The Generalized Composite Commodity Theorem and Food Demand Estimation

A.J. Reed, J.W. Levedahl and C. Hallahan

Reed and Levedahl are senior economists and Hallahan is a statistician with the Economic Research Service, U.S. Department of Agriculture, Washington, DC. Contact person: jareed@ers.usda.gov

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Abstract: This paper reports tests of aggregation over consumer food products and estimates of aggregate food demand elasticities. Evidence that food demand variables follow unit root processes leads us to build on and simplify tests of the Generalized Composite Commodity Theorem found in the literature. We compute food demand elasticities using cointegration applied to a convenient but nonlinear functional form. Estimates are based on consumer reported expenditure data rather than commercial disappearance data.

The Generalized Composite Commodity Theorem and Demand System Estimation

Appropriately specified models of consumer demand are central to market analysis. It has been established that a valid equilibrium for markets characterized by diverse firms or diverse consumer products depends on downward-sloping composite demand curves (Heiner, Braulke, Wohlgenant) or on valid indirect utility functions (Chavas and Cox). Given the large number of consumer food products, food demand and price analysis must be implemented at some level of product aggregation. Improper aggregation can lead to biased estimates of welfare loss associated with public policies, biased estimates of consumer and derived demand elasticities, and misleading tests of market power. Testing for consistent aggregation over food products and estimating aggregate consumer food demand elasticities are the subjects of this paper.

By far the most common justification for aggregation has been separable preferences. One reason for its popularity has been the lack of a viable alternative. Tests of the Composite Commodity Theorem (Hicks, Leontief, Lewbel) are always rejected because it restricts relative prices within a group to remain fixed over time. On the other hand aggregation based on weak separability is often rejected (Diewert and Wales, Eales and Unnevehr).

The Generalized Composite Commodity Theorem (*GCCT*) justifies aggregation under milder conditions (Lewbel). It relaxes the restriction of constant relative prices within groups by strengthening the requirement that independence holds across all groups. Furthermore, the *GCCT* simplifies tests for weakly separable preferences.

This paper tests for valid aggregation of consumer food products and reports estimates of food demand elasticities. As in previous studies we find evidence that food demand variables contain unit roots, so tests for valid aggregates involve tests for spurious regressions. By building on the methodology found in previous studies we simplify tests for valid aggregation. In addition we present estimates of food demand elasticities and tests for weak separability based on recent developments in the theory of nonlinear nonstationary regressions.

Theory

The *GCCT* is a stochastic theory of aggregation over diverse consumer products. It maintains that *n*-elementary share equations are functions of logged elementary prices, **r**, and logged income, z. Following Lewbel let w_i $(i=1,\ldots,n)$ denote the ith elementary budget share and let E denote the mathematical expectations operator. Then $g_i:(\mathbf{r},z) \to w_i$ $(i = 1,\ldots,n)$ such that

$$(1) \qquad w_i = g_i(\mathbf{r}, z) + e_i \qquad \text{where} \quad E(e_i\,|\mathbf{r}, z) = 0 \;\Rightarrow\; E(w_i\,|\mathbf{r}, z) = g_i(\mathbf{r}, z).$$

Since the g_i form a valid elementary demand system, they satisfy adding-up ($\sum g_i = 1$), homogeneity ($g_i(\mathbf{r}\text{-}k, z\text{-}k) = g_i(\mathbf{r}, z)$ for all i), and Slutsky symmetry (i.e., $(\partial g_k/\partial r_j) + (\partial g_k/\partial z)g_j = (\partial g_j/\partial r_k) + (\partial g_j/\partial z)\, g_k$). The compensated demands satisfy negative semi-definiteness.

The theory also maintains the existence of a system of stochastic composite share equations. The M ($< n$) composite shares $W_I \equiv \sum_{i\in I} w_i$ $(I = 1,\ldots,M)$ are functions of logged income z and logged composite prices **R**, or G_I: $(\mathbf{R}, z) \to W_I$ $(I = 1,2,\ldots M)$. In particular,

$$(2) \qquad W_I = G_I(\mathbf{R}, z) + u_I, \quad \text{where} \quad E(u_I\,|\mathbf{R}, z) = 0 \;\Rightarrow G_I(\mathbf{R}, z) = E(W_I\,|\mathbf{R}, z).$$

The orthogonality of the model errors of (*1*) and (*2*) ensure that $g_i(\mathbf{r}, z)$ and $G_I(\mathbf{R}, z)$ are optimal predictors of elementary and composite shares, respectively.

These model errors are related. Following Lewbel, let $G_I^*(\mathbf{r}, z)$ denote the sum of the conditional means of the elementary demands for group I, so that $G_I^*(\mathbf{r}, z) \equiv \sum_{i\in I} g_i(\mathbf{r}, z)$. Also define $\rho_i \equiv r_i - R_I$ as the ith relative price so the vector of all relative prices is $\boldsymbol{\rho}=\mathbf{r}-\mathbf{R}^*$ where $\mathbf{R}^*$ denotes the *n*-vector of group prices with R_I in row i and in every row $i\in I$. This implies

(3) $$u_I = \sum_{i\in I} e_i + G_I^*(\rho+\mathbf{R}^*, z) - G_I(\mathbf{R}, z)$$

which shows the composite model errors are correlated with relative elementary prices.

Lewbel shows that valid aggregation obtains when the vector of all relative prices is distributed independently of the vector of composite prices and income. This implies

(4) $$G_I(\mathbf{R}, z) = \int G_I^*(\mathbf{R}^*+\rho, z)\, dF(\rho)$$

which states the conditional expectation of the *I*th composite share equals an unconditional expectation of sums of the elementary demand functions in the *I*th composite. Lewbel uses *(4)* to obtain three results that relate directly to demand system estimation. First, $G_I(\mathbf{R}, z)$ $(I = 1,2,\ldots M)$ is a valid system of composite demand equations because this system inherits the adding up, homogeneity, and nearly (or in some cases exactly) inherits Slutsky symmetry from the elementary demands. Second, the demand elasticities of $G_I(\mathbf{R}, z)$ are best, unbiased estimates of within-group sums of elementary demand elasticities. Third, *(3)* and *(4)* implies $(G_I^* - G_I)$ is a bias term that arises from aggregation and this term is a function of ρ. Because u_I contains this bias the errors of a composite demand system justified by the *GCCT* will be correlated with relative prices. If instead the demand system is based on weakly separable preferences, $G_I^* = G_I$ so $u_I = \sum_{i\in I} e_I$ and composite demand errors will not be correlated with relative prices.

In time series theory the restriction that ρ_t is distributed independently of $\mathbf{q}_t \equiv [\mathbf{R}_t', z_t]'$ imposes restrictions on the correlation of an infinite number of random variables. For example, if ρ_t and $\boldsymbol{q}_t$ are in-deterministic stationary processes they would satisfy

$$\rho_t = \sum_{s=0}^{\infty} \boldsymbol{C}_s \boldsymbol{V}_{t-s} \qquad \text{and} \qquad \boldsymbol{q}_t = \sum_{s=0}^{\infty} \boldsymbol{D}_s \boldsymbol{V}_{t-s}$$

where $\sum_{s=o}^{\infty} |\boldsymbol{C}_s| < \infty$, $\sum_{s=o}^{\infty} |\boldsymbol{D}_s| < \infty$, and $(\boldsymbol{V}_t\, \boldsymbol{U}_t)$ are *iid* normal with mean zero. In this case independence requires $E(\boldsymbol{V}_t\boldsymbol{U}_s) = 0$ for all (t, s). An implication of vector independence is $E(\boldsymbol{\rho}_t|\boldsymbol{q}_t) = E(\boldsymbol{\rho}_t)$ or that $\boldsymbol{q}_t$ provides no information about $\boldsymbol{\rho}_t$. These restrictions may be difficult or impossible to test.

This may be why tests of the *GCCT* have focused on tests for linear relationships. If *P* denotes the linear projection operator, valid aggregation means $P(\boldsymbol{\rho}_t|\boldsymbol{q}_t)$ is not a linear function of $\boldsymbol{q}_t$ and failure to reject such tests is taken as support for valid aggregation (Lewbel, Davis, Davis, Lin, and Shumway, Asche, Bremmes and Wessells). Because evidence has suggested $\boldsymbol{\rho}_t$ and $\boldsymbol{q}_t$ are often unit root processes, aggregation tests have been based on tests of spurious regressions.

Phillips shows that a simple linear regression constructed from two independently distributed and integrated time series behaves like a model constructed from two non-cointegrated series. That is, Phillips shows correlation between the stationary components of the two series will not affect the asymptotic behavior of the regression. It is significant for our work that Phillips extends this result to multiple regression models, so that if $\rho_{it} = P(\rho_{it}|\boldsymbol{q}_t) + v_{it}$ is a spurious regression, the model behaves as though ρ_i is distributed independently of $\boldsymbol{q}_t$. This suggests tests of the *GCCT* that have been based on a large number of simple regression models can be simplified by basing them instead on multiple regression models.

In particular, if $\boldsymbol{q}_t$ is an integrated vector we can, in a straightforward manner, compute a test of the aggregation scheme. An approach is to compute Engle-Granger tests of no cointegration for each of the individual ρ_i multiple regressions for which ρ_i is an integrated variable, and then follow Davis, Lin, and Shumway. That is, based on the individual tests use the Holm procedure to test the family-wise hypothesis that each integrated element of $\boldsymbol{\rho}$ is jointly spuriously related to $\boldsymbol{q}$. This approach differs from the 'grand test' proposed by Davis because it represents a test of the null of valid aggregation. Moreover, our reading of Huang leads us to conclude that the power problems associated Engle-Granger tests are no more severe for regressions with a large number of regressors than they are for a small number of regressors.

A finding of valid aggregation means a composite demand system is associated with this aggregation scheme. Moreover, if demand variables follow unit root processes, we expect the demand equations to be cointegrated (Karagiannis and Mergos). Because of the interest in demand elasticities, and because a number of useful functional forms used in empirical demand work are nonlinear, it is desirable to apply cointegration methods to nonlinear demand systems.

Cointegration and Nonlinear Share Equation Systems

We maintain that composite shares of a valid demand system are adequately described by the semi-flexible almost ideal (*SAI*) demand system (Moschini). The *SAI* demand system is a re-parameterization of the Almost Ideal (*AI*) demand system (Deaton and Muellbauer). Thus, it describes nonlinear Engle curves, defines community income and exact nonlinear aggregation over consumers, and defines budget shares and income elasticities for income inelastic goods such as food that decline as incomes rise (Moschini). Moreover the *SAI* demand system saves degrees of freedom while maintaining curvature at a point in the data. In this section we show that a version of the nonlinear *SAI* demand system can be estimated using cointegration methods.

Recall that W_I denotes the *I*th composite consumer budget share, z the log of income, and R_J the log of the *J*th composite price. If we let e_i denote the *I*th model error, the *AI* model is

$$(5) \qquad W_I = \alpha_I + \sum_{J=1}^{M} \gamma_{IJ} R_J + \beta_I (z - logP) + e_I \qquad (I = 1, \ldots, M)$$

$$(6) \qquad logP = \alpha_o + \sum_{I=1}^{M} \alpha_I R_J + \tfrac{1}{2} \sum_{I=1}^{M} \sum_{J=1}^{M} R_I R_J$$

with Slutsky-substitution terms

$$(7) \qquad S_{IJ} = [x/(p_I\, p_J)][\gamma_{IJ} + W_I\, W_J - \delta_{IJ}\, w_I + \beta_I\, \beta_J\, (z - logP)]$$

where $\delta_{IJ}=1$ for $I=J$ and $\delta_{IJ} = 0$ for $I\neq J$. Moschini notes that if $\alpha_o = 0$ and price and income variables are deflated by their sample means, then at the sample mean $p_I=x=1$, α_I is the Ith budget share, $\varepsilon_{IJ} = (1/\alpha_I)(\gamma_{IJ} - \beta_I\alpha_I) - \delta_{IJ}$ is a cross-price elasticity of demand, $\eta_I = (\beta_I/\alpha_I) + 1$ is the income elasticity of demand, and the Slutsky substitution terms are

$$(8) \qquad \theta_{IJ} = \gamma_{IJ} + \alpha_I\, \alpha_J - \delta_{IJ}\, \alpha_I.$$

The *SAI* model can be used to estimate demand elasticities conditioned on curvature imposed at the mean (or any other point) of the data. By setting $\mathbf{\Theta} = [\theta_{IJ}] = -T'T$ where $T =[\tau_{IJ}]$ is upper triangular and $\mathbf{\Theta}$ is less than full rank, Moschini restricts the rank of $\mathbf{\Theta}$ by setting the last number of rows equal to zero. For example if $\mathbf{\Theta}$ is a 5-by-5 matrix, setting the last two rows of T to zero restricts $\mathbf{\Theta}$ to a matrix of rank three. Such restrictions allow the parameters of

$$(9) \qquad W_I =\alpha_I + \alpha_I\, R_I - \alpha_I \sum_{J=1}^{M} \alpha_J\, R_J - \sum_{s=1}^{I} \tau_{sI} \sum_{J=s}^{M-1} \tau_{sJ}\, (R_J - R_M) + \beta_I\, z - \beta_I\, logP + e_I \qquad (I=1, \ldots, M-1)$$

$$(10) \qquad logP = \sum_{J=1}^{M} \alpha_J\, R_J - \tfrac{1}{2} [\sum_{J=1}^{M} \alpha_J\, R_J]^{2} + \tfrac{1}{2} \sum_{J=1}^{M} \alpha_J\, (R_J)^2 - \tfrac{1}{2} \sum_{s=1}^{M-1} [\sum_{J=s}^{M-1} \tau_{sJ}\, (R_J - R_M)]^2$$

identify the parameters of *(5)* and *(6)*. Equations *(9)* and *(10)* represent the *SAI* demand model.

The task is to estimate this nonlinear system assuming each share equation is an integrated regression with stationary model errors. The estimator developed by Chang, Park, and Phillips applies to a class of nonlinear, single equation cointegrated models. To describe this class in detail let *(9)* be represented as

$$(11) \qquad W_t = \alpha + q(\mathbf{x}_t\, ,\boldsymbol{\beta}) + e_t$$

where $\boldsymbol{x_t}' = [x_{1t}, x_{2t}, \ldots, x_{kt}]$ is a k-vector of integrated stochastic regressors. Chang, Park, and Phillips show their estimator applies to nonlinear models such as *(11)* when model errors are serially uncorrelated (*i.e.,* e_t is a martingale difference series) and $q(\boldsymbol{x_t}, \boldsymbol{\beta})$ satisfies additive separability. They show that if $q(\boldsymbol{x_t}, \boldsymbol{\beta})$ consists of k additive terms for $\boldsymbol{\beta} = [\boldsymbol{\beta_1}', \boldsymbol{\beta_2}', \ldots, \boldsymbol{\beta_k}']$, additive separability requires

$$q(\boldsymbol{x_t}, \boldsymbol{\beta}) = \sum_{i=1}^{k} q_i(x_{it}, \boldsymbol{\beta_i}).$$

Thus, additive separability permits only one integrated regressor per additive term.

For an *SAI* demand equation, it is seen that if *logP* could be treated as one integrated regressor, *(9)* would satisfy additive separability. The problem lies with *logP*. With the exception of the first term, every term in *(10)* involves two integrated variables, so *(9)* and *(10)* violate additive separability. However, the approximation (Deaton and Muellbauer)

$$\text{(10')} \qquad \log P \approx \sum_{J=1}^{M} W_J R_J$$

does treat *logP* as a single variable so *(9)* and *(10')* satisfy additive separability. Moschini notes that *(9)* and *(10')* represent a valid form of the *SAI* model, and demand system estimation in this study is based on *(9)* and *(10')*.

For integrated regressions, cointegration does not ensure econometric exogeneity. The consequence for estimation of linear, single equation cointegrated regresssions is that *OLS* estimates are biased and even though cointegration means this bias disappears in large samples this bias injects nuisance parameters into the distributions of *OLS* estimates. The result is that standard *t*- and *F*-tests are misleading even in large samples. When econometric exogeneity is achieved, the bias and the nuisance parameters disappear so that *OLS* estimates are normally distributed, and t- and F tests provide correct inference. The fully modified *(FM)* estimator transforms single equation linear models in such a way that the transformed model satisfies econometric exogeneity (Phillips and Hansen).

Chang, Park, and Phillips show similar issues generally arise in nonlinear cointegrated regressions, and so derive an *FM* estimator for single nonlinear cointegrated regressions. To describe their estimator it is asssumed that model errors of *(11)* form a martingale difference series and the regression function, q, satisfies additive separability. Let $\Delta \mathbf{x}_t = \mathbf{v}_t$ and $E(\mathbf{v}_t) = 0$ so $[e_t \;\; \mathbf{v}_t]$ forms a linearly indeterministic stationary vector. The covariance generating function (evaluated at frequency 1) of this vector is

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} = \sum_{k=-\infty}^{\infty} \begin{bmatrix} E(e_t\, e_{t-k}') & E(e_t\, \mathbf{v}_{t-k}') \\ E(\mathbf{v}_t\, e_{t-k}') & E(\mathbf{v}_t\, \mathbf{v}_{t-k}') \end{bmatrix} .$$

Since $\mathbf{v}_t$ is a stationary, serially correlated series it satisfies $\mathbf{v}_t = \sum_{i=0}^{\infty} \boldsymbol{\pi}_i \boldsymbol{\varepsilon}_{t-j}$ where $\boldsymbol{\varepsilon}_t$ is a serially uncorrelated *iid* vector process with $\sum_{i=0}^{\infty}|\boldsymbol{\pi}_i| < \infty$. If the model errors are also stationary and serially correlated they satisfy $e_t = \sum_{i=0}^{\infty} \varphi_i \zeta_{t-i}$ with $\sum_{i=0}^{\infty}|\varphi_i| < \infty$ (where ζ_t may be contemporaneously but not serially correlated with $\boldsymbol{\varepsilon}_t$). In this case

$$\Sigma_{12} = \sum_{k=-\infty}^{\infty} E(e_t\, \mathbf{v}_{t-k}') = \sum_{k=-\infty}^{\infty} E\left(\sum_{i=0}^{\infty} \varphi_i \zeta_{t-i}\right)\left(\sum_{j=0}^{\infty} \boldsymbol{\pi}_j \boldsymbol{\varepsilon}_{t-j-k}\right) \neq 0$$

means exogeneity is violated. However, for serially uncorrelated errors $\varphi_i = 0$ for $i > 0$, and

$$\Sigma_{12} = \sum_{k=-\infty}^{\infty} E(e_{tj}\, \mathbf{v}_{t-k}') = \sum_{k=-\infty}^{\infty} E(\varphi_o\, \zeta_t)\left(\sum_{j=0}^{\infty} \boldsymbol{\pi}_j \boldsymbol{\varepsilon}_{t-j-k}\right) = E(\varphi_o \boldsymbol{\pi}(1)\, \zeta_t\, \boldsymbol{\varepsilon}_t) = E(e_{tj}\, \mathbf{v}_t) \equiv \Sigma_{12}^{0}$$

where $\boldsymbol{\pi}(1) = \sum_{j=0}^{\infty} \boldsymbol{\pi}_j$. Hence violations of econometric exogeneity arise in models with serially uncorrelated errors only because of the presence of contemporaneous correlation between e_t and $\mathbf{v}_t$. Define $\Sigma_{22}^0 = E(\mathbf{v}_t\, \mathbf{v}_t)'$ so that subtracting $\Sigma_{12}^0(\Sigma_{22}^0)^{-1}\, \mathbf{v}_t$ from both sides of *(11)* gives

$$(11') \qquad W_t^* = \alpha + q(\mathbf{x}_t, \boldsymbol{\beta}) + e_t^*$$

where $W_t^* = W_t - \boldsymbol{\Sigma}_{12}^{\mathbf{0}}(\Sigma_{22}^0)^{-1}\, \mathbf{v}_t$, $e_t^* = e_t - \boldsymbol{\Sigma}_{12}^{\mathbf{0}}(\Sigma_{22}^0)^{-1}\, \mathbf{v}_t$ so that

$$\sum_{k=-\infty}^{\infty} E(e_t^*\, \mathbf{v}_{t-k}') = E(e_t\, \mathbf{v}_t') - \boldsymbol{\Sigma}_{12}^{\mathbf{0}}(\Sigma_{22}^0)^{-1} E(\mathbf{v}_t\, \mathbf{v}_t') = \boldsymbol{\Sigma}_{12}^{\mathbf{0}} - \boldsymbol{\Sigma}_{12}^{\mathbf{0}} = \mathbf{0}$$

and econometric exogeneity is obtained. This means nonlinear least squares (*NLS*) estimates of *(11')* are consistent, efficient, and normally distributed. Furthermore, consistent estimates of $\boldsymbol{\Sigma}_{12}^{\mathbf{0}}$ and Σ_{22}^0 obtain by applying *NLS* to *(11)* in the first stage. Chang, Park, and Phillips refer to this estimator as the *efficient non-stationary nonlinear least squares* (*EN-NLS*) estimator.

To estimate demand elasticities of an *SAI* demand model using the *EN-NLS* estimator, it must be expanded to a systems estimator. It should be recognized that when estimating systems of cointegrated regressions a violation of econometric exogeneity means *SUR* or nonlinear *SUR (NSUR)* yield inconsistent estimates.[1] The implication for *FM* system estimation is that single-equation estimators such as *OLS, NLS,* or *EN-NLS* rather than multivariate estimators such as *SUR* or *NSUR* must be used in estimating the model in the first stage. The *SUR* or *NSUR* estimator is then applied to the transformed model in the second stage.

More specifically, represent the *M-1 SAI* composite demand equations as

$$(12) \qquad W_{Jt} = \alpha_J + q_J(\mathbf{x}_t, \boldsymbol{\beta}_J) + e_{Jt} \qquad (J = 1,2,\ldots M-1)$$

[1] The reason is for a system of cointegrated regressions with correlation across the model errors and non-zero correlation with the regressors, information on unit root variables in the system is transmitted to the equations across the system and the *SUR* estimator does not weight that information properly. The result is a bias term associated with a *SUR* or *NSUR* estimator that may not disappear asymptotically (Park and Ogaki).

where W_{Jt} is the Jth composite budget share in time t, $\boldsymbol{x_t}$ is a vector of integrated prices and income, q_J is additively separable, e_{Jt} is the Jth element of the $M\text{-}1$ vector $\boldsymbol{e_t}$ which is drawn from a martingale difference series. Let $\boldsymbol{v_t} = \boldsymbol{\Delta x_t}$, denote the vector of first differences of non-redundant regressors in the system with $E(\boldsymbol{\Delta x_t}) = 0$. Since $\boldsymbol{v_t}$ is a stationary vector series, it satisfies $\boldsymbol{v_t}=\sum_{i=0}^{\infty}\boldsymbol{\pi_i \varepsilon_{t-j}}$ and in general $\boldsymbol{e_t}$ satisfies $\boldsymbol{e_t} =\sum_{i=0}^{\infty}\boldsymbol{P_i}\,\zeta_{t-j}$. But because $E(\boldsymbol{e_t}|\ \boldsymbol{e_{t-1}, e_{t-2}}, \ldots) = 0$, $Var(\boldsymbol{e}) =\boldsymbol{P_o \Lambda P_o}'$ where $\Lambda = E(\zeta_t\,\zeta_t')$ so $\boldsymbol{e_t} = \boldsymbol{P_o}\zeta_t$ where $\boldsymbol{\zeta_t}$ is a serially uncorrelated *iid* vector process that can be contemporaneously but not serially correlated with $\boldsymbol{\varepsilon_t}$. Then

$$\boldsymbol{\Sigma_{12}} =\sum_{k=-\infty}^{\infty} E(\boldsymbol{e_t}\ \boldsymbol{v}_{t-k}') = \sum_{k=-\infty}^{\infty} E(\boldsymbol{P_o}\zeta_t)\ (\sum_{j=0}^{\infty}\boldsymbol{\pi_j}\ \boldsymbol{\varepsilon_{t-j-k}}) = E(\boldsymbol{P_o}\zeta_t\ \boldsymbol{\pi(1)}\ \varepsilon_t) = E(\boldsymbol{e_t}\ \boldsymbol{v_t}) \equiv \boldsymbol{\Sigma_{12}^0}$$

and violation of strict econometric exogeneity in a system of cointegrated regressions derives only from the presence of contemporaneous correlation between $\boldsymbol{e_t}$ and $\boldsymbol{v_t}$ when model errors are serially uncorrelated. Define $\Sigma_{22}^0 = E(\boldsymbol{v_t}\ \boldsymbol{v_t})'$ and let $\omega_{\boldsymbol{120}}^{\boldsymbol{(J)}}$ represent the Jth row of $\boldsymbol{\Sigma_{12}^0}$ so that subtracting $\omega_{\boldsymbol{120}}^{\boldsymbol{(J)}}(\Sigma_{22}^0)^{-1}\boldsymbol{v_t}$ from *(12)* gives

$$(12') \qquad W_{Jt}^* = \alpha_J + q_J\,(\boldsymbol{x_t}, \boldsymbol{\beta_J}) + e_{Jt}^* \qquad (J = 1,2,\ldots, M\text{-}1)$$

where $W_{Jt}^* = W_{Jt} - \omega_{\boldsymbol{12}}^{\boldsymbol{(J)}}(\Sigma_{22}^0)^{-1}\boldsymbol{v_t}$ and $e_{Jt}^* = e_{Jt} - \omega_{120}^{(J)}(\Sigma_{22})^{-1}\ \boldsymbol{v_t}$. This means

$$\sum_{k=-\infty}^{\infty} E(e_{Jt}^*\boldsymbol{v_t}') = E(e_{Jt}\ \boldsymbol{v}_t') - \omega_{120}^{(J)}(\Sigma_{22}^0)^{-1}\ E(\boldsymbol{v}_t\ \boldsymbol{v}_t') = \omega_{\boldsymbol{120}}^{\boldsymbol{(J)}} - \omega_{\boldsymbol{120}}^{\boldsymbol{(J)}} = \boldsymbol{0}$$

and econometric exogeneity is achieved. This means that given consistent estimates of $\boldsymbol{\Sigma_{12}^0}$ *NSUR* estimates of $\boldsymbol{\beta}$ from *(12')* are consistent and normally distributed. It should be noted this nonlinear estimator takes the same form as the *FM* estimator for linear systems (Moon).

There are two more points worth mentioning. First, it is well known that because $\sum_{J=1}^{M} W_{Jt}=1$, the error covariance matrix is singular and the model is estimated using $M\text{-}1$ equations (Berndt and Savin). When cross-equation restrictions are imposed in the first stage of a mean distance estimator (e.g., *SUR*) as they are for the *SAI* model, the estimates will not be invariant to the omitted equation unless one uses a first-stage weight matrix that treats equations symmetrically (see Chavas and Sergerson). The problem is these matrices contain non-zero off diagonal elements and this leads to inconsistent estimates of cointegrated systems for the same reasons *SUR* or *NSUR* yields inconsistent estimates (see footnote 1). In this study we use the identity as the first-stage-weight matrix, and recognize the estimates are consistent but not invariant to the equation omitted.

Second, Chavas and Sergerson note that if the model errors are included in the specification of share equations, as they are in (*12*), they also enter the indirect utility function and so can lead to heteroskedastic errors. They recommend applying a *GLS* transformation that accounts for heteroskedasticity prior to estimation. However, cointegration theory is based on a data-generating process in which partial sums are distributed like continuous time Brownian motion variables (Phillips and Durlauf). This automatically allows for heteroskedastic errors, so the only transformations that are necessary are those ensuring econometric exogeneity.

Empirical Results

This section reports a test of valid aggregation of *19* elementary at-home food products, estimates of composite food demand elasticities, and tests for weakly separable preferences.

We propose the *19* food products be aggregated into the following five at-home food composites. The cereal and bakery composite includes all cereal and bakery products. The meat composite includes beef, pork, other meat, poultry, and fish and seafood. The dairy composite includes fluid milk, butter, cheese, and ice cream. The fruit and vegetable composite includes fresh fruit, fresh vegetables, and processed fruit and vegetables. The other food-at-home composite includes sugar and sweets, fats and oils, non-alcoholic beverages, eggs, and miscellaneous foods. Food-away from home and non-food are treated as valid composites.

Estimates of U.S. quarterly budget shares, after-tax income, and community income are computed from 1982.1 to 2000.4 using weighted sums of household expenditures reported in the diary section of the Consumer Expenditure Survey (*CES*) (U.S. Dept. of Labor), with weights supplied by the *CES*. Quarterly after-tax income is constructed as the U.S. annual estimate divided by four, and nonfood expenditure is the difference between the quarterly after-tax income and the sum of away-from-home and at-home food expenditures. The quarterly budget share for the *I*th composite, w_I, is computed as the ratio of the expenditure for the *i*th good-to-after tax income. If x_h denotes the (weighted) total expenditures for household *h* and k_h denotes the number of members in household *h*, then community income (per capita) associated with *PIGLOG* preferences (Muellbauer) is $x_o = exp[\sum_h x_h \, log(x_h / k_h) / \sum_h x_h]$. If m_x denotes the sample mean of x_o, $z \equiv log\,(x_o / m_x)$ is used in estimation.

Quarterly Laspeyres price indices (1982-84= 1.0), P_J, $J = 1, \ldots, 5$ are constructed for the five at-home categories using the *19* elementary prices, p_j, with expenditure weights constructed from the expenditure data. Logs of mean-deflated prices are used in testing and estimation. Specifically if M_J denotes the sample mean of P_J and m_j denotes the sample mean of p_j, then the *J*th log mean-deflated composite price is log $(P_J / M_J) \equiv R_J$ and the *j*th relative elementary price is log $(p_j/m_j) - R_J \equiv \rho_j$ for every $j \in J$.

Table 1 reports unit root tests on relative prices, composite prices, and community income. They indicate that at the $\alpha = 0.10$ level of significance most of the *27* elements of ρ, $\mathbf{R}$ and *z* follow unit root processes. Both the Dickey-Fuller and the Kwaitkowski, Phillips, Schmidt and Shin tests suggest that community income, six of the seven composite prices, and *9* relative prices follow unit root rather than trend stationary processes. Both tests also suggest the relative price of beef is trend-stationary. The tests conflict for the dairy price index and the remaining *9* relative prices. A test of the Joint Confirmation Hypothesis (*JCH*) of a unit root (Silvestre, Rossello, and Ortuno) confirmed the presence of a unit root in the dairy price index and in *6* of those remaining *9* relative prices. A unit root could not be confirmed for the relative prices of butter, fresh vegetables, and eggs and so these series are considered trend stationary. The results in Table *1* suggest unit root processes generate all composite prices, community income, and *15* of the *19* relative prices.

Table *2* reports the Engle-Granger test statistics (T_k) for each of the *15* integrated relative price regressions. Each is specified as a function of an intercept, a time trend, community income, the five food-at-home, the away-from home, and the nonfood price indices. With the exception of processed fruits and vegetables, the each individual test failed to reject the null of spurious regression. Following Davis, Lin, and Shumway the family-wise test statistic of no cointegration is $max\ |T_k| = 5.989$. For a *10*-percent family-wise significance level, the *(.10/15)* critical point of the distribution of this statistic under the null of no cointegration for each of the tests and for *76* observations is $T^* = 6.952$ (MacKinnon). Since $max\ |T_k| < T^*$ the tests fail to reject the aggregation scheme. This suggests composite demand elasticities for this scheme accurately reflect the elasticities for the products that consumers actually purchase (Lewbel).

The system estimates are computed in three steps. First, compute estimates of $\boldsymbol{\Sigma}_{12}^{0}$ and Σ_{22}^{0} by applying the *NSUR* estimator to the system using the *M-1* identity matrix as the weight matrix in the first stage. Specifically, denote the first-stage residuals as $\boldsymbol{r}_t = [r_{1t}, \ldots, r_{M-1,t}]$, and the vector of first differences of non-redundant regressors (with drift removed) as $\boldsymbol{v}_t$. From these compute the contemporaneous covariance matrix, $\boldsymbol{S}_{22}^{0} = (1/T)\sum \boldsymbol{v}_t^* \boldsymbol{v}_t^{*\prime}$, where $\boldsymbol{v}_t^* = \boldsymbol{v}_t - \boldsymbol{c}\,\boldsymbol{v}_{t-1}$ where $\boldsymbol{c} = (\sum \boldsymbol{v}_{t-1} \boldsymbol{v}_{t-1}')^{-1}(\sum \boldsymbol{v}_{t-1} \boldsymbol{v}_t')$, and the cross-covariance matrix, $\boldsymbol{S}_{12}^{0} = (1/T)\sum(\boldsymbol{r}_t \boldsymbol{v}_t^{*\prime})$.

Second, construct the transformed model as $W_{Jt}^{*} = W_{Jt} - \boldsymbol{s}_{120}(J)\,(\boldsymbol{S}_{22}{}^{0})^{-1}\boldsymbol{v}_t{}'$ where $\boldsymbol{s}_{120}(J)$ denotes the *J*th row of $\boldsymbol{S}_{12}{}^{\boldsymbol{0}}$. Third, apply *NSUR* to the transformed system of cointegrated regressions.

Table 3 presents estimates of composite consumer demand elasticities. They are based on a rank three **Θ** matrix. The results yield relatively large estimates of income elasticities, although such results may be attributed to the very broad definitions of the composites. The results suggest the fruits and vegetable composite is the most price elastic and meat is the least price elastic. Except for the meat composite, the results suggest the food-away-from-home composite is a gross substitute for the at-home food groups, and nonfood is a gross complement for all at-home food groups.

At this point we note that the above estimates are based on consumer-reported expenditure data rather than *USDA*'s computed farm-based commercial disappearance data (e.g., Eales and Unevehr). The problem with using the commercial disappearance data in consumer demand analysis is these data provide information only on the physical amount of farm components in food. By ignoring the value that consumers place on the *mix* of food products, commercial disappearance ignores the fact that the mix of food products purchased has changed over time. Nelson shows that the *CCT* permits composite demand to be decomposed into a physical component and a quality component, where quality is a value measure of the mix of products purchased and where variations in quality reflect changes in the mix of products purchased over time. Reed, Levedahl and Clark show this same decomposition follows from the *GCCT*, and provides evidence that consumers respond to changes in prices and income mostly by adjusting the mix of products purchased. Hence using commercial disappearance data as a proxy for food demand omits this important aspect of consumer demand for food.

Finally, there is interest in checking for weak separability. Under weak separability, the model errors of a composite demand system are not correlated with relative prices. Because the model errors are presumed to be stationary, a test for weak separability reduces to a test that the model errors are uncorrelated with stationary elements of ρ. Table 1 suggests the relative prices for beef (ρ_b), butter (ρ_{bu}), fresh vegetables (ρ_v), and eggs (ρ_e) are stationary. If u_k denotes the residual of *k*th composite demand equation, we estimate

$$(13) \qquad u_{kt} = \pi_{ko} + \pi_{k1}\rho_{bt} + \pi_{k2}\rho_{but} + \pi_{k3}\rho_{vt} + \pi_{k4}\rho_{e\,t}, + \pi_{k5}(\rho_{bt}\rho_{but}) + \pi_{k6}(\rho_{vt}\rho_{e\,t}) + \xi_{ktj}\,.$$

for $k = 1,\ldots, M\text{-}1$ and test the null $\pi_{k1} = \pi_{k2} = \pi_{k3} = \pi_{k4} = \pi_{k5} = \pi_{k6} = 0$. The results presented in Table 4 suggest this aggregation scheme cannot be based on weakly separable preferences.

Conclusions

One part of Lewbel's message is that the Generalized Composite Commodity Theorem may support a number of different aggregation schemes. Another part suggests this theorem could lead to improved estimates of consumer demand elasticities. This paper represents an attempt to address both of these points.

Our results agree with previous studies that suggest data used in food demand analysis are generated from unit root processes so that tests for valid aggregation may reduce to tests for spurious regressions. We build on these studies by applying multi-comparison procedures to multiple rather than simple regression models. This simplifies testing and leads to a straightforward test of the aggregation scheme. Moreover, we choose a popular form to describe this composite food demand system, and show it can be treated as an estimable nonlinear system of cointegrated regressions. The demand elasticities for six broadly defined food categories appear to be reasonable, and tests reject weak separability.

Other results not reported here suggest the elementary food products chosen for this study could have been aggregated differently. While this may be symptomatic of the low power of residual tests for spurious regression, they may also reflect the notion that the stochastic nature of the *GCCT* may support numerous aggregation schemes. This would suggest, for example, that demand and market analysis applied to nutrition-based aggregates such as *USDA*'s food pyramid applies equally well to analysis based on the more traditional farm-based aggregates.

Table 1. Unit Root Tests of Income and Group and Relative Prices

Null hypotheses:	*I(1)*	*I(0)*	*I(1) or I(0)?*
	τ_τ	η_τ	
R (Cereal and Bakery)	-1.611 (8)	0.237 (6)*	I(1)
ρ (cereal)	-1.428 (8)	0.261 (6)*	I(1)
ρ (bakery)	-1.487 (8)	0.260 (6)*	I(1)
R (Meat)	-0.967 (5)	0.205 (6)*	I(1)
ρ (beef)	-3.246 (8)*	0.116 (6)	I(0)
ρ (pork)	-3.014 (6)	0.102 (5)	I(1) (JCH)
ρ (other meat)	-2.206 (5)	0.185 (6)*	I(1)
ρ (poultry)	-2.519 (6)	0.089 (5)	I(1) (JCH)
ρ (fish and seafood))	-2.152 (6)	0.136 (6)*	I(1)
R (Dairy)	-2.217 (6)	0.077 (6)	I(1) (JCH)
ρ (fluid)	-2.575 (3)	0.112 (6)	I(1) (JCH)
ρ (butter)	-4.639 (8)*	0.201 (6)*	I(0) (JCH)
ρ (cheese)	-1.186 (2)	0.208 (6)*	I(1)
ρ (ice cream)	-2.678 (3)	0.111 (6)	I(1) (JCH)
R (Fruits and Vegetables)	-1.751 (5)	0.152 (6)*	I(1)
ρ (fresh fruit)	-1.914 (6)	0.190 (4)*	I(1)
ρ (fresh vegetables)	-3.159 (8)	0.068 (1)	I(0) (JCH)
ρ (proc. Fruit&Veg)	-3.104 (8)	0.118 (1)*	I(1)
R (Other Food at Home)	-1.636 (7)	0.124 (6)*	I(1)
ρ (sugar and sweets)	-2.266 (6)	0.118 (6)	I(1) (JCH)
ρ (fats and oils)	-3.389 (5)*	0.146 (6)*	I(1) (JCH)
ρ (non alcoholic bev)	-2.153 (2)	0.164 (6)*	I(1)
ρ (eggs)	-2.562 (6)	0.054 (5)	I(0) (JCH)
ρ (miscellaneous foods)	-2.160 (8)	0.180 (6)*	I(1)
R (Food Away from Home)	-1.847 (5)	0.290 (6)*	I(1)
R (Nonfood)	-0.928 (3)	0.239 (6)*	I(1)
z (income)	-1.551 (8)	0.194 (4)*	I(1)
10 percent critical values:	τ_τ* = -3.167	η_τ* = 0.119	(τ_τ, η_τ)* = (-3.601,0.073)

Notes: Asterisk (‘*’) denotes rejection of the null at the 0.10 level of significance. The test statistics of the null hypothesis of *I(1)* (τ_τ) are the augmented Dickey-Fuller (1979) (*ADF*) *t*-values of the coefficient on the lagged level variable in the regression of the first-differences on a constant, a time trend, the lagged level and lagged-differences of variables appended to the regression. The number of lags of first differences is reported in parentheses and determined by *SHAZAAM 7.0*. The second column (η_τ) reports test statistics developed by Kwaitkowsi*,* Phillips, Schmidt, and Shin *(KPSS)*. They are sums of squared partial sums of residuals divided by an error variance estimator. The residuals are computed from a model in which the series is regressed on a constant and a time trend, and the error variance estimator is a Bartlett kernel weighted-sum of auto-covariances, with the automatic (Newey-West) bandwidth parameter reported in parenthesis. The third column reports inference based on the Joint Confirmation of a Unit Root, and is used when the tests in the first and second columns conflict (Silvestre, Rossello, and Ortuno). The joint critical values *(-3.601,0.073)* represent the mid-point of critical values for *50* and *100* observations for the *ADF* and the *KPSS* (with Bartlett kernel) tests with trend. They are interpreted as follows. If the value of the *ADF* statistic (column 2) is less (greater) than –3.601and the value of the *KPSS* statistic (column 3) is less (greater) than 0.073 then the series is considered (at the 0.90 level) stationary (integrated). Otherwise the series cannot be confirmed to be a unit root and is therefore considered to be stationary.

Table 2. Individual and Joint Tests of Spurious Regressions

Relative Price Regression	T_k
1. cereal	-4.491 (8)
2. bakery	-4.415 (8)
3. beef	*NC*
4. pork	-3.544 (8)
5. other meat	-3.318 (5)
6. poultry	-3.561 (7)
7. fish and seafood	-2.436 (8)
8. fluid	-3.471 (3)
9. butter	*NC*
10. cheese	-2.390 (7)
11. ice cream	-4.658 (2)
12. fresh fruit	-4.935 (4)
13. fresh vegetables	*NC*
14. processed fruits and vegetables	-5.989 (8)
15. sugar and sweets	-4.600 (8)
16. fats and oils	-4.771 (5)
17. nonalcoholic beverages	-3.273 (6)
18. eggs	*NC*
19. miscellaneous foods	-3.473 (8)

10 percent critical values:

T^* = -5.7381 *(Individual tests)*
T^* = 6.9521 *(Family-wise test)*

Notes: The entries (T_k) are Engle-Granger tests of the null that the *k*th relative price and the vector of composite group prices and income are not cointegrated. The entries are augmented Dickey-Fuller tests of *I(1)* residuals formed from regressing the *k*th relative price on each of the seven integrated group price indices (see Table 1), income, a constant, and a time trend. The number of lagged first difference residuals included (in the residual regression) is reported in parentheses, and is determined by *SHAZAM 7.0*. The *0.10* critical values reported for the individual tests are based on *76* observations and *8* integrated explanatory variables, so that $k = 9$ in MacKinnon. The *0.10* family wise critical value of *6.952* is based on *76* observations, $k = 9$, and the *(0.10/15)* critical point.

Table 3. Composite Demand Elasticities

	Cereal & Bak (R_1)	Meats (R_2)	Dairy (R_3)	Fruits& Vegs (R_4)	Other Food (R_5)	Away (R_6)	NonFood	Income (z)
Cereal & Bakery	**-0.606**	0.036	-0.396	0.399	-0.673	0.182	-0.293	**1.351**
Meat	0.014	**-0.605**	0.257	-0.072	0.180	-0.736	-0.849	**1.810**
Dairy	-0.547	0.589	**-0.861**	-0.143	-1.260	1.321	-1.346	**2.246**
Fruits & Vegetables	0.357	-0.108	-0.089	**-0.979**	-0.237	0.497	-1.042	**1.601**
Other Food (home)	-0.337	0.176	-0.461	-0.125	**-0.741**	0.656	-0.207	**1.038**
Food Away	0.049	-0.337	-0.276	0.154	0.344	**-0.692**	1.173	**1.379**
Nonfood	0.001	-0.002	-0.002	-0.008	-0.003	-0.045	**-0.864**	**0.924**

Table 4. Weak Separability Tests

	Groups	*W*	$P\ (W > \chi^2)$
1.	Cereal and Bakery	59.37	<0.0001
2.	Meat	51.73	<0.0001
3.	Dairy	36.65	<0.0001
4.	Fruit and Vegetables	26.62	<0.0001
5.	Other Food at Home	66.59	<0.0001
6.	Food Away	13.52	0.036

Notes: The entries (*W*) are Wald statistics associated with the null that relative prices are not related to the composite model errors. Each statistic is based on equation *(13)* in the text, and therefore each is distributed chi-square with 6 degrees of freedom. The third column reports probabilities of observing the reported level of *W* under the null of weak separability.

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