Consistent Aggregation in Food Demand Systems

By

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Abstrast: Two aggregation schemes for food demand systems are tested for consistency with the Generalized Composite Commodity Theorem (GCCT). One scheme is based on the standard CES classification of food expenditures. The second scheme is based on the Food Guide Pyramid. Evidence is found that both schemes are consistent with the GCCT.

Consistent Commodity Aggregation in Food Demand Systems

Introduction

Any attempt to estimate a food demand system requires aggregation of elementary products into a manageable number of composite commodities. The challenge is to define the composites so that the estimated system consistently reflects the demand for the elementary products that consumers actually purchase. Incorrect aggregation over products can lead to errors in estimating demand elasticities and imply misleading policy implications.

Assumptions about the separable structure of underlying consumer preferences are the most common justification for maintaining a particular aggregation scheme. However, seperability can impose strong restrictions on consumer preferences that are often difficult to test (Moschini, Moro, and Green; Diewert and Wales). Instead of testing the assumption underlying seperability, a common practice is to heuristically argue in favor of a particular aggregation based on the characteristics of the individual items and the reasonableness of the groupings.

An alternative criterion for aggregating elementary into a valid composite is given by the Hicks-Leontief Composite Commodity Theorem (*CCT*). The CCT requires, however, that the set of relative prices, defined by the ratio of the elementary product price to its corresponding group price index, to remain constant over time (e.g., Nelson, 1991). Empirically, this restriction is generally rejected.

In a seminal paper, Lewbel generalizes the *CCT* by viewing theoretical elementary and composite demand relationships as systems of conditional means (i.e., regression equations). In this setup, Lewbel shows that aggregation under the Generalized Composite Commodity Theorem (*GCCT*) can be obtained under the weaker condition that the vector of relative elementary prices is independent of the vector of group price indices. Specifically, the *GCCT*

relaxes the restriction of constant price ratios within groups by strengthening the requirement that independence holds across all groups.

While this generalization is significant, tests of the *GCCT* that have been proposed do not check all of the conditions implied by vector independence. In an important paper, Davis, *et. al.* propose *family-wise* tests of joint independence between the elementary price ratios of a group and the group's price index. In the case of consumer demand, it is important, however, to check whether such restrictions hold *jointly* across groups and not just within the groups. In this paper, we extend the tests of the *GCCT* to take account of these cross group restrictions.

In this paper, we illustrate this test procedure for two aggregation schemes using quarterly Consumer Expenditure Survey (CES) data from 1984.1 to 1997.4. One is a scheme based on the standard food classification system used in the CES. The second is a scheme based on grouping food items according to their nutritional content and is motivated by the USDA's Food Guide Pyramid.

Theory of the GCCT and Consumer Demand Systems

Consistent aggregation based on the *GCCT* places restrictions on share-based models of composite demand. Elaboration of this theory leads to tests of the *GCCT*.

The *GCCT* maintains the existence of an *n*-vector of elementary demand functions, **g** that maps an *n*-vector of logged elementary retail prices (**r**) and logged income (i.e., total expenditures) 1(*z*) to an *n*-vector of expenditure shares ($w_1, w_2, ..., w_n$). In the *GCCT*, each g_i represents a conditional mean so that if *E* denotes the mathematical expectations operator, there exists elementary demand functions $g_i: (\mathbf{r}, z) \rightarrow w_i$ (i = 1, 2, ..., n) such that

(1) $w_i = g_i(\mathbf{r}, z) + e_i$ where $E(e_i | \mathbf{r}, z) = 0 \implies E(w_i | \mathbf{r}, z) = g_i(\mathbf{r}, z)$.

In particular, **g** satisfies adding-up $(\sum g_i = I)$, homogeneity (for all *i*: $g_i(\mathbf{r}-k, z-k) = g_i(\mathbf{r}, z)$), and Slutsky symmetry (i.e., $(\partial g_k/\partial r_j) + (\partial g_k/\partial z)g_j = (\partial g_j/\partial r_k) + (\partial g_j/\partial z)g_k$). These three conditions are termed integrability conditions (Lewbel) and ensure that (1) derives from a welldefined optimization problem. From the *n*- vector of elementary demands, the task is to find an *M*-vector (M < n) of group or composite demand functions, **G**, that map the *M*-vector of logged group price indices $\mathbf{R} = (R_1, R_2, ..., R_M)$ ' and (log) income, *z*, to the *M*-vector of group expenditure shares ($W_1, W_2, ..., W_M$). It is understood that each element in the group price index vector **R** is constructed as some (continuous) function *h* of the elements of **r** associated with the group (i.e., $R_1 = h_I (r_{i \in I})$). In the theory of the *GCCT*, each G_I is the mean of the *I*th group share conditioned on **R** and *z*. In particular, $G_I: (\mathbf{R}, z) \to W_I$ where $W_I \equiv \sum_{i \in I} w_i$ satisfies (2) $W_I = G_I(\mathbf{R}, z) + u_I$, where $E(u_I | \mathbf{R}, z) = 0 \Rightarrow G_I(\mathbf{R}, z) = E(W_I | \mathbf{R}, z)$

for (I = 1, ..., M).

The theory provides two definitions that link the *n*-system of elementary demand functions **g** to the *M*- system of composite group share functions **G**. First, for any group *K*, the group share is defined as $W_K = \sum_{k \in K} w_k$. Analogously, $G_K^*(\mathbf{r}, z) = \sum_{k \in K} g_k(\mathbf{r}, z)$. Second, ρ_k denotes the deviation between the log of the *k*th elementary price and the log of its group price. Hence if $k \in K$, $\rho_k = r_k - R_K$. In vector notation, $\mathbf{\rho} = \mathbf{r} - \mathbf{R}^*$ in which \mathbf{R}^* is the *n*-vector that has R_K in position *k* and every $k \in K$. These definitions and the above setup imply, by the law of iterated expectations, that

(3)
$$G_K(\mathbf{R}, z) \equiv E(W_K | \mathbf{R}, z) = E\{E(W_K | \mathbf{r}, z) | \mathbf{R}, z)\} \equiv E(G_K^*(\mathbf{R}^* + \mathbf{\rho}, z) | \mathbf{R}, z) \quad (K = 1, ..., M)$$

or $G_K(\mathbf{R}, z) = E(G_K^*(\mathbf{r}, z) | \mathbf{R}, z)$. Equation (3) states that *any* composite share equation that
satisfies (2) is the best predictor of vector sums of elementary demand equations that can be

computed from composite price indices and income. The central question is whether this set of predictors represents a system of composite demand functions.

Lewbel shows that the answer depends on whether the vector ρ is independent of the vector (**R**' *z*)'. From (3) Lewbel notes that vector independence implies

(4)
$$G_{K}(\mathbf{R}, z) = E(G_{K}^{*}(\mathbf{R}^{*}+\boldsymbol{\rho}, z)|\mathbf{R}, z) = \int G_{K}^{*}(\mathbf{R}^{*}+\boldsymbol{\rho}, z) f_{I}(\boldsymbol{\rho}) d\boldsymbol{\rho}.$$

where $f_l(\mathbf{\rho})$ is the marginal probability distribution function of $\mathbf{\rho}$. The first equality is a restatement of (*3*) and the second equality holds when $\mathbf{\rho}$ is independent of (\mathbf{R} ' z)'. Lewbel shows (Theorem 1) that if the second equality of (*4*) is satisfied, the multivariate composite demand system satisfies adding-up and homogeneity. In addition, if elementary demands take on special functional forms, vector independence between $\mathbf{\rho}$ and (\mathbf{R} ' z)' implies Slutsky symmetry holds for composite demand. It is evident from equation (*4*) that the requirement of vector independence places joint restrictions on the conditional mean of the vector $\mathbf{\rho}$ by jointly restricting correlations between all elements of $\mathbf{\rho}$ and all elements of (\mathbf{R} ' z)' (Reed, Levedahl, and Hallahan). *Family-Wise Tests of the GCCT*

For each composite group *K*, the Davis, *et. al.* methodology suggests regressing each ρ_k , $k = 1, 2, ..., n_K$ on the associated group price index R_K (and *z* and a constant and time trend). In the case in which both series are integrated of order 1 or higher, they suggest testing for a spurious regression by performing residual-based tests on each of the n_K model residuals separately. However, since vector independence imposes joint within-group restrictions, they combine the n_k individual tests statistics into a family-wise test using the Holm procedure. In particular they suggest testing the null that the vector ρ_K is independent of R_K (and *z*) by computing the *K*th group's order statistic, $T_K = max_k |T_k|$. The null that ρ_K is independent of R_K (and *z*) is rejected at the α -level of significance if $T_K = max_i |T_i| > |T|_{v,\delta}$. Here v relates to the number of observations used and $\delta = \alpha/n_K$ for n_K one-sided unit root tests. Otherwise, the null that \mathbf{p}_K is independent of R_K is accepted implying that the elementary products of group K represent a valid composite.

This methodology computes each group test independently of all other group tests, so that the correlations among elements of different groups are not considered.¹ Our test procedure uses the same Hohm procedure to define a family-wise test, however, we include all the relative and group price indices in the test.

Assume that $\{\rho_t\}$ and $\{\mathbf{R}_t\}$ are vector time series, and that each vector contains deterministic components (e.g., means, deterministic time trends), **d**. Let $\Re = (\mathbf{R}' z)'$. By the law of iterated expectations it follows that

(5)
$$E(\rho|\mathbf{d}, \mathfrak{R}) = E(\rho|\mathbf{d}) + E\{(\rho - E(\rho|\mathbf{d})|(\mathfrak{R} - E(\mathfrak{R}|\mathbf{d}))\}$$

Consider the case in which all of the elements of the vector ($\rho \Re$) are integrated (of order 1) Write a vector of linear regressions of the form

$$(6) Y = BX + e$$

where $\mathbf{Y} = \boldsymbol{\rho} - E(\boldsymbol{\rho}|\mathbf{d})$ and $\mathbf{X} = \Re - E(\Re|\mathbf{d})$. If \mathbf{Y} and \mathbf{X} are independent then $E\{(\boldsymbol{\rho} - E(\boldsymbol{\rho}|\mathbf{d}))| (\Re - E(\Re|\mathbf{d}))\} = 0$, **B=0**, and model errors are integrated (of order 1). If, on the other hand, the model error are integrated (of order 1), equation (6) would represent a vector of spurious regressions implying that \mathbf{Y} and \mathbf{X} are asymptotically independent (Reed, Levedahl and Hallahan). This suggests that a test of vector independence amounts to a test that the *n*- system of regressions of $\boldsymbol{\rho} - E(\boldsymbol{\rho}|\mathbf{d})$ on $\Re - E(\Re|\mathbf{d})$ is jointly spurious. To test the null of *n* spurious regressions we extend

¹ By testing that each of the n- relative price ratios is independent of the deflated price index is an attempt to account for some cross group correlations

the Holm procedure as described by Davis *et al.* but include all the T_i 's in calculating the order statistic.

Tests of the Standard and Pyramid Aggregations

In this section, two commodity aggregations are tested for consistency with the GCCT. The first is referred to as the *standard* aggregation. This aggregation is defined using the classification of foods in the CES. In total, this aggregation consists of eight composite commodities. Table 1 lists the composition of each composite in the standard aggregation. The second aggregation is referred to as the *pyramid* aggregation. This aggregation is motivated by the USDA's Food Guide Pyramid, and groups the food items according to their nutritional content. The pyramid aggregation tested in this paper is similar to an aggregation scheme used by Ramezani, Rose and Murphy, however, it differs in some important ways. In total, the pyramid aggregate consists of nine composite commodities. Table 2 lists the composition of each composite in the pyramid aggregation. In principle both the standard and the pyramid classification represent the aggregation of the same elementary food items.

Consistency of the standard and pyramid aggregations with the GCCT was tested using the procedure outlined in the previous section. In these tests price indices for each of the composite commodities were built up using the individual CPI food component price series reported by BLS.² Composite price indices were calculated from the individual component prices using (weighted) mean 1997 expenditures as weights. This allowed us to use the greater detailed food expenditure categories incorporated into the CES in 1997. In particular, for

² Several problems associated with data limitations were encountered. The most significant were the changes in the food expenditure categories collected in the CES instituted in 1997. These changes resulted in BLS discontinuing several CPI components that were part of the Old Series. Several component price CPI series that were previously calculated for time periods prior to 1997 are no longer available after this date. In addition, the pyramid aggregated required knowledge of actual prices (such as for peanut butter and dried beans/peas). BLS

example, the protein composite in the pyramid aggregate could be calculated using actual dried beans/peas expenditure. Quarterly data, 1984 to 1997, was used (T=56).

Empirical Results

Results of augmented Dickey-Fuller Unit Root tests for each log composite price indices and each relative price are reported in Table 3 for the standard aggregation and in Table 4 for the pyramid aggregation. The standard aggregation consists of 8 composites and 42 relative prices. The pyramid aggregation consists of 9 composites and 44 relative prices. In both, it is maintained that Food Away from Home group is a valid composite.

For each aggregation, the results suggest that both τ_{μ} and τ_{τ} tests fail, at the 10 percent level of significance, to reject the null that each of the R_{K} series are generated by a unit root. However, the tests for the relative elementary prices are mixed indicating that some of these prices may be stationary. For both aggregations, these results suggest that some elements of ρ are integrated of order 1 and all elements of **R**' are integrated of order 1. A test of a jointly spurious regression of ρ on **R** provides a test of independence between ρ on **R**'. Based on specification given in equation (6), *OLS* residuals from each of the (de-meaned and detrended) regressions of the ρ_k on all the composite price indices were used to compute the Augmented Dickey-Fuller statistics. These statistics were then used to apply the Holm procedure. The critical value of the Holm order statistic for a one-tail family-wise test at the 10 percent level is given by p*(.10, n) = .10/n. The probability of observing a residual ADF test statistic, under the null hypothesis of a spurious regression, that is smaller than the observed

provides these data but generally only back to 1984. We, therefore, limited our aggregation tests to the time period 1984-97.

Holm order statistic was calculated using the approximations given by MacKinnon. Table 5 summarized the results of the family-wise tests for the two aggregations.

Table 5: Family-Wise Tes	t of Vector Independence at th	e 10 Percent Level
Aggregation:	Standard	Pyramid
Observed Holm Order Statistic, T ₀	-5.4052	-5.6454
Prob $(T > T_0)$.0140	.0145
Critical Values	p*(.10, 42)=.0024	p*(.10, 44)=.0023
	p*(.10, 37)=.0027	p*(.10, 42)=.0024

A value of $p^*(.10, n) \le Prob (T>T_0)$ indicates a failure to reject the null hypothesis of familywise vector independence at the 10% level. Probability values for the order statistic were calculated as asymptotic p-values of ADF statistic for the appropriate number of regressors using MacKinnon.

Table 5 show evidence that both aggregation schemes are consistent with the GCCT at conventional significance level of 10 percent. Table 5 includes alternative significance levels for the Holm for each of the aggregations. These levels were obtain after deleting the ρ_i from the family-wise test for which μ_{τ} indicated stationarity. In neither aggregate did this change the value of the order statistic.

Conclusion

Both the standard and pyramid aggregations considered in this paper appear to be consistent with GCCT. The observed P-value associated with the family-wise test (when all the ρ 's are included) is 0.64 for the pyramid and 0.59 for the standard aggregation. Based on an interpretation by DeGroot observed P-value can be interpreted as a degree of belief that the null holds (given diffuse priors). Such a comparison would provide a criterion for evaluating the consistency with GCCT of the two aggregation schemes relative to each other.

Aggregation Scheme	Composites	Elementary Food Items
Standard	Cereal/Bakery (11)	flour/flour mixes; cereals; white bread; other bread; rice/pasta/cornmeal; cookies; biscuits/rolls/muffins; cakes/cupcakes; crackers/cracker products; fresh/frozer bakery products/pie/tarts/turnovers; sweetrolls/coffeecakes/donuts
	Dairy (5)	fresh milk/cream; cheese; ice cream; other dairy; butter.
	Meats (6)	beef; poultry; fish; pork; other meats; eggs.
	Fruits (7)	apples; bananas; oranges; other fresh fruits; frozen fruit/juices; other fruit juices; canned/dried fruits.
	Vegetables (7)	potatoes; lettuce; tomatoes; other fresh vegetables; frozen vegetables; canned beans/corn; other processed vegetables (including dried beans/peas).
	Fats and Oils (3)	margarine, creamer/peanut butter, other fats/salad dressing.
	Other (3)	beverages; potato chips/other snacks; sugars/sweets,
	FAFH (1)	food-away-from-home.

 Table 1: Composition of the Composite Commodities-Standard Aggregation

Aggregation Scheme	Composites	Elementary Food Items
Pyramid	Grains (7)	flour/flour mixes; cereals; white bread; other bread; rice/pasta/cornmeal; biscuits/rolls/muffins; crackers/cracker products
	Dairy (3)	fresh milk/cream; cheese; other dairy products
	Protein (8)	beef; poultry; fish; pork; other meats; peanuts butter/nuts; dried beans/peas; eggs
	Fruit (7)	apples; bananas; oranges; other fresh fruits; frozen fruit/juices; other fruit juices; canned/dried fruits.
	Vegetables (7)	potatoes; lettuce; tomatoes; other fresh vegetables; frozen vegetables; canned beans/corns; processed vegetables (excluding dried beans/peas)
	Sweets (6)	cookies; cakes/cupcakes; fresh/frozen bakery products/pies/tarts/turnovers; ice cream; sugar/sweets; sweetrolls/ coffeecakes/donuts
	Fats and Oils (4)	margarine; butter; other fats/salad dressing; creamer
	Other (2)	beverages; potato chips/other snacks
	FAFH (1)	food-away-from-home.

Table 2: Composition of the Composite Commodities-Pyramid Aggregation

Table 3 : Unit Root Tests—Standard Aggregation

Null Hypothesis: I(1)

$ \begin{array}{cccc} & \tau_{\mu} \\ & & & \\ & \rho(flour)(5) & -1.39 & -1.\\ & & \rho(flour)(5) & -1.92 & -1.\\ & & \rho(rpc)(1) & -2.22 & -3.\\ & & \rho(rpc)(1) & -2.22 & -3.\\ & & \rho(rpc)(1) & -2.22 & -1.\\ & & \rho(rpc)(1) & -0.22 & -1.\\ & & \rho(rpc)(1) & -0.22 & -1.\\ & & \rho(rpc)(1) & 0.02 & -1.\\ & & \rho(rpc)(2) & -1.09 & -2.\\ & & \rho(rpc)(2) & -1.09 & -2.\\ & & \rho(rpc)(2) & -1.09 & -2.\\ & & \rho(rpc)(2) & -2.03 & -2.\\ & & \rho(rpc)(1) & -0.40 & -2.\\ & & \rho(rpc)(1) & -0.40 & -2.\\ & & \rho(rpc)(1) & 0.40 & -2.\\ & & \rho(rpc)(1) & 0.40 & -2.\\ & & \rho(rpc)(1) & 0.94 & -0.\\ & & \rho(rpc)(1) & 0.94 & -0.\\ & & \rho(rpcc)(2) & -2.61* & -3.\\ & & \rho(rpc)(2) & -2.35 & -2.\\ & & \rho(rpc)(2) & -2.25 & -2.\\ & & \rho(rpc)(2) $	Null riypoutesis. 1(1)								
$ \begin{array}{cccccc} R(Meats) (5) & -1.71 & -1.25 & R(Cereal/Bakery) (4) & -1.39 \\ \rho(eggs) (6) & -2.73 & -2.56 & \rho(flour) (5) & -1.92 \\ \rho(pork) (7) & -0.87 & -0.79 & \rho(cereal) (2) & -1.88 \\ \rho(pork) (7) & -1.80 & -1.81 & \rho(ppc) (1) & -2.22 \\ \rho(other) (5) & -1.95 & -2.20 & \rho(w_{-} hread) (0) & -0.02 \\ \rho(poultry) (6) & -2.32 & -2.46 & \rho(o_{-} hread) (0) & -0.02 \\ \rho(poultry) (6) & -1.88 & -1.70 & \rho(piscuits) (6) & -2.01 \\ \rho(poultry) (5) & -0.44 & -1.96 & \rho(piscuits) (6) & -2.01 \\ \rho(rackers) (6) & 10 & -2.04 & \rho(cookies) (7) & -1.26 \\ \rho(marg) (7) & -2.04 & -2.04 & \rho(cookies) (7) & -1.26 \\ \rho(marg) (7) & -2.04 & -2.04 & \rho(cookies) (7) & -1.26 \\ \rho(narg) (7) & -2.62 & -2.61 & \rho(pisc) (4) & -2.03 \\ \rho(other) (1) & -1.87 & -1.87 & \rho(piss) (4) & -2.03 \\ \rho(bev) (0) & -1.79 & -1.73 & \rho(pies) (4) & -2.03 \\ \rho(bev) (0) & -1.79 & -1.73 & \rho(pies) (4) & -2.04 \\ \rho(bev) (0) & -1.79 & -1.73 & \rho(pies) (4) & -2.03 \\ \rho(snecks) (0) & -1.74 & -1.65 & \rho(hutter) (0) & -2.18 \\ \rho(snecks) (0) & -1.74 & -1.65 & \rho(hutter) (0) & -2.61 \\ \rho(snecks) (0) & -1.74 & -1.65 & \rho(hutter) (0) & -2.61 \\ \rho(snecks) (0) & -1.74 & -1.65 & \rho(hutter) (0) & -2.61 \\ \rho(sneck) (0) & -1.74 & -1.65 & \rho(hutter) (0) & -2.61 \\ \rho(snecks) (0) & -1.74 & -1.65 & \rho(hutter) (0) & -2.61 \\ \rho(snecks) (0) & -1.74 & -1.65 & \rho(hutter) (0) & -2.61 \\ \rho(snecks) (0) & -1.74 & -1.65 & \rho(hutter) (0) & -2.61 \\ \rho(snecks) (0) & -1.74 & -1.65 & \rho(hutter) (0) & -2.25 \\ \rho(nter) (5) & -2.25 & \rho(nter) (5) & -2.35 \\ \rho(nter) (5) & -2.22 & \rho(nter) (5) & -2.35 \\ \rho(nter) (5) & -2.22 & \rho(nter) (5) & -2.35 \\ \rho(nter) (5) & -2.22 & \rho(nter) (5) & -2.35 \\ \rho(nter) (5) & -2.22 & \rho(nter) (5) & -1.23 \\ \rho(nter) (5) & -2.35 \\ \rho(nter) (5) & -1.16 & -2.22 & \rho(nter) (5) & -1.23 \\ \rho(nter) (5) & -2.25 & \rho(nter) (5) & -1.23 \\ \rho(nter) (5) & -2.22 & \rho(nter) (5) & -1.23 \\ \rho(nter) (5) & -2.22 & \rho(nter) (5) & -1.23 \\ \rho(nter) (5) & -2.22 & \rho(nter) (5) & -1.23 \\ \rho(nter) (5) & -2.22 & \rho(nter) (5) & -1.23 \\ \rho(nter) (5) & -2.22 & \rho(nter) (5) & -1.23 \\ \rho(nter) (5) & -2.22 & \rho(nter) (5) & -1.23 \\ \rho(nter) (5) & -2.22 & \rho(nter) (5) & -1.23 \\ \rho$	9	. 	ττ		τ ^μ	ττ		μ	τ_{τ}
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$.16	-1.51	R(Meats) (5)	-1.71	-1.25	R(Cereal/Bakery) (4)	-1.39	-1.59
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		2.29	-2.25	p(eggs) (6)	-2.73*	-2.56	$\rho(flour)(5)$	-1.92	-1.16
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		1.63	-2.18	$\rho(\text{beef})(5)$	-0.87	-0.79	p(cereal) (2)	-1.88	-0.08
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		1.87	-2.84	$\rho(\text{pork})$ (7)	-1.80	-1.81	p(rpc) (1)	-2.22	-3.22*
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		1.34	-3.46*	$\rho(other)(5)$	-1.95	-2.20	$\rho(w_bread)(0)$	-0.22	-1.04
$p(poultry) (6) -1.88 -1.70 p(biscuits) (6) -2.01 \\ p(crackers) (6) -1.09 \\ p(crackers) (5) -0.44 -1.96 p(cakes) (5) -1.70 \\ p(crack) (7) -2.04 -2.04 p(cookies) (7) -1.26 \\ p(marg) (7) -2.62* -2.61 p(pies) (4) -1.26 \\ p(other) (1) -1.87 -1.87 p(s_rolls) (0) -2.18 \\ p(bev) (0) -1.79 -1.73 p(s_rolls) (0) -2.18 \\ p(bev) (0) -1.79 -1.73 p(cheese) -0.94 \\ p(sweets) (0) -1.74 -1.65 p(putter) (0) -2.61* \\ p(sweets) (0) -1.74 -1.65 p(butter) (0) -2.61* \\ p(other) (0) -1.74 -1.65 p(butter) (0) -1.23 \\ p(other) (5) -2.35 \\ p(other) (5) -3.03* \\ p(sweets) (2) -1.16 -2.22 p(IC) (0) -3.03* \\ p(sweets) (3) -1.16 p(sweets) (3) p(sweets) (3) -1.16 p(sweets) (3) p(sweets) (3) -1.16 p(sweets) (3) p(sweets) (3) p(sweets) (3) p(sweets$		1.26	-2.56	$\rho(fish)(6)$	-2.32	-2.46	$\rho(o_bread)(0)$	-0.02	-1.69
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	· . ·	1.53	-1.54	p(poultry) (6)	-1.88	-1.70	p(biscuits) (6)	-2.01	-2.25
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		0.75	-2.49				p(crackers) (6)	-1.09	-2.93
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$				R(Fat_Oil) (5)	-0.44	-1.96	p(cakes) (5)	-1.70	-1.17
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	1 I I	1.26	-1.98	$\rho(\text{cr_pb})(1)$	-2.04	-2.04	p(cookies) (7)	-1.26	-2.79
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	· • ·	1.71	-1.71	p(marg) (7)	-2.62*	-2.61	p(pies) (4)	-2.03	-2.15
* $R(Other)(0) -0.24 -2.54 R(Dairy)(1) -0.40$ $\rho(bev)(0) -1.79 -1.73 \rho(cheese) -0.94$ $\rho(snacks)(0) -1.89 -1.88 \rho(fluid)(0) -2.61*$ $\rho(sweets)(0) -1.74 -1.65 \rho(butter)(0) -1.23$ $R(Away)(5) -1.16 -2.22 \rho(IC)(0) -3.03*$	1	3.28*	-3.33*	$\rho(\text{other})(1)$	-1.87	-1.87	$\rho(s_rolls)(0)$	-2.18	-1.83
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		1.17	-3.89*						
* $p(bev)(0)$ -1.79 -1.73 $p(cheese)$ -0.94 p(snacks)(0) -1.89 -1.88 $p(fluid)(0)$ -2.61* p(sweets)(0) -1.74 -1.65 $p(butter)(0)$ -1.23 p(other)(5) -2.35 R(Away)(5) -1.16 -2.22 $p(IC)(0)$ -3.03*	<u>`</u> 1`	1.75	-2.48	R(Other) (0)	-0.24	-2.54	R(Dairy) (1)	-0.40	-2.52
$ \begin{array}{llllllllllllllllllllllllllllllllllll$	<u>`</u> 1`	1.23	-3.38*	$\rho(bev)(0)$	-1.79	-1.73	p(cheese)	-0.94	-0.13
$ \begin{array}{cccc} \rho(sweets) (0) & -1.74 & -1.65 & \rho(butter) (0) & -1.23 \\ & & \rho(other) (5) & -2.35 \\ R(Away) (5) & -1.16 & -2.22 & \rho(IC) (0) & -3.03* \end{array} $	÷г.	1.42	-2.07	$\rho(\text{snacks})(0)$	-1.89	-1.88	p(fluid) (0)	-2.61*	-3.16*
-1.16 -2.22 $\rho(other) (5)$ -2.35 -3.03*	1	·1.14	-2.24	p(sweets) (0)	-1.74	-1.65	$\rho(butter)$ (0)	-1.23	-0.20
-1.16 -2.22 $p(IC)(0)$ -3.03* .							$\rho(other)(5)$	-2.35	
				R(Away) (5)	-1.16	-2.22	p(IC) (0)	-3.03*	•

values associated with the coefficient on the lagged level variable in the regression of the first differenced variable on a constant (τ_{μ}) and on a constant and a time trend (τ_r). The number of lagged differences used is determined by *SHAZAM* as the highest significant lag order (at approximately the 95% confidence interval) from either the autocorrelation or partial autocorrelation function of the first differenced series. The number of lags used to test each price is given in *I(1)* tests that the series is integrated of order 1 against the alternative of trend stationarity. The values reported are augmented Dickey-Fuller (1979) (ADF) tparentheses.

Critical values for the τ_{μ} test 5 percent (-2.898), 10 percent (-2.586); for the τ_{τ} test 5 percent (-3.467), 10 percent (-3.160). * stationary at the 10% level.

Table 5 : Unit Root Tests—Pyramid Aggregation

Null Hypothesis: I(1)

Null Hypothesis: 1(1)	1)							
	τ_{μ}	τ _τ		\mathfrak{L}_{μ}	$\mathfrak{r}_{\mathfrak{r}}$		τ_{μ}	$\mathfrak{r}_{\mathfrak{r}}$
R(Fruit) (6)	-1.16	-1.51	R(Protein) (5)	-1.85	-1.18	R(Grains) (5)	-1.37	-0.93
p(apple) (7)		-2.25	p(eggs) (6)	-2.71*	-2.58	$\rho(flour)(5)$	-1.90	-1.05
ρ (bananas) (4)	-1.63	-2.18	$\rho(\text{beef})(5)$	-1.12	-0.96	p(cereal) (2)	-1.97	-0.45
$\rho(\text{orange})(6)$		-2.84	$\rho(\text{pork})(7)$	-1.58	-1.53	$\rho(rpc)(1)$	-2.08	-2.72
p(o_ff) (4)	-1.34	-3.46*	$\rho(other)(5)$	-1.37	-2.05	$\rho(w_bread)(0)$	-0.15	-0.75
$\rho(fz_fr/juc)(0)$		-2.56	$\rho(fish)(6)$	-2.13	-2.41	$\rho(o_bread)$ (0)	-0.02	-1.20
p(can/dry) (6)	-1.53	-1.54	p(poultry) (6)	-1.89	-1.67	p(biscuits) (6)	-1.93	-1.11
p(o_juc) (6)	-0.75	-2.49	p(pb) (5)	-0.46	-1.56	p(crackers) (6)	-1.22	-3.22*
			$\rho(dr_beans)(2)$	-1.29	-1.89			
R(Vegetable) (6)	-1.24	-1.99				R(Sweets) (5)	-1.41	-1.37
p(potatoes) (7)	-1.72	-1.70	R(Fat_Oil) (2)	-0.63	-3.14	$\rho(pies)$ (4)	-1.43	-2.91
p(lettuce) (6) -3.29*	-3.29*		$\rho(marg)(0)$	-1.55	-0.48	$\rho(s_rolls)(0)$	-0.52	-3.40*
p(tomatoes) (4)	-1.19		$\rho(butter)(0)$	-1.19	-0.40	p(cookies) (7)	-1.99	-0.83
$\rho(o_{fresh}) (6)$	-1.79		$\rho(\text{creamer})$ (0)	-1.55	-0.86	$\rho(cakes)(0)$	-1.04	-3.23*
p(frozen) (2)	-1.22		$\rho(other)(0)$	-1.26	-1.52	p(IC) (5)	-1.62	-1.01
p(process) (7)	-1.58	-				p(s_sweets) (4)	-0.83	-3.07
p(c_crn/bean) (7)-1.12	7)-1.12	-2.23	R(Other) (0)	-0.30	-2.29			
			p(bev) (0)	-1.85	-1.83	R(Dairy) (0)	-0.40	-1.71
R(Away) (5)	-1.16	-2.22	$\rho(\text{snacks})(0)$	-1.86	-1.82	p(cheese) (0)	-1.65	
						$\rho(fluid)(0)$	-2.22	
						p(other)(5)	-2.76*	-2.70

values associated with the coefficient on the lagged level variable in the regression of the first differenced variable on a constant (τ_{μ}) and on a constant and a time trend (τ_{τ}). The number of lagged differences used is determined by *SHAZAM* as the highest significant lag order (at approximately the 95% confidence interval) *I(1)* tests that the series is integrated of order 1 against the alternative of trend stationarity. The values reported are augmented Dickey-Fuller (1979) (ADF) tfrom either the autocorrelation or partial autocorrelation function of the first differenced series. The number of lags used to test each price is given in parentheses.

Critical values for the τ_{μ} test 5 percent (-2.898), 10 percent (-2.586); for the τ_{τ} test 5 percent (-3.467), 10 percent (-3.160). * stationary at the 10% level.

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