A MATHEMATICAL PROGRAMMING MODEL FOR OPTIMAL MANAGEMENT ZONE DELINEATION IN PRECISION AGRICULTURE

By

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A Mathematical Programming Model for Optimal Management Zone Delineation in Precision Agriculture

Abstract. The identification of optimal management zones, including optimal uniform grid size, is a complex issue central to the successful implementation of variable rate input application. A novel economic optimization model is developed and applied that identifies the economically optimal management zone. Variable rate seeding can increase profits and reduce risk.

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Introduction:

One of the most fundamental issues associated with variable rate technology of precision agriculture is how to optimally configure management zones. Management zones are geographic units treated the same for a given area but which are potentially treated separately with respect to input application. A greater degree of accuracy with respect to spatial information regarding the optimal level of input (e.g., fertilizer) is desirable on as fine a scale as feasible. However, the fixed costs per zone or grid (e.g., soil sampling) associated with this greater accuracy will not, at some point, justify the additional costs of this fine scale of management. The decision of how to appropriately delineate economically optimal management zones or grid sizes represents a great opportunity to assist producers in achieving the combined goals of profit maximization and risk management. Nonetheless, this decision likewise presents a daunting and complex problem that challenges researchers, extension specialists, industry leaders and producers alike. Consequently, producers are left facing the difficult decision of how to delineate management zones without suitable guidance.

Alternatively, some producers desire to establish uniform grid size (as opposed to variably sized and shaped management zones) and face the question of the best grid size to use. Although some informal standards regarding the 2.5 acre grid size is often used, decision tools are needed to provide a more robust management zone delineation procedure and uniform grid size level projections for farmers using variable rate application of inputs. The focus of this research is based upon this very fundamental issue of precision agriculture that

is so critical to adequate economic implementation of variable rate application regardless of the production input (e.g., seed, fertilizer, pesticide).

The most appropriate method of responding to this research question is through a multidisciplinary framework representative of key elements of the decision-making environment (including economic, agronomic and engineering aspects). Specifically, a model embodying the decision-making framework of the producer will allow for proper analysis of these questions. A model that allows for the objective of maximization of profits subject to the constraints a producer faces reflects the production environment faced by the farm manager. Furthermore, the consideration of the attitude towards risk possessed by the producer is potentially influential in determining the optimal management zones or grid size. Ultimately, a farmer's decision is driven from the underlying economic (financial and otherwise) consequences of the potential courses of action being considered. In turn, the economic consequences are determined by the underlying production responses of feasible potential courses of action. Therefore, agricultural economic results drive a producer's decisions while the physical agricultural relationships (e.g., agronomic, engineering) provide the foundation for the economic results.

While the specific focus of the empirical application of this project is upon variable rate seeding for a Kentucky corn producer, the techniques developed here will be suitable for a broader audience. This research is especially relevant to all producers who use variable rate technology in that it provides a missing key element for properly assessing alternatives regarding variable rate input application.

Consequently, the long term goal of this proposed research project is to assist crop producers enhance their profitability and risk management by providing procedures and

information that will assist them in determining how to establish management zones.

Accomplishing this goal can be begun by meeting three objectives:

    1) Develop an optimization procedure which will accurately and definitively ascertain the economically optimal delineation of production management zones based on complete data,

    2) Provide an empirical application of this model, and

    3) Perform sensitivity analysis to ascertain how optimal management zones change/respond to fluctuations in the economic decision-making environment.

Background Information:

    Background information in the form of a review of literature can serve to establish a basic framework for the proposed study. Included in this will be a general discussion of economic studies pertaining to precision agriculture. Studies that examine variable rate technology, grid sampling and management zones are then discussed to complete the background information.

    The economic feasibility of precision agriculture is a common underlying question of producers considering its adoption. While the literature regarding the economic issues in the area of precision agriculture is rich with numerous studies, they are broad based and display a substantial number of philosophical discussions rather than quantitative evaluation as is common with new technologies. General philosophical discussions have ranged from historically descriptive (e.g.- Lowenberg-DeBoer; Sonka and Coaldrake) to examining the research opportunities and challenges of the future (e.g.-Weiss). Lowenberg-DeBoer and Swinton conducted a review of the economics of precision agriculture, finding that economic feasibility is dependent upon several factors including many components of the underlying economic,

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agronomic and engineering environment. Precision agriculture has been shown to be profitable, not profitable or inconclusive with mixed results, depending on the crop, inputs and conditions.

In addition to these diverse precision agriculture economic studies, three specific areas are worthy of attention for this research: variable rate technology, grid sampling and management zones. Variable rate technology research has included analysis of such components as nitrogen management (e.g., Thrikawala et al.; Babcock and Pautsch), lime application (e.g., Bongiovanni and Lowenberg-DeBoer) and spatial break-even variability assessment (English, Roberts and Mahajanashetti). Studies rely predominantly upon the use of an assumed level of grid sampling, avoiding the issue of optimal grid size or management zone determination with few exceptions (Thrikawala, Weersink and Kachanoski). Especially germane to this study is the seminal agronomic and cursory economic evaluation on variable rate seeding on corn production in Kentucky (Barnhisel et al.). The authors find that variable rate seeding as based on topsoil depth is agronomically and economically warranted. While some economic research investigates grid sampling issues (e.g., Lenz; Rehm et al.), there is a void in the literature for sound economic models to address optimal grid size which is exceeded only by the apparent lack of economic analysis in the determination of optimal management zone delineation. However, mathematical programming techniques that can appropriately address these issues are possible. One key operations research study involves optimal grouping (Gochet et al.) and may be altered in the formulation of a relevant economic optimization model.

The research proposed in this project lies at the very heart of precision agriculture. It will assist in the establishment of a fundamental framework that will permit analysis of a very germane and basic question currently plaguing the successful implementation of variable rate technology.

Specifically, how does a producer identify the optimal management zone? The innovative model formulation proposed within this research project permits the appropriate economic analysis to be conducted for comparison to the less data intensive and more farmer friendly management zone delineation procedures presently being conceived and tested by others (such as delineation by soil properties or electrical conductivity). Therefore, this research also provides complementary economic comparison among alternative procedures for management zone delineation. Additionally, the techniques include the potential economic assessment of another very important question: What is the optimal uniform grid size? Thus, while some producers will use the variably sized and shaped management zones and others will use uniform sized and shaped grids, optimal determination of both can be handled with the model proposed. Furthermore, this research will provide insights into the establishment of practical and simple decision rules.

Ultimately, this research aims at providing the missing element to permit economic comparison of alternative procedures for management zone delineation and for the improvement of these decision rules. The study proposed herein differs from other research in that it greatly expands optimal management zone delineation.

The overall purpose of this research project of assisting farmers using variable rate technology in identifying management zones may be achieved partially in a three phase approach by accomplishing the three objectives of this study. The first step involves the establishment of the economic analytical framework and is addressed in the next section on model development. The second step involves empirical application of the model. The third step involves the assessment of model results to alternative corn varieties and attitudes towards risk.

Model Development:

The initial focal point of the proposed research project is the establishment of an appropriate tool for economic analysis in assessing the relative performance of proposed alternatives for management zone delineation. This entails the development and formulation of the mathematical programming, constrained optimization economic model. Empirical application requires the estimation of the production response functions underlying the agronomic framework of the analysis as well as general data collection in support of economic investigation.

A mathematical programming model embodying the economic decision framework of a crop producer using variable rate input application may be formulated as a combined mixed integer, nonlinear programming model. Specifically, the model is:

$$MAX \ \bar{Y} - \Phi s^2_y$$

subject to:

$$(1) \quad \sum_{MZ} \sum_{S} \sum_{V} \sum_{P} CORNPROD_{MZ,S,V,P,G,ST} \leq ACRE_{G,ST} \qquad \forall G, ST$$

$$(2) \quad \sum_{MZ} \sum_{S} \sum_{V} \sum_{P} \sum_{G} \sum_{ST} LAB_{S,V,WK} CORNPROD_{MZ,S,V,P,G,ST} \leq TIME_{WK} \qquad \forall WK$$

$$(3) \quad -\sum_{MZ} \sum_{S} \sum_{V} \sum_{P} \sum_{G} \sum_{ST} EXPYLD_{YR,S,V,P,ST} * CORNPROD_{MZ,S,V,P,G,ST}$$

$$+ CORNSALE_{YR} \leq 0 \qquad \forall YR$$

$$(4) \quad \sum_{MZ} \sum_{S} \sum_{V} \sum_{P} \sum_{G} \sum_{ST} INPREQ_{I,P} CORNPROD_{MZ,S,V,P,G,ST}$$

$$- INPPURCH_I \leq 0 \qquad \forall I$$

$$(5) \quad CORNPR * CORNSALE_{YR} - \sum_{MZ} \sum_{S} \sum_{V} \sum_{P} ZCOST * IZ_{MZ,S,V,P}$$

$$- \sum_{I} INPPR_{I} * INPPURCH_{I} - Y_{YR} = 0 \qquad \forall\, YR$$

$$(6) \quad \sum_{YR} \frac{1}{N} Y_{YR} - \bar{Y} = 0$$

$$(7) \quad \sum_{S} \sum_{V} \sum_{P} IZ_{MZ,S,V,P} \leq 1 \qquad \forall\, MZ$$

$$(8) \quad \sum_{G} \sum_{ST} CORNPROD_{MZ,S,V,P,G,ST} - M * IZ_{MZ,S,V,P} \leq 0 \qquad \forall\, MZ,S,V,P$$

$$(9) \quad \sum_{MZ} \sum_{G} \sum_{P} TOTAC_{SHAL} CORNPROD_{MZ,S,V,P,G,DEEP}$$

$$- \sum_{MZ} \sum_{G} \sum_{P} TOTAC_{DEEP} CORNPROD_{MZ,S,V,P,G,SHAL} = 0 \qquad \forall\, S,V$$

$$(10) \quad \sum_{S} \sum_{V} \sum_{P} \sum_{G} \sum_{ST} CONT_{G',G} CORNPROD_{MZ,S,V,P,G,ST} - M * IONE_{MZ} \leq 0 \quad \forall\, MZ,G'$$

$$(11) \quad \sum_{S} \sum_{V} \sum_{P} \sum_{G} \sum_{ST} CORNPROD_{MZ,S,V,P,G,ST} + M * IONE_{MZ} \leq$$

$$M + \sum_{ST} ACRE_{G1,ST} \qquad \forall\, MZ$$

The objective function of this model will be to maximize risk adjusted net farm returns above selected relevant costs in a typical expected value-variance framework. Decision variables will include the following:

$\overline{Y}$ = expected net returns above variable cost (mean across years)

Y_{YR} = net returns above variable cost by year (net returns)

$CORNPROD_{MZ,S,V,P,G,ST}$ = production of corn in management zone MZ under sowing S, variety V, population P at location grid G on soil type ST

$SALES_{YR}$ = quantity of corn sold by year

$INPPURCH_I$ = purchase of input I

$IZ_{MZ,S,V,P}$ = binary (0-no,1-yes) decision variable of whether or not to include sowing S, variety V and population P under management zone MZ

$IONE_{MZ}$ = binary (0-no,1-yes) decision variable of whether or not to include grid in a zone needed for imposing appropriate (spatial continuity) constraints

Constraints include the following:

(1) Land resource limitation

(2) Labor resource limitations by week

(3) Sales balance by year

(4) Input balance by input

(5) Profit balance by year

(6) Expected profit balance

(7 and 8) Limitation of only one production practice per management zone

(9) Balance of field average production practices (non variable rate) across the field

(10) Management zones must either have a contiguous member or

(11) Management zones are limited to only one grid if not contiguous to another grid

coefficients include:

Φ	= Pratt risk-aversion coefficient
$ACRE_{G,ST}$	= Acreage available by grid G and soil type ST with G1 being the largest grid and TOTAC being the sum
$LAB_{S,V,WK}$	= Labor requirements for corn production using sowing S and variety V by week WK
$TIME_{WK}$	= Available field days per week
$EXPYLD_{YR,S,V,C,P,ST}$	= Expected yield of corn by year YR using sowing S, variety V, population P on soil type ST
$INPREQ_{I,P}$	= Input requirement by input for population P
ZCOST	= Cost of sampling to develop a management zone
CORNPR	= Price of crop in dollars less dependent costs (hauling)
$INPPR_I$	= Price of input I
$CONT_{G,G'}$	= A matrix indicating whether or not grid locations are beside each other (1 for the grid in question, -1 if contiguous, 0 otherwise)
M	= A large number following the traditional "Big M" modeling approach

indices include:

S	= Sowing date
V	= Variety
P	= Population
G,G'	= Grid or cell location
MZ	= Management zone
ST	= Soil type (SHAL for Shallow and DEEP)
I	= Input

WK   = Week

YR   = Year

While the actual application initially undertaken is for a Kentucky producer determining

optimal seeding rates by management zone, the economic optimization model may generally apply

to other areas, many enterprises and any possible factor of production (e.g., other fertilizers,

pesticide). In addition to optimal application rates of seeding rate by management zone, the

Kentucky crop producer economic model will incorporate selection of the number, position and

size of management zones using soil depth information from each grid. It should be noted that the

model could allow for the proper identification of the economically optimal management zones for

each individual input applied; the best management zone for one input will not restrict derivation

of the best management zone for the other input.

Production response data for each individual cell (small location of the farm field) will be

required. Specifically, a spatially dependent production function with crop yield as a dependent

variable and seeding rate as a determining factor (independent variables) is needed. Data required

therefore include yield results by grid area. The yield results were taken from Barnhisel et al. and

include yield results for a high (26,000 plants/acre) and low (20,000 plants/acre) population for

shallow and deep soils using a DeKalb and Pioneer variety for a Hardin county farm for the years

of 1993-1995 and are presented in Table 1. The labor requirements per week, input prices and

input requirements per acre were taken from representative Tennessee no-till enterprise budgets

(Gerloff and Maxey). The 1995-1999 Kentucky average season corn price was used ($2.64/bu,

Kentucky Agricultural Statistics 1998-1999). A hauling charge of $0.15/bu was subtracted. The

cost of sampling to establish zones was estimated at $2.00 per zone and assumed to be

reevaluated every ten years. A Shelby county field, also in central Kentucky, was used to depict available acreage by soil depth. A smaller subsection of the field was used given the difficulties of the software in solving the model as discussed later. The available acreage is 0.25 acres per grid given the data is for 100 ft by 100 ft grids. Topsoil depth by location is presented in Table 2 and was broken into shallow (<=6 inches topsoil) and deep (> 6 inches topsoil) soil.

A suitable field days simulation model was used to estimate the number of days suitable for fieldwork. This model relies upon historical weather data and soil water simulation under a modified procedure discussed by Dillon, Mueller and Shearer.   The vector of the field days available appeared as the weekly right-hand side values in the mathematical programming model; the average weekly days available for Shelby county was 5.35 with a standard deviation of 2.68.

The sensitivity of the net returns and the chosen optimal management zone to relevant changes as discussed later is undertaken through alterations in the economic model. Comparison of economic performance and especially the changes in the delineation of optimal management zones will provide insights into how robust this important decision is to fluctuations in the decision-making environment.

The sensitivity of economic results associated with different attitudes towards production risk would also be appropriate. In order to represent the economic decision-making framework including yield risk, an expected value-variance (E-V) framework that incorporates risk-adjusted net returns above selected costs is used. E-V, or mean-variance, analysis is a widely used and accepted method for analyzing risk. Consequently, the ability to configure optimal management zones in order to manage risk will be expressly examined. The sufficiency conditions under which the use of E-V is consistent with expected utility theory include one of the following: (1) normal

distribution (Freund), (2) if the distributions of net returns associated with the decision variable differ only by location and scale (Meyer) or (3) if the utility can be approximated by a quadratic function (Markowitz).

The estimation of the risk aversion coefficient will be undertaken using a procedure developed by McCarl and Bessler. The objective function maximizes the certainty equivalent of net returns which is net returns less the product of Pratt risk-aversion function coefficient and the variance of net returns (σ^2_y). The Pratt risk-aversion function coefficient is a measure of a hypothetical producer's aversion to risk. McCarl and Bessler use a procedure to estimate this coefficient wherein a producer is said to maximize the lower limit from a confidence interval of normally distributed net returns. The resultant general formula for calculating the risk aversion parameter is:

$$\Phi = 2Z_a / S_y$$

where Φ = risk-aversion coefficient, Z_a= the standardized normal Z value of a level of significance and S_y= the relevant standard deviation from the risk-neutral profit maximizing base scenario.

Results and Analysis:

Preliminary results using the model formulated provided managerial insights regarding optimal management zone delineation but also met with difficulties encountered by available software solvers. Initial results for the subsection of the field are presented followed by a discussion of general observations.

The net returns results are shown in Table 3. The risk neutral solution demonstrated the lack of desirability for management zones with a field average approach under one management zone being used. Expected net returns above variable costs were $831.68 for the field with a C.V. (Coefficient of Variation) of 33.13%. The Pioneer hybrid was selected with a low (20,000 plants/acre) plant population level chosen (Table 4). These production practices resulted in an expected corn yield of about 114 bu/acre. The selection of the low plant population for both shallow and deep soils was anticipated given the greater yield performance of this population level on both soil depths for this variety.

When an attitude of risk aversion (assuming a 60% significance level as discussed above for the McCarl and Bessler approach), the expected net returns above variable costs decline slightly to 96.22% of optimal to a level of $800.25 (Table 3). The risk of this strategy is considerably lower than the 33.13% C.V. with a new C.V. of 18.74%. Each logical contiguous grouping of grid locations within the field was selected as a distinct management zone with three total zones being used. Shallow soils employed the low plant population and deep soils used a high (26,000 plants/acre) population level as consistent with Barnhisel et al. as shown in Table 4. The DeKalb variety is selected as a more stable yielding variety albeit with a slight reduction in expected yield (about 113 bu/ac). This indicates that variable rate seeding under optimal management zones offers the potential of production risk reduction under the right circumstances.

Sensitivity experiments which eliminated the Pioneer variety for risk neutral or risk averse cases were conducted to investigate variety dependence of results reflecting the fact that producers face different variety requirements regarding disease resistance and other unique factors. The sensitivity results exactly paralleled the risk averse results for the unrestricted

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scenario (Tables 3 and 4). This demonstrates that the results are variety dependent and that profit maximization would dictate variable rate seeding under appropriate circumstances.

The economic results are obviously heavily dependent on the underlying production functions used. Sensitivity to the yield results was examined by incorporating biophysical simulation results from Dillon, Mueller and Shearer for Shelby county corn production under alternative sowing, maturity length of variety and plant population for shallow and deep soils. Results are not directly presented here but several observations are noted. First, the use of management zones is dependent upon both the sampling cost of establishing an additional zone as well as the comparison of marginal revenue from increased yield to marginal costs of additional inputs as expected. This means that even a low sampling cost may not result in multiple management zones if the marginal costs of seed does not results in adequate yield differences. Results are therefore also sensitive to output (corn) price. Furthermore, there is a complex interaction between nonvariable production practices (variety and sowing date) and variable production practices (seeding rate). The relative portions of soil available, as well as their location, is influential to resulting net returns statistics as well as the optimal management zone delineation and production practice selection.

The initial solution of the model was hampered by its complexity of including mixed integer and nonlinear elements. Even the risk neutral scenario which simplifies into a mixed integer programming model faced difficulties in being solved by the available GAMS software whether using XA or OSL solvers especially with larger problems or greater numbers of permitted management zones. Previous experience of the author and others demonstrate the difficulty of obtaining empirical solutions for mixed integer programs (MIPs) despite a sound

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formulation. Further exploration of mechanisms and alternative formulations and procedures to

assist the solution of larger MIPs is warranted. Nonetheless, this formulation did work

successfully and the potential of this economic model for precision agriculture is substantial.

Summary and Conclusions:

The very concept of precision agriculture is based on the ability to manage factors of

production differently according to spatial variability. This in turn requires a need to identify

appropriate management zones. Therefore, the identification of the economically optimal

management zones and including optimal uniform grid size, is a complex issue central to the

successful implementation of variable rate input application. Nonetheless, while this vastly

important decision has alluded experts, a novel modeling procedure that both identifies the

economically optimal management zone or grid size and permits economic comparison of

alternative decision rules to determine such zones is presented in this research. A multidisciplinary

(agricultural economics, agricultural engineering, agronomy) approach is embedded in a

mathematical (including nonlinear and integer) programming model which maximizes risk adjusted

net returns. Actual comparison of alternative decision rules is possible from the economic model

developed. It is hoped that model results will aid in improving management zone delineation rules

and lead to the development of farm level decision rules including risks faced by producers.

Consequently, the research proposed in this project lies at the very heart of precision agriculture,

especially that of variable rate technology.

Empirical results indicate that variable rate seeding of corn may reduce production risk

and may be profit maximizing depending on underlying conditions. Interactions with other

production practices such as variety and sowing date are critical in the use of variable rate seeding

as well as the optimal management zone delineation. Results are sensitive to the underlying

production function, economic environment (e.g., cost of sampling to establish a new

management zone, output price) and the soil resource available with regard to proportions and

spatial proximity.

The mixed integer, nonlinear model does face some difficulties in being solved by the

software used by the investigator. The large model size and number of management zones seem

especially relevant to this concern. Further improvement regarding alternatives to assist in the

solution of these models is warranted; the formulation does display the potential for substantial

contribution in optimal management zone delineation nonetheless.

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Table 1. Corn Yield Results by Plant Population, Variety and Year (bu/ac)

Seeds/acre	Topsoil Depth[1]	-----DeKalb 646----				----Pioneer 3140----		
		1993	1994	1995		1993	1994	1995
20,000	Shallow	186	88	79		98	92	68
20,000	Deep	133	133	79		150	136	108
26,000	Shallow	84	96	52		79	102	50
26,000	Deep	139	140	115		136	134	112

[1] Shallow refers to $< = 6$ inches, Deep refers to > 6 inches regarding topsoil.

Table 2. Soil Map Depicting Acreage Available by Grid Location and Soil Depth[1]

Row			Column			
---	A	B	C	D	E	F
1	S	D	D	S		
2		S	D	S		
3		S	D	S		
4		S	S	S		
5			D	D	D	D
6			D	D	D	D
7			D	D		

[1] S refers to shallow, D refers to deep. Specifically, shallow refers to $< = 6$ inches, Deep refers to > 6 inches regarding topsoil.

Table 3. Net Returns Results by Risk Attitude and Variety Allowed

Statistic	Both Varieties		No Pioneer	
	Risk Neutral	Risk Averse	Risk Neutral	Risk Averse
Mean ($)	831.68	800.23	800.23	800.23
Percent of Optimal (%)	100.00	96.22	96.22	96.22
Standard Deviation (%)	275.58	149.98	149.98	149.98
C.V. (%)	33.13	18.74	18.74	18.74
Maximum ($)	1062.40	896.60	896.60	896.60
Minimum ($)	526.54	627.43	627.43	627.43

Table 4. Production Practices Results by Risk Attitude and Variety Allowed

Item	Both Varieties		No Pioneer	
	Risk Neutral	Risk Averse	Risk Neutral	Risk Averse
Variety	Pioneer	DeKalb	DeKalb	DeKalb
Shallow Soil Population[1]	Low	Low	Low	Low
Deep Soil Population[1]	Low	High	High	High
Management Zones	One	Three	Three	Three
Corn Yield Average (bu/ac)	113.61	112.94	112.94	112.94

1. Low plant population refers to 20,000 plants/acre and high plant population refers to 26,000 plants per acre. Shallow refers to < = 6 inches, Deep refers to > 6 inches regarding topsoil.