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Transparency, Complementarity and Holdout

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Abstract

This article characterizes the conditions under which holdout (i.e. bargaining inefficiency) may, or may not be significant in a two-sided, one-buyer-many-seller model with complementarity. Our central result is that the severity of holdout (i.e. inefficiency) is critically dependent on three factors, (a) the transparency of the bargaining protocol, (b) the outside option of the buyer, and (c) the marginal contribution of the last seller. We find that although the accepted wisdom that holdout is severe, goes through whenever either the buyer has no outside option, or the bargaining protocol is secret, the holdout problem however is largely resolved whenever either the bargaining protocol is transparent and the buyer has a positive outside option, or if the marginal contribution of the last seller is not too large.

Keywords - Multi-person bargaining, holdout, complementarity, efficiency, secret offers, public offers, Coase theorem, transparency.

JEL Classification Number - C78, D23, D62, L14.

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1 Introduction

Many economic activities involve a single buyer seeking to acquire and combine objects from several sellers, e.g. drug development often requires separate patents, land developers have to combine separate plots of land and firms often purchase assets of other firms. Further, firms often bargain with multiple unions, and, in case of financial distress, with multiple creditors. Coase's (1960) famous railroad example considers a situation where a railroad has to acquire plots of land from several farmers.¹

Given the complementarity inherent in all such activities, received wisdom suggests that the outcome is likely to exhibit *holdout*, with sellers refusing to transact until others have already done so, when commencing production becomes more profitable, allowing the sellers who holdout to extract a greater share of the surplus. In such a scenario holdout is expected to cause inefficiencies, viz. delay, or the implementation of an inefficient project, even, in the presence of strong complementarity, a complete breakdown of negotiation.²

We formalize such interactions as a non-cooperative bargaining problem with one buyer and many sellers, focusing on the tension between the complementarity intrinsic to such a setup and efficiency. Our central result is that the severity of holdout (i.e. inefficiency) is critically dependent on three factors, (a) the *transparency* of the bargaining protocol, (b) the outside option of the buyer, and (c) the marginal contribution of the last seller. We find that although the accepted wisdom that holdout is severe, goes through whenever either the buyer has no outside option, or the bargaining protocol is secret, the holdout problem however is largely resolved whenever either the bargaining protocol is transparent and the buyer has a positive outside option, or if the marginal contribution of the last seller is not too large.

To this end we consider the interaction between one buyer and $n \geq 2$ sellers, all of whom have an object to sell. These objects can be combined to produce value. In

¹Similarly, Cournot analyzed a problem where a brass manufacturer has to buy copper and zinc from two monopoly suppliers.

²In the context of land acquisition, many countries, including the USA, have promulgated *eminent domain* laws (that allow land acquisition for public purposes on payment of compensation), presumably to counter this holdout problem. One of our motivating examples comes from West Bengal, India, where the state government used the Land Acquisitions Act, 1894, to acquire land for building an automobile factory for Nano (the one lakh rupee car) in Singur (West Bengal). It has also been argued by some, e.g. Parisi (2002), that problems like excessive fragmentation can be traced, at least partially, to such holdout problems. In the patents literature, Shapiro (2001) suggests that holdout can be a serious obstacle to R&D.

particular, all sellers have identical objects, with a project using m objects having value $v(m)$. We assume that $v(m)$ is strictly super-additive, with perfect complementarity arising as a special case where $v(m) = 0, \forall m < n$. This formulation allows for different degrees of complementarity, with the buyer being allowed to implement a ‘partial’ project that does not require the use of all objects.

The negotiation process that we consider is a natural extension of the Rubinstein (1982) bargaining model in which the agents, the buyer, as well as the sellers, make simultaneous offers to the other side of the market in alternate periods. Note that this protocol is symmetric in the sense that at no point during the negotiation, is an active seller shut out of the bargaining process. The buyer can choose to exit at any period when he can either implement a partial project (involving the objects collected so far), or opt for an outside option of $C \geq 0$. Further, buyer offers are publicly observable (though we later also allow for secret offers). The question of interest is the possibility of obtaining equilibria that are *asymptotically efficient*, i.e. one where the grand project is implemented with ‘negligible’ delay costs, and moreover buyer payoffs are bounded away from zero.³

In order to focus on the holdout problem more sharply, we begin by analyzing the case of perfect complementarity, so that $v(s) = 0, \forall s < n$. We show that if the outside option is zero then in any equilibrium the buyer payoff is negligible, in that it is bounded above by an amount that goes to zero as the discount factor goes to one. Thus in this case no asymptotically efficient equilibrium exists and the holdout problem is very severe. The result however changes dramatically if the buyer has a positive outside option. In that case there exist equilibria that are asymptotically efficient and the buyer obtains a payoff close to $1/2$.

We then examine a natural alternative to the public offer protocol studied so far, namely one where buyer offers are secret so that a seller only observes the offer being made to her, but not the offers that are being made to the other sellers. In this case holdout is severe with the buyer opting out at the first period itself without collecting any one of the objects, so that there is complete bargaining breakdown.

These results are of interest for several reasons. First, these establish that protocol transparency, as well as the presence of outside options for the buyer, are critical as far

³Given the folk-theorem like results in Chatterjee et al. (1993) and Herrero (1985), the most that we can hope for here is the existence of at least one equilibrium that is efficient, at least asymptotically. See Hyndman and Ray (2007) on this issue though.

as efficiency is concerned.⁴ Second, we identify a class of scenarios where the received wisdom that holdout is severe, goes through. These results thus extend the literature on one-buyer-many-seller bargaining problems with complementarity, viz. Cai (2000, 2003) and Menezes and Pitchford (2004), which concludes that inefficiency is endemic in such setups (see detailed discussion later). Equally interestingly though, we find that efficient equilibria exist whenever offers are public and the buyer has a positive outside option (however small), so that the holdout problem is arguably largely resolved in this case.⁵

The intuition for these results depends on an interplay of two factors. Suppose the buyer has already acquired $n - 1$ of the objects. Then the buyer has a strong incentive to conclude bargaining with the remaining seller also. Thus this seller (and thus *ex ante* all sellers) has some bargaining power. On the other hand, the buyer can opt out, which is a potential source of bargaining power for the buyer. When offers are secret however, the buyer cannot credibly commit to opt out of the game. This is because the buyer can make secret agreements with the other sellers, which may reduce his incentive to opt out. With public offers and a positive outside for the buyer however, such secret agreements are not possible so that threats of opting out are credible. Hence the difference in results.

We then extend the analysis to the case where complementarity is less than perfect, so that $v(s)$ is not necessarily zero for $s < n$. We find that the marginal contribution of the n -th seller, i.e. $1 - v(n - 1)$ plays a critical role in the analysis. Whenever the marginal contribution of the last seller is not too small, in the sense that $v(n - 1) < 1/2$, we find

⁴Interestingly, transparency plays a key role in efficiency calculations in, e.g. the industrial organization literature also. It is also often commonly argued that transparency of governance is a key factor in determining the level of corruption in a society. Stiglitz (1989), for example, shows that greater transparency, in the form of lower search costs, can lead to a reduction in prices in a competitive model. In the context of dynamic oligopoly, the effect is less clear. Greater information sharing among the competing firms may make defection from a collusive outcome easier to detect, thus making collusion easier to sustain. Albaek et al. (1997), for example, show how greater information sharing led to an increase in prices in the Danish ready-mixed cement industry. Vives (2002), among others, examines the incentive for information sharing among competing firms in a static oligopoly. In contrast to Stiglitz (1989) and Vives (2002), however, in the present article transparency refers to the observability of the strategic decisions taken by the agents. Further, in contrast to the dynamic oligopoly literature, in the present article transparency promotes, rather than hinders efficiency.

⁵Interestingly, this view finds some support in the empirical literature on land acquisition. Benson (2005), for example, discusses examples where private railroads managed to collect the required plots without any government intervention. In the Indian context, for example, whereas the Nano project in Singur, West Bengal ran into problems, around the same time there were many instances of trouble free land acquisition by private agents, even in West Bengal.

that the results are analogous to those under perfect complementarity (except for the case when $C < v(n-2)$ and offers are secret). However, when $v(n-1) > \frac{1}{2}$, there is an *efficient* equilibrium where the grand project is implemented in the first period and, moreover, the buyer obtains a payoff of 1. This result holds under both bargaining protocols, public, as well as secret offers. These results show that the assumption that $v(n-1) < 1/2$, provides a precise generalization of the perfect complementarity case.

Moreover, for $v(n-1) > 1/2$, we find that there exist efficient equilibria even when buyer offers are secret. The intuition can be simply stated. If $v(n-1) > 1/2$, then once the buyer reaches an agreement with $n-1$ of the sellers, in any continuation game the buyer must obtain at least $v(n-1)$. In fact, given that $v(n-1) > \frac{1}{2}$, the buyer has a strong incentive to terminate the project immediately and implement $v(n-1)$. because the seller then has a payoff of zero, this consideration considerably reduces the ability of the remaining seller, and *ex ante* of all the sellers, to holdout, allowing the buyer to extract all the surplus from trading.

1.1 Relation to Existing Literature

This article traces its ancestry to one of the most important recent research areas, the theory of coalitional bargaining. Following the seminal work of Rubinstein (1982), as well as the literature on core implementation, researchers have studied the non-cooperative foundations of various cooperative solution concepts, in particular the core and the Shapley value. Whereas Gul (1989) and Hart (1996) are concerned with the Shapley value, Chatterjee et al. (1993), Serrano (1995) and Krishna and Serrano (1996) study implementing the core. Moreover, whereas Chatterjee et al. (1993) consider exogenous, but deterministic bargaining protocols, Okada (1996) examines a model with random proposers. There is also a relatively recent branch of this literature that tries to endogenize the process of coalition formation, e.g. Perry and Reny (1994), Bloch (1996), Ray and Vohra (1997, 1999), and Okada (2000), as well as allow for contractual renegotiation, e.g. Seidmann and Winter (1998), Hyndman and Ray (2007), etc.

Formal treatments of the holdout problem (using game theoretic arguments) were first provided in Eckart (1985) and Asami (1988). The theoretical literature was further developed in Cai (2000, 2003) and Menezes and Pitchford (2004). Like us, Cai (2000) and Menezes and Pitchford (2004) analyze a *cash-offer* model in which the seller is paid immediately after an agreement is arrived at. In contrast, Cai (2003) allows the buyer to offer a *contingent contract* that promises to pay the seller a given amount only when

production is carried out. Whereas Cai (2000) and Menezes and Pitchford (2004) find that inefficiency in the form of delays must occur in equilibrium, Cai (2003) finds that the buyer payoff is arbitrarily close to zero if the number of sellers is large.

Our results extend the literature by clarifying the role of protocol transparency in obtaining efficiency, showing that whether holdout is serious or not depends on three factors, first, the nature of the bargaining protocol, second, the presence of outside options for the buyer, and third, on how complementary the sellers are. Inter alia, we provide a precise characterization of the conditions under which inefficiency, similar to that obtained in Cai (2000, 2003) and Menezes and Pitchford (2004), obtains. This article also contributes to the literature by providing a framework that has some appealing features. First, the bargaining protocol adopted by us is symmetric, second, we allow for outside options (which however can be vanishingly small), as well as general production technologies (which however does include perfect complementarity as a special case).

The rest of the article is organized as follows. Section 2 describes the framework, and, in 2.1, also establishes some preliminary results. Under perfect complementarity, Section 3 examines the case where buyer offers are public, whereas Section 4 examines the case where buyer offers are secret. Section 5 extends the analysis to the case where complementarity is less than perfect. Section 6 concludes. Proofs of some of the propositions and lemmas are collected together in the Appendix.

2 The Framework

A buyer faces $n \geq 2$ sellers. Each of these sellers has an identical object and the buyer has a production technology that combines these objects to produce returns. We begin our analysis with the case of perfect complementarity in which *all* of the objects are needed to produce a positive return. In section 5, we indicate how our analysis can be extended so as to incorporate general production technologies. We normalize units and assume that the combined value of these objects is 1.

The buyer and the sellers bargain over the price of the objects. The bargaining protocol that we use here is a simple variant of the standard Rubinstein (1982) procedure. Time is discrete and continues for ever, so that $t = 1, 2, \dots$. At the start of any period t , there is a set of ‘active’ sellers who are yet to sell their objects. Each period t is divided into three sub-stages.

We begin by describing the first two stages. The first stage of t consists of one side of the market making offer(s) to the other side, followed, in stage two, by the accep-

tance/rejection decisions of the other side. We assume without loss of generality that the buyer makes his offers in odd numbered periods, whereas the sellers make their offers in even numbered ones.⁶

Thus at $t = 1, 3, \dots$, the buyer offers a price vector to the set of sellers active at that point of time. Offers made by the buyer are ‘publicly observable’ in which each seller observes the entire vector of offers made by the buyer.⁷ These sellers then simultaneously decide whether to accept, or reject the offer made to each one. Once a price is agreed upon between the buyer and any seller, the concerned seller immediately receives the agreed upon price and exits the game.

In even numbered periods, $t = 2, 4, \dots$, the sellers who are yet to sell their objects (and we refer to these sellers as the ‘active’ sellers) simultaneously make offers to the buyer. The buyer observes all of these offers and decides which of these offers to accept, if at all.

At the final stage of any period t , either the buyer has obtained all of the objects in which case, the game is over and the buyer gets 1 by implementing the project. If, however, all sellers are yet to sell at this stage, the buyer has to decide whether to continue negotiation or not. If he decides not to continue, we say that the buyer takes up his outside option and we let $C \geq 0$ denote the value of this outside option. Throughout, we assume that $C < 1/2$. If however, the buyer decides to continue negotiation, the game goes on to the next period and the negotiation proceeds with the set of sellers who are yet to sell their objects.

For any given play of the game, the history at the beginning of stage i of date t includes information on all the past offers, the acceptance/rejection decision of the players and the set of active sellers that are yet to sell their objects. All such information are common knowledge among the players.

Finally, we assume that all agents are risk neutral and that they have a common discount factor δ , where $0 < \delta < 1$.

Our focus in this article is on studying sequential equilibria in pure strategies. The central issue addressed here is if, for δ large,⁸ one can support equilibrium in which the

⁶The assumption that the buyer makes an offer to every seller, is without loss of generality because the buyer can always make a negative offer to any seller that is surely to be rejected by the seller.

⁷In section 4, we consider an alternative scenario in which a seller can only observe her component of the offer and does not know what offers are received by other sellers.

⁸The focus on large discount factors is of course standard in the bargaining literature, see e.g. Cai (2000, 2003) and Chatterjee et al. (1993) among many, and merely captures the fact that one is interested in outcomes that obtain when bargaining frictions, namely the gap between subsequent periods, is vanishingly small.

buyer implements the grand project and in which the ‘delay’ cost, if any, is arbitrarily close to zero and in which the buyer makes strictly positive payoff. We term such equilibrium outcomes as ‘asymptotically efficient’.

2.1 Some Preliminary Results

In this sub-section we record some preliminary observations that will be used for the main results in this article. Consider the situation where the buyer has already acquired $n - 1$ of the objects. Because $C < 1/2$, for δ is large, it is straightforward to extend proposition 3.12.2 in Osborne and Rubinstein (1990) to show that the buyer will prefer to reach an agreement with the remaining seller as well, and implement the grand project rather than opt for his outside option. From Rubinstein (1982), it also follows that in such a case, the continuation payoffs of the proposer and the responder are, respectively, $\frac{1}{1+\delta}$ and $\frac{\delta}{1+\delta}$. For any $C \geq 0$, let $\delta(C)$ satisfy

$$\frac{\delta^2}{1+\delta} = C \tag{1}$$

Note that $\delta(C)$ is well defined and is strictly less than 1 because $C < 1/2$. Moreover, at $C = 0$, $\delta(C) = 0$.

Lemma 1 *Fix $\delta > \delta(C)$, and consider any history that starts with exactly one active seller. In any continuation equilibrium of the game*

- (a) *the buyer never opts for the outside option and must continue negotiation,*
- (b) *if t is odd and the buyer has to make an offer, he offers the price $\frac{\delta}{1+\delta}$ which is accepted by the seller, and*
- (c) *if t is even and the seller has to make an offer, the seller asks for $\frac{1}{1+\delta}$, which is accepted by the buyer.*

The next lemma puts a lower bound on a seller’s payoff when the buyer is making acceptable offers to all active sellers.

Lemma 2 *Fix $\delta > \delta(C)$ and consider any history with $m \geq 1$ active sellers at date t , where t is odd. If the equilibrium calls for the buyer to make an acceptable offer to all of the remaining m sellers, then each seller must get at least $\frac{\delta}{1+\delta}$.*

Proof. Because at t , the buyer makes an acceptable offer to all existing sellers, if any of these sellers rejects, then at the end of period t , the buyer would have acquired exactly

$n - 1$ objects. Now from Lemma 1, it follows that the buyer must continue negotiation and the remaining seller will ask for a price of $\frac{1}{1+\delta}$ which will be accepted by the buyer. Thus by rejecting the offer, given that the rest of the sellers are accepting their offers, any seller can assure himself a payoff of $\frac{\delta}{1+\delta}$ and the result follows. ■

Lemma 3 *Let $\delta > \delta(C)$. Fix an equilibrium and a history with $m \geq 1$ active sellers. Then, the continuation payoff of the buyer in this equilibrium is no more than $\frac{1}{1+\delta}$.*

Proof. By Lemma 1, the result is clearly true for $m = 1$. So assume an induction hypothesis that the result is true for histories that begin with $m = 1, \dots, n - 1$ active sellers. Consider now an history that begins with $m = n$ active sellers. Let $Y^*(\delta)$ be the supremum of the buyer's payoff in any continuation equilibrium. If the Lemma is false, then, $Y^*(\delta) > \frac{1}{1+\delta}$ and there is an equilibrium outcome in which an agreement is reached in period t with the buyer getting a payoff arbitrarily close to $Y^*(\delta)$ which in turn is strictly greater than $\frac{1}{1+\delta}$. Because $C < 1/2$, it follows that in this equilibrium outcome, the buyer does not take up his outside option and implements the project. Furthermore, at t , the number of active sellers must be n , otherwise, the induction hypothesis will apply. Now if t is odd, then it is the buyer who is making an acceptable offer to all of the n active sellers. Because $n \geq 2$, by Lemma 2, the buyer's payoff is then no more than $\frac{1-\delta}{1+\delta}$, a contradiction. Therefore, at t , it is the sellers who are making these acceptable offers. Because the payoff to the buyer is more than $\frac{1}{1+\delta}$ and $n \geq 2$, at least one of the sellers is making an offer P_j that is strictly less than $\frac{\delta}{1+\delta}$. Let this seller deviate by asking for a slightly higher price $P_j + \epsilon$. We argue that for ϵ positive but small, the buyer must accept this deviating offer. This is because by accepting all of the offers, the buyer gets a payoff arbitrarily close to $Y^*(\delta)$ whereas if he rejects all/some offers, then his continuation is at most $\delta Y^*(\delta)$. Because $\delta < 1$, there exists $\epsilon > 0$ but small for which the buyer must accept the offer. But seller j then has a profitable deviation. ■

Turning to the sellers, the final lemma gives us an upper bound on the the maximum payoff that a seller can get in an equilibrium.

Lemma 4 *Let $\delta > \delta(C)$ and consider any history ending with $m \geq 1$ active sellers. Then in any continuation equilibrium after this history, the payoff to no active seller is more than $\frac{1}{1+\delta}$.*

Proof. Because of Lemma 1, the result is clearly true for all histories in which there is exactly one active seller. As an induction hypothesis, assume that the result is true after

all histories with m active sellers where $m = 1, 2, \dots, n - 1$ and let $m = n$. If the Lemma is false, the supremum of equilibrium payoff of some seller i is Y_i which is strictly greater than $\frac{1}{1+\delta}$. Therefore, there must exist a continuation equilibrium in which an offer P_i is agreed upon by the buyer and seller i at some date t , with $P_i > \frac{1}{1+\delta}$. Now because of the induction hypothesis, all sellers must be active at this date. Moreover, the negotiation must conclude by the end of period t . Otherwise, negotiation continues in the next period and by Lemma 3, the maximum payoff to the buyer in the continuation game is at most $\frac{1}{1+\delta}$. Because $P_i > \frac{1}{1+\delta}$, the buyer's payoff in this equilibrium is then at most $\frac{\delta}{1+\delta} - P_i$. But this is negative, which is impossible.

Now if t is odd, the buyer is making an acceptable offer to all sellers at this date. By Lemma 2, the payoff to every other seller is no less than $\frac{\delta}{1+\delta}$. If the buyer deviates by offering $P_i - \eta$ to seller i , and an offer of $P_j + \epsilon$, where $(n - 1)\epsilon < \eta$, then all sellers must accept and this will constitute a profitable deviation for the buyer.

Assume then that t is even and it is the sellers who are making acceptable offers P_j resulting in a payoff of $1 - \sum_j P_j$ for the buyer. Suppose the buyer deviates, rejects seller i 's offer, but accepts the rest of the offers. Then in period $t + 1$, the buyer will offer $\frac{\delta}{1+\delta}$ to seller i which, by Lemma 1, must be accepted by seller i . Such a deviation will yield a payoff of $\frac{\delta}{1+\delta} - \sum_{j \neq i} P_j$. Because $P_i > \frac{1}{1+\delta}$, the buyer will be better off from this deviation.

■

Remark 1 *Whereas Lemmas 1-4 have been proved under the assumption that the bargaining protocol involves 'publicly observable' offers, it is easy to check that all of these Lemmas hold even in the 'secret offer' case where each seller could only observe the offer she receives.*

3 The Outside Option and the Holdout Problem

In this section, we prove two results. First, we demonstrate the severity of the holdout problem in the situation when the buyer's outside option is zero. In particular, we show that for any $\delta > 0$ and in any equilibrium, the buyer's payoff can be at most $\frac{1-\delta}{1+\delta}$. When players are sufficiently patient, i.e, δ is close to 1, the buyer's payoff in any equilibrium thus goes to zero. Our second result, however shows that when $C > 0$ (no matter how small), for sufficiently large δ , there are equilibria that are asymptotically efficient and in which the buyer gets a payoff of $\frac{\delta}{1+\delta}$.

Proposition 1 *Suppose that $C = 0$. Then for any $\delta > 0$, in any equilibrium the buyer's payoff is at most $\frac{1-\delta}{1+\delta}$.*

We provide the intuition for the result when $n = 2$. First, we note that if the buyer makes an offer of $\frac{\delta}{1+\delta}$ to each of the sellers, these offers will be accepted by the sellers (by Lemma 4) and thus when $n = 2$, the buyer's payoff can not be less than $\frac{1-\delta}{1+\delta}$, which is strictly positive. Thus, if $C = 0$, the requirement of subgame perfection implies that in any equilibrium, the buyer can never quit negotiation and moreover, because of Lemma 1, any seller who holds out will expect to get at least $\frac{\delta}{1+\delta}$. This explains why $\frac{1-\delta}{1+\delta}$ is the maximum payoff that the buyer can receive if he has to make an acceptable offer to both of the sellers. Of course, this argument is incomplete as it leaves open the possibility of an outcome where the agreement with the sellers takes place sequentially. The proof that follows, however formally rules out such outcomes as well.

Remark 2 *It is important to stress that when $n > 2$, Proposition 1 is critically dependent on our assumption that the sellers are not allowed to randomize in their accept/reject decisions.⁹ To see why this is so, consider the situation with $n \geq 3$ sellers. Assume that in period 1, the buyer makes an offer of $P' < 1/n$ to each seller, where $1 - nP' > C$: each seller accepts this offer with probability r^* strictly less than 1. Furthermore, the buyer continues negotiation with probability one only if no fewer than two sellers have accepted his offer and he opts out otherwise. Given these strategies, if a seller rejects the current offer P' , she now faces the risk of getting zero if at least two other sellers reject because, in that event, the buyer exits the game. On the other hand, when no more than one other seller rejects her offer, this seller's payoff will be strictly higher than P' . It thus follows that there exists a value of r^* for which each seller is indifferent between accepting the offer P' , or rejecting it. We, however, note that such outcomes are necessarily inefficient as with strictly positive probability, the buyer must opt out implementing any project and thus even if mixed strategies are available, for δ close to 1, it is impossible to achieve asymptotic efficiency with probability one and in which buyer gets strictly positive payoff.¹⁰*

Proof of Proposition 1. Let $Y^*(m)$ denote the supremum of the buyer's payoff in any continuation equilibrium following a history with m active sellers. We will prove that when $m \geq 2$, $Y^*(m) \leq \frac{1-\delta}{1+\delta}$.

⁹For $n = 2$, however, the proposition holds even when players are allowed to randomize in their choice of strategies.

¹⁰A formal statement and a proof of this assertion is available upon request.

We first prove this result for $m = 2$. If the claim is false, then there exists an equilibrium in which the buyer's payoff is arbitrarily close to $Y^*(2)$ which in turn is strictly greater than $\frac{1-\delta}{1+\delta}$.

Clearly, in such an equilibrium, all sellers sell and moreover, the payoff to at least one of the remaining active sellers (label her seller 1) must be strictly less than $\frac{\delta}{1+\delta}$. Let t be the date at which an agreement with seller 1 takes place. Clearly by Lemma 1, both sellers must be present at that date by Lemma 1. Now if at t , seller 1 herself was making an offer, she could have asked for a slightly higher price, the buyer could not have rejected because his payoff in this equilibrium was arbitrarily close to $Y^*(2)$ (whereby rejecting he gets at most $\delta Y^*(2)$). Therefore, this offer was made by the buyer. Furthermore, at that date, the buyer could not have made an acceptable offer to seller 2 as well. Because then by Lemma 2, seller 1 could have rejected his offer and obtained $\frac{\delta}{1+\delta}$ in the next period. By Lemma 1, the final agreement then occurs in period $t + 1$ with seller 2 asking for a price of $\frac{1}{1+\delta}$. If P is the offer made to seller 1, then the payoff to the buyer in this equilibrium is given by $Y = -P + \frac{\delta^2}{1+\delta}$ to the buyer. Because Y is arbitrarily close to $Y^*(2)$, which is strictly positive, we have

$$P < \frac{\delta^2}{1+\delta} \quad (2)$$

Now observe that with exactly two sellers remaining, in any continuation equilibrium, the buyer must get a strictly positive payoff. This is because by Lemma 4, the buyer can get a payoff close to $\frac{1-\delta}{1+\delta}$ by making an acceptable offer of $P = \frac{\delta}{1+\delta} + \epsilon$ to both the sellers whenever it the buyer's turn to propose. Because $C = 0$, it follows that with two sellers remaining, the buyer will never exercise his outside option. Now suppose seller 1 deviates and rejects buyer's offer of P and makes a counter offer of $P' = \frac{P}{\delta} + \epsilon$ in the next period. Clearly, acceptance of this offer will be a profitable deviation for seller 1. We now argue that this offer will be accepted by the buyer for ϵ small. If the buyer were to accept seller 1's offer, his overall payoff (from period $t + 1$ perspective) can not be less than

$$Y^a = -\frac{P}{\delta} - \epsilon + \frac{\delta}{1+\delta}.$$

This is because the buyer can always reject seller 2's offer and in $t + 2$ offer $\frac{\delta}{(1+\delta)}$ to seller 2 which, by Lemma 1, will be accepted by seller 2.

On the other hand, if he were to reject this offer, then the maximum that he can obtain is $\delta Y^*(2)$ which by definition is arbitrarily close to δY . It is easy to check that $Y^a > \delta Y$ if and only if $P < \frac{\delta^2}{1+\delta}$ which is true because of (2). This proves that $Y^*(2) \leq \frac{1-\delta}{1+\delta}$.

Assume now as an induction hypothesis that the result be true for all $m = 2, \dots, n - 1$. If the claim is false, then there is an equilibrium in which the buyer obtains all of the

objects, implements the project at date t which gives the buyer a payoff arbitrarily close to $Y^*(n)$ which in turn is strictly greater than $\frac{1-\delta}{1+\delta}$. Because of the induction hypothesis, the number of active sellers m at this date t can be either $m = n$ or $m = 1$.

First consider the case $m = n$. Because $n \geq 2$, it follows from Lemma 2 that the buyer could not have made an acceptable offer at that date because then each seller would have obtained at least $\frac{\delta}{1+\delta}$ and the buyer's payoff will be no more than $\frac{1-\delta}{1+\delta}$. Thus, it is the sellers who are making the offers. But then any seller can ask for a slightly higher price which must be accepted by the buyer.

Finally we argue that the equilibrium cannot involve the buyer reaching an agreement with the single seller, label this seller 1, at the last period. Now at $t - 1$, all n sellers must have been present, otherwise, the induction hypothesis would have applied. Now if t is odd, then at $t - 1$, seller 1 was making an unacceptable offer. If seller 1 at $t - 1$ asked for $P \in (\frac{\delta^2}{1+\delta}, \frac{\delta}{1+\delta})$, the buyer would have accepted this offer. Because this yields him a higher payoff. This would thus have been a profitable deviation for seller 1. Therefore t must be even and by Lemma 1, seller 1 is asking for $P = \frac{1-\delta}{1+\delta}$ at this date. The buyer's payoff from the perspective of period $t - 1$ is thus $\frac{\delta^2}{1+\delta} - \sum_{i \neq 1} P_i$. Because $P_i \geq 0, \forall i$, and the buyer's payoff is strictly greater than $\frac{1-\delta}{1+\delta}$, for every seller $i \neq 1$, we have

$$P_i < \frac{\delta^2}{1+\delta} - \frac{1-\delta}{1+\delta} \quad (3)$$

If any of these sellers (say seller 2) rejects buyer's offer in period $t - 1$, given that the other sellers are accepting their respective offers, next period would begin with exactly two active sellers. In the continuation game, Because $Y^*(2) \leq \frac{1-\delta}{1+\delta}$, it follows that whenever the buyer has to make an offer with two sellers present, he will offer exactly $\frac{\delta}{1+\delta}$ to each of them. Thus, the worst payoff to seller 2 following this deviation is that the agreement takes place in $t + 1$ with the buyer making an offer of $\frac{\delta}{1+\delta}$ to seller 2. Thus, from the perspective of period $t - 1$, seller 2 can assure himself a payoff of $\frac{\delta^3}{1+\delta}$. Because $\delta < 1$, it follows that

$$\frac{\delta^3}{1+\delta} > \frac{\delta^2}{1+\delta} - \frac{1-\delta}{1+\delta} \quad (4)$$

Thus, using equation (3), it follows that $\frac{\delta^3}{1+\delta} > P_2$. This therefore constitutes a profitable deviation for seller 2. ■

3.1 Efficient Equilibria with $C > 0$

For any $C > 0$, let $\delta^0(C)$ satisfy $\frac{\delta^3}{1+\delta} = C$. Because $C > 0$ and $C < 1/2$, $\delta^0(C)$ as defined is strictly less than 1. Moreover Because $\delta < 1$, $\delta^0(C) > \delta(C)$ as defined in equation (1).

Note that δ^0 goes to 0, as C goes to 0.

Proposition 2 *Let $C > 0$ and fix $\delta > \delta^0(C)$. Then, there exists an equilibrium in which the buyer's payoff is exactly $\frac{\delta}{1+\delta}$.*

Comparing Propositions 1 and 2, we find that the results are dramatically different depending on whether the buyer has an outside option (however small), or not. The intuition for this difference arises because with $C = 0$, the buyer can never exercise his outside option. Consequently, and as the proof of Proposition 1 shows, the buyer's maximum payoff in any equilibrium is at most $\frac{1-\delta}{1+\delta}$. This payoff, however, is strictly less than C for sufficiently large δ , whenever C is strictly positive. Hence the buyer has a credible exit option which allows the buyer to extract approximately half of the surplus.

Remark 3 *Note that for δ large, $\frac{\delta}{1+\delta}$ is also the maximum payoff that the buyer can get in any equilibrium. This is true because (a) by Lemma 3, we know that the buyer's payoff in any equilibrium is no more than $\frac{1}{1+\delta}$, and (b) because of Lemma 2, if the buyer were to make an acceptable offer at $t = 1$, each seller must obtain at least $\frac{\delta}{1+\delta}$ which gives a payoff to the buyer that is arbitrarily close to zero for δ large. Thus, in an equilibrium that gives the buyer his maximum payoff, the agreement (with all of the sellers) must be in $t > 1$ and thus the buyer's maximum payoff can be at most $\frac{\delta}{1+\delta}$.*

Proof of Proposition 2. We prove the result here for the case $n = 2$. Appendix A provides the proof for the general case when $n > 2$.

Let $P(\delta)$ satisfy $\frac{\delta}{1+\delta} - P(\delta) = \frac{C}{\delta}$. For $\delta > \delta^0(C)$, it is easy to check that $P(\delta) > 0$.

For all histories that start with exactly one seller, the strategies for the players are given in parts (b) and (c) of Lemma 1. Moreover, at the end of any period, if the buyer has acquired exactly one object, he must continue negotiations.

Consider now any history that starts with two active sellers.

In $t = 1$, the buyer offers zero to both sellers. The first seller accepts an offer P_1 if and only if $P_1 \geq \frac{\delta}{1+\delta}$, whereas the second seller accepts an offer P_2 as long as $P_2 \geq 0$ and $P_1 < \frac{\delta}{1+\delta}$, and rejects any other offer. The buyer continues negotiations unless he has managed to collect both the objects.

For $t > 1$, the strategies depend on whether the game is in phase A, or phase B, with the game starting in phase A at $t = 2$ in case both sellers are active.

Strategies in Phase A.

If $t \neq 1$ is odd, the buyer offers zero to the first seller and $\delta P(\delta)$ to seller 2. The first seller accepts an offer P_1 if and only if $P_1 \geq \frac{\delta}{1+\delta}$. Seller 2 accepts an offer P_2 if and only if $P_2 \geq \delta P(\delta)$ and $P_1 < \frac{\delta}{1+\delta}$ and rejects any other offer.

If t is even, the first seller asks for $\frac{1}{1+\delta}$, whereas seller 2 asks for $P(\delta)$. These offers are accepted by the buyer. If any of the sellers asks for more, the buyer rejects both the offers.

In phase A , at the end of period t , the buyer always continues negotiation if t is odd and he failed to collect less than two objects.

If however, t is even, he will continue negotiation only if at least one of the sellers have deviated and asked for a higher price. He opts out otherwise.¹¹

Transition to Phase B.

If t is even, and one of the sellers deviate from the above strategies and the buyer rejects both of the offers, there will be a transition from phase A to phase B .¹² The state stays in phase B for precisely one period and will revert to phase A in the following period.

Strategies in Phase B. t odd.

The buyer makes an offer of zero to both sellers. Seller 1 accepts an offer if and only if $P_1 \geq \frac{\delta}{1+\delta}$. Seller 2 accepts any non-negative offer if $P_1 < \frac{\delta}{1+\delta}$ whereas if $P_1 \geq \frac{\delta}{1+\delta}$, seller 2 accepts an offer P_2 if and only if $P_2 \geq \frac{\delta}{1+\delta}$.

In stage B , at the end of the period, the buyer continues negotiation only if he has obtained one object. He opts out otherwise.

Observe that given the strategies outlined above for $t \geq 2$ and for any history that starts with both sellers being active, in phase A , the payoff to the buyer is exactly C when t is odd, and it is $\frac{C}{\delta}$, if t is even. Thus, at the end of an odd period, the buyer's discounted payoff by continuing is exactly C . He also gets C by opting out, and thus it is optimal for him to continue in odd periods. If t is even, however and the sellers did not deviate from their equilibrium strategies, phase A will continue and thus if the buyer continues he will get exactly δC whereas by opting out he gets C , thus, he is better off opting out. Finally, if t is even and the state is going to be in phase B , then by rejecting

¹¹Thus, if the sellers did not deviate from their offer strategies but the buyer rejected both of the offers, the buyer will opt out.

¹²It is important to note that the transition to phase B takes place only if at least one of the sellers deviate from their prescribed strategies. By construction, in phase B , it is the buyer who has to make an offer.

all offers today and continuing next period, the buyer expects to get $\frac{\delta^3}{1+\delta}$ which is strictly greater than $\frac{C}{\delta}$ and thus the buyer will be better off continuing negotiation. ■

Remark 4 *Whereas the preceding proposition focuses on establishing the maximum payoff that a buyer can obtain in an equilibrium, it leaves open the question as to whether, it is possible to support (in equilibrium) any buyer payoff in $(C, \frac{1}{2})$? Given the folk theorem like results in Chatterjee et al. (1993) and Herrero (1985), this question is of natural interest. It turns out that a buyer payoff of x at $t = 2$, where $C < x < 1/2$, can be supported as follows. The equilibrium involves the buyer making unacceptable offers at $t = 1$, and the sellers all asking for ϵ , where $1 - (n - 1)\epsilon = x$ at $t = 2$. The buyer accepts all such offers. In case any seller asks for more, the buyer rejects all offers and plays the equilibrium prescribed in Proposition 2.*

Thus there is a range of equilibria all of which are asymptotically efficient. Whereas inefficient equilibria exist as well, the results show that when offers are publicly observable, any inefficiency in outcome must be traced to coordination failures.

4 Holdout Problem: The Secret Offers Case

In this section we consider an alternative bargaining protocol where each seller can observe only her component of the buyer's offer and does not know what offers are received by other sellers. We call this the 'secret offer' case. (Chatterjee and Dutta (1998) refers to this as the telephone bargaining setup.) Whereas the public offer case is very natural (and widely adopted in the coalitional bargaining literature), it may be argued that in the context of holdout the secret offers protocol seems equally natural. It is thus of interest that in this case the results appear to be markedly different. In fact, for any sufficiently large discount factor there is complete breakdown of bargaining with the buyer opting for his outside option.

Proposition 3 *Fix $C \geq 0$. Then there exists $\delta^*(C) < 1$ such that if $\delta > \delta^*(C)$, then in any equilibrium, the buyer opts out in period 1.*

The intuition for this proposition can be understood by considering $n = 2$. First, observe that if the buyer obtains a payoff strictly greater than C , then all sellers must sell. From Lemma 2, it then follows that if the buyer has to make an acceptable offer to both the sellers, then each seller must be given at least $\frac{\delta}{1+\delta}$ leaving at most $\frac{1-\delta}{1+\delta}$ for the

buyer. Of course, for δ close to 1, this payoff is strictly lower than C . It is interesting to note that unlike in the public offer case, when offers are secret, there can not be an equilibrium, where the buyer first makes an acceptable offer to only one of the sellers (say seller 1) and then once it is accepted, negotiates with the the second seller. This is because the buyer can always offer the second seller $\frac{\delta}{1+\delta}$ along with the equilibrium offer to the first seller. Because offers are secret, such an action will not affect the acceptance decision of the first seller. The buyer must be better off from this deviation Because the project is implemented one period earlier and thus saves on the discounting cost. Thus the buyer has the option of implementing the grand project at $t = 1$ (when he obtains $\frac{1-\delta}{1+\delta}$), or opting for his outside option and getting C . For δ close to 1, opting out is the preferred choice.

Proof of Proposition 3. Fix δ and let $Y^*(m)$ denote the supremum of the continuation payoff to the buyer in any equilibrium, starting from a history when m sellers are active.

We first show that $Y^*(m) \leq \max\{\frac{1-\delta}{1+\delta}, C\}$ for $m \geq 2$ and $\delta > \delta(C)$. Note that Because $\delta > \delta(C)$, Lemmas 1-4 hold (see Remark 1).

We first prove this result for $m = 2$. If the claim is false, then there exists an equilibrium in which the buyer's payoff is arbitrarily close to $Y^*(m)$ which in turn is strictly greater than $\max\{\frac{1-\delta}{1+\delta}, C\}$.

Clearly, in such an equilibrium, all sellers sell and moreover, the payoff to at least one of the remaining active sellers (label her seller 1) must be strictly less than $\frac{\delta}{1+\delta}$. Let t be the date at which an agreement with seller 1 takes place. Clearly by Lemma 1, both sellers must be present at that date. Now if at t , seller 1 herself was making an offer, she could have asked for a slightly higher price, the buyer could not have rejected Because his payoff in this equilibrium was arbitrarily close to $Y^*(m)$. Therefore, this offer was made by the buyer. Furthermore, at that date, the buyer could not have made an acceptable offer to seller 2 as well. Because then by Lemma 2, seller 1 could have rejected his offer and obtained $\frac{1}{(1+\delta)}$ in the next period. The proof is now complete because if the buyer were to make an immediate offer of $\frac{\delta}{1+\delta} + \epsilon$ to the second seller along with the prescribed equilibrium offer to seller 1, seller 1 will continue to accept (because offers are secret) and, because of Lemma 4, seller 2 must accept it as well. Because the agreement now takes place in that period itself, the buyer will save on the discounting cost and thus for ϵ small, this must be a profitable deviation. This proves that $Y^*(m) \leq \max\{\frac{1-\delta}{1+\delta}, C\}$ for $m = 2$.

Assume now as an induction hypothesis that the result is true for all $m = 2, \dots, n - 1$ and let $m = n$. If the claim is false, then there is an equilibrium in which the all sellers sell

and the project is implemented at some date t giving the buyer a payoff strictly greater than $\max\{\frac{1-\delta}{1+\delta}, C\}$. Because of the induction hypothesis, the number of active sellers at that date must be either $m = n$, or $m = 1$.

First consider the case $m = n$. Because $n \geq 2$, it follows from Lemma 2 that the buyer could not have made an acceptable offer at that date. Thus, it is the sellers who are making the offers. But then any seller can ask for a slightly higher price which must be accepted by the buyer.

We finally argue that the equilibrium cannot involve the buyer reaching an agreement with a single seller, label this seller 1, at the last period. Now at $t - 1$, all n sellers must have been present, otherwise, the induction hypothesis would have applied. Now if t is odd, then at $t - 1$, seller 1 was making an unacceptable offer and the rest were making acceptable offers. If seller 1 at $t - 1$ asked for $P \in (\frac{\delta^2}{1+\delta}, \frac{\delta}{1+\delta})$, the buyer would have accepted this offer. This would thus have been a profitable deviation for seller 1. Therefore t must be even and at $t - 1$, the buyer is making an acceptable offer to only $n - 1$ sellers. This, however is impossible Because the buyer could have secretly offered $\frac{\delta}{1+\delta}$ to seller 1 along with specified equilibrium offers to the rest of the sellers. All sellers must accept and this will be a profitable deviation for the buyer as he implements the project a period earlier and saves on the discounting cost.

Thus, the continuation payoff to the buyer in any equilibrium is at most $\max\{\frac{1-\delta}{1+\delta}, C\}$ for $\delta > \delta(C)$.

Given C , let $\hat{\delta}(C)$ satisfy $C = \frac{1-\hat{\delta}}{1+\hat{\delta}}$ and let $\delta^*(C) = \max\{\hat{\delta}(C), \delta(C)\}$. For any $\delta > \delta^*(C)$, $C > \frac{1-\delta}{1+\delta}$ and the buyer opts out in the first period itself and obtain C . ■

Propositions 2 and 3 together provide an important insight for the holdout problem: *Holdout is severe when buyer offers are secret, but much less so if buyer offers are transparent, i.e. public.*

5 Holdout Problem in the General Case

In this section, we indicate how to extend our earlier analysis to technologies that allow the buyer to implement a partial project. To this end we assume that each seller has an identical object each that can be combined to generate returns for the buyer. We write $v(s)$ to denote the return to the buyer when a project combining s objects, $0 \leq s \leq n$, is implemented. The *grand project* involves combining all the objects. $v(s)$ is assumed to be non-decreasing in s and we normalize units such that $v(0) = 0$ and $v(n) = 1$.

We assume that $v(s)$ is *strictly super-additive* in that $v(n) > v(s) + v(n - s)$ for any s , where $1 \leq s < n$. Consequently implementing any project, other than the grand project is inefficient. The results in this general case depend critically on whether $v(n - 1)$ is greater than $\frac{1}{2}$ or not. We consider each case in turn.

5.1 $v(n - 1) > \frac{1}{2}$

In this case if the buyer has obtained $n - 1$ objects, in any continuation game with exactly one active seller, the buyer can credibly threaten to exit, resulting in a zero payoff for the remaining seller. Thus, holding out is an extremely costly strategy for an individual seller. This allows the buyer to make an immediate agreement with all of the sellers and in the first period.

Proposition 4 *Suppose $v(n - 1) > \frac{1}{2}$ and $\delta > v(n - 1)$. For both public, as well as secret offers, there exists an equilibrium where the buyer implements the grand project at date $t = 1$ and receives a payoff of 1.*

Whereas the proof can be found in Appendix B, here we outline the structure of the equilibrium profile. At every t even, the sellers all make unacceptable offers, whereas at every t odd, the buyer offers a payoff of zero to all sellers. At t odd, the sellers all accept. If the buyer faces exactly one active seller, he exits the game and implements a project of size $n - 1$. The key step in the argument is Lemma 5 (please see Appendix B) which establishes that after any history with exactly one seller remaining, there in fact exists a continuation equilibrium in which the buyer exits the game and implements a project of size $n - 1$.

5.2 $v(n - 1) < \frac{1}{2}$

In this case, the buyer can never credibly exit the game and implement a project of size $n - 1$ for δ sufficiently large. This is because by continuing and negotiating with the remaining seller, the buyer can assure himself a continuation payoff close to $\frac{\delta}{1+\delta}$ which is strictly greater than $v(n - 1)$ for δ sufficiently large. In fact with general production technology, holdout problem manifests itself only in this case. It is possible to check that for $v(n - 1) < \frac{1}{2}$, Lemma 1-4 hold for δ sufficiently large. Moreover, when offers are public, an exact analogue of Proposition 1 holds; for δ close to 1, there exists an equilibrium that is asymptotically efficient and in which the buyer's payoff is exactly $\frac{\delta}{1+\delta}$.

The results, however are somewhat modified when offers are secret. As the following proposition shows, that unlike in the earlier situation, with a general production technology, there exist equilibrium outcomes that are asymptotically efficient and which gives the buyer a strictly positive payoff.

Proposition 5 (Secret Offer Case) *Suppose that $v(n - 1) < \frac{1}{2}$, then*

- (a) *if $C \geq v(n - 2)$, then there exists $\delta^{**}(C) < 1$ such that for $\delta > \delta^{**}(C)$, in any equilibrium the buyer opts out at $t = 1$.*
- (b) *if $C > v(n - 2)$, then for any $\epsilon > 0$, there exists $\delta(\epsilon)$ such that if $\delta > \delta(\epsilon)$, there exists an equilibrium, in which the buyer implements the grand project at $t = 2$ and in which his payoff is greater than $v(n - 2) - \epsilon$.*

The intuition behind Proposition 5(b) is that with $v(n - 2) > C$, the buyer has the credible threat of implementing a project of size $n - 2$. This allows us to support an asymptotically efficient equilibrium in which no offers are accepted in period 1, but in period 2, all sellers ask for x such that $1 - nx = \delta v(n - 2)$ which are accepted by the buyer and the grand project is implemented at $t = 2$. If any seller asks for more, the buyer rejects all offers and makes acceptable offer to exactly $n - 2$ sellers next period and obtain $v(n - 2)$. The formal proof of the Proposition is provided in Appendix C.

6 Conclusion

This article characterizes the conditions under which holdout (i.e. bargaining inefficiency) may, or may not be significant in a two-sided, one-buyer-many-seller model with complementarity. The central insight is that the transparency of the bargaining protocol, formalized by whether offers are publicly observable or secret, the presence of outside options for the buyer, and the extent of complementarity, play a critical role in generating efficiency. Holdout seems to be largely resolved whenever the bargaining protocol is public and the buyer has a positive outside option, and/or the marginal contribution of the last seller is not too large, but not otherwise.

In a broader context, this article has some implications for the Coase theorem. Whereas it is well known that informational problems can lead to inefficiencies, the literature on coalitional bargaining has identified strategic issues that may, even in the absence of

informational issues, cause the Coase theorem to fail.¹³ Whereas one response to such strategic inefficiency has been to study random bargaining protocols, e.g. Okada (1996), another line of research examines bargaining protocols with renegotiation, Seidmann and Winter (1998), Hyndman and Ray (2007), etc. In this article, however, we examine a *deterministic* (though symmetric) bargaining protocol that does *not* allow for renegotiation. Remarkably enough, even then we find that there is *some* equilibrium that is *asymptotically efficient*, as long as the buyer offers are publicly observable. Further, this holds even with perfect complementarity.

Finally, the analysis in this article focuses on simple ‘unconditional cash offer contracts’ in which if a seller accepts an offer by the buyer, the seller sells the object and exits the game. This leads to the holdout problem. The results are thus strong in that even in the class of these simple contracts, for the public offer case it is possible to support efficiency, as well as a strictly positive payoff for the buyer. Clearly, in such situations, use of complex contracts is unnecessary. For situations where the holdout problem does bite (as in the secret offer case with perfect complementarity) however, it may be of interest to know whether alternative contractual forms may restore efficiency. The answer to this question is in the affirmative. Consider a situation where the buyer can make a ‘conditional offer’ to the sellers. Under a conditional offer, the buyer buys only when every seller agrees to sell, and not otherwise. Such contracts completely mitigate the holdout problem as one can show that for sufficiently patient players, for any $x \in [0, 1] > 0$, it is possible to support an equilibrium where the grand project is implemented in period 1 and the buyer’s payoff is x .¹⁴

7 Appendix

7.1 Appendix A: Proof of Proposition 2 for $n > 2$

We now continue with the proof of Proposition 2 for $n > 2$. We number the sellers 1 through n and given any seller set S , the highest ranked seller is referred to as the first seller, whereas the lowest ranked seller is referred to as the last seller.

We first describe the action profile of the players along the equilibrium path: at $t = 1$, the buyer offers zero to all sellers. All sellers but the first one accept. In period 2, the first

¹³Chatterjee et al. (1993), Bloch (1993) and Ray and Vohra (1997, 1999), among others, have pointed out the role of renegotiation in this context.

¹⁴A proof is available on request.

seller asks for $\frac{1}{1+\delta}$ which is accepted by the buyer and the grand project is implemented at the end of period 2.

For any history with $m = 1$ active seller, the strategy is given by Lemma 1. For any history h_t and $t > 2$ that starts with exactly two active sellers for the first time, the strategies of the players then are given by as outline in the proof of Proposition 2 for $n = 2$.

Assume now that the equilibrium strategy profile has been defined for all histories h_t that start with m active sellers where $m = 1, 2, \dots, n - 1$ and let $m = n$. If t is odd, the buyer offers zero to all active sellers.

To define the acceptance/rejection decision of a seller, consider any arbitrary offer vector (P_1, \dots, P_n) . Consider t odd. If $P_1 \geq \frac{\delta}{1+\delta}$, the first seller accepts P_1 , whereas the acceptance decision of the seller j , where $j \in \{2, \dots, n\}$, is the same as their decision for a history with seller set $\{j, \dots, n\}$, for the offer vector (P_j, \dots, P_n) . On the other hand, if $P_1 < \frac{\delta}{1+\delta}$, the first seller rejects whereas the rest of the sellers accept any non negative offer.

If t is even, every seller asks for a price of 1. The buyer rejects all these offers. Furthermore, if there is an unilateral deviation by one seller who asks for $P > 0$, the buyer continues to reject all of the offers.¹⁵

Finally, if at the end of any period, if the buyer has acquired n objects, the project is implemented. If he has acquired $n - 1$ objects, he continues. In all other cases, he exits and collects his outside option C . ■

7.2 Appendix B: Proof of Proposition 4

The proof of Proposition 4 relies critically on the fact that the buyer can credibly exit the game and implement a project of size $n - 1$. The following lemma does precisely that.

Lemma 5 *Suppose that $v(n - 1) > \frac{1}{2}$ and $\delta > v(n - 1)$. Consider any history that starts at t with exactly one active seller. Then there is a continuation equilibrium such that at every t even, the buyer's payoff is $\frac{v(n-1)}{\delta}$.*

¹⁵To define the strategies of the buyer for any arbitrary history, let $V(t, m)$ denote the continuation payoff of the players at the end of period t when m sellers remain and players play according to the strategies specified above. Consider now a history with $m \geq 3$ sellers and the offer vector (P_1, \dots, P_m) . If the buyer accepts the offers made by the seller set S' with $|S'| = s$, then his continuation payoff is $Y(S') = V(t, m - s) - \sum_{i \in S'} P_i$. Let S^* maximize this payoff and let $s^* = |S^*|$. The buyer accepts the offers in S^* if and only if $Y(S^*) \geq 0$. He rejects all offers otherwise.

Proof of Lemma 5. The proof involves constructing an equilibrium where at every t even, the single remaining seller asks for $\frac{v(n-1)}{\delta}$, which the buyer accepts. The strategies are conditional on whether the game is in either of two phases A, or B, with the game starting in phase A at $t = 1$.

In *Phase A*, at every t odd the buyer offers $\delta - v(n - 1)$ to the seller, and the seller accepts if and only if she obtains at least $\delta - v(n - 1)$. Whereas at every t even, the seller asks for $1 - \frac{v(n-1)}{\delta}$. The buyer accepts if and only if he obtains at least $\frac{v(n-1)}{\delta}$.

In this phase, the buyer always continues negotiation in case an offer is rejected.

Finally, there is *transition to Phase B* if the seller asks for more than $1 - \frac{v(n-1)}{\delta}$ at t even.

In *Phase B*, at every t odd the buyer offers $(1, 0)$, and the seller accepts if and only if she obtains at least 0. Whereas at every t even, the seller offers $(\delta, 1 - \delta)$. The buyer accepts if and only if he obtains at least δ ,

If t is even and an offer is rejected by the buyer, the buyer continues negotiation.

If however, t is odd and the seller rejects buyer's offer, the buyer implements the project of size $n - 1$. In this case, if the buyer fails to implement the project and continues, and further, *Phase B* transits to *Phase A*. ■

Proof of Proposition 4. The proof involves constructing equilibrium profiles such that at every t even, the sellers all make unacceptable offers, whereas at every t odd, the buyer offers a payoff of zero to all sellers. At t odd, the sellers all accept. Because otherwise the other sellers accept, and the buyer exits the game and implements a project of size $n - 1$. We now formally describe the strategies.

For any history that starts with exactly one active seller, the strategies of the players are as specified in the proof of Lemma 5. For any history that starts with m sellers, $m > 1$, the strategies are as follows:

If t is odd, the buyer offers zero to each of the active sellers. Each seller accepts any non-negative offer.

At the end of period t where t is odd, the buyer implements the project only if he has acquired $n - 1$ objects or more. He continues otherwise.

If t is even, each seller asks for $P = 1$. Given any offer vector $P = (P_1, P_2, \dots, P_m)$, let $Z = 1 - \sum_{i \in M} P_i$, where M is the set of active sellers. The buyer accepts every offer if $Z \geq \delta$. If $Z < \delta$, he rejects all offers.

At the end of period t , where t is even, the buyer implements the project only if he has acquired all the objects. He proceeds to the next period otherwise.

Consider a subgame with t odd, where $n - 1$ of the offers have been accepted by the sellers. The buyer's payoff from implementing a project is $v(n - 1)$, whereas if he continues to negotiate, then he obtains $\frac{v(n-1)}{\delta}$ in the next period (Lemma 5), so that opting out immediately is optimal. Further, note that these strategies work for the secret, as well as the public offer game. ■

7.3 Appendix C: Proof of Proposition 5

Proof of Proposition 5. (a) To prove this proposition, one first establishes that for discount factor close to 1, the maximum payoff to the buyer in the secret offer can be no more than $\max\{C, v(n - 2)\}$. The proof for this follows exactly the same arguments as that of Proposition 3. Thus, when $C > v(n - 2)$, in any equilibrium, the buyer must opt out at $t = 1$.

(b) We then argue that when $v(n - 2) > C$, there exists an equilibrium outcome (for δ close to 1) in which along the equilibrium path, the buyer makes an offer that is rejected by every seller. In period 2, each seller then asks for x where $1 - nx = \delta v(n - 2)$. The buyer accepts all these offers and implements the grand project at $t = 2$.

We now formally describe the equilibrium strategy profiles for the players.

Fix a history h_t that ends with the active seller set S and $|S| = m$. When $m = 1$, the strategies are specified in Lemma 1 and thus assume that $m \geq 2$.

If t is odd, the buyer offers zero to all sellers. For $t = 1$, however, these sellers accept an offer P_i if and only if $P_i \geq \frac{1-\delta}{1+\delta}$. For $t \neq 1$, but odd, the first two sellers accept an offer P_i , $i = 1, 2$, if and only if $P_i \geq \frac{\delta}{1+\delta}$. All other sellers accept any nonnegative offer for all $t \geq 3$.

If t is even and $t \neq 2$, each seller asks for P such that $1 - mP = v(n - m)$. Whereas at $t = 2$, each seller asks for P^0 where $1 - nP^0 = v(n - 2)$. The buyer accepts these offers. If any seller asks for more, the buyer rejects all offers.

Finally, consider the implementation decision of the buyer at the end of any period t where the buyer has acquired k objects. The buyer implements the project of size k if and only if $v(k) > 0$. He continues negotiation otherwise.

We observe that given the specified strategies, all sellers in period 1 rejects the buyer's offer. In period 2, each seller offers P^0 and these offers are accepted by the buyer and the grand project is implemented in period $t = 2$.

We note that at $t = 2$, if any seller demands $P > P^0$, the buyer is better off rejecting such an offer. This is because by rejecting all offers at that period, the buyer hopes to get

$v(n - 2)$ next period. Because his offer of zero will be accepted by the last $n - 2$ sellers. Thus, rejection yields a payoff of $\delta v(n - 2)$ to the buyer. If the buyer were to accept the deviating seller's offer of $P > P^0$, his payoff is necessarily less than $\delta v(n - 2)$. Thus, no seller at $t = 2$ has a profitable deviation. It is also straightforward to check that given the acceptance strategies of the sellers, the buyer can not hope to make an acceptable offer to a group of sellers and get a higher payoff.

Finally, it is clear that for δ large, $v(n - 2) > \frac{1-\delta}{1+\delta}$ and thus given the offer/acceptance decision of the sellers, for any $t \geq 3$, it is always optimal for the buyer to implement the project of size $n - 2$ rather than continuing negotiation with the remaining two sellers. ■

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