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Endogenous Growth, Health and the Environment

I. Introduction

Previous models of economic growth with environmental degradation or resource exhaustibility have focused predominantly on the socially optimal, i.e., *efficient*, path of growth, thus omitting an analysis of the *equilibrium* path and how such a path might diverge from the efficient path.[1] As such, they have emphasized the *potential* of an environmentally or resource constrained economy to grow, as it were, by the hands of a central planner, but not the *actual* long run growth path of the economy. Since the latter reflects *market* and/or *institutional* specificity of the economy but the former does not, ignoring the equilibrium path reduces the richness of the analysis under different market and/or institutional structures. Further, by ignoring the equilibrium path as a point of reference little, if any, insight is provided for policy analysis and the question of how the economy may move from the equilibrium to the efficient path. A focus on the role of policy is particularly important, given the magnitude of environmental problems around

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[1] On the exhaustible resource and growth literature see for example, Solow (1974) and Solow and Wan (1976), Hartwick (1978), Ayers (1988). On the environment and growth literature see Foster (1973), D'arge and Kogiku (1973) Krautkraemer (1985) and Smith (1977). [See Pezzey, 1989 for a review]. Although Smith (1989) does emphasize the decentralized interpretations of optimal control theory, the central shortcoming of his paper is that the rate of environmental degradation is specified *exogenously* via a functional form, rather than *endogenously* derived, as in this paper.

the globe, and the urgency it presents to the policy makers[2].

The literature that does focus on how the equilibrium and the efficient paths diverge are the "new growth theories", also known as the theories of "endogenous growth." These theories emphasize the role of human capital, technology, R & D or fiscal policy instruments (e.g., Lucas, 1988; Romer, 1986, 1987, 1990; Barro, 1990, Rebelo, 1990) in economic growth. But the approach taken in the endogenous growth theories has not yet been adopted to the case of environmental or natural resource degradation. This is surprising, because the key to generating a departure of the equilibrium and the efficient paths in the new growth theories is the role of externalities, and externalities are of course at the root of environmental and resource degradation problems.

This paper studies the growth of an economy with environmental degradation, using an endogenous growth approach. Consequently, the paper also contributes to the endogenous growth literature. Growth in this paper is a consequence of the optimizing behavior of infinitely lived atomistic agents where environmental/resource externalities are present. With this approach we are able to derive equilibrium and efficient paths under different scenarios and to shed light on the type and direction of economic policy for sustainable growth.

A second area of contribution is the key role of health. Previous models of growth which address environmental degradation, ignore the impact on the

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[2] Witness for example the UN sponsored Rio De Janeiro meeting in June of 1992. The World Bank will devote its 1992 volume entirely to the environment (World Bank, 1992). One may also cite the Bank's 1991 Progress Report on the environment (World Bank, 1991), or the volume edited by Schramm and Warford (1989) for the Bank, and the World Bank Working paper by John Pezzey (1989).

environmental degradation on health *per se*. Yet, concern over the environmental effects of health is increasingly evident, both by the growth in demand for health and by the increase in the knowledge of environmental substances that affect health, entering through the food chain (See, for example, Caswell, 1991), or through air and water pollution, or leading to a deterioration of the upper atmospheres capacity to filter harmful radiation. At the same time, policies to limit the application of chemicals in food production, the control of affluent and emission discharges from plants and various and other efforts to limit exposure to harmful substances tends to increase production costs. The increase in costs brings into question whether a country, in the presence of policies to address these concerns, can sustain its historic levels of economic growth or whether economic growth is an appropriate welfare index.

Conceptually, because health affects the agents' utility function, and because environmental quality affects health[3] any endogenous growth model which addresses environmental degradation needs to take into account health as a key intermediate variable between environment and the utility function. In the literature that discusses the effects of environment on utility, such a discussion takes place in terms of the direct *amenity* value of the environment, as illustrated for example in the model by Krautkraemer (1985),

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[3]An example of a rigorous micro-level study that documents the impact of "micro-level" environmental effects on health at the household level, such as smoking, is the study by Rosenzweig and Schultz (1983). However, their study is concerned with factors that are subject to choice as in the case of smoking, but not the broad aggregate environmental factors, that are *external* to the household as, for example, in the case of air pollution. The latter is the subject of this paper.

or in the discussion by McConnel (1985). Thus, the effect of environment on utility via health has not been addressed. Although for some simple problems, the amenity effect may be viewed, analytically, as encompassing the health effect, this is not in general the case because of the more complex relation between health and factors other than the environment. This is illustrated in the example of Section IV. In such cases, introduction of the health variable poses analytically distinct issues, that cannot be addressed by a mere re-interpretation of the amenity effect. Moreover, even in the simplest cases, the task of evaluating the amenity value of the environment for the purpose of policy analysis presents great difficulty as reflected, for example, in the contentious area of "contingent valuation" that attempts to gauge "willingness to pay". For this reason, setting environmental standards by health considerations has been advocated as a substitute (e.g. Hueting, 1989).[4]

Three models of the endogenous growth variety are presented. The first two are extremely simple models that emphasize the divergence between the equilibrium and efficient paths when environmental quality is either a consequence of consumption externality (Section II) or production externality (Section III). Health in these models is *not* a crucial variable and acts simply as an intermediary between environment and the utility function. Although certain environmentally harmful products, such as CFCs may be a by-product of both the consumption and the production processes, we shall separate the two sources, studying the consumption induced by-product in Section II, and the production induced by products in Section III. Little is gained by studying the two effects simultaneously.

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[4]For further discussion see S. El Serafy and E. Lutz (1989, pp. 23-38)

The third model (Section IV) constitutes the main part of the paper. This model also emphasizes the divergence between equilibrium and efficient paths; but it also emphasizes the crucial role of health as a *non-trivial* variable in the study of environment and growth. It studies a class of problems, preponderant in the real world, in which health is subject to both the aggregate external effects of environment, as in the first two models, but also a "residue effect" embodied in the product that is consumed. Further, the profit maximizing behavior of producers means that the external and the internal effects are interrelated and subject to trade-offs via a technological frontier. We call the first effect, *disembodied effect* and the second, *embodied effect*. The clearest example of such class of problems is the use of preservatives in food which retard spoilage but which may be potentially carcinogenic (as in the case of Nitrates), versus the use of refrigeration which reduces the need for preservatives, but intensify ozon depletion and are therefore indirectly harmful to health via their environmental effect.

Section V makes some concluding remarks.

II. Environment as a Negative Consumption Externality

The first simple model depicts a disembodied effect of a consumer induced CFC-ozone depletion or a CO_2 emission type of problem. For the present, we abstract both from the producer induced effects as well as the mentioned embodied health effect. Households maximize the utility,

$$W = \int_0^\infty u(C,H)e^{-\rho t}dt \tag{1}$$

where C is a composite consumption good and H is health. Suppose $H = H(E)$, $(H' > 0)$, where E denotes environmental quality, or in terms of the illustrative problem, protection from exposure to harmful radiation. In this section, E is assumed fixed to households, i.e. it is viewed by each household as unaffected by choice of consumption, C, but varies adversely with C in the aggregate, as depicted by a reduction in the extent of radiation-free environment, in the CFC-Ozone illustration. For simplicity, let health be determined linearly by environmental quality, with scales appropriately chosen so that, $H(E) = E$. Then we have:

$$\overline{H} = \overline{E} \quad \text{for each household} \tag{2a}$$

$$H = E = aC^{-b} \quad (a,b>0) \text{ in the aggregate} \tag{2b}$$

where $(^{-})$ in equation (2a) indicates that the variable is exogenous to the individual household. In this and the next section, households are producers and consumers, since separation of the two tasks does not enhance the analysis. The household maximization is subject to the budget constraint:

$$C = f(K) - \dot{K} \tag{3}$$

A utility function of the form,

$$u(CH) = [(CH)^{1-\sigma}-1]/(1-\sigma), \qquad (\sigma > 0) \tag{4}$$

is assumed. This form is customary in dynamic problems, with the simplifying constraint here that C and H have the same weight in the utility function. Assuming a simple production function of the form, $f(K) = AK$, the growth rate of the decentralized economy is:

$$\gamma \equiv \dot{C}/C = \frac{A-\rho}{\sigma} \tag{5}$$

Growth is positive if the productivity of capital (A) exceeds the discount rate, and the presence of health factor makes no difference to the decentralized economy's growth rate because it is ignored by atomistic agents. The transversality condition requires that $A<\rho/(1-\sigma)$.[5] This imposes an upper bound on the productivity of the economy, and also implies that intertemporal substitution be inelastic ($\sigma<1$).

Efficient (Socially Optimum) Path

With the existence of the negative externality as represented by equation (2b), the choice of C and H to maximize (1) yields the socially optimal growth rate:

$$\gamma_s = \frac{A-\rho}{\sigma^*} \qquad \text{where, } \sigma^* \equiv \sigma + (1-\sigma)b. \tag{6}$$

The parameter σ^* may be thought of, as the *effective* intertemporal substitution elasticity. The necessary condition for the stability of this system is that the integrand be concave in $\dot{K}$ (Kamien and Shwartz, 1991, p. 128). This will require that $b<1$,[6] which imposes an upper limit on the

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[5]Using a Current Value Hamiltonian approach, where $\mathcal{H} = U(C(t),H(C(t)) + \lambda[AK(t)-C(t)]$, the transversality condition is that, $\lim_{t\to\infty} e^{-\rho t}\lambda(t)K(t) = 0$. The multiplier λ evolves according the first order condition, $\mathcal{H}_K = \rho\lambda - \dot{\lambda}$, or $\dot{\lambda}/\lambda = (\rho-A)$ which gives $\lambda(t) = \lambda_o e^{(\rho-A)t}$. Substituting for this and for $K(t) = K_o e^{\gamma t}$ into the transversality condition we find this condition is satisfied if $\gamma < A$, or $A < \rho/(1-\sigma)$.

[6]Denote the integrand by V. Then $\partial^2 V/\partial\dot{K}^2 = -e^{-\rho t}H\, u'(.)(1-b)\sigma^*/C < 0$, if $b<1$.

extent of the elasticity of health to consumption. Again, the transversality condition requires that $A< \rho/(1-\sigma^*)$, implying that $\sigma^* < 1$.

Since $\sigma < 1$ from above, from the definition of σ^* we can see that, $\sigma^* > \sigma$.[7] Thus it follows from a comparison of equations (5) and (6), that the efficient growth path is *below* the equilibrium path, and furthermore that it *falls* with higher values of the parameter b: In sum:

$$\sigma < 1 \Longrightarrow \sigma^* > \sigma \Longrightarrow \gamma_s < \gamma \; ; \; \partial\gamma_s/\partial b < 0. \tag{7}$$

To explain, equation (7) suggests that if consumers were able to influence the level C, collectively or via the government, they would *prefer* to increase consumption over time by a *smaller* amount than of they if acted atomistically. This would then slow down the rate of capital accumulation as well.

Path of Environmental Decay

Environmental quality, $\varepsilon = \dot{E}/E$, deteriorates, in equilibrium, at the rate given by equations 2b and 5, and along the efficient path, at the rate given by equations 2b and 6. These are:

$$\varepsilon = -b\gamma = -b\frac{A-\rho}{\sigma} \tag{8}$$

$$\varepsilon_s = -b\gamma_s = -b\frac{A-\rho}{\sigma^*} \tag{9}$$

Equation (8) tells us that along the equilibrium path, environmental quality (and thus health) deteriorates at a rate that is in proportion to the

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[7]Notice that because the transversality condition for the γ_p path means that $\sigma < 1$ which in turn implies $\sigma^* > \sigma$, and because $\sigma^* <1$, is implied by the transversality condition for the γ_s path, it follows that $1>\sigma^*>\sigma$ in this case, so that $\sigma <1$ condition is reproduced again.

size of b. It turns out that environmental quality also deteriorates in proportion to the size of b along the efficient path: Substituting for $\sigma^* = \sigma + (1-\sigma)b$, into equation (9) and differentiating with respect to b, we find that $d\varepsilon_s/db < 0$.

An important result is the fact that because environmental quality deteriorates in proportion to the growth rate of the economy, it must deteriorate at a *slower* rate along the efficient path ($|\varepsilon_s|<|\varepsilon|$), because of the slower growth rate along that path ($\gamma_s<\gamma$). This result suggests that even for a homothetic utility function, maintaining the same level of environmental quality requires continuous investment in environmental quality at a rate given by $|\varepsilon|$ or $|\varepsilon_s|$ (as the case may be). The agents' willingness to pay for the quality of environmental is likely to be greater, *at higher levels of consumption per capita*, in a model based on non-homothetic utility functions that reflect the agents' increasing valuation of the relative significance of the health and environmental factors. In such a model, agents would like consume less of the "bad" environmental effects at higher levels of consumption per capita, along the efficient path, which in turn decreases the rate of environmental degradation along that path. If investments in environmental quality were permitted, agents would also be willing to invest more on the environment relative to the homothetic case. The usefulness of such an observation is transparent in comparing the sustainability of growth in developed *versus* underdeveloped countries.

Finally, it may noted that underlying these results is an important conceptual issue regarding the role of induced technological change that we will address in the concluding section of of the paper.

III. *Environment as a Negative Production Externality*

So far, we have focused on the negative environmental consequences of aggregate consumption. Attention is now placed on a health externality as a product of the production process. In this case, we replace the aggregate heath function in equation (2b) with the function,

$$H = E = \alpha f(K)^{-\beta} \quad (\alpha,\beta>0) \text{ in the aggregate} \tag{10}$$

The health effect to the individual is viewed as *exogenous* and given by equation (2a). Also, assume that f(K) =AK, as before. Thus, the decentralized growth outcome γ remains unchanged, as represented by equation (5). However, the socially optimum path is different. Application of the calculus of variation yields:

$$\gamma_s \equiv \dot{C}/C = \frac{A(1-\beta) - \rho}{\sigma(1-\beta)} \quad . \tag{11}$$

While the second order condition does not constrain β in this case, $\beta > 1$ can still be ruled out since that would imply $\partial\gamma_s/\partial\rho > 0$ which is implausible. Thus, we again focus on the case of $\beta < 1$.

As in the previous model, we find that, $\partial\gamma_s/\partial\beta < 0$, and correspondingly, $\gamma_s < \gamma$, i.e., the efficient growth path is below the equilibrium path. In this case, since accumulation is the source of adverse environmental and health effects the central planner chooses a growth path that implies a slower accumulation rate i.e., one that is slower than the decentralized economy's path. The transversality condition is also satisfied if $A < (1-\sigma)\rho/(1-\beta)$, imposing an upper bound on the productivity of the capital stock.[8] As before,

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[8]To prove this, note that the Current Value Hamiltonian now includes in the objective function, an explicit dependence on K, i.e.,

$\beta < 1$, implies that $\sigma < 1$.

Path of Environmental Decay

Along the equilibrium path, environmental quality deteriorates at the rate given by equations (5) and (10), and along the efficient path it deteriorates at a rate given by equations (11) and (10). These are:

$$\varepsilon = -\beta\gamma = -\beta\frac{A-\rho}{\sigma} \tag{12}$$

$$\varepsilon_s = -\beta\gamma_s = -\beta\,\frac{A(1-\beta)-\rho}{\sigma(1-\beta)} \tag{13}$$

Again, the quality of the environment deteriorates at a *slower* rate along the efficient path ($|\varepsilon_s|<|\varepsilon|$), because of the slower growth rate along that path ($\gamma_s<\gamma$). Beyond this result, the efficient path ε_s does exhibit an interesting property with respect to β, namely the existence of a threshold value of β, say β_o, which *minimizes* ε_s. Differentiating ε_s with respect to β and setting to zero β_o satisfies the equation:

$$A(1-\beta_o)^2 - 2\beta_o\rho + \rho = 0 \tag{14}$$

Further, studying the curvature of $\varepsilon_s(\beta)$ we find that:

$$\partial^2\varepsilon_s/\partial\beta^2|_{\beta_o} > 0 \tag{15}$$

$$\mathcal{H} = U[C(t),\alpha f(K(t))^{-\beta}] + \lambda[AK(t)-C(t)],$$

so that the multiplier λ evolves according to or $\dot{\lambda} = (\rho-A)\lambda - \Omega(t)$, where, $\Omega(t) \equiv \text{const.}C(t)^{1-\sigma}f(K(t))^{\beta(\sigma-1)-1}$. Thus, the growth of λ is *bound* from above, i.e. $\lambda(t) \le \lambda_o e^{(\rho-A)t}$. Thus, an upper bound for the transversality condition, $\lim_{t\to\infty} e^{-\rho t}\lambda(t)K(t) = 0$, may be found which is $\gamma_s < A$, or substituting from (11), $A < (1-\sigma)\rho/(1-\beta)$.

Thus, ε has a local minimum at β_o. This is an interesting result because it points to the existence of trade-offs, along the efficient path, between the two opposite effects of β on environmental quality; A direct worsening effect and an indirect or induced ameliorating effect (via a reduction in the growth rate). This situation is depicted in Figure 1. Note that this means that under the efficient path, the rate of environmental deterioration is *bounded* so that it cannot deteriorate beyond $\varepsilon_m=\varepsilon(\beta_o)$. As shown in the figure, no such trade offs, and thus boundedness, exists along the equilibrium path ε so that a rise in β unambiguously worsens the quality of the environment, which is already deteriorating at a more rapid rate than ε_s.

An important point of both these models is that circumstances exist where maximizing the growth rate is inconsistent with maximizing welfare. In this case, because of the adverse effects of capital accumulation or consumption on utility, the welfare of the society rises more rapidly in a social optimal regime than in a decentralized regime, but growth rate rise *less* rapidly in the former than in the latter regime.

IV. Environment, Health and Growth: Embodied & Disembodied Effects

In the models of the past two sections, household health is completely determined by the quality of its external environment. Thus, no individual household can control its own health directly or individually, but only *indirectly* through influencing the quality of its external environment and *in the aggregate,* through the change in the behavior of all households. Effectively, the atomistic market mechanism fails to maximize utility, i.e, the rate of economic growth attained from the market is not consistent with maximum utility or "quality of life." In this case, a policy must be carried out by some form of institutional innovation that mimics the heavy hand of a central planner.

By contrast, the present section studies a class of problems in which health is subject to both the aggregate external effects of environment, as well as an internal effect that is (partly) endogenous to households, and can be influence through their consumption behavior. Further, the external and the internal effects are subject to trade-offs via technological frontier. We call the first effect, *the disembodied effect* and the second, *the embodied effect*.

An example of such class of problems is the use of preservatives in food which retard spoilage but which may be potentially carcinogenic (as in the case of Nitrates), versus the use of refrigeration which reduce the need for preservatives, but intensify ozon depletion and are therefore indirectly harmful to health. First, note that although the concentration of preserving substances are outside the control of the individual consumers, consumers *can* still control their total ingestion of consumables thereby reducing their detrimental health affects. Thus, even though an external effect remains through consumer's inability to influence the concentration of the residue, the market can still act, through consumer's demand structure, as a mechanism to induce producers to transform the composition of the final product away from inputs that leave traces of contaminants *embodied* in the product. In effect existence of preservatives may be a *hedonic* characteristic of the product consumed. However, the scenario does not end here. For, there are trade-offs in the health consequences of the producers' choice of inputs, which impacts environment and thus health but are *disembodied* from the product consumed, and external to individuals. In the above example, as producers

Figure 1

Environmental Decay in a Simple Endogenous Growth Model

of Production Induced Externality

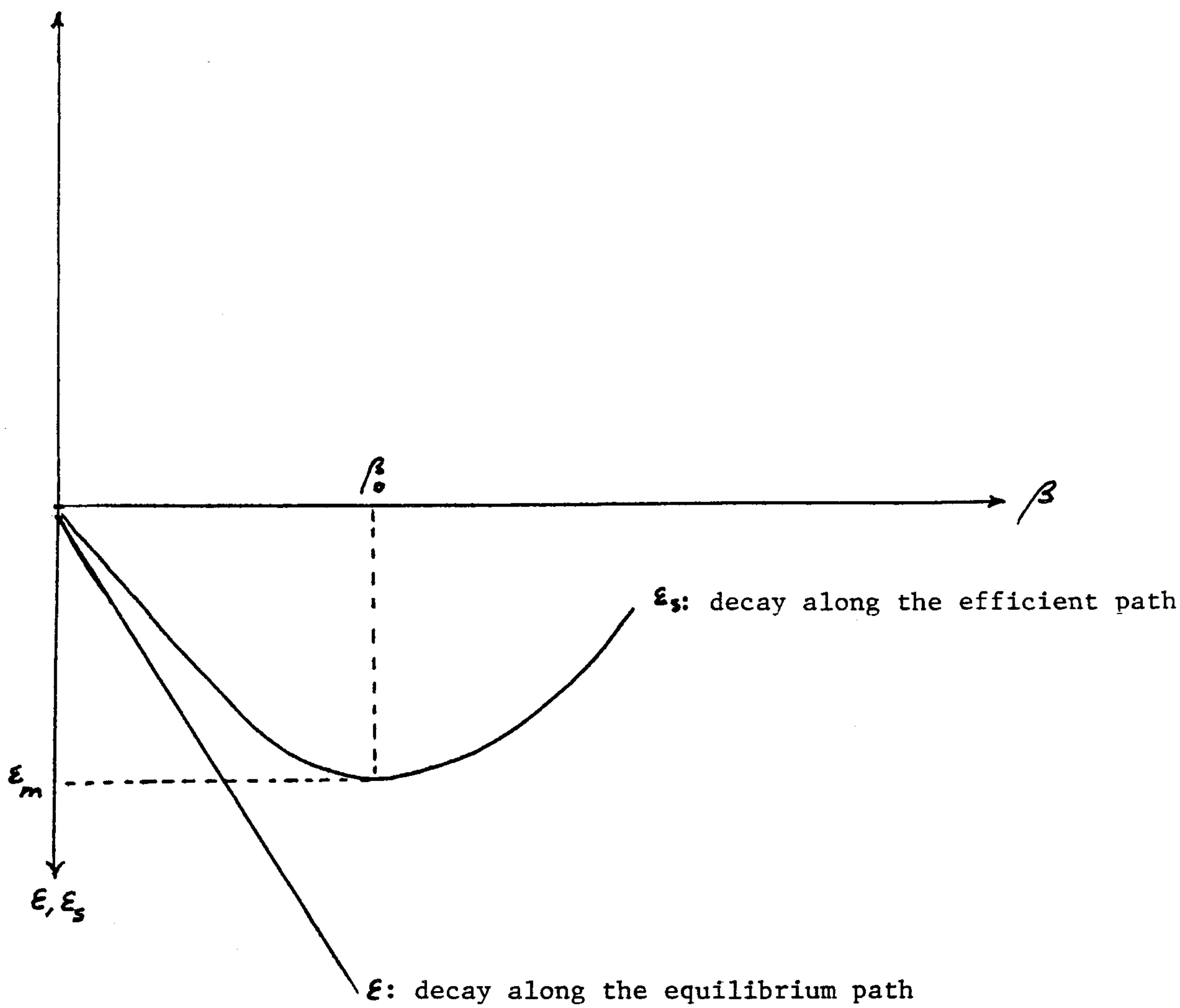
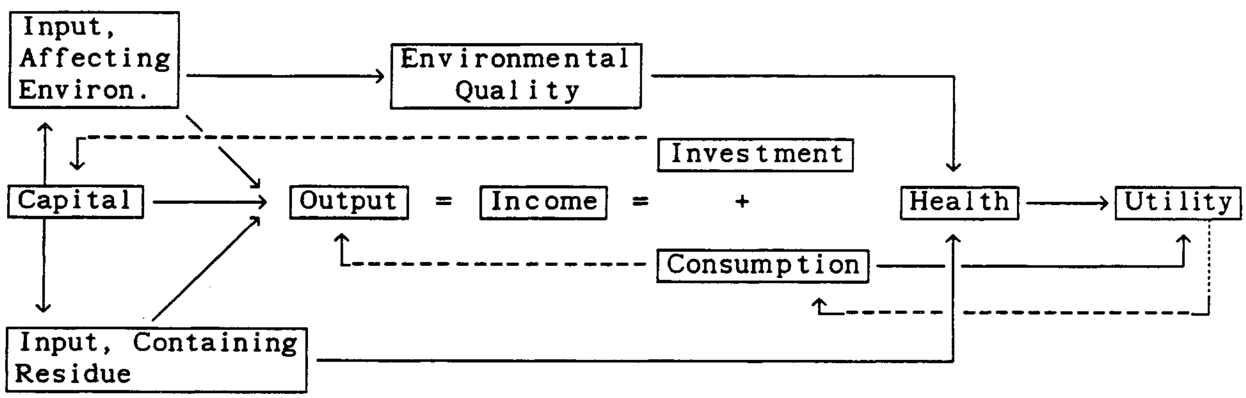


Figure 2

Environment-Health Interactions in a CFC-OZON Type Problem



reduce their use of preservatives, in response to market demand, they may use refrigeration more intensively, thus affecting health adversely via the ozon depletion factor.

Given the above background the question of interest is this: What is the mechanism for sustainable growth in this class of economies and what are the nature of trade-offs involved for the growth path of the economy?

To study this class of problems, the consumption and the production behavior are assumed to be *separated*. Hence, proceeding in the spirit of the framework developed in the previous section, consider an economy with a single composite good, C, that contains traces of an "unhealthy" residue of a given concentration, z. The residue is associated with an input X (e.g., preservatives in food) that is employed in the final production of C. In addition to X, firms also employ another factor, R (e.g. refrigerators), whose use does not pose *direct* health hazards but indirect hazards through its external effect via the environment and thus health. Both X and R are produced via another production process, using capital, and are thus treated as intermediate factors of production. Figure (2) depicts this model.

Consumers maximize (1) subject to the knowledge that H is adversely affected by the overall level of the contaminants consumed and by the environment. Since the ingestion of the substance associated with X is a product of concentration z (fixed to consumers) and consumption C (choice variable), consumers' health can *in part* be influenced via individual C, as discussed above. Environment, however, remains an externality and is thus exogenous to individual consumers, while it is of course impacted adversely in the aggregate by the increased utilization of the input R, as per example above. The health production function is:

$$H = h[\bar{z}C,\ E(\bar{R})] \qquad \text{with } h_1<0, h_2>0\ E'<0. \tag{16}$$

The trade-off in the utility function consists of the positive direct effect of C on utility and its negative indirect effect (via health) on utility. Each consumer faces *given* values of z and R (hence the bar on both), but z and R can vary in the *aggregate*.

Consumers are assumed to own the stock of the firms' capital, renting it to firms, at a rental rate, r. The budget constraint is:

$$PC = rK - \dot{K} \tag{17}$$

where P is the price of the consumer good, C, relative to the capital good, K. Consumers maximize W in equation (1) subject to the constraints (16) and (17).

Firms produce Q amount of the final good, by employing the intermediate factors X, R, that contain the embodied adverse health effects and the disembodied adverse health effect (via environment), and the environmentally neutral factor, K, in a production process represented by constant returns to scale.

$$Q = f(K_Q, R, X) = K_Q^{1-\alpha-\beta} R^{\alpha} X^{\beta} \tag{18}$$

Thus, the concentration ratio, z, is taken from this relation as follows:

$$z = X/Q = \frac{X^{1-\beta}}{K_Q^{1-\alpha-\beta} S^{\alpha}} \tag{19}$$

The intermediate factors X and R are produced by the capital stock in a simple linear (CRS) technology:

$$X = \theta(K_x) = mK_x \tag{20}$$

$$R = \phi(K_R) = nK_R \tag{21}$$

Capital is assumed mobile between sectors X, R and Q. Its supply (stock) at any time t, is determined by consumers (via ownership of the stock of firms) who determine its path K(t) optimally. The allocation of K(t) between the three sectors is determined by equality of its marginal productivity in the two sectors. The externality is that the market provides a sub-optimal solution to this problem because the privately optimal path of K(t) and C(t) does not provide incentives for technical substitution between environmentally, and thus health-neutral, investments (K) and health non-neutral inputs R or X (whether the health effect is direct, as in the case of X, or via the environment, as in the case of R).

Consumer Maximization Problem: The Intertemporal Analysis

To maximize (1) subject to (16) and (17), apply Euler's equation, $\partial V/\partial K - d/dt\partial V/\partial \dot{K}$, to get:

$$(u_C + zh_C u_H)(r-\rho - \dot{P}/P) + [u_{CC} + 2zh_C u_{CH} + (h_C)^2 u_{HH} + h_{CC} u_H)z^2]\dot{c} = 0 \qquad (22)$$

Note that $\dot{P}/P$ allows for change over time in the relative price of C, so that $\dot{P}/P$ is in general nonzero. Rational Expectations is assumed to operate on this variable. Thus, the value of $\dot{P}/P$ is assumed to be correctly perceived by consumers as reflecting the relative inflation of the final good's price in the market. Although prices will be constant and thus $\dot{P}/P = 0$ when the technology exhibits constant returns to scale, $\dot{P}/P$ need not be zero in general. In any case, the determination of this variable must await the general equilibrium considerations.

To carry the analysis beyond this point, we shall need to specify functional forms. Let the utility function be of the form of previous sections:

$$u(CH) = [(CH)^{1-\sigma}-1]/(1-\sigma), \qquad (\sigma > 0) \qquad (23)$$

Also let,

$$H = h[\bar{z}C,E(\bar{R})] = a(\bar{z}C)^{-\theta} E(\bar{R})^{\theta'} = A(\bar{z}C)^{-\theta}(\bar{R})^{-\theta'} \qquad (\theta,\theta'>0) \qquad (24)$$

where for simplicity the adverse effect of the intermediate factor R on the environment, E, is captured by a *inverse* function of R, $E(R) = e/R$ (so that $A = ae^{\theta'}$ is a constant). Using equations (23) and (24) in (22) and simplifying the outcome, we find the growth rate of consumption, for *given* levels of z and R, to be:

$$\gamma_c \equiv \dot{C}/C = \frac{r-\rho-\gamma_p}{\sigma^*} \qquad (25)$$

where $\gamma_p \equiv \dot{P}/P$, and $\sigma^* \equiv \sigma(1-\theta)+\theta$, is now the *effective* intertemporal substitution elasticity.

Now, a few points regarding the range of parameters: First, substituting from (24) into (23), we see that $u(CH) = \Theta(\bar{R}).[Az^{-\theta}C^{1-\theta}]^{1-\sigma}-1\}/(1-\sigma)$, where $\Theta(\bar{R}) = (\bar{R})^{-\theta(1-\sigma)}$. We require that the *net* effect of consumption on utility be positive, so that $\theta < 1$. Also, local concavity of the integrand in (1), requires $\partial^2u/\partial\dot{K}^2 < 0$. This condition is satisfied if $\sigma < 1$.[9] Thus, as in the other two models, the analysis is meaningful only in the range of $\sigma < 1$. Third, With $\sigma^* > 0$, (because $\theta<1$), positive steady state growth rate of consumption requires that $\rho < r - \gamma_p$. On the other hand, to guarantee that the attainable utility W is bounded, the transversality condition (see earlier

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[9] $\partial^2u/\partial\dot{K}^2 = (1/P^2)\{u_{CC}+z^2[h''u_H+h'^2u_{HH}] + 2zh'u_{HC}/P$. Given the specification of the u and h functions, the expression inside the bracket [.] is negative. Also, since h' is negative and $u_{CH} = (1-\sigma)u'$, the last term is negative if $\sigma < 1$. Therefore, $\sigma < 1$ is sufficient but not necessary in this context.

footnotes) requires that $\gamma_c < r$, or that $\rho > r(1-\sigma)(1-\theta) - \gamma_p$. Thus, ρ is bounded from below and above by $r(1-\sigma)(1-\theta) - \gamma_p < \rho < r - \gamma_p$. Interpretation for equation (25) will be given later, following the general equilibrium analysis.

Producer Maximization Problem: The Contemporaneous Analysis

Final goods producers' demand for X and R is determined by the profit maximizing condition of equating the marginal value-product of X and R in Q, from equation (18), to their rewards. Producers are assumed to be atomistic with respect to both factors, facing a price w for factor X, and and s for factor R. Then:

$$X_D = P \frac{\beta}{w} Q \tag{26}$$

$$R_D = P \frac{\alpha}{s} Q \tag{27}$$

As mentioned, the allocation of capital between the sectors X and R and Q is *obtained* by equating its marginal product in each sector, to each other and to the given r,

$$P(1-\alpha-\beta)Q/K_Q = wm = sn = r, \tag{28}$$

Given the linearity of X in K_X, and R in K_R, the X and R sectors' demand for K is entirely elastic while their price w and s is related to the return to capital, r, via equations, $w = r/m$ and $s = r/n$.

Market Clearing Conditions

Equating the supply and demand for X from equations (20) and (26) and for R from equations (21) and (27), we find:

$$X_D = P \frac{\beta}{w} Q = X = mK_x \tag{29}$$

and

$$R_D = P \frac{\alpha}{s} Q = R = nK_R \tag{30}$$

Finally, demand for capital in all three industries must add up to the supply:

$$K_Q + K_x + K_R = K \tag{31}$$

The market clearing condition for Q must require $\dot{K}$ units of the consumption goods C, to be converted to investment goods, so that $\dot{K}$ represents *forgone* consumption. With this in mind, goods market equilibrium requires that:

$$C = Q - \dot{K}/P \tag{32}$$

Comparing this with the representative household budget constraint (equation 17), gives:

$$PQ = rK \tag{33}$$

Not surprisingly, Walras Law holds and the value of output is total factor income.

Contemporaneous Solution

Equations (18),(20), (21), (26), (27), the three equations in (28), equations (29)-(31), and equation (33) can be solved to determine the equilibrium quantities and prices in terms of the stock of capital. These are:

$$X(t) = m\beta K(t) \tag{34}$$

$$R(t) = n\alpha\, K(t) \tag{35}$$

$$K_Q(t) = (1-\alpha-\beta)K(t) \tag{36}$$

$$K_x(t) = \beta K(t) \tag{37}$$

$$K_R(t) = \alpha K(t) \tag{38}$$

$$Q(t) = (1-\alpha-\beta)^{1-\alpha-\beta}(n\alpha)^{\alpha}(m\beta)^{\beta}K(t) \tag{39}$$

$$P = r/(1-\alpha-\beta)^{1-\alpha-\beta}(n\alpha)^{\alpha}(m\beta)^{\beta} \tag{40}$$

$$w = r/m \tag{41}$$

$$s = r/n \tag{42}$$

$$z = \frac{(m\beta)^{1-\beta}}{(1-\alpha-\beta)^{1-\alpha-\beta}(n\alpha)^{\alpha}} \tag{43}$$

Given the specification of the production functions X and Q, the interest rate r is not uniquely determined, i.e. the solution will support any value of r, so that r will be also exogenously determined, a feature of the constant returns technology of the production functions in both sectors.

Equilibrium Growth Path

Given the exogenous determination of r, the prices P and w, as well as the concentration ratio remain time invariant (equations 40 - 43). Thus $\dot{P}/P$ in equation (25) drops out, so that:

$$\gamma_c \equiv \dot{C}/C = \frac{r-\rho}{\sigma^*} \tag{44}$$

Furthermore all quantities in equations (34) - (39) grow at the same rate as the capital stock:

$$\gamma_Q = \gamma_X = \gamma_R = \gamma_{KQ} = \gamma_{Kx} = \gamma_K \tag{45}$$

To relate these to the growth rate of consumption, we return to the budget constraint, equation (17), noting that unlike the earlier analysis where R was viewed fixed by consumers, this time R is a variable in the general equilibrium analysis. We have:

$$\gamma_c = \frac{r\dot{K} - \ddot{K}}{PC} = \frac{r\dot{K} - \ddot{K}}{rK - \dot{K}} \tag{46}$$

Divide the numerator and denominator of the last quotient by K and rewrite $\ddot{K}/K$ as $(\ddot{K}/\dot{K})/(\dot{K}/K)$. Then note that $\dot{K}/K = \gamma_K$. If we assume *steady-state* growth rate, so that all variables grow at a constant rate. Then $\ddot{K}/\dot{K} = \dot{K}/K = \gamma_K$, and γ_K is constant. Then equation (46) becomes:

$$\gamma_c = \frac{r\gamma_K - \gamma_K^2}{r - \gamma_K} = \gamma_K \tag{47}$$

From this equation and equation (45) we see that in steady state all variables grow at the same *constant* rate. Dropping the subscript we denote this rate by γ, where from equation (44):

$$\gamma = \frac{r-\rho}{\sigma^*} \tag{48}$$

Recall that $\sigma^* \equiv \theta + (1-\theta)\sigma$ and that $\sigma < 1$. Thus, the discussion of the socially optimal growth rate in equation (7), now also applies to the case of the equilibrium growth rate (48), with θ replacing b. In particular $\partial\gamma/\partial\theta < 0$, i.e. equilibrium growth declines, the higher are the adverse health consequences of consumption. That the present competitive solution is equivalent to the socially optimum solution of section II (for θ=b) is because knowing θ, individual households can now influence their own level of health, by controlling their total ingestion of C, in manner that is equivalent to the central planner doing so on their behalf in Section II. As it turns out, the divergence between the efficient and the equilibrium path stems from a different source, as outlined below.

Efficient (Socially Optimal Path)

Despite the ability of the consumers to affect their embodied component of their health via the market mechanism, the market will still provide a sub-optimal solution to this problem because the privately optimal path of capital accumulation does not *provide incentives* for technical substitution that can occur between environmentally, and thus health-neutral, investments (K) and health non-neutral inputs R or X (whether or not the health effect is direct, as in the case of X, or via the environment, as in the case of R). In this case, a socially optimal path is equivalent to a *sustainable* growth path, i.e. growth when the above externalities are internalized via some *institutional* mechanism.

In this case, we maximize (1) subject to the health function (16) and the budget constraint (17) with z and R and variables, so that a central planner is also aware of the equations governing the evolution of z and R, via equations (33) and (43). Since z in equation (43) is time-independent, however, its value can be represented by z_0 (as a function of s, r, and technology parameters), which is constant in time. With z_0 a constant and with R(t) given from equation (35) in terms of K(t), the health function of equation (24) is now a function of *both* consumption and capital stock, H = h(C,K). Thus, the Euler equation of the private choice (eq. 22) includes this added dependence of H on K:

$$\begin{aligned}(u_C + z_0 h_C u_H)(r - \rho + P u_H H_K) + P u_H H_K + \\ [u_{CC} + 2z_0 h_C u_{CH} + (h_C^2 u_{HH} + h_{CC} u_H) z_0^2]\dot{C} = 0 \qquad (49)\end{aligned}$$

where $\dot{P}/P$ of equation (22) has also been dropped. Now, with $z = z_o$ and R(t) = $n\alpha K(t)$, from (35), the explicit form of the health function in (24) becomes:

$$H(t) = Az_0 C(t)^{-\theta} [n\alpha K(t)]^{-\theta'} = H_0 C(t)^{-\theta} K(t)^{-\theta'} \qquad (24')$$

where H_0 captures the time-independent terms. Using this function together with the utility function of the previous section, minor simplifications will yield the following equation for the socially optimal path, γ_s:

$$\gamma_s = \frac{r - \rho}{\sigma^*} - \frac{\theta'}{1-\theta}\cdot\frac{r-\gamma_K}{\sigma^*} \tag{50}$$

With $\gamma_K = \gamma_s$, the socially optimum rate is:

$$\gamma_s = \frac{(1 - \xi)r - \rho}{\sigma^* - \xi} \tag{51}$$

where $\xi \equiv \theta'/(1-\theta)$ represents environmental and health externality effects combined. Note that a positive socially optimal growth rate implies that $\xi <$ min $(1,\sigma^*)$. In any case, ξ must be < 1, which implies that $\theta+\theta' <1$. Thus, the adverse elasticity of health to the embodied (residue) and disembodied (environmental) effects of production must be *sufficiently small* and, in any case, less than unity in absolute value for a positive socially optimal growth rate to exist. This is a stronger requirement than that for an equilibrium growth rate to exist. Further, the transversality condition requires that $\gamma^* < r$.[10] From (51) this means that $\rho > r\ (1-\sigma^*) = r(1-\sigma)(1-\theta)$, which in turn is the *same* condition needed for the transversality condition for the case of the *equilibrium* growth path (as $\gamma_P = 0$.)

Comparing γ_s in (51) with γ in (48), two effects emerge. First, compare the term $(1-\xi)r$, present in the numerator of γ_s, with r in the numerator of γ. This effect shows that investments in the X and R sectors of the economy *lower*the growth rate. This is because of the negative impact of both sectors

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[10]The Current Value Hamiltonian in this case involves an objective function that explicitly depends on K(t), so that we can follow the approach of footnote (9).

on the health of consumers. Specifically this means that if consumers were able to influence the levels of X and R, collectively or via the government, they would *prefer* to increase consumption over time by a *smaller* amount than if they acted atomistically. This would also lower the rate of capital accumulation. In effect the return to the capital stock is now less than r because of these two effects. Note that a rise in either θ' or θ increase the size of this externality, ξ, via the numerator.

However, this overstates the negative externality effect, since ξ also appears in the denominator of (51), acting as a positive externality. This second effect may be called an induced external effect. It captures the fact that reduced growth rate of consumption *induces* an increase in capital accumulation in health-neutral investments. The relation of the second effect to capital accumulation can also be seen by the presence of γ_K in the version of γ_s represented by equation (50).

To find the net effect, calculate $\gamma_s - \gamma$:

$$\gamma_s - \gamma = \frac{\xi}{\sigma^*} \frac{r(1-\sigma^*) - \rho}{\sigma^* - \xi} \tag{52}$$

As we have seen from the discussion following equation (51), a positive growth rate for the *efficient* path γ_s means that $\xi < \min(1, \sigma^*)$. This means that $\xi < \sigma^*$ so that the denominator of (52) is positive. Since the numerator is negative (by the transversality condition), it follows that $\gamma_s < \gamma$. Thus, the net effect of the negative externalities that arise because of the adverse health consequences of X and R *dominate*, rendering the welfare maximizing (efficient) growth path smaller than the privately optimal path. This is the same result that was also found in the previous two models.

Impact of θ': From equation (51) and the definition of ξ, we have, $\partial\gamma_s/\partial\theta' = \frac{1}{1-\theta}\, \partial\gamma^*/\partial\xi$. The sign of the latter depends on the sign of the expression, $\Omega \equiv r(1 - \sigma^*) - \rho$, which is negative by the transversality condition. Thus, $\partial\gamma_s/\partial\theta' < 0$, as one would expect.

Impact of θ: Comparing γ with γ_s: In order to gauge the influence of θ on γ_s as against γ, we differentiate γ_s in θ, holding σ^* constant (since σ^* is common in both γ and γ_s) to find, $\partial\gamma_s/\partial\theta|_{\sigma^*} = \theta'/(1-\theta)^2 \partial\gamma_s/\partial\xi$. The sign of this derivative is again negative by the expression, Ω, and thus $\partial\gamma_s/\partial\theta|_{\sigma^*} < 0$. Thus, higher values of θ mean greater divergence between the socially efficient and the equilibrium rate, by reducing further the efficient rate, already lower than the equilibrium rate.

Path of Environmental Decay

As in the other two models, the rate of change of environmental quality is inversely related to growth; so that for the equilibrium path, $\varepsilon = -\theta'\gamma$ and for the efficient path, $\varepsilon^* = -\theta'\gamma_s$. Because, $\gamma_s < \gamma$ environmental quality deteriorates at a slower rate under the efficient path, generalizing results of Section III. Similar conclusions hold for the rate at which health deteriorates, seen by letting $h \equiv \dot{H}/H$, and noting that $h = -(\theta+\theta')\gamma$ versus $h_s = -(\theta+\theta')\gamma_s$.

Finally, the adverse effect of increasing one of the parameters, for example θ', on the rate of environmental deterioration is subject to similar trade-offs, as in Section III, though it is more difficult to determine here weather this rate is also bounded as was the case in that section.

Conclusion:

The approach taken in endogenous growth theories in which economic growth is a result of endogenous decisions by atomistic agents (as opposed to technology-led Solow-growth) is a natural framework to analyze growth and the environment, because environment affects the utility function and thus the decision to consume versus to invest. Further, the clear distinction that such models provide between the *equilibrium* and the *efficient* growth paths, stems from the key role of externalities in the endogenous growth literature, and externalities are of course at the root of any economic analysis of environmental problems. Comparison between the equilibrium and the efficient paths do provide many insights and suggest how policies that ameliorate the adverse health or environmental effects of production or consumption would impact the equilibrium and the socially efficient growth path of the economy.

All models point to a *negative* relation between growth rate (along either equilibrium or efficient paths), and quality of the environment as manifest through health. Although it would have been possible to permit some *exogenous* rate of environmental regeneration or technological change that would allow for growth but would retain the quality of the environment, such addition would have been conceptually trivial, only complicating the algebra but not contributing genuinely to the models of this paper (which in effect take the exogenous rate of technological change to be zero).

Instead, the understanding of how environment and growth can be made compatible and even reinforcing of one another requires an approach based *endogenously induced technological change*. The pioneering works of Ruttan and Hayami (1971) which pointed the way for such investigation in the area of agriculture, can be applied to the area of environment, using the more sophisticated new tools borrowed from the endogenous growth literature. It is most interesting that the authors' reason for why Japanese agriculture took on land saving innovation in the 60's and 70's (because of its land scarcity) finds equal applicability to the Japanese innovation in the area of fossil fuel-saving innovations (because of the scarcity of fossil fuel) which has led to a nearly 50 percent reduction in CO_2 emission over the course of the 1980's while simultaneously enjoying very high growth rates.

The important message of this paper is that in the absence of induced technological innovation in which increased deterioration of the environment would induce fundamental transformation of the knowledge-base, growth may not be environmentally friendly in the long run, even along the efficient path.

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