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Consistent Bargaining

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Abstract

This short paper demonstrates that the equilibrium payoffs of an alternating-offers bargaining game over a unit of surplus converge to equal division provided that the parties are allowed to bargain over all the surpluses generated by the "right" to be the first to make offers. The result obtained in the present paper may provide some "justification" for other division procedures such as the divide-and-choose or the moving-knife mechanisms.

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1. Introduction

A division of one monetary unit, say a perfectly-divisible \$1, in an alternating-offers mechanism generally results in an unequal division. For example, if players' time discount factors are not "too diverse," the player who is endowed with the privilege to be the first to make an offer ends up with more than 50% of the surplus.[1] The natural question that arises from this asymmetric result is why should the second player agree to this bargaining procedure? For this reason, the present paper also "allows" the bargaining parties to allocate the surplus generated by the bargaining procedure itself which treats the bargaining parties asymmetrically.

The above-mentioned asymmetry is of course known for a long time. One "remedy" that has been proposed in the literature is to allow the parties to flip a fair coin at the beginning of each bargaining period for the purpose of restoring equality, see Osborne and Rubinstein (1990, p.53) and Muthoo (1999, p.192). There are two problems associated with the application of this remedy:

(1) Because this lottery mechanism results in a more equal expected division, one should raise the question why do we need the alternating-offers bargaining mechanism to begin with? In fact, once a lottery becomes an integral part of the bargaining procedure, the alternating-offers component can be completely dropped and the parties can simply use a coin to allocate the *entire* surplus. And, even more importantly,

(2) The lottery "remedy" introduces a *foreign element* into the bargaining process, whereby part of surplus is divided by a lottery and some other parts are divided by alternating offers. This bargaining hybrid generates an inconsistency as different surpluses would be divided according to different mechanisms.

The present paper shows that a near equal division is obtained if the bargaining parties utilize one and only one bargaining procedure for the division of the entire surplus, including the surpluses generated by the bargaining procedure itself, which assigns a higher surplus to the player who is endowed with the right to be the first to make an offer. The proximity to equal division would depend on the number of subgames that the parties agree to bargain upon, where each game divides a surplus generated by the privilege of being the first to make an offer in a subsequent game.

The extended bargaining procedure works as follows. Instead of having both parties agreeing on dividing the surplus using only one alternating-offers mechanism, the parties now also bargain over the difference in payoffs generated by assigning the right to be the first to make an offer to one of the parties. Clearly, even the proposed pre-bargaining bargaining game would result in an unequal division of surplus, which the parties may want to bargain

[1] As noted in Muthoo (1999, pp. 53–55), the first-mover advantage vanishes as the time interval between two consecutive offers becomes arbitrarily small. However, this interpretation implies that the bargaining game is conducted between two computers for which the response time is indeed infinetesimal.

over in a second pre-bargaining game, and so on. It is shown that near-equal division is obtained once we add up all the surpluses allocated to each party. The convergence to equal division is extremely fast so one or two pre-bargaining games are sufficient to obtain a near-equal division of surplus.

Section 2 briefly reviews the basic alternating-offers bargaining model. Section 3 extends the bargaining process to include pre-bargaining bargaining stages where the parties bargain over the surpluses generated by the right to be the first to make offers during a series of bargaining games that eventually lead to the standard final bargaining procedure. Section 4 concludes with discussions.

2. The Alternating-offers Bargaining Procedure

This section describes the "basic" bargaining model which serves as the benchmark for the present analysis. Two parties bargain over the division of a perfectly-divisible \$1. In the each even period $t = 0, 2, 4, \ldots$ player 1 offers a division (x_1^t, x_2^t). In each odd period $t = 1, 3, 5, \ldots$ player 2 offers (y_1^t, y_2^t). The first component is the payoff to player 1 and the second is payoff to player 2. Since both parties prefer more to less, offers must satisfy $x_1^t + x_2^t = y_1^t + y_2^t = \$1$. Once being offered, the non-offering party can accept or reject the offer. The game ends immediately with the proposed division of payoffs once an offer is accepted. If an offer is rejected, the game advances one period and the other player makes an offer. Let $0 < \delta < 1$ denote the players' common discount factor.[2]

Rubinstein (1982) proved that in the unique subgame perfect equilibrium (SPE) for this game, the parties make the following offers.

$$x_1^t = y_2^{t'} = \frac{1}{1+\delta} \quad \text{and} \quad x_2^t = y_1^{t'} = \frac{\delta}{1+\delta}. \tag{1}$$

where $t = 0, 2, 4, \ldots$ and $t' = 1, 3, 5, \ldots$. The equilibrium offers (1) are constructed such that they are always accepted by the other party in a SPE. Therefore, player 2 accepts the offer made by player 1 in $t = 0$ and the entire surplus is divided according to (x_1^0, x_2^0).

The important thing to realize is that the division rule (1) allocates different amounts of the pie to the two bargaining parties, who are identical in *all* respects (having identical preferences and the same time discount factor). Formally, we define the surplus of the original game generated by the right to be the first to make an offer by

$$S_0 \stackrel{\text{def}}{=} x_1^0 - x_2^0 = \frac{1}{1+\delta} - \frac{\delta}{1+\delta} = \frac{1-\delta}{1+\delta}. \tag{2}$$

[2] As discussed in Section 4.1, the bargaining procedure described in this paper applies also to unequal discount factors. However, in the general case, unequal division may be a consequence of the players' diverse value of time which are not necessarily related to the first-mover advantage (which is the subject of the present investigation).

Notice that $S_0 = 0$ for $\delta = 1$ and increases to $S_0 = 1$ as δ declines to zero. That is, the first-mover advantage is small for highly patient players, and increases to the entire pie for highly impatient players.

3. Consistent Division of All Surpluses

The surplus defined by (2) can be divided using a *separate* pre-bargaining procedure. In a pre-bargaining game, player 1 offers player 2 part of the surplus generated by the advantage of being the first to make an offer in a subsequent bargaining game. If player 2 accepts, the parties collect the payoffs as prescribed by division rule (1) in the form of transfers between the players (to be added up to all transfers corresponding to the outcomes of all pre-bargaining games). If player 2 rejects, the clock advances one period and it becomes 2's turn to demand from player 1 a compensation for having player 1 be the first to make an offer. If 1 rejects, the clock advances again and it becomes 1's turn to make a proposal, and so on. Thus, the extended bargaining procedure developed in this paper is composed of a (finite or infinite) series of infinite-horizon bargaining subgames. In each subgame, the parties bargain over the surplus generated by the advantage of being the first to make an offer in a subsequent game.

Consider first a single pre-bargaining game in which the parties bargain over the surplus S_0 defined by (2). By *consistent bargaining* I mean that the bargaining procedure used in the final game should also be used in the pre-bargaining game. More precisely, the division rule (1) should also be the division rule for the surplus to be divided in all pre-bargaining games, and not only for the final game. Therefore, applying division rule (1) for the surplus given in (2) implies that out of the final share prescribed to player 1 in the final game, player 1 has to "reimburse" player 2 an amount of $\delta S_0/(1+\delta)$ for the "privilege" to be the first to be making offers. Thus, sums of payoffs from one pre-bargaining game and the final bargaining game of player 1 and player 2 are

$$\pi_1^1 = \frac{1}{1+\delta} - \frac{\delta}{1+\delta} \cdot \frac{1-\delta}{1+\delta} \quad \text{and} \quad \pi_2^1 = \frac{\delta}{1+\delta} + \frac{\delta}{1+\delta} \cdot \frac{1-\delta}{1+\delta}. \tag{3}$$

Note that $\pi_1^1 + \pi_2^1 = 1$ which makes (3) a feasible and efficient allocation.

The division (3) generates a surplus from the "right" to be the first to make an offer how to divide the surplus generated by the "right" to be the first to make an offer in the final bargaining game. Formally, define

$$S_1 \stackrel{\text{def}}{=} \pi_1^1 - \pi_2^1 = \frac{(1-\delta)^2}{(1+\delta)^2} = (S_0)^2\,. \tag{4}$$

Consider now a pre-pre-bargaining game which divides the surplus S_1 according to the rule prescribed by (1). Therefore, the bargaining outcome of this stage is that player 1 should reimburse player 2 an amount of $\delta S_1/(1+\delta)$ for the right to be the first to offer in a subsequent pre-bargaining game. Hence, the total sums of payoffs collected by each player are $\pi_1^2 = \pi_1^1 - \delta(S_0)^2/(1+\delta)$ and $\pi_2^2 = \pi_2^1 + \delta(S_0)^2/(1+\delta)$. Hence,

$$\pi_1^2 = \frac{1}{1+\delta} - \frac{\delta}{1+\delta}\cdot\frac{1-\delta}{1+\delta} - \frac{\delta}{1+\delta}\cdot\frac{(1-\delta)^2}{(1+\delta)^2} \quad \text{and}$$

$$\pi_2^2 = \frac{\delta}{1+\delta} + \frac{\delta}{1+\delta}\cdot\frac{1-\delta}{1+\delta} + \frac{\delta}{1+\delta}\cdot\frac{(1-\delta)^2}{(1+\delta)^2}. \tag{5}$$

Note that $\pi_1^2 + \pi_2^2 = 1$ which makes (5) also both feasible and an efficient allocation.

The payoff allocation (5) generates a surplus from the sequence of bargainable "rights" to have the right, and so on. Formally, define

$$S_2 \stackrel{\text{def}}{=} \pi_1^2 - \pi_2^2 = \frac{(1-\delta)^3}{(1+\delta)^3} = (S_0)^3\,. \tag{6}$$

If the surplus S_2 is to be divided in a pre-pre-pre-bargaining game according to rule (1), player 1 must reimburse player 2 an amount of $\delta(S_0)^3/(1+\delta)$ for the right to be the first. Thus, the total sums of payoffs are given by

$$\pi_1^3 = \frac{1}{1+\delta} - \frac{\delta}{1+\delta}\cdot\frac{1-\delta}{1+\delta} - \frac{\delta}{1+\delta}\cdot\frac{(1-\delta)^2}{(1+\delta)^2} - \frac{\delta}{1+\delta}\cdot\frac{(1-\delta)^3}{(1+\delta)^3} \quad \text{and}$$

$$\pi_2^3 = \frac{\delta}{1+\delta} + \frac{\delta}{1+\delta}\cdot\frac{1-\delta}{1+\delta} + \frac{\delta}{1+\delta}\cdot\frac{(1-\delta)^2}{(1+\delta)^2} + \frac{\delta}{1+\delta}\cdot\frac{(1-\delta)^3}{(1+\delta)^3}. \tag{7}$$

In view of (3), (5), and (7), the total profit of each player in a finite bargaining procedure with n "pre-pre" bargaining subgames is

$$\pi_1^n = \frac{1}{1+\delta} - \frac{\delta}{1+\delta}\cdot\frac{1-\delta}{1+\delta}\cdot\sum_{t=0}^{n-1}\left(\frac{1-\delta}{1+\delta}\right)^t \quad \text{and} \quad \pi_2^n = 1 - \pi_1^n. \tag{8}$$

It can be easily verified that players' profits converge to 0.5 as the number of "pre-pre" bargaining subgames n increases. Formally, $\pi_1^n \longrightarrow 0.5$ and $\pi_2^n \longrightarrow 0.5$ as $n \longrightarrow \infty$.

Finally, as mentioned earlier, even a small number of "pre-pre" bargaining subgames is sufficient to have the parties agreeing on a near-equal division of the pie. To see this, Table 1 displays the payoff to bargaining party 1 as a function of the players' common discount rate δ and the number of "pre-pre" bargaining games n.

Table 1 demonstrates the fast convergence to equal division. Even for highly impatient bargaining parties ($\delta = 0.2$) the difference in payoffs falls below 10% in only three "pre-pre" bargaining rounds. For more patient bargaining parties, the difference falls below 10% already in the second pre-bargainig game. This means that only a small number of "pre-pre" bargaining subgames are needed to get close to equal division.

δ	0.2	0.4	0.6	0.8	0.9
π_1^0	0.8333	0.7143	0.6250	0.5556	0.5263
π_1^1	0.7222	0.5918	0.5313	0.5062	0.5014
π_1^2	0.6481	0.5394	0.5078	0.5007	0.5001
π_1^3	0.5988	0.5169	0.5020	0.5001	0.5000
π_1^4	0.5658	0.5072	0.5005	0.5000	0.5000

Table 1: Proximity to equal division in a small number of "pre-pre" bargaining games.

4. Discussions

Bargaining procedures must be agreed upon by all parties. Thus, the procedure itself should also become an integral part of the negotiation. Therefore, any bargaining mechanism that favors one party may be vetoed out by the other party. The major conclusion to be drawn from this exercise is that unequal divisions tend to be realized only if the parties apply the bargaining procedure over *part* of the surplus. This leaves some surplus to be allocated outside the bargaining process without letting the parties bargain over it directly. Once this surplus becomes an integral part of the bargaining process itself (consistent bargaining in the language of this paper) the final payoffs converge very fast to equal division.

4.1 Discount factors

The strength of the alternating-moves bargaining mechanism is the dependency of the bargaining solution on the players' discount factors (in particular, when one player is more patient than the other). In this respect, it should be pointed out that the procedure proposed in this paper works perfectly well also for unequal discount factors, in which $\delta_1 \neq \delta_2$. The problem, however, is that under unequal discount factors, unequal division *cannot be attributed solely* to the first-mover advantage. That is, unequal division may be realized because one player happens to be more patient than the other. For this reason, the analysis in this paper is confined to equal discount factors. When $\delta_1 = \delta_2 = \delta$, an unequal division is a *direct consequence* of the first-mover advantage (rather than the players' degree of impatience). The first-mover advantage is the subject of the present paper in the sense that the contribution here is to allow players to bargain on all surpluses including those that are generated by first-mover advantages.

4.2 Agenda and bargaining procedures

Perry and Reny (1993) analyze a continuous-time bargaining environment in which players can choose when to make offers, but, each player must wait an exogenously-given amount of time between making offers. Their model generally yields multiple equilibria of divisions. Fershtman (1990) has already demonstrated the importance of choosing an agenda in bargaining problems. The present paper makes the point that the choice of a bargaining procedure (in our case the decision on which player moves first) should be an integral part of the bargaining procedure itself and not be decided by a different mechanism, such as a lottery. In other words, bargaining over procedure is an essential component of the bargaining agenda in every bargaining problem.

My argument for integrating the allocation of surplus generated by the specification of the order of moves into the bargaining procedure is also motivated by general views on how game theoretic models should be constructed in order explain "real-life" behavior. Quoting from Nash (1953, p.129), "The negotiation process must be formalized and restricted, but in such a way that each participant is still able to utilize all the essential strengths of his position." Rubinstein (1991, p.919) discusses a view in which "...a game-theoretic model should include only those factors which are perceived by the players to be relevant. Modeling requires intuition, common sense, and empirical data in order to determine the relevant factors entering into the players' strategic considerations and should thus be included in the model."

Finally, the result obtained in this paper may provide some "justification" for other division procedures such as the *divide-and-choose* or the *moving-knife* mechanisms, see Brams and Taylor (1996) for discussions, references, as well as extensions to more than two players. As it turned out, economists for some reason don't make much use of these mechanisms. The present paper demonstrates that the final division of payoffs under these mechanisms can be approximated by extending the basic alternating-offers bargaining procedure to consistently apply to the division of all surpluses generated by the asymmetric treatment of players in the alternating-offers bargaining procedure itself.

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