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THE NBER/NSF MODEL COMPARISON SEMINAR: AN ANALYSIS OF RESULTS

BY GARY FROMM AND LAWRENCE R. KLEIN

This paper reports comparisons of selected error characteristics and policy multipliers of 11 major econometric models of the U.S. economy. These results were generated by a cooperative effort of the leading model builders under the aegis of the Model Comparison Seminar of the NBER/NSF Conference on Econometrics and Mathematical Economics. Comparisons of turning point performance, error decomposition, alternative policy analyses, and other characteristics are now underway and will be presented in subsequent seminar symposia.

INTRODUCTION

For three years, the leading American model builders (macroeconometric) and proprietors have been meeting regularly in a Seminar for the purposes of designing and implementing uniform applications. Basically, the people intimately concerned with model building and maintenance have been dissatisfied with attempts by third party scholars to use the data underlying the models or generated by the models for their own research purposes—often in the form of model testing. Large scale models are such complicated and delicate mechanisms that they require very careful handling by people who fully understand them. While there is some advantage in having the objectivity of third party researchers at work on the problem, there have been so many unfortunate cases of improper use of materials that the Seminar participants have gathered together for their own study of the problem. Through the interaction of group research with group discipline, and the participation of third-party scholars, it is hoped to achieve the requisite objectivity of comparisons without sacrificing model integrity.

The principal interests of model proprietors in the Seminar work have been focused thus far on error and multiplier analysis. In separate studies, such things as frequency response characteristics, sub-sector performance, specialized policy simulations, optimal control simulations, and other applications have been studied. In the Seminar, we have limited our research to analyses that can readily be made across models, for comparative purposes.[1]

The participating models and proprietors are:

- Bureau of Economic Analysis Model (BEA), A. A. Hirsch, Bruce Grimm, and G. V. L. Narasimham
- Brookings Model, G. Fromm, L. R. Klein and G. Schink
- University of Michigan (MQEM) Model, S. Hymans and H. Shapiro
- Data Resources Inc. (DRI) Model, O. Eckstein, E. Green, and A. Sinai
- Fair Model, Princeton and Yale Universities, R. Fair

[1] For descriptions of each of the models, see *International Economic Review*, Vol. 15, No. 2, June 1974, No. 3, October 1974, Vol. 16, No. 1, February 1975. For comparisons of their structures see G. Fromm, "Implications to and from Economic Theory in Models of Complex Systems," *American Journal of Agricultural Economics*, May 1973, pp. 259-71. Also see the bibliography at the end of this article.

- Federal Reserve Bank of St. Louis Model (FRB, St. Louis), L. Andersen and K. Carlson
- M.I.T., Pennsylvania, S.S.R.C. Model (MPS) A. Ando and R. Rasche
- Wharton Model (Mark III and Anticipations Version), M. D. McCarthy, L. R. Klein, F. G. Adams, G. R. Green, and V. Duggal
- Stanford University (H–C Annual) Model. B. Hickman and R. Coen
- Wharton Annual Model, R. S. Preson
- Cornell University (Liu–Hwa Monthly) Model, T. C. Liu and H. C. Hwa

A principal feature of the present approach to model comparison and testing is the attempt to achieve as much uniformity as seems possible in this area of research. Our collection contains large and small models; annual, quarterly, and monthly models; short and long horizon simulations. It would be both undesirable and unusual if all models were nearly alike.

Still, there are strong similarities among many of the models. With the exception of the monetarist approach of the St. Louis model, all the systems follow a Keynesian Framework in which expenditures depend on income and other variables and production or income are functions of expenditures and factor costs. However, there is considerable variation in detailed specifications and the relative importance accorded financial–real sector interactions in expenditure and portfolio decisions. A limited set of characteristics of the models may be found in Table 1.

TABLE 1

SUMMARY OF SELECTED CHARACTERISTICS OF MODELS

Model	Time Frame	Scale[a]	Disaggregation of Production[b]	Endogenous Financial–Real Interaction[c]
BEA	Quarterly	Medium	Limited	Weak
Brookings	Quarterly	Very Large	Medium	Medium
MQEM	Quarterly	Small	Limited	Weak
DRI–74	Quarterly	Very Large	Medium (Recursive)	Medium
Fair	Quarterly	Small	Limited	None
St. Louis	Quarterly	Very Small	None	Strong
MPS	Quarterly	Large	Limited	Strong
Wharton	Quarterly	Large	Medium	Medium
H–C	Annual	Medium	Limited	Weak
Wharton Annual	Annual	Very Large	High	Medium
Liu–Hwa	Monthly	Medium	Limited	Medium

[a] Based on number of equations: very small = 9 or less; small = 10–49; medium = 50–119; large = 120–199; very large = 200 or more.
[b] Based on sector detail: limited = 2–5 sectors; medium = 6–20 sectors; high = 21 or more sectors.
[c] Based on qualitative judgments on pervasiveness of financial variables in real sector equations and real variables in financial sector equations.

Therefore, the challenging issue is to make comparative sense of standardized applications of differentiated models.

The directives to model proprietors were to,

1. simulate dynamically from fixed initial conditions

1961:1–1967:4	quarterly
1961:1–1967:12	monthly
1961 –1967	annually

2. simulate dynamically from fixed initial conditions beyond sample values
3. calculate fiscal multipliers for changes in non-defense spending or personal income taxes with and without accommodating monetary policies.

In the error calculations for individual variables, we asked for mean-squared error, the variance of error, and the bias computed from the formula,

$$\text{MSE} = \frac{1}{T} \sum_{t=1}^{T} (\hat{X}_t - X_t)^2 = \frac{1}{T} \sum_{t=1}^{T} [(\hat{X}_t - X_t) - (\bar{\hat{X}} - \bar{X})]^2 + (\bar{\hat{X}} - \bar{X})^2$$

$$\text{MSE} = \text{VARIANCE} + \text{BIAS}^2$$

$$\hat{X}_t = \text{forecast value of } X$$

$$X_t = \text{observed value of } X$$

$$\bar{\hat{X}} = \text{mean of } \hat{X} = \frac{1}{T} \sum_{t=1}^{T} \hat{X}_t$$

$$\bar{X} = \text{mean of } X = \frac{1}{T} \sum_{t=1}^{T} X_t$$

In the case of the historical sample period, we standardized the calculations to the period 1961–67, if possible. In some instances, the samples terminated prior to 1967, and the exercise was accordingly translated or truncated. For the extrapolations beyond the sample period, the starting date for the simulations was right after the end of the sample and therefore not uniform across all models. Data limitations made the spans of the extrapolation period differ for each model. Generally, we looked for 8 period lengths of solution for each simulation exercise with a period being a month, quarter, or year. Some models are not structured to run dynamically that long and others are cut short for diverse reasons. Therefore, all simulations are not of equal length, either for solution span or period covered for the different solutions.

The list of variables simulated is:

GNP, nominal
GNP, real, 1958 prices
GNP, implicit price deflator, 1958:100
Unemployment rate
Consumer expenditures, nominal
Consumer expenditures, real, 1958 prices
Nonfarm investment, nominal
Nonfarm investment, real, 1958 prices
Nonfarm inventory investment, nominal
Nonfarm inventory investment, real, 1958 prices

Residential construction, nominal
Residential construction, real, 1958 prices
Short-term interest rate
Long-term interest rate
Nonfarm wage rate
Hours worked per week
Corporate profits before tax and IVA
Money supply (M1)
Employee compensation, nominal
Personal income, nominal.

The small models (Fair and St. Louis) had no information for several of these variables. In other cases, some variables were obtainable from the models; some were not.

Some of the variables in this list, which is merely an extract from the larger list of variables in several of the models, are connected through identities. The first three variables satisfy

$$\frac{\text{GNP (nominal)}}{\text{GNP (real)}} = \text{GNP (deflator)}.$$

All three variables are stochastic, but only two *independent* pieces of information about stochastic performance can be inferred from the error statistics associated with them. Tabulations are given for all three, but they should not be independently interpreted. Also, profits come from a national income—national product identity in some models; in others there are direct profit equations, and the statistical discrepancy is the "residual." In the profits case as well, interpretation should be adjusted to the fact that all the components of income may not be independently estimated.

As a study group we set out with high standards for uniformity; but, as in any practical application, we had to allow many compromises. In the end, we achieved about as much uniformity as we could hope to get from 12 teams of independent scholars—especially in economics.

ERROR ANALYSIS

Before we look into the details of the several models' performance, let us make some overall conclusions on the basis of the error analysis.

1. There are substantial disparities among the different variables studied for simulation error. Smooth, slow moving variables are more accurately simulated than are variables with high variance and large period-to-period fluctuation. Among the components of GNP, the largest element by far is consumption, but on an absolute basis, the errors associated with relatively small magnitudes like fixed investment and inventory investment are as large as the consumption errors. Similarly, on the income side, the errors associated with profits are as large as those associated with wages, although the latter variable is much larger. Also the error in simulating the relatively slow moving long-term interest rate is much

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smaller than the error in simulating the short-term interest rate. Some of the general comments about performance are contradicted for some of the models. These general observations refer to predominant model performance.

2. Simulation error grows with the length of the simulation period; the error in one-period simulations is smaller than the error in two-period simulations which, in turn, is smaller than the error in three period simulations, etc.[2] There are a few exceptions that can be explained by some peculiarities or smallness of sample.

3. There are effectively two regimes—within sample and one in extrapolation. Within sample simulations look very favorable. The error statistics for this group of simulations are about as low as we could expect to realize with "noisy" economic data. If error statistics were actually this small in realistic applications, policymakers would have little to worry about, as far as forecasting precision is concerned. Extrapolation error is, on the other hand, nearly two or three times as large as within-sample simulation error. When one does not have the confines of samples that contain only data to which the model has been "fitted," one is subject to a much wider margin or error. Extrapolation error is just on the borderline of being usable for policy application. There is definitely room for improvement although empirical models with this observed degree of imprecision have proved to be useful in decision-making processes.

4. For central variables like real and nominal GNP, the errors in simulating first differences are smaller than the errors in simulating levels. This is indicative of a significant bias component, which gets "differenced out." In most cases, error accumulation is moderate for simulated first differences.

Table 2 gives results for each model for real and nominal GNP (with first differences, as well), both inside and outside sample periods. The main conclusions (1–4) started above can be seen in this and the succeeding tables. Consider the BEA Model for a start. The GNP error grows from approximately $2.0 billion to about $8.0 or $9.0 billion in 6 quarters; but in first difference form the growth is only from about $2.0 to $4.0 billion. In some models the first difference errors are essentially flat. Also, the increase of extrapolation error over within-sample error is noticeable in every case. It is hard to characterize this growth, but it would not be an understatement to say that error doubles or triples in extrapolation.

The number of extrapolation periods is extremely limited; therefore, firm statements about extrapolation periods cannot be made. More experience will have to be gained with this measure. All models have not been able to provide extrapolation simulations, and the one, two, or three observations for the longest

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[2] These remarks should not be confused with those relevant to error of time-cumulated aggregates; thus the error of one-period change in some variables may be less than the error of total change over many periods. where the latter can be calculated as the sum of all intermediate one-period changes. This kind of cumulation over longer periods of time is used in the paper by Leonall C. Andersen and Keith M. Carlson. "St. Louis Model Revisited," *International Economic Review*. (June, 1964). It was also used in some early error calculations for the Wharton Model. See L. R. Klein and M. K. Evans, *The Wharton Quarterly Econometric Forecasting Model* 2nd enlarged edition. (Philadelphia: Economic Research Unit, University of Penna., 1968.)

extrapolations were sometimes very close. giving a misleading implication of improved forecast accuracy with lengthening horizon in the case of the Wharton Model (first differences) and the Liu–Hwa Model.

There are more striking similarities than differences across models. In the short run, GNP prediction errors for one or two quarters (2–6 months for Liu–Hwa) look very much the same, given the error of measurement of GNP itself in most models. Much larger differences show up in extrapolation. although in change form similarity prevails again. The small models, the Fair Model and the St. Louis Model seem to have rather small GNP errors even in extrapolation, but this may have been a very favorable sample period for them. New economic programs (NEP), shortages, and other rough economic events of later years have been hard on model performance for these two systems. It should be stressed that the Fair model changes every quarter in extrapolation. The up-dating of coefficients is something like the system of "constant adjustments" made in ex ante forecasting, which serve to keep most of the other models much closer to actual values in ex ante forecasting than would be suggested by the extrapolation error calculations in Tables 2–5.

The figures in Table 3 show that consumption errors are of the order of magnitude of GNP errors; they have about the same percentage error as GNP error. Among other leading components of GNP. inventory error is quite large, but it does not grow very much with projection horizon or between within-sample and extrapolation periods. It fluctuates pretty much like an unexplained random variable with a zero mean. It defies systematic explanation in tight-fitting equations and appears in model simulation to be like a disturbance of the system as a whole.

Housing investment and business capital formation have similar error patterns and sizes. They do not grow as much as the consumption error over the simulation horizon, but they have much larger percentage errors than does consumption. In dollar magnitude, the three types of investment (I, II, and IH) contribute more towards total GNP error than does consumption.

In extrapolation, the models got caught up in a highly inflationary environment. The price level error grows considerable with the extrapolation horizon, as does the wage rate projection. The RMSE for the wage rate was remarkably stable over the interpolation simulation horizon. For most models, the short interest rate is subject to larger error than is the long rate. There are only isolated exceptions to this rule for certain periods in a few models.

On the income side of the national income account figures there is a similar classification of stable and volatile items giving rise to a dispersion of error magnitude. Profit error is large relative to the level of profits. The Wharton and BEA Models are exceptional in the extrapolation simulation.

Money supply is a stock variable and therefore slower-moving than components of GNP or personal income. Errors seem to grow only moderately and in some models do not show such large amplification between extrapolation and within-sample periods. Some models, however, by-pass the endogenous treatment of money supply.

The analytical purpose behind this detailed investigation of model comparison is to look for insights into ways of improving upon model performance.

Models that are simulated here without any adjustments in extrapolation do worse and residual variables (unemployment, profit) are better projected in systems that build direct estimates of these variables. Actual forecasts would, in fact, make initial corrective adjustments so that errors would be much smaller in such cases than in the unadjusted extrapolations.

It is not intended to try to infer from this cross-model comparison any best model. No model truly dominates on the basis of the ground rules laid down here. Some are better on one variable; others on different variables. The differences between models are often so small that they are not significant when errors of measurement are taken into account. An improvement in something like GNP simulation would have to be persistently more than $1.0 billion in order to be worth considering, and even that sum is clouded by measurement error. The Wharton anticipations version shows persistently lower errors than does Mark III, but the difference is quite small, at most $0.5 billion. This apparent improvement in error performance is suggestive but by no means definitive.

The Liu–Hwa model, after 8 months has about the same GNP error as many of the models after two quarters, over the sample period. In extrapolation, the Liu–Hwa errors are smaller but the sample is too small to be clearly indicative. One of the brightest hopes for substantial improvement, however, many be in the use of monthly data.

The annual models, after 2 years, show errors that are comparable with quarterly models after 8 quarters. The error of nominal GNP continues to grow after 2 years for the Wharton Annual model, but real GNP errors are quite stable for longer simulations. The Hickman–Coen model simulations are in some cases a bit larger than other model errors for one or two years, but this model's errors stabilize rapidly and do not grow in the third and later years of simulation horizon.

The root mean squared error was decomposed as remarked previously, into a variance and a bias component. The bias component is quite large for some of the main aggregates. That is why the first difference transformation produces markedly smaller errors than for levels of GNP. Other main aggregates such as total consumption or wage payments also have large bias components. Volatile magnitudes such as inventory investment do not have large bias components. In the later, hyperinflationary, period of 1973–74 the tendency to underestimate the price level more than price change is also indicative of a large bias component.

Dynamic Policy Multipliers

Examinations of complete-system solution errors within and beyond sample periods over which parameters are estimated, such as those conducted, are useful for indicating how models perform in unconditional prediction. Given actual values of exogenous variables, such tests reveal whether models yield aggregate economic magnitudes sufficiently close to reality so that the results may be used as reliable inputs for subsequent analysis and policy decisions. However, error statistics generally do not reveal much information about responsiveness of models to shifts in policy variables or parameters. That is, they are of limited value for evaluating conditional forecasting.

TABLE 2

SIMULATION OF GNP IN TWELVE MODELS, ROOT-MEAN-SQUARE-ERROR (BILLIONS OF DOLLARS)

		periods ahead —Within Sample								long run
		1	2	3	4	5	6	7	8	
BEA	GNP$	2.39	4.68	6.57	7.81	8.95	9.99			
	Δ	2.39	3.86	4.15	4.21	4.21	4.50			13.65
	real GNP	1.97	3.99	5.68	6.94	8.12	8.94			4.95
	Δ	1.97	3.28	3.68	3.78	3.76	3.98			9.53
										4.46
Brookings	GNP$	4.08	5.38	5.83	5.85	5.78	5.72	5.66	5.80	
	Δ	4.08	3.29	3.37	3.49	3.72	3.66	3.64	3.66	
	real GNP	3.70	4.66	5.01	5.13	5.19	5.25	5.32	5.57	
	Δ	3.70	2.96	3.13	3.26	3.43	3.38	3.37	3.33	
MQEM	GNP$	3.25	4.72	7.11	8.15	9.15	9.91	10.32	10.08	
	Δ	3.25	5.18	7.51	6.48	6.96	7.21	6.94	6.37	6.51
	real GNP	2.97	4.83	7.11	8.27	9.35	10.14	10.55	10.35	4.29
	Δ	2.97	5.24	6.91	6.20	6.67	6.91	6.63	5.91	9.48
										4.09
DRI	GNP$	4.73	5.82	6.02	6.29	5.78	5.87	6.21	6.33	
	Δ	4.73	4.25	4.28	4.33	4.47	4.52	4.40	4.41	5.24
	real GNP	3.97	4.91	4.78	4.60	4.74	5.58	5.96	6.23	4.34
	Δ	3.97	3.58	3.59	3.57	3.89	3.89	3.79	3.88	6.30
										3.67
Fair	GNP$	2.80	4.12	4.49	4.50	4.00				
	Δ	2.80	3.13	3.47	3.50	3.76				
	real GNP	2.81	4.14	4.32	4.22	3.61				
	Δ	2.81	3.81	3.15	3.33	3.51				
St. Louis	GNP$	3.16	4.51	5.52	6.34	6.93	7.55	8.51	9.60	19.41
	Δ	3.16	3.16	3.16	3.16	3.16	3.16	3.16	3.16	3.16
	real GNP	2.88	4.09	4.77	4.98	4.68	4.33	4.43	4.72	4.34
	Δ	2.88	2.90	2.91	2.92	3.02	3.03	2.90	2.92	2.96
MPS	GNP$	2.65	3.73	5.31	5.22	6.77	7.06	7.36	7.12	
	Δ	2.65	3.62	3.95	3.89	3.91	4.01	4.03	4.08	10.60
	real GNP	2.76	3.60	4.11	4.23	5.46	6.00	6.32	6.38	
	Δ	2.76	3.36	3.45	3.52	3.43	3.60	3.45	3.49	8.20
Wharton (Mark III)	GNP$	2.89	4.60	6.14	6.81	7.20	7.29	7.30	7.16	10.01
	Δ	2.89	4.20	4.07	4.15	4.07	4.09	4.12	4.12	4.16
	real GNP	3.21	4.23	4.65	4.64	4.89	5.12	5.35	5.23	11.93
	Δ	3.21	3.57	3.46	3.68	3.70	3.71	3.70	3.75	4.82
Wharton Anticipations	GNP$	2.82	4.11	5.49	6.18	6.53	6.60	6.67	6.60	12.14
	Δ	2.82	4.03	3.89	3.97	3.92	4.01	4.00	3.96	4.80
	real GNP	2.98	3.65	3.89	3.96	4.36	4.76	5.19	5.70	12.83
	Δ	2.98	3.37	3.27	3.50	3.57	3.64	3.59	3.59	5.12
H–C Annual	GNP$	13.54	13.11	12.74	17.65	16.80	16.69			
	Δ	13.54	10.49	11.23	11.42	10.47	11.18			9.57
	real GNP	9.20	12.77	12.31	13.09	15.50	14.16			10.20
	Δ	9.20	12.18	10.80	11.19	11.22	11.13			10.00
										10.50
Wharton Annual	GNP$	4.96	5.74	10.33	14.32	23.57				
	Δ	4.96	4.27	6.39	9.96	12.71				21.76
	real GNP	6.20	7.08	6.37	8.84	10.87				14.06
	Δ	6.20	7.41	8.44	10.52	10.32				7.21
										9.97
Liu–Hwa (monthly)	GNP$	2.53	2.67	2.95	3.43	3.73	3.82	4.29	4.84	11.66
	Δ	2.53	2.46	2.50	2.44	2.62	2.46	2.48	2.47	2.73
	real GNP	2.23	2.54	2.83	3.31	3.39	3.85	3.62	4.27	11.47
	Δ	2.23	2.48	2.47	2.50	2.60	2.52	2.53	2.49	2.72

periods ahead—Extrapolation								
1	2	3	4	5	6	7	8	Notes
4.30	12.47	18.21	26.78	21.13	19.72			Serial correlations of residuals are used in simulation as estimated in the sample; no other adjustments except for the GM strike, 1964. Extrapolation period is 1969:1-1971:2.
4.30	11.31	6.40	4.04	4.09	4.43			
3.51	9.05	11.54	11.02	8.42	6.83			
3.51	7.93	3.38	3.48	5.56	6.81			
6.74	11.36	16.08	20.94	25.69	29.54	33.18	39.77	No adjustments made to model as estimated for within sample simulation. Period is 1959:1-1965:4. Extrapolation period is 1966:1-1970:4. Extrapolation solution adjusted for average error in last 4 sample periods.
6.74	7.61	8.32	7.94	8.44	7.11	7.08	8.01	
5.86	9.64	13.40	16.41	18.78	20.45	21.24	24.22	
5.86	6.30	6.90	6.47	6.84	5.75	5.81	6.34	
6.04	9.88	12.45	16.49					No adjustments made to model as estimated for within sample simulation. Extrapolation period is 1968:1-1970:4. Extrapolation solution adjusted for average error at end of sample period.
6.04	8.07	8.60	9.35					
5.16	8.38	9.95	12.09					
5.16	6.78	7.45	7.92					
								Model re-estimated in 1974. Within sample simulation. 1962:1-1968:4. Extrapolation not possible with this new version.
2.91	4.35	4.52	6.77	9.89				No adjustment made to model as estimated within sample period, 1962:1-1967:4. but observed values of anticipation variables used as exogenous input. Strike quarters (1964:4, 1965:1, 1965:2) deleted. Coefficients re-estimated every period for extrapolation, 1965:4-1969:4.
2.91	3.76	4.32	4.50	4.49				
3.12	4.74	4.71	5.40	6.61				
3.11	3.15	3.23	3.03	2.98				
10.29	14.88	13.83	11.69	11.15	16.11			No adjustments made to model as estimated for within or outside sample simulations. Extrapolation period is 1970:1-1971:4.
10.29	10.89	11.56	12.62	13.13	10.75			
6.81	8.54	8.36	10.25	8.33	10.86			
6.81	7.04	7.62	8.18	7.77	5.33			
								Serial correlations of residuals are used in simulation as estimated in the sample. no other adjustments.
5.71	17.04	25.09	27.25	34.14	40.35	43.99	46.57	Revised to agree with standard case in Adams, Duggal, Int. Econ. Rev., June 1974. No adjustments made to model as estimated for within or outside sample simulations. Extrapolation period 1970:2-1972:4.
5.71	14.05	10.41	7.40	8.26	6.90	4.43	4.95	
5.02	12.93	17.96	19.35	21.24	21.55	19.73	17.03	
5.02	9.67	6.71	5.00	5.14	3.72	3.57	3.45	
7.07	17.66	23.16	23.49	28.60	34.02	36.79	38.01	No adjustments made to model as estimated for within sample simulations. All anticipatory variables endogenously generated, except for lags. Extrapolation period, 1970:2-1972:4.
7.07	12.39	8.34	6.58	7.14	6.35	4.01	4.03	
5.80	13.00	16.14	16.07	16.56	16.21	14.01	10.65	
5.80	8.80	5.49	5.08	4.87	3.56	3.74	3.66	
								No adjustments made to model as estimated for within sample simulations. 1956-66, by year. Exports, farm inventories and farm residences assumed to be exogenous. Inadequate sample for annual extrapolation.
								No adjustments made to model as estimated for within sample simulations. 1961-1967, by years. Inadequate sample for annual extrapolation.
5.94	5.44	5.92	6.28	6.50	5.09	3.98	4.24	No adjustments made to model as estimated for within or outside sample simulations. sample. 1961:01-1967:12, outside sample 1972:01-1972:12.
5.94	5.80	6.88	5.87	5.65	5.75	5.58	5.63	
5.29	6.19	6.19	7.88	7.76	6.66	7.67	7.38	
5.29	5.53	5.34	5.91	6.15	5.22	5.76	5.74	

TABLE 3

SIMULATIONS OF GNP COMPONENTS IN ELEVEN MODELS, ROOT—MEAN—SQUARE—ERROR (BILLIONS OF DOLLARS)

		periods ahead—within sample							
		1	2	3	4	5	6	7	8
BEA	C$	1.91	2.58	3.08	3.76	4.59	5.24		
	real C	1.89	2.47	3.08	3.84	4.54	4.98		
	I$	1.09	1.84	2.47	3.09	3.63	4.08		
	real I	0.96	1.62	2.17	2.68	3.12	3.47		
	II$	2.25	3.05	3.32	3.43	3.59	3.79		
	real II	2.12	2.86	3.10	3.19	3.33	3.53		
	IH$	0.57	0.77	0.97	0.96	0.94	1.01		
	real IH	0.52	0.64	0.81	0.82	0.82	0.87		
Brookings	C$	2.44	2.77	2.97	2.91	2.88	2.88	2.70	2.74
	real C	2.29	2.58	2.70	2.63	2.61	2.67	2.59	2.72
	I$	0.59	0.84	1.01	1.16	1.19	1.22	1.27	1.43
	real I	0.52	0.78	0.93	1.05	1.11	1.16	1.22	1.37
	II$	1.77	2.26	2.29	2.30	2.41	2.45	2.53	2.59
	real II	1.75	2.23	2.26	2.28	2.39	2.43	2.51	2.57
	IH$	0.53	0.90	0.95	0.93	0.91	0.96	1.00	1.01
	real IH	0.44	0.75	0.77	0.74	0.75	0.80	0.84	0.84
MQEM	C$	1.97	3.05	4.17	5.13	5.80	6.30	6.67	6.80
	real C	2.15	3.20	4.41	5.45	6.08	6.52	6.90	6.88
	I$	1.03	1.73	2.34	2.82	3.22	3.45	3.56	3.37
	real I	0.94	1.52	2.04	2.42	2.70	2.84	2.85	2.68
	II$								
	real II	2.33	2.45	2.57	2.47	2.70	2.93	2.88	2.90
	IH$	0.67	0.91	1.22	1.25	1.27	1.39	1.54	1.60
	real IH	0.59	0.76	1.01	1.05	1.09	1.17	1.28	1.32
DRI	C$	3.35	3.82	3.88	4.17	4.06	4.09	4.06	3.89
	real C	3.06	3.42	3.28	3.10	3.20	3.48	3.65	3.69
	I$	1.62	1.81	2.15	2.20	2.28	2.41	2.51	2.65
	real I	1.40	1.53	1.85	1.94	2.03	2.17	2.29	2.41
	II$	2.08	2.42	2.48	2.56	2.59	2.63	2.74	2.87
	real II	1.91	2.21	2.27	2.35	2.38	2.44	2.55	2.67
	IH$	0.68	1.01	1.38	1.65	1.88	2.19	2.37	2.33
	real IH	0.57	0.80	1.11	1.35	1.59	1.87	2.00	1.93
Fair	C$	1.96	2.33	2.79	2.98	3.17			
	real C								
	I$	0.37	1.17	1.14	1.15	1.11			
	real I								
	II$	2.56	3.43	3.35	3.42	2.96			
	real II								
	IH$	0.56	0.88	1.09	1.12	1.15			
	real IH								
MPS	C$	2.12	2.81	3.24	3.36	3.91	4.42	4.82	4.79
	real C	2.01	2.90	3.80	3.88	4.66	5.15	5.50	5.60
	I$	0.99	1.21	1.34	1.47	1.59	1.68	1.74	1.79
	real I	1.02	1.28	1.51	1.68	1.88	1.98	2.01	1.99
	II$	2.01	2.09	2.14	2.19	2.49	2.48	2.56	2.55
	real II	2.11	2.19	2.25	2.32	2.63	2.62	2.72	2.70
	IH$	0.43	0.63	0.83	0.80	0.81	0.91	1.08	1.22
	real IH	0.49	0.71	0.94	0.89	0.90	1.00	1.21	1.38
Wharton Mark III	C$	1.92	2.53	3.30	3.87	4.23	4.52	4.66	4.80
	real C	2.00	2.46	2.76	2.88	3.00	3.16	3.25	3.22
	I$	1.95	2.23	2.42	2.47	2.51	2.52	2.53	2.52
	real I	1.82	1.97	2.05	2.08	2.14	2.22	2.30	2.37
	II$	3.45	3.64	3.74	3.68	3.72	3.72	3.78	3.86
	real II	3.32	3.50	3.58	3.51	3.53	3.53	3.57	3.65
	IH$	1.84	1.89	2.02	2.14	2.17	2.21	2.24	2.27
	real IH	1.57	1.57	1.67	1.79	1.87	1.95	2.01	2.03

long run	periods ahead—extrapolation							
	1	2	3	4	5	6	7	8
9.05	4.61	9.12	13.41	16.20	18.46	20.09		
5.91	4.26	7.21	9.41	10.02	9.81	9.16		
5.55	2.01	2.32	2.59	4.02	4.85	4.57		
4.10	1.61	1.92	1.84	2.53	2.81	2.28		
3.70	2.59	5.41	6.95	6.45	4.60	2.22		
3.44	2.18	4.50	5.84	5.42	3.86	1.96		
1.59	1.74	3.82	4.44	4.09	3.55	3.09		
1.63	1.14	2.49	2.85	2.51	2.08	1.66		
	5.80	8.45	11.56	13.62	15.50	16.95	17.24	18.56
	5.16	7.19	9.56	10.98	11.73	12.35	11.61	12.04
	3.12	3.79	4.44	5.37	6.66	8.14	9.72	11.31
	2.57	3.37	4.19	5.03	5.97	6.85	7.58	7.94
	3.68	4.68	5.01	5.47	5.05	5.03	4.53	4.23
	3.37	4.27	4.49	4.90	4.48	4.46	3.94	3.66
	1.06	2.38	3.20	3.76	3.20	3.60	3.91	4.63
	0.80	1.83	2.48	2.93	2.97	2.82	2.93	3.30
4.22	4.46	6.87	8.18	10.06				
7.00	4.07	6.54	7.60	8.32				
2.94	2.42	3.54	3.92	5.24				
2.31	2.09	2.12	3.11	4.30				
2.39	2.75	2.52	2.78	2.97				
1.53	0.85	1.23	1.46	1.49				
1.34	0.67	0.97	1.16	1.02				
3.51								
3.81								
2.83								
2.60								
2.83								
2.63								
2.29								
1.92								
	3.27	3.97	4.59	6.15	7.97			
	1.63	1.84	2.05	2.54	3.01			
	3.81	4.77	4.92	4.63	4.84			
	1.06	2.03	3.90	3.67	4.44			
7.12								
4.97								
2.68								
3.21								
3.34								
3.16								
1.60								
1.53								
4.97	5.87	10.43	14.82	17.33	21.64	25.40	28.32	30.35
6.51	2.89	5.04	7.21	8.51	9.31	9.25	8.53	6.97
3.52	4.46	3.41	2.18	1.35	3.50	6.21	8.13	8.83
3.26	2.03	1.65	2.13	2.59	3.58	4.79	5.30	5.17
5.08	4.53	8.85	12.15	12.70	14.20	14.85	14.55	14.76
4.73	3.74	7.38	10.08	10.52	11.62	12.01	11.57	11.50
2.40	4.02	3.62	5.32	7.91	8.80	9.17	9.51	9.68
2.13	3.27	3.29	4.39	6.00	6.59	6.88	7.18	7.39

Notation: C = consumer expenditure
I = nonfarm gross investment in plant and equipment
II = nonfarm inventory investment
IH = residential construction

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TABLE 3 (continued)

		periods ahead--within sample							
		1	2	3	4	5	6	7	8
Wharton Anticipations	C$	1.87	2.30	2.92	3.48	3.85	4.19	4.37	4.55
	real C	1.85	2.11	2.26	2.40	2.61	2.88	3.09	3.11
	I$	1.21	1.48	1.96	2.32	2.39	2.33	2.50	2.57
	real i	1.12	1.33	1.74	2.04	2.10	2.07	2.29	2.43
	II$	2.30	3.60	3.80	3.72	3.84	3.91	4.15	4.28
	real II	2.88	3.46	3.65	3.55	3.66	3.72	3.97	4.09
	IH$	0.53	1.03	1.65	2.00	2.21	2.43	2.55	2.60
	real IH	0.49	0.82	1.35	1.67	1.89	2.15	2.33	2.39
H-C Annual	real C	5.11	6.58	6.60	6.98	8.38	7.79		
	real I	3.39	4.57	4.69	4.75	5.03	4.96		
	real II	2.34	3.10	3.11	3.11	3.16	3.18		
	real IH	1.34	1.92	2.00	2.37	2.49	2.44		
Wharton Annual	C$	2.34	2.66	6.67	10.90	17.99			
	real C	2.98	3.47	3.14	4.08	5.54			
	I$	1.39	2.32	2.72	1.94	2.89			
	real I	1.59	2.37	2.66	2.52	3.85			
	II$	1.93	2.59	2.54	2.74	2.97			
	real II	1.80	2.43	2.37	2.66	2.89			
	IH$	1.44	1.20	2.60	3.78	4.16			
	real IH	1.31	1.00	2.09	2.87	2.68			
Liu-Hwa (Monthly)	C$								
	real C	1.67	1.81	1.97	2.26	2.12	2.50	2.21	2.35
	I$								
	real I	2.33	2.32	2.53	2.35	2.29	2.40	2.55	2.13
	II$								
	real II	2.45	2.60	2.83	2.94	2.84	3.01	2.91	3.27
	IH$								
	real IH	0.40	0.51	0.51	0.65	0.66	0.74	0.73	0.88

For this reason there is keen interest in dynamic multipliers resulting from alternative monetary and fiscal policy actions. This is particularly true during the past few years—when there have been massive shifts in government expenditures, taxes, and monetary policy due to the Vietnamese war and the battle to contain inflation. There has been much debate about desired spending and monetary expansion rates. Some differences in prescriptions have come from differences in goals; others have arisen due to controversy about magnitudes of multipliers.

Discrepancies between multiplier values across models can be attributed to a number of factors, the relative importance of which is yet to be determined. They are listed here in no particular order. First, lack of standardization of variables treated exogenously probably is a major contributor to discrepancies. For example, a model that has an exogenous foreign sector normally will have, other things being equal, higher GNP-foreign sector government expenditure multipliers than a model that makes imports and exports functions of domestic and foreign incomes and prices. Similarly, expenditure multipliers are downward biased when state and local government outlays are taken to be exogenous. Many other such examples could be given, including those from the financial–monetary sector.

Another cause for discrepancies are differences in periods over which multipliers are calculated. Given non-linear relationships between real output, (or capacity utilization or unemployment rates) and prices, increments in nominal or constant dollar fiscal stimulus will reveal different multiplier responses at various

long run	periods ahead—extrapolation							
	1	2	3	4	5	6	7	8
5.59	5.38	9.33	13.16	15.06	18.64	21.66	23.68	24.68
6.46	2.46	4.16	5.87	6.71	6.75	6.04	4.72	2.55
4.52	1.93	2.11	2.68	3.25	2.63	3.62	6.01	6.36
3.88	2.79	2.80	2.03	1.02	0.96	2.59	3.73	3.49
6.50	6.20	6.01	9.82	10.57	12.26	11.93	10.66	10.31
6.07	5.09	5.05	8.18	8.80	10.09	9.74	8.56	8.08
3.57	2.72	1.12	2.85	4.57	5.98	6.50	7.07	7.25
3.14	1.80	1.66	2.87	3.94	4.89	5.30	5.71	5.87
4.93								
4.00								
2.93								
2.70								
16.08								
4.52								
2.50								
3.20								
2.89								
2.76								
4.14								
2.47								
4.34	3.60	3.25	3.36	2.86	3.28	4.38	3.31	3.72
3.10	1.27	1.31	1.39	1.40	1.43	1.40	1.21	1.20
4.60	5.60	5.79	6.53	6.92	6.67	6.64	7.23	7.20
2.34	0.53	0.78	0.90	1.14	1.16	1.19	1.01	1.29

stages of the economy's growth cycle. At high utilization rates and near the peak of the cycle (when potential output gaps are small), real multipliers will be lower than when capacity is less fully utilized. Timing patterns also are affected; real responses are faster and price increases slower at low rather than high utilization rates.

Aside from the degree of exogeneity of a model and the initial conditions at the time exogenous shifts are introduced, the magnitudes of such changes may influence the sizes of multipliers. With a completely linear model, multipliers depend only on the lag structure and parameters which attach to endogenous and exogenous variables.

For example, in matrix notation, a linear system might take the form:

$$A + BY_t + \sum_{j=1}^{p} B_j Y_{t-j} + CZ_t = 0$$

where,

Y = endogenous variables, y_i, $i = 1 \ldots n$

Y_{t-j} = lagged endogenous variables, $y_{i,t-j}$, with lags $j = 1 \ldots p$
Z = exogenous variables, z_k, $k = 1 \ldots m$

A, B, B_j, C = matrices of constant coefficients of orders $1 \times n$, $n \times n$, $n \times p$, and $n \times m$, respectively

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TABLE 4

SIMULATIONS OF MARKET VARIABLES IN TWELVE MODELS, ROOT—MEAN—SQUARE—ERROR

		periods ahead—within sample								long run
		1	2	3	4	5	6	7	8	
BEA	P	0.21	0.33	0.41	0.50	0.55	0.59			1.37
	Un	0.22	0.25	0.31	0.32	0.32	0.34			0.45
	rs	0.19	0.22	0.24	0.23	0.20	0.20			0.21
	rL	0.13	0.17	0.21	0.20	0.21	0.23			0.29
	w	0.03	0.04	0.05	0.05	0.05	0.06			0.11
	h	0.14	0.18	0.21	0.22	0.24	0.24			0.25
Brookings	P	0.20	0.32	0.39	0.44	0.46	0.46	0.48	0.47	
	Un	0.25	0.34	0.38	0.38	0.39	0.40	0.40	0.41	
	rs	0.22	0.22	0.25	0.25	0.25	0.26	0.26	0.26	
	rL	0.10	0.10	0.13	0.16	0.17	0.17	0.18	0.20	
	w	0.01	0.01	0.02	0.02	0.01	0.01	0.01	0.01	
	h	0.11	0.11	0.11	0.11	0.12	0.12	0.12	0.12	
MQEM	P	0.16	0.26	0.36	0.47	0.57	0.65	0.73	0.81	1.13
	Un	0.16	0.32	0.44	0.51	0.53	0.54	0.56	0.54	0.37
	rs	0.20	0.26	0.31	0.37	0.42	0.47	0.56	0.51	0.43
	rL	0.11	0.17	0.21	0.23	0.25	0.27	0.29	0.31	0.34
	w	0.27	0.29	0.43	0.52	0.63	0.73	0.81	0.89	0.43
	h									
DRI	P	0.21	0.34	0.44	0.53	0.57	0.57	0.54	0.57	0.77
	Un	0.20	0.24	0.26	0.27	0.27	0.31	0.33	0.34	0.33
	rs	0.39	0.50	0.52	0.57	0.61	0.61	0.58	0.57	0.59
	rL	0.15	0.22	0.25	0.25	0.23	0.21	0.18	0.18	0.25
	w	0.30	0.40	0.50	0.60	0.60	0.50	0.40	0.40	0.60
	h									
Fair	P	0.18	0.29	0.35	0.37	0.37				
	Un	0.21	0.33	0.43	0.51	0.52				
St. Louis	P	0.19	0.31	0.38	0.45	0.55	0.68	0.82	0.98	3.10
	Un	0.23	0.27	0.32	0.35	0.34	0.24	0.17	0.19	0.33
	rs	0.34	0.36	0.34	0.33	0.36	0.40	0.44	0.50	0.55
	rL	0.16	0.17	0.16	0.16	0.16	0.16	0.19	0.22	0.41
MPS	P	0.21	0.26	0.37	0.47	0.60	0.64	0.64	0.66	1.61
	Un	0.35	0.44	0.48	0.49	0.53	0.60	0.67	0.69	0.44
	rs	0.15	0.17	0.15	0.14	0.16	0.16	0.17	0.15	0.23
	rL	0.11	0.13	0.13	0.15	0.14	0.14	0.12	0.10	0.15
	w	1.06	1.23	2.03	2.31	2.72	2.84	2.79	2.66	4.56
	h	0.11	0.13	0.14	0.15	0.16	0.17	0.16	0.17	0.19
Wharton Mark III	P	0.28	0.31	0.37	0.49	0.62	0.71	0.81	0.92	1.17
	Un	0.21	0.39	0.52	0.57	0.61	0.63	0.65	0.66	0.91
	rs	0.27	0.34	0.36	0.38	0.40	0.41	0.41	0.42	0.42
	rL	0.14	0.22	0.24	0.24	0.28	0.31	0.33	0.35	0.37
	w	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.02
Wharton Anticipations	P	0.29	0.31	0.36	0.48	0.60	0.69	0.78	0.88	1.05
	Un	0.21	0.38	0.50	0.56	0.60	0.63	0.65	0.66	1.05
	rs	0.25	0.33	0.36	0.38	0.40	0.41	0.41	0.41	0.41
	rL	0.14	0.22	0.24	0.25	0.28	0.31	0.34	0.36	0.35
	w	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.02
H-C Annual	P	1.20	1.95	2.36	2.89	3.36	3.39			
	Un	0.88	1.15	1.07	0.92	1.19	1.17			2.32
	rs	0.45	0.42	0.47	0.50	0.56	0.49			1.06
	rL	0.21	0.27	0.33	0.30	0.43	0.44			0.30
	w	0.04	0.06	0.08	0.09	0.11	0.12			0.24
	h	0.01	0.02	0.02	0.02	0.02	0.02			0.05
										0.01

periods ahead—extrapolation								Notes
1	2	3	4	5	6	7	8	
0.25	0.34	0.57	0.98	1.65	2.44			P is measured on a base of 100, Un in percent,
0.35	0.86	1.23	1.39	1.40	1.26			rs in percent, rL in percent, w in $ thousands/
0.87	1.07	1.16	1.19	1.19	1.14			year, h in hours (40 hours standard)
0.42	0.52	0.60	0.68	0.75	0.79			Notation: P = GNP deflator
0.04	0.06	0.10	0.13	0.15	0.17			Un = Unemployment rate
0.15	0.18	0.19	0.22	0.27	0.32			rs = Commercial paper rate
								rL = Bond yield
0.42	0.65	0.80	0.91	0.96	1.08	1.30	1.70	w = Wage rate
0.26	0.51	9.81	1.02	1.16	1.26	1.14	1.21	h = Hours worked
0.43	0.48	0.50	0.52	0.54	0.60	0.70	0.77	rs is the treasury bill rate in percent; rL the
0.28	0.40	0.45	0.51	0.54	0.61	0.69	0.76	treasury bond rate in percent, w in $/hr., and
0.02	0.02	0.03	0.04	0.05	0.07	0.08	0.10	h in hours.
0.17	0.22	0.23	0.30	0.33	0.33	0.31	0.30	
0.39	0.61	0.75	0.86					
0.23	0.52	0.65	0.79					w is an index of private nonfarm
0.54	0.53	0.59	0.66					compensation/man hour, 1967:100.
0.23	0.32	0.38	0.43					
0.44	0.58	0.71	0.83					
0.21	0.39	0.57	0.76	0.97				
0.36	0.68	0.90	1.08	1.23				
0.48	0.81	0.90	0.76	0.71	0.70			
0.22	0.23	0.29	0.36	0.34	0.30			
1.15	1.30	1.36	1.35	1.52	1.41			
0.50	0.52	0.45	0.32	0.34	0.37			
								w is measured in cents
								h is measured in hours/person/week
0.72	0.96	1.04	0.80	1.02	1.41	1.99	2.73	w is measured in $ thousands/yr for the
0.52	1.11	1.63	2.00	2.42	2.83	3.10	3.29	manufacturing sector; h is measured in
0.77	1.10	1.33	1.51	1.57	1.70	1.86	1.93	hours (40 hour standard) for the
0.23	0.35	0.39	0.49	0.56	0.64	0.75	0.89	manufacturing sector.
0.07	0.09	0.11	0.11	0.16	0.22	0.27	0.32	
0.72	0.95	1.01	0.81	1.10	1.54	2.11	2.82	
0.52	1.08	1.53	1.84	2.18	2.51	2.72	2.83	
0.78	1.12	1.36	1.52	1.57	1.69	1.84	1.90	
0.23	0.35	0.40	0.50	0.57	0.64	0.75	0.88	
0.07	0.09	0.11	0.11	0.16	0.22	0.26	0.30	
								w is measured in $/hr.
								h is measured in 1,000 hrs/person/yr.

TABLE 4 (continued)

		periods ahead—within sample								long run
		1	2	3	4	5	6	7	8	
Wharton	P	0.70	0.64	1.52	2.74	3.89				2.99
Annual	Un	0.63	0.97	1.20	1.60	1.92				1.33
	rs	0.14	0.14	0.24	0.38	0.46				0.34
	rL	0.19	0.32	0.42	0.56	0.63				0.54
	w	0.04	0.03	0.05	0.10	0.14				0.11
Liu–Hwa	P	0.29	0.31	0.36	0.36	0.42	0.47	0.46	0.41	2.36
Monthly	Un	0.26	0.35	0.43	0.50	0.59	0.62	0.69	0.69	0.75
	rs	0.12	0.15	0.18	0.19	0.18	0.25	0.25	0.22	0.25
	rL	0.05	0.08	0.12	0.14	0.15	0.15	0.18	0.16	0.55
	w	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.02	0.05
	h	0.21	0.21	0.22	0.22	0.21	0.22	0.23	0.24	0.44

The solution of the system is given by:

$$Y_t = -B^{-1}A - B^{-1} \sum_{j=1}^{P} B_j Y_{t-j} - B^{-1}CZ_t$$

Impact multipliers, first period changes in an element of the column vector Y_t with respect to changes in an element of A or Z_t are, respectively,

$$\frac{dy_{it}}{da_i} = b^{ij}$$

and

$$\frac{dy_{it}}{dz_{kt}} = \sum_{q=1}^{n} b^{iq}C_{qk}$$

where b^{ij} and b^{iq} are the (i,j) and (i,q) elements of B^{-1}. Multi-period impacts of Z_t would depend on summed products of elements of B^{-1}, B_j and C.

However, most econometric models are, to a significant degree, non-linear in variables. For instance, nominal values often are derived by inflating real quantities. Therefore, unless linear approximations are used (which may lead to substantially biased multiplier estimates), numerical methods must be employed to obtain solutions of models and their multipliers.

Finally, the causal nature of models greatly affects multipliers. Reduced from systems generally have vastly different multiplier properties than models which exhibit more complete structural linkages.

The caveats apply to the results shown in Tables 5–8, which report dynamic multipliers over a ten-year interval. Solution periods range from starting near the onset of recessions to the middle of booms. Amounts of exogenous change in non-defense government expenditures vary from $5 billion constant (1958) dollars to $1 billion in current dollars. Tax and monetary shifts are similarly disparate. Thus, lack of standardization hampers intermodel comparisons.

Despite such differences, however, with the exception of the FRB St. Louis Model, there is a fair amount of agreement among quarterly models. Nominal

1	2	3	4	5	6	7	8	Notes
								w is measured in $/hr in the manufacturing sector.
0.54	0.69	0.61	0.73	0.76	0.75	0.82	0.82	
0.30	0.54	0.93	1.00	1.04	1.41	1.17	1.06	
0.27	0.35	0.39	0.39	0.40	0.42	0.42	0.43	w is measured in $ thousands/yr and h in
0.07	0.11	0.13	0.18	0.19	0.22	0.24	0.22	hours (40 hour standard)
0.03	0.03	0.04	0.04	0.03	0.04	0.04	0.04	
0.20	0.20	0.15	0.19	0.12	0.13	0.12	0.13	

GNP-nondefense government expenditure multipliers are around two after four quarters and then generally continue to rise, with slight fluctuations, thereafter. Results for the annual models and Liu–Hwa monthly model are consistent with this pattern.

Much of the sustained multiplier increase is due to pressures on prices, which appear to accelerate as simulation periods are lengthened. Prices continue to rise despite declines in rates of increase or falls in absolute levels of real output, drops in capacity utilization, and higher unemployment rates. Few of the models contain price anticipation variables and, where they are included, it is doubtful that they are strong enough to account for this phenomenon.

These and related effects are mirrored in the results for constant dollar multipliers (Δreal GNP/Δreal expenditures). Conventional textbook expositions generally depict real expenditure multipliers approaching positive asymptotes. But, most of the models here show such multipliers reaching a peak in two or three years and then declining (see Table 5). Multipliers for the MPS model decline to negative values quite early, but not as early as the St. Louis Model. At the end of five to ten years, some of the models show that continued sustained fiscal stimulus has ever-increasing perverse effects.

For models in which the stimulus is introduced in nominal terms, the decline in real expenditure multipliers, in part, is attributable to decreasing the amount of real input. That is, the expenditure increase declines in real terms as prices rise. A concomitant effect of the rise in prices is to lower real values of all other exogenous nominal dollar expenditures or transfers. Moreover, in models where government transfers such as current dollar social security payments are endogenous, insufficient allowance probably is made for Congressional actions to raise benefit levels as inflation erodes real living standards. Thus, when nominal exogenous stimuli are used in solutions of models, unless upward adjustments in outlays are made for endogenous increases in prices, real stimulus falls and multipliers, as conventionally calculated, will tend to decline after a period of time.

There are also other effects at work. These may be illustrated by the Brookings and Wharton Annual model simulations, wherein government expen-

TABLE 5

SIMULATION OF FACTOR INCOME PAYMENTS AND MONEY SUPPLY IN TEN MODELS. ROOT—MEAN—SQUARE—ERROR (BILLIONS OF DOLLARS)

		periods ahead—within sample								long run
		1	2	3	4	5	6	7	8	
BEA	PI	1.96	3.23	4.14	4.80	5.52	6.31			11.41
	W	1.79	2.88	3.55	3.66	3.66	4.09			7.77
	PR	2.44	3.04	4.06	4.82	5.39	5.84			6.86
	MI	0.81	1.38	1.71	1.83	1.94	2.15			3.29
Brookings	PI	1.72	2.39	2.61	2.65	2.43	2.45	2.33	2.50	
	W	1.79	2.63	2.98	3.09	2.95	3.03	3.00	3.05	
	PR	2.34	3.13	3.38	3.55	3.70	3.79	3.82	3.76	
	MI	0.61	1.02	1.29	1.49	1.64	1.74	1.76	1.80	
MQEM	PI	1.86	3.11	4.85	6.07	6.90	7.56	8.28	8.33	4.57
	PR	1.99	2.34	3.09	3.44	4.00	4.36	4.72	5.06	6.01
DRI	PI	4.01	4.64	5.14	5.56	5.03	4.83	4.56	4.40	4.62
	PR	2.17	2.55	2.77	3.12	3.44	3.90	4.11	4.24	4.08
	MI	1.12	1.22	1.33	1.41	1.47	1.49	1.57	1.60	1.30
MPS	PI	2.00	2.66	4.30	4.73	5.72	6.34	6.62	6.51	7.94
	W	1.89	2.47	3.73	4.20	5.18	5.77	5.91	5.77	6.57
	PR	2.63	3.07	3.31	3.03	3.26	3.00	3.01	2.98	3.77
	MI	0.62	1.00	1.34	1.53	1.63	1.66	1.68	1.69	1.64
Wharton Mark III	PI	1.66	2.77	3.88	4.40	4.51	4.63	4.60	4.85	6.74
	W	1.37	2.56	3.66	4.17	4.30	4.37	4.21	4.31	6.92
	PR	2.80	3.05	3.07	3.08	3.47	3.64	3.82	4.00	4.34
	MI	0.77	1.17	1.45	1.64	1.74	1.85	1.96	2.06	2.58
Wharton Anticipations	PI	1.59	2.61	3.65	4.14	4.21	4.30	4.23	4.45	8.39
	W	1.37	2.48	3.51	3.98	4.07	4.12	3.89	3.95	8.53
	PR	2.84	2.98	2.92	2.93	3.31	3.51	3.77	3.95	4.39
	MI	0.78	1.18	1.44	1.62	1.74	1.87	2.00	2.11	2.29
H–C Annual	PI	8.61	8.89	8.63	10.85	11.26	11.33			4.83
	W	7.64	7.81	6.45	8.67	9.57	9.30			6.05
	PR	5.04	4.45	4.78	6.49	5.29	5.35			6.87
	MI	3.16	3.36	3.72	4.16	4.05	4.30			2.44
Wharton Annual	PI	2.47	4.49	9.98	14.11	21.04				18.86
	W	2.46	4.74	9.93	14.13	19.67				17.91
	PR	2.92	3.11	3.44	3.89	3.95				3.37
	MI	2.08	2.76	2.63	2.75	3.37				1.23
Liu–Hwa Monthly	PI	3.86	4.11	4.31	4.32	4.58	4.53	4.27	5.24	7.41
	W	2.02	2.18	2.42	2.65	2.61	2.75	2.91	2.97	8.08
	PR	1.62	2.53	2.91	3.41	3.21	3.88	3.72	3.94	5.72
	MI	0.74	0.91	0.97	1.28	1.08	1.54	1.48	1.84	2.94

diture inputs are stated in real terms. Here, too, real expenditure multipliers rise to a peak and then begin to fall. While there probably are some multiplier feedbacks on some exogenous expenditures and transfers from prices, the primary cause of the fall-offs in multipliers after two years in these models most likely is due to capacity constraints and reductions in rates of increase of business fixed and inventory investment. Only in the Fair model does the real multiplier fail to drop. This model has only a short solution horizon and some nonresponsive anticipatory variables. Although the real multiplier drops in the H–C Annual model, this becomes apparent only after 14 years.

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			periods ahead—extrapolation					
1	2	3	4	5	6	7	8	Notes
2.68	9.44	15.73	19.33	21.81	22.93			Notation: PI = Personal Income
4.00	12.32	19.59	23.60	25.90	26.65			W = Employee compensation
3.17	5.48	4.55	4.17	3.57	4.46			PR = Corporate Profits before tax
2.36	4.59	6.66	8.24	9.47	10.44			MI = Currency and demand deposits
3.43	6.20	9.63	12.67	16.04	19.15	21.50	25.38	Demand deposits without the addition of
3.22	6.24	9.63	12.91	15.86	18.97	20.88	24.36	currency are used for MI. Currency is
3.82	5.70	7.82	10.26	12.68	14.17	15.81	17.50	separately tabulated.
1.23	2.49	3.80	5.31	7.13	9.28	11.59	14.16	
4.87	7.22	7.86	10.92					
4.06	5.20	5.24	5.22					
13.13	22.30	29.54	33.29	41.49	49.16	55.61	61.56	
10.08	18.39	25.14	28.61	36.41	43.84	50.02	55.91	
6.91	9.28	9.00	7.95	7.63	6.82	6.72	7.46	
2.32	2.01	2.28	3.28	4.87	5.86	7.20	8.64	
13.40	22.14	28.60	31.57	38.93	45.76	51.16	55.82	
10.12	18.09	24.21	26.96	34.00	40.55	45.67	50.27	
7.39	9.53	8.25	6.77	6.26	5.46	6.13	7.88	
2.52	2.21	2.14	2.97	4.50	5.32	6.41	7.67	
7.83	8.75	11.96	10.00	7.24	14.67	9.61	10.22	
3.77	3.79	4.46	3.61	3.63	3.56	3.21	3.54	
5.84	5.54	4.25	5.44	5.80	4.29	5.49	5.07	
2.21	1.90	2.83	1.92	2.71	2.12	2.33	2.55	

Economic theory also suggests that declines in real multipliers could be caused by financial stringency if monetary authorities do not curtail rising interest rates by expanding bank reserves so as to support ever higher financial transactions and investment demands. As can be seen by comparing results shown in Tables 6 and 7, an accommodating monetary policy of constant interest rates tends to raise long-term expenditure multipliers but does not alter the basic pattern of movement to a peak and then decline.

Multipliers for decreases in personal taxes are shown in Table 8. In the first few years, nominal GNP-tax multipliers rise more slowly than nominal GNP-

expenditure multipliers, but surpass the latter in the BEA and MQEM models after seven years. Real GNP-tax multipliers in all the models peak after two to three years, but are significantly lower (by 0.3 to 0.9) than real GNP-expenditure multipliers. This is not unexpected. The differences between expenditure and tax multipliers need not necessarily equal unity. They do so only in simplistic balanced-budget models that exclude a multiplicity of leakages and income-expenditure feedbacks. (For a proof, see G. Fromm and P. Taubman; for examples of policy simulations with balanced budget strategies, see V. Duggal.)[3]

Aside from first-round effects in multiplier calculations, government expenditure changes (of a constant average mix) probably are more powerful than personal income tax changes over a period of a few years because shifts in government outlays tend to be more intensive in generating private investment than comparable amounts of personal income tax increase or reduction. This advantage persists in the Brookings, DRI, Wharton Anticipation, H-C Annual, and Wharton Annual models but disappears in the BEA, MQEM, MPS, and Wharton Standard models. In fact, real GNP-real tax multipliers are higher (for some models, less negative) for the latter models after from three to seven years. This occurs because non-linear impacts of capacity constraints and price effects are different in these than in the former models.

There are even more striking disparities between models in multiplier responses to shifts in monetary policy. With the exception of the FRB St. Louis Model (in which demand deposits and currency are augmented), an exogenous increase of either $0.5 billion or $1.0 billion in unborrowed reserves is introduced in each model. As can be seen in Table 9, this has virtually no short-run or long-run effect on nominal GNP in the BEA model and an ever-increasing (at least over five years) in the MPS and Wharton Annual models.[4] In the DRI, Wharton standard, and H-C Annual models, the nominal GNP-nominal money multiplier peaks after two to three years and then begins to decline in a cycling path.

Real GNP-nominal money multipliers reflect these same patterns over the first few years, but because of rises in prices, multipliers are lower thereafter. Prices apparently rise fastest in the DRI and FRB St. Louis models, since real GNP-nominal money multipliers become negative after four or five years.

PROSPECTS

This summary report marks the end of a second phase of comprehensive analysis of American econometric models.[5] In the first phase a number of U.S.

[3] G. Fromm and P. Taubman. *Policy Simulations with an Econometric Model* (Amsterdam: North-Holland. 1967).

V. Duggal. "Fiscal Policy and Economic Stabilization." *The Brookings Model: Perspective and Recent Developments.* eds. G. Fromm and L. R. Klein (Amsterdam: North-Holland, 1975).

[4] In more recent (updated and revised) versions of the BEA model, money multipliers are significantly stronger.

[5] It is encouraging to learn that the format of our research discussions and project planning are attractive to model builders in other environments. Japanese model proprietors have held a similar conference: Canadian model builders have attended our seminars as guests: and European model builders have considered holding similar comparative meetings. All the participants in the U.S. seminars have felt that much was gained in the information exchanges in these model comparison seminars.

models were examined in detail for cyclical content, (1969) followed soon after by a similar examination of price determination, 1970.[6] These two investigations—both conferences—looked carefully into cross-model comparisons for specific characteristics. The Seminar on Model Comparisons in a series of papers in *International Economic Review* (June, 1974, October, 1974, February, 1975) and in the present paper looked at a wider variety of model properties in a cross-section analysis. What remains for future research in this area?

New topics for discussion have enlarged our agenda as follows:

> Turning point analysis
> Ex ante error analysis
> Error decomposition
> Comparative policy simulation
> Added information through model combination

Models perform less well in the neighborhood of critical turning points than along sustained monotonic paths of expansion or decline. Much is to be learned about model performance in seeing whether direction and magnitude of change at peaks and troughs is correctly simulated. A step in turning point research has already been taken by Adams and Duggal and reported in their analysis of the Wharton Model (anticipation version) contained in the *IER* symposium.[7] There was prior consideration of this matter in the 1969 Conference. Now that the U.S. economy is in the midst of a major recession, we are having an unusual opportunity to examine extreme turning points in great detail. When the cycle has completed its course, it will be a good time to look back and see what has been learned about turning point performance.

The Seminar has concentrated attention primarily on sample period and expost extrapolation error. A number of individual model proprietors have been making their own examinations of ex ante forecast error. Additionally, some outsiders have tried to make independent assessments of forecast error. As these parties often lack the familiarity with the models that only the proprietors can acquire in daily use, some of these error calculations encounter the pitfalls pointed out in a general paper at the beginning of the symposium.[8] Accordingly, the participants in the Seminar on Model Comparisons are designing an internal study for the analysis of ex ante forecast errors.

Errors are studied only partly for their own sake; they are most useful as a guide to model improvement by showing where deficiencies occur. To be most helpful in this respect, errors should be decomposed into the parts due to (1) coefficient uncertainty, (2) residual disturbances, (3) errors in forecast input values (initial conditions and exogeneous variables) (4) misspecifications of the

[6] *Economic Models of Cyclical behavior*, ed. B. G. Hickman (N.Y., National Bureau of Economic Research, 1972); *Econometrics of Price Determination*, ed. Otto Eckstein (Washington, D.C.: Federal Reserve Board, 1972).

[7] F. G. Adams and Vijaya Duggal, "Anticipations Variables in an Econometric Model: Performance of the Anticipations Version of Wharton Mark III," *International Economic Review*, 15 (June, 1974).

[8] E. P. Howrey, L. R. Klein, and M. D. McCarthy, "Notes on Testing the Predictive Performance of Econometric Models," *International Economic Review*, 15 (June, 1974), 366–83.

TABLE 6
DYNAMIC MULTIPLIERS: GROSS NATIONAL PRODUCT/GOVERNMENT NONDEFENSE EXPENDITURES

Quarters of change	BEA	Brook-ings	MQEM	DRI-74	Fair	FRB St. Louis	MPS	Wharton Mark III Standard	Wharton Mark III Anticipations	H-C Annual	Wharton Annual	Liu-Hwa Monthly
						ΔGNP/ΔG (current dollars)						
1	1.1	1.8	1.4	1.5	1.1	0.6	1.2	1.2	1.1			0.7
2	1.5	2.3	1.6	1.9	1.4	1.1	1.7	1.5	1.4			1.1
3	1.7	2.7	1.7	2.2	1.5	1.2	2.1	1.7	1.6			1.4
4	1.9	2.8	1.8	2.3	1.6	0.7	2.5	1.8	1.7	2.7		1.6
5	1.9	2.8	1.8	2.3	1.7	0.1	2.8	1.9	1.7		1.5	1.7
6	1.9	2.9	1.8	2.4		0.1	3.0	2.0	1.8			1.8
7	1.9	2.9	1.8	2.5		0.1	3.1	2.1	1.8			1.9
8	1.9	2.9	1.8	2.5		0.1	3.0	2.2	1.8	2.6	2.3	2.0
12	1.9	3.0	2.0	2.5		0.1	2.2	2.4		2.9	2.7	2.4
16	2.1	3.1	2.6	2.3		0.1	1.8	2.6		3.5	3.3	
20	2.5	3.0	3.2	2.5		0.1		2.5		3.7	3.9	
24	2.9	3.1	3.8	2.9				2.3		3.6		
28	3.1	3.3	4.0	3.4				2.3		3.6		
32	3.1	3.4	4.1	3.5				2.3		3.9		
36	3.2	3.7	4.3	3.6				2.8		3.9		
40	3.3	3.8		4.0				3.9		3.9		
						ΔGNP58/ΔG58 (constant dollars)						
1	1.1	1.8	1.4	1.3	1.1	0.5	1.2	1.3	1.1			
2	1.7	2.4	1.6	1.7	1.4	1.0	1.5	1.6	1.4			
3	2.1	2.7	1.7	2.0	1.5	1.0	1.9	1.8	1.6			
4	2.2	2.8	1.7	2.1	1.5	0.5	2.2	2.0	1.7	1.6		
5	2.3	2.8	1.6	2.1	1.6	-0.1	2.3	2.1	1.7		1.7	
6	2.3	2.8	1.5	2.1		-0.2	2.4	2.2	1.8			
7	2.3	2.7	1.4	2.2		-0.2	2.4	2.3	1.8			
8	2.2	2.4	1.4	2.2		-0.2	2.2	2.4	1.8	1.1	1.9	
12	1.8	2.0	1.0	2.0		-0.2	0.7	2.6	1.7	1.4	1.5	
16	1.6	1.5	1.0	1.7		-0.2	-0.5	2.4	1.6	1.8	1.0	
20	1.3	1.2	1.1	1.7				1.9		1.9	0.5	
24	0.03	1.1	1.1	1.9				1.2		1.8		
28	-3.8	1.0	0.7	2.3				0.3		2.0		
32	-7.4	1.0	0.2	2.0				-0.8		2.3		
36	-11.2	0.9	0.1	1.3				-1.9		2.3		
40	-23.2		-0.0	0.7				-3.0		2.4		

TABLE 7

DYNAMIC MULTIPLIERS: GROSS NATIONAL PRODUCT/GOVERNMENT EXPENDITURES WITH COMPENSATING MONETARY POLICY

Quarters of change	BEA	Brookings	MQEM	DRI-74	Fair	FRB St. Louis	MPS	Wharton Mark III		H-C Annual	Wharton Annual	Liu-Hwa Monthly
								Standard	Anticipations			
ΔGNP/ΔG (current dollars)												
1	0.9			1.4			1.2		1.3			
2	1.5			1.8			1.7		1.6			
3	2.0			2.1			2.2		1.9			
4	2.3			2.3			2.6		2.0	2.9	1.6	
5	2.4			2.3			3.0		2.2			
6	2.5			2.5			3.4		2.4			
7	2.5			2.7			3.7		2.5			
8	2.5			2.7			3.9		2.6	3.1	2.5	
12	2.3			2.8			4.4		3.0	3.2	3.2	
16	2.5			2.7			6.0		3.0	3.5	4.1	
20	3.0			2.7					2.8	3.9	5.0	
24	3.5			2.7					2.6	3.8		
28	3.9			2.9					2.4	3.8		
32	4.0			3.3					2.5	3.9		
36	4.1			3.8					3.0	4.0		
40	4.2			4.0					4.2	3.9		
ΔGNP58/ΔG58 (constant dollars)												
1	0.9			1.2			1.1	1.2	1.4			
2	1.5			1.7			1.6	1.6	1.8			
3	1.9			1.9			2.0	1.8	2.1			
4	2.2			2.0			2.3	2.0	2.3	1.7	1.7	
5	2.3			2.1			2.5	2.1	2.5			
6	2.4			2.2			2.7	2.2	2.6			
7	2.3			2.3			2.9	2.3	2.8			
8	2.2			2.4			2.9	2.3	2.9	1.3	2.1	
12	1.7			2.3			2.4	2.2	3.2	1.5	2.0	
16	1.4			2.1			2.4	1.9	3.0	1.8	1.8	
20	1.3			1.9					2.4	2.0	1.5	
24	0.6			1.7					1.6	2.0		
28	-1.1			1.5					0.5	2.1		
32	-2.7			1.4					-0.6	2.2		
36	-5.7			1.5					-1.7	2.4		
40	-9.4			1.1					-2.6	2.5		

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TABLE 8
Dynamic Multipliers: Gross National Product/Personal Taxes

Quarters of change	BEA	Brook-ings	MOEM	DRI-74	Fair	FRB St. Louis	MPS	Wharton Mark III		H-C Annual	Wharton Annual	Liu-Hwa Monthly
								Standard	Anticipations			
ΔGNP/−ΔTP (current dollars)												
1	0.4	1.0	0.6	0.9			0.4	0.4	0.4			0.1
2	0.8	1.4	1.1	1.2			0.9	0.7	0.6			0.2
3	1.1	1.6	1.2	1.2			1.2	0.9	0.7			0.4
4	1.3	1.8	1.2	1.3			1.5	1.0	0.8	1.5	0.8	0.6
5	1.5	1.9	1.3	1.2			1.9	1.2	0.9			0.7
6	1.7	2.0	1.3	1.3			2.2	1.3	1.0			0.8
7	1.8	2.2	1.3	1.3			2.5	1.4	1.0			0.9
8	1.8	2.3	1.4	1.3			2.7	1.5	1.1	1.9	1.8	1.0
12	1.8	2.6	1.7	1.2			3.4	1.7	1.0	2.4	2.2	1.2
16	1.8	2.8	2.1	0.9			3.7	1.5	0.9	2.9	3.9	
20	2.2	2.8	2.6	1.0				1.2		3.1	5.9	
24	2.7	2.9	3.3	1.2				0.7		2.9		
28	3.2	3.2	4.9	1.9				0.4		3.0		
32	3.3	3.5	5.6	3.2				0.4		3.3		
36	3.3	4.0	6.7	4.1				0.7		3.3		
40	3.5	4.7	7.6	4.6				1.6		3.4		
ΔGNP58/−ΔTP58 (constant dollars)												
1	0.4	1.0	0.6	0.9			0.4	0.5	0.4			
2	0.7	1.3	1.1	1.2			0.8	0.8	0.6			
3	1.0	1.5	1.1	1.2			1.1	1.0	0.7			
4	1.2	1.6	1.2	1.3			1.3	1.2	0.8		1.0	
5	1.3	1.6	1.1	1.3			1.6	1.3	0.9			
6	1.4	1.4	1.1	1.3			1.8	1.5	1.0			
7	1.4	1.6	1.1	1.3			2.0	1.6	1.0			
8	1.4	1.6	1.1	1.2			2.1	1.7	1.1		1.4	
12	1.1	1.6	1.1	0.9			2.2	1.9	1.0		0.9	
16	0.8	1.5	1.2	0.6			1.8	1.6	0.9		0.5	
20	0.5	1.3	1.1	0.5				1.1			0.2	
24	0.3	1.2	0.9	0.7				0.6				
28	−0.2	1.2	1.1	0.9				0.3				
32	−0.4	1.3	0.6	0.9				0.1				
36	−0.7	1.4	0.8	0.3				0.2				
40	−1.0	1.5	0.9	0.2				0.6				

TABLE 9
Dynamic Multipliers: Gross National Product/Unborrowed Reserves or Money Stock

Quarters of change	BEA	Brookings	MOEM	DRI-74	Fair	FRB St. Louis	MPS	Wharton Mark III Standard	Wharton Mark III Anticipations	H-C Annual	Wharton Annual	Liu–Hwa Monthly
ΔGNP/ΔM (current dollars)												
1	0.0			0.3		1.2	0.4	1.3				0.0
2	0.0			0.6		2.9	1.1	2.4				0.1
3	0.1			2.0		4.5	2.2	3.4				0.6
4	0.2			4.3		5.3	3.6	4.1		1.5	1.8	1.5
5	0.2			6.2		5.1	5.4	4.8				2.7
6	0.3			7.4		5.1	7.4	5.5				4.3
7	0.5			8.3		5.1	9.5	6.3				5.9
8	0.5			8.8		5.1	11.4	6.8		1.3	7.6	7.4
12	0.8			7.9		5.1	17.4	8.2		0.9	16.0	14.6
16	0.9			4.9		5.1	24.7	8.0		0.1	25.8	
20	1.1			2.4		5.1	36.1	5.9		0.1	43.6	
24	1.3			0.6				3.6		0.2		
28	1.5			-0.1				1.7		0.2		
32	1.5			1.9				1.4		0.1		
36	1.5			1.8				2.1		0.3		
40	1.8			0.4				4.5		0.1		
ΔGNP58/ΔM (constant dollars/current dollars)												
1	0.0			0.3		1.1	0.3	1.4	0.4			0.0
2	0.0			0.5		2.7	1.0	2.6	0.6			0.1
3	0.1			1.9		4.0	2.0	3.6	0.7			0.6
4	0.2			4.1		4.4	3.2	4.5	0.8		2.2	1.4
5	0.2			5.9		4.0	4.8	5.2	1.0			2.7
6	0.3			7.1		3.6	6.4	6.0	1.1			4.5
7	0.4			7.9		3.2	8.1	6.6	1.2			5.7
8	0.4			8.3		2.8	8.4	7.2	1.3		7.4	7.0
12	0.7			6.5		1.2	12.4	8.6	1.3		10.6	10.0
16	0.7			2.8		-0.4	14.5	8.0	1.2		8.2	
20	0.7			-0.1		-1.9	16.1	5.7			5.2	
24	0.6			-1.7				3.6				
28	0.4			-2.5				2.2				
32	0.2			-2.7				2.1				
36	0.0			-4.0				2.3				
40	0.0			-5.1				3.2				

BEA Model: Period 1962–71. Increase of $1 billion in federal nondefense expenditures; proportion due to compensation of government employees based on 1962–71 actual data. $1 billion (1958 dollars) decrease in personal taxes. Increase of $0.5 billion in unborrowed reserves.

Brookings Model: Period 1956:1–1965:4. Increase of $5 billion (1958 dollars) in government expenditures; decrease of $5 billion in personal taxes. Tax multiplier computed as ratio to deflated and undeflated values of $5.0 billion, respectively.

MQEM Model: Period 1962:1–1971:4. $1 billion increase in nondefense expenditures; decrease of $1 billion in personal taxes.

DRI-74 Model: Period 1961:1–1970:4. Increase of $5 billion (1958 dollars) in federal non-defense expenditures. Decrease of $5 billion in personal taxes. Increase of $1.0 billion in unborrowed reserves.

Fair Model: Period 1962:1–1963:1. $1 billion increase in nondefense expenditures; anticipations variables are exogenous. No tax variables in model.

FRB St. Louis Model: Period 1962:1–1966:4. $5 billion increase in nondefense expenditures. Increase of $0.5 billion in M1.

MPS Model: $1 billion increase in exports without accommodating monetary policy and $1 billion decrease in personal taxes. Increase of $0.5 billion in unborrowed reserves.

Wharton Mark III Model: Period 1965:1–1974:4. Increase of $1 billion in nondefense expenditures with average associated change in government wage bill and employment; decrease of $1 billion in personal taxes. Increase of $0.5 billion in unborrowed reserves.

H-C Annual Model: Period 1951–66. Increase of $1 billion in nondefense expenditures. Interest rates are endogenous. Decrease of $1 billion in personal taxes. Increase of $0.5 billion in unborrowed reserves.

Wharton Annual Model: Period 1962–66. Increase of $5 billion (1958 dollars) in nondefense expenditures with average associated change in government wage bill and employment; decrease of $1 billion in personal taxes. Increase of $0.5 billion in unborrowed reserves.

Liu-Hwa Model: Period 1961:01–1964:06. Increase of $1 billion in nondefense spending. Decrease of $1 billion in personal taxes. Increase of $1 billion in unborrowed reserves.

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equation system. The analysis of error is being designed so as to bring these different sources into display for separate measurement.

Although we have not achieved as much model uniformity as we wanted for the calculations discussed in this summary paper, we have come far in this direction. Cross-model comparison has been done only for multiplier and historical error analysis, but the Seminar is now embarking on a new investigation of alternative policy analysis, particularly for the historical phase, 1965–75. Comparable changes in monetary, fiscal, and trade policies, as compared with those actually followed in this period, are being introduced into the several models to see if there is any consensus as to what public authorities might have or should have done to have avoided or mitigated the inflation-recession condition in which we now find ourselves (1974–75). These will be presented in another Seminar symposium.[9]

The different models in this large Seminar collection are all viewing the working of the economy through somewhat different mechanisms—different approximations to reality. Each has some special characteristics, and each has some unusual insight. A combination of model results may prove to be more effective than any one set in interpreting movements in the economy. A study to seek improved or "optimal" combinations of model results is presently being initiated.

[9] Results are to be reported at the December 1975 meetings of the American Economic Association and a summary is to appear in the May, 1976 *American Economic Review*.

These are only some of the findings and lines of research that could be pursued by this unusual Seminar of model builders. As ever, there is much to be done, much more scope for standardization, and much room for improvement—both in model structure and results.

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