THEORETICAL AND EMPIRICAL PROBLEMS IN MODELING OPTIMAL REPLACEMENT OF FARM MACHINES

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Research on the optimal replacement problem has emphasized specification of the theoretically appropriate criterion. Today, the most commonly applied replacement decision theory for machinery assumes that the owner will replace each older machine, "defender," with an identical new machine, "challenger," in accordance with long-run cost minimizing or profit maximizing criteria (i.e., wealth maximization). Perrin (p. 60) summarizes the cost minimization criterion which should be applied: "A machine should be kept another period if the marginal costs of retaining it . . . are less than the 'average' periodic costs of a replacement machine." As Chisholm noted, this criterion is "deceptively simple." Support for Chisholm's observation is evidenced throughout the literature, because acceptance and use of an appropriate criterion has come about slowly.

Samuelson cited an extensive list of writings in forestry and economics in which optimal replacement criteria are partially or wholly incorrect. It includes among others: Boulding's microeconomic text and writings by Hotelling and by Fisher. To this list, the agricultural economics profession can add a number of its own. For example, the *JFE* article by Faris was one of the first to deal with a variety of replacement problems. Faris delineated and applied criteria for replacements occurring within a single production period and for longer term point-input, point-output, and point-input, continuous-output replacements. However, as Chisholm demonstrated, Faris's article erred in that: (1) the opportunity cost on (i.e., interest on) the capital asset was not included in formulating the marginal replacement criterion, and (2) the marginal net revenue was incorrectly defined for purposes of applying an appropriate marginal criterion.[1] Chisholm proceeded to specify a correct marginal-revenue, marginal-cost criterion, and a correct net present value criterion, although he did not discuss their logical linkage (i.e., their equivalence). A major contribution of Perrin's article was to clearly delineate and illustrate equiva-

lence of the two criteria, the same criteria as stipulated by Chisholm. If the marginal criterion is delineated as a pair of inequalities (Chisholm, p. 112; Perrin, p. 65), the optimal replacement age is identical to that from the net present value formulation.

Recently, the net present value (PV) criterion has been more common in the replacement literature than the marginal criterion (e.g., Kay and Rister; Bates et al.; Crane and Spreen). Other than demonstrating the PV criterion with attendant modifications for taxes and inflation, little research has been directed toward applying it to machinery replacement. In applying the criterion to a practical case of machinery replacement, two general problems are encountered. First, there is a problem of what is the correct theoretical specification for a replacement model. Second, there are empirical problems in generating reliable estimates for parameters in the replacement model. This paper explicitly addresses these two problems. In addition, the need for a more powerful analytical method—one which can handle inflation or technical change—is pointed out, along with the outline of potential solutions. Because of misconceptions (past and present) in using theoretically correct criteria, the basic, identical-challenger PV criterion will first be reviewed and related to the standard investment PV formulation.

IDENTICAL-CHALLENGER CRITERION

The optimal replacement age for a machine to be successively replaced by an infinite series of identical challengers can be determined in the discrete case by finding the age (S) which minimizes the absolute value of the expression:

$$(1) \quad PV(S) = [1-(1+r)^{-S}]^{-1} [-M(O) - \sum_{t=1}^{S} (1+r)^{-t}R(t) + (1+r)^{-S}M(S)]$$

[1] There is nothing wrong *per se* with Faris' net revenue definition (and calculations) as total revenue in a given future year less the future value of the investment (establishment) cost and the accumulated future value of annual variable costs. This definition will give the correct net present values and the correct amortized net present values. However, it will not, as Chisholm (pp. 108, 112) notes, give the correct marginal net revenues to use in the marginal replacement criterion. When applying the marginal criterion, subtracting out the compounded value of the investment cost is incorrect. One should calculate only the addition to total revenue less added variable cost for each potential replacement age.

where

$PV(S)$ = Present value for each value of S (units for t may be years or other appropriate time intervals);

$[1-(1+r)^{-S}]^{-1}$ = Present value of a \$1 perpetual annuity received (paid) at the beginning of each and every S years, e.g., S may vary from 3 to 30 years for a tractor replacement problem;

$M(O)$ = New cost of the machine, assumed to be constant for the identical-challenger problem, includes savings (if any) resulting from discounted values of investment tax credits;

$M(S)$ = Remaining (terminal market) value of each machine when replaced, assumed to be constant for the identical-challenger problem; also includes the taxes paid (if any) because of investment tax credit recapture and the taxable gain subject to ordinary and capital gains rates;

$R(t)$ = Costs attributable to the machine during each time period t, including machine maintenance and repairs, insurance, and opportunity costs associated with revenues foregone as a result of breakdown time; also includes tax savings due to depreciation;

r = the appropriate periodic after-tax discount rate.

Frequently, only costs are considered, as in expression 1. Even though a cost-minimizing criterion is shown, the opportunity cost of revenues foregone because of untimely breakdowns must be considered. Expression 1 can also be used in a profit-maximizing sense, by selecting the age S, for which the value of PV is a maximum. In this case, the R(t) includes revenues and thus represents periodic net cash flows.[2] It should be noted that, in the identical-challenger case, revenue-cost streams for all successive challengers are assumed to be identical to those associated with the current machine (i.e., expectations are constant with respect to each challenger).

Notice that the terms inside the brackets of expression 1 are exactly the same as a standard capital investment problem for a fixed planning horizon of S years, viz., (1) M(O) denotes the value of the machine at t = 0, i.e., the initial value; (2) $\sum_t (1+r)^{-t}R(t)$ denotes discounted revenues and/or costs expected during each machine's life; and (3) $(1+r)^{-S}M(S)$ denotes the terminal value of the machine discounted to t = 0. The perpetuity factor $[1-(1+r)^{-S}]^{-1}$ converts the standard capital investment criterion to one that allows determination of the optimal replacement timing.

Note that replacement is simply a special type of a mutually exclusive investment decision. Replacement decisions are mutually exclusive in the time periods of ownership for the sequence of a specific project-type; whereas, standard investment decisions are mutually exclusive in the project-type for the same time period. Expressing the value of R(t), M(O), and M(S) at t = 0 evaluates the standard investment at the beginning of each series of S periods. This value is then treated as the amount of payment of an annuity paid every S years. The perpetuity factor is multiplied by this value in order to find the PV of an infinite stream of such payments.

RESEARCH NEEDS FOR PARAMETER INFORMATION

The identical-challenger model is conceptually valid and, given relatively accurate empirical estimates of its parameters, it can be effectively employed to make replacement decisions for certain situations.[3] However, "accurate empirical estimates" imply that several empirical problems must be confronted in conducting research on optimal replacement of farm machines. Some of the major problems are: (1) realistically estimating the cost of machine maintenance and repairs (M & R) over time; (2) accurately estimating remaining values of the machine [M(S) in expression 1, also denoted as RV in the text]; and (3) determining opportunity costs of untimely breakdowns.

The order of discussion in this section follows the order of these three problems. Each problem type is described and discussed in light of previous empirical work. Differences in optimal replacement ages, determined by applying model 1, are illustrated for 55-horsepower diesel tractor requiring a \$14,600 investment (Table 1). A real discount rate of 3 percent and an income tax bracket of 25 percent are assumed for the analyses.

Perhaps the most serious empirical problem results from the lack of data on the time incidence of maintenance and repair (M & R) costs. Researchers continue to rely heavily on formulas

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[2] Note that if the net revenue streams are positive numbers, the sign on the term must be positive.

[3] Testimony to the relevance of identical-challenger models is evident on two fronts: (1) the literature in economics and finance (cited in part earlier in the paper) has contained a number of articles during the past 60–100 years seeking to correctly (logically) interpret and use such a model in decision making regarding durable assets and, more specifically, regarding machinery replacement; and (2) there are a number of industrial and agricultural firms (usually corporations) currently using such a model to aid in replacement decisions.

TABLE 1. Optimal Replacement Ages for a Farm Tractor with Comparative Repair, Remaining Value, and Breakdown-Time Formulas[a]

Repair Formula	Remaining Value Formula	
	Peacock-Brake, 3b	McNeill, 3c
Breakdown-time formula 4b, Exponent = 1.4173		
TAR, 2a	8	7
Exponent = 1.5	9 ($52)	8 ($614)
	7 ($196)	9 ($1,200)
TAR, 2b	7	5
Exponent = 2.033	8 ($1,316)	7 ($1,202)
	6 ($1,618)	6 ($1,579)
Breakdown-time formula 4b, Exponent = 1.6		
TAR, 2a	7	3
Exponent = 1.5	5 ($116)	5 ($647)
	6 ($866)	4 ($1,624)
TAR, 2b	5	3
Exponent = 2.033	6 ($1,948)	4 ($2,503)
	7 ($2,364)	5 ($2,599)

[a] The age (years) which minimizes the present value (cost) is shown first for each repair-remaining value breakdown combination, followed by the age for which the PV (cost) is second lowest and third lowest. The added PV (added cost) of not selecting the minimum age is shown in parentheses beside each age. Formulas, as numbered, are presented in the narrative.

from the *Agricultural Engineers Yearbook* or similar sources. Prior to 1979, their formula for tractors was:

$$(2a) \quad TAR\% = (0.12) [(100) X/12,000]^{1.5}$$

where TAR denotes total cumulative repair costs expressed as a percentage of the tractor's original cost, X equals the accumulated hours of use, and 12,000 equals the estimated lifetime use ("wear-out life"). Formulas for other machines are similar power functions, all having exponents around 1.5. With such a function, it is probable that major overhauls, usually necessary in later years, would be grossly underestimated. Any exponent less than 2.0 means that annual M & R will increase smoothly at a decreasing rate. Hunt (pp. 69–71) takes note of these shortcomings and presents study results for two other formulas. However, neither covers machine use beyond 4,000 hours for tractor or comparable lives for other machines.

Starting in 1979, the *Agricultural Engineers Yearbook* presents TAR percentages for 1970 conditions, with exponents around 2.1. Specifically, the formula for diesel tractors is:

$$(2b) \quad TAR\% = (100) (.012) (X/1000)^{2.033}$$

where TAR and X are defined as in expression 2a. Repair cost estimates [as part of R(t)] for 2a relative to 2b do, as expected, result in different optimal replacement ages. For the $14,600 tractor example, stipulated above, use of formula 2b rather than 2a shortens the optimal replace-

ment age by one or two years, depending upon the remaining value and breakdown formula that is selected (see Table 1 for details). Thus, it is apparent that replacement results are fairly sensitive to the repair estimates used.

Ideally, the appropriate M & R function should be specific to the farm for which the replacement decisions are to be made. Of course, most farms do not have sufficient M & R data on which to base replacement decisions and, thus, must look to estimates made available from research. In developing M & R functions, the function should be able to account for factors specific to the situation for which the estimates will be used. Therefore, the function should logically account for the size and make of the machine, as well as the type of activity in which the machine is engaged. Also, the function should allow for significant ups and downs in yearly expenditures in order to capture the appropriate timing of major expenditures. At least a third-degree polynomial or some transcendental function would seem plausible. Perhaps a spline function approach would be more practical. Again, however, the crux of this problem is a lack of data on M & R for tractors and other major farm machines for specific situations over an extended number of years.

Formulas for estimating RV's for tractors and other machines are available in the *Agricultural Engineers Yearbook*, from research by Peacock and Brake, and from recent research in Canada by McNeill. The respective tractor formulas [sources in brackets] are given as follows:

$$(3a) \quad RV(1) = 68(0.92)^{(Age\ in\ Years)} \ [Agricultural\ Engineers\ Yearbook,\ 1976,\ p.\ 324]$$

$$(3b) \quad RV(2) = 65.6 - 4.1^{(Age\ in\ Years)} \ [Peacock\ and\ Brake,\ 1970]$$

$$(3c) \quad RV(3) = (100)e^{-0.4299 - .0436\ Age + .0691\ Condition} \ [McNeill,\ 1979]$$

where RV = percentage of the original cost.

The usefulness of these formulas for specific decision-making situations can be questioned. Original research underlying the engineering formula is not documented. The engineering formula shows RV at the end of year 1 to be only 63 percent of a tractor's original cost. But, during the 1970s, one-year-old tractors frequently resold for more than the original list price. Such a large difference seemingly cannot be explained only by inflation. Other variables should be included in the formula. The other two formulas exhibit similar deficiencies.

To show the impact of alternative RV estimates, optimal replacement ages for the $14,600, 55-horsepower tractor were determined using two formulas to estimate M(S) in model 1, viz., McNeill's formula, 3c, compared to Peacock-

Brake's formula, 3b. Optimal ages for 3c are from one to four years earlier than optimal replacement ages for 3b—the exact difference depending on the estimate of M & R and downtime opportunity costs (Table 1). As seen from this comparison, RV estimates can have a significant impact on replacement decisions.

As Peacock and Brake recommended, an extensive study of RVs is needed for tractors and for other major farm machines. Resultant formulas seemingly should account not only for the machine's age, but also the machine size (e.g., tractor horsepower), shifts in farmers' demand due to changes in their cash flow, differences in demand for different machine makes, inflationary effects, and so on.

Determining opportunity costs of breakdowns actually consists of two steps. First, the amount of downtime that one expects to occur must be estimated. Then, in order to place a value on the downtime, the amount of revenue foregone must be determined.

Accumulated downtime functions for tractors are available in the *Agricultural Engineers Yearbook* (1979, p. 254) and are given as follows:

(4a) $B = 0.0000021 \, X^{1.9946}$ (Spark Ignition)

(4b) $B = 0.0003234 \, X^{1.4173}$ (Diesel)

where B is the accumulated amount of downtime in hours, and X is the accumulated usage in hours. Again, neither the data nor the methods used in estimating the parameters are well documented. No downtime formulas are available for other machinery and equipment, although some rules regarding downtime are stated for selected machinery and equipment.

To illustrate the effect of added downtime costs on optimal replacement ages, formula 4b was modified by changing the exponent from 1.4173 to 1.60. As expected, depending on M & R and remaining value formulas, this change lowers the optimal replacement age from one to four years (Table 1).[4] These comparative results are based on the assumption that each hour of downtime causes revenues to decline $30. Again, the results indicate a need for developing downtime formulas that account for the specific situation for which downtime estimates are needed.

Valuing revenues foregone because of breakdowns is related to the weaknesses in directly applying the identical-challenger specification. First, the effects of inflation and technological change may cause the assumptions of an identical-challenger model to be unrealistic. The second problem is one of simultaneity—that is, not considering investment and production alternatives within the constraint set when making replacement decisions. These two weaknesses are addressed in the remaining sections.

PROBLEMS IN MODELING INFLATION AND TECHNOLOGICAL CHANGE

The assumption underlying model 1 of identical challengers may be reasonable for determining optimal replacement of trees or for the aging of wines—problems often studied by those interested primarily in theoretical properties of the basic marginal analysis replacement model. However, this assumption is often not true for farm machinery replacement situations. Improved fuel usage, lower repairs or other improvements in machine performance resulting from certain technical changes may be reasons for considering replacement. Or, perhaps more likely, inflation could continue to lead to changes in decision makers' expectations regarding machine prices, periodic cash flows (including effects of changed tax payments), and discount rates. Expanding what has been reported in the literature (e.g., Perrin, pp. 62, 63), modifications to the identical-challenger model can result in a PV model that is considerably more realistic for these cases.

Although unlikely, it is possible that inflation could have an impact uniformly on all parameters that affect the replacement decision. Watts and Helmers (as well as some financial management texts) demonstrate that, for the standard net present value case, proportional changes in costs and revenues will not change the PV amounts and, thus, the investment decision. By simply taking this argument a step further, the assumption of equal inflationary impacts means that, for the replacement problem, the infinite series of values will not change. Hence, uniform-impact inflation will not make the identical-challenger model invalid. The replacement decision remains unaffected, because the minimum (or maximum) PV amount remains unchanged.

The foregoing argument about proportional inflation-ary impacts partially breaks down on an after-tax basis. Bates et al. studied the tax effect of inflation on tractor replacement. They assumed uniform and constant relative inflationary impacts on all cost and revenue streams, and concluded that the tax impact of inflation may lengthen the optimal replacement age. This conclusion was based on the fact that inflation (1) lowers the real value of the tax shield of depreciation and (2) increases the tax due on the disposition of the tractor. Although the income tax may change the replacement result, it does not invalidate the logic of the identical-challenger model itself. Uniform and constant inflationary impacts imply only that the real tax effects of inflation will remain identical for each machine rotation or replacement.

However, neither inflation nor technical change is seldom so well behaved as to cause

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[4] The results shown in Table 1 are quite consistent with those derived by Kay and Rister when they hypothetically accounted for downtime opportunity costs. Of course, their article focused on tax effects and, consequently, employed only the Peacock-Brake remaining value formula, along with only one repair formula. For a 3-percent discount rate and 25-percent tax rate, they show optimal replacement to occur after 7 years. For essentially the same formulas and data, we show 8 years (Table 1).

uniform and constant impacts on all components of costs and revenues. This is especially true for technological change, which usually implies savings of costs relative to revenues. When costs and revenues are affected unequally, whether from inflation or from technological change, assumptions of the identical-challenger model become too restrictive. The logical validity of the identical-challenger model breaks down because relative changes occur among streams of cash flows from one machine to another.

One approach to modeling situations in which the relative costs and revenues change from machine to machine is to segment the model to reflect various expected cash flows of each machine. A separate present value term is evaluated for each segment. The number of segments in this modified PV approach depends on the planning horizon for each decision maker and, perhaps more importantly, on the reliability of information on which expectations are formed about future values of the model's basic parameters when interfaced with appropriate technical-change and inflation parameters. The final term or segment in the model represents the value of cash flows for the infinite series of replacements beyond the machines for which specific cash flows can be delineated. This last segment approximates the effect of perpetual replacement, thereby avoiding a more drastic truncating effect.

The following model illustrates a situation where, at t = 0, the decision maker will be able to delineate expectations regarding specific technical and/or market price changes. Expectations are assumed to be formed at t = 0 with the first three machines being non-identical in cash flows, and all future machines having cash flows identical to machine 3. This model is general in that it allows for inflation, individual-component price, and discount rate changes within the first two series of cash flows and between the first two series and all consecutive series. Technical changes can be reflected by changing values of M(O), M(S), and R(t) when using prices at t = 0. The model is specified as

(5) $\text{PV}(S^*) =$

$$[-M(1) - \sum_{t=1}^{S1} (R_{t1} \prod_{i=1}^{t} \frac{(1+g_{i1})}{(1+r_{i1})(1+f_{i1})}) +$$

$$M(S1) \prod_{t=1}^{S1} \frac{(1+h_{t1})}{(1+r_{t1})(1+f_{t1})}] +$$

$$\frac{1}{\prod_{t=1}^{S1} (1+r_{t1})(1+f_{t1})} [-M(2) \prod_{t=1}^{S1} (1+k_{t1}) -$$

$$\prod_{t=1}^{S1} (1+g_{t1}) \sum_{t=1}^{S2} (R_{t2} \prod_{i=1}^{t} \frac{(1+g_{i2})}{(1+r_{i2})(1+f_{i2})}) +$$

$$\prod_{t=1}^{S1} (1+h_{t1})M(S2) \prod_{t=1}^{S2} \frac{(1+h_{t2})}{(1+r_{t2})(1+f_{t2})}] +$$

$$\frac{[1-(1+r)^{-S3}]^{-1}}{\prod_{t=1}^{S1} (1+r_{t1})(1+f_{t1}) \prod_{t=1}^{S2} (1+r_{t2})(1+f_{t2})} [-$$

$$M(3) \prod_{t=1}^{S1} (1+k_{t1}) \prod_{t=1}^{S2} (1+k_{t2}) -$$

$$\prod_{t=1}^{S1} (1+g_{t1}) \prod_{t=1}^{S2} (1+g_{t2}) \sum_{t=1}^{S3} \frac{R_{t3}}{(1+r)^t} +$$

$$\prod_{t=1}^{S1} (1+h_{t1}) \prod_{t=1}^{S2} (1+h_{t2}) \frac{M(S3)}{(1+r)^{S3}}]$$

where

M(1), M(2) and M(3) denote the new costs of machines 1, 2, and 3, respectively, expressed in t = 0 values.

M(S1), M(S2) and M(S3) denote the remaining values of machines 1, 2, and 3, respectively expressed in t = 0 values. These terms, similar to the identical-challenger model (expression 1), also reflect the taxes paid (if any) resulting from investment tax credit recapture and the taxable gain subject to ordinary and capital gains rates.

R(t1), R(t2) and R(t3) denote the respective yearly costs of machine maintenance and repairs, insurance, and opportunity costs associated with revenues foregone because of machine breakdowns expressed in t = 0 values. These terms also reflect expected tax savings from depreciation or cost recovery accounting.

S1, S2 and S3 denote respective optimal replacement ages for machines 1, 2, and 3, respectively, such that $S1 \lessgtr S2 \lessgtr S3 \lessgtr S$. These are the variables that must be solved simultaneously in order to determine the optimal PV.

r_{t1}, r_{t2} and r denote annual real discount rates for machines 1, 2, and 3, respectively, with $r_{t1} \lessgtr r_{t2} \lessgtr r$ and all rs > 0.

f_{t1}, f_{t2} and f_{t3} denote annual general inflation rates for machines 1, 2, and 3, respectively, with $f_{t1} \lessgtr f_{t2} \lessgtr f_{t3}$ and all fs $\lessgtr$ 0.

k_{t1}, k_{t2}, and k_{t3} denote annual rates of price changes for the new market cost of machines 1, 2, and 3, respectively, with $k_{t1} \lessgtr k_{t2} \lessgtr k_{t3}$ and all ks $\lessgtr$ 0.

g_{t1}, g_{t2}, and g_{t3} denote annual rates of price

changes for machine operating costs—$R(t1)$, $R(t2)$, and $R(t3)$, respectively, with $g_{t1} \lessgtr g_{t2} \lessgtr g_{t3}$ and all gs $\lessgtr 0$.

h_{t1}, h_{t2}, and h_{t3} denote annual rates of price changes for the remaining (used) market value of machines 1, 2, and 3, respectively, with $h_{t1} \lessgtr h_{t2} \lessgtr h_{t3}$ and all hs $\lessgtr 0$.

This type of discrete model maintains the essential logical structure of marginal replacement theory. Information needs are similar to more sophisticated models such as those formulated using control theory or dynamic programming. Within each segment, the numerator of each term is expressed in nominal dollar units; thus, division by appropriate products of $(1+r)$ $(1+f)$ results in real dollar values at $t = 0$. Real-dollar values of the basic model parameters—M values and R(t) values—can be obtained from the decision maker based on his prior experiences or estimated from formulas empirically derived, such as those delineated in the foregoing section. The terms in the third segment representing the series of perpetual replacements are given in real dollars at $t = S1 + S2$. Therefore, this last segment embodies the accumulation of expected technical and market changes (at $t = S1 + S2$) into the infinite series of identical cash flows representing the perpetual replacements.

The model can be expanded to handle any number of non-identical cash-flow series. However, as a practical matter, expressing the model in three segments should be adequate. Expansion beyond three segments makes the model somewhat cumbersome because of the number of terms, and makes the value of using additional machine-specific cash flows questionable relative to forecast accuracy.

To demonstrate the logic of model 5, consider the following example. Suppose at the beginning of a current tractor's life one expects a technical innovation that will decrease repairs, breakdown, and insurance costs by a uniform percentage, β, throughout each challenger's life. Here, the model is expressed in two, rather than three, segments and the individual rates of price change—g, h, and k—are assumed to equal the general inflation rate—f—for corresponding years. Hence,

$R(t2) = [1 + \beta/100][R(t1)]$ for each $t > S1$.

Consistent with this innovation, as new and used tractor markets tend toward efficiency, one also expects certain percentage increases—α and γ—in machine market values, i.e.,

$M(2) = [1 + \dfrac{\alpha}{100}][M(1)]$ for each $t > S1$, and

$M(S2) = [1 + \dfrac{\gamma}{100}] [M(S1)]$ for each $t > S1$.

For purpose of comparison with the model 1 results, assume $R(t1)$ to reflect the 1979 Agricultural Engineers repair formula, 2b above, and the accumulated downtime function, 4b above. Also, assume M(S1) to be determined via the Peacock-Brake remaining value formula, 3b above. Recall that the optimal replacement age S, using model 1 for this situation, was 8 years (Table 1). The following comparative optimal replacement ages were calculated, using model 5 for selected values of β, α and γ:

β	α	γ	S1 (Defender)	S2 (Challenger)
			(Years)	
−20	+10	+15	7	7
−30	+10	+20	7	7
−40	0	+20	5	7

Note that large percentage changes are needed in annual cash flows and in market values of the tractor in order to cause a substantive change between replacement ages, S1 and S2, for such a "one-time" expected change in technology. Only when $R(t1)$ is assumed to decline by 40 percent, coupled with no change in the tractor's original cost and a 20-percent increase in its remaining market value, is there any substantive change between the replacement age of the defender and challenger; S1 = 5 and S2 = 7.

Similar sorts of examples could be shown for inflation scenarios. In essence, there is no difference in using model 5 to estimate the effects of inflation or technical change. In the case of technical change, one is using the model to make decisions that must incorporate expected changes in real ($t = 0$) values of machine operating costs—R(t) values—and the resultant expected changes in real values of new and used machine prices. In the case of inflation, one is using the model to make decisions that must incorporate expected changes in general inflation rates and corresponding expected annual rates of change in nominal values of new and used machine prices and machine operating costs. For either inflationary or technological changes, the impact on the replacement decision depends on two factors which are not independent: (1) relative changes in the annual cash flow amounts within a cash flow series for a specific machine, and (2) relative changes in the value of the stream of cash flows among the series of machines. Thus, the impact of inflation or technological change on the optimal replacement age of the current machine (the defender) becomes primarily an empirical problem in that the result depends on the specific time-incidence pattern of the change in relative cash flows. Based on the foregoing technical-change example, it appears that annual rates of specific-component price changes—k, g, and h values—must be fairly significant relative to corresponding annual changes in the general price level—f values—in order to cause significant shifts in optimal replacement timing.

SIMULTANEOUS ASPECTS
OF REPLACEMENT

Even with good estimates of repairs, remaining values, and breakdown formulas, coupled with appropriate specification of non-identical challenger models used to account for technical changes or inflation, a basic theoretical weakness in replacement decision modeling still remains for most farm situations. Generally, this weakness results from the lack of simultaneous considerations. Several aspects of replacement decisions depend on simultaneous situations.

First, the choice of replacement should be made from among the set of several potentially feasible replacements. Ideally, rather than automatically considering any particular technologically improved machine as the appropriate challenger (as implied by the non-identical challenger criterion), the decision model should consider all feasible challengers. A technologically improved challenger does not necessarily imply that it is the most economically efficient one for each given situation.

Another aspect is that the replacement decision should occur in conjunction with production decisions, because farm machinery decisions and production decisions are mutually dependent. That is, replacement decisions depend on production opportunities, but production opportunities, in turn, depend on machinery replacement decisions. This implies that what seems to be an optimal replacement decision may not be optimal at all; rather, the decision is optimal only for a specific production situation. The need for simultaneous decisions for investments and production is closely related to the need for making such decisions within a constrained framework. This is true since most farm planning situations have internally and externally imposed constraints. Production-replacement decisions should be made within such constraints to maximize the value (wealth) of the firm, or, equivalently, to minimize the opportunity costs associated with employing these resources. A key consideration within the constrained production-replacement decision is that of opportunity cost of breakdowns. When breakdowns reduce the amount of production that would have otherwise occurred, an opportunity cost is incurred. Such a breakdown can be viewed as reducing machinery capacity. Thus, the opportunity cost of such a breakdown is the implicit value of the lost machinery capacity, which can be appropriately determined only when production decisions and production revenues are determined, and total machinery capacity is known.

Another important example in considering replacement decisions subject to constraints is that of a funds constraint, i.e., pure funds (capital) rationing. When this situation occurs, the discount rate should be determined simultaneously with the capital budget. Thus, an optimal decision may be one that delays replacement in order to accept other investment projects that will achieve a higher firm value. That is, the replacement decision should be coordinated with other investment decisions. Several other examples could be cited for which simultaneity is needed, but the foregoing should suffice in pointing out its importance.

One approach to modeling replacement decision within a constrained framework may be a mathematical programming model—more specifically, a multiperiod mixed integer programming model. A mixed integer model could account for simultaneous aspects over the time period modeled, thus allowing for fairly specific evaluation of expectations for these periods. The problem of simultaneity has been dealt with in the context of the standard production-investment problem, assuming no lumpiness of investment projects (Boehlje and White). However, programming methods have not been extended to analyze the problem of replacement of farm machinery. Two problems inherent in replacement decisions probably have precluded its use: (1) replacement of machinery can be analyzed only if integer activities are possible, and (2) an infinite horizon cannot be explicitly modeled with programming methods.

The integer activity requirement can be overcome with the improved mixed integer algorithms that have been developed over the past decade. But the problem of an infinite horizon remains. Thus, solving the infinite horizon problem in such a way that the simultaneous aspects of mathematical programming can be used, at least for time periods near the decision period, would represent a significant breakthrough in decision models for farm machinery and equipment replacement.

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