Empirical Performance of Alternative Option Pricing Models for Commodity Futures Options

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Abstract

The central part of pricing agricultural commodity futures options is to find appropriate stochastic process of the underlying assets. The Black's (1976) futures option pricing model laid the foundation for a new era of futures option valuation theory. The geometric Brownian motion assumption girding the Black's model, however, has been regarded as unrealistic in numerous empirical studies. Option pricing models incorporating discrete jumps and stochastic volatility have been studied extensively in the literature. This study tests the performance of major alternative option pricing models and attempts to find the appropriate model for pricing commodity futures options.

Keywords: futures options, jump-diffusion, option pricing, stochastic volatility, seasonality

Introduction

Proper model for pricing agricultural commodity futures options is crucial to estimating implied volatility and effectively hedging in agricultural financial markets. The central part of pricing agricultural commodity futures options is to find appropriate stochastic process of the underlying assets. The Black-Scholes (1973) option pricing model laid the foundation for a new era of option valuation theory. The geometric Brownian motion (GBM) assumption girding the Black-Scholes model, however, has been regarded as unsatisfactory by many researchers. Empirical evidence clearly indicates that many underlying return series display negative skewness and excess kurtosis features (see a review in Bates, 1996b) that are not captured by Black-Scholes. In addition, while volatility of the underlying process is assumed to be constant in the Black-Scholes model, implied volatilities from the Black-Scholes model often vary with the strike price and maturity of the options (e.g. Rubinstein, 1985, 1994).

Impacts of new information may cause discrete jumps in the underlying process. Merton (1976) derives an option pricing formula for a general case when the underlying asset process is generated by a mixture of both continuous and jump stochastic processes. But the jump risk is assumed diversifiable and therefore nonsystematic. Bates (1991) provides an option pricing model on a jump-diffusion process with systematic jump risk to show that the Crash of '87 was predictable.

The stochastic volatility processes have been widely studied in the literature. Hull and White (1987) give a closed-form solution for the price of a European option based on the assumption of zero-correlation between stochastic volatility and stock price. They find that Black-Scholes model frequently overprices options and the degree of overpricing increases with the time to maturity. Heston (1993) derives a closed-form

solution for the price of a European call based on Fourier inversion methods. But his model allows any degree of correlation between stochastic volatility and spot asset returns. He finds that correlation between volatility and the spot price is important for explaining return skewness and strike-price biases in the Black-Scholes model.

Bates (1996a) further extends Bates' (1991) and Heston's (1993) models to price options on combined stochastic volatility/jump-diffusion (SVJD) processes under systematic jump and volatility risk. He finds that stochastic volatility alone cannot explain the "volatility smile" of implied excess kurtosis except under implausible parameters of stochastic volatility, but jump fears can explain the smile. Bates (2000) refines Bates (1996a) model by incorporating multifactor specification in stochastic volatilities and time-varying jump risk to explain the negative skewness in post- '87 S&P 500 future option prices. Bakshi, Cao and Chen (1997) develop a closed-form European option pricing model that admits stochastic volatility, stochastic interest rates, and jump-diffusion process. They find that incorporating stochastic interest rates does not significantly improve the performance of the SVJD model. More general and complicated models that incorporate jumps both in volatility and in the underlying have also been developed, such as those in Duffie, Pan and Singleton (2000) and Eraker, Johannes and Polson (2003).

As for pricing options on commodity futures, Hilliard and Reis (1999) use transaction data of soybean futures and options on futures to test out-of-sample performance of Black's (1976) and Bates' (1991) jump-diffusion models. Their results show that Bates' model outperforms Black's model. Richter and Sørensen (2002) set up a stochastic volatility model with the inclusion of the seasonality and convenience yield. Koekebakker and Lien (2004) extend Bates' (1991) model by including seasonal and maturity effects in a deterministic volatility specification. None of these studies incorporate both

jump component and stochastic volatility. This void is filled in the present study.

The objective of this study is to test the performance of the most widely used option pricing models and to investigate the appropriate model for pricing commodity futures options. The option pricing models include Black's (1976) model, Bates (1991) Jump model, Heston's (1993) stochastic volatility (SV) model, and stochastic volatility jump diffusion (SVJD) model. The data used are three years of intradaily corn futures and options on futures data. The next section provides an introduction to several major option pricing models. The third section discusses the data sample and the estimation method. The fourth section presents the empirical results, while the fifth section concludes.

Option Pricing Models

Two assumptions are maintained in the following option pricing models:

1. Continuously compounded risk-free rate, r, is assumed constant.[1]
2. Markets are frictionless: there are no transaction costs or taxes, trading is continuous, all securities are divisible, and there are no restrictions on short selling or borrowing.

Black's Model

In Black's (1976) model, the price movement of commodity futures follows a geometric Brownian motion:

$$\frac{dF}{F} = \mu dt + \sigma dZ \tag{1}$$

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[1]Empirical findings suggest that option pricing is not sensitive to the assumption of a constant interest rate. For example, Bakshi, Cao and Chen (1997) find that incorporating stochastic interest rates does not significantly improve the performance of the model with constant interest rates.

where F is futures price; Z is a standard Brownian motion with $dZ \sim N(0, dt)$; μ is the expected rate of return on futures; and σ is the annualized volatility of the futures price, which is assumed to be constant.

The closed-form formulae for call option price (C) and put option price (P) are:

$$\begin{align}
(2) \qquad C &= e^{-r\tau}[F_t N(d_1) - XN(d_2)] \\
(3) \qquad P &= e^{-r\tau}[XN(-d_2) - F_t N(-d_1)]
\end{align}$$

where

$$\begin{align}
d_1 &= \frac{\ln(F_t/X) + \sigma^2\tau/2}{\sigma\sqrt{\tau}} \\
d_2 &= d_1 - \sigma\sqrt{\tau}
\end{align}$$

and F_t is the futures price at current time t, X is strike price, τ is the time to maturity of the option, and $N(\cdot)$ is cumulative probability distribution function for a standard normal distribution.

Bates' Jump Model

In Bates (1991) jump-diffusion model, the stochastic differential equation with possibly asymmetric, random jumps is given by:

$$(4) \qquad \frac{dF}{F} = (\mu - \lambda\bar{k})dt + \sigma dZ + kdq$$

where:

μ is the rate of return of the futures price;

σ is the constant volatility of the futures price;

Z is a standard Brownian motion;

λ is the annual frequency of jumps;

k is the random percentage of price change conditional on a jump occurring that is log-normally, identically, and independently distributed over time, with unconditional mean $\bar{k}$ and $\ln(1 + k) \sim N(\ln(1 + \bar{k}) - \frac{1}{2}\delta^2, \delta^2)$;

q is a Poisson counter with intensity of λ so that $\mathrm{Prob}(dq = 1) = \lambda dt, \mathrm{Prob}(dq = 0) = 1 - \lambda dt$.

The risk neutralized stochastic process is:

$$(5) \qquad \frac{dF}{F} = -\lambda^* \bar{k}^* dt + \sigma dZ + k^* dq^*$$

where

λ^* is risk-adjusted frequency of jumps;

k^* is the risk-adjusted random percentage of price change conditional on a jump occurring with $E(k^*) = \bar{k}^*$ and $\ln(1 + k^*) \sim N(\ln(1 + \bar{k}^*) - \frac{1}{2}\delta^2, \delta^2)$;

q^* is a Poisson counter with intensity λ^*;

σ and δ are the same as in the actual process.

A European call futures option (C) is priced at its discounted expected value:

$$(6) \qquad \begin{aligned} C &= e^{-r\tau} \sum_{n=0}^{\infty} Prob^*(n\ jumps) E_t^*[\max(F_{t+\tau} - X, 0)|n\ jumps] \\ &= e^{-r\tau} \sum_{n=0}^{\infty} [e^{-\lambda^*\tau}(\lambda^*\tau)^n/n!][F_t e^{b(n)\tau} N(d_{1n}) - X N(d_{2n})], \end{aligned}$$

where

$$b(n) = -\lambda^* \bar{k}^* + n \ln(1 + \bar{k}^*)/\tau,$$

$$d_{1n} = [\ln(F_t/X) + b(n)\tau + \frac{1}{2}(\sigma^2\tau + n\delta^2)]/(\sigma^2\tau + n\delta^2)^{\frac{1}{2}},$$

and

$$d_{2n} = d_{1n} - (\sigma^2\tau + n\delta^2)^{\frac{1}{2}}.$$

A European put futures option has an analogous formula:

$$(7) \qquad \begin{aligned} P &= e^{-r\tau} \sum_{n=0}^{\infty} Prob^*(n\ jumps) E_t^*[\max(X - F_{t+\tau}, 0)|n\ jumps] \\ &= e^{-r\tau} \sum_{n=0}^{\infty} [e^{-\lambda^*\tau}(\lambda^*\tau)^n/n!][X N(-d_{2n}) - F_t e^{b(n)\tau} N(-d_{1n})]. \end{aligned}$$

Heston's Stochastic Volatility Model

Heston's (1993) stochastic volatility (SV) model assumes that the futures price and volatility of the futures price obey the stochastic processes:

$$
\begin{aligned}
\frac{dF}{F} &= \mu dt + \sqrt{V}dZ \\
dV &= (\alpha - \beta V)dt + \sigma_v \sqrt{V}dZ_v \\
cov(dZ, dZ_v) &= \rho dt
\end{aligned}
$$

(8)

where:

μ is the rate of return of the futures price;

V is the variance term;

σ_v is the volatility of volatility;

ρ is the correlation of the two standard Brownian motions, i.e. $cov(dZ, dZ_v) = \rho dt$.

The risk neutralized stochastic processes are:

$$
\begin{aligned}
\frac{dF}{F} &= bdt + \sqrt{V}dZ^* \\
dV &= (\alpha - \beta^* V)dt + \sigma_v \sqrt{V}dZ_v^* \\
cov(dZ^*, dZ_v^*) &= \rho dt
\end{aligned}
$$

(9)

where b is cost-of-carry (r for non-dividend stock options, 0 for futures options); β^* and α/β^* are the speed of adjustment, and long-run mean of the variance; and the parameters α, σ_v, and ρ in the risk-neutral processes are the same as in the actual processes.

Closed-form solutions for valuing a European call option and a European put option are given as:

$$
\begin{aligned}
C &= e^{-r\tau}[F_t P_1 - X P_2]
\end{aligned}
$$

(10)

$$
\begin{aligned}
P &= e^{-r\tau}[X(1 - P_2) - F_t(1 - P_1)]
\end{aligned}
$$

(11)

where P_1 and P_2 are probabilities analogous to the cumulative normal probabilities under Black's model and derived from their characteristic functions by using Fourier inversion methods. The probabilities P_1 and P_2 are a special case of their counterparts in the stochastic volatility jump diffusion model below.

Stochastic Volatility Jump Diffusion Model

The stochastic volatility jump diffusion (SVJD) processes increase flexibility as compared to the above three models by incorporating both jumps and movement of volatility:

$$
\begin{aligned}
(12) \qquad \frac{dF}{F} &= (\mu - \lambda \bar{k})dt + \sqrt{V}dZ + kdq \\
dV &= (\alpha - \beta V)dt + \sigma_v \sqrt{V}dZ_v \\
cov(dZ, dZ_v) &= \rho dt \\
prob(dq = 1) &= \lambda dt, \quad \ln(1 + k) \sim N(\ln(1 + \bar{k}) - \frac{1}{2}\delta^2, \delta^2)
\end{aligned}
$$

where:

μ is the rate of return of the futures price;

λ is the annual frequency of jumps;

k is the random percentage of price change conditional on a jump occurring that is log-normally, identically, and independently distributed over time, with unconditional mean $\bar{k}$;

q is a Poisson counter with intensity of λ;

V is the variance term conditional on no jump occurring;

σ_v is the volatility of volatility;

ρ is the correlation of the two standard Brownian motions, i.e. $cov(dZ, dZ_v) = \rho dt$;

q and k are uncorrelated with each other or with Z and Z_v.

In a representative agent production economy, risk neutral processes of futures price are given by

$$
\begin{aligned}
(13) \qquad \frac{dF}{F} &= -\lambda^* \bar{k}^* dt + \sqrt{V}dZ^* + k^* dq^* \\
dV &= (\alpha - \beta^* V)dt + \sigma_v \sqrt{V}dZ_v^* \\
cov(dZ^*, dZ_v^*) &= \rho dt \\
prob(dq^* = 1) &= \lambda^* dt, \quad \ln(1 + k^*) \sim N(\ln(1 + \bar{k}^*) - \frac{1}{2}\delta^2, \delta^2)
\end{aligned}
$$

where β^* and α/β^* are the speed of adjustment, and long-run mean of the variance; and the parameters α, σ_v, δ, and ρ in the risk-neutral processes are the same as in the actual processes.

Bates (1996a) shows that European options, with an exercise price of X and time to maturity of τ, are priced as the expected value of their terminal payoffs under the risk neutral probability measure:

$$(14) \qquad\qquad C = e^{-r\tau}[F_t P_1 - X P_2]$$
$$(15) \qquad\qquad P = e^{-r\tau}[X(1 - P_2) - F_t(1 - P_1)].$$

The probabilities, P_1 and P_2 can be obtained using the Fourier inversion formulae:

$$(16) \qquad\qquad P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{\varphi_j(i\Phi)e^{-i\Phi x}}{i\Phi}\right] d\Phi \qquad (j = 1, 2)$$

where $x = \ln(X/F_t)$, Re denotes the real part. φ_j are the characteristic functions for P_1 and P_2 with the exact expressiones as:

$$(17) \qquad \varphi_j(\Phi|\Theta, \tau) = \exp\{C_j(\tau; \Phi) + D_j(\tau; \Phi)V + \lambda^*\tau(1 + \bar{k}^*)^{\mu_j + \frac{1}{2}}$$
$$\times[(1 + \bar{k}^*)^\Phi e^{\delta^2(\mu_j\Phi + \frac{\Phi^2}{2})} - 1]\} \qquad (j = 1, 2)$$

where

$$(18) \qquad C_j(\tau; \Phi) = -\lambda^*\bar{k}^*\Phi\tau - \frac{\alpha\tau}{\sigma_v^2}(\rho\sigma_v\Phi - \beta_j - \gamma_j)$$
$$- \frac{2\alpha}{\sigma_v^2}\ln\left[1 + \frac{1}{2}(\rho\sigma_v\Phi - \beta_j - \gamma_j)\frac{1 - e^{\gamma_j\tau}}{\gamma_j}\right],$$

$$(19) \qquad D_j(\tau; \Phi) = -2\frac{\mu_j\Phi + \frac{1}{2}\Phi^2}{\rho\sigma_v\Phi - \beta_j + \gamma_j\frac{1 + e^{\gamma_j\tau}}{1 - e^{\gamma_j\tau}}},$$

$$(20) \qquad \gamma_j = \sqrt{(\rho\sigma_j\Phi - \beta_j)^2 - 2\sigma_v^2(\mu_j\Phi + \frac{1}{2}\Phi^2)},$$

$$(21) \qquad \mu_1 = +\frac{1}{2}, \ \mu_2 = -\frac{1}{2}, \ \beta_1 = \beta^* = \rho\sigma_v, \text{ and } \beta_2 = \beta^*.$$

Note that if the jump parameters (λ^*, $\bar{k}^*$, and δ) are set zero, this model becomes stochastic volatility model. Therefore, the SVJD model nests the SV model as a special case. The integral in equation (16) is solved using numerical integration methods.

Data and Estimation Method

Three years of intradaily transactions data for corn futures and corn futures call options[2] traded on the Chicago Board of Trade (CBOT) were used. The data consist of the time and price of every transaction for the period of January 2001 to December 2003. The CBOT corn futures contracts are available for March, May, July, September, and December expiration dates. American-type options are traded on all the contracts. The total sample consists of 18 corn futures contracts.

Several filters are applied to construct the synchronous futures and futures options prices. First, weekly data rather than daily data are used in order to reduce computational burden and to avoid the microstructure issues such as the day-of-the-week effect and limits of daily price change. Wednesday (or Tuesday if Wednesday is not available) is selected as having the fewest trading holidays. Second, options transactions are matched with the nearest underlying futures within 4 seconds. If no matching futures price is obtained within 4 seconds, this option observation is discarded. Third, the options with time-to-maturity less than 10 trading days are deleted to avoid maturity effects. Fourth, corn options with price less than 2.5 cents are deleted. Fifth, options with price lower than their intrinsic value (i.e. Call<[Futures-Strike]) are deleted to eliminate the observations with arbitrage opportunity. The resulting data set includes 8,995 Wednesday observations. The average daily number of options matched is 59.2.

The exogenous variables in the sample data are observed transaction option price, C, strike price, X, futures price, F, time to maturity of options, τ, and instantaneous interest rate, r. Wednesday 3-month Eurodollar deposits rates are used for the risk-free discount rate.

Besides the exogenous variables obtained from the data set, the above option pricing models require different parameters as inputs. For Black's option pricing model, the only unobservable input is the volatility term, σ; for Bates91 model, the unobservable inputs include volatility, σ, and structural parameters $\Theta=(\lambda^*, \bar{k}^*, \text{ and } \delta)$; for SV model, they are volatility $\sqrt{V}$ and structural parameters $\Theta=(\alpha, \beta^*, \sigma_v, \text{ and } \rho)$; for SVJD model, besides the volatility, inputs also include seven structural parameters $\Theta=(\lambda^*, \bar{k}^*, \delta, \alpha, \beta^*, \sigma_v, \text{ and } \rho)$.

In principle, econometric methods can be applied to estimate the parameters since the stochastic processes are known. However, the requirement of a very long time series of futures prices makes this approach inconvenient. Alternatively, a very practical approach is to calculate the implied parameters using the market option prices and observable inputs in the option pricing formulae. Specifically, the implied parameters in the option pricing formulae are obtained by minimizing the sum of squared pricing

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[2]Call options are selected because they are more liquid than put options and therefore can represent the very liquid contracts.

errors of all options for each day in the sample data set.

$$(22) \qquad (\widehat{v}, \widehat{\Theta}) = \arg \min_{v, \Theta} \sum_{j=1}^{N_t} [C_{tj} - \widehat{C_{tj}}(v, \Theta)]^2$$

where N_t is the number of options used for date t; C_{tj} is the j-th observed market option price on date t; $\widehat{C_{tj}}$ is the model determined option price with observed exogenous inputs; v is instantaneous volatility for date t (i.e., σ for Black's and Jump models, and $\sqrt{V}$ for SV and SVJD models); Θ is the vector of structural parameters for Jump, SV, and SVJD models. For Black's model there are no structural parameters. Then, the volatility term and structural parameters can be obtained by non-linear least square estimation.

This procedure can result in an estimate of implied volatility and structural parameters for each day. As discussed in Bates (1991), it is potentially inconsistent with the assumption of constant parameters when deriving option pricing models, because the implied parameters are not constrained to be constant over time; but a chronology of parameter estimates and some stylized facts for future specification of more complicated dynamic models could thereby be generated through this estimation procedure. The objective function in equation (22) for implied parameter estimation has been used by several others including Bates(1991), Bakshi, Cao and Chen (1997), Hilliard and Reis (1999), and Koekebakker and Lien (2004).

Results

In-Sample Pricing Fitness

These four option pricing models are estimated: Black's (1976) model, Bates' (1991) jump-diffusion model, Heston's (1993) stochastic volatility (SV) model, and stochastic volatility jump diffusion (SVJD) model. Table 1 reports the Wednesday average and standard error of each estimated volatility/parameter as well as the Wednesday average root mean squared errors. First, the in-sample root mean square errors of Bates91, SV, and SVJD are considerably lower than those of Black76 model. SVJD yields lowest RMSEs, while Bates91 gives lower RMSEs than SV.

Second, the structural parameter estimates for the SVJD model indicate that the jumps and mean-reverting stochastic volatility are both important. Though the mean level of jump size ($\bar{k}^*$) is quite low, the jump frequency (λ^*) is significant. This may be

due to the relative brevity of the sample period, which makes it hard to detect a salient jump pattern. The average annual jump frequency in Bates91 (1.29 times per year) is higher than that in SVJD (0.63 times per year), possibly because Bates91 relies more on jump parameters to explain the futures price movement, while SVJD model also allows volatility to change over time. This also can be an explanation that the jump magnitude of Bates91 is bigger than that of SVJD. The speed of volatility adjustment, β^* is 0.97 in SV and 2.06 in SVJD , which compares with values of 1.15 and 2.03 in Bakshi, Cao and Chen (1997) for S&P 500 call options. The long run means of mean-reverting volatility process, which are measured by $\sqrt{\alpha/\beta^*}$, are 0.26 and 0.24 in SV and SVJD, respectively. Note that in addition to the variance term (V), both the variation of mean jump size and jump frequency (i.e. δ and λ) affect the variation of the continuously compounded rate of return. In fact, $\delta^2\lambda^*$ is the instantaneous conditional return variance per year attributable to jumps in the risk-neutral processes. The implied volatility from SVJD is $\sqrt{V + \delta^2\lambda^*}$ rather than $\sqrt{V}$ alone so that the average value of implied volatility in SVJD is 0.2397. The consistently negative estimates of ρ in SV and SVJD indicate that implied volatility and rate of return of futures price are negatively correlated. This means that the implied distribution of the rate of return of the underlying asset perceived by option traders is negatively skewed.

Third, the implied volatilities from Black76, Heston93 and SVJD (with average values of 0.2419, 0.2507, and 0.2397) are very close. This finding is consistent with Bakshi, Cao and Chen (1997). They explain that option prices are sensitive to the volatility input and thus even small differences in volatility can result in significantly different pricing results. The implied volatilities from Bates91 are considerably lower. This finding is consistent with Koekebakker and Lien (2004) for soybean futures call options. One explanation is that the estimates of jump frequency and size are both quite high in Bates91. So Bates91 may treat the variation of volatility as jumps, which further decreases the average level of estimates for implied volatility.

Out-of-Sample Pricing Performance

One may argue that there might be an overfit problem because the number of parameters increases along these four models. Therefore, out-of-sample testing is performed. Specifically, the previous day's (Tuesday's) data are used to estimate the volatility/parameters, and then Tuesday's estimates and Wednesday's data are used to predict Wednesday's option prices based on the two models, separately. Then we subtract the model-determined price from its observed counterpart to compute the pricing error. This procedure is repeated for every call and each day in the data sample, to obtain the average root mean squared pricing errors and their associated standard deviations.

Note that this procedure does not constitute a true out-of-sample test in the usual

sense, since Wednesday's volatility and structural parameters are assumed to be unchanged from Tuesday's. However, the out-of-sample testing here is pricing out-of-sample options rather than forecasting options prices. The latter involves not only an estimate for the volatility and structural parameters but a forecast for the exogenous variables such as price of the underlying asset and instantaneous interest rate. Therefore, the testing is equivalent to testing the stability of parameters. This procedure is consistent with previous approaches in the literature (e.g. Bakshi, Cao and Chen, 1997; Hilliard and Reis, 1999).

Bates91, SV, and SVJD yield lower RMSEs than Black76 in 137, 93, and 121 Wednesdays respectively out of 150 Wednesdays in our data sample. The means of RMSEs are shown in Table 2. Tuesday parameter estimates are not reported because they are similar to their Wednesday's counterparts. Bates91 yields roughly as low out-of-sample RMSEs as SVJD, though one may expect the opposite to hold because of the large number of additional structural parameters in SVJD.

Conclusion

This study investigates the improvement over the Black's model from allowing for discrete jump and stochastic volatility in pricing futures options. Although there are several studies that have examined the performance of Bates91, SV, and SVJD models on different asset classes, little research has compared the performance of these major option pricing models for agricultural commodity futures options. This study fills this void by testing the in-sample and out-of-sample performance of Black76, Bates96, SV, and SVJD models using CBOT intradaily corn futures and options on these futures.

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Table 1: Parameter Estimates for Alternative Models

Parameters	Black76	Bates91	Heston93	SVJD
v	0.2419 (0.0720)	0.1369 (0.0379)	0.2507 (0.0462)	0.2289 (0.0585)
λ^*		1.2930 (0.5764)		0.6261 (0.0988)
$\bar{k}^*$		0.1152 (0.0743)		-0.0237 (0.0623)
δ		0.1042 (0.1207)		0.0775 (0.0293)
α			0.0663 (0.0267)	0.1207 (0.0727)
β^*			0.9719 (0.0524)	2.0554 (0.1015)
σ_v			0.4131 (0.0584)	0.3837 (0.0489)
ρ			-0.5612 (0.0352)	-0.5787 (0.0624)
RMSE	1.0859	0.7284	0.8904	0.7123

Note: Four models are estimated: Black's (1976), Bates (1991) jump-diffusion model, Heston's (1993) stochastic volatility (SV), and stochastic volatility jump diffusion (SVJD) models. v is instantaneous volatility (i.e., σ for Black's and Jump models, and $\sqrt{V}$ for SV and SVJD models). The structural parameters, λ^*, $\bar{k}^*$, δ, are the frequency, magnitude, and variation coefficient of the magnitude of the jump component, respectively. The other structural parameters, β^*, α/β^*, and σ_v, are the speed of adjustment, long-run mean, and the variation coefficient of the stochastic volatility term V. And ρ is the correlation of the two standard Brownian motions in SV and SVJD models. Average root mean squared errors RMSEs are reported for each model. The unit of the RMSEs is cent since corn futures option prices are in cents. Standard deviations of the estimates are in parentheses.

Table 2: Out-of-sample Average Root Mean Squared Errors

Black76	Bates91	Heston93	SVJD
1.1604	0.8798	1.1365	0.8886

Note: The unit is cent.