

## ECONOMETRIC METHODOLOGY II : STRENGTHENING TIME SERIES ANALYSIS

R.F. Townsend<sup>1</sup>

*This article reviews some of the recent methodology developed for the analysis of time series data stressing that the statistical properties of the individual series need to be analysed to avoid spurious regressions. A convergence of econometric methodology is entertained with specific focus on cointegration and error correction models which allows the testing of long run relationships between variables and allows for a more dynamic structure than some of the previous models that appear in the literature. An example of this is the commonly used partial adjustment model in supply analysis which is nested in the less restrictive error correction model. Tests can be performed on the validity of these restrictions. These models have a wide application in agricultural economic analysis.*

### EKONOMETRIESE METODOLOGIE II : VERSTERKING VAN TYDREEKS-ANALISE

*Hierdie artikel verskaf 'n oorsig van resente metodologie wat ontwikkel is vir die ontleding van tydreeksdata. Die metodologie beklemtoon dat die statistiese eienskappe van individuele tydreeks eers ontleed moet word om valse regressies te verhoed. Die samevloeiing van ekonometriese metodologie word beskryf met spesifieke fokus op koïntegrasie en foutkorreksie-modelle. Hierdie modelle maak dit moontlik om langtermyn verhoudings tussen veranderlikes te toets en maak voorsiening vir 'n meer dinamiese struktuur as vorige modelle in die literatuur. 'n Voorbeeld hiervan is die algemeen gebruikte parsieële aanpassingsmodel in aanbodanalise wat in samehang met die minder beperkende foutkorreksiemodel gebruik word. Die geldigheid van hierdie beperkinge kan getoets word. Hierdie modelle het wye toepassingsmoontlikhede in landbou-ekonomiese analise.*

#### 1. INTRODUCTION

This article presents the second part of a two part series on econometric methodology. The first part appeared in the September 1997 issue of *Agrekon*. The focus of the earlier discussion was on alternative methodologies for analysing data and the role of theory and observation in each approach. The recent literature has seen a convergence of some of these methodologies. The Sim's vector autoregressive approach and the Hendry approach (discussed in the preceding paper) have, to some extent, been united in the cointegration literature (see Engle and Granger, 1991). Since its introduction in the mid-

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<sup>1</sup> Department of Agricultural Economics, Extension and Rural Development, University of Pretoria.

1980s there has been a tremendous growth in both the theoretical aspects and the application of cointegration. Many econometricians have regarded this as the most important recent development in empirical modelling. A contributing factor to its widespread adoption is the simplicity of the concept and application. Despite these developments the appearance of this methodology in the South African economic literature, particularly in agricultural economics, remains minimal.

This paper will attempt to discuss and clarify some aspects of this approach which needs to be taken into consideration when analysing time series data. Determining the statistical properties of data series will be examined in the next section focusing on the simple concepts of unit roots and difference and trend stationarity. The concept of cointegration will then be discussed in section 3 which is followed by a derivation of the error correction model in section 4. Alternative tests for unit roots and cointegration will then be presented in section 5 and 6 respectively. Recent development in causality testing will be examined before the conclusion.

## **2. STATIONARITY**

When using time-series data to derive estimates of the parameters in an equation it is necessary, prior to estimation, to determine the statistical properties of each series for the variables in the equation. Conventional asymptotic theory for least squares estimation assumes stationarity of the variables and if this is not the case then these individual series may need to be converted into a stationary process in order to derive meaningful (non-spurious) results.

An example of spurious relationship is shown in Table 1 where a regression of two 'hypothetical' trended (random walk) series, based upon independently generated series of random numbers, can produce significant results. Table 1 presents the results of 100 regressions from randomly generated series. Fifty seven percent of the regressions produced significant relationships at the 1% level. These results have fairly profound implications. Regressing two unrelated series on one another has a high probability of being significant. Whilst demonstrating the point by empirical example it has been shown theoretically that application of ordinary least squares (OLS) to non-stationary series yields biased and inconsistent results.

Most time-series data tend to exhibit a trend over time, as shown in Figure 1, and are thus non-stationary (the mean and variance of the series are not constant) in the levels. As shown, a regression of two unrelated trended

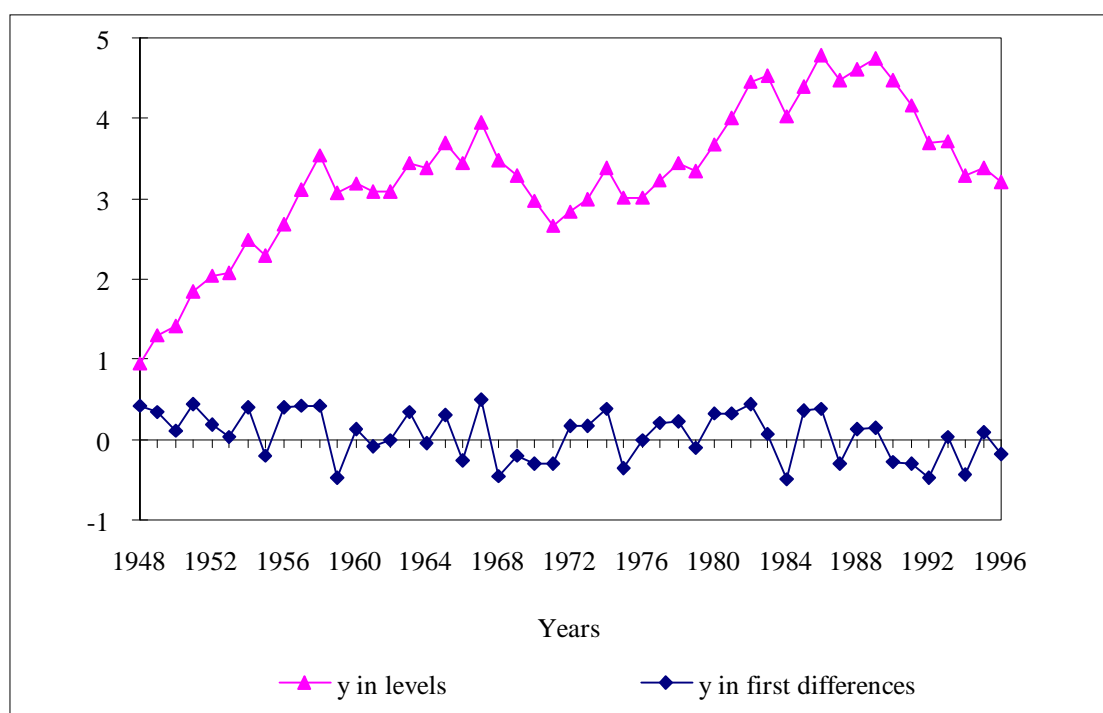
**Table 1: Frequency distribution of absolute t-statistic from 100 regressions of unrelated random walks**

Significance level	t-Statistic	Number of cases (%)
>10%	$t < 1.671$	26
10%	$1.671 < t < 2.009$	8
5%	$2.009 < t < 2.678$	9
1%	$t > 2.678$	57

variables may produce significant results with a high  $R^2$  value thus leading to 'nonsense' (Yule, 1926) or 'spurious' regressions (Granger and Newbold, 1987). To identify the statistical properties of the individual time series a very simple data-generating process can be specified as

$$y_t = \rho y_{t-1} + u_t \tag{1}$$

The current values of  $y$  depend on the last period's value  $y_{t-1}$  plus a disturbance term,  $u_t$  which includes other random influences.  $y_t$  will be stationary if  $|\rho| < 1$ , and if  $\rho = 1$  then  $y_t$  will be non-stationary. Banerjee *et al* (1993) use alternative values of  $\rho$  to generate figures for 200 observations have



**Figure 1: A difference stationary variable**

a unit root and  $y_t$  can be transformed into a stationary series by taking first differences, which is also shown in Figure 1, ( $y$  in first differences). The number of times a variable needs to be differenced in order to induce stationarity depends on the number of unit roots it contains. In this case,  $y_t$  is referred to as a *difference stationary* process since it is stationary after differencing. If a series is differenced ( $d$ ) times before it becomes stationary, then it contains  $d$  unit roots and is said to be integrated of order  $d$ , denoted  $I(d)$ .

$y_t$  in the levels is a non-stationary series,  $y_t$  in first differences ( $\Delta y_t = y_t - y_{t-1} + u_t$ ) is stationary.

If the data-generating process contains a non-zero intercept, then equation (1) can be written as

$$Y_t = b + \rho y_{t-1} + u_t \quad (1)$$

if  $\rho=1$ ,  $y_t$  will follow a stochastic trend, that is, it will drift upwards or downwards depending on the sign of  $\beta$ , taking first differences of  $y_t$ ,  $\Delta y_t = \beta + u_t$ .

Finally, consider the following data-generating process

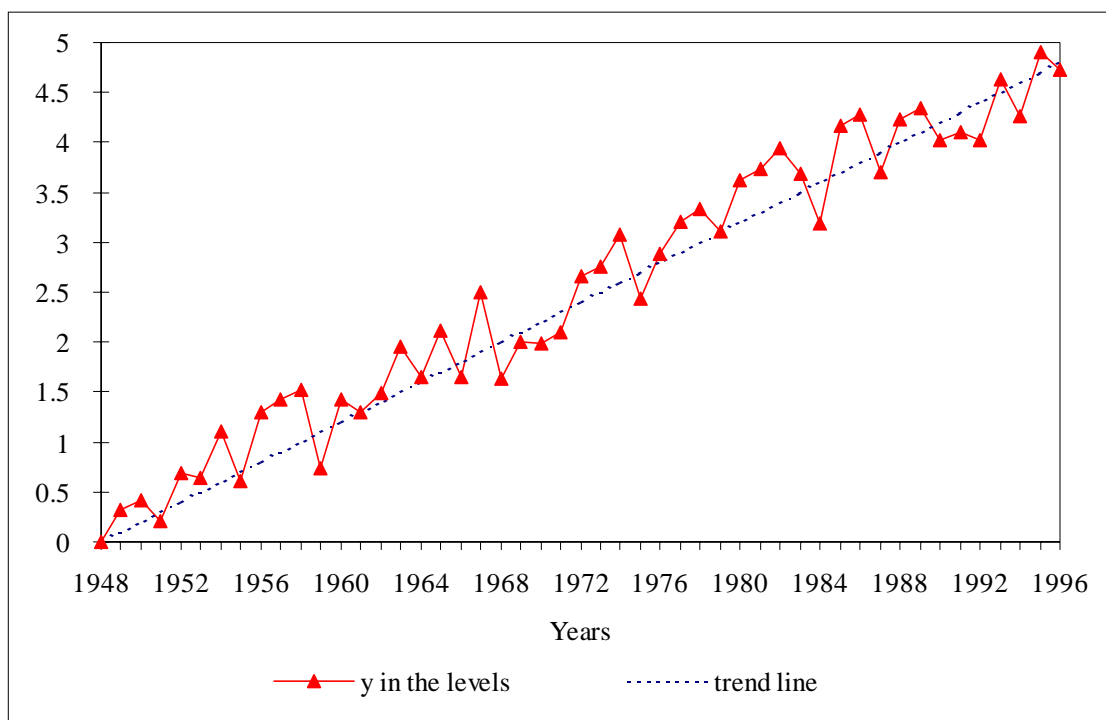
$$Y_t = \alpha + \beta t + u_t \quad (3)$$

$t$  is a deterministic trend and  $u_t$  is stationary.  $y_t$  is said to be *trend stationary* indicating that the data has a deterministic trend and that deviations from this trend are stationary as shown in Figure 2.

### 3. COINTEGRATION

In order to obtain meaningful results from a regression of two trended variables, cointegration needs to be established. The concept of cointegration states that if there exists a long-run relationship between variables, then deviations from the long-run equilibrium path should be bounded, and if this is the case then the variables are said to be cointegrated. In the short run variables may drift apart due to seasonal factors. However, in the long run economic forces such as the market mechanism or government intervention will bring them together again.

Consider two time series  $y_t$  and  $x_t$  which are both integrated of the same order  $I(d)$ . A static regression is run and tested to see if linear combinations of



**Figure 2:** A trend stationary variable  $y_t = \beta t + u_t$  (in this case  $\beta=0.1$ ).

the variables are themselves integrated of the same order as the individual variables.

$$Y_t = \alpha + \beta x_t + u_t \tag{4}$$

In general, any linear combination of the two series will also be  $I(d)$ ; i.e. the residual  $u_t$ , from regressing  $y_t$  on  $x_t$  is  $I(d)$ . If, however, the disturbance  $u_t$  is of a lower order of integration  $I(d-b)$  where  $d > b > 0$ , then Engle and Granger (1987) define  $y_t$  and  $x_t$  as cointegrated of order  $(d,b)$  (this can be expressed as  $CI(d,b)$ ). Thus, if  $y_t$  and  $x_t$  were both  $I(1)$ , and  $u_t$  was  $I(0)$  then the two series would be cointegrated of order  $CI(1,1)$ .

In equation (4),  $\beta$  measures the long-run relationship between  $y$  and  $x$ , and  $u$  is the divergence from the equilibrium path. If there is a stable long-run relationship between  $y_t$  and  $x_t$ , then the divergence,  $u_t$ , should be bounded which implies that  $u_t$  should be a stationary process. The relevance of cointegration in a particular model will be discussed in the next section.

#### 4. AUTOREGRESSIVE DISTRIBUTED LAGS, ERROR CORRECTION AND PARTIAL ADJUSTMENT

Disequilibrium is inherent in many economic relationships due to the inability of economic agents to adjust to new information instantaneously. There are often substantial costs of adjustment, which result in the current value of the dependent variable  $y$  being determined not only by the current value of some explanatory variable,  $x$ , but also by past values of  $x$ . In addition, as  $y$  evolves through time in reaction to current and previous values of  $x$ , past values of itself will also enter the short-run model. Consider a very simple dynamic model

$$Y_t = a_0 + g_0x_t + g_1x_{t-1} + a_1y_{t-1} + u_t \quad (5)$$

where  $u_t$  is white noise ( $IN(0, \sigma^2)$ ).  $\gamma_0$  denotes the short-run response of  $y_t$  to a change in  $x_t$ . The long-run effect if the model were in equilibrium is

$$y_t = a_0 + b_1x_t \quad (6)$$

Thus, in the long run, the elasticity between  $y$  and  $x$  is  $\beta_1 = (\gamma_0 + \gamma_1)/(1 - \alpha_1)$  (see appendix). This model requires that the variables are stationary and, as already noted, that for many time series this is not the case. A common approach taken is to model the variables in first differences. However, this loses information about the long-run relationships. To overcome this problem the levels terms can be reinstated in the differences specification to form the error correction model (ECM). Engle and Granger (1987) suggest that, if there is cointegration, then the error correction model is a valid representation (Alogoskoufis & Smith, 1991, provide a critique on the ECM).

The notion of 'error correction' was introduced by Davidson *et al* (1978) in the context of a consumption function. This was a way of capturing adjustments in a dependent variable which depended not on the level of the explanatory variable, but on the extent to which an explanatory variable deviated from an equilibrium relationship with the dependent variable. Equation (4) shows two variables in stable equilibrium, but in reality this may never be observed to hold. The discrepancy,  $y_t - \alpha - \beta x_t$ , or  $u_t$ , contains useful information since, on average, the system will move towards equilibrium. If  $y_{t-1} - \alpha - \beta x_{t-1}$  represents the previous disequilibrium, then the discrepancy should be useful as an explanatory variable for the next direction in the movement of  $y_t$  (Banerjee *et al* 1993). Thus, the error correction model incorporates this discrepancy as an explanatory variable.

The ECM can be derived as a simple reparameterisation of equation (5) and can be represented as (see appendix for derivation)

$$Dy_t = g_t Dx_t - (1 - a_1)[y_{t-1} - b_0 - b_1 x_{t-1}] + u_t \tag{7}$$

where  $\beta_0 = \alpha_0/1-\alpha_1$ ,  $\beta_1 = (\gamma_0 + \gamma_1)/(1-\alpha_1)$ . The ECM has several advantages over the autoregressive distributed lag model. It incorporates both short-run and long-run effects where  $\gamma_0$  captures the short-run effect on  $y$  of the changes in  $x$ ,  $\beta_1$  accounts for the long-run equilibrium relationship between  $y$  and  $x$ .  $[y_{t-1} - \beta_0 - \beta_1 x_{t-1}]$  is the divergence from the long-run equilibrium. Thus, at any time equilibrium holds the divergence will be equal to zero.  $(1-\alpha_1)$  measures the extent of correction of such errors by adjustment in  $y$ ,  $(1-\alpha_1)$  is negative and less than one, meaning that the correction is back towards equilibrium. As all the variables in the ECM are stationary, standard regression techniques are valid. In order to derive the partial adjustment model from the ECM,  $\gamma_0$  in equation (7) needs to be constrained to zero (see appendix). This allows us to test the partial adjustment model over the ECM using a Wald test of zero restrictions on the difference terms.

## 5. UNIT ROOT TESTS

In order to determine whether a series is stationary or non-stationary tests for the presence of a unit root need to be performed. Some common tests for unit roots will be discussed here, namely the Dickey-Fuller test (DF) and the Augmented Dickey-Fuller test (ADF) proposed by Dickey-Fuller (1981) and the Cointegrating Durbin Watson test proposed by Sargan and Bharagava (1983).

### 5.1 The Dickey-Fuller test

The Dickey-Fuller (DF) and the Augmented Dickey-Fuller (ADF) test can be presented as

$$\Delta y_t = \alpha + \beta t + (\rho - 1)y_{t-1} + \sum_{i=1}^n \lambda_i \Delta y_{t-i} + e_t \tag{8}$$

$\Delta y_t$  is the first difference of  $y$ .  $\alpha$  allows for a non-zero intercept or drift component.  $t$  is included to allow for a deterministic trend as  $y_t$  may be trend-stationary. The first three terms on the right-hand side of equation (8) show the Dickey-Fuller test format. The null hypothesis is that  $y_t$  has a unit root ( $H_0: \rho=1$ ) against a stationary alternative ( $H_a: \rho < 1$ ). The advantage of expressing this regression as equation (8) is that this is equivalent to testing  $(\rho-1)=0$  against  $\rho < 0$ . The Dickey-Fuller test is appropriate for series generated by an AR(1) process. If, however,  $y_t$  follows an AR( $p$ ) process where  $\rho > 1$ , the error term will be

autocorrelated to compensate for the misspecification of the dynamic structure of  $y_t$ . Autocorrelated errors will invalidate the use of the DF distributions which are based on the assumption that  $e_t$  is white noise. The Augmented Dickey-Fuller includes additional difference terms on the right-hand side of the equation to account for this problem,  $n$  is large enough to make  $e_t$  white noise.

As the underlying data-generating process is unknown, the general form shown in equation (8) is used to test for a unit root, deterministic trend and drift. However, having unnecessary nuisance parameters (constant and trend terms) will lower the power of the tests against stationary alternatives. Perron (1988) put forward a sequential testing procedure shown in Table 2. A similar process is also discussed in Harris (1995).

Tests (1), (2) and (3) test the individual variables in equation (8) for significance. Normal  $t$ -test critical values cannot be used as the critical values for these tests are non-standard and have been derived from Monte Carlo simulation. The null hypothesis for these tests are that the true value of the coefficient is zero, so a large  $t$ -ratio suggests a rejection of the null. (4) and (4a) are joint tests. (4) tests whether the series has a unit root ( $\rho-1=0$ ), no trend ( $\beta=0$ ) and no drift ( $\alpha=0$ ). If this test is rejected a joint test (4a) as to whether the series has a unit root but no trend can be used. If the null of no trend has been accepted it can be removed from the equation and further tests for a unit root can be conducted. This provides test statistics for unit root tests with no nuisance parameters (non-significant trend term). (5) and (6) are individual variable  $t$ -tests on the presence of a unit root and a drift respectively and (7) is a joint test of a unit root and drift. The source of the appropriate critical values is shown in the last column. Similarly, if the drift term is non-significantly different from zero it can be removed so there are no nuisance parameters in the test (8).

Another common test is the Durbin-Watson test proposed by Sargan and Bargava (1983) (sometimes known as the integrating regression Durbin-Watson). This requires the regression of a variable  $y$  on a constant  $c$ .

$$Y_t = c + \epsilon_t \quad (9)$$



**Table 2: Testing procedure using the DF/ADF tests**

Step and model	Null hypothesis	Test statistic	Critical values
(1) $\Delta y_t = \alpha_1 + \beta_1 t + (\rho_1 - 1)y_{t-1} + \sum \lambda_i \Delta y_{t-1} + u_t$	$(\rho_1 - 1) = 0$	$t_\tau$	Fuller (1976, p.373)
(2) $\Delta y_t = \alpha_1 + \beta_1 t + (\rho_1 - 1)y_{t-1} + \sum \lambda_i \Delta y_{t-1} + u_t$	$\beta = 0$	$t_{\beta\tau}$	Dickey-Fuller (1981) Table III p.1062
(3) $\Delta y_t = \alpha_1 + \beta_1 t + (\rho_1 - 1)y_{t-1} + \sum \lambda_i \Delta y_{t-1} + u_t$	$\alpha = 0$	$t_{\alpha\tau}$	Dickey-Fuller (1981) Table II p.1062
(4) $\Delta y_t = \alpha_1 + \beta_1 t + (\rho_1 - 1)y_{t-1} + \sum \lambda_i \Delta y_{t-1} + u_t$	$(\rho_1 - 1) = \beta_1 = \alpha = 0$	$\Phi_2$	Dickey-Fuller (1981) Table V p.1063
(4a) $\Delta y_t = \alpha_1 + \beta_1 t + (\rho_1 - 1)y_{t-1} + \sum \lambda_i \Delta y_{t-1} + u_t$	$(\rho_1 - 1) = \beta_1 = 0$	$\Phi_3$	Dickey-Fuller (1981) Table VI p.1063
(5) $\Delta y_t = \alpha_2 + (\rho_2 - 1)y_{t-1} + \sum \lambda_i \Delta y_{t-1} + u_t$	$(\rho_2 - 1) = 0$	$t_\mu$	Fuller (1976)
(6) $\Delta y_t = \alpha_2 + (\rho_2 - 1)y_{t-1} + \sum \lambda_i \Delta y_{t-1} + u_t$	$\alpha_2 = 0$	$t_{a\mu}$	Dickey-Fuller (1981) Table I p.1062
(7) $\Delta y_t = \alpha_2 + (\rho_2 - 1)y_{t-1} + \sum \lambda_i \Delta y_{t-1} + u_t$	$(\rho_2 - 1) = \alpha_2 = 0$	$\Phi_1$	Dickey-Fuller (1981) Table IV p.1063
(8) $\Delta y_t = (\rho_3 - 1)y_{t-1} + \sum \lambda_i \Delta y_{t-1} + u_t$	$(\rho_3 - 1) = 0$	$t$	Fuller (1976)

If  $y$  has a unit root then in equation (9)

$$\epsilon_t = r\epsilon_{t-1} + v_t \tag{10}$$

$r$  is equal to unity.  $v$  is normally distributed with zero mean and constant variance. The null hypothesis that  $\epsilon$  has a unit root is tested against the alternative that it follows a first order Markov process with an absolute value of less than one. The conventional Durbin-Watson statistic is used, values significantly greater than zero lead to rejection of the null, the critical values are provided by Sargan and Bargava (1983).

## 6. COINTEGRATION TESTS

### 6.1 The single equation approach

In the discussion on cointegration, if two time series  $y_t$  and  $x_t$  are integrated of order  $d$ ,  $I(d)$ , then if a linear combination of the two series is integrated of order less than the individual variables, they are cointegrated; i.e. if the disturbance term ( $u_t = y_t - \alpha - \beta x_t$ ) is integrated of order,  $I(d-b)$  where  $b > 0$ . To test the null hypothesis that  $y_t$  and  $x_t$  are not cointegrated in the Engle-Granger framework is to directly test whether the error term in the cointegrating regression is  $I(0)$ , i.e. stationary, or  $I(1)$ . The cointegration test for single equations are similar to the test for order of integration as discussed in the previous section. Using equation (8), the presence of a unit root in the error term can be determined, i.e.  $u_t$  in the cointegrating regression in equation (4). Thus equation (8) can be rewritten as

$$\Delta u_t = \alpha + \beta t + (\rho - 1)u_{t-1} + \sum_{i=1}^n \lambda_i \Delta u_{t-i} + v_t \tag{11}$$

The question of the inclusion of the trend or difference terms in the regression equation depends on whether a constant or trend appear in the cointegrating regression. On the basis of Monte Carlo experimentation, Hansen (1992) has shown that, irrespective of whether  $u_t$  contains a deterministic trend or not, including a time-trend in the DF/ADF test will result in a loss of power leading to an under-rejecting of the null hypothesis of no cointegration when it is false. In this test it is not possible to use the standard DF tables of critical values. The reason for this is that OLS chooses the smallest sample variance, even if the variables are not cointegrated, making the error term as stationary as possible. The OLS estimation minimises the (sum of squares) deviations of  $u_t$  from the OLS regression line obtained from  $y_t = \alpha + \beta x_t + u_t$  that is, OLS obtains  $\beta$  that

will minimise the variance. The standard DF distribution would tend to over-reject the null. The distribution of the test statistic is also affected by the number of regressors ( $n$ ) included in the cointegrating regression. Thus, different critical values are needed as  $n$  changes, as well as with sample size changes. MacKinnon (1991) provides a set of critical values for this based on response surfaces.

Restricting any analysis of non-stationary time series to a single-equation framework also precludes identifying two or more cointegrating relationships. The OLS approach provides no guarantee that a unique cointegrating vector has been estimated. Moreover, the existence of a second relationship implies that the coefficients in equation (4) do not have a ready interpretation. In order to investigate this issue, a system approach must be conducted. Thus, the DF, ADF and CRDW tests have been superseded by the Johansen Maximum Likelihood estimation method (Johansen 1988, Johansen & Juselius 1990).

## 6.2 Cointegration with multiple equations : The Johansen approach

This approach allows the estimation of all the cointegrating relationships and constructs a range of statistical tests to test hypotheses about how many cointegrating vectors there are and how they work in the system. Johansen (1988) proposed a general framework for considering the possibility of multiple cointegrating vectors and this framework also allows questions of causality and general hypothesis tests to be carried out in a more satisfactory way.

The procedure begins by defining a VAR of a set of variables  $X_t$ ,

$$X_t = \pi_1 X_{t-1} + \dots + \pi_k X_{t-k} + e_t \quad t = 1, \dots, T \tag{12}$$

if there are four variables in the model then this becomes a four-dimensional  $k$ -th order vector autoregression model with Gaussian errors.  $X_t$  is a vector of all relevant variables and  $k$  is large enough to make the error term white noise. The length of the lag can be determined by the Akaike Information Criteria (AIC) or the Schwarz Criteria (SC) as mentioned in part one of this two part series. In this form the model is based on minimal behavioural assumptions on the economic phenomenon of interest. The VAR model can be reparameterised in error correction form as (Cuthbertson *et al* 1993)

$$\Delta X_t = \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \Pi X_{t-k} + e_t, \quad t = 1, \dots, T$$

where 
$$\begin{matrix} \Gamma = [(\mathbf{I} + \pi_1), (\mathbf{I} + \pi_1 + \pi_2), \dots, (\mathbf{I} + \pi_1 + \dots + \pi_k)] \\ \Pi = \mathbf{I} - \pi_1 - \pi_2 - \dots - \pi_k \end{matrix} \tag{13}$$

$x_t$  is the vector of all relevant variables and  $I$  is the identity matrix. The procedure involves the identification of the rank of the matrix  $\Pi$ . The heart of the Johansen procedure is simply to decompose  $\Pi$  into two matrices  $\alpha$  and  $\beta$  both which are  $N \times r$  such that

$$\Pi = \alpha\beta' \quad (14)$$

The rows of  $\beta$  may be defined as the  $r$  distinct cointegrating vectors, i.e. the cointegrating relationships between the non-stationary variables, and the rows of  $\alpha$  show how these cointegrating vectors are loaded into each equation in the system. The loading matrix, therefore, effectively determines the causality in the system. Johansen (1988) gives a maximum likelihood estimation technique for estimating both matrices and he outlines suitable tests which allow us to test the number of distinct cointegrating vectors which exist, as well as to test hypothesis about the matrices. Testing restrictions on  $\beta$  in equation (14) allows tests on parameter restrictions on the long-run properties of the data. By testing restrictions on the  $\alpha$ -matrix the direction of causality within the model can be tested. Thus the Johansen approach allows the estimation of multiple cointegrating regressions and tests of parameter restrictions. The ability to conduct causality tests within this systems approach will be discussed in more detail.

In previous studies of causality the common method used was the Granger causality test formalised by Granger (1969). This test was developed within an implicit framework of stationarity and as many of the variables are non-stationary or integrated of order one ( $I(1)$ ), a common approach was to perform the test in first differences. This test has since been widely used. However, there has been some uncertainty with respect to testing for Granger-causality in cointegrated systems. MacDonald & Kearney (1987) indicate that this model may be mis-specified by Granger (1969). More recently, Mosconi & Giannini (1992) and Hall & Wickens (1993) have developed estimation and testing procedures for causality within systems of integrated variables, which exhibit cointegration. Hall & Wickens (1993) use a more restrictive definition of causality than Mosconi & Giannini (1992), involving only the long-run conditions. They suggest that a sufficient, but not necessary condition for weak (long-run) causality is given by a simple restriction on the Johansen loading matrix,  $\alpha$  in  $\Pi = \alpha\beta'$ . If the  $\alpha$  matrix has a complete column of zeros then no cointegrating vector will appear in a particular block of the model, thus indicating no causal relationship. Expanding out equation (13) for a two variable case gives

$$\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = \begin{bmatrix} \gamma_{11} \\ \gamma_{21} \end{bmatrix} \cdot \begin{bmatrix} \Delta x_{1t-1} \\ \Delta x_{2t-1} \end{bmatrix} + \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \cdot \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} x_{1t-1} \\ x_{2t-1} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} \quad (15)$$

If  $\alpha_{11} \neq 0$ ,  $\alpha_{12} \neq 0$  and  $\alpha_{21} = \alpha_{22} = 0$  the causality runs from  $x_2$  to  $x_1$  and there is no feedback to  $x_2$ . Bi-directional causality requires  $\alpha_{11} \neq 0, \alpha_{12} \neq 0, \alpha_{21} \neq 0, \alpha_{22} \neq 0$  and no causal relationship requires  $\alpha_{11} = \alpha_{12} = \alpha_{21} = \alpha_{22} = 0$ . The restrictions are validated by direct Wald tests on the loading parameters (Hall and Milne 1994, Caporale & Pittis, 1995).

This brief description shows that the Johansen approach provides a more flexible system for determining long run relationships and allows explicit tests on the long parameters of the model. This approach contains characteristics of both the flexible structure of the Sims approach and rigorous testing of the Hendry approach to provide a practical method for time series analysis.

## 7. CONCLUSION

The purpose of this paper was to give a brief introduction to cointegration and error correction models as there has been a noticeable absence of these time series considerations in the South African agricultural economics literature. In order to develop the credibility of results and avoid the possibility of spurious regressions (estimating a significant relationship when in fact non is present) cointegration can be used.

Indeed, recent literature has shown a trend towards a convergent methodology combining the Hendry and Sims approach. The main focus has been on cointegration in time series analysis which is used to test equations for spurious regressions. When analysing time series data consideration must be given to the statistical properties of the individual series analysed, cointegration allows testing of long run relationships between variables and error correction models have proved useful by allowing a more dynamic structure than some of the previous models. This allows a more rigorous approach at testing and estimating relationships between variables.

Policy advice based on results from econometric modelling which doesn't recognise or take account of these considerations could be misleading. The results from such models at best ignore important information about the underlying (statistical and economic) processes generating the data, and at worst leads to nonsensical (or spurious) results. Thus, when analysing time series data it is a responsibility of the applied researcher to test for the properties of these data and if unit roots are present (and evidence suggests that they generally are) to use the appropriate modelling procedures.

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## APPENDIX

Deriving the ECM from the ADL shown in equation (5)

$$y_t = \alpha_0 + \gamma_0 x_t + \gamma_1 x_{t-1} + \alpha_1 y_{t-1} + u_t \quad (16)$$

Subtracting  $y_t$  from both sides

$$y_t - y_{t-1} = \alpha_0 + \gamma_0 x_t + \gamma_1 x_{t-1} - y_{t-1} + \alpha_1 y_{t-1} + u_t \quad (17)$$

combining the  $y_{t-1}$  terms

$$\Delta y_t = \alpha_0 + \gamma_0 x_t + \gamma_1 x_{t-1} - (1 - \alpha_1) y_{t-1} + u_{t-1} \quad (18)$$

Subtracting  $\gamma_0 x_{t-1}$  from both sides

$$\Delta y_t - \gamma_0 x_{t-1} = \alpha_0 + \gamma_0 x_t - \gamma_0 x_{t-1} + \gamma_1 x_{t-1} - (1 - \alpha_1) y_{t-1} + u_t \quad (19)$$

Factorising by  $\gamma_0$  and the taking  $\gamma_0 x_{t-1}$  to the right-hand side

$$\Delta y_t = \alpha_0 + \gamma_0 \Delta x_t + (\gamma_0 + \gamma_1) x_{t-1} - (1 - \alpha_1) y_{t-1} + u_t \quad (20)$$

Rearranging

$$\Delta y_t = \gamma_0 \Delta x_t + \alpha_0 - (1 - \alpha_1) y_{t-1} + (\gamma_0 + \gamma_1) x_{t-1} + u_t \quad (21)$$

The error correction form shown in equation (7)

$$\Delta y_t = \gamma_0 \Delta x_t - (1 - \alpha_1) [y_{t-1} - \beta_0 - \beta_1 x_{t-1}] + u_t \quad (22)$$

where  $\beta_0 = \alpha_0 / (1 - \alpha_1)$ ,  $\beta_1 = (\gamma_0 + \gamma_1) / (1 - \alpha_1)$ .

### Deriving the partial adjustment model

Constraining the difference terms on the right-hand side of the ECM in equation (21) to zero in yields

$$y_t - y_{t-1} = \alpha_0 - (1 - \alpha_1) y_{t-1} + (\gamma_0 + \gamma_1) x_{t-1} + u_t \quad (23)$$

Add  $y_{t-1}$  to both sides and expanding out



$$y_t = \alpha_0 + y_{t-1} - y_{t-1} + \alpha_1 y_{t-1} + (\gamma_0 + \gamma_1)x_{t-1} + u_t \quad (24)$$

This gives

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + (\gamma_0 + \gamma_1)x_{t-1} + u_t \quad (25)$$

If  $\alpha_1=1-\delta$  and  $(\gamma_0+\gamma_1)=\delta\beta$  then equation (25) can be rewritten in the more common partial adjustment notation as

$$y_t = \alpha_0 + (1-\delta)y_{t-1} + \delta\beta x_{t-1} + u_t \quad (26)$$

where  $\beta=(\gamma_0+\gamma_1)/(1-\alpha_1)$ .