CORE
Metadata, citation and similar papers at core.ac.uk
Provided by Research Papers in Economics

Seminar Paper No. 729

SOCIAL SECURITY AND THE EQUITY PREMIUM PUZZLE

by

Conny Olovsson

INSTITUTE FOR INTERNATIONAL ECONOMIC STUDIES
Stockholm University

Seminar Paper No. 729

The Welfare Gains of Improving Risk Sharing in Social Security

by
Conny Olovsson

Papers in the seminar series are published on the internet
in Adobe Acrobat (PDF) format.
Download from http://www.iies.su.se/

Seminar Papers are preliminary material circulated to
stimulate discussion and critical comment.

March 2004
Institute for International Economic Studies
S-106 91 Stockholm
Sweden

Social Security and the Equity Premium Puzzle

Conny Olovsson*

March 16, 2004

Abstract

This paper shows that social security may be an important factor in explaining the equity premium puzzle. In the absence of shortselling constraints, the young shortsell bonds to the middle-aged and buy equity. Social security reduces the bond demand of the middle-aged, thereby restricting the possibilities of the young to finance their equity purchases. Their equity demand increases as does the average return to equity. Social security also increases the covariance between future consumption and the equity income of the young. The effect on the equity premium is substantial. In fact, a model with social security and borrowing constraints can generate a fairly realistic equity premium.

JEL Classification G12 H55

Keywords: Asset prices, the equity premium puzzle, social security

1 Introduction

In 1985, Mehra and Prescott shed light on the question as to why the historical equity premium is so high. They showed that the equity premium generated in a representative consumer framework is 0.35 percent at most, in contrast to the historical 6 percent in the U.S.[1] This paper takes a life-cycle approach and analyzes whether the pay-as-you-go (PAYGO) social security system could be an important factor in the explanation of the so-called equity premium puzzle. The analysis is partly motivated by the equity premium having been substantially higher since the introduction of the current U.S. PAYGO system in 1935. According to Mehra and Prescott (2003), the premium in the U.S. was 3.92 percent for the period 1889-1933, and 8.93 percent for the period 1934-2000.

*Institute for International Economic Studies, Stockholm University, 106 91 Stockholm, Sweden, e-mail: conny@iies.su.se.
I am grateful for comments from Kjetil Storesletten, Mats Persson and Per Krusell. Thanks to Christina Lönnblad for editorial assistance. Financial support from Jan Wallander's and Tom Hedelius' Foundation is gratefully acknowledged.

[1] Mehra and Prescott (1985).

The potential importance of social security is mainly due to the considerable size of the program: in 2001, social security was the major source of income (providing at least 50 percent of total income) for 65 percent of the aged beneficiaries, and it was the only source of income for 20 percent of the aged population in the U.S.[2] In addition, social security is a non-traded asset containing several elements of aggregate risk, thereby affecting the financial behavior of both taxpayers and retirees.

This paper sets up a simple three-period overlapping generations (OLG) model, where agents work for two periods and retire in the third, and shows that social security unambiguously increases the equity premium. Two social security arrangements with defined benefits are considered in this paper. First, the U.S. system where agents receive wage-indexed benefits based on their past income and second, the more standard arrangement in macroeconomic models with constant and completely safe benefits. The effect of social security on the equity premium is found to be substantial: up to 83 percent higher with social security than without. Since equity in the model is a claim to all risky capital in the economy, a realistic equity premium might be about 3 percent.[3] The model with wage-indexed benefits based on past income, borrowing constraints and a relative risk aversion (RRA) coefficient of 6 generates an equity premium of 2.31, which is rather close to the target.

It turns out that the exact effect of social security on the equity premium to a large extent depends on whether agents are subject to shortselling constraints. In the absence of such constraints, social security influences the equity premium by changing the relative demand for equity. When agents do not receive social security, the young shortsell bonds and buy the bulk of the equity stock.[4] Their marginal valuation of equity is therefore decisive in the pricing process, and since they value equity highly, the return to it is low. The middle-aged mainly hold bonds, but the difference in expected return between the two assets is small. When agents do receive social security, the need to save for retirement is reduced. In addition, since social security is a relatively safe asset, it reduces the motive to hold bonds for retirement. Consequently, middle-aged agents' demand for bonds goes down. The lower demand for bonds effectively

[2] Fast Facts and Figures about Social Security (2003).

[3] Following Constantinides et al (2002).

[4] The large equity demand of the young is explained by the fact that they will receive some wage income in the next period, wheras the middle-aged will receive zero exogenous income. The covariance between future consumption and equity income is thus initiallty lower for the young than for the middle-aged. In the trading process, the middle-aged demand bonds and the young demand equity to reduce this discrepancy.

restricts the possibilities of the young to finance their equity purchases by shortselling bonds. Therefore, they demand less equity and the price of equity goes down (and the return goes up).

The main influence of social security in the presence of shortselling constraints is due to the fact that social security significantly increases the covariance between future consumption and equity income for the young.[5] At the same time, the covariance between future consumption and equity income for the middle-aged is relatively unaffected by social security. The higher covariance is first of all due to the arrangement with defined benefits featuring contra-cyclical taxes. More specifically, the government may be forced to increase taxes when wage income is low, and vice versa. Since equity and wage income are highly correlated, the young find equity to be a worse hedge in economies with social security, since it does not pay off when really needed. Their marginal valuation of equity will therefore be lower, whereas the marginal valuation of the middle-aged remains relatively unchanged. The result is a lower equilibrium price and a higher average return.

This paper adds to the literature on the importance of life-cycle effects for the equity premium. Earlier findings are that the equity premium is generally high when the equity is mainly priced by the middle-aged who face a high covariance between their future consumption and equity income. The marginal valuation of equity is low for this group, resulting in a high expected return to equity. A natural question is then why the equity would be priced by the middle-aged. Constantinides et al (2002) argue that the young would like to shortsell bonds and invest in equity, but financial frictions such as shortselling constraints may prevent them from this. Storesletten et al (2001), on the other hand, argue that the young actually choose not to hold equity, since they find it too risky. This paper offers an explanation in-between the two.[6] As in Constantinides et al (2002), the young would like to shortsell bonds and invest in equity. However, social security reduces the middle-aged agents' demand for bonds and thus, the amount of bonds the young can shortsell. As a result, the young choose to reduce their equity demand.

Finally, even though the model with social security and shortselling constraints may be viewed as

[5] Recall that the Consumption Capital Asset Pricing Model (C-CAPM), predicts that the price of an asset should depend on the covariance between future consumption and the return to that asset. An asset paying off in states of high marginal utility of consumption commands a higher price than one paying off in states of low marginal utility of consumption. Moreover, marginal utility of consumption varies inversely with consumption, implying that the asset paying off in states of low levels of consumption command a higher price than the asset paying off in states of high levels of consumption.

[6] Both Constantinides et al (2002) and Storesletten et al (2001) abstract from social security.

successful, neither shortselling constraints nor social security alone found to generate an equity premium of realistic size. This paper is also related to Abel (2003), who analyzes the price of capital in the presence of social security; and to the large literature on the effects of social security on individual and aggregate savings.[7]

2 The Economic Model

2.1 The Consumers

The basic model used is an overlapping-generations, pure exchange economy where each generation lives for three periods as young, middle-aged and old, respectively. Each generation is modeled by a representative consumer. There is one consumption good in each period, which perishes at the end of the period. Wages, consumption, dividends and coupons as well as bond and equity prices are denominated in units of the consumption good. The wage income process in the economy is stochastic and exogenous, so I am abstracting from the labor-leisure trade-off. The index $i = 0, 1$ and 2 is used to denote the young, the middle-aged and the old, respectively. A consumer born in period t receives the wage income $w_{t,0} > 0$ when young, $w_{t+1,1} > 0$ when middle-aged and a social security benefit $\widehat{\varphi}_{t+2,2} \geq 0$ when old.

There are two types of securities in the economy, bonds and equity. Both are in unit supply and infinitely lived. The bond is default-free and pays a fixed coupon $b > 0$ in every period in perpetuity. With this set-up, the bond can be considered as a proxy for long-term government debt. The *ex coupon* bond price in period t is denoted by q_t^b, and it is the price of the claim to the coupon b paid in perpetuity beginning in period $t + 1$.

The equity is a claim to a dividend stream in perpetuity and pays a net dividend d_t in period t. The *ex dividend* price of equity in period t is denoted by q_t^e, and it is the price of the claim to a perpetual dividend stream, beginning with period $t + 1$. Equity is here considered as the total sum of the claims to firms and real estate.

The consumer born in period t has zero endowment of assets. This consumer makes a portfolio

[7] See, for instance, Feldstein (1974, 1996), who argue that social security crowds out private savings; and Hugget (1996), Hugget & Ventura (2000) and Domeij & Klein (2002) who argue that social security can explain a large degree of the heterogeneity in saving rates between households, and thereby many of the features of the wealth distribution.

decision $z_{t,0} = \left(z^b_{t,0}, z^e_{t,0}\right)$ when young; adjusts this decision to $z_{t+1,1} = \left(z^b_{t+1,1}, z^e_{t+1,1}\right)$ when middle-aged; and sells the portfolio in period $t+2$, when old. As usual, a negative position in bonds or stocks denotes a short position in that asset.

In period t, the young consume $c_{t,0}$, the middle-aged consume $c_{t,1}$ and the old consume $c_{t,2}$. Denote the tax rate by θ_t, and the social security benefit received by the old by φ_t. The budget constraints in period t are then given by

$$c_{t,0} + z^b_{t,0} q^b_t + z^e_{t,0} q^e_t + \leq w_{t,0} \left(1 - \theta_t\right) \tag{1}$$

$$c_{t,1} + z^b_{t,1} q^b_t + z^e_{t,1} q^e_t \leq w_{t,1} \left(1 - \theta_t\right) + z^b_{t-1,0} \left(q^b_t + b\right) + z^e_{t-1,0} \left(q^e_t + d_t\right) \tag{2}$$

$$c_{t,2} \leq \widehat{\varphi}_t + z^b_{t-1,1} \left(q^b_t + b\right) + z^e_{t-1,1} \left(q^e_t + d_t\right), \tag{3}$$

where $(1), (2)$ and(3) are the budget constraints faced by the period t young, middle-aged and old, respectively, and $\widehat{\varphi}_t$ are net social security benefits.

Period utility takes the following form

$$u\left(c\right) = \frac{c^{1-\gamma} - 1}{1 - \gamma}, \tag{4}$$

where $\gamma > 0$ is the constant coefficient of relative risk aversion. For an agent of age $i \in \{0, 1\}$, the Bellman equation to the consumer's problem is then given by

$$V_i \left(z_{t,h}\right) = \max_{c_t, z^b_{t,i}, z^e_{t,i}} u\left(c_{t,i}\right) + \beta E_t V_{i+1} \left(z_{t+1,i+1}\right), \tag{5}$$

subject to the relevant budget constraints given by (1)-(3), where β is the subjective discount factor.

It is also assumed that

$$V_{t,3} \equiv 0 \ \forall t,$$

which implies that the old do not buy any assets (and that altruistic bequests are ruled out).

Aggregate income is defined as

$$y_t = w_{t,0} + w_{t,1} + b + d_t. \tag{6}$$

Following Constantinides et al (1998), the definition of income includes the (constant) coupon payment on government debt.[8]

2.2 The Government Sector

Three different social security arrangements are considered in this paper, beginning with the benchmark case with no social security. Naturally, the government sector is then completely abstracted from, so both taxes and benefits are set to zero. Second, I deal with the U.S. Social Security System with wage-indexed benefits based on past income and third and finally, the case with completely safe benefits. The last two cases are discussed and modeled in detail below.

2.2.1 The U.S Social Security System

The U.S. Social Security System is basically a PAYGO system, where the government collects taxes and pays out wage-indexed benefits, based on past income to the currently old. The system features defined benefits. More specifically, there are basically three important factors determining the social security benefits received by agents in the U.S.: the agent's *average income*, *the replacement ratio* and the evolvement of *average wages*.[9] The first factor is important because the level of benefits when reaching the retirement age is based on lifetime earnings. For each worker, there is a calculation of the worker's Average Indexed Monthly Earnings (AIME) - an average of a worker's earnings over the best 35 years of her career. One period is assumed to be 20 years in the model and each worker supplies labor for two periods, i.e. a total of 40 years. For simplicity, averages are computed over these 40 years

[8] This is somewhat non-standard, but the assumption allows me to completely abstract from the government and thus from taxes to finance the debt in the no-social security version. In any case, the interest on government debt in the U.S. is about 3 percent of GDP and the calibration remains essentially unchanged notwithstanding if the definition of GDP includes the term b.

[9] See, for instance, the *Social Security Handbook.*

rather than just the 35 best years. The average income history of an agent aged i is denoted by $\psi_{t,i}$. Formally, $\psi_{t,2} \equiv \frac{w_{t-2,0}+w_{t-1,1}}{2}$ is the average (pre-taxed) lifetime earnings of the period t old. The second determinant of the social security benefits is the replacement ratio, i.e. the rate at which social security replaces past earnings. This parameter will be denoted by η. Finally, the rate of return on social security is related to the aggregate wage-index, i.e., the evolvement of aggregate labor income. More specifically, an individual's earnings are indexed to the average wage level at the time of retirement.[10] The variability of this index over time influences the variability of benefits, and its comovement with other assets will be important for portfolio composition. The (pre-taxed) social security benefit of an agent retiring in period t is then

$$\varphi_t = \eta \psi_{t,2} \widehat{W}_t \tag{7}$$

where

$$\widehat{W}_t = \frac{W_t}{E\left[W\right]}. \tag{8}$$

Thus, if the wage rate in period t is above its unconditional mean, benefits are increased proportionately.

The government is required to balance its budget in each period. Budget balance in combination with defined benefits implies that the tax rate will be a stochastic variable, taking on whatever value is needed to keep the government budget in balance. In contrast to the complex means by which taxable benefits are determined in the U.S., benefits are, for simplicity, taxed at the period-t income tax rate θ_t. The government budget constraint is then

$$\theta_t \left(w_{t,0} + w_{t,1} + \varphi_t\right) \geq \left(1 - \theta_t\right) \varphi_t. \tag{9}$$

It is then straightforward to use (7), (9) and the fact that the government balances the budget to

[10] Actually, an individual's earnings are indexed to the average wage level 2 years prior to the year of eligibility, i.e., when the agent attains the age of 62.

write the tax in period t as

$$\theta_t = \frac{\eta\psi_{t,2}\widehat{W}_t}{w_{t,0} + w_{t,1} + 2\eta\psi_{t,2}\widehat{W}_t}. \tag{10}$$

2.2.2 Safe Benefits

A natural benchmark may also be the case with completely safe benefits, more specifically, a setting where all old agents receive the same benefit and, consequently, do not face any uncertainty in their exogenous income. This is probably the most common assumption when modeling social security in macroeconomic models. Since a set-up with defined benefits is considered, taxes are stochastic, and since benefits are assumed to be constant, they cannot be taxed. Instead, retirees receive a benefit equal to the unconditional expected value of the after-taxed benefit in the model with wage-indexed benefits based on past income. The completely safe benefit is given by

$$\widehat{\varphi}_t = \varphi = E\left[\left(1 - \theta_t^{W.I}\right)\eta\psi_{t,2}\widehat{W}_t\right], \tag{11}$$

where $\theta_t^{W.I}$ is the period-t tax rate in the economy, with wage-indexed benefits. Finally, taxes paid by the young and the middle-aged are then given by

$$\theta_t^S = \frac{\varphi}{w_{t,0} + w_{t,1}}. \tag{12}$$

2.3 Shortselling Constraints

Implications for equity prices are analyzed both with and without shortselling constraints. First, Constantinides et al (2002) have argued that shortselling constraints may have important implications for asset prices, and the equity premium in a life-cycle setting. Second, borrowing constraints can be motivated on grounds of realism: it is a well known fact that human capital alone does not collateralize major loans in modern economies, for reasons of moral hazard and adverse selection. The borrowing-unconstrained economies, on the other hand, may be viewed as natural benchmark cases. In short, in borrowing-constrained economies, agent $i \in \{0, 1\}$ is subject to the following two additional constraints

$$z_{t,i}^{b} \geq 0 \text{ and } z_{t,i}^{e} \geq 0 \ \forall t. \tag{13}$$

3 Equilibrium

Instead of specifying joint stochastic processes for wage income and dividends $(d_t, w_{t,0}, w_{t,1})$, I specify processes for aggregate income and wage income $(y_t, w_{t,0}, w_{t,1})$. More specifically, the joint process of the detrended aggregate income and the wage income of the young and the middle-aged is modeled as a time-stationary process. Since each of these variables will only be allowed to take on a finite number of values, the triple $(y_t, w_{t,0}, w_{t,1})$ can be represented by the state variable $s_t = j$, $j \in J$. This variable is modeled as a Markov process with a non-degenerate, unique, stationary probability distribution with the transition probability matrix $\pi = (\pi_{ij})$, and is referred to as the "income state". It is then straightforward to represent a stationary equilibrium, wherein decisions made in period t and prices in period t are functions of the current income state j, and the *one period lag* of the investment decisions of the middle-aged in period-t $z_{-1} \equiv \left(z_{-1}^{b}, z_{-1}^{e}\right)$. However, in the economy with wage-indexed benefits based on past income, agents also need to keep track of the average income history of the currently old $\psi_2(j_{-1}, j_{-2})$ and middle-aged $\psi_1(j, j_{-1})$.

A stationary equilibrium of this economy is given by time-invariant decision rule for agents' asset holdings, $z_0^b(j, z_{-1}, \psi_2, \psi_1)$, $z_1^b(j, z_{-1}, \psi_2, \psi_1)$, $z_0^e(j, z_{-1}, \psi_2, \psi_1)$ and $z_1^e(j, z_{-1}, \psi_2, \psi_1)$, such that the following conditions hold**:**

- The decision rules solve agents' maximization problem, given by (5)
- $z_0^b + z_1^b = 1$ and $z_0^e + z_1^e = 1$
- The government budget constraint (9) is satisfied with equality

The first condition ensures that each consumer's consumption and investment policy maximizes her expected utility from the set of admissible policies (while taking the price process as given) and the second condition ensures that the bond and equity markets clear in all periods. Note that by Walras' law, condition (2) ensures that goods markets clear.

Naturally, in the presence of shortselling constraints, the following additional conditions must also have to be satisfied: $z_i^b \geq 0$ and $z_i^e \geq 0 \qquad i = 0, 1, 2.$

4 Numerical Computation of the Equilibrium

In the calibration, y will only be allowed to assume four different values. In addition, since it is computationally costly to keep track of income histories, w_0 and w_1 are only allowed to take on two values each. More specifically, I assume the following aggregate structure:

$$J = \begin{cases} \left(\overline{y} - v\right), \left(\overline{w}_i \left(1 - \zeta\right)\right) \\ \left(\overline{y} - \frac{v}{4}\right), \left(\overline{w}_i \left(1 - \zeta\right)\right) \\ \left(\overline{y} + \frac{v}{4}\right), \left(\overline{w}_i \left(1 + \zeta\right)\right) \\ \left(\overline{y} + v\right), \left(\overline{w}_i \left(1 + \zeta\right)\right) \end{cases} \quad i = 0, 1,$$

where $\overline{y}$ is the average aggregate income and $\overline{w}_i$, the average wage income of an agent aged i. Since wages of the young and middle-aged both can be high and low, it follows that ψ_2 is a variable with four possible values. Finally, for a given realization of $j \in J$, the variable ψ_1 can attain two different values, resulting in a total of 32 different exogenous states.[11] The stochastic process is assumed to be *i.i.d.* over time. Although it is well established that aggregate productivity shocks are highly autocorrelated at annual and quarterly frequencies, there does not seem to exist any conclusive evidence indicating such positive serial correlation at generational frequencies (i.e. 20-30 year periods).[12] As a benchmark, aggregate shocks in the model are therefore uncorrelated across time. Moreover, it is assumed that $\pi_1 = \pi_4$ and $\pi_2 = \pi_3$.

Since no closed-form solutions for portfolio policy functions and pricing functions exist, these functions are approximated by B-splines of order 4 (i.e. piecewise cubic polynomials). The system of equations is solved with a Gauss-Jacobi approach.

[11] The numerical approximation then features more than 40000 unknowns.

[12] This assumption is also in line with several other papers analyzing OLG-models with two or three periods. See, for instance, Bohn (1999) and Smetters (2002).

5 Calibration

The preference parameters are the RRA coefficient, γ and the subjective discount factor, β. Results are presented for the values $\gamma = 4$ and 6 of the RRA coefficient. The discount factor is set to 0.44 for a period of 20 years. This corresponds to an annual subjective discount factor of 0.96, which is standard in the macro-economic literature.

In the U.S., the current payroll tax is 12.4 percent, and the replacement ratio is 44 percent.[13] The ratio of retired individuals to working people is somewhat higher in the model (0.5) than in the U.S. today (0.25), making it difficult to simultaneously match the payroll tax and replacement ratio. I set out to match the empirically observed replacement ratio of 44 percent, i.e. $\eta = 0.44$. The expected tax rate needed to finance this ratio is $E(\theta) = 0.152$.

There are 6 additional parameters to be determined: $\overline{y}, v, \overline{w}_0, \overline{w}_1, \zeta, \pi_1$.[14] These parameters are chosen to satisfy the following 6 target statistics.

(*i*) **The average share of income going to labor**, $E\left(\frac{w_0+w_1}{y}\right)$. This statistic was set to be in the interval $0.66 - 0.75$, consistent with US historical experience.

(*ii*) **The average share of the wage income going to the young** $\frac{w_0}{w_1}$. Actual income profiles from Storesletten et al (2003) are used to calibrate this statistic to be 0.75.[15] Since $w_{t,0}$ and $w_{t,1}$ are assumed to be perfectly correlated, the ratio $\frac{w_0}{w_1}$ equals 0.75 in each period.

(*iii*) **The average share of income going to interest on government debt**, $\frac{b}{E(y)}$. This was set at 0.03, consistent with US historical experience.

(*iv*) **The coefficient of variation of the 20-year aggregate income,** $\frac{\sigma(y)}{E(y)}$. This statistic is rather problematic to calibrate since even a century-long time series only provides five non-overlapping observations, resulting in large standard errors of the point estimates. I follow Constantinides et al (2002), and calibrate this statistic to be 0.25.

[13] See, for example Mchale (1999).

[14] Since $\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$, the transition matrix is uniquely defined by π_1.

[15] Profiles are estimated from PSID data.

(*v*) **The coefficient of variation of the 20-year aggregate wage** $\frac{\sigma(W)}{E(W)}$. This vital statistic represents another calibration challenge for **the reasons** mentioned above. Against this background, I once more stay close to Constantinides et al (2002), and calibrate the coefficient of variation of the 20-year aggregate income to be 0.20.

(*vi*) **The cross-correlation of aggregate labor income and aggregate income** $corr\,(y_t, w_t)$. When the production process is explicitly modelled, the standard assumption is the Cobb-Douglas production function with its constant capital and labor shares. With a single productivity shock**,** the implications are that labor, capital and aggregate income are all perfectly correlated. The Cobb-Douglas production function has strong empirical support in the long run. According to Baxter and Jermann (1997) and Bohn (1999), the returns to capital and labor are highly correlated in the long run, thereby supporting the Cobb-Douglas assumption with constant factor shares. With these findings in mind, I present results for $corr\,(y_t, w_t) = 0.90$ and 0.97.

Implied parameter values are stated in the Appendix. Since the equity in the model is the claim not just to corporate dividends but to all risky capital in the economy, a realistic equity premium is, according to Constantinides et al (1998) about 3 percent.

6 Simulation Results

All economies are simulated for 20000 periods and results from these simulations are presented in this section[16] The mean return of an asset is defined as 100 x [{mean of the 20-year holding period return}$^{1/20}$-1]. The standard deviation of the equity or bond return is defined as 100 x [std{20-year holding period return}$^{1/20}$]. The mean premium of equity return over the bond return is defined as the difference of the mean return on equity and the mean return on the bond. The standard deviation of the premium of equity return over the bond return is defined as 100 x [std{20-year equity return} - std{20-year bond return}$^{1/20}$].

[16]The case with $corr\,(y_t, w_t) = 0.97$ is presented in the Appendix. All economies were simulated for 21000 periods, but the first 1000 periods were discarded.

Table 1: Simulation statistics

	$\gamma = 4$, CORRELATION $(W, Y) = 0.9$		
	NO SHORTSELLING CONSTRAINTS		
	NO SOCIAL SECURITY	SOCIAL SECURITY	
	-	SAFE	WAGE-INDEXED
MEAN EQUITY RET.	6.59	10.73	10.08
STD OF EQUITY RET.	5.1	5.98	5.33
MEAN BOND RET	5.77	9.6	8.93
STD OF BOND RET.	4.54	5.36	4.68
MEAN PRM/BOND	0.82	1.13	1.15
STD PRM/BOND	1.55	2.00	1.97
EQUITY HELD BY THE YOUNG (%)	92	48	57
COV(c_1, EQUITY RET.)	2.25	2.30	2.36
COV(c_2, EQUITY RET.)	2.47	3.89	2.07

Table 2: Simulation statistics

	$\gamma = 6$, CORRELATION $(W, Y) = 0.9$		
	NO SHORTSELLING CONSTRAINTS		
	NO SOCIAL SECURITY	SOCIAL SECURITY	
	-	SAFE	WAGE-INDEXED
MEAN EQUITY RET.	6.33	12.66	11.51
STD OF EQUITY RET.	6.52	8.28	7.20
MEAN BOND RET	5.40	10.96	9.84
STD OF BOND RET.	6.02	7.75	6.68
MEAN PRM/BOND	0.93	1.70	1.67
STD PRM/BOND	1.34	2.00	1.93
EQUITY HELD BY THE YOUNG (%)	128	52	63
COV(c_1, EQUITY RET.)	1.99	2.12	2.17
COV(c_2, EQUITY RET.)	3.49	4.41	2.43

The first important observation across all cases reported in the tables is that social security raises the equity premium. More specifically, the equity premium is up to 83 percent higher with wage-indexed benefits than without social security when agents are not borrowing constrained, and up to 37 percent when they are. Tables 1 to 4 show that social security increases the equity premium through two main channels: a *demand* effect and a *valuation* effect.

The main influence of social security on the equity premium in the absence of shortselling constraints is due to the demand effect, which is seen from the fact that social security considerably influences the relative demand for equity. In the absence of social security, the young shortsell bonds and hold between 90-130 percent of the total equity stock.[17] Their marginal valuation of equity is thus decisive

[17] The large equity demand of the young is explained by the fact that they will receive some wage income in the next period, wheras the middle-aged will receive zero exogenous income. The covariance between future consumption and equity income is thus initiallty lower for the young than for the middle-aged. In the trading process, the middle-aged demand bonds and the young demand equity to reduce this discrepancy.

Table 3: Simulation statistics

	$\gamma = 4$, CORRELATION $(W, Y) = 0.9$		
	SHORTSELLING CONSTRAINTS		
	NO SOCIAL SECURITY	SOCIAL SECURITY	
	-	SAFE	WAGE-INDEXED
MEAN EQUITY RET.	6.54	10.76	10.04
STD OF EQUITY RET.	3.7	5.17	4.21
MEAN BOND RET	4.99	9.16	8.30
STD OF BOND RET.	3.01	4.46	3.39
MEAN PRM/BOND	1.56	1.60	1.74
STD PRM/BOND	1.57	2.03	1.99
EQUITY HELD BY THE YOUNG (%)	12	15	13
COV(c_1, EQUITY RET.)	0.41	0.97	0.72
COV(c_2, EQUITY RET.)	3.93	3.62	3.48

Table 4: Simulation statistics

	$\gamma = 6$, CORRELATION $(W, Y) = 0.9$		
	SHORTSELLING CONSTRAINTS		
	NO SOCIAL SECURITY	SOCIAL SECURITY	
	-	SAFE	WAGE-INDEXED
MEAN EQUITY RET.	6.35	12.72	11.43
STD OF EQUITY RET.	4.14	6.90	5.32
MEAN BOND RET	4.62	10.50	9.12
STD OF BOND RET.	3.51	6.25	4.56
MEAN PRM/BOND	1.73	2.22	2.31
STD PRM/BOND	1.50	2.08	2.05
EQUITY HELD BY THE YOUNG (%)	11	11	9
COV(c_1, EQUITY RET.)	0.09	0.49	0.31
COV(c_2, EQUITY RET.)	4.65	4.31	4.12

in the price process, and since they value equity highly, the return to equity is low in this economy. The middle-aged mainly hold bonds, but the difference in expected return between the two assets is small (less than 1 percent). When agents receive social security, they need not to save as much for retirement. Consequently, the middle-aged agents' bond demand is significantly reduced. The lower bond demand effectively restricts the possibilities of the young to finance their equity purchases. As a result, the young demand less equity and the price of equity goes down (and the return goes up). In fact, social security reduces their equity demand by up to as much as 60 percent. The effect on the equity premium is substantial: it is up to 83 percent higher with wage-indexed benefits, than without social security. However, wage-indexed and safe benefits generate the same equity premium in every important respect. Moreover, social security does not influence the covariances between future consumption and equity income in any significant way. Basically, the covariance increases somewhat for the young, and decreases somewhat for the middle-aged.

The main influence of social security in the presence of shortselling constraints is due to the valuation effect. This can be seen from the share of equity held by the young being close to constant, indicating that the relative demand for equity is almost unaffected by social security.[18] However, social security significantly increases the covariance between future consumption and equity income for the young - up to 6 times, as in table 9 in the Appendix. At the same time, the covariance between future consumption and equity income for the middle-aged is relatively unaffected by social security. This is true for both wage-indexed and safe benefits. The higher covariance is first of all due to defined benefits featuring contra-cyclical taxes. More specifically, taxpayers are (at least sometimes) forced to pay higher taxes when wage income is low and vice versa. Since equity and wage income are highly correlated, the young find equity to be a worse hedge in economies with social security because it does not pay off when really needed. Their marginal valuation of equity is therefore lower, whereas their marginal valuation of the middle-aged remains relatively unchanged. The result is a lower equilibrium price and a higher average return. Second, contra-cyclical taxes make taxpayers' consumption more volatile, which makes asset prices more volatile since the price agents are willing to pay, will vary more between states. Notice that both bond and equity returns are most volatile in economies with safe benefits (where taxes are always contra-cyclical), which also increases the covariance. As a result, agents demand a higher premium for holding equities.[19]

The conclusion is that social security unambiguously increases the equity premium. None of the model economies above generate a premium equal to 3 percent. However, the economy with borrowing constraints, wage-indexed benefits based upon past income and an RRA coefficient of 6 has a premium of 2.31 percent, which is fairly close to the target of 3 percent. Moreover, the equity is never priced exclusively by the middle-aged when the model is calibrated with realistic income profiles. Instead, the price of equity is always determined in the trading process between the young and the middle-aged. Finally, social security seems to exacerbate the risk-free rate puzzle, since agents receiving social security

[18] It is still a fact that the young find the equity very attractive and would like to invest in it. However, since their wage income is lower in the present than in the future, they choose not to buy when prevented from borrowing.

[19] Naturally, this effect is more pronounced when they are more risk averse.

value marginal savings less, which leads to lower asset prices and higher returns.

7 Conclusions

This paper finds that the pay-as-you-go social security system may be an important factor in the explanation of the famous equity premium puzzle. The effect of social security on the equity premium is substantial: it is up to 83 percent higher with than without social security. In addition, a model with wage-indexed benefits based on past income, borrowing constraints and an RRA coefficient of 6 can actually generate a fairly realistic equity premium.

In the absence of shortselling constraints, social security influences the equity premium by changing the relative demand for equity. When agents do not receive social security, the young shortsell bonds and buy the bulk of the equity stock. The middle-aged mainly hold bonds, but the difference in expected return between the two assets is small. When agents receive social security, the need to save for retirement, especially in bonds, is reduced. Consequently, the middle-aged agents' demand for bonds goes down. A lower bond demand implies that the amount of bonds the young can shortsell to the middle-aged is reduced. As a result, the young demand less equity and the price of equity goes down (and the return goes up).

The main influence of social security in the presence of shortselling constraints is due to the fact that social security significantly increases the covariance between future consumption and equity income for the young. At the same time, the covariance between future consumption and equity income for the middle-aged is relatively unaffected by social security. The higher covariance is first of all due to the arrangement with defined benefits featuring contra-cyclical taxes. More specifically, the government may be forced to increase taxes when wage income is low, and vice versa. Since equity and wage income are highly correlated, the young find equity to be a worse hedge in economies with social security, since it does not pay off when really needed. Their marginal valuation of equity will therefore be lower, whereas the marginal valuation of the middle-aged remains relatively unchanged. The result is a lower equilibrium price and a higher average return.

This paper adds to the literature on the importance of life-cycle effects for the equity premium. Earlier findings are that the equity premium is generally high when the equity is mainly priced by the middle-aged who face a high covariance between their future consumption and equity income. The marginal valuation of equity is low for this group, resulting in a high expected return to equity. A natural question is then why the equity would be priced by the middle-aged. Constantinides et al (2002) argue that the young would like to shortsell bonds and invest in equity, but financial frictions such as shortselling constraints may prevent them from this. Storesletten et al (2001), on the other hand, argue that the young actually choose not to hold equity, since they find it too risky. This paper offers an explanation in-between the two.[20] As in Constantinides et al (2002), the young would like to shortsell bonds and invest in equity. However, social security reduces the middle-aged agents' demand for bonds and thus, the amount of bonds the young can shortsell. As a result, the young choose to reduce their equity demand.

References

[1] Abel, Andrew B. "The Effects of a Baby Boom on Stock Prices and Capital Accumulation in the Presence of Social Security." *Econometrica* 72 (2), March 2003, pp. 551-578.

[2] Baxter, Marianne and Jermann, Urban J. "The International Diversification Puzzle is Worse than You Think." *The American Economic Review* 87 (1), 1997, pp. 170-180.

[3] Bohn, Henning. "Should the Social Security Trust Fund Hold Equities? An Intergenerational Welfare Analysis". *Review of Economic Dynamics* 2, July 1999, pp. 666-697.

[4] Constantinides, George M., Donaldson, John B. and Mehra, Rajnish. "Junior Can't Borrow: A New Perspective on the Equity Premium Puzzle." *Quarterly Journal of Economics* 117 (1), 2002, pp. 269-2976.

[5] Domeij David and Klein Paul. "Public Pensions: To What Extent Do They Account for Swedish Wealth Inequality?" *Review of Economic Dynamics* 5(3), July 2002, pp. 503-534.

[6] Fast Facts and Figures about Social Security, Social Security Administration, 2003.

[20] Both Constantinides et al (2002) and Storesletten et al (2001) abstract from social security.

[7] Feldstein, Martin. "Social Security, Induced Retirement, and Aggregate Capital Accumulation." *Journal of Political Economy* 82 (5)1974, pp. 905-926.

[8] Feldstein, Martin. "Social Security and Saving: New Time Evidence." *National Tax Journal* 49 (2) June 1996, pp. 151-164

[9] Huggett, Mark "Wealth Distribution in Life-Cycle Economies." *Journal of Monetary Economics* 38, 1996, pp 469-494.

[10] Hugget, Mark and Ventura Gustavo. "Understanding Why High Income Household Save More than Low Income Households." *Journal of Monetary Economics* 45, 2000 pp. 361- 97.

[11] McHale, John "The Risk of Social Security Benefit Rule Changes: Some International Evidence." NBER working paper 7031, 1999.

[12] Mehra, Rajnish and Prescott, Edward C. "The Equity Premium: A Puzzle."*Journal of Monetary Economics* 15 (2), 1985, pp. 145-161.

[13] ——. "The Equity Premium in Retrospect. NBER working paper 9525, 2003.

[14] Smetters, Kent A., "Trading with the Unborn: A New Perspective on Capital Income Taxation" NBER working paper 9412, 2002.

[15] The Social Security Handbook, Social Security Administration, SSA Publication No. 65-008. 2004.

[16] Storesletten, Kjetil; Telmer, Chris I and Yaron Amir "The Risk Sharing Implications of Alternative Social Security Arrangements." Carnegie-Rochester Conference Series on Public Policy, 1999, vol. 50, pp. 213-259.

[17] —."Consumption and Risk Sharing over the Life Cycle." 2003. Forthcoming in *Journal of Monetary Economics*.

A Appendix

A.1 The Benchmark Calibration

The calibration in the benchmark case with corr $(W, Y) = 0.9$ is given by

Table 5: PARAMETER VALUES

$\overline{y}$	v	$\overline{w}_0$	$\overline{w}_1$	ζ	π_1
10	2.8	2.89	3.86	0.2	0.187

.

A.2 Tables

The results from the simulations with a higher correlation between wages and income are presented in the tables below.

Table 6: Simulation statistics

	$\gamma = 4$, CORRELATION $(W, Y) = 0.97$		
	NO SHORTSELLING CONSTRAINTS		
	NO SOCIAL SECURITY	SOCIAL SECURITY	
	-	SAFE	WAGE-INDEXED
MEAN EQUITY RET.	6.71	10.81	10.12
STD OF EQUITY RET.	5.25	5.94	5.34
MEAN BOND RET	5.98	9.80	9.09
STD OF BOND RET.	4.73	5.40	4.79
MEAN PRM/BOND	0.72	1.01	1.03
STD PRM/BOND	1.32	1.73	1.69
EQUITY HELD BY THE YOUNG (%)	98	45	58
COV(c_1, EQUITY RET.)	2.23	2.17	2.28
COV(c_2, EQUITY RET.)	2.55	4.25	2.08

Table 7: Simulation statistics

	$\gamma = 6$, CORRELATION $(W, Y) = 0.97$		
	NO SHORTSELLING CONSTRAINTS		
	NO SOCIAL SECURITY	SOCIAL SECURITY	
	-	SAFE	WAGE-INDEXED
MEAN EQUITY RET.	6.66	12.86	11.68
STD OF EQUITY RET.	6.52	8.33	7.25
MEAN BOND RET	5.84	11.40	10.24
STD OF BOND RET.	6.06	7.86	6.79
MEAN PRM/BOND	0.82	1.46	1.45
STD PRM/BOND	1.17	1.69	1.65
EQUITY HELD BY THE YOUNG (%)	127	51	63
COV(c_1, EQUITY RET.)	1.76	1.99	2.01
COV(c_2, EQUITY RET.)	3.87	4.80	2.56

Table 8: Simulation statistics

	$\gamma = 4$, CORRELATION $(W, Y) = 0.97$		
	SHORTSELLING CONSTRAINTS		
	NO SOCIAL SECURITY	SOCIAL SECURITY	
	-	SAFE	WAGE-INDEXED
MEAN EQUITY RET.	6.64	10.89	10.17
STD OF EQUITY RET.	3.73	5.20	4.20
MEAN BOND RET	5.32	9.55	8.67
STD OF BOND RET.	3.09	4.59	3.49
MEAN PRM/BOND	1.32	1.34	1.50
STD PRM/BOND	1.32	1.74	1.72
EQUITY HELD BY THE YOUNG (%)	11	15	13
COV(c_1, EQUITY RET.)	0.48	1.12	0.83
COV(c_2, EQUITY RET.)	3.88	3.37	3.27

Table 9: Simulation statistics

	$\gamma = 6$, CORRELATION $(W, Y) = 0.97$		
	SHORTSELLING CONSTRAINTS		
	NO SOCIAL SECURITY	SOCIAL SECURITY	
	-	SAFE	WAGE-INDEXED
MEAN EQUITY RET.	6.53	13.01	11.73
STD OF EQUITY RET.	4.24	7.06	5.42
MEAN BOND RET	5.03	11.11	9.71
STD OF BOND RET.	3.63	6.48	4.77
MEAN PRM/BOND	1.5	1.90	2.02
STD PRM/BOND	1.22	1.76	1.74
EQUITY HELD BY THE YOUNG (%)	10	11	8
COV(c_1, EQUITY RET.)	0.11	0.65	0.41
COV(c_2, EQUITY RET.)	4.72	4.06	3.88

SEMINAR PAPER SERIES

The Series was initiated in 1971. For a complete list of Seminar Papers, please contact the Institute.

2003

717. *Assar Lindbeck*	Improving the Performance of the European Social Model - The Welfare State over the Life Cycle.
718. *Harry Flam*	Turkey and the EU: Politics and Economics of Accession.
719. *Assar Lindbeck*	An Essay on Welfare State Dynamics.
720. *Gino A. Gancia*	Globalization, Divergence and Stagnation.
721. *Dirk Niepelt*	Tax Evasion Dynamics.
722. *Dirk Niepelt*	Intra-Generational Conflict: The Role of Balanced Budget Rules.
723. *Peter Svedberg and John E. Tilton*	The *Real* Real Price of Nonrenewable Resources: Copper 1870-2000.
724. *Peter Svedberg*	World Income Distribution: Which Way?
725. *Assar Lindbeck and Dennis Snower*	The Firm as a Pool of Factor Complementarities
726. *Davide Ticchi and Andrea Vindigni*	Endogenous Constitutions

2004

727. *Conny Olovsson*	Why do Europeans Work so Little?
728. *Conny Olovsson*	The Welfare Gains of Improving Risk Sharing in Social Security
729. *Conny Olovsson*	Social Security and the Equity Premium Puzzle

ISSN 0347-8769
Stockholm, 2004
Institute for International Economic Studies