | 3,127 | 18,958 | 3,341 | 308 | 2,778 | 34,295 |
| 27 | 3,052 | 18,958 | 3,267 | 308 | 2,702 | 34,295 |
| 28 | 2,943 | 18,958 | 3,159 | 308 | 2,593 | 34,295 |
| 29 | 2,768 | 18,958 | 2,987 | 308 | 2,478 | 34,295 |
| 30 | 2,445 | 18,958 | 2,667 | 308 | 2,478 | 34,295 |
| 31 | 2,442 | 18,958 | 1,858 | 308 | 2,478 | 34,295 |
| 32 | 2,442 | 18,958 | 147 | 308 | 2,478 | 34,295 |

rejected. On the basis of these tests the coefficients of Regression 3 were adopted for the economic production function.

The production function was re-estimated using stock estimates derived on the basis of alternative estimates of initial biomass, $\mathrm{x}_{0}$. The estimate of 91,000 tonnes was found to be at the low end of the range of estimates of $x_{0}$ which are consistent with the observed harvest record. Estimates of $101,000,111,000$ and 121,000 tonnes were used to generate stock values for the production function estimation. In each of these production function estimations the hypotheses that the coefficient on effort and on stock were unity could not be rejected on the basis of the $\log$ likelihood ratio tests. The effect on the various net present value estimates of
setting the coefficient on stock in the production function equal to unity is reported for purposes of comparison.

## Stock Estimates

Using the production function estimates of A and z together with the other parameter values chosen, the optimal and open-access equilibrium stock estimates can be calculated. The optimal stock estimates are obtained by solving equation (3) for a range of interest rates, while the open-access stock is estimated from a zero profit equilibrium condition. The "safe" stock level is calculated from the yield per recruit function.

## Optimal Stock

Using the functional forms and parameter values described above, equation (3) can be solved for the optimal stock corresponding to a range of real rates of interest. Sustainable harvest can be calculated by solving equation (1) for the harvest level corresponding to the equilibrium fish stock:

$$
\begin{equation*}
h=x-(x-G(x)) e^{-\delta(x)} \tag{13}
\end{equation*}
$$

By substituting the stock-recruitment equation (4), with $x_{t}=x_{t-\gamma}$ into equation (13) and differentiating, the stock corresponding to maximum sustainable yield can be calculated as the solution to:

$$
\begin{equation*}
0=1+\mathrm{e}^{-\delta(x)}\left((\mathrm{x}-\mathrm{G}(\mathrm{x})) \delta_{\mathrm{x}}-\left(1-\mathrm{G}_{\mathrm{x}}\right)\right) \tag{14}
\end{equation*}
$$

The maximum sustainable yield is obtained by substituting $\mathrm{x}_{\text {msy }}$ into (13). The optimal stock estimates and corresponding sustainable harvest levels for a range of interest rates, together with the MSY estimates, are reported in Table 3.

It can be seen from Table 3 that the optimal stock is sensitive to the rate of interest. At a 1 per cent real rate of interest around $22 \%$ of biomass is retained, whereas at $6 \%$ only about $12 \%$ is retained. Real rates of interest are generally believed to be in the $4-6 \%$ range which in Table 3 correspond to less than $15 \%$ of virgin biomass.

Use of the real rate of interest to compute the net present value of investing in the fishery assumes that the prices and costs used to calculate profit do not change relative to the general price level. There are reasons to believe that this assumption will not be satisfied. It is generally assumed that the income elasticity of demand for fish exceeds unity and that the relative price of fish will rise over time. Furthermore as orange roughy stocks are mined down the price of this species relative to others will rise unless it has perfect substitutes as implied by the earlier assumption of perfectly elastic demand. It is also likely that there will be further technical change in the orange roughy fishery which will lead to lower harvesting costs through an increase in catchability. On the other hand, the unit cost of effort is likely to rise as labour and fuel costs rise. These anticipated changes can be modelled by allowing the price of roughy, $p$, to rise at rate $k_{1}$, the catchability coefficient, A, to rise at rate $k_{2}$, and the unit cost, $c$, to rise at rate $k_{3}$ over time. If $k_{1}=k_{3}-k_{2}$, then profit will grow in real terms at rate $k_{1}$. (If the relationship
$\mathrm{k}_{1}=\mathrm{k}_{3}-\mathrm{k}_{2}$ does not hold the maximization problem becomes nonautonomous and there may be multiple equilibria (see Clark and Munro (1975)).

A constant rate of growth of profit can be incorporated in the analysis by reducing the discount rate which forms the discount factor, $\alpha$, in equation (2) by the amount of the rate of growth of price. The use of a relatively low real rate of interest to capitalize net revenue streams from primary production has a precedent in agriculture where observed low current annual returns as a percentage of land value are thought to be explained partly by anticipated technical changes leading to decline in production costs.

Using a real rate of interest of $3 \%$ the optimal stock is around $17 \%$ of virgin biomass. The sustainable yield corresponding to this stock level is around 2400 tonnes per year. These estimates are not highly sensitive to the value of the estimated coefficient on stock in the economic production function. For example, at a $3 \%$ real rate of interest the optimal stock and sustainable yield corresponding to the stock coefficient value of unity are 21,884 and 2489 tonnes respectively. As can be seen by comparison with the values reported in Table 3, this change in the value of the stock coefficient has little effect on the results.

## Open-access Equilibrium

In open-access equilibrium the stock will be fished down to the level at which unit harvesting cost equals price: $\mathrm{x}_{\infty}=(\mathrm{c} / \mathrm{pA})^{1 / 1-z}$. The equilibrium values of $\mathrm{x}_{\infty}$ for the alternative estimates for A and z are reported in Table 3 as optimal stock estimates as $r \rightarrow \infty$. As expected for a schooling fishery the open-access equilibrium is very low at 308 tonnes. It is doubtful whether this stock level is sustainable. When the coefficient on stock in the economic production function is set equal to unity, the open-access equilibrium stock level rises to 3664 tonnes, which is still a dangerously low level.

## The Sustainable Stock Equilibrium

For there to be a sustainable yield, the stock must be viable at the level corresponding to that yield. A rule-of-thumb which has been applied to the New Zealand orange roughy fishery is the " $\mathrm{F}_{0.1}$ Rule": fishing mortality should be set at a rate at which the slope of the yield per recruit function is one tenth of its slope at the origin (see Gulland and Boerema (1973)). The yield per recruit is given by:

$$
\begin{equation*}
(\mathrm{Y} / \mathrm{R})=\int_{\gamma}^{\mathrm{t}_{\mathrm{m}}} \mathrm{Fe}^{-(\mathrm{m}+\mathrm{F})(\mathrm{t}-\gamma)} \mathrm{W}(\mathrm{t}) \mathrm{dt} \tag{15}
\end{equation*}
$$

where

$$
\begin{aligned}
\gamma & =\text { age of recruitment to the fishery } \\
\mathrm{t}_{\mathrm{m}} & =\text { maximum age achieved } \\
\mathrm{F} & =\text { fishing mortality } \\
\mathrm{m} & =\text { natural mortality } \\
\mathrm{W}(\mathrm{t}) & =\text { weight for age relationship. }
\end{aligned}
$$

Clark and Francis (1990) use the weight for age relationship implied by equations (5) and (6) in their application of the $\mathrm{F}_{0.1}$ Rule to the Challenger Plateau stock.

Using the values already selected for $\gamma, m, L_{\infty}, a_{o}, u$ and $v$, and the value $t_{m}$ $=70$ years, chosen by Clark and Francis, the following equation can be solved numerically for $\mathrm{F}_{0.1}$ :

$$
\begin{equation*}
\mathrm{d}(\mathrm{Y} / \mathrm{R}) / \mathrm{dF}=\left.(0.1)(\mathrm{d}(\mathrm{Y} / \mathrm{R}) / \mathrm{dF})\right|_{\mathrm{F}=0} \tag{16}
\end{equation*}
$$

The solution value reported by Clark and Francis is 0.075 . On this basis the recommended annual harvest from the St Helens stock is 2478 tonnes which corresponds to an equilibrium stock of around 34295 tonnes, which is in excess of the MSY stock of 27172 tonnes. The MSY stock is around $25 \%$ of biomass and according to the $\mathrm{F}_{0.1}$ Rule this stock level provides an inadequate safety margin.

## The Approach Path and Net Present Values

A most rapid approach path from the virgin biomass, $\mathrm{x}_{\mathrm{o}}$, to the optimal stock, $\mathrm{x}^{*}$ is optimal when price is unaffected by level of harvest, and the unit cost of harvest is unaffected by the level of harvest chosen. It has been argued that these conditions are satisfied for the St Helens fishery for the observed range of harvests. The highest observed harvest of around 15,000 tonnes per annum can be regarded as the maximum rate at which the price of fish and the unit cost of effort can be considered constant. This rate, $\mathrm{h}_{\text {max }}$, will be treated as the maximum rate in the linear control optimization.

The time-path of harvest implied by the most rapid approach is not entirely straightforward. Consider, as an example, the approach from $x_{o}$ to $x^{*}$. At a harvest rate of 15,000 tonnes the stock will be fished down from $x_{0}$ to $x^{*}$ over a period $\mathrm{t}^{*}<\gamma$ years. During the period $0<\mathrm{t}<\gamma$ recruitment to the stock is determined by the virgin biomass $x_{0}$. Since $G\left(x_{0}\right)>G\left(x^{*}\right)$ recruitment will be in excess of the level required to maintain $x=x^{*}$. This means that harvest, $\hat{h}$, over the period $\mathrm{t}^{*}<\mathrm{t}<\gamma$ can be higher than the long-run sustainable harvest, $\mathrm{h}^{*}$ :

$$
\begin{equation*}
\hat{\mathrm{h}}=\mathrm{h}^{*}+\mathrm{e}^{-\delta\left(\mathrm{x}^{*}\right)}\left(\mathrm{G}\left(\mathrm{x}_{\mathrm{o}}\right)-\mathrm{G}\left(\mathrm{x}^{*}\right)\right) \tag{17}
\end{equation*}
$$

For the period $\gamma<\mathrm{t}<\gamma+\mathrm{t}^{*}$ recruitment is still determined by a stock level which is higher than $x^{*}$ and sustainable harvest is in the range $\hat{h}>h>h^{*}$. The harvest and stock levels implied by this most rapid approach path to $\mathrm{x}^{*}, \mathrm{x}_{\infty}$, and $x_{0.1}$ are reported in Table 4. Figure 2 illustrates the nature of the approach.

Under open access conditions the harvest rate would be unconstrained and a more rapid approach to $x_{\infty}$ than that implied by $h_{\max }$ would be possible. This could have the effect of increasing net present value by bringing harvests forward, on the one hand, and reducing net present value by increasing the unit cost of effort and reducing the price of orange roughy on the other. A nonlinear profit function together with a model of the entry and exit dynamics of the fishery would be required to analyse this question which is beyond the scope of the present paper.

The profit function is evaluated for the harvest and stock level reported for each of the years for each of the three exploitation paths on the assumption that harvest is taken instantaneously at the reported stock level for the period. Net present values, at a $3 \%$ discount rate, are calculated on the assumption that the profit from the equilibrium phase is in perpetuity. The net present values corresponding to the three stocks, $\mathrm{x}^{*}, \mathrm{x}_{\infty}$, and $\mathrm{x}_{0.1}$ are: $\$ 180$ million, $\$ 122$ million, and
(a) Approaches to alternative stock equilibria

(b) Time-path of harvest in the approach to $\mathrm{X}^{*}$


Figure 2. Approach paths.
$\$ 165$ million respectively. If the coefficient on stock in the economic production function is set at unity, the three net present value estimates become $\$ 172$ million, $\$ 116$ million and $\$ 163$ million respectively.

## Conclusions

The model developed in this paper can be used to answer three sets of questions relating to the management of the St Helens orange roughy stock: first, will an unregulated open-access fishery result in the commercial extinction of the stock, and if so, what will be the cost of this outcome? second, what is the equilibrium stock which maximizes expected net present value? and, third, what is the cost of ensuring a high probability that the exploitation of the stock is sustainable? It is assumed that the real rate of interest is $3 \%$, which is towards the low end of the accepted range to allow for the likelihood that real returns from the fishery will rise over time.

Based on the NPV estimates, the answers to the three questions are: openaccess will result in commercial extinction and the cost of this outcome relative to the maximum expected net present value is around $\$ 58$ million; the maximum expected net present value of the fishery is around $\$ 180$ million; and the cost of a conservative strategy with respect to sustainability (the $\mathrm{F}_{0.1}$ rule) is around $\$ 15$ million.

This paper has used information derived from the New Zealand experience to analyse management options for the recently discovered St. Helens stock. There are probably other Australian stocks to be exploited: for example, on the Cascade Plateau near the south-eastern corner of the Australian Fishing Zone, and in the Great Australian Bight. Exploratory fishing is currently being conducted off Chile and Namibia, and France is exploring in the North Atlantic. Over the next decade additional orange roughy fisheries are likely to develop in both the Northern and Southern Hemispheres. The information and analysis presented in this paper can contribute to developing management policies which can be put in place before excess disinvestment in the stocks occurs.

## Appendix

Derivation of the Growth Rate $\mathrm{g}(\mathrm{x})$ and an Approximation to the Survival Rate $\delta(\mathrm{x})$.

The problem is to describe the form of the relationship between instantaneous net survival rate $\delta$ (growth less natural mortality) and stock size x . The approach used here is to calculate both $\delta$ and x as a function of fishing mortality F . A simple functional form for $\delta(\mathrm{x})$ is then fitted to a set of $(\delta, \mathrm{x})$ pairs calculated across a range of values of F .

$$
\begin{equation*}
\delta \text { as a function of } \mathrm{F} \tag{1}
\end{equation*}
$$

As noted in the survival section above, Schnute (1989) has shown that the growth component in the survival term is related to growth at mean weight, i.e.

$$
\mathrm{g}(\mathrm{x})=\ln \left(\mathrm{W}^{\prime} / \mathrm{W}\right)
$$

where $W$ is mean weight and $W^{\prime}$ is the weight which a fish of weight $W$ at time $t$ will grow to by time $t+1$.

Mean weight for a given F can be calculated by

$$
\begin{aligned}
\mathrm{W}_{\mathrm{F}} & =\Sigma\{\mathrm{N}(\mathrm{t}) \cdot \mathrm{W}(\mathrm{t})\} / \Sigma \mathrm{N}(\mathrm{t}) \\
& =\Sigma\left\{\mathrm{e}^{-(\mathrm{m}+\mathrm{F}) \mathrm{t}} \cdot \mathrm{~W}(\mathrm{t})\right\} / \Sigma \mathrm{e}^{-(\mathrm{m}+\mathrm{F}) \mathrm{t}}
\end{aligned}
$$

where summation is over ages $t$ from $\gamma$ to infinity. The corresponding age $t_{F}$ at mean weight can be calculated by substituting $\mathrm{W}_{\mathrm{F}}$ in equation (6) and solving for $t$. $W_{F}^{\prime}$ is then found by substituting $t_{F}+1$ for 1 for $t$ in equation (6). Finally, the growth component $g_{f}$ is calculated as

$$
\mathrm{g}_{\mathrm{F}}=\ln \left(\mathrm{W}_{\mathrm{F}}^{\prime} / \mathrm{W}_{\mathrm{F}}\right)
$$

and the net survival term $\delta$ as a function of F is given by

$$
\begin{equation*}
\delta_{\mathrm{F}}=\mathrm{g}_{\mathrm{F}}-\mathrm{m} \tag{A1}
\end{equation*}
$$

Let $\mathrm{x}_{\mathrm{F}}$ be the equilibrium recruited biomass for a given level of fishing effort $F$. Then

$$
\begin{aligned}
x_{F} & =\sum_{t=\gamma}^{t_{m}} N_{t} W(t) \\
& =\sum_{t=\gamma}^{t_{m}} \operatorname{Re}^{-(M+F)(t-\gamma)} W_{\propto}\left[1-e^{-K\left(t-t_{0}\right)}\right]^{v}
\end{aligned}
$$

where $\mathrm{N}_{\mathrm{t}}=$ number at age t , and
$W_{\infty}=u \cdot L_{\infty}{ }^{\nu}$. Recruitment R is given by

$$
\mathrm{R}=\frac{\mathrm{ax}_{\mathrm{F}}}{\left(1+\left(\mathrm{b} / \mathrm{x}_{\mathrm{o}}\right) \mathrm{x}_{\mathrm{F}}\right)} \cdot \frac{1}{\mathrm{~W}_{\gamma}}
$$

where $\mathrm{W}_{\gamma}=$ weight at recruited age $\gamma$. Then

$$
\mathrm{x}_{\mathrm{F}}=\frac{\mathrm{W}_{\infty}}{\mathrm{W}_{\gamma}} \frac{\mathrm{ax}_{\mathrm{F}}}{\left(1+\left(\mathrm{b} / \mathrm{X}_{\mathrm{o}}\right) \mathrm{x}_{\mathrm{F}}\right)} \cdot \mathrm{e}^{(\mathrm{M}+\mathrm{F}){ }_{2} \mathrm{~S}_{\mathrm{F}}}
$$

where $S_{F}=\sum_{t=\gamma}^{t_{m}} e^{-(M+F) t}\left[1-e^{-K\left(t-t_{0}\right)}\right]^{v}$

Rearranging terms,

$$
\begin{equation*}
\mathrm{x}_{\mathrm{F}}=\left[\frac{\mathrm{W}_{\infty}}{\mathrm{W}_{\gamma}} \mathrm{e}^{\left.(\mathrm{M}+\mathrm{F}) \gamma \mathrm{aS}_{\mathrm{F}}-1\right] /\left(\mathrm{b} / \mathrm{x}_{\mathrm{o}}\right)}\right. \tag{A2}
\end{equation*}
$$

$$
\begin{equation*}
\text { An approximation to } \delta(x) \tag{3}
\end{equation*}
$$

By filtering various simple functional forms to $(\delta, x)$ pairs generated from (A1) and (A2), an adequate approximation $\left(R^{2}\right.$ regression $\left.=0.993\right)$ was given by the equation:

$$
\delta(\mathrm{x})=-0.032\left(\mathrm{x} / \mathrm{x}_{\mathrm{o}}\right)^{1 / 2}
$$

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