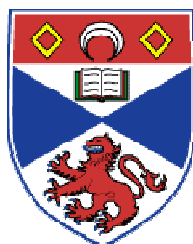# CENTRE FOR DYNAMIC MACROECONOMIC ANALYSIS
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Using time-varying VARs to diagnose the source of
'Great Moderations': a Monte Carlo analysis[*]

Richard Harrison   Haroon Mumtaz
Bank of England   Bank of England
Tony Yates Bank of England

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ABSTRACT

In this paper, we assess the ability of time-varying VAR models to correctly diagnose the source of 'Great Moderations' generated in simulations of a learning model. We find that, in general, they can. For example, in data sets with Great Moderations generated by good policy, the VAR correctly identifies a downward shift in the policy disturbance. And it shows that if the policy behaviour associated with the latter part of the sample (during which policy is conducted well) are applied to the earlier part of the sample, the implied variances of output, inflation and interest rates would have been much lower. An important caveat to our results is that they appear to be sensitive to the method used to identification of monetary policy shocks. When we identify monetary policy shocks using a Cholesky decomposition, the VAR provides quite clear evidence in favour of the correct explanation for our simulated Great Moderations When sign restrictions are used to identify the monetary policy shocks, conclusions from the counterfactual experiments are less precise. The contrast between our results and previous work based on Monte Carlo evidence using RE models suggests that the ability of VARs to correctly diagnose the source of the Great Moderation may be dependent on the nature of the expectations-formation process in the private sector.

1 Introduction

One of the starkest empirical facts in macroeconomics is the dramatic nature of the changes in the time series properties of inflation and real activity since the 1970s. Most industrialised countries experienced a 'Great Inflation' period of high and volatile inflation during the 1970s, followed by a 'Great Moderation' in volatilities to a period of low and stable inflation that has been dubbed the 'Great Stability' in studies of the UK data.[1] These changes in the level and volatility of inflation were, in many countries, accompanied by changes in its persistence. And similar patterns have also been observed in measures of real activity such as GDP. Figure 1 documents these facts for the United Kingdom.[2]
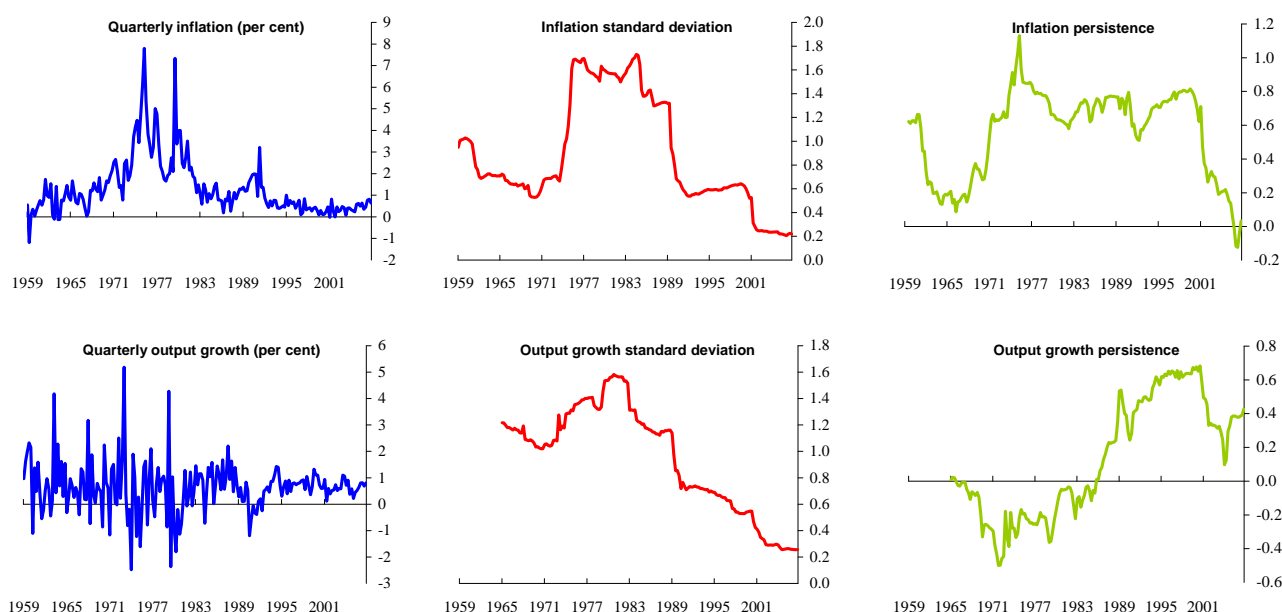
Unsurprisingly these facts have spawned a huge literature investigating the possible causes of the changes in the time series of macroeconomic data.[3] Broadly speaking, the literature has sought to identify the relative contribution of three factors: good luck; favourable structural change; and better policy. The 'good luck' explanation is simply that the shocks hitting economies in recent decades may have been smaller and less volatile than the shocks hitting those economies in the 1970s and early 1980s.[4] Favourable structural change – for example,

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[1]See, for example, Bernanke (20 Feb 2004) for the United States, or King (2007) for the United Kingdom.

[2]Inflation data is based on the RPI measure prior to 1976, when it is spliced to the RPIX series. Output is GDP at market prices. Standard deviations and persistence (sum of first four autocorrelation coefficients) are based on rolling samples of 40 quarters.

[3]This literature is surveyed by Velde (2004). But some notable contributions are: Stock & Watson (2002), Cogley & Sargent (2005), Cogley & Sargent (2002).

[4]Policymakers themselves recognise that good luck may have played a part in the Great Moderation.

greater competition in goods and factor markets – could have reduced the extent to which the macroeconomy is sensitive to shocks of a given variance. And improved frameworks for and conduct of monetary and fiscal policies may have allowed policymakers to respond more effectively to stabilise the economy in the face of the shocks hitting it.

This paper is concerned with the extent to which econometric investigations may confuse changes in the expectations process (associated with improvements in monetary policy) with changes to the process driving macroeconomic shocks. Such confusion might lead researchers to falsely conclude that good luck is the primary explanation of the Great Moderation. This conjecture is not new. As Bernanke (2004) noted:

> ... changes in monetary policy could conceivably affect the size and frequency of shocks hitting the economy, at least as an econometrician would measure those shocks. This assertion seems odd at first, as we are used to thinking of shocks as exogenous events, arising from "outside the model," so to speak. However, econometricians typically do not measure shocks directly but instead infer them from movements in macroeconomic variables that they cannot otherwise explain. Shocks in this sense may certainly reflect the monetary regime.

Some have interpreted Bernanke's conjecture in terms of a change in the monetary policy reaction function from one that responded 'too weakly' to inflationary pressures in the 1970s and early 1980s to one that was much more responsive thereafter.[5] The ideas is that a policy response to inflation that is 'too weak' can give rise to multiple equilibria in rational expectations models: the equilibrium paths for inflation and activity are 'indeterminate'. In such cases, inflation and activity can be driven by so-called sunspot shocks that are unrelated to the fundamental disturbances to demand and supply. When monetary policy becomes more responsive to inflation, the equilibrium paths of inflation and real activity are uniquely determined and the effects of sunspot shocks are eliminated. Benati & Surico (2006) generate data from simulations in which monetary policy behaviour exogenously changes from a reaction function that generates indeterminate equilibria to one that ensures a unique determinate equilibrium. They then ask whether an econometrician would correctly diagnose the source of their simulated Great Moderations. They find that standard econometric approaches would tend to diagnose 'good luck' instead of correctly diagnosing 'good policy' since the change in monetary policy regime alters the transmission mechanism of shocks.

Bernanke (2004) and King (2005) are explicit about this. The issue is how much of a contribution this has made.

[5]Subsample estimates of monetary policy reaction functions by [Clarida et al] suggest that the responsiveness of nominal interest rates to inflation increased in the 1980s and 1990s. The study of [Lubik and Schorfeide] found similar results when estimating a small New Keynesian model over subsamples of US data.

In this paper, we also use simulated data to test whether econometricians can correctly diagnose the cause of changes in their time series properties. But our simulated data are generated from a model characterised by private sector learning. This model captures the idea that changes in monetary policy behaviour can change the properties of the economy through the expectations formation process when those expectations are not formed rationally. Non-rational expectations may be seen as both emprically plausible and useful when thinking about monetary policy. Indeed, Bernanke (2007) himself argues that "many of the most interesting issues in contemporary monetary theory require an analytical framework that involves learning by private agents and possibly the central bank as well".

Several recent papers have also examined the extent to which econometric techniques are able to uncover changes in the time series properties of data that are generated by non-rational expectations models. Perhaps the closest exercise to ours is that of Milani (2007). Milani's interpretation of Bernanke's conjecture in very similar to ours. Milani studies how data generated by a model of adaptive learning would be interpreted by an empirical model that was agnostic about expectations. Our exercise differs from his in two respects. Our model of learning is less sophisticated. Milani includes endogenously time-varying gain to capture the idea that the amount by which agents discount past data will depend on their view of the usefulness of that data for predicting the future. Our empirical strategy is, however, a little more advanced. While Milani looks for whether the econometrician will detect spurious ARCH/GARCH effects in the volatility of shocks to regression equations linking inflation and the output gap to a constant and one lag, the VARs we estimate incorporate time-variation in both the propagation parameters and the shock variances. We also identify monetary policy shocks from the other shocks.

Brazier *et al.* (2008) illustrate how endogenous changes in expectations-formation can alter the inflation process such that an econometrician will interpret these switches as fluctuations in the volatility of shocks to inflation, even though the underlying volatility of shocks is no different. They suppose that the econometrician runs very simple autoregressions for inflation. Aoki & Kimura (2008) study how a model of two-way learning between the private sector and the central bank could account for the Great Moderation. He notes how the resolution, through learning, of uncertainty about private sector perceptions of the inflation target by the central bank, and uncertainty about the inflation target by the private sector can lead to a reduction in the estimated persistence of inflation as measured by simple autoregressions for inflation.

Our paper proceeds in three steps. First, we set out out a simple New Keynesian model of the economy in which agents' expectations are formed by 'constant gain' learning about the structure of the economy. This specification means that agents discount past data more

heavily than if they made use of the entire history of data. Putting a relatively high weight on recent data makes sense if the correlation and persistence properties of the variables in the economy change over time – as is the case in Great Moderation episodes. We simulate the model many times under each of two scenarios. In the 'good luck' scenario, we reduce the variance of the shocks to demand and pricing at the midpoint of the sample, which proxies a reduction in the variance of the shocks hitting the economy. In the 'good policy' scenario, we reduce the variance of shocks to the monetary policy rule at the midpoint of the sample, proxying an improvement in monetary policy behaviour (fewer shocks to interest rate setting). Both scenarios induce changes in the time series properties of output, interest rates and inflation, which agents learn about gradually over time. We describe the model and simulation output in Section 2.

Second, we describe the econometric tools that we will use. We use a time-varying VAR framework to diagnose the causes of the observed changes in the time series properties of a data set. The time-varying VAR approach permits both the coefficient matrices and the covariances of the innovations to evolve (as random walks) over time. We use two alternative assumptions to identify the shocks to the VAR that provide alternative (estimated) separations of monetary policy and other shocks. The first identification assumption is a standard ordering assumption (Cholesky decomposition) and the second is based on sign restrictions on the impulse response functions. The setup of the VAR is described in detail in Section 3.

The third step is to confront the time-varying VAR approach with the data generated by the learning model: we do this in Section 4. The question we pose is whether an econometrician confronted with data from the 'good luck' and 'good policy' scenarios and armed with the time-varying VAR technology would be able to correctly identify the sources of the changes in the time series properties of the data. To do so we estimate VAR models on many data sets produced by each scenario. We then analyse the properties of the shock processes that we identify and the impulse responses of the VAR with respect to those shocks. We also conduct counterfactual experiments as follows. We split each sample and impose the properties of certain shocks in the second (more stable) subsample on the first (less stable) subsample. So for example, we can impose the properties of the identified monetary policy shocks from the second subsample on the first to see what would have happened in the first subsample under the assumption that monetary policy shocks had the same properties as in the second subsample If the first subsample looks very different when the policy shocks from the second subsample are applied, then this can be interpreted as evidence that 'good policy' was the driver of the change in time series properties. Similarly, we can impose the properties of the non-policy shocks from the second subsample on the first to examine the 'good luck' explanation These experiments mirror those performed on actual UK data by Benati & Mumtaz (2006).

Our results suggest that the effects of better monetary policy on expectations can be correctly identified as 'good policy' by a time-varying VAR approach, when expectations formation is described by adaptive learning. These results contrast with those of Benati & Surico (2006), who find that, time-varying VARs are unable to correctly identify 'good policy' in data generated by a rational expectations model under the assumption that policy changes from using an 'indeterminate' policy rule to using a 'determinate' rule. This suggests that the ability or otherwise of time-varying VARs to identify the contributions to the Great Moderation depends on the nature of the expectations and learning processes that characterise private sector behaviour.

In Section [x] we conclude by asking how one should we read our results in light of the fact that studies using time-varying parameter VARS on actual data tend to diagnose Great Moderations as being generated by good luck We argue that one interpretation is that our results lead us to put less weight on Bernanke's scepticism about the ability of empirical tools to correctly diagnose the Great Moderations caused by good policy. We find that appropriately identified time-varying parameter VARs are able to correctly uncover the true source of Great Moderations simulated from a model in which policy-induced changes in expectations formation are important. But we also argue that our findings may lead us to put less weight on the notion that the Great Moderation can be explained using a simple model with adaptive learning: our model is perhaps too stylised to replicate the patterns in time series data that characterise the Great Moderations we observe in actual macroeconomic data. And given the extreme flexibility of the time-varying parameter VAR, our Monte Carlo experiments may not provide a stern enough test of the technique. It must surely remain a possibility that good luck played a role in the Great Stability. The Governor of the Bank of England himself acknowledged that "Lady Luck smiled on us" in producing what he called the NICE (non-inflationary-continuously-expanding) decade of the 1990s. But how much this was a factor is an open question.

2   An adaptive learning model for simulating Great Moderations

2.1   The model

We simulate Great Moderations using a New Keynesian model modified so that expectations are formed by constant gain, adaptive learning. Since the model has been discussed and motivated exhaustively elsewhere, our discussion here is extremely brief. Linearized versions of the model's three equations, with all variables expressed as log deviations from steady state,

comprise an inflation equation; an aggregate demand or IS equation; and a policy rule.

$$\pi_t - \mu\pi_{t-1} = \beta E_t\left[\pi_{t+1} - \mu\pi_t\right] + \kappa x_t + \varepsilon_{\pi,t}, \tag{1}$$

Note that our inflation equation includes a term in lagged inflation. This term can be justified by the assumption that price setting is governed by a Calvo (1983) contracting scheme and that firms that do not reoptimise their prices index them to an average of lagged inflation (with weight μ) and trend inflation.[6] We set this weight to be 0.1, since we are including other sources of persistence, via the shocks, and via learning.

$$x_t = E_t x_{t+1} - \sigma^{-1}(ir_t - E_t\pi_{t+1}) + \varepsilon_{x,t}, \tag{2}$$

$$ir_t = \varphi_\pi \pi_t + \varphi_x x_t + \varepsilon_{ir,t}, \tag{3}$$

Policy is therefore set using a Taylor rule. We set the output gap coefficient to be 0.5 and the inflation coefficient to be 1.5. These coefficients are familiar benchmark values and help to ensure stability in our model.

We assume that our shocks follow persistent processes:

$$\varepsilon_{\pi,t} = \rho_\pi \varepsilon_{\pi,t-1} + \sigma_{z_\pi} z_{\pi,t} \tag{4}$$

$$\varepsilon_{x,t} = \rho_x \varepsilon_{x,t-1} + \sigma_{zx} z_{x,t} \tag{5}$$

$$\varepsilon_{ir,t} = \rho_{ir} \varepsilon_{ir,t-1} + \sigma_{z_{ir}} z_{ir,t} \tag{6}$$

We collect together the variables that agents have to forecast in the vector Y:

$$Y_t = \begin{bmatrix} \pi_t \\ x_t \\ ir_t \end{bmatrix}$$

and we assume that agents form their expectations by projecting from past values, thus:

$$E_t(Y_t) = \Psi_t Y_{t-1}$$

Agents' forecasting coefficients are updated each period according to the following recursion:

$$\begin{aligned} \Psi_t &= \Psi_{t-1} + \gamma R_t^{-1}(Y - \Psi_{t-1}Y_{t-1}) \\ R_t &= R_{t-1} + \gamma(R_{t-1} - Y_{t-1}Y'_{t-1}) \end{aligned}$$

Note that here we are simply substituting out for terms in $E_t Y_t$ using agents' VAR-based forecasts. Preston (2005) refers to this as the 'Euler equation' approach to learning models,

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[6]See [Woodford].

and notes that it arguably does some violence to the underlying microfoundations of the model. We side-step the issue in the hope that this approach nevertheless provides an adequate approximation to the behaviour that would characterise a sticky-price learning model.[7]

We simulate using the following calibration of our model:

$$
\begin{array}{ll}
\beta & 0.99 \\
\kappa & 0.2 \\
\mu & 0.1 \\
\sigma & 5 \\
\rho_\pi & 0.4 \\
\rho_x & 0.4 \\
\rho_r & 0.9 \\
\varphi_\pi & 1.5 \\
\varphi_x & 0.5 \\
\gamma & 0.03
\end{array}
$$

In common with other researchers using learning models of this type, we are forced to use what is known as a 'projection facility' to guarantee stability of our learning model. Our facility is one that allows agents to use the updated forecast coefficients provided that the modulus of the maximum eigenvalue of Ψ_t is less than unity. If this condition fails, agents are assumed to carry forward last period's forecast coefficients to this period. This facility is not itself enough to guarantee that the model is stable. But, together with our calibration of the gain at 0.03 – a value comparable to that used by other researchers in this literature – it ensures that explosive simulations are relatively rare.

2.2 Designing 'better policy' and 'good luck' induced Great Moderations

We simulate two sets of 1000 Great Moderations, one that are brought about by events that we characterise as 'better policy', the second brought about by 'good luck'. For each simulation, we simulate the model for 1000 periods, and then extract the final 160 periods.[8] We think of our model as a quarterly model, so 160 periods is meant to match the roughly 40 years

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[7]There is a debate about the importance of this simplification. [Harrison and Taylor] compare the behaviour of a simple New Keynesian model with non-rational expectations solved using the Euler equation approach and Preston's 'long horizon expectations' method. They find that the models behave quite similarly as long as the deviation from rational expectations is relatively small.

[8]Prior to the 1000 periods of model simulation, we 'burned in' the shocks for 3000 periods to eliminate pseudo-randomness in these draws.

of post-war history that are focused on in studies of the Great Moderation, with the first 20 being characterised by a 'Great Immoderation' and the final 20 featuring an improvement of one sort or other. To construct our 'good luck' Great Moderations, after period 80 we halve the variance of the shocks to the inflation and aggregate demand equations from 0.2 to 0.1. Our 'better policy' simulations are based on cutting the variance of the shocks to the monetary policy equation from 0.2 in the first 80 periods, to 0 in the final 80 periods.

Table 1: Properties of the DGP

	Subsample 1			Subsample 2		
	Median	lower	upper	Median	lower	upper
DGP with change in the volatility of the monetary policy shock						
Standard deviation						
Inflation	0.23	0.20	0.27	0.22	0.19	0.26
Output Gap	0.28	0.23	0.34	0.23	0.20	0.26
Interest Rate	0.39	0.34	0.47	0.32	0.28	0.38
AR1 Coefficient						
Inflation	0.61	0.48	0.72	0.56	0.42	0.69
Output Gap	0.67	0.52	0.77	0.54	0.41	0.65
Interest Rate	0.55	0.37	0.69	0.50	0.35	0.66
DGP with change in the volatility of the non-policy shocks						
Standard deviation						
Inflation	0.35	0.27	0.50	0.18	0.14	0.25
Output Gap	0.29	0.24	0.36	0.15	0.13	0.19
Interest Rate	0.54	0.43	0.75	0.32	0.26	0.41
AR1 Coefficient						
Inflation	0.77	0.65	0.87	0.76	0.65	0.86
Output Gap	0.67	0.55	0.77	0.65	0.55	0.75
Interest Rate	0.69	0.53	0.84	0.55	0.33	0.73

Table 1 presents some basic statistics calculated using the two DGP's across the two subsamples. The estimated standard deviation of the endogenous variables is lower in the second subsample with this difference greater in the dataset generated under the good luck hypothesis. There is little evidence for a change in persistence as measured by the AR(1) coefficient.

3 Monte Carlo experiments

In this section we explain the Monte Carlo experiments that we perform. In Section 3.1 we briefly outline the key elements of the experiment design, before outlining the specification and estimation of the time-varying VAR model in Section 3.2. We explain the 'counterfactual' simulations in Section 3.3.

3.1 Experiment design

The design of the Monte Carlo experiments is based on the following three steps:

Step 1. Use the model in equations (1) to (3) as the data generating process and simulate two data-sets from the model under different assumptions about the change in policy and non-policy shocks (as described in Section 2.2).

Step 2. Estimate a time-varying VAR with stochastic volatility using each dataset and conduct counterfactual experiments to determine the source of any estimated change in the volatility of the endogenous variables.

Step 3. Repeat the above 500 times

3.2 The time-varying VAR: specification and estimation

We estimate a time-varying VAR(1) (TVP-VAR) with stochastic volatility using each of the generated datasets. The choice of the TVP-VAR as an empirical approximation of the data reflects that fact that (a) it has been the model of choice in several recent studies on the great moderation (see for e.g. Cogley & Sargent (2005) and Primiceri (2005)) and (b) that it is a flexible device to model time-variation in the dynamics and volatilities incorporated in the DGP.

Specifically, we estimate the following time-varying parameter VAR:

$$Z_t = \mu_t + \sum_{l=1}^{L} \phi_{l,t} Z_{t-l} + v_t \tag{7}$$

where Z contains generated data on inflation, output and the interest rate. We fix the lag length L=1. The parameters of the model $\beta_t = \{\mu_t, \phi_{l,t}\}$ evolve as random walks

$$\beta_t = \beta_{t-1} + \eta_t$$

The covariance matrix of the innovations v_t is factored as

$$VAR(v_t) \equiv \Omega_t = A_t^{-1} H_t (A_t^{-1})' \tag{8}$$

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The time-varying matrices H_t and A_t are defined as:

$$H_t \equiv \begin{bmatrix} h_{1,t} & 0 & 0 \\ 0 & h_{2,t} & 0 \\ 0 & 0 & h_{3,t} \end{bmatrix} \qquad A_t \equiv \begin{bmatrix} 1 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 \end{bmatrix} \tag{9}$$

with the $h_{i,t}$ evolving as geometric random walks,

$$\ln h_{i,t} = \ln h_{i,t-1} + \nu_t$$

Following Primiceri (2005) we postulate the non-zero and non-one elements of the matrix A_t to evolve as driftless random walks,

$$\alpha_t = \alpha_{t-1} + \tau_t , \tag{10}$$

and we assume the vector $[\, v_t', \, \eta_t', \, \tau_t', \, \nu_t']'$ to be distributed as

$$\begin{bmatrix} v_t \\ \eta_t \\ \tau_t \\ \nu_t \end{bmatrix} \sim N\left(0, V\right), \text{ with } V = \begin{bmatrix} \Omega_t & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & G \end{bmatrix} \text{ and } G = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix} \tag{11}$$

The model in equations 7 to 11 is estimated using Bayesian methods. A detailed description of the prior distributions and the sampling method is given in Appendix A. Here we summarise the basic algorithm which involves the following steps:

1. Use the first 30 observations in the sample to set priors and starting values[9].

2. Conditional on a draw for the VAR coefficients β_t, sample the VAR covariance matrix. This involves the following draws:

   - The off-diagonal elements of the covariance matrix α_t are simulated by using the multi-move Gibbs sampler in Carter & Kohn (2004)

   - The volatilities of the reduced form shocks H_t are drawn using the Metropolis-Hastings scheme introduced in Jacquier *et al.* (2004).

3. Conditional on the draw for H_t and α_t draw the VAR coefficients β_t. This is carried using the methods in Carter & Kohn (2004).

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[9]This implies that the effective sample (used for estimation) consists of 130 periods with a structural break imposed in period 50.

4. Conditional on the draw for H_t, α_t and β_t draw the hyperparameters Q, S and G from their respective distributions.

5. Go to step 2.

We take 20000 draws from these conditional posterior distributions and retain the last 1000 draws for inference. This choice of the total number of draws is dictated primarily by the need to keep the monte-carlo experiment computationally tractable. Using code written in the OX^{TM} programming language, 20000 draws from the posterior are obtained in approximately 25 minutes implying that one set of monte carlo experiments (with 500 replications) takes about 10 days to complete.

3.2.1 Identifying monetary policy shocks

We identify monetary policy shocks based on two identification schemes. Firstly, as in Primiceri (2005) we use a Choleski decomposition of Ω_t with the interest rate ordered last. Secondly, following Benati (n.d.) we use sign restrictions to identify the monetary policy shock. Our identification strategy imposes the restriction that the contemporaneous impact of the monetary policy shock be non-negative on the interest rate and non-positive on inflation. In selecting this 'agnostic' identification strategy we follow Uhlig (2005). In addition, the agnostic nature of the identification scheme substantially reduces the computational burden[10]. This identification scheme is implemented as follows. We compute the time-varying structural impact matrix, $A_{0,t}$, via the procedure recently introduced by Rubio et $al.$ (2005). Specifically, let $\Omega_t = P_t D_t P_t'$ be the eigenvalue-eigenvector decomposition of the VAR's time-varying covariance matrix Ω_t, and let $\tilde{A}_{0,t} \equiv P_t D_t^{\frac{1}{2}}$. We draw an $N \times N$ matrix K from the $N(0,1)$ distribution. We take the QR decomposition of K. That is we compute Q and R such that $K = QR$. We then compute a candidate structural impact matrix as $A_{0,t} = \tilde{A}_{0,t} \cdot Q'$. If $A_{0,t}$ satisfies the sign restrictions we keep it. Otherwise we draw another matrix K and recompute $A_{0,t}$.

3.3 Outline of the counterfactual experiments

We evaluate the performance of the TVP-VAR in two ways. Firstly, we assess if the parameter estimates accurately reflect the changes in the dynamics and the volatility of the endogenous

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[10]In particular, we found it very difficult to impose a contemporaneous fall in both the output gap and inflation (in response to an increase in interest rates) in the second half of the sample with the algorithm repeatedly getting stuck for a long period of time.

variables built in to the DGP. Secondly, we investigate if we can accurately identify the monetary policy shock and correctly infer its importance in bringing about any estimated changes in dynamics or volatilities. Reflecting the methods adopted in Primiceri (2005), Sims & Zha (2006) and Benati (n.d.) we carry out this second exercise through counter-factual experiments. In particular, we identify the monetary policy shock in the estimated TVP-VAR. We then impose the volatility of the monetary policy shock and parameters of the interest rate equations estimated in the second half of the sample on the first half of the sample. This counterfactual sequence of VAR parameters and shock volatilities is used to re-estimate the unconditional volatility of inflation, the output gap and the interest rate. This is then compared to the actual estimates of these volatilities to gauge if the TVP-VAR can correctly identify the source of the structural change. For example, when data is generated under the good policy scenario and if the change in the volatility of the policy shocks is accurately captured by the TVP-VAR, then we would expect the counterfactual estimate of the variance of inflation, output gap and the interest rate to be substantially below the actual estimate.

4 Results

This section presents the main results from our Monte Carlo experiments. We describe the results using data from simulations in which Great Moderations are generated by improvements in monetary policy in Section 4.1. Then in Section 4.2 we describe the results using data from simulations in which Great Moderations are generated by good luck. Each of these sections begins by describing the properties of the posterior estimates, before moving on to discuss the structural

4.1 Detecting 'Great Moderations' generated by good policy

4.1.1 A summary of the posterior

Here we describe our estimates of volatility and persistence obtained using the estimated TVP-VAR. Note that the figures below present the distribution of these moments obtained across the 500 replications. In other words, we save the median estimate for each estimate of the TVP-VAR and present the median and the 90% confidence interval estimated over the 500 Monte Carlo replications.

The top 3 panels of figure 1 presents the estimates for the elements of H_t (see equation 9). There is little change in the volatility of the shock to the output and inflation equation. In contrast, there is strong evidence of a decrease in the volatility of the shock to the interest rate equation after period 50, with the estimated volatility close to zero. The top right panel
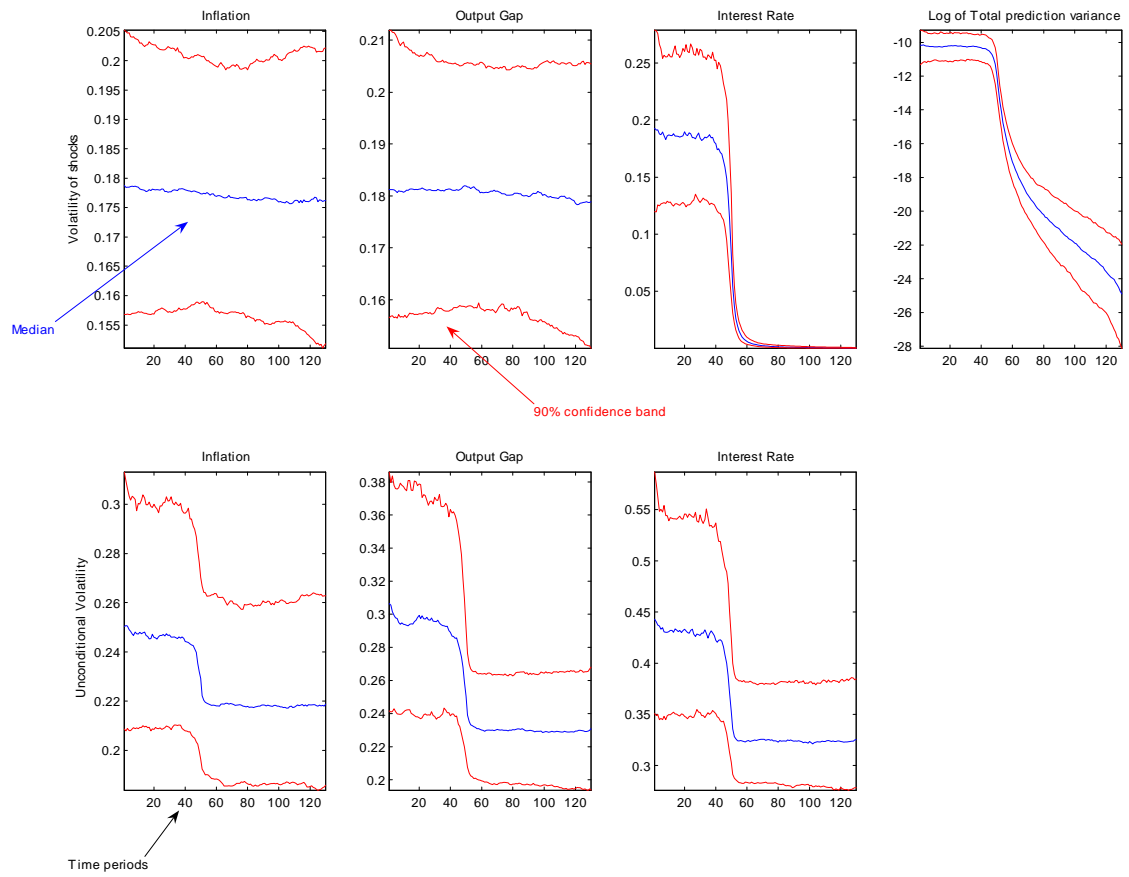
Figure 1: Estimated volatility for Great Moderations generated by good policy

presents the estimated prediction variance. This is defined as $\ln \det (\Omega_t)$ and measures the total shock variation in the VAR system. The results suggest a sharp decline in the total prediction variance after period 50. The bottom panel of the figure presents the estimates of the unconditional volatility of each variable. We approximate the unconditional volatility as $\int_{\varpi} f_{t|T}(\omega)$, where the spectrum $f_{t|T}(\omega)$ is calculated as

$$f_{t|T}(\omega) = s(I_3 - \phi_{t|T}e^{-i\omega})^{-1}\frac{\Omega_{t|T}}{2\pi}\left[(I_3 - \phi_{t|T}e^{-i\omega})^{-1}\right]' s' \tag{12}$$

where I_3 denotes a 3×3 identity matrix , s is a selection vector that picks out the coefficients associated with the ith variable in the VAR and ϖ denotes the frequency. The estimated unconditional volatility of all three variables declined after period 50. Note, however that the estimated confidence intervals are quite wide possibly indicating that the evidence on the decline of volatility provided by the TVP-VAR is not clear cut. This is, however, consistent with the fact that the degree of the reduction in volatility seen in this DGP is relatively small (see table 1).

Figure 2 presents the estimated normalised spectral density of each variable at $\varpi = 0$. We define the normalised spectrum as $\frac{f_{t|T}(\omega)}{\int_{\varpi} f_{t|T}(\omega)}$ and use this as a measure of persistence (see for e.g. Cogley & Sargent (2005)). Figure 2 suggests some evidence for a change in the persistence of inflation and the output gap with the spectral density lower after period 50.

4.1.2 Structural estimates

Figure 3 plots the estimated standard deviation of the monetary policy shock obtained using the two identification schemes. In both cases, this standard deviation is calculated as

$$\left(s\left(A_{0,t}^{-1\prime}\Omega_t A_0^{-1}\right)s'\right)^{0.5}$$

where A_0 denotes the structural impact matrix obtained using either the Choleski decomposition or the sign restrictions algorithm of Rubio *et al.* (2005) and s denotes a selection vector. Figure 3 suggests two interesting conclusions. First, regardless of the identification scheme used, the TVP-VAR correctly identifies a fall in the volatility of the monetary policy shock after period 50. However, the degree of reduction in the standard deviation of monetary policy shocks is smaller when sign restrictions are used to identify the shock. Secondly, the standard deviation of the monetary policy shock is substantially smaller (in the first 50 periods) when the sign restriction scheme is used. This suggests that the role played by the monetary policy shock may potentially be under-estimated when sign restrictions are used to identify the shock.

Figure 4 plots the estimated median time-varying impulse response functions with the top panel displaying the estimates under recursive identification and the bottom panel displays
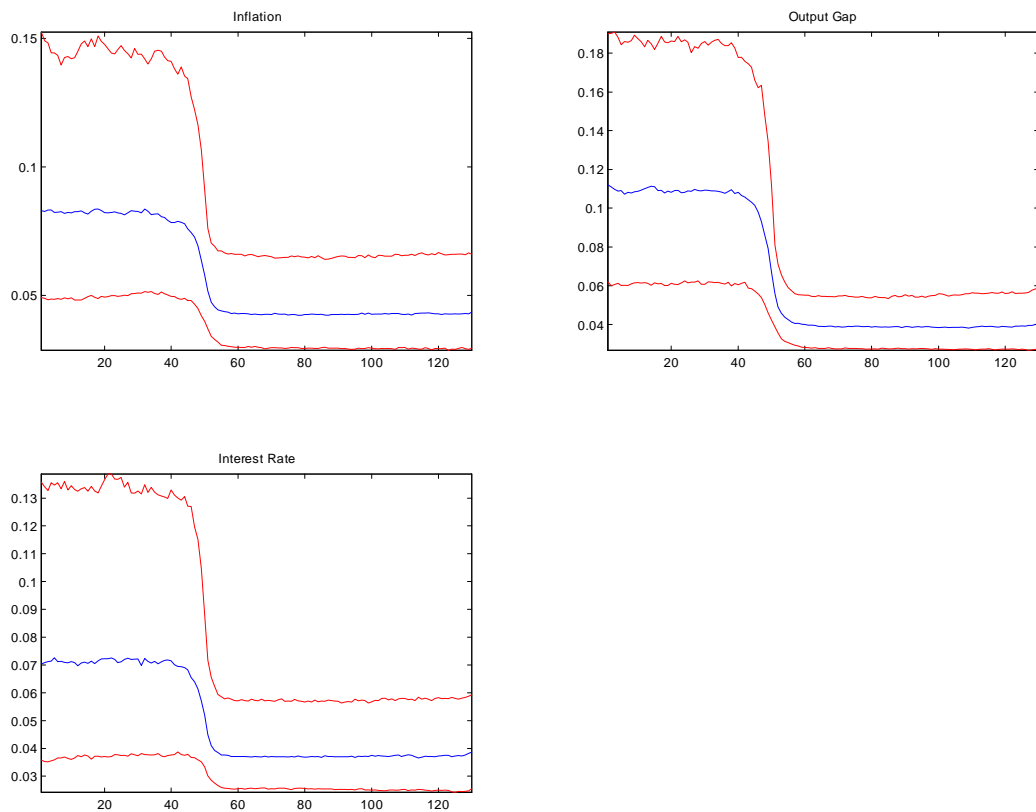
Figure 2: Estimated spectrum at frequency zero for Great Moderations generated by good policy
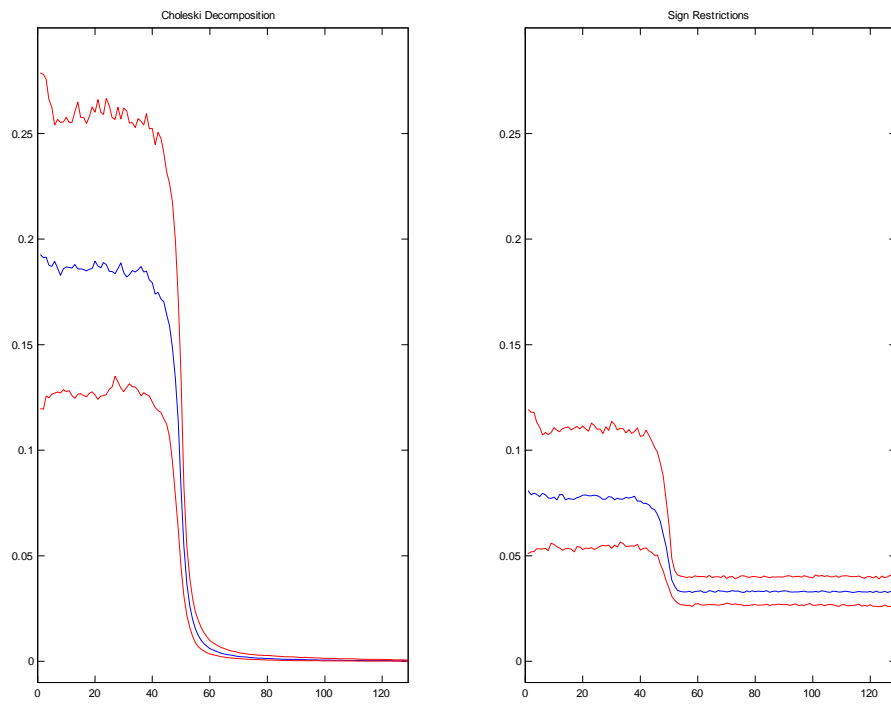
Figure 3: Estimates of standard deviation of the monetary policy shock for simulated Great Moderations generated by good policy
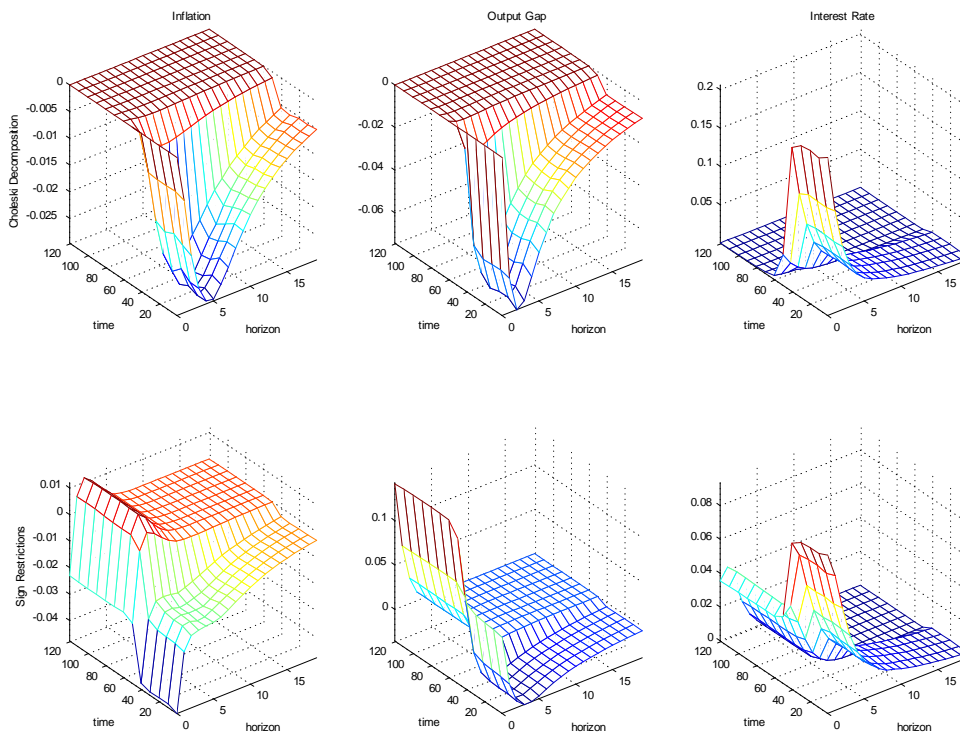
Figure 4: Impulse response to a monetary policy shock for simulations of Great Moderations generated by good policy

those obtained under the sign restriction scheme. Both estimates share one feature: the magnitude of the response of the output gap and inflation depends entirely on the magnitude of the monetary policy shock with little evidence of variation in the transmission mechanism. However in their other features the responses are starkly different. Under the Cholesky scheme, a contractionary policy shock decreases inflation and output. Under sign restrictions, the inflation response displays a price puzzle in the second subsample. Similarly, the response of the output gap is positive in the second sub-sample. These anomalies suggest that the minimal restrictions incorporated in this scheme may not be enough to correctly identify the monetary policy shock in this setting.

4.1.3 Counterfactual experiments

Counterfactual experiments form a key piece of evidence in several recent papers that deal with the great moderation (see for e.g. Primiceri (2005), Sims & Zha (2006) and Benati (n.d.)). In our Monte Carlo experiment we further analyse the ability of the TVP-VAR to diagnose the causes of the great moderation by investigating the performance of two counterfactual experiments.

Under the first experiment, we impose the elements of the monetary policy rule estimated after period 50 over the entire sample. In other words, we combine the coefficients of the monetary policy equation and the volatility of the monetary policy shock from the "good policy period" with the estimated non-policy blocks of the TVP-VAR coefficients and the covariance matrix. We refer to the first experiment as the *policy counterfactual*. In the second experiment we combine the non-policy shocks from the second sub-sample (i.e. after period 50) with the estimated policy block of the VAR. We refer to this experiment as the *non-policy counterfactual*.

In both cases we use these counterfactual sequence of VAR parameters and shocks to re-estimate the unconditional volatility of the three endogenous variables. We then compare these counterfactual estimates with those presented in the bottom panel of figure 1. We report results for the both counterfactual experiments under each of our identification schemes.

In practical terms, the counterfactual experiments involve the following steps for each Monte Carlo replication

1. For Gibbs iteration i, divide the sequence of drawn β_t, A_t and H_t into the two subsamples S_1 and S_2 (where the second subsample S_2 starts after period 50). Take a random draw from the sequences β_t, A_t and H_t from S_2. Call these $\tilde{\beta}$, $\tilde{A}$ and $\tilde{H}$.

2. Construct the covariance matrix $\tilde{\Omega}$ using $\tilde{A}$ and $\tilde{H}$ and then find the structural impact matrix $\tilde{A}_0$ using either the Choleski decomposition or the sign identification. Use this

to calculate the variance of the structural shocks as $\tilde{H}^* = \tilde{A}_0^{-1'}\tilde{\Omega}\tilde{A}_0^{-1}$ where $\tilde{H}^*$ is a 3×3 diagonal matrix with the variance of monetary policy shock as the last element.

3. Using the structural impact matrix $A_{0,t}$ obtained at each date t, estimate the volatility of structural shocks at each date $H_t^* = A_{0,t}^{-1'}\Omega_t A_{0,t}^{-1}$.

4a. In the first counterfactual experiment construct a new sequence of the VAR covariance matrix $\bar{\Omega}_t$ by replacing the last element of H_t^* with the last element of $\tilde{H}^*$ (this forces the volatility of the monetary policy shock to equal the estimate in S_2) and combining with $A_{0,t}$. In addition, replace the coefficients of the interest rate equation in β_t with those from $\tilde{\beta}$. Call this counterfactual sequence of VAR coefficients $\bar{\beta}_t$.

4b. In the second counterfactual experiment construct a new sequence of the VAR covariance matrix $\bar{\Omega}_t$ by replacing the first two element of H_t^* with the first two element of $\tilde{H}^*$ (this forces the volatility of the non-policy shock to equal the estimate in S_2) and combining with $A_{0,t}$.

5. Using $\bar{\Omega}_t$ and $\bar{\beta}_t$ (for experiment 1) construct the spectral density using equation 12 and estimate the unconditional volatility.

6. Repeat for Gibbs iteration i=1...100.

Counterfactual experiments using Cholesky identification Figure 5 presents results for the *policy counterfactual* using the Choleski decomposition to obtain $A_{0,t}$ and $\tilde{A}_0$. The black lines in the top three panels represent the estimated unconditional standard deviation (also shown in figure 1). The blue and the red lines represent the distribution of the unconditional volatility under the assumption that the elements of the monetary policy rule estimated after period 50 prevail over the entire sample. It is clear from these figures that under this counterfactual assumption, the median estimate of volatility is substantially lower in the first 50 periods and remains flat over the sample period. This suggests that in this experiment the TVC-VAR correctly assigns the cause of the great moderation to a change in the elements of the monetary policy rule. The bottom panel of the figure tries to assess the significance of the difference in the counterfactual and actual estimate of the unconditional volatility. In particular, it shows the estimated mean probability that the counterfactual estimate of volatility is less than the actual estimate. This probability is close to 90% for all three variables in the first 50 periods of the counterfactual experiments and then drops to around 50% over the rest of the sample. Therefore, there is a high probability that the TVC-VAR correctly diagnoses the cause of the great moderation in this experimental setting.
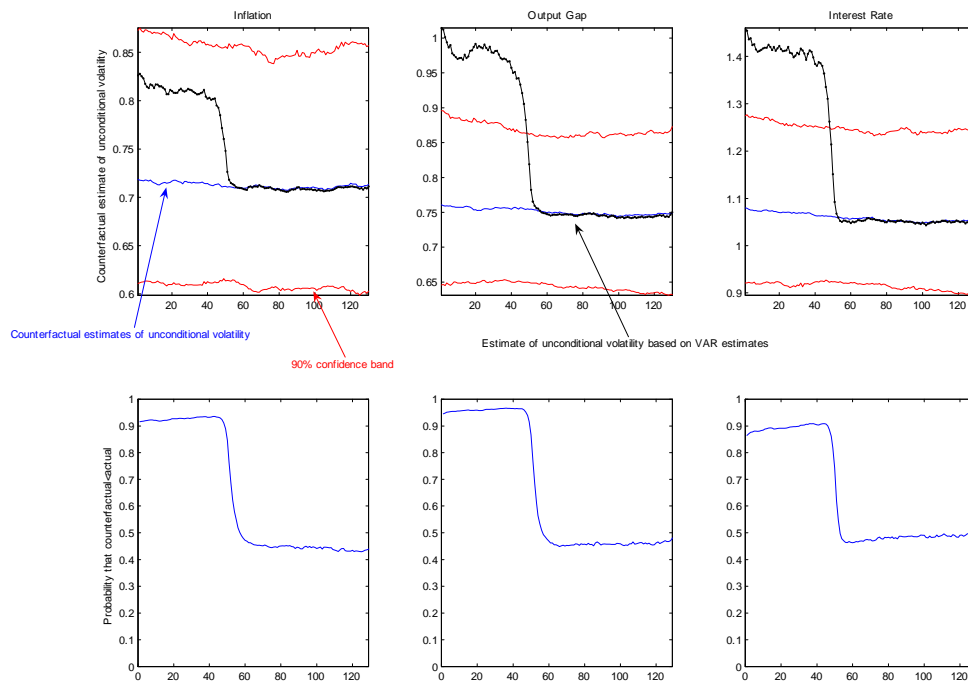
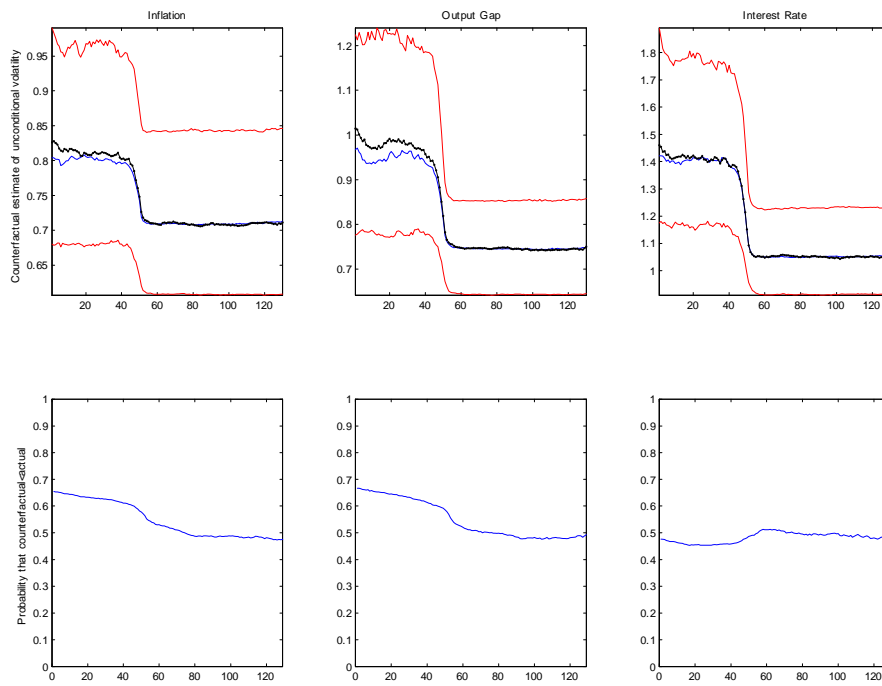Figure 5: Policy counterfactual (Choleski identification; data generated by good policy)

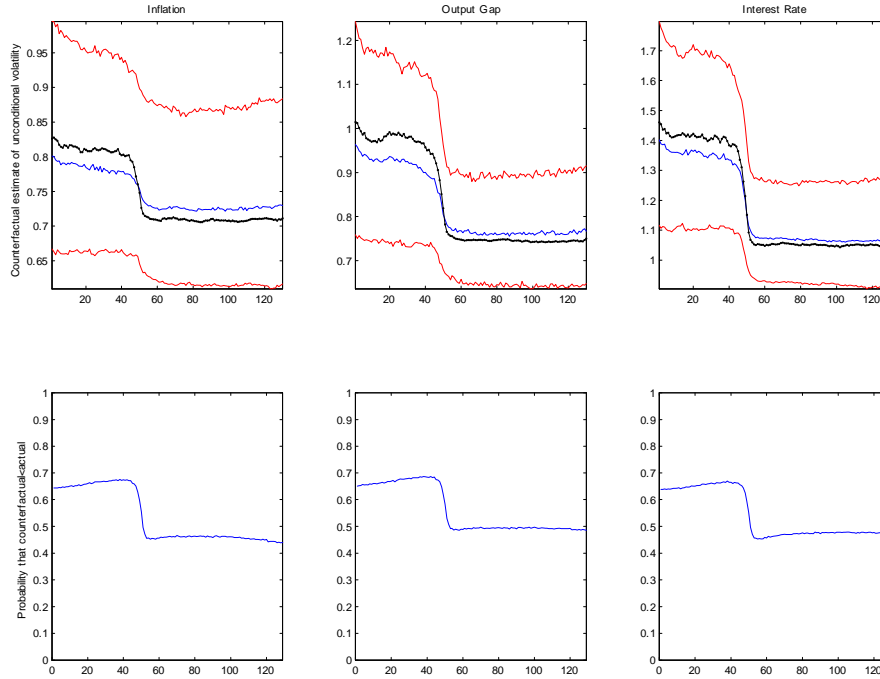Figure 6: Non-policy counterfactual (Choleski decomposition; data generated by good policy)

Figure 7: Policy counterfactual (sign identification; data generated by good policy)

Figure 6 shows the results for the *non-policy counterfactual* using the Choleski identification scheme. The figure suggests that if the volatility of the non-policy shocks estimated after period 50 is imposed on the sample, there is very little change in the estimated volatility of the endogenous variables. The probability that the counterfactual estimate of volatility is less than the actual estimate is substantially lower than that depicted in figure 5.

Counterfactual experiments using sign restriction identification Figure 7 presents the estimates for the *policy counterfactual* when the sign identification scheme is used. In contrast to Choleski identification (see 5), the results are far less clear. Although the median counterfactual estimates of volatility are lower than the actual ones, the difference is smaller in magnitude and the estimated probabilities shown in the bottom panel are only around 70% in the first half of the sample.

Figure 8 presents results from the *non-policy counterfactual* when sign restrictions are used to identify the shocks. The counterfactual estimates are quite close to the actual estimate of unconditional volatility. Therefore as with the Choleski scheme, the TVC-VAR does not lead to an overestimate of the importance of non-policy shocks.
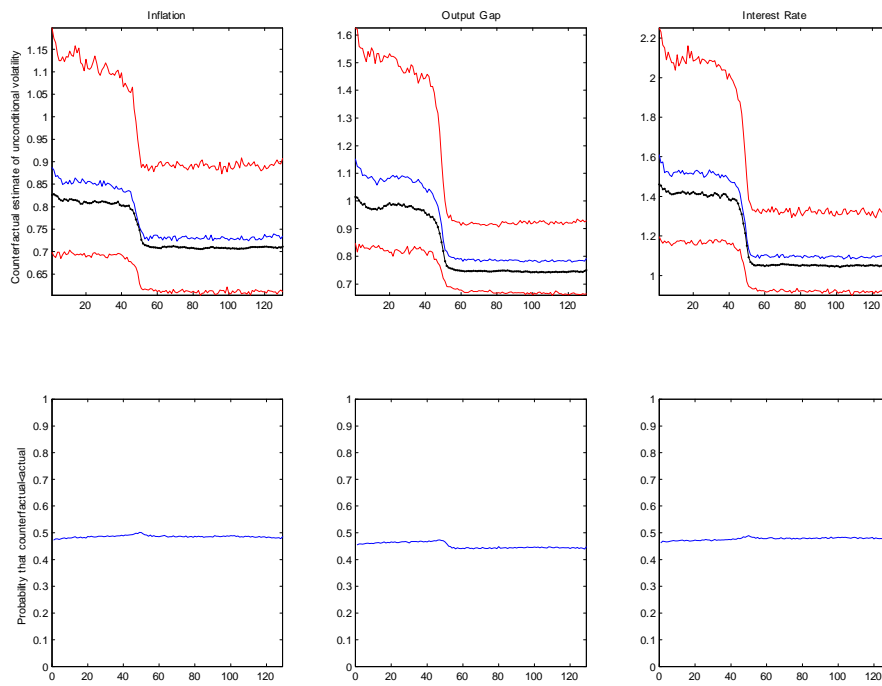
23

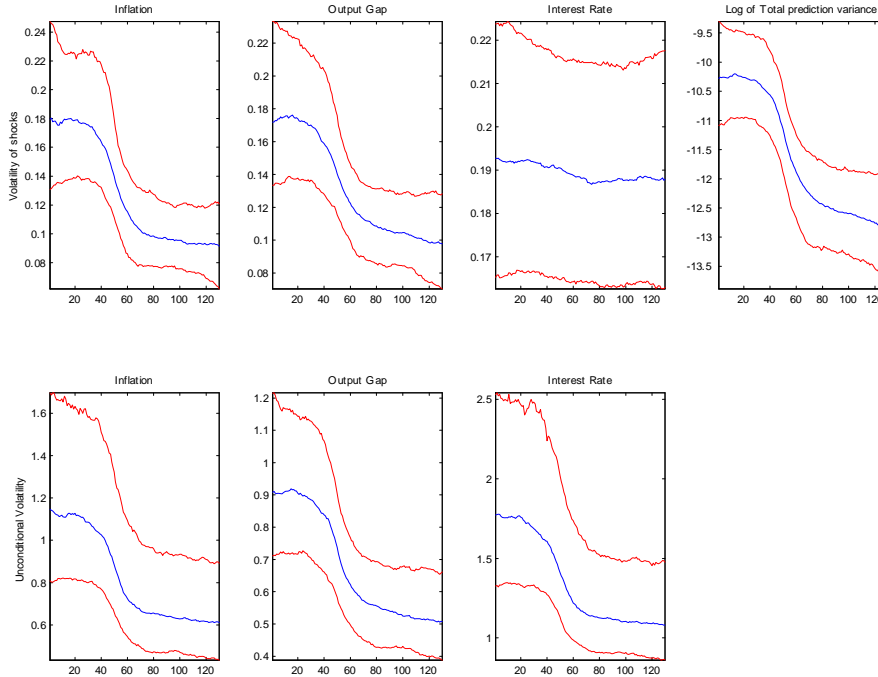Figure 8: Non-policy counterfactual (sign restrictions; data generated by good policy)

Figure 9: Estimated volatility for Great Moderations generated by good luck

4.2 Detecting 'Great Moderations' generated by good luck

4.2.1 A summary of the posterior

Figure 9 plots the estimated stochastic volatility (top panels) and the unconditional volatility. The estimates indicate a change in the volatility of the orthogonalised shocks to the inflation and output equation with the volatility falling after period 50. In contrast the shock to the interest rate equation shows little change in its volatility. The bottom panel of the figure shows the unconditional volatility is estimated to decline for all three variables.

Figure 9 shows that the TVC-VAR produces little evidence to support that the persistence of the endogenous variables has changed. This is in line with the simple AR(1) coefficients reported in Table 1.

4.2.2 Structural results

In this subsection we present results based on the structural version of the TVC-VAR. As before, we identify the shocks based on a Choleski decomposition and on sign restrictions.

Figure 11 displays the standard deviation of the monetary policy shock obtained using the two identification schemes. The two estimates display little time-variation and this feature
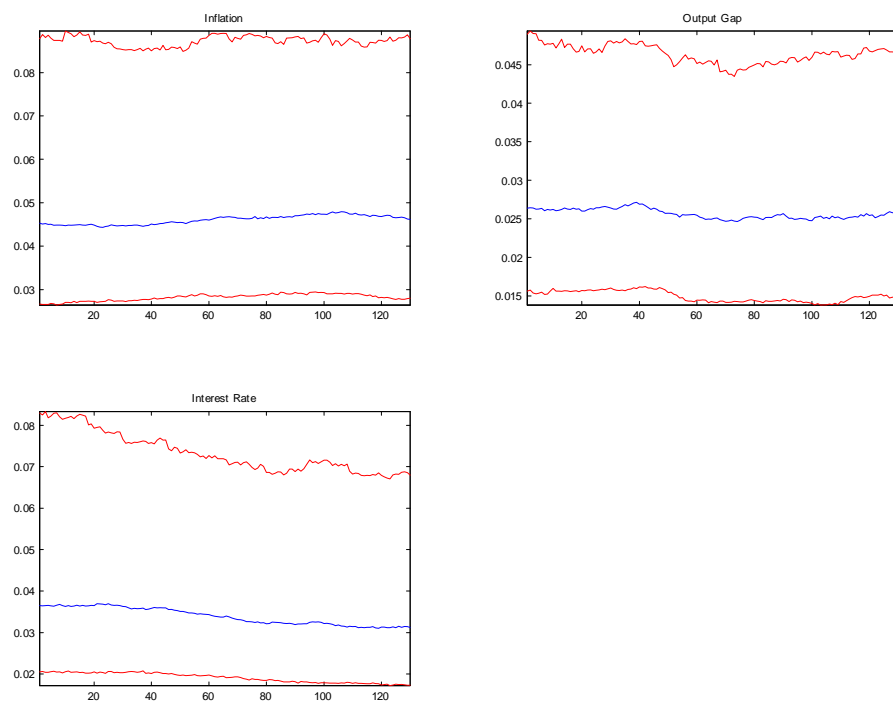
25

Figure 10: Normalised spectrum at frequency zero for Great Moderations generated by good luck
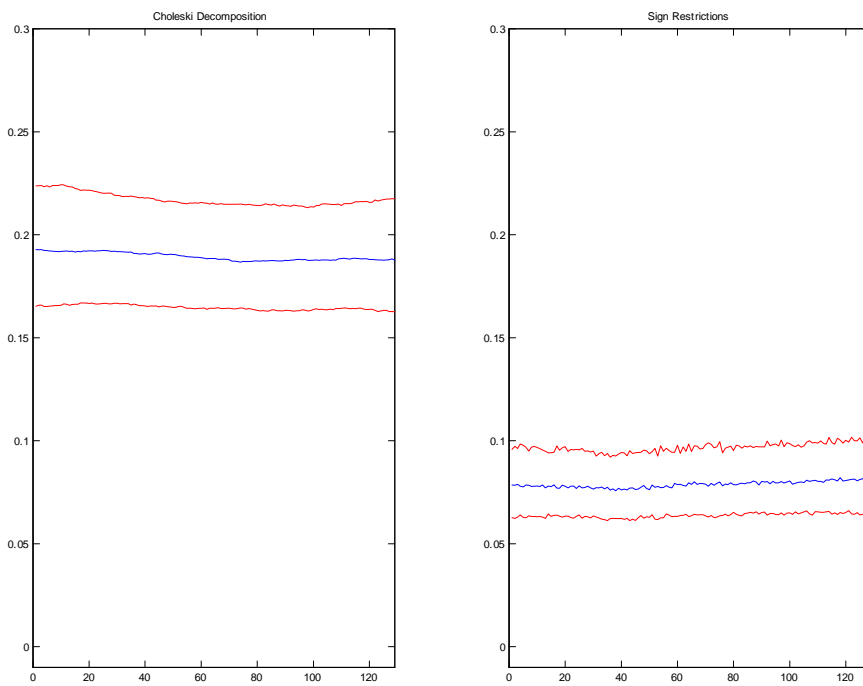
Figure 11: Estimates of standard deviation of the monetary policy shock for simulated Great Moderations generated by good luck
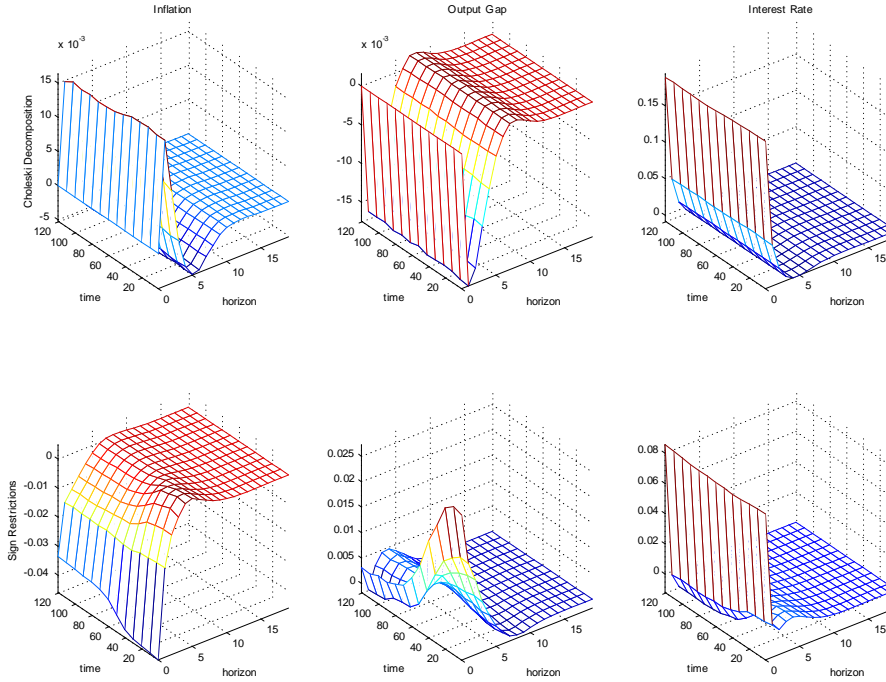
Figure 12: Impulse Responses to a monetary policy shocks

is consistent with the DGP. However as in the previous experiment (see figure 3) the sign identification scheme produces a substantially smaller estimated for the standard deviation of the monetary policy shock suggesting again that this identification scheme assigns a less important role to the policy shock.

Figure 12 presents the responses to a monetary policy shock under both identification scheme. The Choleski decomposition scheme produces a large price puzzle with inflation positive for one period after the shock. The response of output under the sign restriction scheme is positive. Note also that the responses under the sign restriction scheme suggest a dampening of the transmission mechanism, with the inflation and output response become weaker towards the end of the sample period. In summary, the impulse response functions from the two identification schemes are quite unclear and contradictory suggesting that the TVC-VAR is unable to provide a clear picture of time-varying structural dynamics in this DGP.
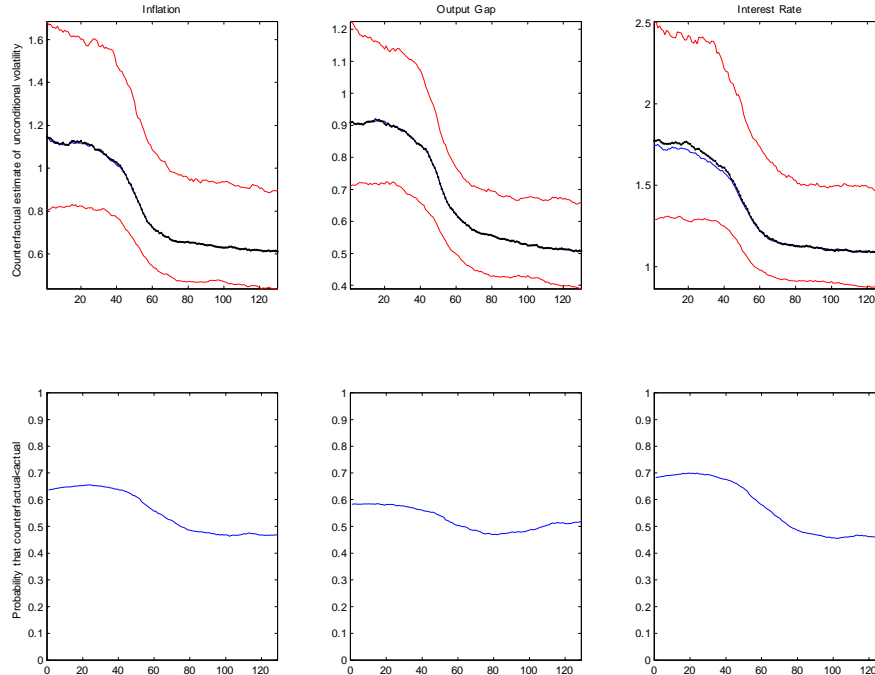
Figure 13: Policy counterfactual (Choleski identification; data generated by good luck)

4.2.3 Counterfactual experiments

Counterfactual experiments using Cholesky identification In this section we again assess the ability of the TVC-VAR to diagnose the factors behind the structural change imposed in this DGP.

The top panels of figure 13 present the estimates of the *policy counterfactual* (the unconditional volatility calculated under the assumption that the elements of the policy rule estimated after period 50 prevails over the entire sample). The figure shows that the counterfactual estimates (blue lines) are virtually indistinguishable from the actual estimate (black line). The bottom panels show the probability that the counterfactual estimate is less than the actual estimate hovers around 0.5.

The results for the *non-policy counterfactual* in figure 14 are quite different. This figure presents results from the counterfactual experiment where the volatility of non-policy shocks estimated after period 50 are imposed over the entire sample. The counterfactual estimates of volatility are lower than the actual estimates suggesting that the TVC-VAR correctly assigns the structural break to non-policy shocks. This observation is further re-enforced by the fact that the estimated probability shown in the bottom panel is around 90% before period 50.
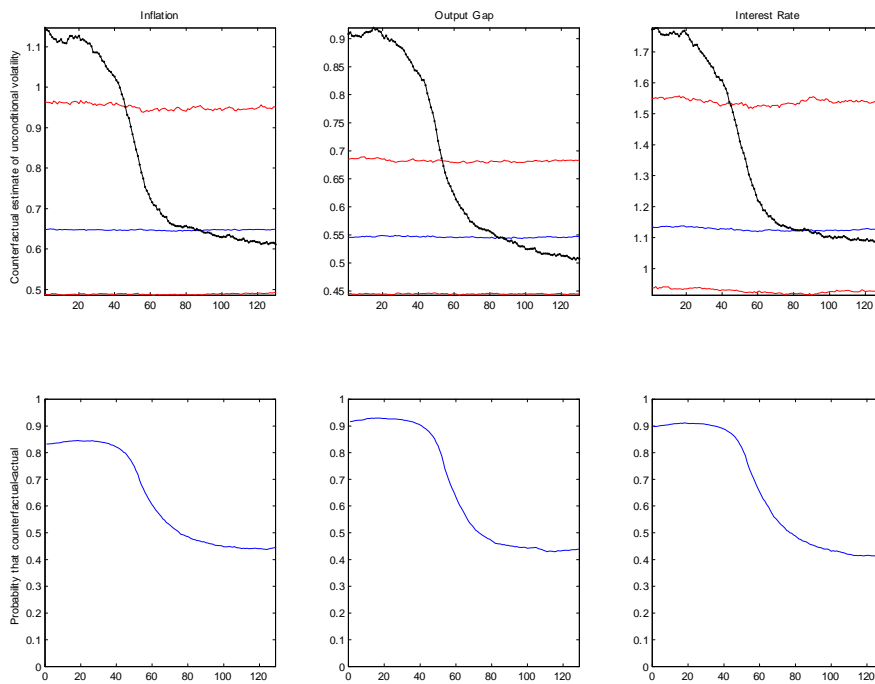
Figure 14: Non-policy counterfactual (Choleski decomposition; data generated by good luck)
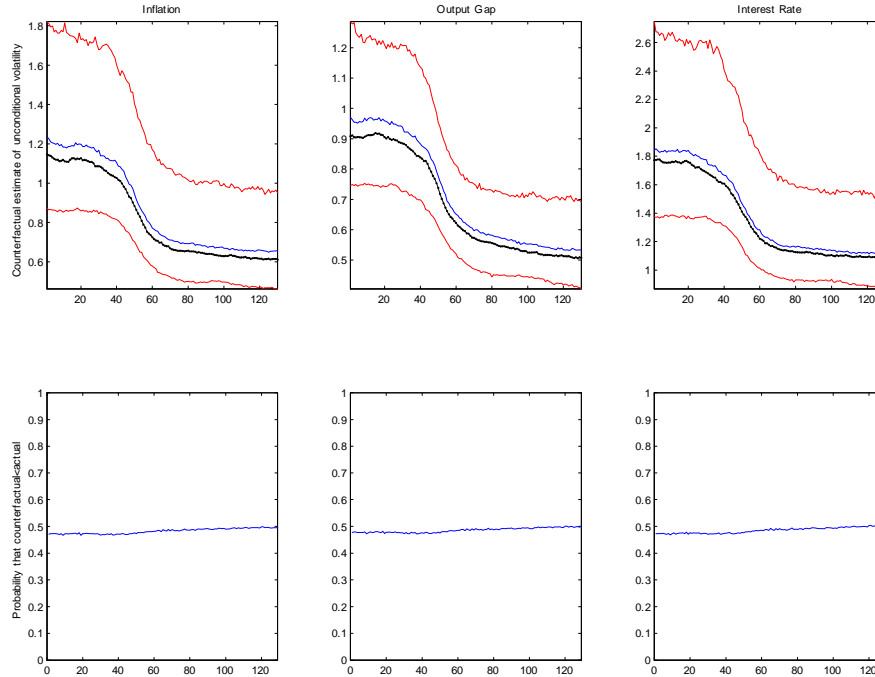
Figure 15: Policy counterfactual (sign identification; data generated by good luck)

Counterfactual experiments using sign restriction identification   Figures 15 and 16 present the results from the counterfactual experiments using the sign identification scheme. The results point to the same conclusions as the Choleski identification. When the volatility of the policy shocks estimated after period 50 is imposed over the entire sample, the resulting estimate of unconditional volatility is little different from the ctual estimate. In contrast, when the volatility of 'good' non-policy shocks is imposed over the entire sample, the counterfactual volatility estimates are lower than actual estimates. Note, however, that under the sign identification scheme the results are less precise. In particular, in the *non-policy counterfactual* , the probability that the counterfactual volatility is lower than actual volatility is only around 70% in the first half of the sample. This suggests that there is some non-trivial probability that the TVC-VAR (with sign restrictions) erroneously assigns importance to structural shifts in the volatility of policy shocks.

4.3   Summary and discussion of the results

Table [x] below, summarises some of the information contained in the lower panels of Figures 5–8 and 13–16.
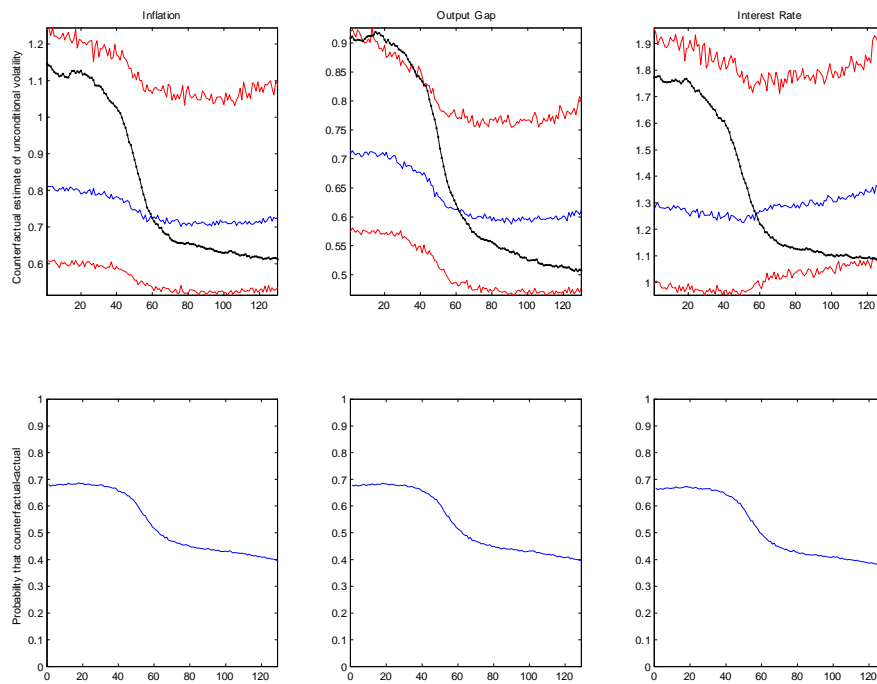
31

Figure 16: Non-policy counterfactual (sign identification; data generated by good luck)

			Great Moderations generated by:			
			Good policy		Good luck	
			Detection probabilities		Detection probabilities	
Counterfactual	Identification	Variable	Subsample 1	Subsample 2	Subsample 1	Subsample 2
Policy	Cholesky	Inflation	0.92	0.46	0.63	0.50
		Output gap	0.96	0.47	0.63	0.50
		Interest rate	0.89	0.49	0.46	0.49
	Sign	Inflation	0.66	0.46	0.49	0.49
		Output gap	0.67	0.49	0.47	0.44
		Interest rate	0.65	0.47	0.47	0.48
Non-policy	Cholesky	Inflation	0.64	0.50	0.83	0.50
		Output gap	0.57	0.50	0.91	0.50
		Interest rate	0.68	0.50	0.90	0.50
	Sign	Inflation	0.47	0.49	0.67	0.45
		Output gap	0.48	0.49	0.67	0.45
		Interest rate	0.47	0.49	0.66	0.43

The table splits our experiments into four large panels. The columns define the conditions under which the simulated data were generated: by good policy or good luck. The rows define the counterfactuals: policy and non-policy respectively. Within each column we report 'detection probabilities' for two subsamples. Subsample 1 is the period prior to the simulated Great Moderations and subsample 2 is the remainder of each simulation. The detection probabilities are simple the average values of the lines in the bottom rows of 5–8 and 13–16 and are reported for each variable under each of our two identification schemes.

Loosely speaking, if the TVP-VAR performs well in diagnosing the true cause of our simulated Great Moderations, we would expect to see more marked differences in the detection probabilities across subsamples for the 'on diagonal' panels of the table. This is most clearly the case when the shocks are identified using the Cholesky decomposition, which is unsurprising given the discussions in previous subsections.

In summary, when the simulated data sets contain Great Moderations generated by a fall in the variance of policy shocks, the TVC-VAR does well in recovering the source of the structural change. This is particularly true when the shocks are identified using a Cholesky scheme. The results from the sign restriction identification are more mixed. This is primarily because under this identification scheme the estimated change in the volatility of the monetary policy shock is smaller than under Cholesky identification. When the simulated data contain Great Moderations generated by good luck, we again find that the TVC-VAR performs reasonably

well in diagnosing the cause of the structural change. And again, these results depend, to an extent, on the identification scheme used. When sign restrictions are used to identify the monetary policy shocks, conclusions from the counterfactual experiments are less precise.

5   Conclusions

In this paper we construct simulated data from a New Keynesian model with learning for each of two scenarios. The first scenario generates 'Great Moderations' by reducing the variance of monetary policy shocks ('good policy'). The second scenario generates them by reducing the variance of non-policy shocks ('good luck'). We then ask if time-varying VAR models can correctly identify the source of the Great Moderations in data sets from both scenarios. We find that, in general, they can. For example, in data sets with Great Moderations generated by good policy, the VAR correctly identifies a downward shift in the policy disturbance. And it shows that if the policy shocks associated with the latter part of the sample (during which policy is conducted well) are applied to the earlier part of the sample, the implied variances of output, inflation and interest rates would have been much lower. Likewise, for data sets generated by 'good luck', the VAR correctly identifies a downward shift in the variance of non-policy shocks. And counterfactual experiments correctly identify the role of non-policy shocks in determining the changes in the variances of output, inflation and interest rates observed in the data sets.

An important caveat to our results is that they appear to be sensitive to the method used to identify monetary policy shocks. When we use a Cholesky decomposition, the VAR provides quite clear evidence in favour of the correct explanation for our simulated Great Moderations When sign restrictions are used, however, the conclusions from the counterfactual experiments are somewhat less precise.

Our results suggest that the effects of better monetary policy on expectations can be correctly identified as 'good policy' by a time-varying VAR approach, when expectations formation is described by adaptive learning. These results contrast with those of Benati & Surico (2006), who find that, time-varying VARs are unable to correctly identify 'good policy' in data generated by a rational expectations model under the assumption that policy changes from using an 'indeterminate' policy rule to using a 'determinate' rule. This suggests that the ability or otherwise of time-varying VARs to identify the contributions to the Great Moderation depends on the nature of the expectations and learning processes that characterise private sector behaviour.

Since econometric analysis of actual data using time-varying parameter VAR models – such as Benati & Mumtaz (2006) – suggests that the Great Moderation was generated by 'good

luck', how should we read our results? One interpretation is that they lead us to put less weight on Bernanke's scepticism about the ability of empirical tools to correctly diagnose the Great Moderations caused by good policy. Bernanke argues that changes in the expectations formation process caused by improvements in monetary policy may appear as 'good luck' in the data. We use a model with private sector learning so that changes in monetary policy behaviour will have an effect on the economy via changes in the expectations formation process. Even so, we find that appropriately identified time-vary parameter VARs are able to correctly uncover the true source of Great Moderations simulated from this model.

Of couse, this leads us to another interpretation: our findings may lead us to put less weight on the notion that the Great Moderation can be explained using a simple model with adaptive learning. While the model incorporates plausible learning behaviour by private sector agents, it is perhaps too stylised to replicate the patterns in time series data that characterise the Great Moderations we observe in actual macroeconomic data. Given the extreme flexibility of the time-varying parameter VAR, one could argue that our Monte Carlo experiments generate data that is 'easy' for the econometric technique to analyse. One caveat, of course, is that the results are sensitive to the way that economic structure is imposed on the VAR: that is, the way that monetary policy shocks are identified. This suggests that more exotic data generating mechanisms, for example models featuring switches in expectations behaviour such as those analysed by Milani (2007) and Brazier *et al.* (2008) may prove a sterner test for the time-varying VAR approach. We leave this conjecture for future work.

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A   Appendix: Priors and Estimation

A.1   Priors

Elements of $\phi_{l,t}$

The starting value for $\phi_{l,t}$ is derived by estimating a fixed coefficient VAR on the first 30 observations in each simulated dataset. The prior for the matrix Q (which essentially controls time-variation) is described below

Elements of H_t

Let $\hat{v}^{ols}$ denote the OLS estimate of the VAR covariance matrix estimated on the pre-sample data (first 30 observations). The prior for the diagonal elements of the VAR covariance matrix (see 9) is as follows:

$$\ln h_0 \sim N(\ln \mu_0, I_3 \times 10)$$

where μ_0 are the diagonal elements of the cholesky decomposition of $\hat{v}^{ols}$.

Elements of A_t

The prior for the off diagonal elements A_t is

$$A_0 \sim N\left(\hat{a}^{ols}, V\left(\hat{a}^{ols}\right)\right)$$

where $\hat{a}^{ols}$ are the off diagonal elements of $\hat{v}^{ols}$, with each row scaled by the corresponding element on the diagonal. $V\left(\hat{a}^{ols}\right)$ is assumed to be diagonal with the diagonal elements set equal to 10 times the absolute value of the corresponding element of $\hat{a}^{ols}$.

Hyperparameters

The prior on Q is assumed to be inverse Wishart

$$Q_0 \sim IW\left(\bar{Q}_0, T_0\right)$$

where $\bar{Q}_0$ is assumed to be $var(\hat{\phi}^{OLS}) \times 10^{-4} \times 3.5$ and T_0 is the length of the pre-sample used for calibration.

The prior distribution for the blocks of S is inverse Wishart:

$$S_{i,0} \sim IW(\bar{S}_i, K_i)$$

where $i = 1..5$ indexes the blocks of S. $\bar{S}_i$ is calibrated using $\hat{a}^{ols}$. Specifically, $\bar{S}_i$ is a diagonal matrix with the relevant elements of $\hat{a}^{ols}$ multiplied by 10^{-3}.

Following Cogley & Sargent (2005) we postulate an inverse-gamma distribution for the elements of G,

$$\sigma_i^2 \sim IG\left(\frac{10^{-4}}{2}, \frac{1}{2}\right)$$

A.2 Simulating the Posterior Distributions

Time-Varying VAR coefficients

The distribution of the $\phi_{l,t}$ is linear and Gaussian:

$$\varphi_{l,T}|Z_{i,t}, \Xi \sim N\left(\phi_{l,T|T}, P_{T|T}\right)$$
$$\phi_{l,t}|\phi_{lt+1}, Z_{i,t}, \Xi \sim N\left(\phi_{lt|t+1, \phi_{lt+1}}, P_{t|t+1, F_{t+1}}\right)$$

where $t = T - 1, ..1$, Ξ denotes a vector that holds all the other VAR parameters and:

$$\phi_{lT|T} = E\left(\phi_{lT}|Z_t, \Xi\right)$$
$$P_{T|T} = Cov\left(\phi_{lT}|Z_t, \Xi\right)$$
$$\phi_{lt|t+1} = E\left(\phi_{lt}|Z_t, \Xi\right)$$
$$P_{t|t+1} = Cov\left(\phi_{lt}|Z_t, \Xi\right)$$

As shown by Carter & Kohn (2004) the simulation proceeds as follows. First we use the Kalman filter to draw $\phi_{lT|T}$ and $P_{T|T}$ and then proceed backwards in time using:

$$\phi_{t|t+1} = \phi_{t|t} + P_{t|t} P_{t+1|t}^{-1}\left(\phi_{t+1} - \phi_t\right)$$

$$P_{t|t+1} = P_{t|t} - P_{t|t} P_{t+1|t}^{-1} P_{t|t}$$

Elements of H_t

Following Cogley & Sargent (2005), the diagonal elements of the VAR covariance matrix are sampled using the methods described in Jacquier *et al.* (2004).

Element of A_t

Given a draw for ϕ_t the VAR model can be written as

$$A_t'\left(\tilde{Z}_t\right) = u_t$$

where $\tilde{Z}_t = Z_t - \sum_{l=1}^{L} \phi_{l,t} Z_{t-l} = v_t$ and $VAR\left(u_t\right) = H_t$. This is a system of equations with time-varying coefficients and given a block diagonal form for $Var(\tau_t)$ the standard methods for state space models described in Carter & Kohn (2004) can be applied.

VAR hyperparameters

Conditional on Z_t, $\phi_{l,t}$, H_t, and A_t, the innovations to $\phi_{l,t}$, H_t, and A_t are observable, which allows us to draw the hyperparameters—the elements of Q, S, and the σ_i^2—from their respective distributions

A.3 Convergence

As mentioned above, the total number of Gibbs sampling replications employed in each monte carlo replication are limited to 20,000 in order to keep the experiment computationally tractable. We assess convergence of the Gibbs sampler by constructing cumulative means of the key model parameters over the retained draws.
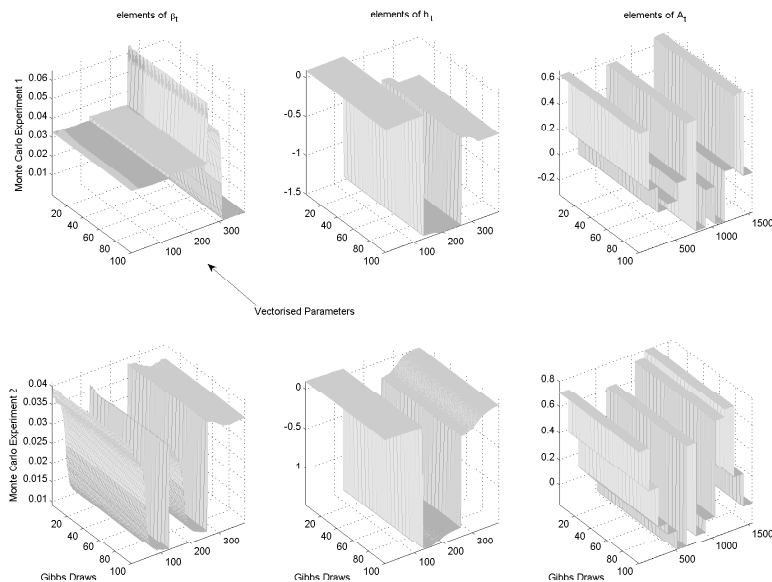
Cumulative means of Gibbs draws.

The figure above plots the average estimate of the cumulative means (i.e. averages over the Monte-Carlo draws). The cumulative means are computed for the sequence of vectorised TVC-VAR parameters over every 10 retained Gibbs draws. The figure shows that there is little fluctuation in these mean estimates. This provides some evidence for convergence of the Gibbs sampler (on average over the Monte-Carlo replications).

ABOUT THE CDMA

The **Centre for Dynamic Macroeconomic Analysis** was established by a direct grant from the University of St Andrews in 2003. The Centre funds PhD students and facilitates a programme of research centred on macroeconomic theory and policy. The Centre has research interests in areas such as: characterising the key stylised facts of the business cycle; constructing theoretical models that can match these business cycles; using theoretical models to understand the normative and positive aspects of the macroeconomic policymakers' stabilisation problem, in both open and closed economies; understanding the conduct of monetary/macroeconomic policy in the UK and other countries; analyzing the impact of globalization and policy reform on the macroeconomy; and analyzing the impact of financial factors on the long-run growth of the UK economy, from both an historical and a theoretical perspective. The Centre also has interests in developing numerical techniques for analyzing dynamic stochastic general equilibrium models. Its affiliated members are Faculty members at St Andrews and elsewhere with interests in the broad area of dynamic macroeconomics. Its international Advisory Board comprises a group of leading macroeconomists and, ex officio, the University's Principal.

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PAPERS PRESENTED AT THE CONFERENCE, IN ORDER OF PRESENTATION:

Title	Author(s) (presenter(s) in bold)
Robust Learning Stability with Operational Monetary Policy Rules	**George Evans (Oregon and St Andrews)** co-authored with Seppo Honkapohja
Who pays for job training?	**Parantap Basu (Durham)**
Electoral Uncertainty and the Deficit Bias in a New Keynesian Economy	**Campbell Leith (Glasgow)**
Sacrifice Ratio or Welfare Gain Ratio? Disinflation in a DGSE monetary model	**Guido Ascari (Pavia)**
Estimating insurance and incentive effects of labour market reforms	**Klaus Waelde (Glasgow)**
Using time-varying VARs to diagnose the source of 'Great Moderations': a Monte Carlo analysis	**Tony Yates (Bank of England)**
The Optimum Quantity of Money with Gold Reserves	**Max Gillman (Cardiff)**
Managing Disinflation Under Uncertainty	**Eric Schaling (Pretoria)**
Factor demand linkages and the business cycle: interpreting aggregate fluctuations as sectoral fluctuations	**Sean Holly (Cambridge)**
Monetary and Fiscal Rules in an Emerging Small Open Economy	**Paul Levine (Surrey)** with joint with Nicoletta Batini (IMF and Surrey) Joe Pearlman (London Met)
Can a pure Real Business Cycle Model explain the real exchange rate?	**Patrick Minford (Cardiff)**
Macroeconomic Implications of a Key Currency	**Matthew Canzoneri (Georgetown)**
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