

Convergence, Inequality and Education in the Galor and Zeira Model

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This short paper analyses a simple extension to the model of Galor and Zeira (1993). I show that the result of club convergence holds under a much more continuous and much more realistic assumption of the education function. In order to achieve this result, the hypothesis of a fixed cost in education assumed in the original model has been replaced by the assumption that individuals can choose exactly how much to invest. It is also assumed that this investment positively affects the productivity of the individual which, in turn, influences his salary. [JEL Classification: D31, D63, I20, O47]

1. - Introduction

In the last twenty years, the convergence of national incomes *per capita* has been the subject of intense debate in economic literature. The controversy¹ principally focuses on the validity of the following two, competing hypotheses²:

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¹ See GALOR O. (1996) for more details about the debate.

² There is also a third hypothesis "The absolute convergence" - which suggests that *per capita* incomes of countries converge one to another in the long run

1) “The conditional convergence hypothesis” — which suggests that *per capita* incomes of countries identical in their structural characteristics (for example preferences, technologies, rates of population growth, *etc.*) converge towards one another in the long-run independently of their initial conditions.

2) “The club convergence hypothesis” (polarisation, persistent poverty, and clustering) — which suggests that *per capita* incomes of countries identical in their structural characteristics converge towards one another, providing their initial conditions are also similar.

In other words, the conditional convergence hypothesis is linked to the idea that each economy is characterized by a unique, globally stable, steady state equilibrium, whereas the club convergence hypothesis suggests the existence of multiple, locally stable, equilibriums, which means that countries with similar structural characteristics converge towards the same equilibrium provided that their initial conditions are similar as well.

The hypothesis of conditional convergence belongs to the oldest conception of growth (that of neoclassical models) which derives from the application of the basic Solow model to the comparison of countries. These models, characterized by diminishing marginal productivity of capital and by constant returns to scale, suggest that among countries with similar technologies, preferences, *etc.*, the lower the levels of output *per capita*, the higher the growth rates³ are.

Although the neoclassical paradigm represents one of the pillars of the growth theory, these models can not explain (endogenously) the condition of persistent poverty which affects most developing countries. Indeed, if the conditional convergence hypothesis was able to exhaustively represent the phenomenon of growth, the empirical data should show higher growth rates in developing countries compared to developed countries. Not only has this not happened, but also the growth of many poor countries

regardless their structural characteristics. But this hypothesis has been rejected by the economists in light of the empirical results of BARRO R.J. (1991) and QUAH D. (1996).

³ See MANKIW N.G. *et AL.* (1992) for supporting evidence for the conditional convergence hypothesis.

has been negative. Thus, the difference between national incomes seems to increase rather than decrease⁴.

An example used in the literature, first by Lucas (1993) and then by Benabou (1996), is the comparison of the miraculous growth of South Korea and the stagnation experienced by the Philippines. Although these two countries were similar with respect to all major economic aggregates in early 1960s, over the next quarter century South Korea had an average growth of 6% while the Philippines stagnated at about 2%. Only lately it has been noticed that initial conditions were in fact quite different: the income distribution in the Philippines was much more unequal.

In order to account for these divergences in income *per capita* highlighted by the empirical studies, following Romer's famous 1986 article an array of so-called endogenous growth models came into being. These models promote a new concept of growth, based on increasing returns to scale⁵. According to these models, the power of capital — human capital, physical capital and capital of knowledge — is high where the level of capital is already plentiful, and low where the capital is scarce. Behind this conviction, there is the idea that knowledge can be broadened through investments and thus create several positive externalities for society. This suggests that the lower the level of capital in a country, the lower the growth rate expected. This interpretation of the growth phenomena gives a reasonable explanation of how many countries can become a victim of poverty traps from which it is very difficult to escape. Alas, the empirical evidence based on time series does not seem to support the endogenous growth model's predictions, suggesting that in the real world there are decreasing return to scale rather than increasing return to scale⁶.

Thus, the issue for economists becomes to understand how it

⁴ See EASTERLY W. (2001) for a complete report of all the empiric studies which support this result.

⁵ See ROMER P.M. (1990); GROSSMAN G. - HELPMAN E. (1991) and AGHION P. - HOWIT P. (1992) for the endogenous growth models based on investment in R&D, ARROW K.J. (1962) for the Learning by Doing type of models, and REBELO S.T. (1991) for the AK models.

⁶ See JONES C.I. (1995); (1999).

is possible to generate club convergence in the neoclassical paradigm of growth. In other words, researchers must find a way to make the standard growth models allow for the heterogeneous behaviour of agents and countries, so that they can do different things not only in the short run but in the long run as well. The first attempt in this new research stream has been made by Aziaridis and Drazen (1990), who use non-convexities to add heterogeneous behaviour in the saving function of the agents in the neoclassical growth models. Nowadays we know that heterogeneity can be added in many different ways, but it was not so evident at that time⁷. Galor (1996) indeed underlines how the hypothesis of multiple long-run *equilibria* can, in fact, be perfectly consistent with the neoclassical paradigm if we allow for heterogeneous agents. Specifically, if the neoclassical growth models are augmented so as to capture additional significant elements such as human capital, income distribution and fertility, within the hypothesis of capital market imperfections and non-convexities of technology, they will generate club convergence. Several articles — amongst which; Galor and Zeira (1993), Aghion and Bolton (1996), Benabou (1996), Durlauf (1996) and Quah (1996) — demonstrate that in the presence of capital market imperfections and of a fixed cost for the production of human capital (or of a final article of manufactured goods), the initial distribution of wealth significantly affects the economic activity both in the short and long run⁸.

This short paper analyses a simple extension to the model of Galor and Zeira (1993). I show that the result of club convergence still hold under the assumption of a much more continuous education function. In order to achieve this result, the hypothesis

⁷The same result can be achieved for example through asymmetric information (PIKETTY T., 1997), the rule of redistribution policies, *etc.* For a detailed summary see BENABOU R.J. (1996). For an example which applies to the model of Galor and Zeira see MOAV O. (2002) who replaces non-convex technology with a convex bequest function, or by introducing fertility choice in a framework of a trade-off between the quantity of child and the quality of their life. (MOAV O., 2005).

⁸Actually more recent studies have shown that neoclassical growth model can generate club convergence even if the credit markets are perfect. See MOOKHERJEE D. - RAY D. (2003) and TETSUYA N. (2006).

of a fixed cost in education assumed in the original model has been replaced by the assumption that individuals can choose exactly how much to invest. It is also assumed that this investment positively affects the productivity of the individual which, in turn, influences his salary.

Intuitively these new assumptions are much more realistic because, generally, there are many different opportunities of investment in education. For example the choice between a public or a private university, or the choice to follow a more or less specialized course, or even the choice between studying abroad or not, *etc.* Then, it is obvious that generally, someone with a better quality of education will end up in a higher paid job. This is due to the fact that — at least from the employer's eyes — someone with a higher level of education is more productive⁹.

2. - The Model

Consider a small open economy with a single good. The good can be either consumed or invested. The good can be produced by two technologies, one which uses skilled labour and capital and the other using unskilled labour only. Production in the skilled sector is the following:

$$(1) \quad Y_t^s = F(K_t, H_t)$$

where Y_t^s , K_t and H_t , are respectively the level of output, physical capital, and generic human capital in time t . It is worthy of note that $H_t = \bar{a}_t L_t^s$, that is the level of generic human capital is given by the product of the sector's average productivity \bar{a}_t with the number of skilled workers L_t^s . F is a concave production function with decreasing return to scale.

Production in the unskilled sector is described by:

⁹ What really matters for the employer is the productivity of the employee. However before the latter has been hired his productivity is not directly observable by the former. The level of education is therefore used as a signal of productivity.

$$(2) \quad Y_t^n = w_n L_t^n$$

where Y_t^n and L_t^n are output and unskilled labour input respectively, and $w_n > 0$ is the marginal productivity in this sector.

Individuals in this economy live for two periods. They can either work as unskilled for both periods or invest in education in the first period and work in the skilled sector in the second period. The amount invested in human capital in terms of monetary cost is given by h , and unlike the original model of Galor and Zeira, is not fixed but endogenously produced by the model.

I assume that the investment in education positively affects the productivity of individual i , (or similarly his part of human capital¹⁰) defined as:

$$(3) \quad a_i = 1 + h_i^\beta$$

where $0 < \beta < 1$, which implies a productivity function with decreasing return to scale with respect to the investment in education.

Each individual has one parent and one child, which means that the population is constant and equal to L in every generation. Individuals care about future generations and leave them bequests. In line with the original model, I assume that individuals consume in their second period life only¹¹. Formally, we can express the individuals' preferences as following:

$$(4) \quad u = \alpha \log c + (1 - \alpha) \log b$$

where c is consumption in the second period and b is the bequest left to the child. Thus, each individual is born with the same preferences and abilities, they only differ in their initial wealth, that is how much they have inherited from their parents.

Capital is assumed to be perfectly mobile so that individuals

¹⁰ Indeed: $a_i = H_i = (1 + h_i^\beta)$ where H_i is the part of human capital owned by individual i .

¹¹ It is a very strong assumption but it is sometimes used in Overlapping generations models.

and firms have free access to the international capital markets. The world interest rate is equal to $r > 0$ and is constant over time. Individuals can lend any amount at this rate. However, if they want to borrow, they must pay a higher interest rate: $i > r$. This difference between the lending and the borrowing rate comes from the assumption that individuals can evade debt payments by escaping to other places. Therefore, there are some monitoring costs for the bank in order to guarantee the reimbursement of the debt. These costs are recovered by charging the borrowers a higher interest rate. These costs create what the authors call "credit market imperfections"¹².

Unlike individuals, firms are unable to evade debt, due to reasons such as immobility, reputation, *etc.* and therefore can borrow at the lenders' interest rate r ¹³. As we are dealing with a small open economy, r is an exogenous variable in the model. This means that firms take it as given and choose the level of capital accordingly, that is to equalize the marginal revenue with the marginal costs of capital:

$$(5) \quad f'(k) = r$$

where

$$k = \frac{K}{\bar{a}L^s} = \frac{K}{H}$$

is capital per unit of efficiency (or similarly the factor intensity of the economy). Since r is constant, k also will be constant.

I further assume that both the goods market and the labour market are perfectly competitive. This implies that firms have a profit equal to zero. Thus, the salary of a single individual i is defined as:

$$(6) \quad (1 + h_i^\beta)w_s$$

¹² The results still hold under different types of credit market imperfections, such as asymmetric information, as long as borrowing is costly.

¹³ As argued by the authors in the original paper, this assumption does not affect the results, but simply reflect that generally individuals are more credit constrained than firms.

where w_s is a function of capital per unit of efficiency and as a consequence is fixed¹⁴.

The salary is thus given by the product of a component determined by the exogenous parameters of the models, that is the technology and the interest rate, with the productivity of the individual (or similarly his part of human capital). This means that each individual will have a different wage according to his investment in education.

3. - Endogenous Investment in Human Capital

It is obvious that the log-linearity of the utility function implies that each individual has the same propensity to consume (and thus to leave bequest). In other words, it is independent from the initial wealth and is equal to a fixed share of income.

Let us now describe the individual optimal decisions with respect to h . There are three possible states of the world for each individual: he can decide not to invest in education and thus work as unskilled for both periods; he can decide to invest in education more than his initial wealth and thus be a borrower; he can decide to invest less than his initial wealth and thus be a lender.

The individual lifetime utility in the three states of the world is:

$$(7) \quad U(x) = \log[w_n(2+r) + (1+r)x] + \varepsilon \quad \text{unskilled}$$

$$(8) \quad U(x, h) = \log[w_s(1+h^\beta) - (1+i)(h-x)] + \varepsilon \quad \text{borrower}$$

$$(9) \quad U(x, h) = \log[w_s(1+h^\beta) + (1+r)(x-h)] + \varepsilon \quad \text{lender}$$

where $\varepsilon = \alpha \log \alpha + (1-\alpha) \log (1-\alpha)$. The individual decision will rely upon the comparison between these three utilities.

¹⁴ For the formal derivation see *APPENDIX 1*.

By maximising (8) and (9) we get respectively¹⁵:

$$(10) \quad h_b = \left(\frac{\beta w_s}{1+i} \right)^{\frac{1}{1-\beta}}$$

$$(11) \quad h_l = \left(\frac{\beta w_s}{1+r} \right)^{\frac{1}{1-\beta}}$$

We can immediately see that the borrower's optimal investment in education is less than the lender's optimal investment in education. The reason is that the borrower has an additional cost to have access to education given by the difference between the borrowing and the lending interest rate. Therefore, the unique assumption of credit market imperfections is sufficient to create a first separation in the society, because the rich have an easier access to education and as a consequence, are more likely to work in the skilled sector. It is interesting to study if this separation still holds in the long run; we will consider this in the fourth section. For now let us concentrate on the short run *equilibria*.

The individual faces the following strategic problem: given his initial wealth x , he has to figure whether it is profitable to invest in education, and if so, decide between operating as a borrower or as a lender.

Let us start by comparing (7) and (9). The individual will decide to invest in education only if:

$$(12) \quad w_s(1+h^\beta) - h(1+r) \geq w_n(2+r)$$

The first thing worthy of note is that the condition stated by (12) does not depend directly on wealth, but indirectly, in the sense that it is only defined for $x \geq h$.

However given the exogenous parameters of the model, that is the production function, the value of β , r and w_n , it can happen

¹⁵ For the formal derivation see APPENDIX 2.

that equation (12) never holds. In that case no one invests in education¹⁶. Hence there is no physical capital K and an excess supply of loans prevail. This drives the world interest rate down until (12) is satisfied for some values of h .

The left hand side of equation (12) has a maximum at

$$h = h_l = \left(\frac{\beta w_s}{1+r} \right)^{\frac{1}{1-\beta}}$$

Let us define h_* as the amount of investment in education which satisfies (12) with equality. Given the argument above, in equilibrium: $h_* \leq h_l$ ¹⁷.

Consider the most general case, that is when the constraint is not binding. First, remember that (12) applies only for lenders, in other words when $x \geq h$. So we can infer that individuals whose inherited wealth is at least as high as h_l will decide to invest h_l in education rather than studying as unskilled for both periods. Those who inherit between h_* and h_l will invest in education exactly equal to their initial wealth since the derivative of the lender's income is positive at h less than h_l .

What happens when the initial wealth is less than h_* ? Those people, since they can't be lenders, must choose between being borrower and being unskilled. At this point it becomes essential to distinguish between these two cases: $h_b < h_*$ or $h_b \geq h_*$.

Let us start by considering the second case, which as we will see, is more interesting.

Individuals whose initial wealth is equal to $x < h_* \leq h_b$, will decide to work in the unskilled sector if and only if:

$$(13) \quad w_n(2+r) + x(1+r) > (1+h_b^\beta)w_s - (h_b - x)(1+i)$$

¹⁶ It is worthy of note that we are assuming: $w_s < w_n(2+r)$. This assumption although plausible, is crucial for the validity of the results.

¹⁷ Given the concavity of the productivity function, it is clear that for a given value of the LHS of (12) there will two values of h (one higher and one lower than h_l) which satisfy the equation with equality. The lower value will never be chosen in equilibrium since it is a dominated strategy: the same level of income can be achieved by a smaller investment in education! Hence, in the analysis that follows, we will not consider as solutions all the values $h > h_l$.

where the RHS of (13) is the income function of the borrower at its maximum value. We can immediately notice that unlike equation (12), equation (13) directly depends on the initial wealth.

To be more precise, inequality (13) holds only if:

$$(14) \quad x < f = \frac{w_n(2+r) + h_b(1+i) - (1+h_b^\beta)w_s}{i-r}$$

Thus, individuals who inherit less than the threshold value f prefer to work as unskilled.

Rearranging equation (14) we get:

$$(15) \quad x < f = h_b - \frac{(1+h_b^\beta)w_s - h_b(1+r) - w_n(2+r)}{i-r}$$

where the last term in the RHS must be positive since the income function for borrowers is defined only at $x < h_b$. In other words, if it is profitable to borrow only when the initial wealth is higher than f , but this threshold value is higher than h_b , no one will borrow since the borrowing condition requires $x < h_b$. This corresponds to the first of the two cases mentioned above. Indeed if $h_b < h_*$, we know from equation (12) that

$$\frac{(1+h_b^\beta)w_s - h_b(1+r) - w_n(2+r)}{i-r}$$

is negative. As a result the borrowing state disappears. But as we have already said, this drives r down which in turn drives h_* down until $h_b \geq h_*$. The latter corresponds to the first case, where

$$\frac{(1+h_b^\beta)w_s - h_b(1+r) - w_n(2+r)}{i-r}$$

is positive. Hence in equilibrium: $f < h_b$ ¹⁸.

Summarizing the results above:

- if $x < f$, the individual will work as unskilled.
- if $f \leq x < h_*$, the individual will borrow an amount equal to $h_b - x$.

¹⁸ Let us assume for sake of simplicity that: $f < h_*$. This assumption does not change the fundamental results of the model but only the threshold values.

— if $h_b \leq x \leq h_l$, the individual will invest in education exactly equal to his initial wealth.

— if $x > h_l$, the individual will lend an amount equal to $x - h_l$.

We still need to analyse what happens when $h_* \leq x < h_b$. In this case the individual can choose between investing in education exactly equal to his initial wealth and achieving an income equal to $(1 + x^\beta)w_s$, or borrowing an amount equal to $h_b - x$ to invest h_b in education and achieving an income equal to $(1 + h_b^\beta)w_s - (h_b - x)(1 + i)$. From what has been shown up until now, it is clear that the more x approximates to h_* , the more likely is the income of the borrower to be larger than the income achieved by investing exactly the initial wealth. Whereas the opposite is more likely to be true the more x approximates to h_b . Thus we can infer that there is a value of x between h_* and h_b , that we will call h_{**} , which will make the individual indifferent between borrowing $h_b - x$ to invest h_b in education, and investing exactly x in education. The exact value of h_{**} depends obviously on the exogenous parameters of the model. Hence at $x = h_{**}$, the following condition holds:

$$(16) \quad (h_b - x)(1 + i) = (h_b^\beta - x^\beta)w_s$$

Equation (16) simply states that for such an amount of initial wealth, the costs of borrowing match exactly the additional income achieved from investing more than x in education. As a result, individuals who inherit $h_* \leq x < h_{**}$ will become borrowers, whereas individuals who inherit $h_{**} \leq x < h_b$ will invest x in education (being neither borrowers nor lenders).

Hence, the optimal strategy that couples each value of x with an amount h invested in education is the following:

— if $x < f$, the optimal investment in education is equal to 0.

— if $f \leq x < h_{**}$, the optimal investment in education is equal to h_b .

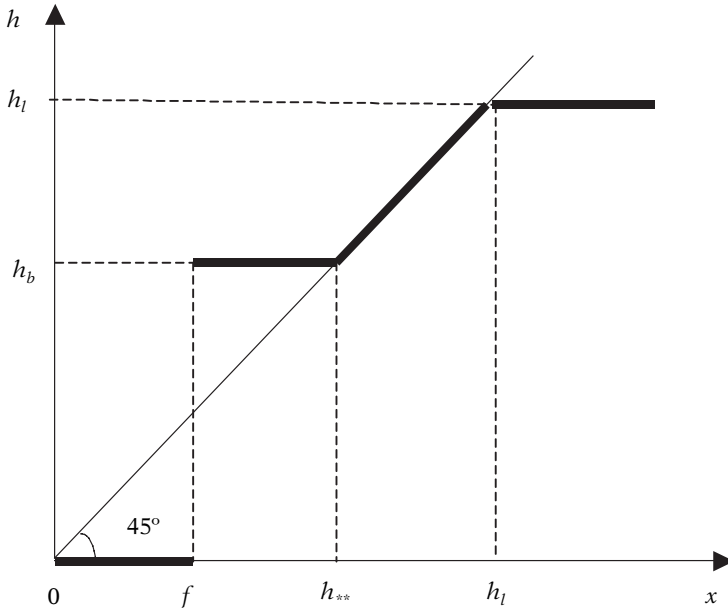
— if $h_{**} \leq x \leq h_l$, the optimal investment in education is equal to x .

— if $x > h_l$, the optimal investment in education is equal to h_l .

The relationship between the initial wealth and the investment in education is underlined in Graph 1:

GRAPH 1

RELATIONSHIP BETWEEN INITIAL WEALTH
AND INVESTMENT IN EDUCATION



The implications in the short run are the same as in the original model. Even abandoning the hypothesis of indivisibility of the investment in education, the initial distribution of wealth still determines how much individuals invest in education, as well as their consumption and their bequest. As a consequence, it also determines aggregate output. This result is due to the assumption that borrowing capital is costly, since these costs are sufficient to generate different levels of access to the credit market according to the each one's initial wealth.

However, this short run analysis loses relevance if the distribution of wealth is ergodic, that is if every initial distribution of wealth converges in the long run to the same distribution of wealth.

In their paper, Galor and Zeira show that the assumption of

non convexities in technologies at the individual level (indivisibility of h) leads to multiple long run *equilibria* in the distribution of wealth.

We will see in the next section that these results still hold when h is divisible.

4. - Long Run Dynamics

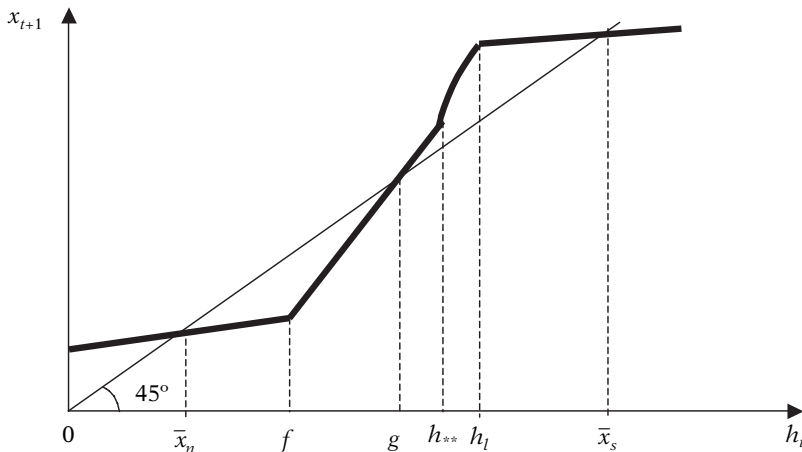
As in the original model, the distribution of wealth not only determines equilibrium in period t , but also determines next period distribution of inheritances:

$$(17) \ x_{t+1} = b_t = (1-\alpha)y_t = \begin{cases} (1-\alpha) [w_n(2+r) + x_t(1+r)] & \text{if } : x_t < f \\ (1-\alpha) [(1+h_b^\beta)w_s - (h_b - x_t)(1+i)] & \text{if } : f \leq x_t < h_{**} \\ (1-\alpha) [(1+x_t^\beta)w_s] & \text{if } : h_{**} \leq x_t \leq h_l \\ (1-\alpha) [(1+h_l^\beta)w_s + (x_t - h_l)(1+r)] & \text{if } : x_t > h_l \end{cases}$$

Equation (17) is described in Graph 2:

GRAPH 2

LONG RUN EQUILIBRIA



It is clear from Graph 2 that there are three *equilibria* \bar{x}_n , g , \bar{x}_s , of which only two \bar{x}_n and \bar{x}_s , are stable. Indeed the slopes of the income functions are the same as in the original model, where it was assumed that:

$$(18) \quad (1-\alpha)(1+r) < 1$$

$$(19) \quad (1-\alpha)(1+i) > 1$$

Equation (18) guarantees that the process of bequest from generation to generation is stable and does not explode. On the other hand, equation (19) suggests that the monitoring costs are high enough to make unstable.

Individuals who inherit less than f work as unskilled and so do their descendants. Their inheritances converge to a long run level \bar{x}_n :

$$(20) \quad \bar{x}_n = \frac{1-\alpha}{1-(1-\alpha)(1+r)} w_n(2+r)$$

Individuals who inherit more than f invest in education but not all their descendants will remain in the skilled sector. The critical point is:

$$(21) \quad g = \frac{(1-\alpha)[h_b(1+i) - w_s]}{(1+i)(1-\alpha) - 1}$$

Individuals who inherit less than g , even if they invest h_b in education, after some generations their descendants will work as unskilled and their wealth will converge to \bar{x}_n .

Individuals who inherit more than g , invest in education and so do their descendants. Their wealth converge to:

$$(22) \quad \bar{x}_s = \frac{1-\alpha}{1-(1-\alpha)(1+r)} (1+h_l^\beta)w_s - h_l(1+r)$$

As in Galor and Zeira the long run level of average wealth is:

$$(23) \quad \bar{x}_s - \frac{L_t^g}{L} (\bar{x}_s - \bar{x}_n)$$

where L_t^g is the population who inherit less than g . Therefore, equation (23) depends on the initial distribution of wealth. The higher the number of individuals who inherit more than g , the higher will be the number of descendants who will converge to \bar{x}_s , the higher will be the long-run level of average wealth. The same initial average of wealth concentrated on a little minority will make the economy converge to \bar{x}_n .

5. - Implications

These results have three different but related implications:

— The club convergence hypothesis still hold under the assumption of a much more continuous education function. In other words, incomes *per capita* of two countries with similar preferences and technologies converge towards one another provided that their initial conditions are similar. In this model the initial conditions refer to the initial distribution of wealth, thus to the concept of equity.

— Even under the assumption of a much more continuous education function, inequality has a negative effect in the aggregate level of income, in both the short and long run¹⁹.

— Even under the assumption of a much more continuous education function, we can generate a non ergodic distribution of wealth. In fact, this model suggests that inequalities tend to persist over time. Countries with high initial inequalities, will maintain an unequal distribution of wealth. This result permits us to give a new interpretation of the empirical data which show a negative relation between inequality and wealth, suggesting that distribution of income is more equal in rich countries than in developing countries. Contrary to the famous curve of Kuznets (1955), which explains this correlation through the existence of different stages on the development path of a country²⁰, this mod-

¹⁹ For more information about the relation between inequality and growth see GALOR O. - MOAV O. (2004).

²⁰ In other words, according to Kuznets, the differences between poor countries and rich countries are due to the fact that these countries belong to differ-

el attributes this dissimilarity to the fact that countries can converge to different long run *equilibria*.

6. - Extensions and Discussion

Let us now focus on the theoretical implications emerging from the conclusions drawn so far in this paper, and more generally from the hypothesis of club convergence.

First of all, the model explains the differences between national income *per capita* through the initial conditions and especially through the initial distribution of wealth. Indeed, according to this model, the permanent crisis of developing countries can be due to an unequal distribution of wealth which does not allow the poorest people to sufficiently invest in education. This has caused the country's level of human capital to decrease over time, eroding individuals' wealth. This model can also explain why international donations from developed countries to developing countries did not achieve the expected results: donations are potentially effective provided that they are equally distributed among the population and that they are actually invested in human capital rather than spent on consumption²¹. Thus, according to this model, poor countries are likely to remain poor, whereas rich countries have in common a large initial investment in human capital.

Furthermore it is worthy of note that equation (23) decreases as L_t^g/L , the fraction of population who does not invest enough in human capital to improve their economic status, increases. This suggests two things: first, that is not a large population *per sé* — as frequently argued — which hinders growth, but rather the *ratio* between the skilled population and the total population. It is clear that, the amount of skilled individuals being equal, in a country with a larger population — if we assume that investments in

ent stages of development. This theory implies, thus, that in the last stage of development/growth all countries will have a similar, more equal, distribution of income (conditional convergence hypothesis).

²¹ For more information about failures of international donations see EASTERLY W. (2001); (2006).

human capital create positive externalities for the society — it will be more difficult to create the externalities necessary to take the economy out of the crisis. From this point of view, smaller countries are more efficient thanks to easier coordination and to a lesser dispersion of resources. Secondly, aggregate initial wealth being equal; growth will be higher where income is more equally distributed. Again, the negative relation between inequality and growth is supported, at least in the field of human capital.

If we assume that investments in human capital create positive externalities for the society, the model is able to explain the phenomenon of “brain drain”. Indeed, individuals who potentially have the ability and the possibility to invest in education may not be able to get their investment returns, if the society in which they live has a low average level of human capital. This happens because a country with a low percentage of skilled/ educated persons is less likely to offer a high quality education and institution, plus there could be less demand for skilled workers. It is clear that in this case, individuals will find it more rewarding to move to a rich country; where the level of human capital is high, and where they will get a better education and later will receive a salary more adequate to their high competences. This extension is equivalent, for example, to the assumption in the model that the maximum investment h that an individual can make in education depends not only on his own initial wealth but also the other’s initial wealth. Indeed, it is less likely to found a top level university where there is a low demand for education.

Furthermore, it is possible — if we still assume positive externalities and interdependence of wealth — that from a collective point of view, it is more suitable if all individuals study, even if it means eventually renouncing the maximization of their own profit. This would allow an increase in the average level of human capital of the country and convergence to the rich equilibrium. Supposing that this could happen, because the individual members of a society are rational, forward looking, and care enough about future generations to renounce a part of their own consumption in order to guarantee a better quality of life for their offspring, the problem of the lack of coordination among indi-

viduals would make it very difficult to happen. In fact, we can presume that individuals would agree to the making of such sacrifices, if indeed they had expectations — and here arises the important role of expectations — that other individuals would, in turn, invest in education: because individual investment has an insignificant impact on growth. What, however, are their expectations based upon? Clearly, the answer is upon the country's average level of wealth. Now, it is evident that in developing countries where most of the population is poor, individuals will certainly not expect a large aggregate investment in education, and so will decide not to risk²² investing in human capital. That is how vicious circles are created. Poor countries have expectations of remaining poor, and this puts an end to individuals having the necessary incentives to invest/risk in their future. This circle could be interrupted by coordination among individuals. Indeed, they could commit²³ themselves to investing in human capital regardless of their initial wealth. Clearly such a coordination of the population is very difficult to realize, especially in developing countries where the population is large and the mass media has little diffusion.

Finally, I would like to highlight how all the implications just discussed only depend on two assumptions: a non convex technology, and credit market imperfections. Indeed if individuals could borrow at rate r , every member of the society would be investing in education the same amount: h_t ²⁴. The distribution of wealth would become ergodic and there would be no longer multiple long run *equilibria*. In other words, the extent to which the initial distribution of wealth determines the long run equilibrium depends on the degree of openness of the credit markets. A coun-

²² Investing in human capital in spite of an initial wealth below the threshold level is a risk since individuals exchange a secured part of their income of today with a potential part of their income of tomorrow.

²³ The commitment must be binding since there is the so called problem of "time inconsistency": once each individual is certain that the population will invest in education — and so that the average level of human capital will increase — since the personal investment has a slight impact, each individual will be induced to deviate from the commitment in order to maximize his income.

²⁴ Indeed equation (12) would apply to all individuals, including borrowers.

try which starts with large income inequalities but which guarantees equal opportunities to all individuals, will reduce these inequalities and converge to the rich equilibrium. On the other hand, a country with small income inequalities but where the access to the credit market is allowed to rich people only — for example because loans require large monetary warrants — will increase these inequalities over time and converge to a lower national income *per capita* compared to its potential level. Hence this model underlines the essential role of the credit market in influencing a country's economy in the short as well as in the long run. It therefore becomes natural to question the importance in this context of the creation of microcredit²⁵. Although still far from guaranteeing equal opportunities, microcredit is contributing to a reduction in the gap of opportunities between the rich and the poor, by giving the latter access to the credit market from which they were left out.

All these arguments lead us to a conclusion which, in my opinion, is very important: the market alone does not necessarily create growth. The latter can require government interventions aimed at promoting equal opportunities. Indeed, we must take into account that, in the reality represented by this model, a unique policy of equal redistribution of income is sufficient to facilitate the access to education for all individuals. Similarly, the government could either implement a policy aimed at facilitating access to the credit market, or offer scholarships to the poorest.

Finally, as already mentioned, these government interventions — and international aid programmes — must not be restricted to a simple allocation of funds but must also be accompanied by measures of control aimed at guaranteeing that these funds are actually invested in human capital. In other words, these policies of intervention should create the right incentives²⁶ for individuals to make choices which then promote growth.

²⁵ For more information about the microcredit see MUHAMMAD Y. (2001).

²⁶ For more details about the theory that — “Individuals respond to incentives” — see EASTERLY W. (2001).

7. - Conclusions

In this paper, I have tried to extend the model of Galor and Zeira to show how, even under the assumption of a much more continuous education function, the initial distribution of income continues to influence economic activity both in the short and in the long run. This result enables us to extend the results of Galor and Zeira (1993) to situations closer to reality; such as the opportunity to decide how much to invest in education. Thus, the hypothesis of club convergence still hold for every discretionary value of h , not only when it is a fixed value.

Escaping from these poverty traps is possible only through government interventions. Since long run aggregate income will be higher if it is more equally distributed initially, government interventions through the policy of income redistribution, is not only advantageous but sometimes even crucial.

The primary role played by initial conditions in determining the long-term equilibrium must make us wonder how much chance accounts for the fate of a country. If a natural disaster hits a group of the population, inequalities will emerge, and if the government does not respond promptly, these inequalities will last over time reducing the long run income *per capita*.

This model, although it is consistent with the neoclassical paradigm, succeeds in reproducing economies which are intrinsically unstable.

APPENDIX 1

Given the complexity of the analysis, I will justify equation (6) through a specific example. Let us consider the following Cobb-Douglas production function for each firm j ²⁷:

$$Y_j = K_j^\delta H_j^{1-\delta}$$

Profit maximisation implies:

$$\begin{aligned}\frac{\delta Y_j}{\delta K_j} &= \delta K_j^{\delta-1} H_j^{1-\delta} = r \\ \frac{\delta Y_j}{\delta H_j} &= (1-\delta) K_j^\delta H_j^{-\delta} = w_s\end{aligned}$$

where r is the remuneration of each unit of physical capital, and w_s is the remuneration of each unit of human capital.

Each firm in perfect competition pays each unit of input the value of its marginal productivity.

In equilibrium, the total demand for K_j and H_j from the firms will be equal to the total supply offered by:

$$\sum_j K_j = \sum_i K_i$$

where j denotes each firm and i denotes each individual. Similarly for human capital:

$$\sum_j H_j = \sum_i H_i$$

It can be inferred that the factorial intensity chosen by each firm is equal to the factorial intensity of the entire economy:

²⁷ For sake of simplicity I removed the temporal notation.

$$\frac{K_j}{H_j} = \frac{\sum_i K_i}{\sum_i H_i} = \frac{K}{H} = k$$

Hence:

$$\begin{aligned} \frac{\delta Y_j}{\delta K_j} &= \delta k^{\delta-1} = r \\ \frac{\delta Y_j}{\delta H_j} &= (1-\delta)k^\delta = w_s \end{aligned}$$

Firm's income distribution is based on the quantity of input owned by each individual. Human capital distribution among individuals depends on individual investment in education and will be remunerated accordingly. More specifically, if individual i who invested h_i , owns a part $H_i = (1+h_i^\beta)$ of the aggregated human capital

$$H = \sum_i (1+h_i^\beta)$$

his remuneration will be proportional. Thus, the wage paid to each individual i is: $H_i w_s = (1+h_i^\beta)w_s$, which matches equation (6) in my model.

The argument can be easily extended to all homogeneous linear production functions.

APPENDIX 2

— The maximisation problem of the borrower is given by:

$$\text{Max}_h : y = (1 + h^\beta)w_s - (h - x)(1 + i)$$

We get:

$$\begin{aligned} \frac{\delta y}{\delta h} &= w_s * \beta h^{\beta-1} - (1 + i) = 0 \\ h &= h_b = \left(\frac{\beta * w_s}{1 + i} \right)^{\frac{1}{1-\beta}} \\ \frac{\partial^2 y}{\partial h^2} &= w_s \beta (\beta - 1) h^{\beta-2} < 0 \end{aligned}$$

The second derivative bears out that it is a maximum.

— The maximisation problem of the lender is given by:

$$\text{Max}_h : y = (1 + h^\beta)w_s + (x - h)(1 + r)$$

Solving:

$$\begin{aligned} \frac{\delta y}{\delta h} &= w_s * \beta h^{\beta-1} - (1 + r) = 0 \\ h &= h_l = \left(\frac{\beta * w_s}{1 + r} \right)^{\frac{1}{1-\beta}} \\ \frac{\partial^2 y}{\partial h^2} &= w_s \beta (\beta - 1) h^{\beta-2} < 0 \end{aligned}$$

The second derivative bears out that it is a maximum.

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