A Theory of Economic Development with Endogenous Fertility*

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A Theory of Economic Growth

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Abstract

In this paper I build a unified model of economic growth to account for the time-series evolution of output, fertility and population in transition through the industrialization of an economy. Specifically, I merge the unified growth models *à* la Galor and Weil (2000) and Hansen and Prescott (2002) to capture the importance of human capital formation, fertility decline and the transition from agriculture to industry in transition from stagnation to growth. Moreover, I also incorporate young adult mortality in the model. Initially, the aggregate human capital and return to education are low and the mortality rate is high; therefore parents invest in quantity of children. Once sufficient human capital is accumulated and mortality rates are reduced thanks to increasing life expectancy, with the activation of the modern human capital intensive sector, parents start to invest in quality of their children. The simulation of the model economy improves upon the quantitative performance of the existing literature and successfully captures the evolution of fertility, population and GDP of the British economy between 1750 and 2000.

Keywords: Industrial Revolution; Malthusian Growth; Economic Development; Demographic Change.

JEL Classification Numbers: J10, O11, O41, N10, N30.

1 Introduction

The process of industrialization or in broader terms economic development can be categorized in three stages [Galor and Weil (1999, 2000), Hansen and Prescott (2002), Galor (2005)]: The first stage is called the Malthusian stage, where low (or no) population growth goes hand in hand with low (if any) growth in output per capita. In the second stage of development, called the post-Malthusian stage, technological progress rises and both output per capita and population grow, meaning that the growth rate of output is higher than the growth rate of population. Finally, there is the modern stage[1] where output per capita continues to grow whereas the population growth is low (if any).

Even though there are no strictly defined time periods for the three stages, the Malthusian stage accounts for most of history up to the end of 1700's quite well. Galor and Weil (1999, 2000) and in particular Galor (2005) characterize this stage as one with little education and human capital, low productivity, and high gross reproduction rate but much lower net reproduction rate (due to high mortality), in turn leading to low population growth. The industrial revolution, starting roughly sometime between $1760-1840$ [Floud and McCloskey (1994)], lead to the second stage the characteristics of which lasted up to the 20th century. The fertility rate did not decrease much in the transition [Galor and Weil (1999, 2000) and Galor (2005)], but the higher reduction in mortality (or increase in life expectancy)[2] lead to an increase in population. However, the growth rate of output was higher than the growth rate of population, so in this stage output per capita increased and living standards improved, contrary to the well-known predictions of the Malthusian growth theory. Finally, the modern stage, in which population growth rates started to decline, began approximately in the first half of the 20th century. The main characteristics of this stage are low fertility

[1]Galor and Weil (2000) call these stages Malthusian, post-Malthusian and modern growth regimes, respectively. Hansen and Prescott (2002) talk about stages which are only differentiated by the Malthus and Solow production functions.

[2]This is also documented in Nerlove and Raut (2003) and in Clark (2005).

and mortality, increased level of education and human capital, and high productivity growth. The characteristics of this stage, along with the previous ones, is well documented by Galor and Weil (1999, 2000), Hansen and Prescott (2002), Doepke (2004), Galor (2005), Bar and Leukhina (2010), and more recently in Galor (2010).

The main purpose of this paper is to build a unified model of economic growth and demographic change which can account for the characteristics of growth in output and population in transition through the process of economic development in United Kingdom as described in Lucas (2002). The model constructed in this paper is a combination of the Malthusian and Solow growth models with an additional human-capital-intensive production function which allows for spill-over effects. It is a standard general equilibrium growth model with overlapping generations and endogenous fertility decision. On the production side there are two different technologies which differ in their total factor productivities (TFP) and use of factors. The first one, called the *primitive* technology is assumed to employ effective labor (the product of number of workers, the portion of time devoted to work by each worker and the level of human capital that each worker possesses), reproducible capital and a fixed amount of land. The second technology, titled the *modern* production function, does not use land as an input, but employs effective labor and capital only, and also allows for spill-over effects. Human capital for each worker depends on the education of the worker, determined by his parents and the rate of technological change as in Galor and Weil (2000) and Lagerlof (2006). Moreover, I also introduce mortality into the model by assuming that each generation of households may live up to 2 periods, however only a fraction of them, depending on the young-adult mortality rate, survive to the second period. With the help of this specification, in equilibrium I am able to obtain a formula for optimal fertility level as a function of technological improvement, mortality and education. Once the model is simulated, initially, only the primitive sector is active, the aggregate human capital and return to education are low and the mortality rate is high; therefore parents have more incentives to invest in quantity of children. Once sufficient human capital is accumulated and mortality rates are reduced, with the activation of the modern human capital intensive sector, parents have more incentives to invest in quality of their children.

The numerical exercise, I present at the end of the paper, reflects the characteristics of the three periods discussed in the beginning. The simulation is done for nine periods corresponding to 300-350 years.[3] Assuming that the model economy starts in the early 18th century, I track the evolution of the variables of the economy up to the end of the 20th century. The model generates series for output, output per capita, fertility and different than the fertility level and growth of population which successfully match the data from the British economy.

This paper is related to various other works in the literature. In accounting for the transition, the model embodies elements from Tamura (1996), Galor and Weil (2000), Stokey (2001), Hansen and Prescott (2002), Lagerlof (2006), and Bar and Leukhina (2010). Moreover, the representative agent's maximization problem with endogenous fertility is similar to the one used in unified growth theory by Galor and Weil (2000) and many others.

Among the related literature, Galor and Weil (2000), Hansen and Prescott (2002) deserve more discussion as they are closely related to the present study.

Galor and Weil (2000) is the main point of departure of the model with respect to individual decision making and the production of human capital. They present a one-sector OLG model with endogenous technological progress and fertility to account for the evolution of output, population and technology. [4] The present study, even though largely consistent with their results, extends their paper with important modifications and differences. Specifically, as Galor (2005) also mentions, the analysis of Galor and Weil (2000) do not explicitly incorporate the structural transformation from a primitive technology to a modern one. In

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[3]As Hansen and Prescott (2002) also do, I assume that each period in the OLG model economy corresponds approximately to 35-40 years.

[4]There are also many studies such as Lagerlof (2006), Weisdorf (2006), and Strulik and Weisdorf (2008) which used the Galor-Weil model as their benchmark.

my paper however, this transformation explicitly exists and contrary to Hansen and Prescott (2002), it is related to human capital accumulation. This is one of the key mechanisms generating the evolution of population in the model. Furthermore, adding young-adult mortality to the model, helps to account for different behavior of fertility and population growth rates in the data. Finally, the present study also complements Galor and Weil (2000) by quantitatively accounting[5] for the evolution of output, population and fertility in UK through and after the industrial revolution. In summary, the present study nicely fits the ideas proposed in Galor and Weil (2000) and complements the related literature.

Hansen and Prescott (2002) is the another point of departure of this study, especially for the production side of the model. Similar to the present study, they develop an OLG model with two sectors in which the economy shifts from an agricultural sector to an industrial sector in the course of economic development.[6] However, unlike other unified growth theories and the model presented in this paper, population growth is simply assumed to be a function of growth in consumption; thereby lacking micro-foundations for factors behind its transition. Moreover, human capital formation, that appears to be one of the central forces in the unified growth literature, is absent in Hansen and Prescott (2002). As Galor (2005) also argues, such a reduced form analysis does not identify the economic factors behind the process of technological change, as well as the forces behind the demographic dynamics. The main value added of the present study on the other hand is filling in the gap in Hansen and Prescott (2002) by incorporating human capital formation with micro-foundations and endogenous population dynamics in the model. This allows to better identify the economic factors behind the evolution of output and population, as well the factors behind the process of technology

[5]Lagerlof (2006) is another example of a quantitative study in this regard.

[6]Tamura (2002) presents a model where human capital accumulation causes the economy to switch from agriculture to industry endogenously. Different than this paper, I look at a shorter period, therefore my model performs better in terms of explaining the short-run fluctuations in the data after the industrial revolution. Moreover, I also incorporate young adult mortality into the analysis.

change. Specifically, it shows that human capital plays a central role in sustaining the rate of technological progress in the industrial sector and in generating the demographic transition.

The rest of this paper is organized as follows: In the next section, I discuss some empirical facts from United Kingdom to motivate our model. In section 3, I present the model economy, defines a competitive equilibrium and solves it. Simulation of the model economy in its transition through the three stages is then presented in Section 4. Finally, I offer concluding remarks in Section 5. The appendix presents an easy proof of proposition 2 of section 3.

2 Empirical Motivation

The claim that the economic history can be analyzed in three periods can be easily observed when one looks at historical data. One can see the different characteristics of the three periods by looking at GDP, GDP per capita and population figures. Figure 1 below[7] illustrates the behavior of the population of the United Kingdom after 1700. The increase in the level of population in the long-run is obvious. But more important is the slope of this curve, namely, the growth rate of population over time.

Figure 1 about here

Figure 2 shows the population growth rate derived from the data in figure 1. Even though there are some fluctuations, the trend is that the growth rate jumps from a very low level to a higher level after the start of industrial revolution and then decreases over the long-run

[7]The data for population are obtained from Wrigley and Schofield (1989) and Wrigley et.al.(1997). In an earlier draft of the paper I also used data presented in Floud and McCloskey (1994) and Maddison (2007). One important notice should be made at this moment for all data used throughout the paper. To be able to make better comparison with the simulation, all empirical data presented here averaged out for 35 year periods from 1716 to 1996, e.g. in the following figure the population level in for 1951 is not the actual population in that year, but is the average of population between 1916 and 1951. One exception is for 1716 where the average is taken from 1701 to 1716. Data from different sources listed above do not differ significantly, especially once this averaging is applied.

almost to its original level. Excluding the fluctuations, and looking at the trend, this picture confirms the demographic transition in the three different stages which we hypothesized in the previous section.

Figure 2 about here

There are various reasons why population statistics follow such patterns. Decomposing the growth rate of population to observe the fertility and mortality rates can be a step towards that purpose. For that purpose, figure 3 below documents the evolution of the gross reproduction rate (GRR) and the average life expectancy in England.[8] Gross reproduction rate, which was slightly above 2 before the industrial revolution, jumps to almost 3 in the 1820's but decreases thereafter up to almost 1 at the end of the 20th century. In the OLG-model economy which we will discuss in the next section, the mortality rate will be the probability that the representative agent born at period t will die before $t+1$, which has no counterpart in the data. Therefore, throughout the simulation, we will assume that the average life expectancy documented in figure 3.2 has a negative relationship with the mortality rate in our model, even though the form of this relationship is unknown. (A specific functional form will be assumed to capture this relation later in the paper.) For now, the data shows that the average life expectancy increases uninterruptedly after the industrial revolution. Notice that the increase in GRR and life expectancy positively affects population growth. But when the GRR starts to decrease in time, the population continues to grow as the life expectancy becomes higher. Towards the end of the 20th century, the growth in the life expectancy ceases and GRR decreases (almost to 1) which accounts for the slowdown in the population growth rate.

Figure 3 about here

[8]GRR data is taken from Clark (2005) and from Office of National Statistics, and life expectancy data from Arora (2001) and the Human Mortality Website: www.humanmortality.org

Figure 4 looks[9] at GDP and GDP per capita in the United Kingdom. The increasing trend of both variables after the industrial revolution is obvious. As discussed in the introduction, prior to the industrial revolution, the growth in GDP is balanced by the growth in population, so that the growth in GDP per capita is low (if any). But in the second stage both variables start to grow uninterruptedly.

Figure 4 about here

As a summary of these figures, we can conclude that the three stages which are discussed in detail in the previous section are observable from the documented data above. Now I can build a model to explain these observations.

3 The Model

3.1 Households' Problem

Overlapping generations live for 2 periods. A young household born in period t has the following utility function:

$$\log c_t^t + \beta(1 - \xi_t) \log c_{t+1}^t + \gamma n_t^{1-\epsilon} h_{t+1}(e_{t+1}) \tag{1}$$

Here c_t^t is consumption of the young household in period t, whereas c_{t+1}^t is its consumption when old. ξ_t is the probability that the young household does not survive period t. Besides its own consumption, the representative household can choose the number of children it is going to have, n_t, and the amount of education it should invest for its children, e_{t+1}. γ and ϵ are simply parameters which show the level of altruism the household has towards its children.

[9] Data after 1870 is taken from the Office of National Statistics. Data before that is generated from the data presented in Broadberry et.al. (2010).

Human capital evolves according to the following equation:

$$h_{t+1}(e_{t+1}, g_t) = \psi(e_{t+1}, g_{t+1}) \tag{2}$$

where g_{t+1} is the rate of average technological progress which will be defined more in detail with technology. I further assume that ψ satisfies $\psi_e > 0$, $\psi_{ee} < 0$, $\psi_g < 0$, and $\psi_{gg} > 0$. The first two conditions indicate that education increases the level of human capital but at a decreasing rate. For the other two conditions, the assumption is that faster technological progress erodes human capital by making knowledge obsolete, however at a decreasing rate.[10]

Throughout the simulation, I will assume the following functional form for the human capital accumulation function.

$$h_{t+1}(e_{t+1}, g_{t+1}) = \psi(e_{t+1}, g_{t+1}) = \frac{a + be_{t+1} - g_{t+1}}{a + be_{t+1} + g_{t+1}}. \tag{3}$$

This form obviously satisfies the four properties listed above.[11]

At any period t, the young agent born at t can spend his income for consumption, c_t buying capital, k_{t+1} or land l_{t+1}. He earns rent from his capital and land next period. Notice that the depreciation rate is assumed to be equal to 1. The agent's labor income at period t depends on the wage rate w_t, the level of human capital that the agent possesses $h_t(e_t)$, and the amount of time that he spends working, z_t. The more he spends his time for work, the less is the amount of education he can provide for his n_t children. Parameters a and b

[10] Galor and Weil (2000) make a further assumption, namely $\psi_{eg} > 0$. The intuition is that technological progress increases the return to education or that the erosion of human capital due to technological change decreases with education. As Lagerlof (2006) also emphasizes, this assumption is sufficient but not necessary to generate the result that e_{t+1} is increasing in g_{t+1}.

[11] Notice that a similar function is also used by Lagerlof (2006). Moreover, this function also satisfies the fifth property that, $\psi_{eg} > 0$, if I restrict ϵ to be above some threshold level. In the simulation exercise below this assumption will hold anyhow.

represent the time cost of raising children.[12] (In the simulation, they will be assumed to be fixed numbers.) The agent does not work at $t+1$.

Accordingly, the households' budget and time constraints are given by

$$c_t^t + k_{t+1} + p_t l_{t+1} = w_t h_t(e_t) z_t \tag{4}$$

$$c_{t+1}^t = r_{K,t+1} k_{t+1} + (p_{t+1} + r_{L,t+1}) l_{t+1} \tag{5}$$

$$z_t + n_t(a + b e_{t+1}) = \bar{z}, \tag{6}$$

where p_t stands for the relative price of land.

3.2 Technology

The model I present in this paper is an OLG model with 2 different technologies. The *primitive* sector employs land, effective labor and physical capital to produce output. The second sector, called the *modern* sector, does not employ land. The production functions are given by:

$$Y_{P_t} = A_{P_t} K_{P_t}^{\alpha_P} H_{P_t}^{\theta_P} L_{P_t}^{1-\alpha_P-\theta_P} \tag{7}$$

$$Y_{M_t} = A_{M_t} \eta(S_t) K_{M_t}^{\alpha_M} H_{M_t}^{1-\alpha_M} \tag{8}$$

The variables A_i, Y_i, K_i, H_i and L_i refer to TFP, output, physical capital, effective labor, and land in sector $i \in \{P, M\}$. I also assume that $A_{P_t} = A_P^t$ and $A_{M_t} = A_M^t$. This means that TFP in both sectors grow at an exogenous rate.

Remember that g_t is defined to be the rate of technological progress of the economy. With these two production functions in hand,

$$g_{t+1} = \frac{A_{t+1} - A_t}{A_t}, \tag{9}$$

where A_{t+1} is simply be a weighted average of $A_{P_{t+1}}$ and $A_{M_{t+1}}$, i.e.

[12] Robinson (1997) provides a very detailed survey of this literature.

$$A_t = \frac{Y_{P_t} A_{P_t} + Y_{M_t} A_{M_t} \eta(S_t)}{Y_t}, \tag{10}$$

where $Y_t = Y_{P_t} + Y_{M_t}$. So even though, TFP in the two sectors grow at exogenous rates A_P and A_M, the aggregate TFP A_t is a function of various endogenous variables of the model.

Throughout the model, land does not depreciate and is fixed at 1. Since only the primitive sector employs land, this will imply that $L_{Pt} = 1$ for any period t.

Consistent with the names of the production function, the modern sector will be capital intensive and effective-labor intensive compared to the primitive sector. Therefore, throughout the paper it will be the case that $\alpha_P < \alpha_M$ and $\theta_P < 1 - \alpha_M$.

The modern sector exhibits spill-over effects which are represented by the function $\eta(S_t)$, where $\eta'(S_t) > 0$, $\eta''(S_t) < 0$, and $S_t = N_t h_t$ is the total level of human capital in the economy. Notice that this specification is not new in the literature.[13]

Since the depreciation rate for physical capital is assumed to be 1, the feasibility constraint of the economy[14] is given by

$$C_t^t + C_t^{t-1} + K_{t+1} = Y_{Pt} + Y_{Mt}. \tag{11}$$

For simplicity it will be convenient to assume that the same firm operates in each sector alone. Given values for A_i, w, r_K, r_L, and S_t, this firm solves the following maximization problem subject to the production functions

$$max \quad Y_i - wH_i - r_K K_i - r_L L_i \qquad i \in \{P, M\} \tag{12}$$

[13]See Romer (1986) or Wang and Xie (2004)

[14]The implicit simplifying assumption made here is that capital in possession of the young who do not survive to the next period is automatically transferred to those who survive.

3.3 Equilibrium and Characterization

Given N_0, k_0,and ξ_t (and assuming that $L_t = 1$ for all t), a competitive equilibrium in this economy is defined to be sequences of household allocation $\{c_t^t, c_t^{t+1}, k_{t+1}, l_{t+1}, z_t, n_t, e_{t+1}\}$, firm allocations $\{K_{M_t}, K_{P_t}, H_{M_t}, H_{P_t}, Y_{M_t}, Y_{P_t}\}$ and prices $\{p_t, w_t, r_{K,t}, r_{L,t}\}$ such that given prices:

1. Households maximize their utility subject to the budget constraints specified above.

2. The representative firm maximizes its profits subject to the production functions.

3. Market clearing conditions hold. Specifically:

$$H_{M_t} + H_{P_t} = H_t = z_t h_t N_t \tag{13}$$

$$S_t = h_t N_t \tag{14}$$

$$L_{P_{t+1}} = L_{t+1} = l_{t+1} N_t = 1 \tag{15}$$

$$K_{M_t} + K_{P_t} = K_t = k_t N_{t-1} \tag{16}$$

$$C_t^t + C_t^{t-1} + K_{t+1} = Y_{M_t} + Y_{P_t} \tag{17}$$

$$N_{t+1} = n_t N_t. \tag{18}$$

Here are some theorems that are worth to state before solving for the competitive equilibrium:

Proposition 1: *For any wage rate w and capital rental rate r_K, the firm finds it profitable to operate in the primitive sector. This implies that $Y_{P_t} > 0$ for all t.* **Proof.** The proof of this proposition is in Hansen and Prescott (2002). ■

Proposition 2: *Given a wage rate w and capital rental rate r_K, maximized profit per unit of output in the modern sector is positive if and only if*

$$A_{M_t} > \frac{1}{\eta(S_t)} (\frac{r_K}{\alpha_M})^{\alpha_M} (\frac{w}{1-\alpha_M})^{1-\alpha_M}. \tag{19}$$

Proof. The proof of proposition 2 is presented in the appendix. ■

To make use of these propositions, in some period t one should first calculate

$$w_t = A_{P_t} \theta_P K_t^{\alpha_P} H_t^{\theta_P - 1} \tag{20}$$

and

$$r_{K_t} = A_{P_t} \alpha_P K_t^{\alpha_P - 1} H_t^{\theta_P} \tag{21}$$

If the condition of proposition 2 does not hold under these prices, then these are the equilibrium wage and capital rental rate. If proposition 2 holds, then these are not equilibrium prices; instead, one should use the following system of equations:

$$w_t = A_{P_t} \theta_P K_{P_t}^{\alpha_P} H_{P_t}^{\theta_P - 1} = A_{M_t} \eta(S_t)(1 - \alpha_M) K_{M_t}^{\alpha_M} H_{M_t}^{-\alpha_M} \tag{22}$$

and

$$r_{K,t} = A_{P_t} \alpha_P K_{P_t}^{\alpha_P - 1} H_{P_t}^{\theta_P} = A_{M_t} \eta(S_t)(\alpha_M) K_{M_t}^{\alpha_M - 1} H_{M_t}^{1-\alpha_M}. \tag{23}$$

In each period t, using these equalities and the market clearing conditions, it is straightforward to calculate K_{P_t}, H_{P_t} K_{M_t} and H_{M_t}.

Now consider the households' maximization problem: First notice that from the first-order conditions one directly obtains an expression for e_{t+1} which directly determines h_{t+1}

$$e_{t+1} = (\lambda g_{t+1} - \frac{a}{b}), \tag{24}$$

where $\lambda > 0$ is a constant, namely a function of some parameters of the model. [15]

First-order conditions also yield:

$$p_{t+1} = p_t r_{K,t+1} - r_{L,t+1}. \tag{25}$$

Moreover, the budget constraint implies

[15] Specifically $\lambda = \frac{\frac{1}{1-\epsilon} + \sqrt{1 + (\frac{1}{1-\epsilon})^2}}{b}$

$$N_t(w_t z_t h_t - c_t^t) - p_t = K_{t+1}, \tag{26}$$

and when I combine the budget constraint and first-order conditions I obtain

$$c_t^t = \frac{w_t h_t z_t}{1 + \beta(1 - \xi_t)}. \tag{27}$$

Lastly, from first-order conditions one can derive

$$n_t^\epsilon = \frac{\gamma(1-\epsilon)h_{t+1}z_t}{(1+\beta(1-\xi_t))(a + be_{t+1})}. \tag{28}$$

Equations (28) and (6) yield a system of 2 equations and and 2 unknowns: n_t and z_t. Given values of the parameters and ξ_t, it is straightforward to solve for both of them. Careful examination of equation (28) reveals that n_t also depends on the rate of technological progress through e_{t+1}. Everything being equal, this captures the Malthusian idea that technology may limit population growth as in Kremer (1993).

Notice that N_t is the number of young agents(or workers) at any time t, whereas population at t is given by this number plus the number of old agents at time t, i.e.

$$\pi_t = N_t + (1 - \xi_{t-1})N_{t-1}. \tag{29}$$

So the population growth rate from t to $t+1$ is given by

$$\frac{\pi_{t+1} - \pi_t}{\pi_t} = \frac{N_{t+1} + (1-\xi_t)N_t - (N_t + (1-\xi_{t-1})N_{t-1})}{N_t + (1-\xi_{t-1})N_{t-1}}. \tag{30}$$

4 Simulation

Notice that, given the parameters and the sequences of $\{A_{M_t}, A_{P_t}\}_{t=0}^{t_n}$, initial capital stock, and initial number of young agents, $(K_0, N_0$ respectively), the initial price of land p_0 and the mortality ξ_t, all equilibrium allocations can easily be calculated. One complication is that, to compute p_0, I use a numerical (recursive) shooting algorithm similar to one used in Hansen and Prescott (2002). Moreover, notice that g_{t+1} (which is one of the determinants of e_{t+1} and hence of z_t) depends on A_{t+1} the value of which is unknown in period t because it depends on shares of the two sectors in period $t+1$. This requires using the numerical shooting algorithm to accurately obtain g_{t+1} in period t. I will describe the process in more detail below.

Before starting the discussion of the simulation exercise there is one more task: Choosing values for various parameters of the model. Most of the chosen parameters are consistent with the existing literature. Table 1 below documents the values chosen for the key parameters of the benchmark model with mortality.

Table 1: Values for Basic Parameters in the Benchmark Model

Parameter	Description	Value
A_P	TFP in the primitive sector	1.032
A_M	TFP in the modern sector	1.518
α_P	Capital share in the primitive sector	0.1
θ_P	Effective Labor share in primitive sector	0.6
α_M	Capital share in modern sector	0.4
β	discount rate	1
γ	altruism	0.675
ϵ	altruism	0.49
a	fixed cost of each child	0.15
b	educational cost of each child	1
$\bar{z}$	total amount of time	20

My choice of A_P, A_M, α_P, θ_P, and α_M is from Hansen and Prescott (2002). Moreover, the values of a and b are from Lagerlof (2006). I calibrated ϵ and γ to match the GRR's and population growth rates in 1716 and 1751. Finally, I normalized $\bar{z}$ to a value of 20.

For the modern sector the form of the spill-over effect is assumed to be given by

$$\eta(S_t) = \frac{S_t + \nu}{S_t + 1}, \tag{31}$$

where ν is less than 1. First, notice that this specification of the function satisfies the desired properties stated above. Furthermore, since the initial conditions are chosen such that the modern sector is idle at $t = 0$, this requires that $\nu < 0.41$[16]. Various values are experimented for its value, and the reported simulation of the benchmark model takes it to be equal to 0.2.

Moreover, I need values for ξ_t, which is the probability that the household does not survive to the second period. The evolution of the average life expectancy in UK is plotted in figure 3.2. Assuming that each period in the model corresponds to a period of 35 years and the life expectancy in UK is normally distributed with the mean values plotted in figure 3.2 and a standard deviation of 25 years[17], I can calculate ξ_t. With this I now have all information to do the simulation. To clearly understand the effect of ξ_t on the model, I run two simulations. In one of them I feed in ξ_t's I calculate from the data into the model in the way I describe above. In the second simulation, denoted by "model without mortality", I assume that there is no mortality whatsoever, i.e. $\xi_t = 0$.

The simulation basically works as follows:

Since I assume that the economy initially is in the steady state with the primitive production function, g_0=g_{-1}=$A_P - 1$.[18] Therefore, I also have e_0 and h_0. Given ξ_0, A_P, A_M, K_0, N_0, and p_0, I can then calculate e_1, h_1, n_0 and z_0 provided that I know g_1. However, g_1 depends on whether proposition 2 holds in period 1 or not. Now, if proposition 2 does not hold in period 1, then g_1 is simply equal to $A_P - 1$. In this case I can calculate e_1, h_1, n_0 and

[16]For all other value of ν the modern sector is active at $t = 0$

[17]I should notice that the choice of the variance is somewhat arbitrary here; however since I assume a constant variance, it only affects the level of ξ_t, not it's trend, whereas the mean (average life expectancy) is time variant and also affects the evolution of ξ_t.

[18]Notice that when $A_t = A_{P_t}$ and $A_{t+1} = A_{P_{t+1}}$, then $g_{t+1} = \frac{A_P^{t+1} - A_P^t}{A_P^t} = A_P - 1$

z_0. However, if proposition 2 holds in period 1. Then, I cannot assume that $g_1 = A_P - 1$, because since this means that the modern sector is activated A_1 will not equal A_{P_1}. Instead, it will equal a weighted average of A_{P_1} and A_{M_1}. To calculate the weights of this average, I use a shooting algorithm and guess the weights of the primitive and modern sectors in total production in period 1 and calculate all the above mentioned variables accordingly, including the output weights in period 1[19]. If my guess of the weights is above or below the calculated weights, I update my guess and recalculate. Using this algorithm, I simulate the model economy for 9 periods from $t = 0$. Each period represents 35 years, as the idea is to simulate the transition of population and output from the beginning of the 18th century up to the third millennium.

Below I present the results of the simulation:

Figure 5 presents the evolution of the population in the model simulations together with the data. As evident from the figure, the model with variable ξ_t closely follows the evolution of the population in the data, whereas the model without mortality is underestimates the level of population . Moreover, one can further evaluate the model's performance to account for the population by looking at figure 6.

Figure 5 about here

In the benchmark model with mortality, the population starts to grow at an increasing rate after the industrial revolution but then its growth rate declines, as it is the case in the data. One reason why the population increases at an increasing rate is that the mortality rate ξ_t decreases, as the life expectancy goes up. Increasing life expectancy is also the crucial factor behind the gradual reduction in the population growth. That is also why the population growth declines steadily in the model without mortality.

Figure 6 about here

[19]Notice that, calculating output weights in period 1 requires knowledge of z_1 and e_2 which in turn requires knowledge of g_2 etc. Therefore, what I actually guess is a output-weight vector from periods 0 to t.

Next, I plot the fertility rates n_t in figure 7. Notice that in the benchmark model the fertility increases first (which is the other reason why the population increases at an increasing rate) but then sharply decreases in the following periods almost to 1. On the other hand, in the model without mortality, the fertility rate steadily declines and underpredicts it's counterpart in the data.

Figure 7 about here

In figure 8, I observe what happens to output and output per capita, respectively. Here, I didn't draw the output simulation without the mortality per se since there isn't any significant difference between both model simulations. Notice that output slowly increases from period 0 but with a parallel increase in the population, output per capita remains stagnant. With the industrial revolution this situation changes and both variables increase together.

Figure 8 about here

Figure 9 shows the average rate of technological progress (g_t) and the fraction of time spent for each child again in the benchmark model. They follow the same pattern as the latter is an increasing function of the former. Time spent for each child goes from a level of 2% up to almost 21% of total available time of the parent. This is to explain the increase in the education and human capital of children.

Figure 9 about here

Lastly, figure 10 illustrates the evolution of the shares of the primitive and the modern sectors. The primitive sector never shuts down but becomes very insignificant after the fifth period of the model, whereas the modern sector slowly becomes the dominant sector of the economy.

Figure 10 about here

5 Concluding remarks

In this paper I built a unified model of economic growth to account for the time-series evolution of output, fertility and population in transition through the industrialization of the British economy. For this purpose, I merged the models presented in Galor and Weil (2000) and and in Hansen and Prescott (2002) to capture the importance of human capital formation, fertility decline and the transition from agriculture to industry in transition from stagnation to growth. Furthermore, I also incorporated young adult mortality in my model which allowed to differentiate the behavior of fertility and population in certain periods. This way, the model captures explicitly the shift from a primitive to a modern sector in the transition from stagnation to growth, without assuming away human capital formation and the endogenous determination of population and fertility. Moreover, the presented simulations of the model economy significantly improve upon the quantitative performance of the existing literature by successfully capturing the evolution of fertility, population and GDP of the British economy between 1750 and 2000.

One extension of the present model can be made by endogenizing the mortality rate ξ_t. Considering that life expectancy is foremost affected by leaving standards, one way of doing this is assuming that the mortality rate is some decreasing and convex function of output per capita.

Moreover, the model economy can also be used to quantitatively investigate behavior of relevant variables in different economies. In this regard, similar simulations can be performed to explain data from various other European countries, but lack of data might be a serious issue here.

Appendix

Here I provide the proof of the proposition 2.

Proof.

First notice that the modern production function is given by

$$Y_{M_t} = A_{M_t}\eta(S_t)K_{M_t}^{\theta}N_{M_t}^{1-\theta}. \tag{32}$$

Given w and r_K I can write the profit function (for simplicity of notation drop time and modern sector subscripts) as

$$Y - wN - r_K K. \tag{33}$$

The profit per unit is then

$$1 - w\frac{N}{Y} - r_K\frac{K}{Y}. \tag{34}$$

If I multiply the reciprocal of (33) by N we obtain

$$N/Y = \frac{1}{A\eta(S_t)}(\frac{N}{K})^{\theta}, \tag{35}$$

and similarly multiplying the reciprocal of (33) by K I obtain

$$K/Y = \frac{1}{A\eta(S_t)}(\frac{N}{K})^{\theta-1}. \tag{36}$$

Substituting (36) and (37) into (35), I get

$$1 - \frac{w}{A\eta(S_t)}(\frac{N}{K})^{\theta} - \frac{r_K}{A\eta(S_t)}(\frac{N}{K})^{\theta-1}. \tag{37}$$

Now, maximizing this function with respect to N and K, I obtain the following FOCs

$$-\frac{w}{A\eta(S_t)}K^{-\theta}\theta N^{\theta-1} + \frac{r_K}{A\eta(S_t)}K^{1-\theta}(1-\theta)N^{\theta-2} = 0 \tag{38}$$

$$\frac{w}{A\eta(S_t)}K^{-\theta-1}\theta N^{\theta} - \frac{r_K}{A\eta(S_t)}K^{-\theta}(1-\theta)N^{\theta-1} = 0. \tag{39}$$

Both of these first order conditions separately imply the same thing which is

$$\frac{w}{1-\theta}N = \frac{r_K}{\theta}K \tag{40}$$

or

$$\frac{N}{K} = \frac{r_K(1-\theta)}{w\theta}. \tag{41}$$

Now what needs to be done is show that

$$1 - \frac{w}{A\eta(S_t)}(\frac{N}{K})^{\theta} - \frac{r_K}{A\eta(S_t)}(\frac{N}{K})^{\theta-1} > 0 \tag{42}$$

if and only if inequality (18) is satisfied. To prove this, it is enough to show that (18) and (38) are equivalent.

To show this, I take (38) which immediately becomes

$$1 > \frac{w}{A\eta(S_t)}(\frac{N}{K})^{\theta} + \frac{r_K}{A\eta(S_t)}(\frac{N}{K})^{\theta-1}. \tag{43}$$

Now using (42) this becomes

$$1 > \frac{w}{A\eta(S_t)}(\frac{r_K(1-\theta)}{w\theta})^{\theta} + \frac{r_K}{A\eta(S_t)}(\frac{w\theta}{r_K(1-\theta)})^{1-\theta} \tag{44}$$

or

$$A\eta(S_t) > w(\frac{r_K(1-\theta)}{w\theta})^{\theta} + r_K(\frac{w\theta}{r_K(1-\theta)})^{1-\theta} \tag{45}$$

or

$$A\eta(S_t) > w^{1-\theta}r_K^{\theta}(1-\theta)^{\theta}\theta^{-\theta} + w^{1-\theta}r_K^{\theta}(1-\theta)^{\theta-1}\theta^{1-\theta} \tag{46}$$

or

$$A\eta(S_t) > (\frac{r_K}{\theta})^{\theta}(\frac{w}{1-\theta})^{1-\theta}(1-\theta+\theta) \tag{47}$$

which is simply

$$A > \frac{1}{\eta(S_t)}(\frac{r_K}{\theta})^{\theta}(\frac{w}{1-\theta})^{1-\theta} \tag{48}$$

■

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Figure 1: Population

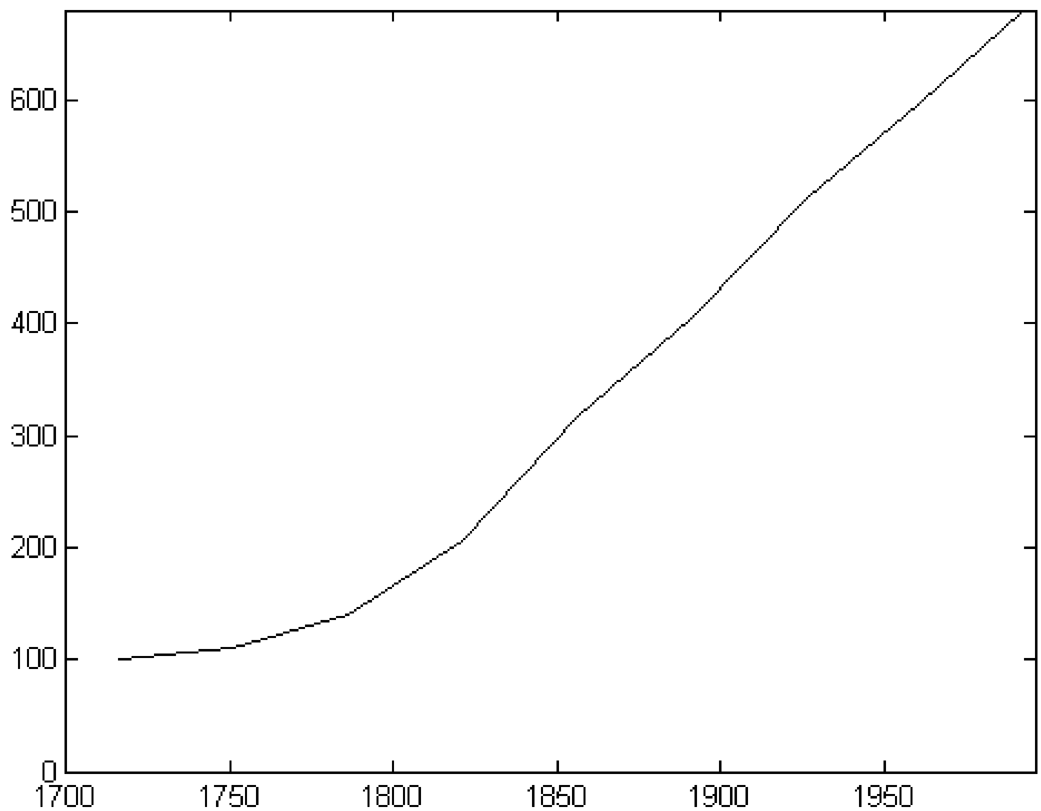


Figure 2: Population Growth Rate

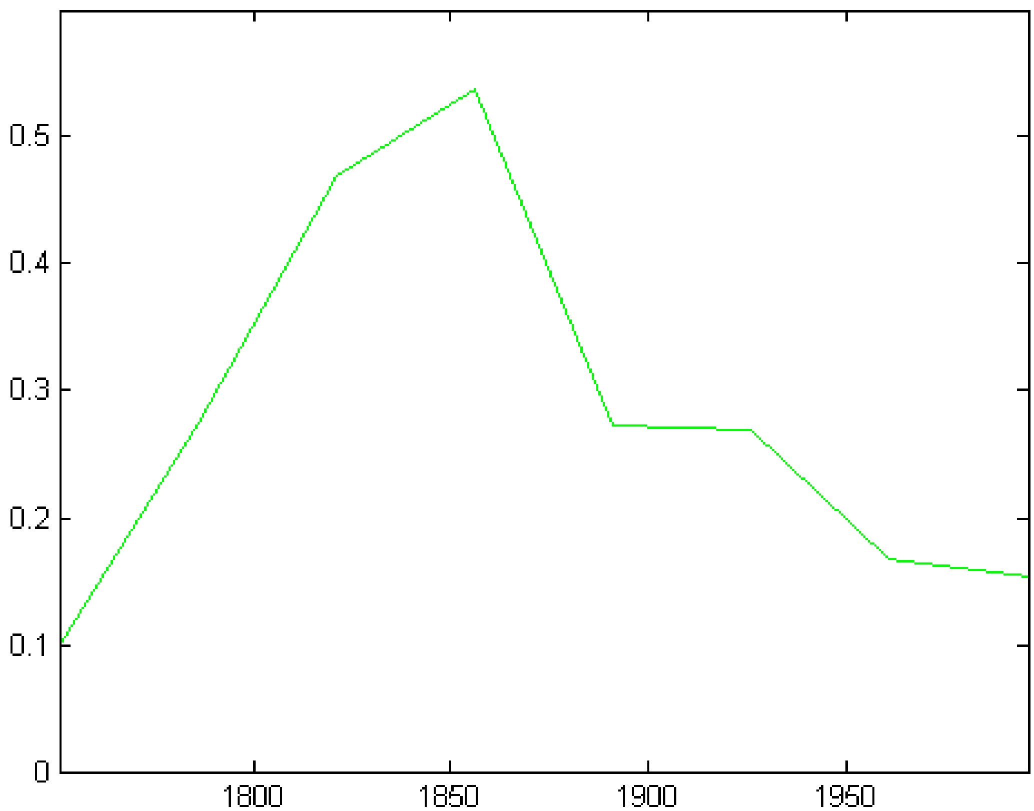


Figure 3.1: Gross Reproduction Rate

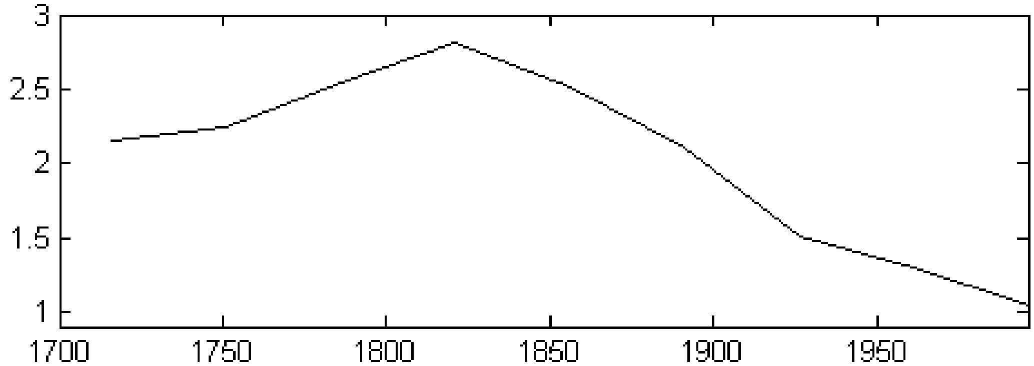


Figure 3.2: Average Life Expectancy

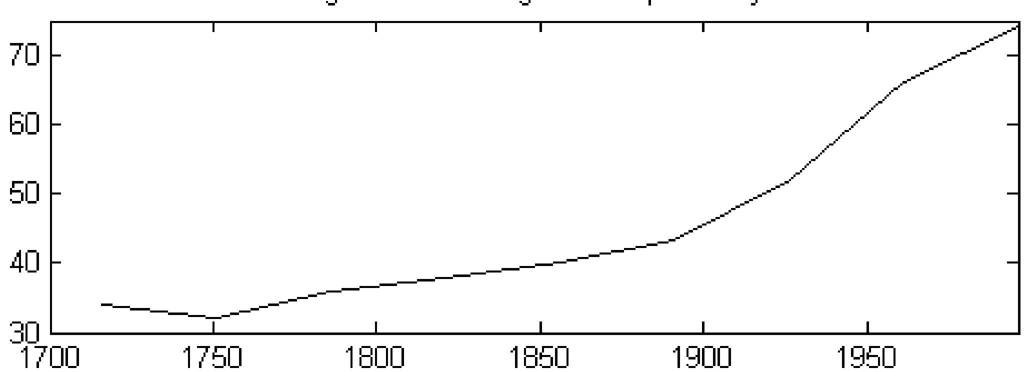


Figure 4.1: GDP

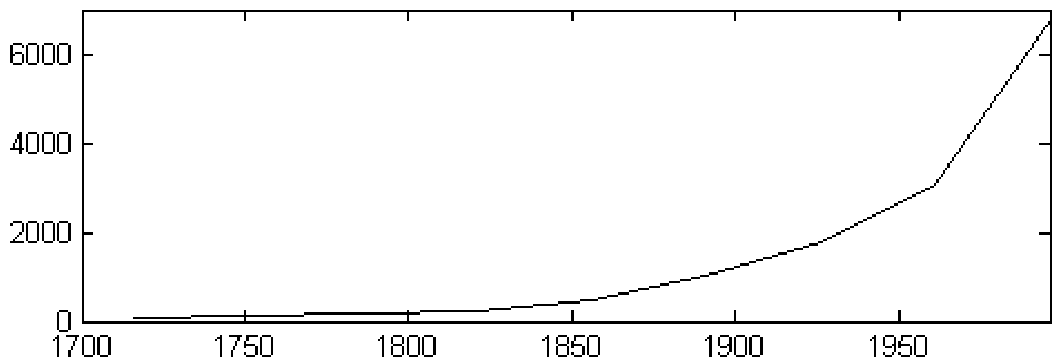
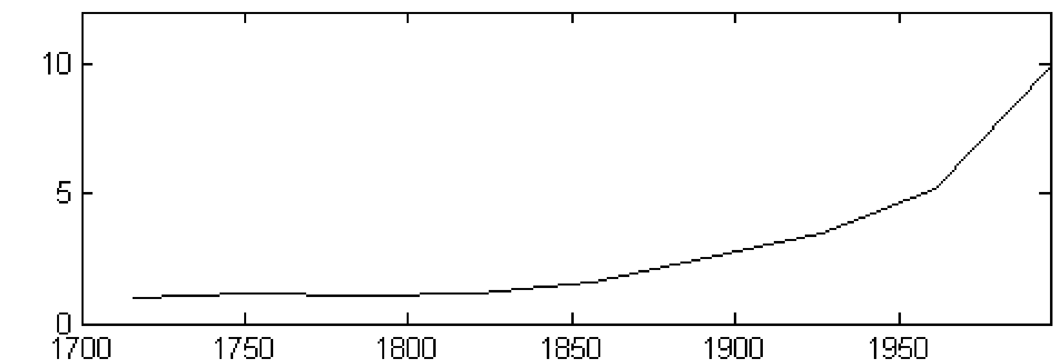


Figure 4.2: GDP per capita



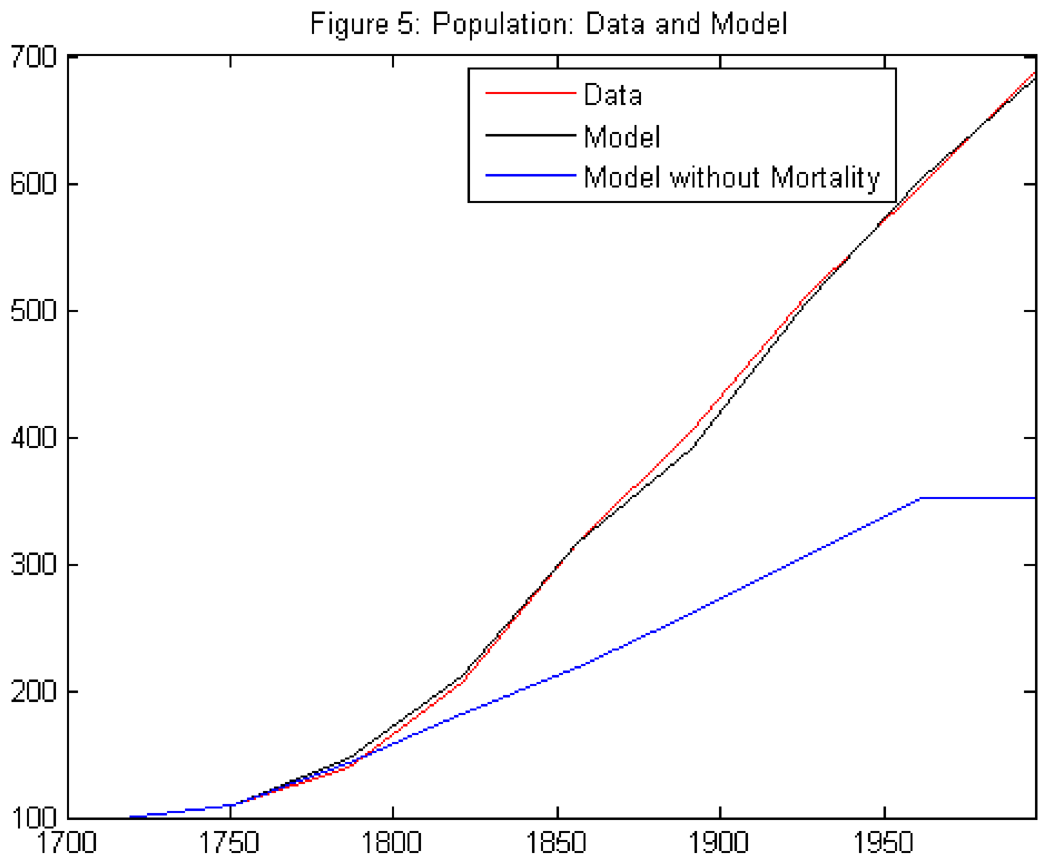


Figure 5: Population: Data and Model
700
600
500
400
300
200
100
1700
1750
1800
1850
1900
1950
Data
Model
Model without Mortality

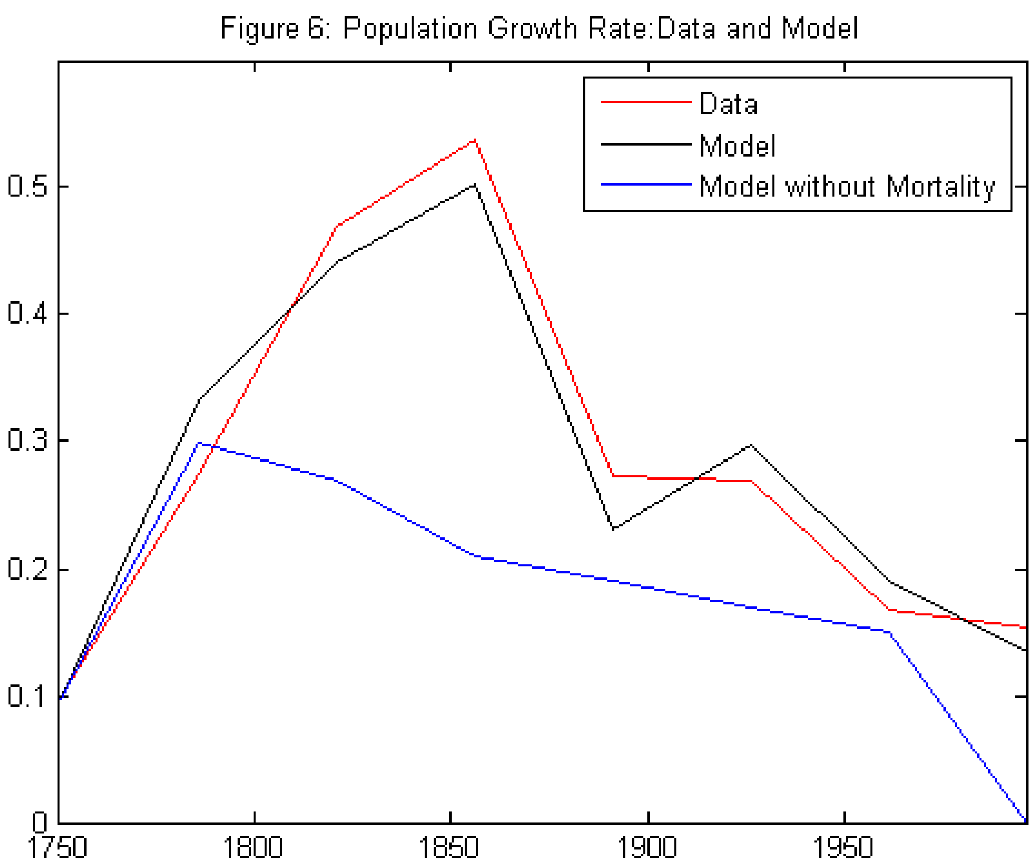


Figure 6: Population Growth Rate:Data and Model
Data
Model
Model without Mortality
0.5
0.4
0.3
0.2
0.1
0
1750
1800
1850
1900
1950

Figure 7: GRR: Data and Model

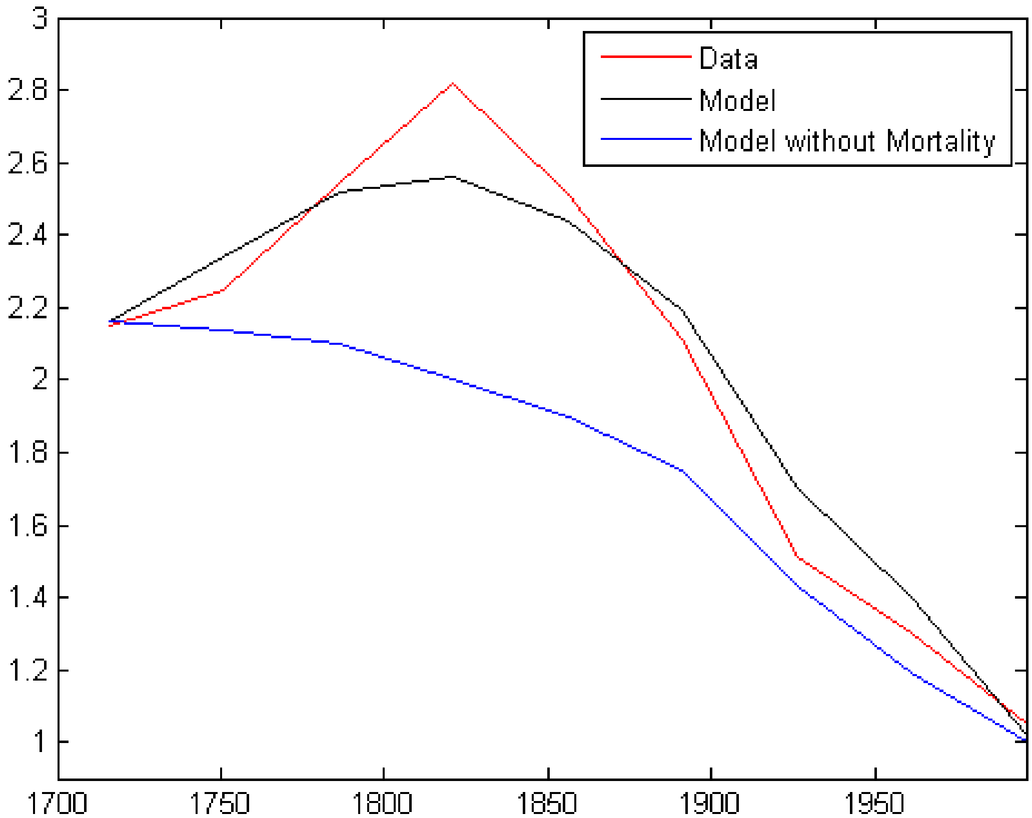


Figure 8.1: Output: Data and Model

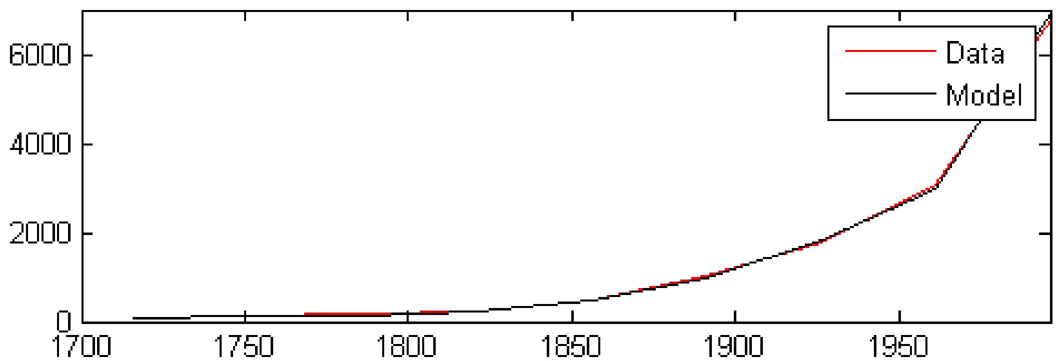
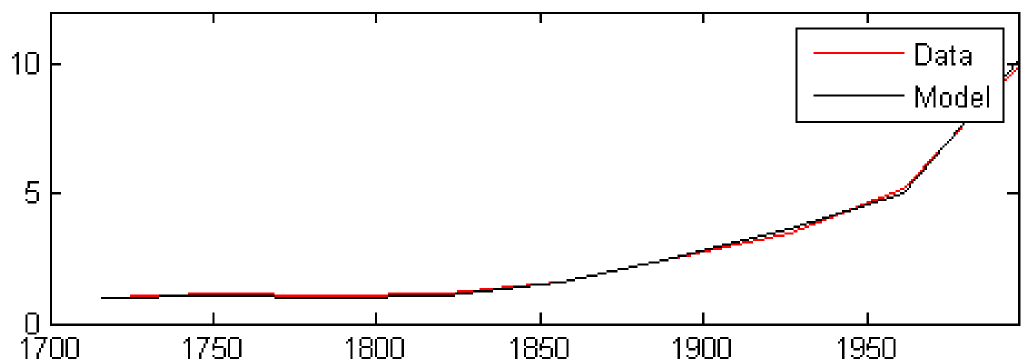


Figure 8.2: Output per capita:Data and Model



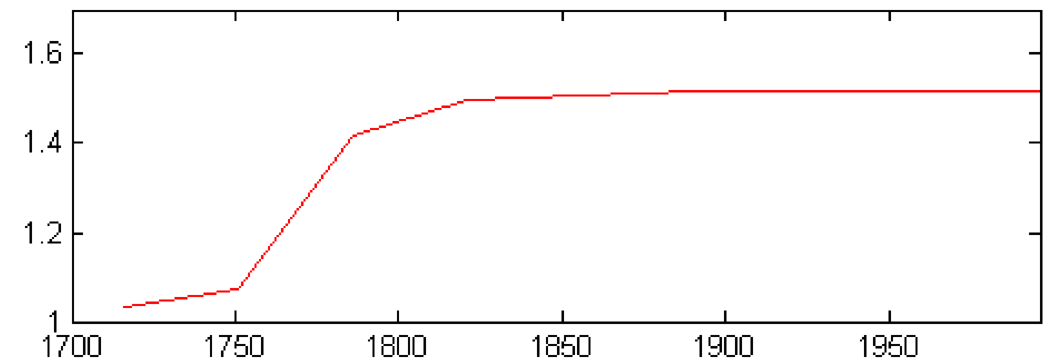


Figure 9.1: Growth rate of Average Technological Progress

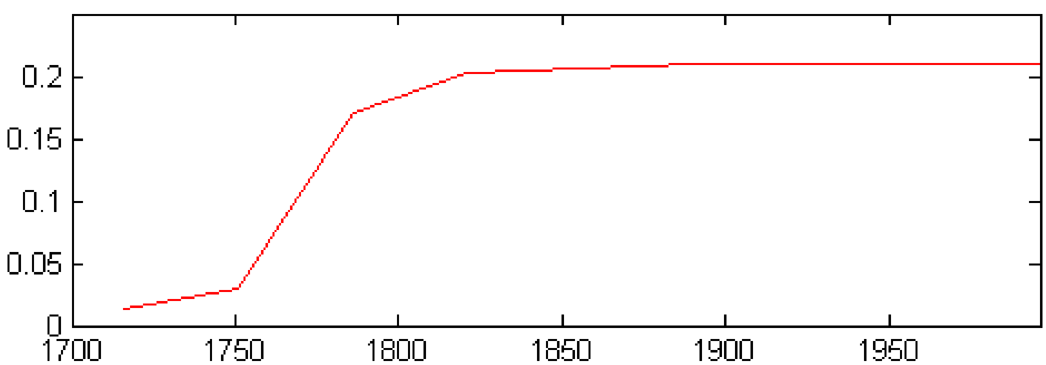


Figure 9.2: Fraction of Total Time Spent for Children

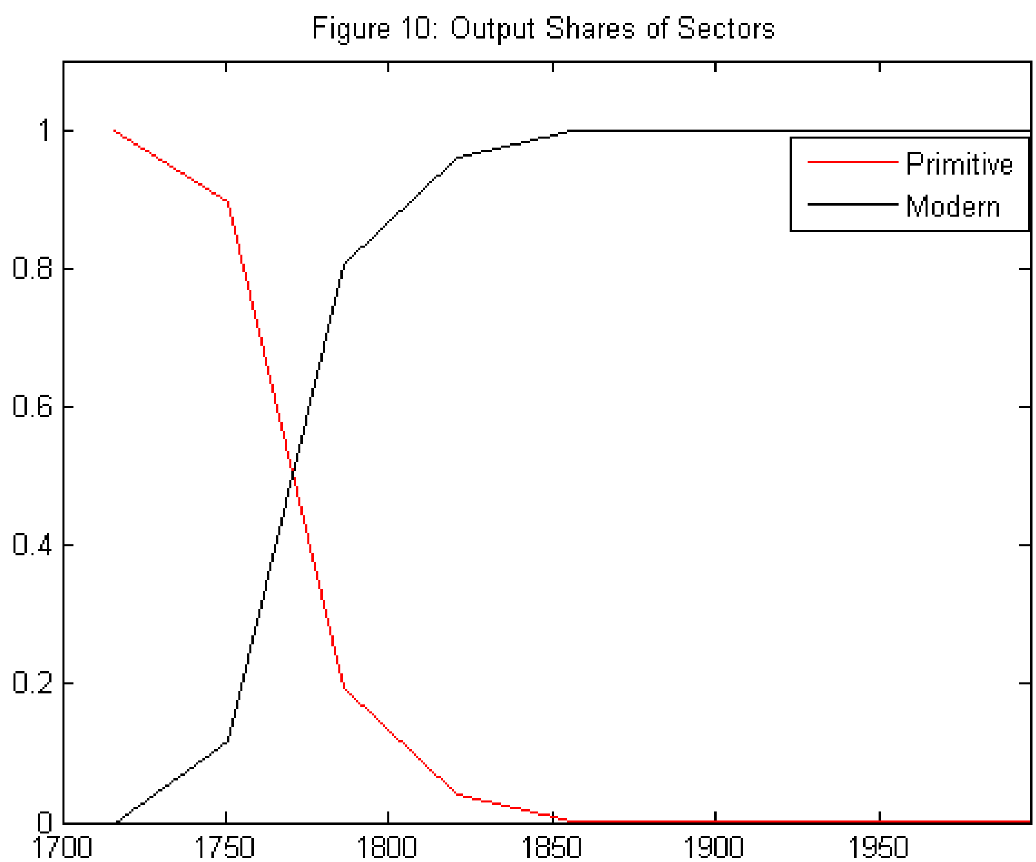


Figure 10: Output Shares of Sectors
Primitive
Modern
1
0.8
0.6
0.4
0.2
0
1700
1750
1800
1850
1900
1950