Hitotsubashi University

Global COE Hi-Stat Discussion Paper Series 139

Research Unit for Statistical and Empirical Analysis in Social Sciences (Hi-Stat)

Multidimensional Poverty Rankings based on Pareto Principle: A Practical Extension

Maki Michinaka
Takahiro Ito

May 2010

Hi-Stat Discussion Paper

Hi-Stat
Institute of Economic Research
Hitotsubashi University
2-1 Naka, Kunitatchi Tokyo, 186-8601 Japan
http://gcoe.ier.hit-u.ac.jp

Multidimensional Poverty Rankings based on Pareto Principle: A Practical Extension

Michinaka, Maki *and Takahiro Ito †

March 30, 2010

Abstract

This paper proposes a ranking method of multidimensional poverty and extends it aiming to enhance its practical utility. While our original ranking method that assumes non-comparability among different dimensions of poverty succeeds in eliminating some implicit arbitrariness in existing ranking, it also confronts a disadvantage that a non-negligible number of objectives (countries) are ranked at the same level. In order to improve this disadvantage, we propose an extended ranking method, where we allow the data to have a certain range of bandwidth. The introduction of bandwidth improves the usefulness of our ranking in the sense that it decreases the number of countries with the same rank. In addition, a simulation exercise shows that this extension also improves the robustness of the ranking against measurement errors.

*Ph.D. Candidate, Graduate School of Economics, Hitotsubashi University. e-mail: ed020404@g.hit-u.ac.jp

†Assistant Professor, Institute of Social and Economic Research, Osaka University

1 Introduction

This paper focuses on issues of multidimensional poverty measurement and ranking. A multidimensional poverty approach regards poverty not only as an economical connotation but also as a multifaceted one including various non-economical factors such as health, education, social exclusion and safety.

Literature on multidimensional poverty measurement can be traced back to early contributions like the physical quality of life index by Morris (1979), the deprivation index by Townsend (Townsend et. al. 1989), and the quality of life index by Dasgupta and Weale (1992). However, only in recent years have a number of studies tried to establish theory-based conceptions and methods to measure multidimensional poverty, based on the pioneering works.

Existing studies in this field can be broadly classified into two strands: statistical and non-statistical approaches. In the former, some sort of latent variable models are often employed. A latent variable model regards multidimensional poverty as an unobserved endogenous variable determined by several exogenous variables such as social, political and institutional factors. This kind of statistical analysis enables us to investigate the causal relationship among different dimensions of poverty.[1]

On the other hand, non-statistical approaches can be divided into a further two subcategories: the fuzzy set approach and the multidimensional poverty ordering approach. The former explicitly takes into account the vagueness of multidimensional poverty. The

[1]For more details on this topic, see Krishnakumar and Ballon (2008), Asselin (2009) and Kuklys (2005).

terms 'the poor' and 'the non-poor' may bring some ambiguity, even in the context of uni-dimensional (e.g. income-based) poverty. Despite this, a number of studies dichotomize the poor and non-poor by a sole poverty line. The fuzzy set approach aims to capture this ambiguity by employing so-called 'membership functions' that describe the degree of poverty, and succeeds in dealing with the dynamics of poverty (Qizilbash, 2006; Betti et al., 2008). The multidimensional poverty ordering approach is inspired by a pioneering work on the characterization of poverty index by Sen (1976). This approach consists of two stages. In the first, who is poor and to what extent are determined. For a set of individuals, the subset to which the poor belong is defined and the level of poverty for the set is expressed as an index value. Such poverty indices are usually characterized by an axiomatic basis, with aggregation of the shortfalls of the poor falling below a certain poverty line. The next stage provides some kinds of ranking rules to order sets of individuals in accordance with the level of multidimensional poverty. The majority adopt stochastic dominance criteria or its applications (Chakravarty and Bourguignon 2002; Tsui, 2002; Velez and Robles, 2008).

With these studies taken as the starting point, this paper proposes an alternative ranking methodology for multidimensional poverty. Whereas our method can be classified as a multidimensional poverty ordering approach, the approach in this paper can be distinguished from others. Our approach is based on the significant assumption that we allow the non-comparability of one dimension of multidimensional poverty with another. This reflects the implicit belief that we can never compare the value of poverties over dimensions because a distinct dimension represents a distinct aspect of poverty. However, this belief also highlights a practical disadvantage of the ranking, whereby many objectives are ranked at the same level. In typical rankings, one objective corresponds to one rank, but multiple objectives may have equivalent rank in our approach. Consequently, the ranking yielded by our approach is possibly coarser than other typical rankings in the sense that many non-comparable objectives remain. Due to this disadvantage, dominance order ranking is subject to the criticism that it lacks practical utility, despite successfully eliminating implicit arbitrariness in existing measures.

In order to alleviate the coarseness of the ranking, we propose an extended ranking method, where we allow the data to have a certain range of bandwidth. The introduction of bandwidth is also interpretable as neglecting a certain range of differences between the data, and doing this turns many countries from non-comparable to comparable. Thus, the extended ranking method can improve the usefulness of the ranking in the sense that it decreases the number of countries with the same rank. In addition, this extension has a secondary effect: the extended method of ranking is more robust to measurement errors than the original method, since allowing the data to have a bandwidth is equivalent to presuming that the data have measurement errors. We will confirm this by conducting a simulation exercise.

The rest of this paper is organized as follows. The next section reviews the framework of the dominance order ranking and its extension. Section 3 examines the ranking results derived from the original and extended method, and shows the result of a simulation exercise. We conclude in the final section.

2 Dominance Order Ranking and Its Practical Extension

2.1 Reviewing the dominance order ranking

Before proceeding to explain the extended method of the dominance order ranking proposed by Michinaka (2009), we review the concept of Michinaka's ranking method and its advantages and drawbacks.[2] Let us assume that the level of multidimensional poverty for each country is expressed by the multidimensional development profile, which is a bundle of the values of multiple indicators representing the level of poverty, such as GDP per capita, infant mortality rate, and adult literacy rate. These indicators are common among all objectives (e.g. individuals, countries or societies) to be ranked. We also assume that the value of each indicator is a real positive number. Note that the basis for the information in our approach is the degree of development, although most existing approaches use the degree of deprivation for the same basis, based on some sort of poverty lines. This is why we refer to a 'multidimensional development profile' instead of 'multidimensional poverty profile.' As stated in the previous section, we eliminate the implicit arbitrariness included in all poverty lines.

[2]Michinaka (2009) proposes three different ranking methods based on the concept of the Pareto dominance: minimal order ranking (MINOR), maximal order ranking (MAXOR) and Pareto dominance order ranking (PDOR). In what follows, unless otherwise noted, we use the term 'dominance order ranking' to refer to 'MAXOR.'

The dominance order ranking is formulated as follows. Let C be the set of countries and I be the set of the poverty indicators. The number of elements in C and I is denoted by $\sharp C$ and $\sharp I$, respectably. The level of multidimensional development for any countries in C is expressed as $f(c) = (f_c^i)_{i \in I}$ where $f(\cdot)$ is a mapping that assigns the $\sharp I$-dimensional poverty level to a country c in C.

Regarding binary relations determining a ranking, we let $\succeq$ denote the binary relation on C that means 'at least as developed as,' defined as $c \succeq \hat{c} :\Leftrightarrow \forall c, \hat{c} \in C \ \& \ \forall i \in I, \ f_c^i \geq f_{\hat{c}}^i$. Corresponding to this binary relation, we now define the three binary relations on C; (1) $\succ$ means 'strictly more developed than' and is defined as $\forall c, \hat{c} \in C, \& \ \forall i \in I, c \succ \hat{c} :\Leftrightarrow \ f_c^i \geq f_{\hat{c}}^i \ \& \ \exists f_c^i$ such that $f_c^i > f_{\hat{c}}^i$, (2) $\sim$ means 'as developed as' and is defined as $c \sim \hat{c} :\Leftrightarrow \forall c, \hat{c} \in C, \& \ \forall \ i \in I, f_c^i = f_{\hat{c}}^i$, and (3) $\bowtie$ means 'non-comparable' and is defined as $c \bowtie \hat{c} :\Leftrightarrow \forall c, \hat{c} \in C, \exists i \in I$ such that $f_c^i > f_{\hat{c}}^i \ \& \ \exists j \in I$ such that $f_c^j < f_{\hat{c}}^j$. $\succ$ and $\sim$ are the asymmetric and symmetric factors of $\succeq$, and $\bowtie$ is an incomparability relationship corresponding to $\succeq$: namely $c \bowtie \hat{c} \Leftrightarrow \neg(c \succeq \hat{c}) \ \& \ \neg(\hat{c} \succeq c)$. [3] Since the binary relation $\succ$ describes Pareto dominance, if $c \succ \hat{c}$ $(\forall c, \hat{c} \in C)$, then c is interpreted as dominating $\hat{c}$.

Using the above binary relations, we now define the dominance order ranking. First of all, we define a maximal set of X as follows:

$$\overline{M}(X, \succ) = \{x \mid x \in X \ \& \ \text{there is no } y \in X \text{ such that } y \succ x\}$$

Utilizing the concept of maximal sets, the dominance order ranking is generated by repeating the following steps:

[3] The symbol $\neg$ denotes the negation of a logical statement.

(step 1) Make the maximal set on C, and call it $\overline{M}_1$, and define the (relative) complement of $\overline{M}_1$ in C ($C\backslash\overline{M}_1$) as C_1.

(step 2) Again, make the maximal set $\overline{M}_2$ on C_1, namely,

$\overline{M}_2(C_1, \succ) = \{c \mid c \in C_1 \;\&\; \text{there is no } \hat{c} \in C_1 \quad \text{such that } \hat{c} \succ c\}$. and define $C_1\backslash\overline{M}_2)$ as C_2.

(step 3) In the same manner, make the maximal set $\overline{M}_r$ on C_{r-1} until $C_{r-1}\backslash\overline{M}_r = \emptyset$.

Consequently, this procedure yields a sequence of maximal sets, namely, $\overline{M}_1$, $\overline{M}_2$, $\ldots$, $\overline{M}_r$, $\ldots$, $\overline{M}_R$. The subscript r of $\overline{M}_r$ corresponds to the rank of the countries belonging to the maximal set.

Thus, the processes of making a dominance order ranking are equivalent to that of making a partition of a set. These processes require no aggregation or indexation of different development indices. In this sense, the dominance order ranking succeeds in eliminating the implicit arbitrariness associated with the aggregation, and this is attributed to the fact that the ranking is based solely on the ordinal relationship between the values of indices.

However, owing to this fundamental non-comparability between different indices, the dominance order ranking has the drawback of 'tie-full tendency.' It means many countries are ranked at the same rank, and things will worsen as the dimension of development or poverty indices ($\sharp I$) increases.[4]

Due to this disadvantage, the dominance order ranking is subject to the criticism that it lacks practical utility even though it is convincing, less arbitrary and intuitively

[4] Regarding other advantages and disadvantages, see Michinaka (2009).

understandable. In the next subsection, we propose a method to improve the drawback of the tie-full tendency.

2.2 Allowing a Bandwidth of Data

For simplicity's sake, consider a case where there are only two indices ($\sharp I = 2$) denoted by x and y. Figure 1 depicts the way of the dominance order ranking. Focusing on country D in the first panel, the tie-full tendency is related to the shaded square areas lying to the northwest and southeast of D. We refer to these areas as 'non-comparable areas' of D, since countries B, C, E and G in these areas are non-comparable to D. The tie-full tendency is mainly attributed to these non-comparable areas, and consequently, reducing the area is largely equivalent to improving the tie-full tendency,

In fact, there are several ways to reduce the area. For instance, approaches admitting a cardinality among values of multiple indicators, like the Human Development Index (HDI), mean arbitrary weights are placed on each indicator. Consequently, any pairs of $f(c) = (f^i_c)_{i \in I}$ and $f(c) = (f^i_{\hat{c}})_{i \in I}$ for all $c, \hat{c} \in C$ are comparable since $f(c) = (f^i_c)_{i \in I}$ for all $c \in C$ can be a scalar as an aggregated index value (in short, there is no non-comparable area).

One of the ways to decrease in a dimension of the non-comparable areas, while maintaining the advantage of the dominance order ranking, is to allow the data of the indicators to have a certain range of bandwidth (the second panel of Figure 1). This is also interpretable as neglecting a certain range of differences between the values of indicators, or equivalent to presuming that the data have measurement errors. Considering the fact that country-level data potentially contain a certain level of measurement errors, allowing data to have a bandwidth (as d^x and d^y in the second panel) can be justified to some extent and is also plausible from a practical perspective. As the figure shows, doing this makes the area decreased, and means countries C and E can escape from the non-comparable areas of D.

At the same time, however, this approach also has a weakness: the existence of the bandwidth generates an area within which all values are regarded as indifferent. In the second panel of Figure 1. this is depicted as the area bounded by the solid line and referred to as 'indifference area' of D. Due to this area, the country F is reclassified from the category of comparable to indifferent.

Thus, the introduction of a bandwidth has an advantage and disadvantage: whereas the number of countries reclassified from the category of non-comparable (i.e. $c \bowtie \hat{c}$) to comparable (i.e. $c \succ \hat{c}$ or $\hat{c} \succ c$), denoted by $\sharp M$, increases, the same applies to that moving from comparable to indifferent (i.e. $c \sim \hat{c}$), as denoted by $\sharp D$. Regarding $\sharp M$ and $\sharp D$ as the benefit and cost of introducing a bandwidth, an optimal bandwidth for index i can be obtained as the solution to the following maximization problem:

$$\widehat{d^i} = \arg\max\{\sum_{c \in C} \sharp M_c(d^i) - \sharp D_c(d^i)\}$$

In this paper, we allow the bandwidth to vary among countries by setting $d_c^i = f_c^i \times r_i$ (but r_i is common for all countries), and choose an optimal r_i in the same manner.

Subsequently, for all $c,\ \hat{c} \in C$ and $i \in I$, f_c^i and $f_{\hat{c}}^i$ are regarded as equivalent if $|f_c^i - f_{\hat{c}}^i| \ \leq d_c^i$. In other words, if $|f_c^i - f_{\hat{c}}^i| \ \leq d_c^i$, then the development level of c and that of $\hat{c}$ are regarded as indifferent. In the next section, we present the ranking result obtained through this procedure and compared with the result of the standard dominance order ranking.

3 Ranking Results and a Simulation Exercise

3.1 Results of the ranking methods

In this section, we show the ranking results obtained through the dominance order ranking and the extended ranking. We adopt the data used to calculate the HDI, which is one of the most consulted multidimensional poverty measures. The HDI is a composite index consisting of four indicators; life expectancy at birth, the adult literacy rate, the combined gross enrolment ratio for primary, secondary and tertiary schools,,and GDP per capita. The data of these indicators for 182 countries were used to calculate the HDI in 2009.

Using this HDI 2009 data, we show two ranking results generated by the ranking methodologies proposed in the previous section, namely, the dominance order ranking and the extended dominance order ranking (See Table 1). Concerning the extended ranking, the calculated result of the optimal value of r is 0.1073. While the HDI ranking in 2009 for 182 countries is a complete ranking from the first (Norway) to the 182nd (Niger), a number of countries are ranked identically in terms of both the dominance order ranking and the extended dominance order ranking. Consequently, the former manages to rank the 182 countries into only seventeen groups from first to last place. In this ranking, twenty-two countries are ranked into the top bracket (the rank of 69, namely the seventh place group) and at the least, a country (the rank of 182nd, namely the bottom place group). While the latter still sees several countries ranked the same, it succeeds in decreasing the numbers. The extended ranking ranks 182 countries to forty groups. Only nine countries are ranked at the top (the rank of 9th, namely the second place group) and at the opposite end, a country (the rank of 181st and 182nd, namely the bottom and next group). In other words, the extended ranking succeeds to improve the practical utility in the sense that it alleviates the coarseness of the original dominance order ranking.

As stated in the previous section, this extension brings both benefit and cost to the original ranking. The benefit is the fact that neglecting of slight difference among data values possibly changes some binary relations non-comparable to comparable. Conversely, the cost of this neglect also possibly changes some binary relations from comparable to indifferent. For an example of the former case, see the Czech Republic and Albania ranked 43rd in the dominance order ranking. The level of multidimensional poverty of the former is $(f^i_{CR})_{i \in I} = (76.4., 99.0, 83.4, 24144)$ while that of the latter is $(f^i_{ALB})_{i \in I} = (76.5, 99.0, 67.8, 7041)$. These countries are ranked the same due to only a slight (0.1) difference in the value of life expectancy. The introduction of bandwidth will mean this slight difference can be neglected, while the ranks of these countries in extended ranking are quite different from each other (34th and 93rd). Likewise, for an instance of the latter, see Portugal ranked 34th in the dominance order ranking with

$(f^i_{POR})_{i \in I} = (78.6., \ 94.9, \ 88.8, \ 22765)$ dominates Chez Republic so that the former is ranked prior to the latter. Meanwhile, the introduction of bandwidth changes the binary relation on these countries from comparable to indifferent. Consequently, the ranks of these countries are the same (34th) in the extended ranking.

Our results shows that when we allow approximately a 10% difference in data value, the practical utility of the dominance order ranking is maximized, namely, the number of countries that have the same rank is minimized. It seems natural that we assume the existence of measurement error in any dataset. In particular, it is difficult to collect precise datasets in developing countries. With this in mind, acceptance of an error range of plus or minus 10% does not seem a quite unreasonable assumption.

3.2 A Simulation Exercise

As mentioned earlier, our extension has a secondary effect, whereby the extended method of ranking is more robust to measurement errors than the original method. To see this, a simulation exercise is implemented.

First of all, we assume that $\ln f^i_c = \mu^i_c + \epsilon^i_c$, rather than the true value μ^i_c, is observed, where ϵ^i_c is a random error. The random error may come from the measurement or other resources, and has i.i.d. $N(0, {\sigma_i}^2)$. Hence, we regard f^i_c as a log-normal random variable with mean $\exp(\mu^i_c + \frac{\sigma^2_i}{2})$ and variance $\exp(2\mu^i_c + \sigma^2_i)\{\exp(\sigma^2_i) - 1\}$.

We now consider the following measure ρ that indicates the extent to which the true value μ^i_c explains the observed value $\ln f^i_c$: $\rho = \frac{E[\mu^i_c - \bar{\mu}^i]^2}{E[\ln f^i_c - \bar{\mu}^i]^2}$. This measure, which is similar to the coefficient of determination (regression R^2) when regressing $\ln f_c^i$ on μ_c^i, ranges between zero and one, and as it assumes a larger value, the error ϵ_c^i has less influence on the observed value $\ln f_c^i$. Subsequently, an unbiased and consistent estimator of σ_i^2 for each ρ is calculated by:

$$\widehat{\sigma}_i^2 = \frac{\sum(\ln f_c^i - \mu_c^i)^2}{N} = \frac{\sum[(\ln f_c^i - \bar{\mu}^i) - (\mu_c^i - \bar{\mu}^i)]^2}{N} = \frac{(1-\rho)\sum(\ln f_c^i - \bar{\mu}^i)^2}{N}, \forall i \in I \; and \; \forall \rho$$

Using this $\widehat{\sigma}_i^2$, we simulate 100 runs of a (hypothetical) true value of f_c^i, denoted by z_c^i:

$$z_{c,t}^i = \exp\left(\mu_{c,t}^i + \frac{\widehat{\sigma}_{i,\rho}^2}{2}\right) = \exp\{(\ln f_c^i - \epsilon_{c,t}^i + \frac{\widehat{\sigma}_{i,\rho}^2}{2})\} = f_c^i \times \exp(-\epsilon_{c,t}^i + \frac{\widehat{\sigma}_{i,\rho}^2}{2}),$$

where the additional subscript t means the t-th trial, and $\epsilon_{c,t}^i$ is drawn at random from $N(0, \widehat{\sigma}_{i,\rho}^2)$. In each trial, using the hypothetical data z_c^i, we obtain ranking results based on the standard procedure and our 'with-bandwidth' procedure, and investigate the extent to which ranking results are sensitive to hypothetical (measurement) errors.

Figure 2 shows the results of the simulation exercise. As a measure to indicate the robustness of the ranking results, we employ the Spearman's and the Kendall's rank correlation coefficients between the ranking result, using the actual data and hypothetical data respectively. As the coefficients are close to unity, the results are interpreted as being robust to measurement errors. The figure shows that the coefficients of the extended method are significantly bigger than those of the original method. which indicates that our 'with-bandwidth' method is more robust to measurement errors than the original ranking method.

4 Conclusion

This paper proposed a ranking named the dominance order ranking, which is a method for ranking the levels of multidimensional poverty and extended it in order to improve its practical utility. This extended ranking is much finer than the original ranking. In addition, the extended ranking is more robust to measurement errors than the original.

While the dominance order ranking succeeds in eliminating some implicit arbitrariness in existing multidimensional poverty rankings, it has the disadvantage of a number of objectives being ranked the same. In other words, a number of objectives remain non-comparable. Due to this disadvantage, the dominance order ranking is criticized as lacking practical utility.

To enhance the practical utility, we introduce a new aspect that allows a certain range of measurement error in the data we use. Neglecting a slight disparity in the data value possibly decreases the number of countries with the same rank. We select the range of measurement errors that maximizes practical utility in the sense of minimizing the number of countries with the same rank. When we allow a difference of approximately 10.73% among data values, the practical utility is maximized and the number of ranks in ranking increased to forty from seventeen. This extension also enhanced the robustness to error in data and is shown by a simulation exercise.

Ackowledgement

We appreciate the support of the Global COE Program on Research Unit for Statistical and Empirical Analysis in Social Science (Hi-Stat), by the Institute of Economic Research and the Department of Economics at Hitotsubashi University.

References

Asselin, Louis-Marie (2009) *Analysis of Multidimensional Poverty: Theory and Case studies*, New York: Springer.

Betti, Gianni, Cheli, Bruno, Lemmi, Achille and Vijay Verma (2008) "The fuzzy set approach to multidimensional poverty: the case of Italy in the 1990s," in Kakwani, N and J. Silber (ed) (2008) *Quantitative Approaches to Multidimensional Poverty Measurement,* Basingstoke : Palgrave Macmillan, pp. 30-48.

Chakravarty, Satya R. and François Bourguignon (2002) "Multi-dimensional poverty orderings," *DELTA Working paper*, No. 2002-22.

Dasgupta, Partha and Martin Weale (1992) "On Measuring the Quality of Life," *World Development,* Vol. 20, No. 1, pp. 119-131.

Velez, Carlos Eduardo and Marcos Robles (2008) "Determing the parameters of axiomatically derived multidimensional poverty indices: An application based on reported well-being in Colombia," in Kakwani, N and J. Silber (ed) (2008) *Quantitative Approaches* *to Multidimensional Poverty Measurement,* Basingstoke : Palgrave Macmillan, pp. 210-225.

Krishnakumar, Jaya and Paola Ballon (2007) "Estimating Basic Capabilities: A Structural Equation Model Applied to Bolivia," *World Development,* Vol. 36, No. 6, pp. 992-1010.

Kuklys, Wiebke (2005) *Amartya Sen's capability approach : theoretical insights and empirical applications: Studies in choice and welfare, Vol. 2,* Berlin : Springer.

Michinaka, Maki (2009) "Multidimensional Poverty Rankings based on Pareto Principle," mimeo.

Morris, Morris. D. (1979) *Measuring the Condition of the World's Poor: The Physical Quality of Life Index,* Pergamon Policy Studies, No. 42, New York : Pergamon Press.

Qizilbash, Mozaffar (2006) "Philosophical Accounts of Vagueness, Fuzzy Poverty Measures and Multidimensionality," in Lemmi, A. and G. Betti (ed.) *Fuzzy Set Approach to Multidimensional Poverty Measurement; Economic Studies in Inequality, Social Exclusion and Well-Being, Vol. 3* New York: Springer, pp. 9-28.

Sen, Amartya K. (1976) "Poverty: An Ordinal Approach to Measurement," *Econometrica,* Vol. 44, No. 2, pp. 219-231.

Townsend, Peter, Phillimore, Peter and Alastair Beattie (1989) *Health and deprivation : inequality and the North,* London : Routledge.

Tsui, Kai-yuen (2002) "Multidimensional Poverty Indices," *Social Choice and Welfare,* Vol. 19, No. 1, pp. 69-93.

United Nations Development Programme (2009) *Human Development Report 2009,* Gordonsville: Palgrave Macmillan.

Table 1: Dominance Order Ranking, Extend Dominance Order Ranking and Human Development Index

DOR	EDOR	HDI		country	life expectancy at birth		adult literacy rate		combined gross enrolment ratio		GDP per capita (PPP$)	
		rank	value		rank	value	rank	value	rank	value	rank	value
1	1	11	0.960	Luxembourg	24	79.4	1	99.0	16	94.4	2	79485
1	1	19	0.951	Liechtenstein	26	79.2	1	99.0	41	86.8	1	85382
1	3	1	0.971	Norway	12	80.5	1	99.0	8	98.6	5	53433
1	9	2	0.970	Australia	5	81.4	1	99.0	1	100.0	22	34923
1	9	3	0.969	Iceland	3	81.7	1	99.0	13	96.0	19	35742
1	9	4	0.966	Canada	11	80.6	1	99.0	7	99.3	18	35812
1	9	7	0.963	Sweden	8	80.8	1	99.0	17	94.3	16	36712
1	9	16	0.955	Denmark	34	78.2	1	99.0	1	100.0	17	36130
1	18	9	0.960	Switzerland	3	81.7	1	99.0	49	82.7	13	40658
1	23	10	0.960	Japan	1	82.7	1	99.0	42	86.6	26	33632
1	23	24	0.944	Hong Kong, China (SAR)	2	82.2	76	94.6	88	74.4	11	42306
12	3	33	0.910	Qatar	48	75.5	86	93.1	57	80.4	3	74882
12	5	5	0.965	Ireland	19	79.7	1	99.0	10	97.6	10	44613
12	5	13	0.956	United States	28	79.1	1	99.0	21	92.4	9	45592
12	5	23	0.944	Singapore	14	80.2	79	94.4	46	85.0	7	49704
12	5	30	0.920	Brunei Darussalam	38	77.0	73	94.9	74	77.7	6	50200
12	9	6	0.964	Netherlands	17	79.8	1	99.0	11	97.5	14	38694
12	9	14	0.955	Austria	16	79.9	1	99.0	27	90.5	15	37370
12	9	35	0.903	United Arab Emirates	37	77.3	99	90.0	107	71.4	4	54626
12	18	8	0.961	France	7	81.0	1	99.0	15	95.4	25	33674
12	18	12	0.959	Finland	22	79.5	1	99.0	1	100.0	23	34526
12	18	15	0.955	Spain	9	80.7	53	97.9	12	96.5	27	31560
12	18	31	0.916	Kuwait	36	77.5	78	94.5	100	72.6	8	47812
12	23	18	0.951	Italy	6	81.1	47	98.9	23	91.8	29	30353
12	23	20	0.950	New Zealand	15	80.1	1	99.0	1	100.0	32	27336
12	30	28	0.934	Andorra	12	80.5	1	99.0	127	65.1	12	41235
27	9	17	0.953	Belgium	22	79.5	1	99.0	17	94.3	21	34935
27	23	21	0.947	United Kingdom	25	79.3	1	99.0	34	89.2	20	35130
27	23	22	0.947	Germany	18	79.8	1	99.0	37	88.1	24	34401
27	23	25	0.942	Greece	29	79.1	60	97.1	1	100.0	31	28517
27	30	26	0.937	Korea (Republic of)	26	79.2	1	99.0	9	98.5	35	24801
27	30	27	0.935	Israel	9	80.7	60	97.1	33	89.9	34	26315
27	51	51	0.863	Cuba	33	78.5	1	99.0	1	100.0	95	6876
34	30	29	0.929	Slovenia	34	78.2	1	99.0	20	92.8	33	26753
34	34	34	0.909	Portugal	31	78.6	73	94.9	35	88.8	42	22765
34	34	38	0.902	Malta	20	79.6	89	92.4	54	81.3	39	23080
34	34	39	0.895	Bahrain	47	75.6	105	88.8	28	90.4	30	29723
34	39	32	0.914	Cyprus	20	79.6	56	97.7	75	77.6	36	24789
34	39	37	0.903	Barbados	38	77.0	1	99.0	19	92.9	48	17956
34	39	118	0.719	Equatorial Guinea	168	49.9	113	87.0	133	62.0	28	30627
34	44	55	0.847	Libyan Arab	64	73.8	114	86.8	14	95.8	57	14364
34	58	44	0.878	Chile	32	78.5	66	96.5	50	82.5	59	13880
43	34	36	0.903	Czech Republic	42	76.4	1	99.0	48	83.4	37	24144
43	34	40	0.883	Estonia	74	72.9	1	99.0	25	91.2	43	20361
43	39	43	0.879	Hungary	69	73.3	47	98.9	30	90.2	46	18755
43	39	46	0.870	Lithuania	91	71.8	1	99.0	22	92.3	49	17575
43	44	50	0.865	Uruguay	43	76.1	53	97.9	26	90.9	70	11216
43	51	41	0.880	Poland	48	75.5	1	99.0	39	87.7	53	15987
43	58	49	0.866	Argentina	53	75.2	57	97.6	36	88.6	62	13238
43	72	54	0.854	Costa Rica	30	78.7	70	95.9	98	73.0	73	10842
43	81	73	0.814	Dominica	40	76.9	108	88.0	65	78.5	83	7893
43	93	70	0.818	Albania	41	76.5	1	99.0	122	67.8	93	7041
53	44	42	0.880	Slovakia	56	74.6	1	99.0	56	80.5	45	20076

Table 1(continued)

DOR	EDOR	HDI		country	life expectancy at birth		adult literacy rate		combined gross enrolment ratio		GDP per capita (PPP$)	
		rank	value		rank	value	rank	value	rank	value	rank	value
53	44	47	0.868	Antigua and Barbuda	84	72.2	1	99.0	45	85.6	47	18691
53	44	59	0.843	Saudi Arabia	77	72.7	117	85.0	65	78.5	40	22935
53	44	64	0.837	Trinidad and Tobago	110	69.2	49	98.7	137	61.1	38	23507
53	44	68	0.826	Belarus	111	69.0	1	99.0	28	90.4	74	10841
53	51	48	0.866	Latvia	83	72.3	1	99.0	30	90.2	51	16377
53	51	52	0.856	Bahamas	71	73.2	71	95.8	103	71.8	44	20253
53	51	56	0.846	Oman	48	75.5	118	84.4	118	68.2	41	22816
53	51	57	0.845	Seychelles	76	72.8	92	91.8	52	82.2	50	16394
53	51	82	0.804	Kazakhstan	130	64.9	1	99.0	24	91.4	72	10863
53	58	45	0.871	Croatia	44	76.0	49	98.7	77	77.2	52	16027
53	61	53	0.854	Mexico	44	76.0	87	92.8	58	80.2	58	14104
53	64	58	0.844	Venezuela (Bolivarian Republic of)	66	73.6	72	95.2	44	85.9	65	12156
53	64	60	0.840	Panama	48	75.5	83	93.4	59	79.7	67	11391
53	64	61	0.840	Bulgaria	72	73.1	52	98.3	51	82.4	69	11222
53	72	78	0.806	Peru	73	73.0	102	89.6	37	88.1	85	7836
69	61	62	0.838	Saint Kitts and Nevis	84	72.2	55	97.8	96	73.1	56	14481
69	61	71	0.817	Russian Federation	122	66.2	1	99.0	53	81.9	55	14690
69	64	63	0.837	Romania	80	72.5	57	97.6	61	79.2	64	12369
69	64	103	0.755	Gabon	144	60.1	115	86.2	55	80.7	54	15167
69	69	66	0.829	Malaysia	58	74.1	91	91.9	105	71.5	61	13518
69	69	75	0.813	Brazil	84	72.2	99	90.0	40	87.2	79	9567
69	72	65	0.834	Montenegro	61	74.0	67	96.4	86	74.5	66	11699
69	77	69	0.821	Saint Lucia	66	73.6	75	94.8	77	77.2	77	9786
69	77	85	0.796	Ukraine	116	68.2	1	99.0	32	90.0	94	6914
69	81	77	0.807	Colombia	77	72.7	88	92.7	63	79.0	81	8587
69	84	72	0.817	Macedonia (the Former Yugoslav Rep.	58	74.1	62	97.0	113	70.1	80	9096
69	84	80	0.806	Ecuador	55	75.0	94	91.0	73	77.8	91	7449
69	87	74	0.813	Grenada	52	75.3	69	96.0	96	73.1	92	7344
69	87	76	0.812	Bosnia and Herzegovina	54	75.1	65	96.7	114	69.0	87	7764
69	87	86	0.787	Azerbaijan	107	70.0	1	99.0	124	66.2	84	7851
69	93	93	0.772	Belize	46	76.0	134	75.1	67	78.3	96	6734
69	96	84	0.798	Armenia	66	73.6	1	99.0	85	74.6	100	5693
69	101	89	0.778	Georgia	96	71.6	1	99.0	81	76.7	110	4662
69	101	98	0.769	Tunisia	64	73.8	130	77.7	83	76.2	90	7520
69	107	99	0.768	Tonga	92	71.7	1	99.0	70	78.0	120	3748
69	107	105	0.751	Philippines	96	71.6	83	93.4	60	79.6	124	3406
69	124	110	0.737	Occupied Palestinian Territories	69	73.3	82	93.8	67	78.3	135	2243
91	69	79	0.806	Turkey	92	71.7	106	88.7	109	71.1	63	12955
91	72	81	0.804	Mauritius	88	72.1	112	87.4	79	76.9	68	11296
91	72	125	0.694	Botswana	160	53.4	122	82.9	111	70.6	60	13604
91	77	67	0.826	Serbia	63	73.9	67	96.4	86	74.5	75	10248
91	77	83	0.803	Lebanon	90	71.9	102	89.6	70	78.0	76	10109
91	84	87	0.783	Thailand	113	68.7	81	94.1	70	78.0	82	8135
91	87	100	0.766	Jamaica	92	71.7	116	86.0	69	78.1	98	6079
91	93	96	0.770	Jordan	81	72.4	93	91.1	64	78.7	107	4901
91	96	95	0.771	Maldives	102	71.1	62	97.0	108	71.3	104	5196
91	96	113	0.729	Bolivia	128	65.4	96	90.7	43	86.0	117	4206
91	101	94	0.771	Samoa	98	71.4	49	98.7	90	74.1	113	4467
91	101	109	0.739	Turkmenistan	132	64.6	1	99.0	92	73.9	106	4953
91	107	102	0.759	Sri Lanka	61	74.0	95	90.8	116	68.7	116	4243

Table 1 (continued)

DOR	EDOR	HDI		country	life expectancy at birth		adult literacy rate		combined gross enrolment ratio		GDP per capita (PPP$)	
		rank	value		rank	value	rank	value	rank	value	rank	value
91	114	107	0.742	Syrian Arab Republic	58	74.1	121	83.1	125	65.7	112	4511
91	114	115	0.727	Mongolia	122	66.2	59	97.3	61	79.2	125	3236
91	119	114	0.729	Guyana	119	66.5	1	99.0	47	83.9	127	2782
91	119	117	0.720	Moldova	115	68.3	1	99.0	104	71.6	131	2551
91	124	116	0.725	Viet Nam	57	74.3	98	90.3	130	62.3	129	2600
91	131	120	0.710	Kyrgyzstan	117	67.6	1	99.0	76	77.3	141	2006
110	81	88	0.782	Iran (Islamic Republic of)	101	71.2	123	82.3	95	73.2	71	10955
110	87	97	0.769	Suriname	112	68.8	97	90.4	89	74.3	86	7813
110	87	129	0.683	South Africa	164	51.5	108	88.0	80	76.8	78	9757
110	96	91	0.772	Saint Vincent and the Grenadines	98	71.4	107	88.1	115	68.9	89	7691
110	96	92	0.772	China	74	72.9	85	93.3	116	68.7	102	5383
110	101	90	0.777	Dominican Republic	81	72.4	104	89.1	94	73.5	97	6706
110	101	104	0.754	Algeria	84	72.2	133	75.4	93	73.6	88	7740
110	107	101	0.761	Paraguay	92	71.7	76	94.6	101	72.1	114	4433
110	107	106	0.747	El Salvador	100	71.3	125	82.0	91	74.0	99	5804
110	107	123	0.703	Egypt	108	69.9	149	66.4	82	76.4	103	5349
110	114	112	0.732	Honduras	89	72.0	120	83.6	84	74.8	119	3796
110	119	119	0.710	Uzbekistan	117	67.6	64	96.9	99	72.7	133	2425
110	124	124	0.699	Nicaragua	77	72.7	129	78.0	101	72.1	130	2570
110	134	127	0.688	Tajikistan	120	66.4	1	99.0	110	70.9	145	1753
124	107	108	0.741	Fiji	113	68.7	79	94.4	105	71.5	115	4304
124	114	111	0.734	Indonesia	105	70.5	90	92.0	118	68.2	121	3712
124	114	128	0.686	Namibia	143	60.4	108	88.0	123	67.2	105	5155
124	119	122	0.704	Guatemala	106	70.1	138	73.2	112	70.5	111	4562
124	119	143	0.564	Angola	178	46.5	147	67.4	126	65.3	101	5385
124	124	130	0.654	Morocco	104	71.0	162	55.6	138	61.0	118	4108
124	124	132	0.619	Bhutan	126	65.7	167	52.8	152	54.1	108	4837
124	131	121	0.708	Cape Verde	102	71.1	119	83.8	120	68.1	126	3041
132	124	126	0.693	Vanuatu	108	69.9	128	78.1	130	62.3	122	3666
132	124	142	0.572	Swaziland	179	45.3	127	79.6	141	60.1	109	4789
132	131	136	0.601	Congo	159	53.5	126	81.1	144	58.6	123	3511
132	136	131	0.651	Sao Tome and	129	65.4	111	87.9	120	68.1	149	1638
132	139	138	0.586	Myanmar	137	61.2	101	89.9	149	56.3	168	904
137	134	134	0.612	India	134	63.4	150	66.0	138	61.0	128	2753
137	136	133	0.619	Lao People's Democratic Republic	132	64.6	139	72.7	142	59.6	136	2165
137	139	137	0.593	Cambodia	142	60.6	132	76.3	145	58.5	144	1802
137	139	141	0.572	Pakistan	122	66.2	164	54.2	174	39.3	132	2496
137	139	156	0.514	Lesotho	180	44.9	124	82.2	135	61.5	151	1541
137	144	135	0.610	Solomon Islands	125	65.8	131	76.6	162	49.7	146	1725
137	144	144	0.553	Nepal	121	66.3	160	56.5	140	60.8	166	1049
137	144	145	0.543	Madagascar	145	59.9	143	70.7	136	61.3	167	932
137	144	147	0.541	Kenya	158	53.6	136	73.6	142	59.6	150	1542
137	152	162	0.489	Timor-Leste	140	60.7	168	50.1	129	63.2	174	717
137	152	164	0.481	Zambia	181	44.5	144	70.6	128	63.3	153	1358
137	159	146	0.543	Bangladesh	126	65.7	165	53.5	158	52.1	156	1241
137	159	157	0.514	Uganda	163	51.9	136	73.6	130	62.3	164	1059
137	163	160	0.493	Malawi	162	52.4	142	71.8	134	61.9	173	761
151	136	140	0.575	Yemen	135	62.5	158	58.9	151	54.4	134	2335
151	139	153	0.523	Cameroon	165	50.9	146	67.9	156	52.3	137	2128

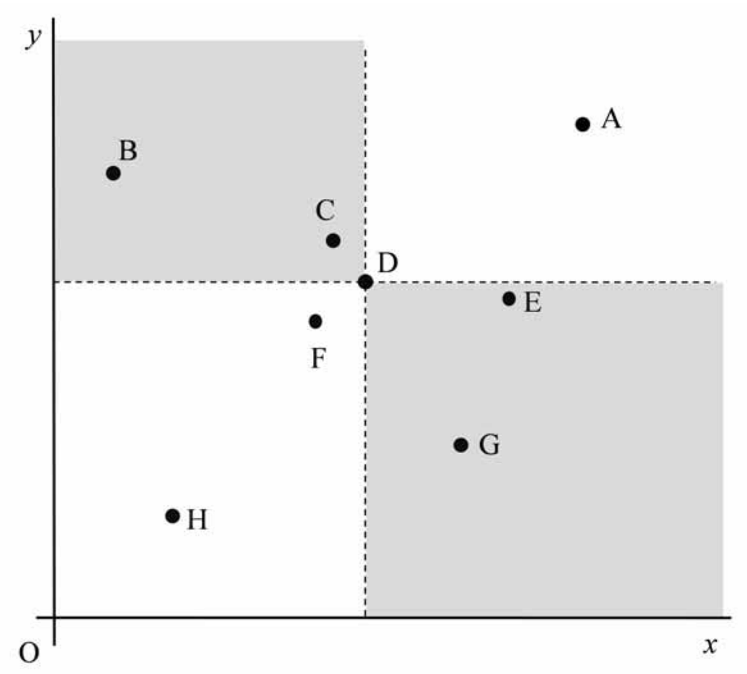
Table 1 (continued)

DOR	EDOR	HDI		country	life expectancy at birth		adult literacy rate		combined gross enrolment ratio		GDP per capita (PPP$)	
		rank	value		rank	value	rank	value	rank	value	rank	value
151	144	155	0.520	Djibouti	155	55.1	145	70.3	182	25.5	140	2061
151	144	158	0.511	Nigeria	173	47.7	141	72.0	154	53.0	142	1969
151	152	139	0.576	Comoros	130	64.9	134	75.1	169	46.4	160	1143
151	152	149	0.532	Haiti	138	61.0	155	62.1	158	52.1	159	1155
151	152	150	0.531	Sudan	147	57.9	156	60.9	173	39.9	138	2086
151	152	151	0.530	Tanzania (United Republic of)	156	55.0	140	72.3	147	57.3	158	1208
151	152	152	0.526	Ghana	152	56.5	151	65.0	148	56.5	154	1334
151	168	169	0.442	Liberia	147	57.9	163	55.5	146	57.6	180	362
161	144	148	0.541	Papua New Guinea	140	60.7	159	57.8	172	40.7	139	2084
161	144	154	0.520	Mauritania	151	56.6	161	55.8	160	50.6	143	1927
161	159	165	0.472	Eritrea	146	59.2	154	64.2	178	33.3	178	626
161	163	159	0.499	Togo	136	62.2	166	53.2	153	53.9	171	788
161	163	161	0.492	Benin	138	61.0	174	40.5	155	52.4	155	1312
161	163	167	0.460	Rwanda	169	49.7	152	64.9	157	52.2	169	866
161	168	172	0.402	Mozambique	172	47.8	171	44.4	150	54.8	170	802
161	168	176	0.389	Congo (Democratic Republic of the)	174	47.6	148	67.2	166	48.2	182	298
161	174	174	0.394	Burundi	167	50.1	157	59.3	164	49.0	181	341
170	159	163	0.484	Cote d'Ivoire	150	56.8	169	48.7	175	37.5	147	1690
170	163	166	0.464	Senegal	154	55.4	173	41.9	171	41.2	148	1666
170	168	168	0.456	Gambia	153	55.7	172	42.5	168	46.8	157	1225
170	168	173	0.396	Guinea-Bissau	175	47.5	153	64.6	176	36.6	179	477
170	174	170	0.435	Guinea	149	57.3	178	29.5	163	49.3	161	1140
170	174	171	0.414	Ethiopia	157	54.7	176	35.9	164	49.0	172	779
170	178	181	0.352	Afghanistan	182	43.6	181	28.0	161	50.1	165	1054
177	168	175	0.392	Chad	170	48.6	177	31.8	177	36.5	152	1477
177	174	179	0.369	Central African Republic	177	46.7	170	48.6	180	28.6	175	713
177	178	178	0.371	Mali	171	48.1	182	26.2	167	46.9	163	1083
177	178	180	0.365	Sierra Leone	176	47.3	175	38.1	170	44.6	176	679
177	181	177	0.389	Burkina Faso	161	52.7	179	28.7	179	32.8	162	1124
182	182	182	0.340	Niger	166	50.8	179	28.7	181	27.2	177	627

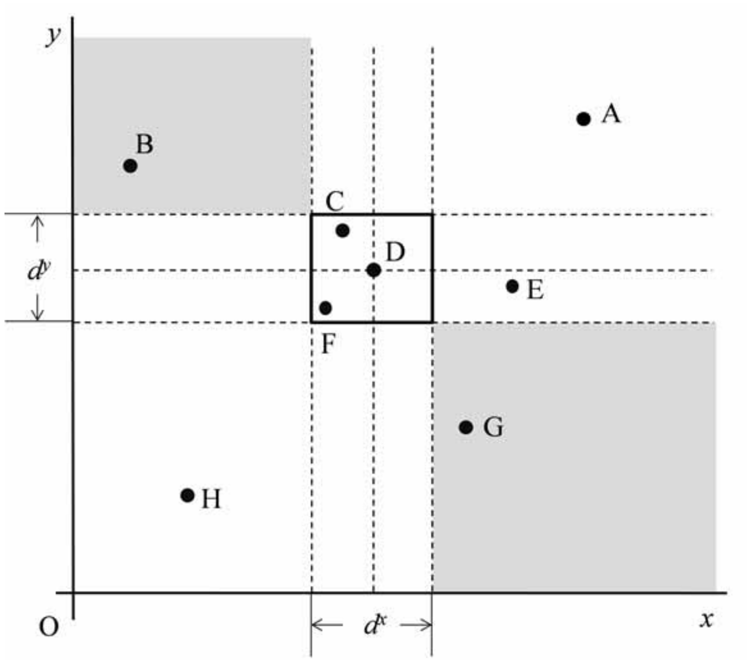
Notes:
1. DOR: the dominance order ranking EDOR: the extended dominance order ranking
HDI: the Human Development Index
2. This table was made by the author based on the data on the Human Development Report 2009 (UNDP 2009).
3. The HDI rank is determined using HDI values to the sixth decimal point.
4. The most of developed countries do not maintain the statistics of adult literacy rate, and the UNDP applies 99.0% to these countries. To keep a consistency with these countries, the author applied 99.0% to the countries that achieved over 99.0% adult literacy rate.
5. Though the value of combined gross enrolment ratio of some countries are over 100.0 in the HDR 2009, the author applied 100.0 to these countries.

Figure 1: Illustrative drawing of the dominance order ranking method

A: Comparable and non-comparable areas

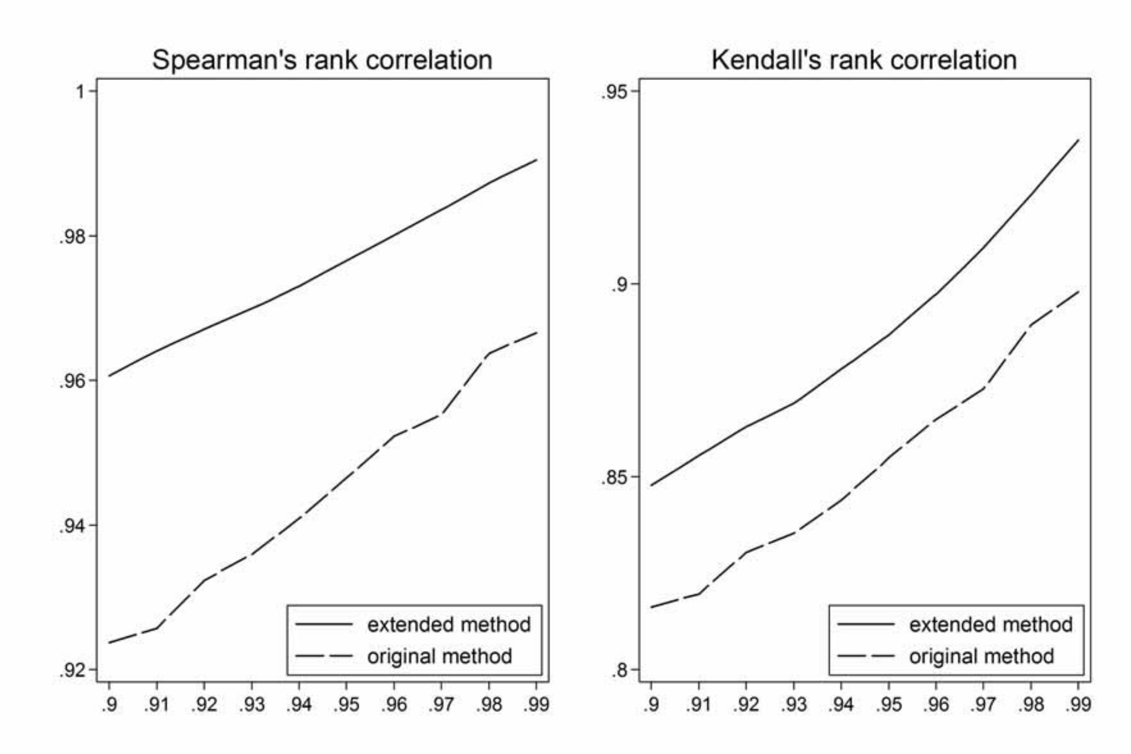


B: Introduction of a bandwidth



Note: *y* and *x* on the vertical and horizontal axes are indicators representing the level of poverty, and capital letters A to H indicate countries. The shaded areas in both panels are 'non-comparable areas' of country D, and the area bounded by the solid line in the second panel is referred to as 'indifference area' of D.

Figure 2: Results of a simulation exercise



Note: Correlation coefficients are on the vertical axis, and ρ, which indicates the influence of the hypothetical error, is on the horizontal axis. Simulations are implemented at 0.01 unit intervals for $\rho \in [0.9, 0.99]$.