Samuelson's Full Duality and the Use of Directed Acyclical Graphs:
The Birth of Causally Identified Demand Systems

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Authors: Matthew Stockton, Oral Capps, Jr., and David Bessler

Affiliation: **Ph.D. student, Professor and Southwest Dairy Marketing Endowed Chair, and Professor Department of Agricultural Economics, Texas A&M University**

Mailing Address: **Dr. Oral Capps, Jr.**
Department of Agricultural Economics
330C Blocker
Texas A&M University
College Station, TX 77843-8491

Telephone Number: (979) 845-8491

Email: **ocapps@tamu.edu**

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Abstract

To date, mixed demand systems have been all but ignored in empirical work. A possible reason for the scarcity of such applications is that one needs to know a priori which prices and quantities are endogenous in the mixed demand system. By using a directed acyclical graph (DAG), causal relationships among price and quantity variables are identified giving rise to a causally identified demand system (CIDS). A statistical comparison is made of the traditional Rotterdam model with a Rotterdam mixed demand system identified through the use of a DAG. In this analysis, the respective Rotterdam demand systems consist of five products: steak, ground beef, roast beef, pork, and chicken.

Introduction

John Maynard Keynes once wrote in a letter to R. F. Harrod, dated July 4, 1938, "Progress in economics consist almost entirely in a progressive improvement in the choice of models...Economics is the science of thinking in terms of models joined to the art of choosing models which are relevant to the contemporary world."

This paper is primarily concerned with making appropriate choices for the models commonly applied to demand estimation. A scientifically sound methodology is proposed to help as a guide in the use of the full duality as it relates to empirical application.

While economic theory provides the basis used to construct empirical models it is often left to the individual analyst justify the appropriate use of the chosen model. The correct assignment of the appropriate structure to any given problem is paramount to empirical model development and it is at this cross roads that theory and application meet. In many cases the choice of model to use to do the empirical work is not completely specified by the theory and must be justified in an ad hoc fashion. For example theory may indicate which variables a model should include but say nothing of the functional form to fit the model. It is the empirical or applied scientist who must push the methodology to compensate the theory.

The study of demand and consumer behavior is one of the most studied areas of economics, and yet there are still many debates on methodology. Two debates that come to mind are the functional form issue (Davis), and the inverse or directly specified system debate (reference would be good here). Some of the discussion, especially about appropriate choice and specification of a specific model could be answered if the relationships reflected in the data were known. If the modeler knew the true nature of the process whereby the data was generated he would simply apply the specification that was representative of that process. Unfortunately in economics we have been left without the benefit of the causal identities that comes from using experiments. Most of the economic data is secondary, or observational in nature and true experimentation is unavailable to help clarify the process where cause and effect can be identified clearly.

This paper aims to propose a tool that may help to clarify the muddied waters created by secondary and observation information.

When an understanding of the causal relationships of the data are not available there is a tendency to be cautious in approach and to use only models that are widely accepted and have been traditionally applied. This of coarse is a good thing but it has a tendency to limit the use of theory. For example, parts consumption theory has been left virtually unapplied in empirical work. Starting with Samuelson it has been recognized that demand could be determined by three possible specifications. Most often economists have used a singularly dependent model, either one direct demand which is quantity dependent, or two an inverse demand which is price dependent. The third type of demand is a mixed type where demand is both quantity and price dependent. The literature is full of singularly dependent demand estimates, most being quantity dependent, with fewer being price dependent, with the use of the mixed demand, both price and quantity dependent, limited to a single handful.

The few examples of mixed demand are rare and are strictly justified by rhetorical means. The justifications for those few mixed systems published in the literature are based on sound economic reasoning but have no empirical clout and have thus far failed to win many converts to a wide use of the mixed demand cause. What is interesting however is the recognition by Chavs, Moschini and Vissa, pioneers of mixed demand specifications, of the potential value of the mixed demand system. All three authors recognized the potential of putting the mixed demand specification into the toolbox of the blue collar economist. Chavas referring to the potential value of mixed demand, stated that, " Since both prices and quantities are endogenous variables at the market level, the rational for using quantity dependent demand functions in this context mostly disappears." Chavas blazed the trail between Samuelson's Full duality and theoretical implication of that duality. He provides the behavioral implications that are consistent with consumer theory for a mixed demand specification. Moschini and Vissa further support the implementation and use of the mixed demand specification and declare the mixed demand specification as having "obvious potential". They provide the model specification necessary to cope with the direct and indirect utility functions to operationalize the theory of mixed demand. However as much as this enables empirical economist to move forward with implementation of mixed demand it does not provide a method of determining when to appropriately apply full duality. If mixed demand is to ever become the powerful tool its conceptors envisioned it to be, it must have a theoretically consistent, and empirically viable basis. The intent of this paper is to propose such a theoretically and empirically sound basis.

Literature

The event of mixed dependent demand relationships although recognized as possible, has been rarely applied. In 1965 Paul Samuelson introduced his piece in Econometrica about full duality demonstrating that utility maximization was possible under a mixed utility framework, both price and quantity dependent. However Samuelson directed his theory to only the idea of a rationed system in equilibrium. In 1983 Jean Paul Chavas built on Samuelson's work by showing that compensated mixed demand is identical to compensated conditional demand. Mixed demand has a Slutsky type relationship, including symmetry and negative semi-definite matrices, with symmetrical cross effects except in sign. Chavas also showed that conditions of homogeneity, and adding up could be imposed on a mixed demand system making it as he says, "very attractive for the investigation of consumption decisions since it is obtained without sacrificing the elegance of the theory". Thus Chavas was the first to show that a mixed system or as Samuelson called it "Full Duality" is consistent with accepted demand theory and was the bridge between the two polar cases of the singularly dependent, price and quantity derived demands. However it was not until Moschini and Vissa in 1993 that the first formally published use of a mixed demand system is used in an empirical application. In their use of a mixed demand system, Moschinni and Vissa study the retail price and quantity relationships in the Canadian meat market. They justify the application of the mixed system based on an appeal of rationality. Moschinni and Vissa indicated the mixed system was necessary since chicken, a supply controlled product, was one part of a three product market that included two other freely traded commodities, beef and pork. However plausible the justification, no other evidence was used to substantiate the application of a mixed demand system. While there is no objection to them using this mixed demand approach it is a subjective application and leaves room, no matter how small, for argument about the appropriateness of the application.

Some have speculated for the lack of mixed demand applications. Moschini and Vissa speculate that so few applications of Samuelson's full duality stem from the condition that the mixed system requires a simultaneous knowledge of indirect and direct utility functions necessary to specify the system. However, after the introduction of there solution, directly specifying the demand system, which has been available for more than 10 years, it has yet to be applied by anyone else. This unfortunately leaves the Moschini and Vissa paper falling short of the authors expectations. While being a big step forward in implementation it is apparently not the only hold back to the common use and application of mixed demand systems. However several positive things do come from Moschini and Vissa's work, a demonstration of Chavas' theoretically sound model, and the validation that a mixed demand system results in different estimates of parameters and elasticities then the singularly dependent system. This still leaves the question about the limited application of full duality unanswered.

The real problem with applying full duality is not theoretical, but is empirical. What is needed is a bridge between theoretical concepts and empirical application. Does such a bridge exist?

Judea Pearl in his 1990 book "Causality" boldly states the substance of what theories are made "casual machinery that underlies and propels our understanding of the world". Representative models are in effect the illustration of the causes on the effects. Therefore, to fully apply demand theory it is necessary to determine the model's causes and effects. Until recently no scientific and theoretically tenable methodology had been accessible to combine with economic theory that reveals the causal relationships from observational or non-experimental data. Recent work in statistics and computer science has developed techniques to determine causality from simple statistical information. Specific application of this new theory is through an algorithm known as PC-algorithm, which creates a directed acyclical graph (DAG) from correlation and conditional correlations between variables. The primary focus of this paper is to demonstrate how this new theory of causality can be applied to clarify the not so clear waters left in the wake of theoretical gaps. Had Samuelson, Chavas, and Moschini and Vissa been able to provide the road map that connects the lofty peaks of theory to the valley's of empiricism, by being able to identify causality, they may well have seen the fulfillment of their expectations for mixed demands.

Methodology

Overview

The creation of the Directed Acyclical Graph (DAG) specified demand system or more generally Causally Identified Demand System (CIDS) requires the model to be created in a series of stages or steps. The first step, as with most econometric procedures, requires the screening and cleaning of the data. The second step entails putting the data into the appropriate form so the DAG software used to determine causal relationships may be applied. In this case the proper data form is a correlation matrix of the variables as they are to be estimated in the demand system.

The causal relationships, represented by the DAG procedure, are then used as a guide in specifying the left-hand and right-hand side variables of the equations in the demand system, which may or may not be a singularly dependent or mixed demand system. The CIDS model is then estimated.

The resulting CIDS estimates are then transformed into elasticities, which will be statistically compared singularly and adversely with the elasticities calculated from the estimates of a singularly dependent demand system (SDDS). It may happen that the CIDS and SDDS models may be identical in which case a comparison would be unnecessary. In this later case the extra work of applying the CIDS methodology only serves to enhance the robustness of the estimated model.

Data

The data used in these model is a subset of the data used in a paper published in the American Journal of Agricultural Economics, August 1989 by Oral Capps, Jr.. This scanner data contained 138 weekly observations of various Randall Stores located in Houston, Texas from January 1986 to June 1987. Of the many (UPC) coded items, five where selected as a group. This group called meat was comprised of three beef products, steak, ground beef, roast beef, and two general product types, pork, and chicken. This data was ideal for the purposes of this study. Scanner data is panel type data, exhibiting both cross sectional and time series properties.

The DAG (Directed Acyclical Graph)

A directed acyclical Graph (DAG) is a picture representing cause and effect among a set of variables. To completely explain the mathematics behind such a representation is beyond the intent and scope of this paper. However a general explanation of the DAG and its empirical identification will be given here to facilitate understanding of the results. For more details the reader is referred Pearl or Sprites, Glymour and Scheimes.

DAG is a graphoid that represents causal relationships between variables. It contains no cyclical paths. See Figure 1. Arrows are used to show cause and effect flows. These do not flow in a cyclical manner. $A \longrightarrow B \longrightarrow A$

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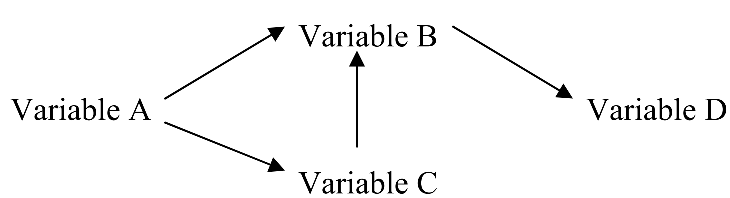
DAG's are constructed from statistical information among and between variables. The software, TETRADII, employs PC- algorithm which is a sequential series of steps that uses correlations and conditional correlations of the variables and the Fisher Z statistic to determine causal relationships. (Sprites, Glymour and Scheimes)

Three key assumptions are made to make this method of directing edges possible, causal sufficiency, Markov conditioning, and the faithfulness condition. Causal sufficiency, relates to the completeness of the variables being considered for directing. If some variable is missing such that it does, in fact, cause two or more variables included in the analysis then the output graph from PC-algorithm will be misleading. The Markov

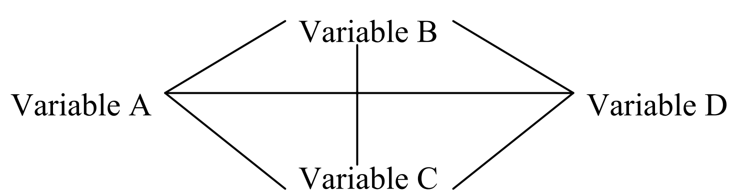
Figure 1. A Directed Acyclical Graph



condition, is that the probability generating variables in the analysis can be written in terms of the conditional probabilities where they are only conditional with respect to there causal parents. Any effect a grandparent has on a grandchild is through the parent. The third assumption, the faithfulness condition, relates to correlation and edges. A zero correlation or conditional correlation is observed between two variables it is because there is no edge connecting these variables. It is not the case that fundamental structural parameters are of the specific magnitude such that the algebraic combination of them cancel one another.

To help explain the DAG process, in this case PC algorithm, a simple set of four variables, ABCD, will be used to illustrate. The first step in the algorithm is to take all variables and assume that each are connected to each of the others by drawing a line or edges between each and all variables. See figure #2.

Figure 2. A Complete Undirected Graph



The connections in figure 2 are represented by lines referred to as undirected edges, without an arrow or direction, representing no causality between variables. Each of the relationships between each pair of variables is tested for statistically significant correlation, those pairs that do not have a statistically significant correlation, using a Fisher Z test are removed. When the pair wise comparison is complete and all statistically insignificant edges have been removed, edges are tested conditionally on other connected variables.

The conditional correlation between all groups of threes that are connected, have edges, is then used to direct the remaining edges. In this step the concept of D-separation, is used in conjunction with its relationship to causal forks, inverted forks and causal chains and provides the information to direct the undirected edges. For example the variables A, B, and C connected by two edges, one between A and B, the other between B and C, has only four possible combinations of directed edges. See figure 3.

Figure 3. Causal Relationships Among Triples, A, B, C.

$A \leftarrow B \rightarrow C$ Causal Fork	$A \rightarrow B \rightarrow C$ Causal Chain Right	$A \rightarrow B \leftarrow C$ Inverted Fork
	$A \leftarrow B \leftarrow C$ Causal Chain Left	

Of the four possible causal paths, only one can be directly directed by conditioning on the common variable. This case occurs when the common variable B, is conditioned on the correlation between the two adjacent variables A and C, causes a connection between these adjacent variables, that is the adjacent variables are **not** D-separated, but are D-connected. D-connected variables have a 1 to 1 representation with a causal path type. A set of two variables, A and C, are D-connected by a common variable, B, have an inverted fork representation, arrows pointing from the two adjacent variables to the common variable as shown in Figure 4.

Figure 4. Inverted Fork

$A \leftarrow B \rightarrow C$
Causal Chain Left

If however when the common variable, B, is conditioned on and the correlation between the adjacent variables, A and C, disappears then these variables are D-separated by the common variable, B, and any other of the three causal path conditions exist. See Figure 5.

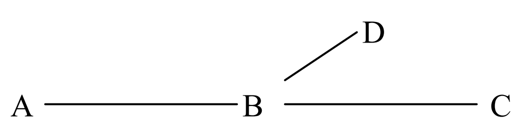
Figure 5. DAGs Where Conditioning in the Middle Variable Removes Partial Correlation Between Outside Variables.

A ← B → C	A → B → C	A → B ← C
Causal Fork	Causal Chain Right	Causal Chain Left

In a large set of variables, where variables can be part of more then one set of three variables, combinations of the directed edges can be used to direct other edges from among those which remain undirected.

To clarify how directed edges might direct undirected edges consider our small group of four variables, A, B, C, and D. Assume the whole set of four variables has been found to have edges between A and B, B and C, C and D, and D and B, all of these edges are yet undirected. See figure 6.

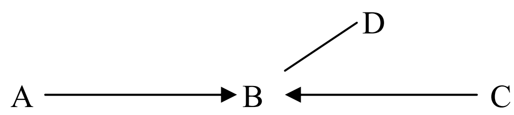
Figure 6. Hypothetical DAG with removed but Undirected Edges



Taking a set of three variables, such as the set ABC with B as the common variable and A and C as the adjacent variables, it is found by conditional correlation that

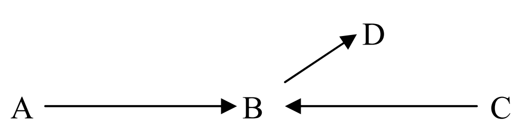
B, D-connects A and C, therefore the edges are directed as an inverted fork with arrows pointing away from A and C towards B. See Figure 7.

Figure 7. Hypothetical DAG, first step B D-connects A and C



With further conditioning it is found that all other sets of three are D-separated, implying that no more edges may be directly directed and those sets remain undirected by conditional correlation. However, the information that the set ABD is D-separated given B allows the edge between B and D to be directed. Since the edge between A and B is pointing towards B the only possible way the edge between B and D can be directed is from B to D (given we know the ABD is D-separated given B. The set ABD is a D-separated conditioning on B which implies three possibilities, but of the these three possibilities the only one possible is a causal chain right directed towards D from B and from A towards B. See figure 8.

Figure 8. Hypothetical DAG, Directed result of D-separation of ABD given B.



Not all edges may be directed in all cases, and may require some further use of statistical techniques, such as the use of SIC or AIC Criteria to direct those undirected edges. The key point here is that the direction of the edges depends on the different relationships between various sets of variables and there D-separation and D-connection.

The Demand Model

The primary purpose of developing the demand models in this case is to extend the currently followed techniques and to enhance the use of theory in the application of empiricism. Therefore, it is necessary to understand how the SDDS and a mixed demand CIDS model compare.

Normally when a demand system is identified the pertinent products are grouped by relationship justified by the separability assumption. The separability assumption allows for the isolation of individual groups of products from all other commodities and the substitution of total expenditure for income. The variables in a mixed demand system are specified in a like manner, the difference being the causal relationship between specific prices and quantities.

In this case a system for meat demand was specified with five individual products, three beef, steak, ground beef and roast, a general category for pork, and chicken. A directly specified demand system was applied after the fashion of Moschini and Vissa.

The Rotterdam model, which is a directly specified demand system and whose application is consistent with the use of scanner data, is the model of choice. The use of the Rotterdam model allowed for the imposition of the three classical conditions of homogeneity, symmetry, and adding-up. Two models were generated, a mixed model, a result of the causal identification process, and a normally specified model for comparison, one without the identification process. Both the mixed and the normal demand systems were estimated as non-linear systems and adjusted for autocorrelation using Shazam 9.0 software. A full mathematical representation of the standard model is shown in figure 1 and the mixed system model can be found in figure 2. A more complete description and derivation and explanation for estimating and calculating elasticities using the mixed demand system can be found in Moschini and Vissa's 1993 AJAE paper. A general description of the elasticity estimates for the standard and mixed system can be found in appendix A.

Figure 1. Rotterdam Quantity Dependent Demand System Model (SDDS)

$$\omega_I \, dln\ q_i = \beta_i \, dln\ y + \sum^{n}_{j=1} C_{ij} \, dln\ p_j$$

where

$\omega_i \eta_i = B_i$,

$\omega_i \varepsilon_{ij} = C_{ij}$,

$\omega_i =$ budget share of the ith commodity

η_i expenditure elasticity of the ith commodity

ε_{ij} = compensated elasticities of the ith quantity of the jth price

Figure 2. Rotterdam Mixed Demand equations by (CIDS)

$$\omega_I \, dln\ q_i = \boldsymbol{\alpha_i} \, dln\ y + \sum^{m}_{j=1} \boldsymbol{\tau_{ij}} \, dln\ p_j + \sum^{n}_{k=m+1} \boldsymbol{\gamma_{ik}} \, dln\ q_k$$

$$\omega_k \, dln\ p_k = \boldsymbol{\mu_k} \ dln\ y + \sum^{m}_{j=1} \boldsymbol{\lambda_{kj}} \, dln\ p_j + \sum^{n}_{s=m+1} \boldsymbol{\delta_{ks}} \, dln\ q_s$$

Where (i)is the quantity dependent variables, (j) the counter for the quantity dependent variable in the ith and kth equations, (k) the price dependent variables and (s) the counter for the parables dependent variables in the kth equations. In this model m = 4, and n = 5, i = 1 to 4, j = 1 to 4, k = 5, and s = 5

$$\omega_i \eta_j = \boldsymbol{\alpha_I}$$

$$\omega_i \varepsilon_{ij} = \boldsymbol{\tau_{ij}}$$

$$\omega_i \psi_{ik} = \boldsymbol{\gamma_{ik}},$$

$$\omega_k \theta_k = \boldsymbol{\mu_k}$$

$$\omega_k \rho_{kj} = \boldsymbol{\lambda_{kj}}$$

$$\omega_k \psi_{ks} = \boldsymbol{\delta_{ks}}$$

$\omega_i =$ budget share of the ith quantity dependent commodity.

$\omega_k =$ budget share of the kth price dependent commodity.

$\eta_i =$ expenditure elasticity of the ith commodity.

$\varepsilon_{ij} =$ compensated “quantity” elasticities of the ith quantity of the jth price.

$\psi_{ik} =$ diversion elasticity of the kth quantity on the ith quantity.

$\theta_k =$ price expenditure elasticity of the kth price.

$\rho_{kj} =$ price competition elasticity of the jth price on the kth price.

$\psi_{ks} =$ price flexibility of the sth quantity on the kth price.

Prior to building the mixed demand system the TETRADII program was used to determine a DAG for all price, quantity, and expenditure variables. TETRADII uses an algorithm as previously described in the DAG procedure section. All of the variables used in the TETRADII program were in the same form as they were when used in the Rotterdam Models. Both the CIDS, and the SDDS models used all of the same variables enabling a direct comparison of the results.

One of the advantages of using a model such as the Rotterdam model is that the elasticities are calculated at a specific point. Given that the two models were built using the same data set and have the same common variables it was possible to make a statistical comparison of the parameter estimates as well as the elasticities at the same specific points. As is common in many cases the comparison between the two models was evaluated at the means.

Results

Causal Identification (DAG)

The Rotterdam model requires that there be variables for price, quantity and expenditure. The five meats required a single expenditure variable and five each of the price and quantity variables. All 138 weekly observations were used to obtain the eleven by eleven correlation matrix. See Table 1.

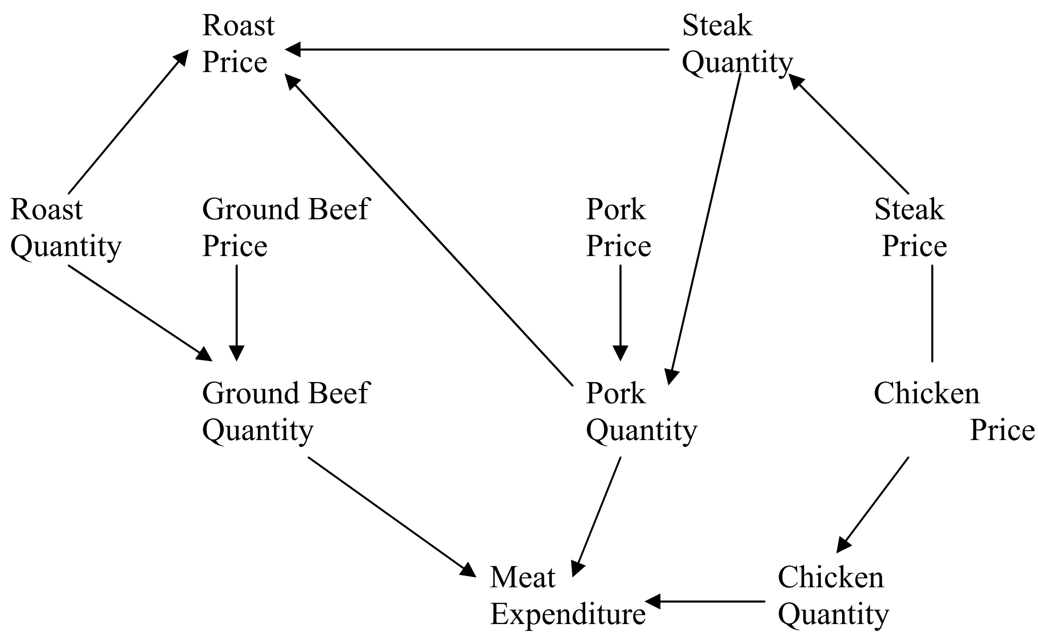
Table 1. Correlation Matrix

	Expend-iture	Price Ground	Quantity Ground	Price Roast	Quantity Roast	Price Steak	Quantity Steak	Price Chicken	Quantity Chicken	Price Pork	Quantity Pork
Expend	1										
P Ground	0.0654	1									
Q Ground	0.3009	-0.776	1								
P Roast	0.0338	0.2145	-0.229	1							
Q Roast	0.1357	-0.145	0.348	-0.912	1						
P Steak	-0.064	-0.182	0.1472	0.434	-0.482	1					
Q Steak	0.4429	0.1485	0.148	-0.277	0.4768	-0.824	1				
P Chicken	-0.125	-0.233	0.1244	-0.318	0.2828	-0.269	0.1845	1			
Q Chicken	0.7279	0.244	0.0186	0.3196	-0.231	0.2383	0.042	-0.705	1		
P Pork	-0.146	-0.167	0.1718	-0.434	0.4394	-0.265	0.2106	0.0086	-0.126	1	
Q Pork	0.8043	0.1605	0.0893	0.3458	-0.221	0.2352	0.1089	0.0504	0.5317	-0.531	1

The resulting DAG which is formed at the .20 significance level, the recommended statistical level of a sample this size (Sprites, Glymour and Scheimes), is shown in Figure 9.

All of the price arrows that are pointing to quantities indicate price is casual to quantity. In all cases except one is price causal to quantities, the exception is roast, where quantities are causal to price. The expenditure variable is a sink, only has arrows directed in. The contributing variable to the expenditure sink are quantity of ground beef, quantity of chicken, and quantity of pork. This relationship would indicate that expenditures are a result of quantity purchases

Figure 9. DAG for the five commodity CIDS Model



. This result may be due to the nature of the type of data. An individual shopper's expenditure is a result of quantities, which in turn are sensitive to relative prices. Another interesting occurrence is the fact that the price of roast is also a sink of the quantity of pork, quantity of steak and quantity of roast. The fact that steaks and roast are fabricated, in many cases from the same part of the beef carcass, and that many cuts of pork are roast or chops that may substituted for beef roast or steaks and that the quantity of roast is causal to quantity of ground beef leads to the possibility that this sink relationship allows roast price to act as a residual stabilizer for red meat. It is important to keep in mind the circumstances under which the data were created. In this case the data comes from the store level information and therefore have results that reflect the time and space relationships of the individual stores. An increase in roast quantities at the grocery store level, beef being perishable and roast being the cheapest cut type, would likely cause beef roast price to decline to clear the market. Another possible explanation for this relationship would be the use of beef roasts as a leader item, where large quantities are moved cheaply to attract shoppers who may make other purchases.

Increased pork quantities would have the reverse affect on beef roast prices. A larger quantity of pork would reduce the freezer space and increase the price of roast by limiting quantities of beef roast available for sale. In the case of a sale on pork, increased quantity of pork and a higher roast price would be helpful to clear out the pork. Increased steak quantities would have a positive effect on roast beef prices. Beef carcass parts can be either fabricated into steak or roast, if the number of steaks fabricated increases the number of roast fabricated would decline therefore increased steak quantities would create less roast and prices would increase. As is true in the case of pork a sale on steak could also be accompanied by higher prices of beef roast. Making the relative price difference between beef roast and steak can be accomplished by either lowering steak price, or raising roast price, or both, thus encouraging increased sales of steak quantities.

Parameter Estimates and Elasticities

The CIDS model differs from the SDDS model in one variable, beef roast. The CIDS model is a mixed demand system with the variable representing beef roast as being quantity dependent. The variable representing beef roast as well as all the other ten variables are price dependent in the SDDS model. See figures 1 and 2.

The statistical analysis for all coefficient and elasticity estimates were preformed at the five percent (5%) significance level.

A summary of the expenditure coefficients for both the SDDS (B1-B5) and CIDS (α1-α4, and μ5) models are all statistically significant. See Table 2 and 3.

The SDDS model coefficients (C11- C25, C51-C55) for the equations ground beef, steak, and beef roast were all statistically significant. The SDDS model equation for the quantity of chicken coefficients (C32, C33, and C35), were all statistically significant. The chicken equation coefficients for price of ground beef and the price of pork (C31, C34), which had a p-values of .941 and .061 respectively were not statistically significant. The coefficients of C42, C44, and C45 in the quantity of pork equation in the SDDS were statistically significant, the price of ground beef and price of chicken coefficients (C41, C43) with p-values of .419 and .126 however were not. Again please refer to Table 2.

The CIDS model has coefficients of statistical significance for (τ11-τ25, τ32-τ34, τ44, g15-g45, and λ54) with the coefficient for ground beef equation and chicken coefficient (τ31), beef roast equation and pork coefficient (τ44), pork equation and ground beef, steak, chicken (λ51-λ53), and beef roast (δ55) being insignificant.

Table 2. Equations	SDSS Coefficient Estimates Total Meat Expenditure	Ground Beef, j=1	Beef Steak, j=2	Chicken General, j=3	Pork General, j=4	Beef Roast, j=5
$\omega_i\, dln\, q_i =$	$\beta_i\, dln\, y$	$C_{ij}\, dln\, p_j$	$C_{ij}\, dln\, p_j$	$C_{ij}\, dln\, p_j$	$C_{ij}\, dln\, p_j$	$C_{ij}\, dln\, p_j$
Ground Beef, i = 1	B1	C11	C12	C13	C14	C15
Coeffeicent Estimates	**0.101**	**-0.283**	**0.049**	**0.034**	**0.071**	**0.027**
P-Values	0	0	0	0.0002	0	0.0001
Beef Steak, i = 2	B2	C21	C22	C23	C24	C25
Coeffeicent Estimates	**0.109**	**0.049**	**-0.381**	**0.057**	**0.125**	**0.039**
P-Values	0	0	0	0	0	0
Chicken General, i = 3	B3	C31	C32	C33	C34	C35
Coeffeicent Estimates	**0.255**	-0.001	**0.019**	**-0.318**	0.025	**0.018**
P-Values	0	0.94171	0.03625	0	0.06827	0.00052
Pork General, i = 4	B4	C41	C42	C43	C44	C45
Coeffeicent Estimates	**0.487**	-0.014	**0.033**	-0.021	**-0.512**	**0.022**
P-Values	0	0.419	0.0424	0.1268	0	0.0118
Beef Roast, i = 5	B5	C51	C52	C53	C54	C55
Coeffeicent Estimates	**0.048**	**0.023**	**0.035**	**0.026**	**0.046**	**-0.18**
P-Values	0	0.0011	0	0	0	0

It should be noted that all coefficients were tested at the average budget share of the commodities. In creating the non-linear systems the algebra used to manipulate the right hand sides when gathered by terms including budget shares as part of the coefficients.

The budget shares are part of the estimated coefficient value and account for variation that is not normally associated with a symmetrical matrix of coefficients. A full summary of the algebra results and specification of the elasticity conversion equations is found in appendix B.

Table 3. Equations	**CIDS Coefficient Estimates** Expenditure Corrficients	Ground Beef, j = 1	Beef Steak, j = 2	Chicken General, j = 3	Pork General, j = 4	Beef Roast, k = 5
$\omega_i\, dln\ q_i =$	$\alpha_i\, dln\ y$	$\tau_{ij}\, dln\ p_i$	$\tau_{ij}\, dln\ p_i$	$\tau_{ij}\, dln\ p_j$	$\tau_{ij}\, dln\ p_j$	$\gamma_{ik}\, dln\ q_k$
Ground Beef, i = 1	$\alpha 1$	$\tau 11$	$\tau 12$	$\tau 13$	$\tau 14$	$\gamma 15$
Coefficient Estimates	**0.115**	**-0.28**	**0.06**	**0.037**	**0.067**	**-0.009**
P-Value	0	0	0	0.0001	0.0003	0.0193
Beef Steak, i = 2	$\alpha 2$	$\tau 21$	$\tau 22$	$\tau 23$	$\tau 24$	$\gamma 25$
Coefficient Estimates	**0.124**	**0.061**	**-0.362**	**0.061**	**0.115**	**-0.011**
P-Value	0	0	0	0	0	0.00264
Chicken General, i = 3	$\alpha 3$	$\tau 31$	$\tau 32$	$\tau 33$	$\tau 34$	$\gamma 35$
Coefficient Estimates	**0.25**	0	**0.03**	**-0.32**	**0.04**	**-0.01**
P-Value	0	0.6504	0.0045	0	0.0079	0.0224
Pork General, i = 4	$\alpha 4$	$\tau 41$	$\tau 42$	$\tau 43$	$\tau 44$	$\gamma 45$
Coefficient Estimates	**0.499**	-0.021	0.018	-0.015	**-0.485**	-0.006
P-Value	0	0.2552	0.3279	0.2948	0	0.3673
$\omega_i\, dln\ p_k =$	$\mu_k\, dln\ y$	$\lambda_{kj}\, dln\ p_j$	$\lambda_{kj}\, dln\ p_j$	$\lambda_{kj}\, dln\ p_j$	$\lambda_{kj}\, dln\ p_k$	$\delta_{ks}\, dln\ q_s$
Beef Roast, k = 5	$\mu 5$	$\lambda 51$	$\lambda 52$	$\lambda 53$	$\lambda 54$	$\delta 55$
Coefficient Estimates	**0.012**	**0.011**	**0.013**	**0.014**	0.012	**-0.04**
P-Value	0.008	0.006	0.0006	0	0.1655	0

Both the CIDS and the SDDS models compensated and uncompensated own price elasticity estimates were all negative and significantly different from zero. All own price elasticities for both the compensated and uncompensated demands were greater in absolute value than one, making them elastic. See Table 4 and 5.

Table 4

Compensated Elasticities
"Hicksian"

Commodity	Ground Beef		Beef Steak		Chicken General		Pork General		Beef Roast	
Model	SDDS	CIDS	SDDS	CIDS	SDDS	CIDS	SDDS	CIDS	SDDS	CIDS
Ground Beef	-1.155	-1.138	0.327	0.38	0.252	0.262	0.424	0.415	0.152	0.11
P-values	0	0	0	0	0	0	0	0	0	0
Compared P's *	0.093		0.001		0.007		0.187		0.746	
Beef Steak	0.302	0.351	-1.448	-1.37	0.332	0.345	0.622	0.583	0.193	0.123
P-values	0	0	0	0	0	0	0	0	0	0
Compared P's *	0.001		0.001		0.001		0.278		0.158	
Chicken General	0.257	0.267	0.369	0.381	-1.187	-1.204	0.394	0.444	0.168	0.141
P-values	0	0	0	0	0	0	0	0	0	0
Compared P's *	0		0		0		0		0.016	
Pork General	0.391	0.378	0.621	0.578	0.355	0.397	-1.605	-1.456	0.238	0.114
P-values	0	0	0	0	0	0	0	0	0	0.084
Compared P's *	0		0		0		0		0.544	
Beef Roast	0.467	0.304	0.642	0.374	0.504	0.393	0.793	0.349	-2.406	-1.821
P-values	0	0.001	0	0	0	0	0	0.107	0	0
Compared P's *	0.879		0.25		0.649		0.02		0	

* Compared P's are the result of the statistical comparison of the SDDS and CIDS elasticities as being equal.

All compensated cross price elasticities for both CIDS and SDDS models were positive for all five commodities and significantly different from zero, except two. See Table 4. The two insignificant cross price elasticities were for pork and beef roast, and beef roast and pork, for only the CIDS specification with p-values of .084 and .107 respectively. The positive nature of all the cross price elasticities, are an indication that all goods in both models are net substitutes.

For compensated elasticities the SDSS and CIDS model were not significantly different from each other nine out of twenty five times and therefore different fourteen out of twenty five times.

From the table of uncompensated elasticities it can be seen that the first two commodities ground beef and steak have the same statistical significance status as the compensated elasticities, all are positive and significant for both the SDDS and CIDS model, indicating gross substitutes. See Table 5

Table 5

Uncompensated Elasticities
" Marshallian "

Commodity	Ground Beef		Steak		Chicken		Pork General		Beef Roast	
Model	SDDS	CIDS	SDDS	CIDS	SDDS	CIDS	SDDS	CIDS	SDDS	CIDS
Ground Beef	**-1.256**	**-1.254**	**0.218**	**0.255**	**0.153**	**0.149**	**0.314**	**0.289**	**0.119**	**0.072**
P-values	0	0	0	0	0	0	0	0	0	0.018
Compared P's *	0.153		0.125		**0.009**		0.068		**0.009**	
Steak	**0.201**	**0.237**	**-1.557**	**-1.493**	**0.234**	**0.234**	**0.513**	**0.459**	**0.16**	**0.086**
P-values	0	0	0	0	0	0	0	0	0	0.003
Compared P's *	0.137		0.791		**0.003**		**0.009**		**0**	
Chicken	-0.003	0.011	**0.086**	**0.104**	**-1.442**	**-1.454**	0.111	**0.167**	**0.083**	**0.058**
P-values	0.942	0.797	0.036	0.01	0	0	0.068	0.015	0.001	0.022
Compared P's *	**0**		**0**		**0**		**0**		**0**	
Pork	-0.058	-0.082	**0.135**	0.079	-0.084	-0.053	**-2.092**	**-1.955**	**0.092**	-0.036
P-values	0.419	0.259	**0.042**	0.256	0.127	0.379	0	0	0.012	0.594
Compared P's *	0.58		**0**		**0**		**0**		**0.003**	
Roast	**0.318**	**0.268**	**0.48**	**0.334**	**0.359**	**0.357**	**0.632**	0.31	**-2.454**	**-1.833**
P-values	0.002	0.005	0	0.001	0	0	0	0.151	0	0
Compared P's *	**0.034**		**0.001**		**0.037**		**0**		**0**	

* Compared P's are the result of the statistical comparison of the SDDS and CIDS elasticities as being equal.

Six cross price elaticities were found to be negative but insignificant, chicken and ground beef of SDDS model, pork and ground beef, pork and chicken of both the SDDS and CIDS models, and pork and beef roast from the CIDS model. However, none of the six gross complementary cross price elasticitics are statistically different from zero, leaving all other goods as gross substitutes. Four of the remaining positive cross price elasticities are not statistically significant, one from the SDDS specification chicken and pork, and three from the CIDS model, roast beef and pork, chicken and ground beef, and pork and steak. Of all the commodities pork has the most cross price elasticities that are negative and statistically insignificant. Two of the cross price elasticities for both the Hicksian and Marshallian elasticities were the same, pork and beef roast, and beef roast and pork. Of the ten insignificant uncompensated cross-price elasticities six were insignificant for both SDSS and CIDS specifications, while three were only insignificant for the CIDS model, and only one for the SDSS model.

The CIDS model generates more elasticity estimates that are statistically insignificant eight of the twelve for both the compensated and uncompensated elasticities, then the SDDS model. Remembering that which ever model is found to be appropriate would also be the model whose statistical results would be most appropriate.

A comparison of the statistical difference of the estimated elasticities between the SDDS and CIDS, at the mean budget share and price show some interesting differences. The statistical p-values are recorded in tables 4 and 5 as “Compared P’s”. Of the five own price compensated elasticities only one is not statistically different between the two models, ground beef. This implies that a different magnitude in quantity response to an own price change is predicted depending on which model was used to estimate the elasticities, except in the case of ground beef. Of the five uncompensated own price elasticities, between the SDDS and CIDS models, two were not significantly different from each other, ground beef and steak. Three of the five own price quantity responses would be predicted as being different. Twelve of the twenty, sixty percent, of the compensated cross price elasticities were statically different from each other. The eight statically insignificant compensated cross price elasticities are for, ground beef and pork, ground beef and roast, steak and pork, steak and beef roast, pork and beef roast, beef roast and ground beef, beef roast and steak, and beef roast and chicken. The uncompensated cross price elasticities sixteen out of twenty, eighty percent, of the elasticities as being statically different from each other. The four uncompensated cross price elasticities that were not statistically different from each other were, ground beef and steak, ground beef and pork, steak and ground beef, and pork and ground beef. These results indicate that the SDDS and the CIDS models are more statically different then they are alike, with the compensated elasticities being statically different sixteen out of twenty five estimates, or 64% different and the uncompensated elasticities being different nineteen out of twenty five estimates, or 76% different. Overall the difference in elasticity estimates between the SDDS and CIDS model was thirty five times out of fifty or 70% different.

Discussion

To briefly summarize the CIDS model and the SDDS model not only produce different coefficient estimates, but these different coefficients lead to elasticity estimates that are on average significantly different 70% of the time. The SDDS model produced four insignificant coefficients, while the CIDS model had six insignificant coefficients; three of those six coefficients were the same for both models.

What clearly can be shown is that specification between the SDDS and CIDS model result in mostly different results. The question yet remaining is under what conditions should we apply the CIDS model verses the SDDS. Given the assumptions where by the DAG was created and are valid the identified system would be more preferred. By using a technique such as directed acyclical graphs a demand system can be justified that makes complete use of the consumer theory, Samuelson Full Duality becomes a viable reality. No longer does the applied economist have to rely solely on the merits of a specialized set of circumstances which are no longer without empirical teeth, but through an judicious application of causality add strength to the use of a specific demand specification.

The final question as to the effectiveness of causality in identifying demand specification far from complete, and further work needs to be done to validate this preliminary outcome. But what is significant is that this work is an attempt at crossing bridges. There are those on the side of the valley who are strongly convinced in the power of structural models and those on the other who are just as convinced of the power of information derived from the data. This work is aimed at combining them both, the theory for structure and information from the data to build a better mousetrap. As Keynes said "Progress in economics consist almost entirely in a progressive improvement in the choice of models." This application although basic in concept is what empirical economics so desperately needs. It should be remembered that in using the CIDS methodology assumption are made, just as in any model, it behooves all to remember as with all models the limitations those assumption place on the model, the old adage "buyer beware" would be an appropriate warning label.

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