

HEURISTIC AND OPTIMUM SOLUTIONS IN ALLOCATION PROBLEMS

Iulian Mircea¹
Radu R. Șerban²

Abstract

In this paper we present some models and algorithms for solving some typical production planning and scheduling problems. We present the Resource-Constrained Project Scheduling Problem (RCPS) and algorithms for the determination of maximal couplings with minimal arch length in the graph attached to an allocation problem, and for the determination of the solution of Dirichlet problem and of the potential-voltage problem which appear in a production planning. We develop a model for allocating work among potential VO partners, taking into account fixed and variable work costs and transportation costs.

Keywords: Dirichlet's problem, conex graph, maximal coupling, RCPS problem, virtual organization, allocation problem. **JEL Classification:** C610, L230

Introduction

Due to the variance of products and the fluctuation of production load, the aims is difficult be achieved by traditional planning logic. The improvement techniques look at scheduling as a combinatorial optimization problem, start with any solution, and try to fiind an optimal or near-optimal solution by iterative improvements. Neural networks too have been investigated for solving scheduling problems. Also, a large number of developed heuristic search techniques. Task rescheduling plays an important role in the performance and robustness of distributed manufacturing systems with dynamic failure patterns.

Today, the virtual organizational structure is emerging. A virtual organization (VO) can be defined as a temporary network of companies quickly coming together to exploit fast-changing opportunities. Several factors are driving businesses toward the use of the virtual organizational structure. First, the pace of business is continually increasing with shorter product life cycles requiring quicker response to market opportunities. Second, the cost of market entry is often smaller than previously, especially in the information services and other technology-driven industries. Third, corporations are now driven more by customer demands than by internal needs. And finally, there is an increased need for globalization to remain competitive. Here, we attempt the problem of selecting VO partners in a virtual organization breeding environment (VBE).

Also, we present some models for production planning problems and industrial scheduling applications, and also, some algorithms for solving some typical problems.

¹ PhD, Associate Professor, Economic Informatics Department, Academy of Economic Studies, Bucharest, Romania

² PhD, Associate Professor, Spiru Haret University of Bucharest

The different models for production planning problems can broadly be classified into two distinct categories: Monolithic Production Planning (MPP) and Hierarchical Production Planning (HPP) models [1]. The major difference between these models is the existence of structural levels in the HPP models which reduce the variance of data, the complexity of production planning problem, and split it up into more or less independent subproblems integrated by several interfaces. The Resource-Constrained Project Scheduling Problem (RCPS) is interesting for two reasons: 1) it is the core problem of many real-life industrial scheduling applications, where resources can handle several tasks at a time; 2) it is an academic problem for which there exists a variety of algorithms and benchmarks, and new algorithms can be tested. For solving the RCPS and for optimization of nonlinear functions was developed the algorithm Particle Swarm Optimization (PSO) [see Kennedy (1999)].

The end-to-start precedence constraints between the activities can be modeled by an acyclic graph. In this context we will use the following notations and theoretical results on graphs. Let $G = (X, U)$ be a finite and conex graph, where X is the set of knots and U is the set of arches. For $A \subset X$, we denote by ω_A^+ the subset of the arches (x_i, x_j) with $x_i \in A$, $x_j \notin A$, and by ω_A^- the subset of the arches (x_i, x_j) with $x_i \notin A$, $x_j \in A$. The function $\varphi: U \rightarrow \mathbf{R}$ is a flow compatible to the surpluses $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_m)$ in the graph G if $\varphi(u) \geq 0$, $\forall u \in U$ and $\sum_{u \in \omega_{x_i}^-} \varphi(u) - \sum_{u \in \omega_{x_i}^+} \varphi(u) = \sigma_i$, $\forall x_i \in X$. Let

$\gamma = (u_1, u_2, \dots)$ be a cycle in G with the arches separated in two sets $A_1(\gamma)$ and $A_2(\gamma)$, as their sense (direction) coincides or does not coincide with the sense of crossing the graph. The real valued function π defined on U is a tension or potential difference in G if the following condition is fulfilled $\sum_{u \in A_1(\gamma)} \pi(u) - \sum_{u \in A_2(\gamma)} \pi(u) = 0$, $\forall \gamma$. By analogy to

the theory of electric networks, we consider a function r defined on U , where the value $r_j = r(u_j)$ is called the resistance of the arch u_j , and the value $c_j = \frac{1}{r_j}$ is called

capacity (capability, power) of the arch u_j . Because the flow can be assimilated with the intensity, for each arch u_j we have $r_j \cdot \varphi_j = \pi_j$, where $\varphi_j = \varphi(u_j)$ and $\pi_j = \pi(u_j)$.

For π the given tension in G , we will define on X a function p called the graph potential: we choose an arbitrary knot x_i and we associate to it the number $p_i = 0$, then we move to an adjacent knot x_k and we associate to it the number $p_k = p_i - \pi(x_i, x_k)$, and the procedure continues through all the knots in X and we define $p(x_k) = p_k$, $\forall x_k \in X$.

Too as well, a potential function is p' defined in a similar way to p by $p'_i = 0$ and $p'_k = p'_i + \pi(x_i, x_k)$. We denote by S the matrix of incidences of the graph G , by R the diagonal matrix of the resistances, and by C the diagonal matrix of the capacities. The

matrix S has the elements s_j^i , where $s_j^i = \begin{cases} 1, & \text{if } u_j \in \omega_{x_i}^+ \\ -1, & \text{if } u_j \in \omega_{x_i}^- \\ 0, & \text{otherwise} \end{cases}$. The matrix R has the

elements $r_j^i = \begin{cases} r_j, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$, and C has the elements $c_j^i = \begin{cases} c_j, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$.

Theorem 1. The following statements hold true:

- (a) $S \cdot \varphi = \sigma$; (b) $R \cdot \varphi = \varphi \cdot R = \pi$;
 (c) $C \cdot \pi = \varphi$; (d) $S^T \cdot p = \pi$ and $S^T \cdot p' = -\pi$.

If G is a transportation network, i.e. it has a first vertex (the entry) and a last vertex (the exit), then $S \cdot C \cdot S^T \cdot p = \sigma$. If in a finite, conex and without loops graph, these vertexes do not exist, they can be introduced fictitiously being conjoined to the entry and exit vertexes of the graph by infinite resistance arches. The mathematical model of the materials (primary objects) distribution leads to the following problem, the so called Dirichlet's problem: for $G = (X, U)$ a conex and without loops graph, σ the array of surpluses having the coordinates in the vertexes $x_i \in X$ and r a resistance on U , one looks for a flow φ compatible to σ such that $\varphi \cdot r$ is a tension in G . If $r_j \geq 0, \forall j$, then the solution exists and it is unique [7].

Partner selection in a virtual organization

When a VBE identifies a business opportunity, it has to determine a 'good' VO configuration for meeting the identified customer need; this is essentially an optimization problem that can be formulated as a mixed integer linear programming model. We develop a model for allocating work among potential VO partners, taking into account fixed and variable work costs, transportation costs etc. Let $M = \{1, 2, \dots, m\}$ denote the set of candidate partners in the VO. The project tasks are denoted by $N = \{1, 2, \dots, n\}$. We note: $C_{i,j}$ capacity of candidate i on task j (more general $C_{i,j}$ is distribution of capacity, $c_{i,j}^k$ k -th element of $C_{i,j}$, without loss of generality, it can be assumed that $c_{i,j}^k$ are sorted in descending order so that $c_{i,j}^1 = \max_k c_{i,j}^k$); g_i fixed cost of candidate i work on the project; $g_{i,j}$ fixed cost of candidate i work on task j of the project; $t_{a,b}$ unit transportation cost between candidates a and b ; $v_{i,j}$ variable cost of candidate i work on task j ; w_j workload of task j which is measured in relevant units (e.g. person days); $r = (r', r'')$ denote a pair of tasks such that the output of task r' must be at the same location where task r'' is carried out; R denote the set of all such pairs, $\delta_{r',r''}$ quantity of transportation required between tasks r' and r'' . Variables: $x_{i,j}$ candidate i work

allocation on task j ; y_i takes value one if i is selected into the VO, zero otherwise; $y_{i,j}$ takes value one if i performs work on task j , zero otherwise, the matrix $Y = (y_{i,j})$; $s_{a,b}$ takes value one if both candidates a and b are selected into the VO, zero otherwise; $s_{a,b}^r$ takes value one if candidates a and b perform tasks r' and r'' , respectively, and transportation is required between tasks r' and r'' , zero otherwise (where $r = (r', r'')$). Therefore, for any given pair of tasks $r = (r', r'')$, we have:

$$s_{a,b}^r = \begin{cases} 0, & \text{if } y_{a,r'} = 0 \text{ or } y_{b,r''} = 0 \\ 1, & \text{if } y_{a,r'} = 1 \text{ and } y_{b,r''} = 1 \end{cases}, \quad \forall r \in R, a, b \in M, c_{a,r'}^1 \geq w_{r'} \text{ and } c_{b,r''}^1 \geq w_{r''},$$

where this definition applies for all pairs of candidates a, b such that a is capable of performing task r' and b can perform task r'' . The total transportation costs can be written as $\sum_{r \in R} \delta_{r',r''} \cdot t_{a,b} \cdot s_{a,b}^r$ (the third term of the our objective function).

In the objective function, the first term ($\sum_{i=1}^m g_i \cdot y_i$) is the sum of fixed costs due to the

addition of partners to the VO, while the second term ($\sum_{j=1}^n \sum_{i=1}^m (g_{i,j} \cdot y_{i,j} + v_{i,j} \cdot x_{i,j})$)

covers the fixed and variable costs due to the work that the partners perform on their respective tasks. This function is flexible in that some costs can be ignored if they are irrelevant. The decision variable is the work-allocation matrix X having the elements $x_{i,j}$.

Our optimization model is

$$\min_X COST(X) = \sum_{i=1}^m g_i \cdot y_i + \sum_{j=1}^n \sum_{i=1}^m (g_{i,j} \cdot y_{i,j} + v_{i,j} \cdot x_{i,j}) + \sum_{r \in R} \delta_{r',r''} \cdot t_{a,b} \cdot s_{a,b}^r.$$

$$\text{subject to } \sum_{i=1}^m x_{i,j} \geq w_j, \quad \forall j \in N, 0 \leq x_{i,j} \leq c_{i,j}^1, \quad \forall i \in M, j \in N,$$

$$\frac{\sum_{j=1}^n x_{ij}}{\sum_{j=1}^n w_j} \leq y_i \leq \sum_{j=1}^n x_{ij}, \quad y_{ij} \geq \frac{x_{ij}}{c_{ij}^1}, \quad \forall i \in M, j \in N, s_{a,b}^r \leq \frac{y_{a,r'} + y_{b,r''}}{2},$$

$$s_{a,b}^r \geq y_{a,r'} + y_{b,r''} - 1, \quad \forall r \in R, y_i \in \{0,1\}, y_{ij} \in \{0,1\}, \forall i \in M, j \in N.$$

Production planning and scheduling problems

In the detailed production scheduling problem, there is a set of projects PRO to be executed within the scheduling horizon. Each project $P \in PRO$ is characterized by an earliest start time est_P and a latest finish time lft_P . The project P comprises a set of non-

preemptive tasks T_p . Resource-Constrained Project Scheduling Problem (RCPSP) is defined by a set of tasks T and a set of resources R . Each task $t \in T$ has a fixed duration d_t . Each task t requires one unit of the renewable cumulative resource $r(t) \in R$ during the whole length of its execution. The capacity of the resource r is denoted by $q(r)$, which means that r is able to process at most $q(r)$ tasks at a time. Tasks that belong to the same project can be connected by end-to-start precedence constraints. The precedence constraint $i \rightarrow j$ states that task i must end before the start of task j , i.e. $end_i \leq st_j$. The solution of a RCPSP instance consists of determining valid start times st_t for the tasks such that all temporal, precedence, and resource constraints are satisfied and some objective function is minimized. We assume that all the raw materials of a project have to be on stock by the start time of the project. Each task i is performed on some resources k of which it requires a given amount r_{ik} . Each resource k is available in a given quantity R_k . The problem is solved when one has found a set of starting dates S_i such that:

$$\text{for all precedence constraints with } i \rightarrow j : S_i + d_i \leq S_j \quad (1)$$

$$\text{for all resources } k, \text{ and all time } t : \sum_{\{i | S_i \leq t \leq S_i + d_i\}} r_{ik} \leq R_k \quad (2)$$

The purpose is to minimize the latest end time ($S_i + d_i$) over all tasks i .

The problems of transfer and allocation are very important in distribution production. There are ways of solving the transfer problem: methods based on graphs (in particular, we have the Hitchcock's problem for the existence of a flow compatible with the given surpluses in a graph).

D). The transfer problem can be stated as: given the surpluses $\sigma_i (i = 1, 2, \dots, n)$ located in the vertexes $x_i \in X$ and the numbers $v_j, j = 1, 2, \dots, m$, assigned to the arches $u_j \in U$, where $G = (X, U)$ is the graph that models the transfer, one requires the potential $p = (p_1, p_2, \dots, p_n)$ satisfying the conditions: (i) $p_k - p_i \leq v_j$ for $u_j = (x_i, x_k) \in U$ and (ii) the scalar product $\langle \sigma, p \rangle$ should be minimal.

For $u_j = (x_i, x_k) \in U$ we replace the number v_j by v_k^i in order to point out the vertexes. The potential p is said to be compatible if $p_k - p_i < v_k^i$. Modeling through a transportation network allows to find a maximal flux saturating the exit arches. The graph G can be transformed into a transportation network $R = (X_R, U_R)$ in this way:

- add an entry $a \notin X$ and an exit $b \notin X$;
- introduce the entry arches (a, x_i) , where $x_i \in X$ has $\sigma_i > 0$, and we denote by P their set;
- introduce the exit arches (x_i, b) , where $x_i \in X$ has $\sigma_i < 0$, and we denote by Q their set;

- $X_R = X \cup \{a, b\}, U_R = U \cup P \cup Q$, and we put on each arch $u_j \in U_R$ the

$$\text{capacity } c(u_j) = \begin{cases} c_j = \sigma_i, & \text{when } u_j \in P \\ c_j = -\sigma_i, & \text{when } u_j \in Q \\ +\infty, & \text{when } u_j \in U \end{cases}$$

Let φ be the flow which extends in R the one defined on G , where $\varphi_j = y_k^i$ for $u_j = (x_i, x_k) \in U$. Consequently, we have $\varphi_j = c_j$ where $u_j = (a, x_i) \in P$ and $\varphi_j = c_j$ when $u_j = (x_i, b) \in Q$. The network is chosen such that the sought flow will saturate its entry and exit arches, so $\sum_{u_j \in P} c_j = \sum_{u_j \in Q} c_j$.

The problem of transfer is stated, in its canonical form, like: given a transportation network with entry a and exit b , with capacities c_j on the entry arches and c_j on the exit arches, one seeks for a maximal flow φ such that:

(i) $\varphi_j \geq 0, \forall j$;

(ii) $\varphi_j = c_j$ for $u_j \in \omega_a^+$ and $\varphi_j = c_j$ for $u_j \in \omega_b^-$;

(iii) $\sum_{u_j \in \omega_{x_i}^+} \varphi_j - \sum_{u_j \in \omega_{x_i}^-} \varphi_j = 0, i = 1, 2, \dots, n$;

(iv) $\sum_{u_j \in U} v_j \cdot \varphi_j$ must be minimal when the numbers v_j assigned to the arches

$u_j \in U$ are known.

Eliminating the arches for which $p_k - p_i < v_k^i$ one gets from R a partial network R' .

Theorem 2. If the compatible potential p in the associated partial network R' induces the fact that the maximal flow does not saturate the exit arches, then it can be found in G another compatible potential p' such that $\langle p', \sigma \rangle < \langle p, \sigma \rangle$. If in R' the maximal flux saturates the exit arches, then the scalar products $\langle p, \sigma \rangle$ and $\langle v, \varphi \rangle$ are minimal.

According to Theorem 2, the next algorithm (ALG1) solves the general transfer problem:

Step 1. Introduce in G a tension whose components verify the inequalities $\pi_j \leq v_j, j=1, 2, \dots, m$, almost with quality. From this tension it is deduced a potential p .

Step 2. The graph G is transformed into a transportation network R in which the arches with $\pi_j < v_j$ are suppressed, obtaining a network R' .

Step 3. Looks for the maximal flux φ in R' .

Step 4. If φ does not saturate the exit arches, the potential must be improved and the algorithm resumes to Step 2. So, in G we have that φ is the solution of the transfer problem and $\langle \sigma, p \rangle$ is the solution of the conex problem on the potential.

The transfer problem can be brought to a Hitchcock's problem which is modeled by a simple graph (the set of vertexes is $X \cup Y$, X and Y disjoint, and the set of arches is $U \subset \{(x, y) \mid x \in X, y \in Y\}$). Any subset of U which does not skip over a vertex is called coverage of the graph. A problem is to find a minimal coverage (according to cardinality, the number of arches in the coverage).

II). The allocation problem: in a distributed production process (environment) composed of m units M_1, M_2, \dots, M_m are to be performed n manufacturing processes L_1, L_2, \dots, L_n . Each unit can execute only certain work processes and to each unit a single work is allocated. We denote by G the set of arches (i, j) associated to the possibility of assigning work process L_j to unit M_i . To every arch $(i, j) \in G$, a constant number v_{ij} is attached (v_{ij} can be regarded as a cost or a profit) together with a variable y_{ij} , y_{ij} is one if unit M_i executes L_j , zero otherwise.

The mathematical model of the allocation problem is

$$\begin{aligned} & \min(\max) \sum_{(i,j) \in G} y_{ij} \cdot v_{ij} \\ \text{subject to } & \sum_{j=1}^n y_{ij} \leq 1, \quad i = \overline{1, m}, \quad \sum_{i=1}^m y_{ij} \leq 1, \quad j = \overline{1, n}, \quad \sum_{(i,j) \in G} y_{ij} = \min(m, n) \end{aligned}$$

One looks for the values y_{ij} which satisfy the above constraints. The solution is placed on the arches of a coupling within the simple associated graph (L, P, G) , where $L = \{L_1, L_2, \dots, L_n\}$, $P = \{M_1, M_2, \dots, M_m\}$. If $m < n$, then the number of the arches for the coupling is m (maximal coupling of P in L). If $m > n$, then the number of the arches for the coupling is n (maximal coupling of L in P). If $m = n$, the coupling is of L on P and it is maximal. Therefore, the allocation problem is equivalent to the problem of choosing among the maximal couplings within the graph those for which the sum of the numbers v_{ij} attached to their arches are minimal. If in the objective function we look for the maximum, putting $V = \max_{(i,j) \in G} v_{ij}$ and $t_{ij} = V - v_{ij}$, then

$\sum_{(i,j) \in G} y_{ij} \cdot v_{ij} = V \cdot \sum_{(i,j) \in G} y_{ij} - \sum_{(i,j) \in G} y_{ij} \cdot t_{ij} = V \cdot \min(m, n) - \min \sum_{(i,j)} y_{ij} \cdot t_{ij}$ and the maximum problem comes to a minimum problem. We will consider that the relationship defined by G is surjective ($m = n$), hence the inequalities from the constraints become equalities.

The solution of the problem can be done by attaching a simple graph and determining a maximal coupling with minimal lengths of the arches. This solution uses the following heuristic procedure (ALG2).

Step 1. The set of arches of the graph which models the problem is divided into classes with the same $v_{i,j}$, ordered in ascending order.

Step 2. We start with an arch from the first class, we exclude its adjacent arches from the next classes, and then we continue choosing the arches for $v_{i,j}$ in ascending order and excluding the adjacent arches from the next classes to form a maximal coupling W_0^s with n arches.

Step 3. We compute $S_s = \sum_{(i,j) \in W_0^s} v_{i,j}$ and we keep the couplings with minimum sum (they represent the solutions of the problem).

The algorithm for determining all maximal couplings (ALG3):

Step 1. Construct the tree containing the maximal couplings attached to Y :

- From the root knot (denoted by 0 and forming level 1) we set arches connecting level 1 knots (arches $(0, j)$, where the knots j verify $y_{1,j} = 1$);

- The construction of the tree for the levels $2, 3, \dots, n$ is done as follows: for i from 2 to n , for every knot k on level $i-1$ and for every j with $y_{i,j} = 1$, which is not a knot on the branch from the root 0 to the knot k , one adds in the tree the knot j on level i and the arch (k, j) .

Step 2. Run through all the branches (root 0 – final knot) of the tree constructed at Step 1. Each branch of length n defines a maximal coupling composed of the arches (*level, knot*), starting with level 1 and finishing with level n .

Thus, solving the allocation problem means generating the graph that models the problem, determining all the maximal couplings (using ALG3) and selecting those with minimal sum of the values, obtaining in this way all the solutions of the problem.

Numerical illustration and final remarks

For optimal selection partners (four partners and six tasks) in VO, we have fixed cost

$$\text{matrix (in monetary units) } G = \begin{pmatrix} 15 & 20 & 0 & 30 & 10 & 25 \\ 20 & 25 & 15 & 35 & 0 & 20 \\ 12 & 24 & 10 & 33 & 12 & 18 \\ 18 & 18 & 12 & 30 & 9 & 27 \end{pmatrix}, \text{ variable cost matrix}$$

$$V = \begin{pmatrix} 5 & 7 & 0 & 10 & 4 & 5 \\ 6 & 8 & 5 & 8 & 0 & 4 \\ 3 & 4 & 4 & 5 & 3 & 4 \\ 6 & 5 & 3 & 4 & 5 & 3 \end{pmatrix}, \text{ capacity matrix}$$

$$C = \begin{pmatrix} 80 & 90 & 0 & 100 & 150 & 70 \\ 90 & 70 & 80 & 50 & 0 & 100 \\ 120 & 100 & 90 & 60 & 70 & 80 \\ 150 & 40 & 120 & 30 & 90 & 50 \end{pmatrix} \text{ and the workload vector}$$

$$W = (250 \ 180 \ 150 \ 120 \ 200 \ 140).$$

The optimal selection is $X = \begin{pmatrix} 80 & 40 & 0 & 0 & 130 & 0 \\ 0 & 0 & 0 & 30 & 0 & 90 \\ 120 & 100 & 30 & 60 & 70 & 0 \\ 50 & 40 & 120 & 30 & 0 & 50 \end{pmatrix}$, the minimal cost is

4616 monetary units.

Also, we present three short examples in which the solutions are obtained using the algorithms ALG1-3 described above.

Example 1. Let us consider the graph given by $(Y, V) = \begin{pmatrix} 1 & 1 & 1 & 0 & 5 & 4 & 3 & \infty \\ 0 & 1 & 1 & 1 & \infty & 2 & 6 & 3 \\ 1 & 0 & 0 & 1 & 3 & \infty & \infty & 5 \\ 0 & 1 & 0 & 1 & \infty & 4 & \infty & 6 \end{pmatrix}$. With ALG2 we get $W_0^1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$,

$W_0^2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$, $W_0^3 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, $W_0^4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$ the maximal

couplings, with $S_1 = 14$, $S_2 = 13$, $S_3 = 19$, $S_4 = 20$.

Example 2. Let us consider the graph given by $(Y, V) = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 3 & 4 & \infty & \infty & 4 \\ 1 & 1 & 0 & 1 & 0 & 5 & 2 & \infty & 3 & \infty \\ 0 & 1 & 1 & 0 & 1 & \infty & 4 & 2 & \infty & 1 \\ 1 & 0 & 1 & 0 & 1 & 2 & \infty & 1 & \infty & 1 \\ 1 & 0 & 1 & 1 & 0 & 3 & \infty & 3 & 2 & \infty \end{pmatrix}$. Applying ALG3 we get the following

12 maximal couplings:

$$W_0^1 = \{(1,1), (2,2), (3,3), (4,5), (5,4)\}, \text{ with } S_1 = 10,$$

$$W_0^2 = \{(1,1), (2,2), (3,5), (4,3), (5,4)\}, \text{ with } S_2 = 9,$$

$$W_0^3 = \{(1,1), (2,4), (3,2), (4,5), (5,3)\}, \text{ with } S_3 = 14,$$

$$W_0^4 = \{(1,2), (2,1), (3,3), (4,5), (5,4)\}, \text{ with } S_4 = 14,$$

$$W_0^5 = \{(1,2), (2,1), (3,5), (4,3), (5,4)\}, \text{ with } S_5 = 13,$$

$$W_0^6 = \{(1,2), (2,4), (3,3), (4,5), (5,1)\}, \text{ with } S_6 = 13,$$

$$W_0^7 = \{(1,2), (2,4), (3,5), (4,1), (5,3)\}, \text{ with } S_7 = 13,$$

$$W_0^8 = \{(1,2), (2,4), (3,5), (4,3), (5,1)\}, \text{ with } S_8 = 12,$$

$$W_0^9 = \{(1,5), (2,1), (3,2), (4,3), (5,4)\}, \text{ with } S_9 = 16,$$

$$W_0^{10} = \{(1,5), (2,2), (3,3), (4,1), (5,4)\}, \text{ with } S_{10} = 12,$$

$$W_0^{11} = \{(1,5), (2,4), (3,2), (4,1), (5,3)\}, \text{ with } S_{11} = 16,$$

$$W_0^{12} = \{(1,5), (2,4), (3,2), (4,3), (5,1)\}, \text{ with } S_{12} = 15.$$

Applying algorithm ALG2 we obtain the coupling with the minimal sum W_0^2 .

Example 3. For the Dirichlet's problem, let us consider the entries

$$S = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 \end{pmatrix}, r = \begin{pmatrix} \frac{1}{2} & 1 & \frac{1}{4} & \frac{1}{3} & 1 \end{pmatrix} \text{ and } \sigma = (38 \quad -2 \quad -16 \quad -20). \text{ We}$$

get the potential $p = (6 \quad 4 \quad 2 \quad 0)$, the tension $\pi = (2 \quad 2 \quad 4 \quad 6 \quad 2)$ and the flux $\varphi = (4 \quad 2 \quad 16 \quad 18 \quad 2)$.

If for the Dirichlet's problem we have that: the surpluses are given only in certain vertexes of the graph, the number of given components of the vectors σ and p is equal to the power of the set of vertexes, and the resistances r_j of the arches are nonnegative for every j , then the problem has a unique solution for graphs which are or can be brought to transportation networks. A conclusion for such a problem is that the given potentials are not necessarily settled in the vertexes in which the surpluses are not known. The flows and the tensions do not have necessarily all components integer numbers.

For partner selection in VO under normal conditions, we can assume that infrastructure is reliable and therefore a transportation partner is available. However, if selecting the right transportation partner is crucial to the success of the project, then each $r \in R$ can be associated with a new task of the project. Thus, the selection of partners for these tasks is done similarly to other tasks. In this way, the decision maker can also cater for possible risks related to transportation. Nevertheless, work performed in collaboration causes transaction costs that would not exist if one entity performed the job. At the partner selection phase of VO creation it is unrealistic to estimate the transaction costs that arise during the VO life-cycle. Therefore, it is more practical to study non-monetary indicators that influence the size of transaction costs over the VO life-cycle. One such indicator is the number of past collaboration activities between partner candidates. It is reasonable to assume that the more the companies have collaborated earlier, the better they know each other's ways of action, which reduces the transaction costs of collaboration. When used as partner selection criteria in VO configuration, we refer to these indicators as network preparedness criteria. The network preparedness criteria differ from traditional selection criteria. The measurement of inter-organizational performance is more viable in the management of a VBE than in an "open universe" of organizations. This is because the VBE members collaborate repeatedly, which permits the collection of data about inter-organizational performance. Thus, considerations such as trust, success of past collaboration, and congruence between organizational culture and objectives can be employed as potentially useful criteria for VO partner selection.

References

- [1] Brucker P., Drexl A., Mohring R., Neumann K., Pesch E. (1999) Resource-constrained project scheduling: Notation, classification, models, and methods, *European Journal of Operational Research*, 112(1):3–41.
- [2] Covrig M., Șerban R., Titan E., Șerban R. R., (2010) Production Planning Models for a Virtual Organization, *Proceedings of the International Conference on Applied Economics (ICOAE 2010)*, Athens, Greece, August 26-28, 2010, 645-652.
- [3] Davis E., Patterson J.,(1975) A comparison of heuristic and optimum solutions in resource-constrained project scheduling, *Management Science*, 21:944–955.
- [4] Gourgand M., Grangeon N., Norre S. (2004) A contribution to the stochastic Flow Shop scheduling problem, *European Journal of Operational Research*,151(2): 415-433.
- [5] Kennedy J. (1999) Effects of neighborhood topology on particle swarm performance, *Proceedings of the IEEE Congress on Evolutionary Computation*, 1931-1938.
- [6] Markus A., Vancza J., Kis T., and Kovacs A. (2003) Project scheduling approach to production planning, *CIRP Annals – Manufacturing Technology*, 52(1):359–362.
- [7] Mircea I., Covrig M., Ciobanu E., Serban R.R., Florian VI. (2010), On Some Models and Algorithms for Production Planning and Scheduling, *Revista Română de Informatică și Automatică*, 20(1), 107-120
- [8] Mircea I., Șerban R., Todose D. (2008) Linear programming models applied in selection of partners in a virtual organization, In *Proc. of the 4th International Conference of ASECU – Development: Cooperation and Competitiveness*, Academy of Economic Studies, Bucharest.
- [9] Todose D., Țițan E., Șerban R.R., (2010) Integrated Production Planning; Application to Virtual Organization, *Proceedings of the 17th International Economic Conference - IECS 2010 „The economic world’ destiny: crisis and globalization?”*, „Lucian Blaga” University of Sibiu, The Faculty of Economic Sciences, May 13-14, 2010, Sibiu.
- [10] Valls V., Ballestin F., Quintanilla S. (2005) Justification and RCPSP: A technique that pays, *European Journal of Operational Research*, 165:375-386.