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SEARCH FATIGUE

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Bruce Ian Carlin and Florian Ederer
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ABSTRACT

Consumer search is not only costly but also tiring. We characterize the intertemporal effects that search fatigue has on oligopoly prices, product proliferation, and the provision of consumer assistance (i.e., advice). These effects vary based on whether search is all-or-nothing or sequential in nature, whether learning takes place, and whether consumers exhibit brand loyalty. We perform welfare analysis and highlight the novel empirical implications that our analysis generates.

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1 Introduction

It is now well-accepted in economics that search is costly, unless you are lucky enough to be a μ-type (i.e., a shopper or an expert with zero search costs). Search may take place sequentially (e.g., Lippman and McCall, 1976; Reinganum, 1979; Stahl, 1989; Stahl, 1996), by examining a fixed sample of options (e.g., MacMinn, 1980), or via an all-or-nothing approach (e.g., Salop and Stiglitz, 1977; Varian, 1980). Producer and consumer behavior in these settings have been widely studied, and equilibria typically share two standard features: price dispersion and heterogeneous sophistication. Baye, Morgan, and Scholten (2006) provide a thorough review of this literature.[1]

The fact that search is tiring (and can affect future behavior) has been overlooked. Fatigue from searching in one period may affect the costs and incentives to become informed in future periods. In this paper, we characterize oligopoly behavior cognizant of consumers' tendency to become fatigued from search. In contrast to standard search models, accounting for fatigue leads to time-varying prices and consumer assistance (i.e., advice) as well as product proliferation. We also explore how learning, brand loyalty and the type of search that occurs impacts oligopoly outcomes.

In our setting, firms offer products for sale over an infinite horizon. In each period, every firm offers one product of fixed value and has discretion to offer other products with zero value. Prices, consumer assistance, and the length of the product line are set competitively and optimally. We begin by considering a single consumer who purchases one item in each period from one of the firms and rationally chooses whether to search for the best alternative, taking into account the impact that fatigue from search has on future behavior. We consider three types of search: all-or-nothing search, sequential search in which the consumer incurs a cost for each additional firm she visits, and sequential search in which the consumer pays a fixed cost to have the freedom to sort products systematically.

This initial analysis yields the following findings. Whereas a monopolist always chooses to produce only the valuable product, firms competing in an oligopoly usually engage in socially wasteful product proliferation. But the amount of product proliferation that occurs depends on the

[1]See also Diamond (1971), Stiglitz (1979), Weitzman (1979) Braverman (1980), Braverman and Dixit (1981), Salop and Stiglitz (1982), Burdett and Judd (1983), Carlson and McAfee (1983), and Rob (1985).

search technology. When search is sequential and the consumer incurs an incremental cost per firm she visits, the equilibrium mimics that of Diamond (1971) in which no search occurs and there is no product proliferation. This is not surprising because all firms enjoy local monopoly power. However, when all-or-nothing search is used or when the consumer pays a fixed cost for the freedom to search sequentially, product proliferation arises. Interestingly, in the all-or-nothing search setting, the average amount of product proliferation for each firm is monotonically decreasing in the number of firms. In contrast in the fixed cost sequential search setting, firms maximally expand their product lines in a uniform way, independent of the number of firms in the market. We show that this is the most severe form of product proliferation.

When there is a monopolist or search occurs with incremental costs, prices do not vary across time and are set at monopoly levels. However, when all-or-nothing search or fixed cost sequential search occurs, time-varying prices and consumer assistance arise. In those periods when consumers are fatigued, prices evolve at monopoly levels and consumer assistance is highest. During periods of search, prices are lower, but no consumer assistance is offered.

We then extend our model to include consumer learning and to allow firms to alter their product lines dynamically. In this extension, we show that product proliferation is time-varying because the firms extend their product lines during search and only offer the product of fixed value when search does not take place. In this setting, providing assistance is a dominated strategy because the firm could instead lower the number of extraneous products that they offer and save the cost of production. Also, since learning takes place over time, which is modeled as a time-varying cost function, product proliferation becomes more severe as time progresses. Intuitively, the firms need to increase their product lines to assure that the consumer continues to require rest intermittently. This implies that redundant product proliferation and industry maturity are positively correlated.

Finally, we conclude our analysis by considering that a mass of consumers search for the best alternative. At any time, there is a fraction of rested consumers and a group that is fatigued from previous search. The equilibrium that we derive exhibits features that are similar to the one consumer case: time-varying prices and consumer assistance. Several additional findings arise. First, the firms play a mixed strategy in prices that is time varying depending on the fraction of fatigued consumers. The distribution is similar to that in Varian (1980) and Stahl (1989), except that it varies period to period. Also, like Rosenthal (1980), the distribution of prices depends on the number of firms: the stronger the degree of competition the more likely it is that firms choose to set higher prices. Finally, brand loyalty does not cause our results about time-varying price distributions and consumer assistance to disappear. Rather, brand loyalty affects the distribution that the firms choose in equilibrium. When there is less loyalty, the firms compete more vigorously in the market, which causes the lower bound on the price distribution to decrease, but leads firms to place more probability weight on higher prices.

The results in this paper apply to markets in which consumers search intermittently and repeatedly for the best alternative: money management, travel, shipping services, loans and credit terms, to name a few. It also applies to industries that share consumers (e.g., banking and insurance) who search intermittently for related products. To our knowledge, the empirical implications that we highlight throughout the paper are new to the search literature. Testing them empirically is the subject of future research.

Our paper adds to a growing literature in which firms can make search more difficult, either via obfuscation (Spiegler, 2006; Ellison and Wolitzky, 2011; Carlin and Manso, 2011), complexity (Carlin, 2009), or shrouding attributes (Gabaix and Laibson, 2006). In contrast, to previous studies, we endogenize both the consumer's search choice and the firms strategic decisions in a dynamic framework. Additionally, we consider issues such as consumer assistance, product proliferation, and brand loyalty, which are all new considerations in this literature.

To ease the reader's search for our salient ideas and to prevent fatigue, we provide the following outline for the paper. In Section 2, we describe the setup of our model of search fatigue. Section 3 analyzes monopoly and oligopoly behavior in several search settings. Section 4 explores time-varying product proliferation and learning. Section 5 reconsiders our analysis with a continuum of consumers and heterogeneous fatigue. We also analyze the effect that brand loyalty has on search fatigue and oligopoly behavior. Section 6 concludes. All proofs are contained in Appendix A. Some other model variations are explored in Appendix B.

2 The Model

Consider a market in which n firms, indexed by $j \in N = \{1, ..., n\}$, offer a line of goods to a single consumer over an infinite horizon. Time evolves in discrete periods indexed by $t \in \{1, \ldots, \infty\}$, and utility (i.e., profits or consumer surplus) is discounted by $\delta = \frac{1}{1+r}$ per period. Each firm makes three strategic choices during the game: the length of its product line, prices for all of its products, and how much assistance to offer the consumer when she visits its store. The timing of the game is described below and is depicted in Figure 1.

The length of each firm's product line is denoted by ℓ_j and $L \equiv \sum_{j=1}^{n} \ell_j$. Each product line must contain a single product of fixed value $\bar{q}$. We refer to this as the *special* product. Each firm has discretion to offer additional products of zero value and incurs an initial cost of κ for each additional product it adds. As such, the average quality of each firm's products is $\frac{\bar{q}}{\ell_j}$. In our analysis, we start by considering the case in which the product line decision occurs once at the beginning of time. Subsequently, in Section 4 we analyze the case in which firms can alter their product lines in each period t.

In each period, every firm chooses prices for each of its products. Let p_j^1 denote the price of firm j's special product and denote the other prices by p_j^m, where $m \in \{2, ..., \ell_j\}$.

The consumer purchases one product in each period and chooses whether to search for the best alternative. Search may be all-or-nothing or sequential in nature. We analyze both of these cases and compare the equilibrium outcomes. Before making a search decision, the consumer observes how many firms are in the market and how many total products are offered. In period t, the consumer's search cost is $c(x_{t-1})$, where x_{t-1} is the number of products she examined in the prior period. We assume that $x_0 = 0$, that $c(\cdot)$ is increasing in its argument, and that $c(\infty) = \infty$. Therefore, when the consumer chooses whether to search, she has to take into account the effect her decision has on her choice next period as well as the resulting firm behavior. To make the analysis economically meaningful, we assume that $0 < c(0) < \bar{q}$.

If the consumer chooses not to search, she is randomly allocated to one of the L products in the market with equal probability. Unless the firm offers assistance, the consumer can only decide

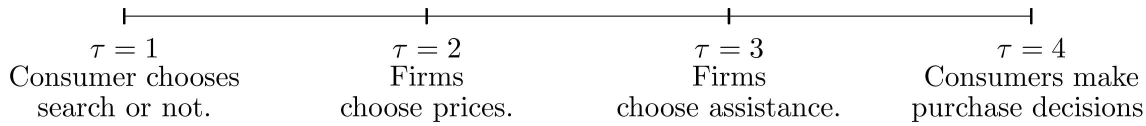

Figure 1: Search fatigue game. At the beginning of the game, each firm chooses the length of its product line. Then, in each period $t \in \{1, 2, \ldots\}$, the choices of the consumer and firms take place according to this timeline. First, at $\tau = 1$, the consumer decides whether she wants to search. At $\tau = 2$, the firms choose their prices. At $\tau = 3$, when the consumer visits a particular firm, it chooses whether to provide consumer assistance or not. Finally, at $\tau = 4$, the consumer chooses whether to proceed with her purchase or walk away.

whether or not buy this randomly chosen product at the price offered by the firm.[2]

Define J_t as the number of stores that a consumer visits in period t. When a consumer visits a firm's store and there are multiple products to choose from, the firm may offer assistance $a_j \in \{0, 1\}$ at no cost. When $a_j = 1$, the consumer is directed to the special product, even if the consumer was randomly allocated to another product offered by the firm. In this case, the consumer saves on the search costs incurred at that store. When a firm chooses $a_j = 0$, the consumer is left to make choices on her own without assistance and without a decrease in future search costs. As such, the effective x_t with advice may be computed as

$$x_t = L - \sum_{k \in J_t} a_k \ell_k. \tag{1}$$

The timing of the game is as follows. At the beginning of the game $t = 0$, each firm chooses ℓ_j. Then, each period $t \in \{1, 2, \ldots\}$, is divided into four parts. At $\tau = 1$, the consumer first observes how many firms and products are in the market and decides whether to search. Then, at $\tau = 2$ the firms set prices for each of their products. At $\tau = 3$, if the consumer visits firm j's store, it chooses whether to offer consumer assistance. Finally, at $\tau = 4$, the consumer makes her purchase decision. The consumer makes a purchase only if her utility is weakly greater than walking away.

[2] As will soon become apparent, if we pose an alternative model with inertia, our results are unchanged. Specifically, if the consumer remains with the same firm when she does not search, our results on product proliferations, time-varying prices, and consumer assistance remain. See Remark 3.

3 Results

We define a subgame perfect equilibrium by the tuple of vectors $(\ell^*, F_t^*(p), a_t^*, s_t^*)$. The vector $\ell^* \equiv [\ell_1^*, \ldots, \ell_n^*]$ lists the product line choices that the firms choose at $t = 0$. The vectors $F_t^*(p)$ and a_t^* are the time-varying equilibrium pricing and advice strategies that the firms use. As we will show shortly, in the one consumer case, pricing entails pure strategies, whereas with heterogeneous fatigue (i.e., Section 5), the firms utilize mixed strategies. Finally, s_t^* is the time-varying search decision by the consumer.

Going forward, we make the assumption that

$$\kappa < \frac{\delta \bar{q}}{n^2(\bar{L} - 1)}. \tag{2}$$

This condition is sufficient to assure participation of each of the firms. It simplifies the analysis because it allows us to abstract away from the coordination problems that arise in joint production problems. Indeed, it assures that the initial cost of adding products is not too high, and that all firms consider producing extra products.

3.1 Monopoly

We begin by analyzing the case where $n = 1$, which will serve as a benchmark of comparison to the oligopoly case.

Suppose the consumer chose to search in the current period t. In that case she selects the product that yields the highest utility. Given that the monopolist can extract all the surplus from the transaction, he sets prices to make the consumer indifferent between buying the special product and walking away. The equilibrium price for special product is $p^1 = \bar{q}$ while all other prices are $p^j > 0$ for $j \neq 1$. The monopolist chooses $a = 0$ and its per period profits are equal to $\bar{q}$. The consumer earns zero utility from purchasing the special product and she incurs the cost of search.

Now suppose the consumer does not search in period t. The monopolist can choose $a = 1$ and direct her toward the special product. In this case, the monopolist's per period profits are $\bar{q}$. The consumer again earns zero utility from purchasing the special product, but she does not incur the cost of search.

The consumer receives a surplus of $-c(\ell)$ per period when she searches and 0 when she does not search. Hence, for any ℓ the consumer never searches. As a result, the monopolist maximizes profits by producing only one product, namely the special product, $\ell = 1$.

The following proposition summarizes this result.

Proposition 1. *(Monopoly) Suppose that a firm is a monopolist in the market ($n = 1$). The equilibrium outcome vector $(\ell^*, F_t^*(p), a_t^*, s_t^*)$ is such that the monopolist produces one product, $\ell^* = 1$, $p^{1,*} = \bar{q}$ for all t, no search takes place, $s_t^* = 0$, and no assistance is given, $a_t^* = 0$. Consumer surplus is zero and the firm's expected discounted profits are*

$$\Pi^* = \frac{1}{1-\delta}\bar{q}. \tag{3}$$

Before moving to the analysis of oligopoly competition, it is important to note a few straightforward observations. First, there is no product proliferation in the monopoly case. Since the monopolist collects all of the rents from production, there is no reason to offer suboptimal products. Second, there is no consumer search; since there is only one firm and one valuable product, no effort is wasted on search and there is no fatigue. Thus, the monopoly case is most efficient from a welfare standpoint. Since prices are merely transfers, zero costs are expended on extra product proliferation and sorting products. This efficiency is not retained in the oligopoly setting.

3.2 Oligopoly

In what follows, we consider the two types of search in turn. In both cases we solve by backward induction period by period.

3.2.1 All-or-Nothing Search

In this search setting, if the consumer elects to search in period t she pays $c(x_{t-1})$, and she becomes completely informed about the products and prices offered by every firm in period t. Recall that x_{t-1} is the number of products that she examined in the prior period without assistance, $x_0 = 0$, $c(\cdot)$ is increasing in its argument, $0 < c(0) < \bar{q}$, and $c(\infty) = \infty$.

Let us first suppose that the consumer chooses to search and that a total of L products and n special products are offered in the market. Given that the consumer searches, the firms set prices such that $p_j = 0$ for all products due to Bertrand competition. The consumer selects one of the special products from one of the n firms and earns a surplus of $\bar{q} - c(x_{t-1})$, while all the firms earn zero profits. It is important to note that there is no inherent benefit for the manager at any of the firms to provide consumer assistance.

Now, suppose that the consumer does not search. It is straightforward to show that each firm sets prices $p_j^1 = \bar{q}$ for all j and, to ensure that the special product is always purchased, $p_j^m > 0$ for all j and $m \neq 1$. Further, it is clear that if the consumer arrives at firm j, it is optimal for the manager to choose $a_j = 1$ so that the consumer correctly identifies the special product. Otherwise, the consumer might try to purchase a suboptimal product $m > 1$ yielding the firm zero profit. With assistance, the consumer purchases the special product and earns zero surplus, but the firm earns profits equal to $\bar{q}$.

Given this purchase behavior the firms all strictly prefer that the consumer does not search as they earn zero profits when search occurs and they have a positive expected profit $\frac{\bar{q}}{n}$ in each period when no search occurs. Given that $c(0) < \bar{q}$, the consumer is always willing to search in the current period if she did not search in the previous period. However, if the consumer is sufficiently fatigued, she will not search and this preserves rents for the firms in the market. In the context of our model, the consumer is willing to search if and only if $\bar{q} > c(L)$ and rests otherwise. Hence, because $c(\cdot)$ is monotonically increasing, there exists a unique threshold $\bar{L}$ above which the consumer is unwilling to search in the current period if she searched in the previous period without assistance. More formally, $\bar{L}$ is the smallest integer such that $\bar{q} \leq c(\bar{L})$.

Proposition 2. *(All-or-Nothing Search) Suppose that n firms compete in a dynamic all-or-nothing search setting and that (2) is satisfied. The equilibrium outcome tuple $(\ell^*, F_t^*(p), a_t^*, s_t^*)$ is such that $L^* = \bar{L}$ and*

(i) In all odd number periods $(t = 1, t = 3, \dots)$, the consumer searches, $s^* = 1$, $a_j^* = 0$ for all j, and $p_j^{m,*} = 0$ for all $j \in N, m \in \ell_j$.

(ii) In all even number periods $(t = 2, t = 4, \dots)$, no consumer search occurs, $s^* = 0$, $a_j = 1$ for all j, $p_j^{1,*} = \bar{q}$ for all j, and $p_j^{m,*} > 0$ for all $j \in N, m \in \ell_j$ such that $m > 1$.

Each firm earns discounted expected profits equal to

$$\Pi_j^* = \frac{\delta \bar{q}}{n(1-\delta^2)} - (\ell_j^* - 1)\kappa, \tag{4}$$

and the consumer's expected discounted surplus is

$$U^* = \frac{1}{1-\delta^2}\bar{q}. \tag{5}$$

According to Proposition 2, search, prices, and consumer assistance are time-varying. The firms have an incentive to produce enough products to ensure that a consumer who searches becomes fatigued. By construction, the consumer searches in the first period because her search cost is $c(0)$ and she stands to gain $\bar{q}$. However, because $L^* = \bar{L}$, her second period search cost $c(\bar{L})$ is greater than $\bar{q}$ whence it is optimal for her to rest during the second period. In this case, the firm that she goes to has monopoly power and directs her to the special product.

None of the firms have a unilateral incentive to deviate from this equilibrium. First, no firm has a reason to give a searching consumer any assistance. If one firm did so, then the consumer would not get as tired during search and would search again in the subsequent period: because $c(\bar{L}-1) < \bar{q}$, assistance would decrease the firm's expected profits from $\frac{\bar{q}}{n}$ to zero. Therefore, no firm will offer the consumer assistance when she is actively searching. In contrast, when the consumer is not searching and is randomly paired with a particular firm, that firm will offer assistance to direct the consumer to the special product that yields revenue $\bar{q} > 0$. Much like the monopoly case, the firm has a strong incentive to assure that the consumer ignores products that have no value for the consumer.

Second, no firm has a reason to change its product line unilaterally, given the behavior of the other firms. Suppose $L = \bar{L}$. If a particular firm j deviates and produces more products than ℓ_j^*, it incurs an extra cost of κ per additional product, but it does not change its expected revenues: this is not a profitable deviation. Suppose that the firm deviates and produces any fewer products than ℓ_j^*. The best possible deviation would be to produce only the special product and to avoid paying $(\ell_j^* - 1)\kappa$. In this case, the expected profit drops from $\frac{\bar{q}}{n}$ to zero. But because (2) holds, this is never a profitable deviation.

It is also straightforward to show that while $\bar{L}$ is unique, a unique equilibrium does not exist. For example, suppose $n = 4$ and $\bar{L} = 9$. There are many permutations in which the firms can produce that will constitute an equilibrium. As long as $\bar{L}$ products are produced by all the firms in the market, there is no reason for any one firm to deviate.

3.2.2 Sequential Search

We now consider two types of sequential search, one in which the consumer pays an incremental cost for each additional firm that she visits and one with a fixed cost that the consumer pays to search firms sequentially. In both cases when the consumer searches a firm, she examines all products at the firm.

Incremental cost of search Suppose in each period that the consumer is randomly paired with a firm and then pays $c(x_{t-1})$ for each additional firm she visits. As before, x_{t-1} is the number of products that the consumer examined in the prior period without assistance, and the previous assumptions regarding the search cost continue to hold.

The following proposition characterizes the equilibrium of this game.

Proposition 3. *(Incremental Sequential Search) Suppose that n firms compete in a dynamic incremental sequential search. Then, no search takes place, $\ell_j^* = 1$ for all j, no assistance is needed, $p_j^1 = \bar{q}$ for all t, consumer surplus is zero, and the firms' expected profits are*

$$\pi^* = \frac{1}{(1-\delta)n}\bar{q}. \tag{6}$$

The result in Proposition 3 mimics that in Diamond (1971). Since the consumer faces a fixed but positive search cost, the equilibrium involves uniform monopoly pricing. The consumer has no incentive to pay a cost to search since all products and prices are the same. Given that the consumer does not search, the firms have no incentive to add superfluous products to their lines. The equilibrium is similar to that in Proposition 1, which is not surprising: the incremental search costs endows each firm with local monopoly power so that there is no incentive for socially wasteful product proliferation.

Fixed cost of sequential search Suppose at the beginning of each period, the consumer chooses whether to pay $c(x_{t-1})$. If she does so, she can search sequentially with no incremental cost for doing so. If not, she is randomly paired with one firm as in the all-or-nothing search setting. As before x_{t-1} is still the number of products that the consumer examined in the prior period without assistance, and the previous assumptions regarding the search cost continue to hold.

Proposition 4. *(Fixed Cost Sequential Search) Suppose that n firms compete in a dynamic fixed cost sequential search game. Suppose that (2) holds. Then, there exists an equilibrium $\ell_j^* = \bar{L}$ for all j such that*

(i) In all odd number periods ($t = 1, t = 3, \dots$), the consumer searches, $a_j^* = 0$ for all j, and $p_j^{m,*} = 0$ for all $j \in N, m \in \ell_j$.

(ii) In all even number periods ($t = 2, t = 4, \dots$), no consumer search occurs, $a_j = 1$ for all j, $p_j^{1,*} = \bar{q}$ for all j, and $p_j^{m,*} > 0$ for all $j \in N, m \in \ell_j$ such that $m > 1$.

Each firm earns discounted expected profits equal to

$$\Pi_j^* = \frac{\delta \bar{q}}{n(1-\delta^2)} - (\bar{L}-1)k_p, \tag{7}$$

and the consumer's expected discounted surplus is

$$U^* = \frac{1}{1-\delta^2}\bar{q}. \tag{8}$$

The proof and intuition of Proposition 4 follows the same logic as Proposition 2, except that it leads to more severe product proliferation. Since the pricing during periods of search yields a Bertrand paradox, the consumer only visits one firm. In this case, each firm must produce $\bar{L}$ products to induce sufficient fatigue so that the consumer rests intermittently. Compared to the all-or-nothing search setting in which there are $\bar{L}$ products in the market, $n\bar{L}$ products are made available when sequential search is used. This maximizes socially wasteful product proliferation and leads to a drop in welfare.

Remark 1. *Fixed-sample search in which the consumer compares the products from k firms may also be analyzed in the same fashion (e.g., MacMinn, 1980). For any $\frac{\bar{L}}{k}$ that is integer valued, the* *case in which $\ell_j^* = \frac{\bar{L}}{k}$ leads to equilibrium that is qualitatively similar to that in Propositions 2 and 4. The condition in (2) is sufficient and the degree of product proliferation L_k is monotonically decreasing in k such that $\bar{L} < L_k < n\bar{L}$.*[3]

Remark 2. *It is straightforward to show that the same results would hold with a unit mass of identical consumers who all begin the game with $x_0 = 0$ (i.e., they are initially rested). We will show this to be the case in the Section 5 when we consider heterogenous fatigue.*

Remark 3. *Our assumption about the random choice of product whenever the consumer does not search can also be relaxed. It is straightforward to show that if instead of random choice, a consumer who does not search in period t is allocated to the same product that she purchased in period $t-1$, all of our results still hold. If the consumer purchases the same product again when she does not search, the price is equal to $\bar{q}$.*

4 Time-Varying Product Proliferation and Learning

So far, we have restricted firms to make their product line choices once and not adapt to consumer search. In this section, we build on our all-or-nothing search framework to relax that assumption. Additionally, we have assumed that the consumer does not learn from period to period. Implicitly, we have restricted our analysis to a consumer that already has expertise in buying the products under study. In what follows, we reconsider this assumption and allow the consumer to learn as time evolves.

Consider the setup in Section 2 with two modifications. First, let us suppose that each firm chooses $\ell_{j,t}$ at the beginning of each period $t \in \{1, \dots, \infty\}$. The cost of producing non-valuable products is still κ, but this cost is incurred in each period the extra products are offered. The timing for the game is portrayed in Figure 2.

Second, let us suppose that the consumer's cost function is time-varying. Specifically, assume that $c_t(z) > c_{t+1}(z)$ for all t and any z. This captures the idea that during each period, the consumer learns at least something new about the market under study. However, we make no assumption regarding the importance of what she learns.

[3] Analytical results for k-sample search are available upon request from the authors.

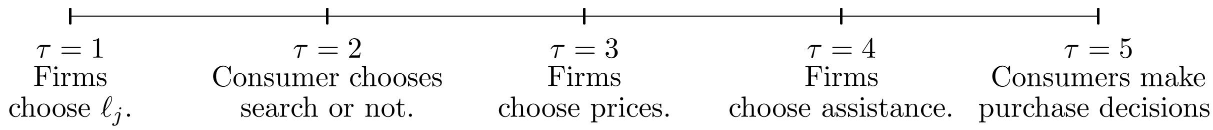

Figure 2: Search fatigue game with time-varying product lines. In each period $t \in \{1, 2, \ldots\}$, the choices of the consumer and firms take place according to this timeline. At $\tau = 1$, each firm chooses the length of its product line. Then, at $\tau = 2$, the consumer decides whether she wants to search. At $\tau = 3$, the firms choose their prices. At $\tau = 4$, when the consumer visits a particular firm, it chooses whether to provide consumer assistance or not. Finally, at $\tau = 5$, the consumer chooses whether to proceed with her purchase or walk away.

Given these assumptions the threshold $\bar{L}$ to induce intermittent search behavior by the consumer will be time-varying. We denote this dependence by $\bar{L}_t$ and specify $\bar{L}_t$ as the smallest integer such that $\bar{q} \leq c_t(\bar{L}_t)$.

Proposition 5. *(Recurring product line choices and learning) Suppose that n firms compete in a dynamic all-or-nothing search setting and that at every time t*

$$\kappa < \frac{\delta \bar{q}}{n^2(\bar{L}_t - 1)}. \tag{9}$$

Then, there exists an equilibrium tuple $(\ell^*, F_t^*(p), a_t^*, s_t^*)$ such that $L_t^* = \bar{L}_t$ and

(i) In all odd number periods ($t = 1, t = 3, \ldots$), the consumer searches, $s^* = 1$, $a_j = 0$ for all j, and $p_j^{m,*} = 0$ for all $j \in N, m \in \ell_j$.

(ii) In all even number periods ($t = 2, t = 4, \ldots$), no consumer search occurs, $s^* = 0$, $a_j^* = 0$ for all j, $\ell_j^* = 1$ for all j, and $p_j^{1,*} = \bar{q}$ for all j.

The results in Proposition 5 contrast with those in Proposition 2 in three ways. First, product proliferation is time-varying. In periods in which the consumer searches, the firms produce enough products to tire her out. However, during periods in which she rests, no product proliferation takes place. Second, consumer assistance is a superfluous consideration. That is, it is a dominated strategy for any firm to produce extra products during even number periods and grant assistance. Instead, it is optimal for them to only produce the special product and save on the costs of proliferation. Third, Proposition 5 and the construction of our model implies that aggregate product proliferation will increase in time as the consumer becomes more experienced. This implies that empirically we should observe more redundant product proliferation in markets that are more seasoned.

The condition in (9) is related to that in (2) except that $\bar{L}$ is time-varying and increasing. However, it is important to point out that we are also implicitly assuming that the consumer's cost function doesn't get too small as time went on. If it did so that $\kappa > \frac{\delta \bar{q}}{n^2(\bar{L}_t - 1)}$, then there would exist a critical time after which search would occur in every period. That is, for a finite time, prices and product proliferation would vary with time. After that, product proliferation would stop and prices would evolve at Bertrand prices.

5 Aggregate Time-Varying Fatigue

In this section, we extend our analysis to consider time-varying fatigue in which a fraction of consumers begins the game rested and the remainder is too fatigued to search in the initial period. We derive an equilibrium with a time-varying distribution of prices, product proliferation, and consumer assistance. Following that, we explore the additional consideration of brand loyalty. Indeed, some consumers may have switching costs or inertia, and we analyze how this affects our analysis.

5.1 Heterogeneous Fatigue

Consider an all-or-nothing search environment in which μ_t is the fraction of consumers who choose to search in any period t and $1 - \mu_t$ is the fraction who is randomly paired with a firm. The setup is otherwise unchanged from Section 2. The firms initially choose ℓ_j and then the timing proceeds according to the timeline in Figure 1. For ease, we make three simplifying assumptions. First, we assume that $c(0) = 0$ so that consumers will incur zero current search costs in a period if they are rested. Second, we assume that κ is positive but arbitrarily small.[4] Third, we assume that the

[4] A condition like that in (2) could be calculated for the case of heterogeneous fatigue. Rather than express it explicitly, any arbitrarily low value for κ will assure that we avoid the coordination problems that arise in joint production problems.

firms can distinguish μ_t-types when they visit their stores. As we discuss shortly, this assumption is not necessary to derive a symmetric equilibrium, but makes our characterization easier. Indeed, in Appendix B we consider an alternative model in which firms cannot detect types during visits.

Proposition 6. *(Heterogeneous Fatigue) Suppose that n firms compete in a dynamic all-or-nothing search game in which μ_t of the consumers search at any time t. Then, there exists an equilibrium $(\ell^*, F_t^*(p), a_t^*)$ where $L = \bar{L}$ and all firms set their prices according to a continuous, monotonically increasing, time-varying distribution function $F_t(p)$ over the support $[p_t^*, \bar{q}]$. The distribution function is computed as*

$$F_t(p) = 1 - \left[\frac{(\bar{q}-p)(1-\mu_t)}{np\mu_t}\right]^{\frac{1}{n-1}}, \tag{10}$$

with lower bound

$$p_t^* = \frac{\bar{q}}{\frac{n\mu_t}{1-\mu_t}+1}. \tag{11}$$

In equilibrium, $\mu_t = \mu_0$ for all even periods $t \in \{2, 4, \ldots\}$ and is equal to $1-\mu_0$ otherwise.

The lower bound p_t^* is monotonically decreasing in μ_t and n. The function $F_t(p)$ is monotonically increasing in μ_t and decreasing in n for all p. As $n \to \infty$, $p_t^* \to 0$ and $F_t(p) \to 0$ for all p.

According to Proposition 6, the fraction μ_0 search during $t = 1$ and get the best deal in the market. Their payoff is $\bar{q} - p_{min}$. The firms provide no assistance to searching customers and as such they are fatigued after analyzing $\bar{L}$ products. Given their fatigue, they do not search at time $t = 2$. In contrast, each consumer in the fraction $1 - \mu_0$ does not search and is randomly paired with a firm. They each receive assistance and all purchase the special product from the firm that they visit. Their payoff is also almost surely positive and depends on their firm's draw from $F^*(p)$. Since they do not search, however, they become rested at time $t = 2$ and search during that period. Therefore, in each period the roles of the consumers switch. As such, there will be time-varying price distributions that take values in one of two states, alternating each period depending on μ_0 and $1 - \mu_0$.

Not surprisingly, in each period, the firms play a mixed strategy equilibrium with regard to prices. This result is consistent with previous all-or-nothing search models (e.g., Salop and Stiglitz, 1977; Varian, 1980) and price dispersion results from the impossibility of a pure strategy equilibrium. In this case, however, the distribution of prices varies over time due to the heterogeneous and intermittent fatigue of the consumers. Only if $\mu_0 = \frac{1}{2}$ will $F_t(p)$ be constant over time. As μ_0 departs from one-half will there be increasing variation across periods. In the extreme, when μ_0 is one, the firms will set prices as they did in Section 3.2.1, that is alternating between Bertrand competition and monopoly prices. Therefore, Proposition 6 nests a result in Proposition 2 as a special case when consumers are homogeneous.

According to our comparative statics results, with more firms (higher n), the lower bound of the distribution decreases, consistent with more competition. However, $F(p)$ also decreases, which implies that firms tend to put more probability on setting higher prices. In the limit as $n \to \infty$, even though the lower bound converges to zero, the firms all choose $p_j = \bar{q}$ almost surely. This is not surprising since increasing competition makes it less likely to be the low-price firm in the market. This effect was first described by Rosenthal (1980), that is the ability of competition to induce rising prices.

Remark 4. *The assumption that the firms observe which consumers are searching is not necessary to derive an equilibrium in the spirit of Proposition 6. In Appendix B, we derive one in the case when the firms share the burden of producing $\bar{L}$ products equally. Compared to the case in Proposition 6, the lower bound of the support is lower, but $F_t(p)$ is lower for all p. This change arises because each firm can no longer direct non-searching searching consumers to the special product.*

5.2 Brand Loyalty

So far in our analysis, we have made the somewhat unrealistic assumption that consumers (may) switch products every period and start afresh. Now, we relax that assumption and suppose that consumers can develop brand loyalty, either due to inertia, switching costs that are outside the model, or relationships with someone at the firm.

Consider the same setup as Section 5.1 except that at any time t, a fraction $1-\lambda_t$ of consumers have brand loyalty. We assume that $1-\lambda_t$ is evenly distributed among the firms, so that no firm has a brand advantage. Of the remaining λ_t fraction of consumers, a proportion μ_t are rested and search during period t. The remaining $1-\mu_t$ fraction of λ_t are each randomly paired with a firm.

Proposition 7. *(Brand Loyalty) Suppose that n firms compete in a dynamic all-or-nothing search game in which $1-\lambda_t$ of the consumers have brand loyalty at any time t. Then, there exists an equilibrium $(\ell^*, F_t^*(p), a_t^*)$ where $L=\bar{L}$ and all firms set their prices according to*

$$F_t(p) = 1 - \left[\frac{(\bar{q}-p)(1-\lambda_t\mu_t)}{np\lambda_t\mu_t}\right]^{\frac{1}{n-1}}, \tag{12}$$

over the support $[p_t^*, \bar{q}]$ with lower bound

$$p_t^* = \frac{\bar{q}}{\frac{n\lambda_t\mu_t}{1-\lambda_t\mu_t}+1}. \tag{13}$$

The lower bound p_t^* is monotonically decreasing in λ_t. The function $F_t(p)$ is monotonically decreasing in λ_t for all p.

Much of the structure of the equilibrium with brand loyalty is unchanged from the results in Proposition 6. The fraction of searching consumers alternates in each period, the firms use a mixed strategy when choosing prices, and firms help non-searching consumers (including those with brand loyalty).

The distribution of prices is the only dimension that changes with brand loyalty. In periods of high brand loyalty (low λ), the lower bound of the distribution is higher, but the firms place less probability weight on high prices. When there is low brand loyalty, there is more competition for searchers and p_t^* is lower. However, given a lower chance of being the low-priced firm, all firms weight higher prices more within $F_t(p)$. This implies that there is likely to be more price dispersion in periods of low brand loyalty than when there is more brand loyalty.

6 Conclusion

In this paper, we analyze how fatigue affects consumer behavior and oligopoly outcomes. Our model variations focus on one industry and generally we find that fatigue induces product proliferation and time-varying prices and consumer assistance.

Our analysis would apply equally well to a case in which a consumer searches for goods in multiple industries. In the context of our model, as long as there is a probability that the consumer searches for an item in each industry in each period, firms should still find it optimal to produce multiple products, and prices (or price distributions) should be time varying. As such, we believe that our model's empirical predictions can be extended across industries as well, especially those in which consumers search for related items (e.g., loans, investments, and insurance).

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Appendix A

Proof of Proposition 1.

The proof proceeds by backward induction for each period. Denote the product offering of the monopolist by $\ell \geq 1$.

Suppose the consumer searches in period t and the consumer chooses the product with the highest utility. To induce her to buy the special product, for any ℓ the monopolist optimally sets prices $p^1 = \bar{q}$ and $p^j > 0$ for $j \neq 1$ and earns a profit of $\bar{q}$. The consumer's surplus is equal to $-c(x_{t-1}) \leq 0$.

Suppose the consumer does not search in period t. The consumer is willing to buy the product to which she was allocated as long as it offers non-negative utility. For any $\ell > 1$, the monopolist optimally chooses $a = 1$ and sets prices $p^1 = \bar{q}$ and $p^j > 0$ for $j \neq 1$. As such, its profits are $\bar{q}$ and the consumer earns zero surplus.

Thus, in every period the consumer prefers not to search for any ℓ. The monopolist maximizes profits by producing only one product, namely the special product, and earns

$$\Pi^* = \frac{1}{1-\delta}\bar{q}. \tag{A1}$$

Given that only one product is produced, $a = 0$ for all t. ∎

Proof of Proposition 2.

The proof proceeds by backward induction for each period. There are L products offered in the market and n special products.

Suppose the consumer searches in period t and the consumer selects the product with the highest utility. Bertrand competition between the firms offering identical products leads to $p_j^m = 0$ for all j and m. To show this, suppose that $p_j^m = 0$ for all j and m, and that firm j deviates by setting $p_j^m > 0$ for any m. For $m \geq 2$, the payoff to the consumer from purchasing such a product from firm j would be negative, which would lead to no sale. For $m = 1$, if $p_j^1 > 0$, the consumer would purchase the special product from another firm. Therefore, increasing prices is not a profitable deviation. In this case, the consumer selects one of the special products from one of the n firms and earns a surplus of $\bar{q} - c(x_{t-1})$. All firms earn zero profits.

Suppose the consumer does not search in period t. The consumer is willing to buy the product to which he was allocated as long as it offers non-negative utility. If the consumer is randomly allocated to firm j and $\ell_j > 1$, the firm optimally chooses $a_j = 1$. For any ℓ_j, each firm j optimally sets prices $p^1 = \bar{q}$ and $p^j > 0$ for $j \neq 1$ and its profits are $\bar{q}$. All other firms earn zero profits. Each firm's expected per period profit is equal to $\frac{\bar{q}}{n}$. The consumer earns zero surplus.

Hence, the consumer searches in period t, if and only if $\bar{q} > c(x_{t-1})$. If the consumer did not search in period $t-1$ (i.e., $x_{t-1} = 0$), she searches in period t because $c(0) < \bar{q}$. Let $\bar{L}$ be the smallest integer such that $\bar{q} \leq c(\bar{L})$. If $L < \bar{L}$, the consumer searches in every period and each firm earns zero discounted expected profits. If $L \geq \bar{L}$, the consumer searches in all odd number periods ($t = 1, t = 3, \dots$) and does not search in all even number periods ($t = 2, t = 4, \dots$). In this case, each firm j earns discounted expected profits equal to

$$\Pi_j^* = \frac{\delta \bar{q}}{n(1-\delta^2)} - (\ell_j - 1)\kappa, \tag{A2}$$

and the consumer's expected discounted surplus is

$$U^* = \frac{\bar{q}}{1-\delta^2}. \tag{A3}$$

Suppose that in equilibrium all other $n-1$ firms choose to produce a total of $x < \bar{L}$ products. We now show that firm j prefers to produce $\bar{L} - x$ products to deter search in all even number periods. The assumption in (2) assures this to be the case. Further, when the consumer searches in odd-numbered periods, firm j has a strict incentive to set $a_j = 0$ since $c(\bar{L} - 1) < \bar{q}$.

Suppose that the total number of products offered in the market is equal to $\bar{L}$. If a particular firm j produces $\ell_j^* + 1$ instead of ℓ_j^* products, it incurs an extra cost κ and thus reduces its expected discounted profits. Now, suppose that the firm only produces the special product. It avoids paying $(\ell_j^* - 1)\kappa$ in product line costs, but its (expected) per-period profit drops to zero. Because (2) holds, this is never a profitable deviation.

Hence, there is a unique equilibrium number of products $L = \bar{L}$. ■

Proof of Proposition 3.

We prove our claim using an induction argument. The logic follows that in Diamond (1971). We start by conjecturing that each firm only produces the special product and then show that this is the case in equilibrium.

Suppose that in period t the consumer has already searched $n-1$ firms and has received price quotes from each of them. Define P_{n-1} as the set of prices quoted to the consumer previously and $\underline{p}^{n-1}$ as the minimum element of P_{n-1}. Suppose that the consumer has chosen to visit firm n. Firm n's optimal strategy is to set a price $\underline{p}^{n-1} - \epsilon$ and earn profits of $\underline{p}^{n-1} - \epsilon$. However, looking forward and reasoning back, since $\epsilon < c(x_t)$, the consumer will choose not to visit firm n.

Now consider the pricing behavior of the $(n-1)^{th}$ firm, given that the consumer has chosen to search for a price from this firm. Define P_{n-2} and $\underline{p}^{n-2}$ accordingly. It is optimal for $p_{n-1} = \underline{p}^{n-1} - \epsilon$. If the consumer stops searching, firm $n-1$ earns $\underline{p}^{n-1} - \epsilon$. If the consumer continues searching, firm $n-1$ earns zero, but as we proved this will never happen. Therefore, looking forward and reasoning back, the consumer will not search the $(n-1)^{th}$ firm because $\epsilon < c(x_t)$. By induction, the consumer arrives at the first firm it samples and does not search further. As such, $p_j^1 = \bar{q}$ for all j.

Now, we can prove our claim that $\ell_j = 1$ for all j. Suppose that instead that $\ell_j > 1$ for a particular firm j. Suppose that the consumer visits this firm first. The consumer will identify the special product as the weakly optimal product to purchase. Indeed, as long as $p_j^m > 0$ for $m \geq 2$, the consumer will not consider the alternatives. As such, all non-valuable products will not benefit for the firm. Now, suppose that the consumer visits another firm $i \neq j$ first. By the previous argument, the consumer will not choose to search further for a special product from another firm. Certainly, the extra non-valuable products at other firms does not induce search. Therefore, making $\ell_j > 1$ is a dominated strategy.

Given this, $\ell_j = 1$. In each period t, $p_j^1 = \bar{q}$ for all j, and $s_t = 0$. Consumer surplus is zero and each firm's expected profits are

$$\Pi^* = \frac{1}{(1-\delta)n}\bar{q}. \tag{A4}$$

■

Proof of Proposition 4.

The proof proceeds by backward induction for each period. There are L products offered in the market and n special products.

Suppose the consumer searches in period t and the consumer selects the product with the highest utility. Bertrand competition between the firms offering identical products leads to $p_j^m = 0$ for all j and m. To show this, suppose that $p_j^m = 0$ for all j and m, and that firm j deviates by setting $p_j^m > 0$ for any m. For $m \geq 2$, the payoff to the consumer from purchasing such a product from firm j would be negative, which would lead to no sale. For $m = 1$, if $p_j^1 > 0$, the consumer would purchase the special product from another firm. Therefore, increasing prices is not a profitable deviation. In this case, the consumer selects one of the special products from one of the n firms and earns a surplus of $\bar{q} - c(x_{t-1})$. All firms earn zero profits.

Suppose the consumer does not search in period t. The consumer is willing to buy the product to which he was allocated as long as it offers non-negative utility. If the consumer is randomly allocated to firm j and $\ell_j > 1$, the firm optimally chooses $a_j = 1$. For any ℓ_j, each firm j optimally sets prices $p^1 = \bar{q}$ and $p^j > 0$ for $j \neq 1$ and its profits are $\bar{q}$. All other firms earn zero profits. Each firm's expected per period profit is equal to $\frac{\bar{q}}{n}$. The consumer earns zero surplus.

Hence, the consumer searches in period t, if and only if $\bar{q} > c(x_{t-1})$. If the consumer did not search in period $t-1$ (i.e., $x_{t-1} = 0$), she searches in period t because $c(0) < \bar{q}$. Let $\bar{L}$ be the smallest integer such that $\bar{q} \leq c(\bar{L})$. If $L < \bar{L}$, the consumer searches in every period and each firm earns zero discounted expected profits. If $L \geq \bar{L}$, the consumer searches in all odd number periods ($t = 1, t = 3, \dots$) and does not search in all even number periods ($t = 2, t = 4, \dots$). In this case, each firm j earns discounted expected profits equal to

$$\Pi_j^* = \frac{\delta \bar{q}}{n(1-\delta^2)} - (\ell_j - 1)\kappa, \tag{A5}$$

and the consumer's expected discounted surplus is

$$U^* = \frac{\bar{q}}{1-\delta^2}. \tag{A6}$$

Each firm j optimally chooses $\ell_j = \bar{L}$. Producing $\bar{L}+1$ instead of $\bar{L}$ products incurs an extra cost κ without any added benefit. The firm will also not produce $\ell_j < \bar{L}$ because (2) holds. Hence, there is a unique equilibrium in which number of products in the market is $L = n\bar{L}$. ■

Proof of Proposition 5:

The proof proceeds by backward induction for each period. There are L products offered in the market and n special products.

Suppose the consumer searches in period t and the consumer selects the product with the highest utility. Bertrand competition between the firms offering identical products leads to $p_j^m = 0$ for all j and m. To show this, suppose that $p_j^m = 0$ for all j and m, and that firm j deviates by setting $p_j^m > 0$ for any m. For $m \geq 2$, the payoff to the consumer from purchasing such a product from firm j would be negative, which would lead to no sale. For $m = 1$, if $p_j^1 > 0$, the consumer would purchase the special product from another firm. Therefore, increasing prices is not a profitable deviation. In this case, the consumer selects one of the special products from one of the n firms and earns a surplus of $\bar{q} - c(x_{t-1})$. All firms earn zero profits.

Suppose the consumer does not search in period t. The consumer is willing to buy the product to which he was allocated as long as it offers non-negative utility. If the consumer is randomly allocated to firm j and $\ell_j > 1$, the firm optimally chooses $a_j = 1$. For any ℓ_j, each firm j optimally sets prices $p^1 = \bar{q}$ and $p^j > 0$ for $j \neq 1$ and its profits are $\bar{q}$. All other firms earn zero profits. Each firm's expected per period profit is equal to $\frac{\bar{q}}{n}$. The consumer earns zero surplus.

Hence, the consumer searches in period t, if and only if $\bar{q} > c_t(x_{t-1})$. If the consumer did not search in period $t-1$ (i.e., $x_{t-1} = 0$), she searches in period t because $c_t(0) < \bar{q}$. Let $\bar{L}_t$ be the smallest integer such that $\bar{q} \leq c_t(\bar{L}_t)$. If $L < \bar{L}_t$, the consumer searches in every period and each firm earns zero discounted expected profits. If $L \geq \bar{L}_t$, the consumer searches in all odd number periods ($t = 1, t = 3, \dots$) and does not search in all even number periods ($t = 2, t = 4, \dots$). In this case, each firm j earns discounted expected revenues equal to

$$R_j^* = \frac{\delta \bar{q}}{n(1-\delta^2)}, \tag{A7}$$

and the consumer's expected discounted surplus is

$$U^* = \frac{\bar{q}}{1-\delta^2}. \tag{A8}$$

Now, we can observe that it is a dominated strategy for any firm to choose $\ell_j > 1$ and $a_j = 1$ in even periods. Instead, each firm optimally chooses $\ell_j = 1$ and $a_j = 0$ to save the cost of κ per extra product.

In each odd period, suppose that the $n-1$ firms produce z_t products. The optimal choice of $\ell_{j,t}$ is $\bar{L}_t - z_t$. If it produces $\bar{L}_t - z_t + 1$ instead, it incurs an extra cost κ but does not enjoy any benefit for doing so. Now, suppose that the firm only produces the special product. It avoids paying $(\bar{L}_t - z_t - 1)\kappa$ in product line costs, but its (expected) per-period profit drops to zero. Because (9) holds, this is never a profitable deviation.

Finally, when the consumer searches in odd-numbered periods, each firm j has a strict incentive to set $a_j = 0$ since $c_t(\bar{L}_t - 1) < \bar{q}$.

Hence, there is a unique equilibrium number of total products offered in every odd period $L = \bar{L}_t$. ■

Proof of Proposition 6.

Outline of proof: The proof proceeds by backward induction. In each period, we first consider consumer buying behavior and the consumer assistance offered by each firm conditional on identifying which consumers are searching. Following that, we consider the firms' pricing strategies. Working backward, we then consider the search decision by consumers. Finally, we show the existence of an equilibrium ℓ^*.

Step One: Buying behavior and consumer assistance
At any time t, μ_t-type consumers identify all products and prices in the market and choose the one that gives the highest payoff. By construction, $x_t = L$, so that their next period search cost is $c(L)$. When a firm identifies a searching consumer, it chooses $a_j = 0$ as there is a cost to lowering future search costs and no benefit to giving assistance. At time t, $(1-\mu_t)$-type consumers are randomly paired with a firm. In this case, the firm will offer $a_j = 1$ and direct the consumer to the product that is most profitable for the firm. As we will show shortly, in equilibrium this is the special product.

Step Two: Pricing
First let us consider the price of the special product and assume that the firm always directs $(1-\mu_t)$-types to this product. Eventually, we will show that this is indeed always optimal in equilibrium.

Define J^* as the set of firms who quote the lowest price for the special product and n_{j^*} as the number of firms in J^*. Then, the payoff function for each firm $j \in N$ is

$$\max_{p_j \in [0,\bar{q}]} \quad \pi_j(p_j) = p_j Q_j, \tag{A9}$$

where the expected demand Q_j is calculated as

$$Q_j = \frac{\mu \;\; 1_{\{j \in J^*\}}}{n_{j^*}} + \frac{1-\mu}{n},$$

Given this, the payoff to each firm is continuous, except when its price is the lowest and equal to at least one of its competitors.

We prove existence of a symmetric mixed-strategy equilibrium by appealing to Theorem 5 in Dasgupta and Maskin (1986). Using their notation, let $A_j = [0, \bar{q}]$ be the action space for firm j and let $a_j \in A_j$ be a price in that space. As such, A_j is non-empty, compact, and convex for all j. Define $A = \times_{j \in N} A_j$ and $a = (a_1, \ldots, a_n)$. Let $U_j : A \to \mathbb{R}$ be defined as the profit function in (A9). Define the set $A^*(j)$ by

$$A^*(j) = \{(a_1, \ldots, a_n) \in A | \exists i \neq j \; s.t. \; p_j = p_i\}$$

and the set $A^{**}(j) \subseteq A^*(j)$ by

$$A^{**}(j) = \{(a_1, \ldots, a_n) \in A | \exists i \neq j \; s.t. \; p_j = p_i = p_{min} > 0\}.$$

As such, the payoff function U_j is bounded and continuous, except over points $\bar{a} \in A^{**}(j)$. The sum $\sum_{j \in N} U_j(a)$ is continuous since discontinuous shifts in demand from informed consumers between firms at points in $A^{**} = \times_{j \in N} A^{**}(j)$ occur as transfers between firms who have the same low price in the industry. Finally, it is straightforward to show that $U_j(a_j, a_{-j})$ is weakly lower semi-continuous. Since any time $p_i = p_j = p_{min}$, firm i and j share the demand, there exists a $\lambda \in [0,1]$ large enough such that

$$\lambda[(p_j - \epsilon)\mu + \frac{(p_j - \epsilon)(1-\mu)}{n}] + (1-\lambda)\frac{(p_j - \epsilon)(1-\mu)}{n} \geq \frac{p_j \mu}{2} + \frac{p_j(1-\mu)}{n}], \tag{A10}$$

for ϵ arbitrarily small. Rearranging and letting $\epsilon \to 0$ yields

$$\lambda p_j \mu \geq \frac{p_j \mu}{2}, \tag{A11}$$

which is true for all $\lambda > 0$. Therefore by Theorem 5 in Dasgupta and Maskin (1986), there exists a symmetric mixed-strategy equilibrium for this subgame, conditional on the firms always directing non-searching consumers to the special product.

Now, we can prove properties about $F^*(p)$, again conditional on the firm always directing consumers to the special product.

i. Continuity: Suppose that there did exist a countable number of mass points in the distribution of $F^*(p)$. Then, we can find a mass point p' and an $\epsilon > 0$ such that $f^*(p') = a > 0$ and $f^*(p' - \epsilon) = 0$. Now consider a deviation by firm j to choose $\hat{F}(p)$ such that $\hat{f}(p') = 0$ and $\hat{f}(p' - \epsilon) = a$. Since $E[\pi_j(p)]$ using $F^*(p)$ is strictly less than using $\hat{F}(p)$, this would be a profitable deviation. Therefore, in equilibrium, no mass points can exist.

ii. Strict monotonicity (Increasing): Suppose there exists an interval $[p_a, p_b]$ within $[0, \bar{q}]$ such that $F(p_b) - F(p_a) = 0$. Then, for any $\hat{p}$ such that $p_a < \hat{p} < p_b$, $[1 - F(\hat{p})]^{n-1} = [1 - F(p_a)]^{n-1}$. Since $\hat{p}[1 - F(\hat{p})]^{n-1} > p_a[1 - F(p_a)]^{n-1}$ and $\hat{p}[1 - (1 - F(\hat{p}))^{n-1}] > p_a[1 - (1 - F(p_a))^{n-1}]$, then there exists a profitable deviation. Thus, $F(p_b) - F(p_a) \neq 0$ for any interval $[p_a, p_b]$ within $[0, \bar{q}]$.

Given continuity and strict monotonicity, we can write the symmetric $F(p)$ explicitly. For any price p that a firm may choose,

$$\pi_j(p) = p\mu[1 - F(p)]^{n-1} + \frac{p(1-\mu)}{n}. \tag{A12}$$

Since each firm needs to be indifferent between setting an price over a support $[p^*, \bar{q}]$, we can write

$$p\mu[1 - F(p)]^{n-1} + \frac{p(1-\mu)}{n} = \frac{\bar{q}(1-\mu)}{n}. \tag{A13}$$

Rearranging yields the expression in (10). We can then solve

$$p^*\mu + \frac{p^*(1-\mu)}{n} = \frac{\bar{q}(1-\mu)}{n} \tag{A14}$$

for p^*, which yields (11). Finally, inspecting (11), it is clear that $p^* > 0$ for any $\mu < 1$. Therefore, the firms will always direct non-searching consumers to the special product since they don't make positive profits by selling alternative products.

The comparative statics in Proposition 6 are derived by straightforward differentiation. Taking the derivative of $\left[\frac{1}{n}\right]^{\frac{1}{n-1}}$ with respect to n yields

$$\left[\frac{1}{n}\right]^{\frac{1}{n-1}} \left[\frac{n \ln n - n + 1}{n(n-1)^2}\right],$$

which is positive for all $n \geq 2$. Therefore, $1 - F(\hat{p})$ is strictly increasing in n. Taking the limit

$$\lim_{n\to\infty} \left[\frac{1}{n}\right]^{\frac{1}{n-1}} \to 1.$$

Step Three: Consumer Search Decision

It is straightforward that in each period, any consumer with $x_{t-1} = 0$ will search, since $c(0) = 0$. For the consumers with $x_{t-1} > 0$, they will search if and only if

$$\bar{q} - E[p_{min}|F(p)] > c(x_{t-1}). \tag{A15}$$

Step Four: Firms Choice of Product Lines

Given that $c(\cdot)$ is strictly increasing in its argument, there exists an $\bar{L}$ such that $\bar{q} - E[p_{min}|F(p)] < c(\bar{L})$, so the consumer does not search. With the condition (2), proving that any ℓ^* that induces $L = \bar{L}$ follows the same logic as in Proposition 2. Given this, $\mu_t = \mu_0$ for all even periods $t \in \{2, 4, \ldots\}$ and is equal to $1 - \mu_0$ otherwise. ■

Proof of Proposition 7.

The proof follows with the exact same logic as the Proof of Proposition 6. The only difference is the computation of $F_t(p)$.

In any period t, for any price p that a firm may choose,

$$\pi_j(p) = p\mu\lambda_t[1 - F_t(p)]^{n-1} + \frac{p\lambda_t(1-\mu)}{n} + \frac{p(1-\lambda_t)}{n}. \tag{A16}$$

Since each firm needs to be indifferent between setting an price over a support $[p^*, \bar{q}]$, we can write

$$p\mu\lambda[1 - F_t(p)]^{n-1} + \frac{p\lambda_t(1-\mu)}{n} + \frac{p(1-\lambda_t)}{n} = \frac{\bar{q}(1-\mu)}{n} + \frac{\bar{q}(1-\lambda_t)}{n}. \tag{A17}$$

Rearranging yields the expression in (12). We can then solve

$$p^*\mu + \frac{p^*(1-\mu)}{n} + \frac{p^*(1-\lambda_t)}{n} = \frac{\bar{q}(1-\mu)}{n} + \frac{\bar{q}(1-\lambda_t)}{n} \tag{A18}$$

for p^*, which yields (13).

The comparative statics regarding λ_t in Proposition 7 are derived by straightforward differentiation. ■

Appendix B

B.1 Heterogeneous Fatigue: Fatigue Private Knowledge

Consider the setup in Section 5.1, except that the firms can distinguish μ_t-types when they visit their stores and firms share the burden of L^* in equilibrium when one exists. This latter assumption is not necessary for an equilibrium to exist, but makes characterization easier. Otherwise, the equilibrium may involve asymmetry in mixed strategies.

Proposition B1. *(Heterogeneous Fatigue) Suppose that n firms compete in a dynamic all-or-nothing search game in which μ_t of the consumers search at any time t. Then, there exists an equilibrium $(\ell^*, F_t^*(p), a_t^*)$ where $L = L^*$ and all firms set their prices according to a continuous, monotonically increasing, time-varying distribution function $F_t(p)$ over the support $[p_t^*, \bar{q}]$. The distribution function is computed as*

$$F_t(p) = 1 - \left[\frac{(\bar{q} - p)(1 - \mu_t)}{Lp\mu_t}\right]^{\frac{1}{n-1}}, \tag{B19}$$

with lower bound

$$p_t^* = \frac{\bar{q}}{\frac{L\mu_t}{1-\mu_t} + 1}. \tag{B20}$$

In equilibrium, $\mu_t = \mu_0$ for all even periods $t \in \{2, 4, \ldots\}$ and is equal to $1 - \mu_0$ otherwise.

The lower bound p_t^* is monotonically decreasing in L and the function $F_t(p)$ is monotonically increasing in L for all p.

Proof:

The proof follows with the exact same logic as the Proof of Proposition 6. The only difference is the computation of $F_t(p)$.

In any period t, for any price p that a firm may choose,

$$\pi_j(p) = p\mu[1 - F(p)]^{n-1} + \frac{p(1-\mu)}{n}\frac{1}{\ell} = p\mu[1 - F(p)]^{n-1} + \frac{p(1-\mu)}{n}\frac{1}{\frac{L}{n}}. \tag{B21}$$

Since each firm needs to be indifferent between setting an price over a support $[p^*, \bar{q}]$, we can write

$$p\mu[1 - F(p)]^{n-1} + \frac{p(1-\mu)}{L} = \frac{\bar{q}(1-\mu)}{L}. \tag{B22}$$

Rearranging yields the expression in (B19). We can then solve

$$p^*\mu + \frac{p^*(1-\mu)}{L} = \frac{\bar{q}(1-\mu)}{L} \tag{B23}$$

for p^*, which yields (B20).

The comparative statics regarding L in Proposition B1 are derived by straightforward differentiation. ■