

Optimal management of a forest/wildlife system with bilateral externalities

Richard Ready
Department of Agricultural Economics and Rural Sociology
Pennsylvania State University
University Park, PA 16802 USA

Olvar Bergland and Eirik Romstad
Department of Economics and Social Sciences
Agricultural University of Norway
NO-1432 Aas, Norway

To be presented at the annual meetings of the American Agricultural Economics Association,
Chicago, Illinois, August 2001.

This research was funded by a grant from the Norwegian Research Council. The authors thank Hans Fredrik Hoen for useful comments and suggestions. Copyright 2001 by the authors. All rights reserved. Readers may make verbatim copies of this document for non-commercial purposes by any means, provided that his copyright notice appears on all such copies

Contact Information:

Richard Ready
Department of Agricultural Economics and Rural Sociology
The Pennsylvania State University
University Park, PA 16802
Tlf: 814-863-5575
Fax: 814-865-3746
E-mail: rready@psu.edu

Optimal management of a forest/wildlife system with bilateral externalities

Abstract

Browsing by large ungulate wildlife species damages valuable trees, but increases the productivity of the wildlife stock. The optimal management of trees and wildlife involves a tradeoff between these two impacts, as well as correct timing of tree harvests from each forest stand. For a model of moose and pine in Norway, it is found that harvest of adjacent forest stands should be adjusted over time to bring the stands into synchrony. Doing so maximizes the moose productivity boost that occurs immediately following harvest of mature trees, and protects the young trees from overbrowsing by spreading out the browsing pressure among more young trees. This result is, however, sensitive to the specification of the relationship between forage availability and moose productivity.

Keywords: optimal forest rotation, wildlife management, browsing

Optimal management of a forest/wildlife system with bilateral externalities

Several studies have explored how forest management should be adjusted to take into account externalities generated by the forest (Hartman 1976, Riitters, Brodie and Hann 1982, Swallow et al 1990, Tahvonen and Salo 1999). An important example of such an externality is the provision of wildlife habitat. Some species of wildlife benefit from large stands of older trees, while others benefit from having at least some younger stands. When the rate of external benefits generated by the forest depends on the age of the forest stand, the total net present value from an infinite series of rotations can be non-convex in rotation length (Swallow et al 1990). This means that there may exist more than one local optimum when choosing the rotation length.

These studies have all modeled the externalities generated by the forest stand as unilateral. However, wildlife populations that use the forest can also have positive or negative impacts back on the forest. This is the case for moose and pine in Norway. Younger stands of pine provide important winter browse for moose, increasing the productivity of the moose stock. In return, the moose browse down the pine, slowing growth and decreasing eventual timber harvest values. The optimal management strategy for a single forest stand takes into account both externalities, and can take one of two forms, a long-rotation strategy that focuses primarily on timber production or a short-rotation strategy that focuses on moose production (Ready, Bergland and Romstad 2000).

A second group of authors have investigated interactions among multiple forest stands, determining when it is preferable to synchronize harvest of adjacent stands, and when it is preferable to increase the difference in harvest times across stands (Swallow and Wear 1993; Swallow, Talukdar and Wear 1997; Koskela and Ollikainen 1999; Tahvonen and Salo 1999).

This issue is relevant when considering browsing of forest stands, because wildlife will move to exploit vulnerable stands. The optimal management of a given stand therefore depends on the condition of adjacent stands.

In this paper we develop a multi-stand model of pine trees browsed by a stock of moose, and characterize its optimum. We then empirically investigate whether, when adjacent stands are of different age, the optimal strategy is to bring the stands into synch, or to increase the difference in cutting times.

Moose and Pine in Norway

The population of the European moose (*Alces alces alces*) in Norway has increased dramatically since the second world war. While the annual harvest varied between 1000 and 2000 animals between 1890 and 1945, it has steadily increased since 1945, so that in the 1990's it exceeded 35000 animals per year (Andersen and Sæther 1996). One reason for the increase is a simultaneous increase in the amount of land covered by forests. A second reason is that harvest is now targeted more toward males and younger animals resulting in a stand with a larger fraction of older females. This makes the stock more productive, in particular in its utilization of winter feed.

Scotch pine (*Pinus silvestris*) is one of the three most important commercial tree species in Norway. Commercial forests are predominantly privately owned as part of family farms, and the scale of individual operations is small. The average individual area harvested is only 1.4 ha. This creates a patchy environment ideal for moose habitat.

In winter, when snow covers the moose's preferred forage, young pine trees are an important food source (Sæther et al 1992). Moose tends to browse the tips of the pine twigs, usually

limiting themselves to the previous season's new growth, which is softer and more easily digested. This has two impacts on the tree. First, it slows the tree's growth, delaying eventual harvest. Second, if severe, it can kill the tree or deform it such that it loses commercial value. Once the tree reaches a height where the moose can no longer reach its top branches, further browsing has little impact on the tree's growth or value. In areas with higher moose densities, young pines are browsed so heavily that up to 3.5% of the available forest area is effectively lost to pine production (Solbraa 1998).

While the presence of moose is clearly damaging to pine production, the presence of young pines provides a productivity boost for the moose population. This is especially true in years with deeper snow cover, when young pine may be the only suitable food available. With good winter forage, female calves can grow faster and reach sexual maturity faster, increasing the population's productivity rate, and potential for harvest. Thus, every time a stand is harvested the moose stock that lives in that area receives a boost in productivity. Under Norwegian wildlife law, the state determines how many animals can be shot each season from each management area. This quota is then given to the land owners in the management area. Land owners form a hunting association, and decide among themselves how to harvest and share the quota. Thus, the increase in moose productivity translates into increased take of moose, a benefit to the members of those hunting associations.

During the winter season, when pine is an important forage for moose, moose in Norway tend to limit their range. However, the winter range of a moose is large relative to the area included in a typical even-age forest stand. A single moose therefore has access to several forest stands, of different age. This has important implications for how intensively a single stand of vulnerable

age is browsed. The browsing intensity depends not only on the density of moose in the area, but also on whether other stands in the area are also in the vulnerable stage.

In the next section, a dynamic multi-stand model of the moose/pine system is developed, and the conditions for socially optimal moose and tree management are derived. This model is then simulated using realistic parameter values, and used to investigate whether the bilateral externalities between moose and pine favor synchronization of adjacent forest stands, or divergence in tree harvest timing.

A Model of Forests and Moose

Forest and Moose Dynamics

The forest is divided into K sections. Prior to the beginning of the problem, the K sections were planted at times t_0^1, \dots, t_0^K so that at time t , their ages are $s^1, \dots, s^K = t - t_0^1, \dots, t - t_0^K$. The sizes of the stands are z^1, \dots, z^K , with total Z .

The standing volume of lumber in tree stand k , per unit area (hectare), is measured by $x^k(t)$, so that the total volume is given by $x^k(t) \cdot z^k$. For older trees, this measures the saleable volume after harvest. For young trees, this should be thought of more as an index of biomass than as an actual measure of volume. When a stand is harvested, natural regeneration supplies an initial biomass of x_0^k per hectare, which may differ among stands due to differences in the natural productivity of the land. Tree growth adds to $x(t)$ over time, but browsing by moose decreases $x(t)$.

A tree's vulnerability to browsing depends on its size. It is high during the early years after a harvest, but decreases sharply when the tree reaches a height of about 3 meters, and moose can no longer reach the top branch. Define the vulnerability of the tree stand as $v^k(t) = v(x^k(t))$. This

function should approach 1 for small trees, and 0 for larger trees. The actual amount of pine trees available for forage in stand k at time t is then $x^k(t) \cdot v(x^k(t))$ per hectare.

The browsing pressure that a given stand will experience depends on 1) its vulnerability, 2) the number of moose in the area, 3) the vulnerability of other stands in the area, and 4) the amount of browse other than young pines available in the area. In addition to young pines, moose can browse other forage species, which have a density f^0 per unit area, so that the total amount of forage in the area is $f^k(t) = x^k(t) \cdot v^k(t) + f^0$ per hectare. The area-weighted mean amount of winter forage available in the area is then

$$\bar{f}(t) = \frac{1}{Z} \sum_{k=1}^K z^k \cdot f^k(t)$$

For simplicity, it is assumed that f^0 is the same across all stands.

How will moose distribute themselves among the K tree stands? Stands with more forage are likely to contain more moose. One modeling approach is to assume that the moose locate themselves so that the density of moose per unit of forage available is constant. If the average density of moose in the entire area is $m(t)$, then each stand will have

$$m^k(t) = m(t) \cdot \frac{f^k(t)}{\bar{f}(t)}$$

moose in it per hectare. If moose target pine and other forage equally, then the young pines in stand k will receive a browsing pressure

$$b^k(t) = m^k(t) \cdot \frac{x^k(t) \cdot v^k(t)}{f^k(t)} = m \cdot \frac{x^k(t) \cdot v^k(t)}{\bar{f}}$$

At lower moose densities, the dominant impact of browsing is lost biomass (i.e. delayed growth).

Tree biomass per unit area changes over time according to

$$\dot{x}^k(t) = F^k(x^k(t), b^k(t))$$

where the function $F^k(\cdot)$ includes both natural tree growth and loss of biomass due to browsing.

At higher moose densities, however, moose can damage trees to the point where they lose commercial value or even die. Let $d^k(t)$ measure the percent of the tree stand area that is undamaged. Then $d^k(t_0) = 1$, and $d^k(t)$ decreases over time due to moose damage, as a function of the size of the trees and the browsing pressure they receive:

$$\dot{d}^k(t) = D(x^k(t), b^k(t))$$

While, technically, $d^k(t)$ can never go below 0, it will be assumed that it never gets small enough that that constraint becomes important.

At any given point in time, the moose population (density) is given by $m(t)$. Moose stock increases due to natural growth, which depends on the amount of forage available, and decreases due to harvest, so that

$$\dot{m} = g(m(t), \bar{f}(t)) - h$$

where $g(\cdot)$ is a growth function, and h is harvest. The moose stock per unit area, $m(t)$, is measured as "moose equivalent units", i.e. number of moose, ignoring individual size.

Economics and Control

The value of killing one moose is j . This includes both the value of the meat and the recreational value to the hunter. The wildlife management agency gets to choose moose harvest, h , which is measured as a rate of harvest, and is limited to the interval $h \in [0, m(t)]$.

The per hectare net value of harvesting the standing lumber from stand k at time t is $d^k(t) \cdot r(x^k(t))$. The function $r(x^k(t))$ is the net revenue the owner would receive from harvesting one hectare of forest with tree volume $x^k(t)$, and zero tree mortality due to moose damage. The

net revenue function includes both harvesting costs and any replanting costs. This function will be highest for mature timber, and possibly negative for very young timber.

Tree stand K is harvested at times t_1^k, t_2^k, \dots . In the atomistic solution, the forest owner gets to choose cutting times. In the social-planning solution, cutting times are chosen to maximize social net benefit. The discount rate for both the tree owner and for society is δ .

Optimization

There are then two sets of control variables: the rate at which moose are harvested, and the timing of the tree harvests. Society's objective function includes the present value of net profits from each timber cutting plus the value of the moose harvest. The maximization problem is then

$$\max_{t_i^k, h} W = \sum_{k=1}^K \sum_{i=1}^{N_k} \left[z^k \cdot r(x^k(t_i^k)) \cdot d^k(t_i^k) \cdot e^{-\delta t_i^k} \right] + \int_0^{\infty} j \cdot h(t) \cdot e^{-\delta t} dt$$

subject to

$$\begin{aligned} x^k(0) &= x_0^k \\ \dot{x}^k &= F^k(x^k(t), b^k(t)) \quad \text{when } t \neq t_i^k \\ x^k(t_i^{k+}) &= x_0 \\ m(0) &= m_0 \\ \dot{m} &= g(m(t), \bar{f}(t)) - h \\ d^k(0) &= d_0^k \\ \dot{d}^k(t) &= D(x^k(t), b^k(t)) \\ d^k(t_i^{k+}) &= 1 \end{aligned}$$

This gives us the following state variables: $m(t)$, $x^k(t)$, and $d^k(t)$ for $k=1, \dots, K$. The vulnerability indices $v^k(t)$, browsing pressure measures $b^k(t)$, and forage availability measure $f^k(t)$ are all functions of the $x^k(t)$ and $m(t)$, and so do not represent new state variables.

Optimal Moose Harvest

The solution follows the approach of Seierstad and Sydsæter (1987), except that current-valued Hamiltonian and costate variables are used. The current-valued Hamiltonian for this problem, with time suppressed for each state variable, is given by

$$H = j \cdot h + \lambda_m (g(m, \bar{f}) - h) + \sum_{k=1}^K \lambda_x^k \cdot F(x^k, b^k) + \sum_{k=1}^K \lambda_d^k \cdot D(x^k, b^k)$$

The first order condition for moose harvest is given by:

$$\begin{aligned} \frac{\partial H}{\partial h} = j - \lambda_m \leq 0 \quad h(t) \geq 0 \quad h(t) \cdot (1 - \lambda_m) = 0 \\ \Rightarrow \lambda_m \geq j \end{aligned}$$

The costate variable λ_m measures the current value of having one more moose in the stock at time t . The current value of harvesting a moose at time t is always j . Any time λ_m is larger than j , it makes sense to invest any extra moose you get into the stock. If $\lambda_m = j$, then you can harvest any extra moose. Thus, if the moose population is larger than the singular solution, you harvest down to the singular solution immediately. If the moose population is smaller than the singular solution, you stop all harvest to allow the population to grow up to the singular solution. While technically h must be non-negative, for simplicity that restriction is ignored here. The practical reality is that hunting does not occur continuously, but in annual lumps. In the simulations presented later the stock adjustments needed are usually small relative to the annual harvest, so that the restriction on h is never binding over an entire hunting season.

The adjoint equation for moose is

$$\dot{\lambda}_m = \delta \cdot \lambda_m - \frac{dH}{dm} = \delta \cdot \lambda_m - \lambda_m \frac{\partial g(m, \bar{f})}{\partial m} - \sum_{k=1}^K \left[\lambda_x^k \frac{dF(x^k, b^k)}{dm} + \lambda_d^k \frac{dD(x^k, b^k)}{dm} \right]$$

At the singular solution, so that $\lambda_m = j$, $\dot{\lambda}_m = 0$, this rewrites as

$$\delta = \frac{\partial g(m, \bar{f})}{\partial m} + \sum_{k=1}^K \frac{\lambda_x^k}{j} \frac{dF(x^k, b^k)}{dm} + \sum_{k=1}^K \frac{\lambda_d^k}{j} \frac{dD(x^k, b^k)}{dm}$$

which can, at least implicitly, be solved for the optimal value of m .

This last result is similar to the standard arbitrage result that (with no harvest cost effects) the growth rate of a renewable stock should be set equal to the interest rate. The difference is two extra terms for each forest stand, which capture the impact of the moose stock on future tree harvests, through the browsing pressure measure, b^k . The ratios λ_x^k / j and λ_d^k / j trade off the value of more wood in stand k with the value of more moose. Both of these extra terms are negative for each stand, so that $m(t)$ is set at a level where $\partial g / \partial m > \delta$. The impact of browsing on forest growth rates therefore implies that moose populations should be held lower than they would be without browsing impacts.

Optimal Tree Harvest Timing

The adjoint equation for x is

$$\dot{\lambda}_x^k = \delta \lambda_x^k - \frac{\partial H}{\partial x^k} = \delta \lambda_x^k - \lambda_m \cdot \frac{\partial g(m, \bar{f})}{\partial \bar{f}} \cdot \frac{d\bar{f}}{dx^k} + \sum_{j=1}^K \left[-\lambda_x^j \frac{dF(x^j, b^j)}{dx^k} - \lambda_d^j \frac{dD(x^j, b^j)}{dx^k} \right]$$

When evaluating the derivatives in the adjoint function for x , it is important to remember that a change in x^k will change both f^k and \bar{f} , which will directly influence $g(\cdot)$, and indirectly influence $F(\cdot)$ and $D(\cdot)$ for each stand through the associated change in b^k .

The adjoint equations for d is simply

$$\dot{\lambda}_d^k = \delta \cdot \lambda_d^k - \frac{\partial H}{\partial d_k} = \delta \cdot \lambda_d^k$$

Because d does not feed back into any other process, the only reason you want more d is so you can sell more wood at harvest. The current value of more wood at harvest increases at the discount rate, so that the present value $\lambda_d^k \cdot e^{-\delta t}$ remains constant over time.

At each tree cutting time, the state variables x^k and d^k jump instantaneously. In addition, the wildlife management agency may need to adjust the moose stock instantaneously as well. The size of the jumps in x^k and d^k are fixed by the biology of the trees. As for the moose jump, we know that $\lambda_m = j$ both before and after the jump (if $h > 0$). The size of the jump in the moose stock is therefore just enough to bring λ_m back to the right level, as defined by the arbitrage condition.

Finally, the following conditions¹ must be satisfied immediately before the i^{th} harvest of tree stand k , that is at time t_i^{k-} ,

$$\lambda_x^k(t_i^{k-}) = z^k \cdot d^k(t_i^{k-}) \cdot \frac{\partial r(x^k(t_i^{k-}))}{\partial x^k(t_i^{k-})}$$

$$\lambda_d^k(t_i^{k-}) = z^k \cdot r(x^k(t_i^{k-}))$$

yielding end points for the differential equations controlling λ_x and λ_d . These end points make economic sense. The shadow values for these two state variables are simply the extra revenues at cutting time from a marginal increase in x^k or d^k immediately prior to harvest.

Finally, we turn to the timing of the cutting. At harvest, society receives a discrete payoff equal to the net cash flow generated by the timber sale, plus the value of the instantaneous adjustment in the moose population. The necessary condition² for the optimal choice of the timing of a tree harvest is

$$H(t_i^{k+}) = H(t_i^{k-}) - \delta \cdot \left[z^k \cdot r(x^k(t_i^{k-})) \cdot d^k(t_i^{k-}) + j \cdot (m(t_i^{k-}) - m(t_i^{k+})) \right]$$

¹ These are generated by applying Seierstand and Sydsæter's equation (74). See also their page 208, paragraph C.

² This is generated by applying Seierstand and Sydsæter's equation (77).

This arbitrage conditions states that the gain from delaying the cut, which is equal to the rate of payoff prior to the cut minus the opportunity cost of delaying the discrete payoff from the cut, must equal the gain from not delaying the cut, which is equal to the rate of payoff after the cut. Note that if it at the start of the problem, if the gains from cutting exceed the gains from delay, it is optimal to cut immediately.

The equations of motion and first order conditions for both the moose management problem and the tree harvest timing problem are summarized in Table 1.

Numerical Simulation

To simulate this system, specific functional forms must be chosen for the $g(\cdot)$, $F(\cdot)$, $D(\cdot)$, $v(\cdot)$ and $r(\cdot)$ functions, and then parameter values must be chosen.

First, for the moose growth equation, we assume that the moose stock grows logistically, with growth rate and carrying capacity that depend on the amount of young pine available:

$$g(m(t), f(t)) = r_m(\bar{f}) \cdot m \cdot \left[1 - \frac{m}{G_m(\bar{f})} \right]$$

Two forms for the relationships between the growth parameter and carrying capacity and forage availability will be explored. The linear model takes the form

$$\begin{aligned} r_m(\bar{f}) &= r_m^0 + r_m^1 \cdot \bar{f} \\ G_m(\bar{f}) &= G_m^0 + G_m^1 \cdot \bar{f} \end{aligned}$$

where r_m^0 and G_m^0 are the growth rate and carrying capacity of an area with no winter browse available, and r_m^1 and G_m^1 are the slopes of linear functions between the these parameters and amount of browse available. This linear form may not be realistic across all levels of browse availability. In particular, it may be the case that winter survival is critically dependent on

having some minimum amount of forage availability, but that additional forage above that minimum gives much smaller increases in moose productivity. A relationship that can capture decreasing returns to forage availability is called here the log model, given by

$$\begin{aligned} r_m(\bar{f}) &= r_m^0 + r_m^1 \cdot \ln(\bar{f} - f_{\min}) \\ G_m(\bar{f}) &= G_m^0 + G_m^1 \cdot \ln(\bar{f} - f_{\min}) \end{aligned}$$

where f_{\min} represents some minimum forage requirement for overwinter survival, and values of r_m^0 , r_m^1 , G_m^0 , and G_m^1 are different from those used in the linear model.

Next, trees grow logistically and are browsed at a rate that depends on how many moose are in the stand and how susceptible the stand is to browsing:

$$\dot{x}^k = F^k(x^k, b^k) = r_p^k \cdot x^k \cdot \left[1 - \frac{x^k}{G_x^k} \right] - c \cdot b^k$$

where c is the amount of forage consumed by one moose, r_p^k is the tree (pine) growth parameter for stand k , and G_p^k is the maximum possible tree volume per hectare for stand k . The rate at which tree material is browsed is then $c \cdot b^k$.

Young trees tolerate some browsing without longterm effects. Trees die or become irretrievably damaged because they are browsed at too high a rate. A larger tree can tolerate more browsing than a smaller tree. One possible form for the tree mortality function is then

$$\dot{d}^k(t) = D(x^k, b^k) = -\gamma \cdot \left(\frac{b^k}{x^k} \right)^\sigma$$

With this form, the rate of tree death declines slowly at first, as tree biomass increases, and then rapidly when trees reach the size where moose cannot reach the top branches. With $\sigma > 1$, the marginal damage from one more moose is increasing in the number of moose. A value of $\sigma = 2$ will be handy computationally, and appears reasonable.

The stand vulnerability index, $v^k = v^k(x^k)$ starts out close to 1 when x^k is small, and then decreases toward 0 as x^k increases. A simple form that does this is

$$v^k(x^k) = 1 - \frac{1}{1 + \exp(\alpha_v - \beta_v x^k)}$$

The net revenue received by the land owner per unit area will likely be negative for very young stands (to cover clearing costs), but will increase as the stand matures, but at a decreasing rate.

One form that fits available data on timber prices and harvest costs well is:

$$r(x^k) = \alpha_0 + \frac{\alpha_1}{1 + \exp(\alpha_2 + \alpha_3 \cdot \ln(x^k))}$$

Parameter Values

Table 2 lists the parameter values chosen for the baseline simulations in this paper.

Parameters for the pine growth equation and the revenue function were estimated based on data from Norwegian commercial forests with productivity grade H11.0.³ The value of a moose was estimated from a contingent valuation study of Norwegian moose hunters (Sødal 1989).

Parameters for the vulnerability function were chosen so that vulnerability falls most rapidly when the tree volume reaches about 15 m³ per hectare. For the pine growth function used here, that occurs when the stand reaches an age of about 10 years.

Parameters in the $D(\cdot)$ function were chosen so that mortality over a stand's life matches approximately actual measured damage seen in heavily browsed areas of Norway. However, little is known about the quantitative relationship between moose density and tree mortality. Similarly, the moose growth rate and carrying capacity were set such that simulation results

³ Data for pine growth and timber prices was taken from Handbok for Planlegging i Skogbruket, published by the Agricultural University of Norway.

matched measured densities and calving rates, but data on the relationship between the moose growth rate and carrying capacity and the availability of forage is not available.

Thus, the results provided here should be viewed as illustrative, not as guidelines for moose management in Norway.

Simulation Results

Simulations were performed with the Gauss add-on package SIMGAUSS, using a variable-step Runge-Kutta method. An initial moose population of 0.05 animals per ha is assumed, but is immediately adjusted according to the relevant arbitrage condition. Two forest stands of equal size are included in the simulation. At the beginning of the simulation, both forest stands contain volume equal to the long-run maximum possible (G_x). Forest stand 1 is harvested at time 0. Forest stand 2 is harvested after a fixed amount of time. Thus, after forest stand 2 is harvested, we have two stands of unequal age. The initial difference in ages is called the lag between the two stands. The research question is whether, over time, the optimal strategy is to push the two stands “into synchrony,” or further “out of synchrony.”

The model can be either partially or fully optimized. For a given series of cutting times, a partial optimization involves finding the correct values of the adjoint variables λ_x and λ_d for each forest stand and for each harvest rotation. These adjoint variables then determine the correct time path for moose harvests, to maintain the optimal moose density over the lifetimes of the forests. In the simulations presented below, this approach is used to determine the fixed rotation length that gives highest social benefits, under the restriction that the lag between stands is held constant. More broadly, the optimal set of cutting times can be found. These can then be

compared to the best fixed rotation length. In all simulations, optimal cutting times are determined to the nearest 0.1 years.

Optimal Moose Management

Within each forest rotation, the wildlife management agency sets moose harvests in order to maintain the moose stock at the socially optimal level. This level responds to changes in forage availability, changes in tree vulnerability, and to the adjoint variables λ_x and λ_d . Moose management and tree growth are interdependent only when the trees are vulnerable. The optimal moose stock size when trees are large is constant over time, and is a function only of the moose stock growth parameter, the moose stock carrying capacity, and the discount rate. For the parameters used here, this “no interaction” optimal stock size is 0.05125 animals per hectare. This is the optimal moose density when no young trees are vulnerable to browsing.

It is during the time when the trees are young that the important interactions between moose and trees occur. During that time, the optimal management of moose stocks involves a tradeoff between two competing objectives. On the one hand, moose are more productive when young trees are present (have higher growth rate and larger carrying capacity). This effect means that it is desirable to have larger moose stocks when the trees are young, to take advantage as much as possible of that higher productivity. However, having higher moose stocks when moose are young slows tree growth, and kills some young trees.

Depending on the relative importance of these two effects, the optimal moose stock size when trees are young could be higher than or lower than the “no interaction” optimal stock size. If moose are very highly valued relative to trees, or if the damage to trees is small, then the optimal moose stock size will be higher when trees are young. If, however, trees are highly

valued relative to moose, or if the damage to trees is very high, then the optimal moose stock size will be lower when the trees are young. For a more complete discussion of the factors influencing moose management decisions, see Ready, Bergland, and Romstad (2000).

Optimal Harvest Timing

We first determine the fixed rotation that gives highest social benefits, given that moose are managed optimally for that rotation pattern. If the two stands are initially in synch, i.e. the initial lag=0, or if the initial lag is less than 7 years, the fixed rotation length that generates highest total benefits from moose and pine is 93.6 years. If the initial lag is greater than 7 years, the best fixed rotation length is 93.7 years.

Next we determine whether this fixed rotation strategy maximizes total social benefits, or whether it is preferably to bring the two stands into synch, or to push the two stands further out of synch. If the initial lag is 0, i.e. if the forests are already in synch, then the optimal strategy is to keep them in synch - the fixed rotation strategy is the welfare maximizing strategy. If the two stands start with a lag, however, the welfare maximizing strategy is to adjust the rotation length for each stand so that the lag between the stands shrinks over time. For example, if the initial lag between the two stands is 1 year, the optimal strategy is to harvest stand 1 at stand age 93.9, and harvest stand 2 at time 94.2, when the stand age is 93.2, so that the lag after one rotation shrinks to 0.3 years. The second harvest for stand 1 would occur at stand age 93.7, while the second harvest for stand 2 would occur at stand age 93.4, both of which occur at time 187.6. With an initial lag of 1 year, then, the optimal strategy is then to bring the two stands into synch over the course of two rotations.

Figure 1 shows the optimal lag between harvests in the $i+1^{\text{st}}$ rotation, given the lag between harvests in the i^{th} rotation. A 45% line is included, so that the degree of convergence in each rotation can be determined. The amount by which we want to reduce the lag each rotation depends on the size of the lag at the beginning of that rotation. Both the moose productivity boost and browsing damage depend on the amount of available pine forage, $x^k \cdot v(x^k)$. This peaks at about stand age 7 or 8. It is not surprising, then, that the degree to which the optimal harvest times converge is highest with lags of this duration. If the lag between the stands is between 5.5 and 8.8, the optimal strategy is to reduce the lag by 1.2 years during the next rotation. For initial lags less than 5.5 years, or greater than 8.8 years, the optimal strategy is to reduce the lag by less than that amount. For very high lags, greater than 15 years, the optimal strategy is again the fixed-rotation strategy. With these lags, the two stands are never vulnerable at the same time, and so do not influence each other.

So, if the initial lag is 0, the optimal strategy is to continue managing the forest as one large stand, with a common harvest time. If the initial lag is greater than 15 years, the optimal strategy is to manage the forests independently, and not worry about interactions between the stands. If the initial lag is between 0 and 15 years, the optimal strategy is to gradually bring the forest stands into synchrony.

Factors Motivating Stand Convergence

Why is synchronizing the stands the preferred strategy? The only interaction between the two forest stands is through the moose stock, so the answer must lie in the interactions between pine and moose. There are two interactions between moose and pine - the increase in moose productivity that comes from increased forage availability (captured by parameters R_m^{-1} and G_m^{-1}),

and the damage to pine growth and survival caused by browsing (captured by parameters γ and c). Both of these interactions are sensitive to the amount of pine that is available for browsing in each stand, $x^k \cdot v(x^k)$. Having both stands vulnerable at the same time has two effects. First, it spreads out the browsing pressure over a larger area. Second, it shortens the amount of time that forage is available to the moose, but concentrates that forage.

The impact of the first effect is clear. From the perspective of a single stand of trees, it is better to be vulnerable to browsing at the same time that neighboring stands are vulnerable, spreading the browsing pressure over a larger area. Magnifying this tendency is the fact that tree mortality occurs as a squared function of browsing pressure. These effects can be isolated by removing the moose productivity enhancement (setting R_m^1 and G_m^1 equal to zero). When only the browsing damage effects are included in the model, the optimal strategy is still to bring the two stands into synch over time, showing that the browsing damage effects alone are enough to bring the two stands into convergence. Further, either component of browsing damage, taken by itself, is enough to generate this result. When the impact on the growth rate of pine is removed, by setting c equal to 0, the mortality effect is sufficient to encourage convergence. Conversely, when the mortality effect is removed, by setting γ equal to zero, the growth rate effect alone encourages convergence.

The impact of the second effect is less clear. From the perspective of the moose stock, is it better to have all of the pine forage available at the same time, or spread out over a longer period? The answer depends on the exact form of the relationship between forage availability and moose productivity. In the linear model, for a fixed moose stock, the rate of growth of the stock increases linearly with the amount of forage available. Absent any adjustment to the moose stock, there is no advantage to concentrating the forage into a short period of time.

However, when the wildlife management agency's behavior is included, the relationship becomes non-linear. Absent a strong browsing effect, the wildlife management agency should increase moose stocks when forage availability is high. This enhances the moose productivity effect. The result is a non-linear, convex relationship between forage availability and moose production. It is therefore preferable to concentrate the forage into a short period of time, taking advantage of the greater-than-proportional increase in moose productivity. Indeed, if the moose productivity effect is isolated (by setting γ and c equal to zero), the optimal strategy is still to bring the two stands into synch, showing that the moose productivity effect alone is enough to encourage convergence.

What if the relationship between forage availability and moose productivity is concave - that is, there are decreasing marginal productivity returns to forage? This was investigated using the log model for moose productivity effects. The parameters were adjusted so that the marginal increase in productivity from an increase in forage was the same at $\bar{f}=0$ as with the linear model. The specific parameter values used were $R_m^0=1.05$, $R_m^1=0.03$, $G_m^0=0.105$, $G_m^1=0.003$, and $f_{\min}=9$. The browsing damage effect was eliminated (γ and c equal to zero). The result is that, for lags less than 15 years, the optimal strategy is to *increase* the lag between the two rotations. Even if the initial lag is zero, so that the forest is comprised of one even-age stand, the optimal strategy is to split the stand into smaller pieces, and bring those smaller stands out of synch over time. The benefit from doing so is that the moose productivity boost that occurs after each rotation will stretch over a longer period of time.

Summary and Conclusions

The relationship between moose and pine is bilateral - young pine enhances moose growth, while moose damage young pine. Both of these relationships have implications for forest management.

The browsing damage impact unambiguously encourages synchronization of nearby forest stands. This allows the browsing pressure to be spread out over a larger area during the time when the trees are vulnerable to damage, keeping the pressure below the level where serious harm is done. It is important, though to manage the moose stock correctly during this vulnerable period, to take advantage of the enhanced moose productivity without allowing too much damage to occur.

The implications of the moose productivity impact are more equivocal. If the marginal increase in moose productivity from additional forage is increasing in the amount of forage available, then the optimal strategy is to concentrate the forage into a short period of time, by cutting adjacent forest stands at the same time. This results in a ten year period with very high forage availability, and even higher moose productivity, followed by a long time with no productivity boost. If the marginal increase in moose productivity from additional forage is decreasing in the amount of forage available, then the optimal strategy is to stretch out the forage over as long a period as possible. This is done by spacing out harvests of adjacent stands, so that there is at least one stand providing forage for as long a period as possible.

Clearly, the actual solution will vary depending not only on the form of the relationship chosen for modeling the moose productivity impact, but also on factors such as the amount of non-pine forage available and other factors limiting moose stock size. Whether forest stands should be brought into synch, or pushed out of synch, may well vary from location to location,

particularly due to differences in the quality of the land, as reflected in differences in the growth rate of the pine trees.

References

- Andersen, Reidar, and Bernt-Erik Sæther. 1996. Elg i Norge: Biologi, Atferd og Forvaltning N.W. Damm & Son – Teknologisk Forlag.
- Hartman, R. 1976. “The Harvesting Decision when a Standing Forest has Value.” Economic Inquiry 14:52-58.
- Institute for Forest Economics and Institute for Forest Taxation, Agricultural University of Norway. 1987. Handbok for Planlegging i Skogbruket Oslo: Landbruksforlaget.
- Koskela, E., and M. Ollikainen. 1999. “Optimal Public Harvesting Under the Interdependence of Public and Private Forests.” Forest Science 45(2):259-271.
- Ready, Richard C., Olvar Bergland, and Eirik Romstad. 2000. “Optimal Management of Two Interlinked Renewable Resources: Moose and Pine in Norway.” Discussion paper # D-23/2000, Department of Economics and Social Sciences, Agricultural University of Norway, Aas, Norway.
- Riitters, Kurt, J. Douglas Brodie, and David W. Hann. 1982. “Dynamic Programming for Optimization of Timber Production and Grazing in Ponderosa Pine.” Forest Science 28(3):517-526.

Seierstad, Atle, and Knut Sydsæter. 1987. Optimal Control Theory with Economic Applications
Amsterdam: North Holland.

Solbraa, Knut. 1998. Elg og Skogbruk: biologi, økonomi, beite, taksering, forvaltning. Biri,
Norway: Skogbrukets kursinstitutt.

Swallow, Stephen K., Peter J. Parks, and David N. Wear. 1990. “Policy-Relevant
Nonconvexities in the Production of Multiple Forest Benefits” Journal of Environmental
Economics and Management 19(3):264-280.

Swallow, Stephen K., Piyali Talukdar, and David N. Wear. 1997. “Spatial and Temporal
Specialization in Forest Ecosystem Management under Sole Ownership” American Journal
of Agricultural Economics 79(May):311-326.

Swallow, Stephen K., and David N. Wear. 1993. “Spatial Interactions in Multiple-Use Forestry
and Substitution and Wealth Effects for the Single Stand” Journal of Environmental
Economics and Management 25(2):103-120.

Sæther, Bernt-Erik, Knut Solbraa, Dag Petter Sødal, and Olav Hjeljord. 1992. The Final Report
from the project “Moose – forest – society”. Research Report # 28. Norwegian Institute for
Nature Research.

Sødal, Dag Peter. 1989. Economic Valuation of Moose Hunting Report # 1/89, Institute for Forest Economics, Agricultural University of Norway.

Tahvonen, Olli, and Seppo Salo. 1999. "Optimal Forest Rotation with *in Situ* Preferences." Journal of Environmental Economics and Management 37(1):106-128.

Table 1. Summary of the social-planning solution

time path	Fixed Value	Equation of Motion or F.O.C.
m	$m(0) = \tilde{m}$	$\dot{m} = g(m, \bar{f}) - h$
h		given by $\delta = \frac{\partial g(m, \bar{f})}{\partial m} + \sum_{k=1}^K \frac{\lambda_x^k}{j} \frac{dF(x^k, b^k)}{dm} + \sum_{k=1}^K \frac{\lambda_Q^k}{j} \frac{dD(x^k, m)}{dm}$
x^k	$x^k(0) = \tilde{x}^k,$ $x^k(t_i^k) = x_0^k$	$\dot{x}^k = F^k(x^k(t), b^k(t))$ when $t \neq t_i^k$
λ_m	$\lambda_m = j$ when $h > 0$	$\dot{\lambda}_m = 0$ when $h > 0$
d^k	$d^k(0) = \tilde{d}^k$ $d^k(t_i^k) = 1$	$\dot{d}^k = D(x^k(t), b^k(t))$
λ_x^k	$\lambda_x^k(t_i^{k-}) = d^k \cdot \frac{\partial r(x^k)}{\partial x^k}$	$\dot{\lambda}_x^k = \delta \cdot \lambda_x^k - \lambda_m \cdot \frac{\partial g(m, \bar{f})}{\partial \bar{f}} \cdot \frac{\partial \bar{f}}{\partial x^k} + \sum_{j=1}^K \left[-\lambda_x^j \frac{dF(x^j, b^j)}{dx^k} - \lambda_Q^j \frac{dD(x^j, b^j)}{dx^k} \right]$
λ_d^k	$\lambda_d^k(t_i^k) = r(x^k)$	$\dot{\lambda}_d^k = \delta \cdot \lambda_d^k$
t_i^k		$H(t_i^{k+}) = H(t_i^{k-}) - \delta \cdot [z^k \cdot r(x^k) \cdot d^k + j \cdot (m(t_i^{k-}) - m(t_i^{k+}))]$

Table 2. Parameter Values

symbol	value	Definition
δ	0.025	discount rate
J	5400	value of killing one moose
G_p	403.737	max pine biomass
r_p	0.0469	pine growth rate
x^0	9.8095	initial pine biomass
c	0.5	quantity forage eaten by one moose
r_m^0	0.75	moose baseline growth rate ^a
r_m^1	0.03	increase in moose growth rate ^a
G_m^0	.075	moose baseline carrying capacity ^a
G_m^0	.003	increase in moose carrying
α_v	15	intercept parameter in $v(x)$ function
β_v	1	slope parameter in $v(x)$ function
γ	500	scale parameter in D function
σ	2	exponent in D function
α_0	-1000	intercept in price function
α_1	125007	parameter in price function
α_2	16.026	parameter in price function
α_3	-2.7775	parameter in price function
f^0	10	baseline forage available

^a Values shown are for the linear model. See text for sensitivity analysis using the log model

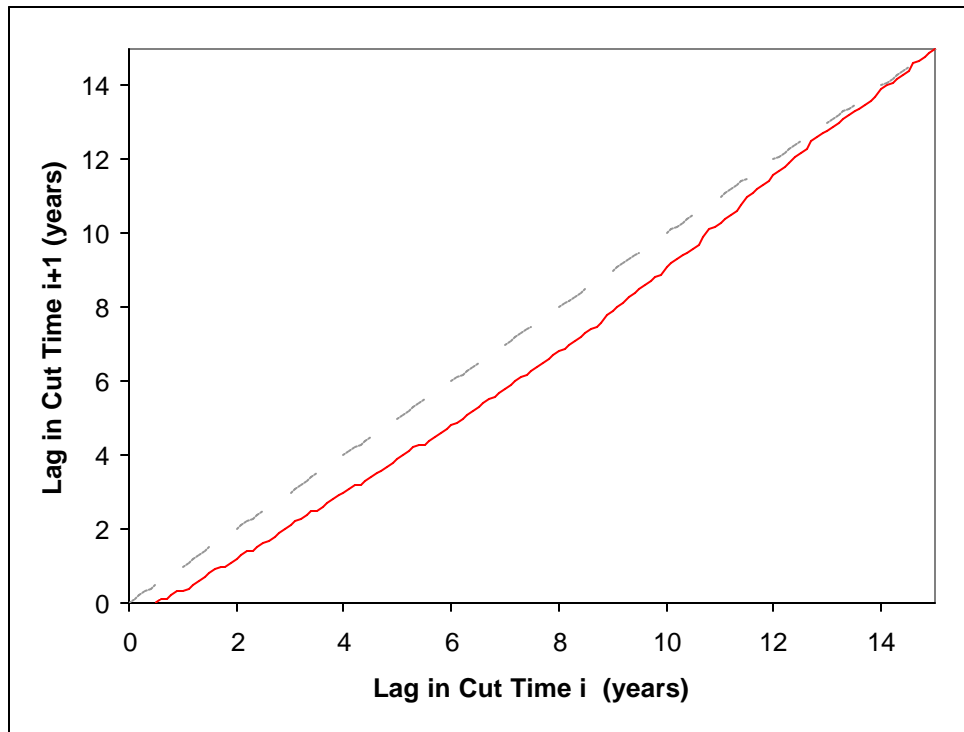


Figure 1. Optimal adjustment of the lag between two adjacent stands.