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On the Effect of Technological Progress on Pollution: A New Distortion in an Endogenous Growth Model

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Abstract

We derive a model of endogenous growth with physical capital, human capital, and technological progress through quality-ladders. We introduce welfare-decreasing pollution in the model, which can be reduced through the development of cleaner technologies. From the quantitative analysis of the model, we show clear evidence that the new externality from technological progress to pollution considered in this model is sufficiently strong to induce underinvestment in R&D as an outcome of the decentralized equilibrium. An important policy implication of the main result of this article is a justification to subsidize the research in cleaner technologies.

JEL Classification: O13, O15, O31, O41, Q50

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1 Introduction

Economic growth is often associated with significant environmental problems, since it is typically accompanied by increases in natural resource use and in undesirable pollutant emissions. The environmental damages unleashed by economic development are not only harmful per se, but also diminish future growth prospects through the degradation of basic productive assets, such as soil, water and the atmosphere, that are essential for human activities, thus calling into question the sustainability of such growth. It has long been clear that one way out of this conundrum is to develop new technologies, especially those that bring positive economic productivity effects and are also environmentally friendly.

Initially, technical progress was incorporated into growth models exogenously, which showed its potential benefits but didn't explain how it occurred. More recently there has been a proliferation of studies of endogenous technical change, which analyse the interaction between choices in the dynamic economic system and technological development. Some of these studies include environmental variables in the analysis. Smulders [28] is an excellent non-technical summary of the evolution of technology's role in growth models with natural resources, which points out that the form of technological change is crucial and that, given the costs of technological improvements, appropriate regulation is essential to ensure that such change will continue "at a sufficient rate and in the right direction" (pg. 172).[1]

Löschel [?] presents an overview of economic models of environmental policy that incorporate technological change, both exogenous and endogenous. The author emphasizes the overwhelming evidence for endogenous technological change, especially over a long time horizon, although he highlights the complexities inherent to the inclusion of such endogeneity in policy models. A more recent survey, focusing on models for climate policy analysis, is Gillingham, Newell, and Pizer [10].

As Jaffe, Newell, and Stavins [15] make clear, both technological innovation and pollution are characterized by market failures leading to a number of well-known externalities. The negative effects of pollution fall (wholly or partly) on third parties that

[1] Some authors recognize that not all technology is good for the environment. For instance, both Bovenberg and Smulders [5] and Ikazaki [14] consider distortions in models with pollution-augmenting technological change. More recently, Cunha-e-Sá, Leitao and Reis [7] develop a model which distinguishes clean and dirty technological change.

are not involved in the pollution-producing decision, thus creating an environmental externality. As for technology, there are knowledge externalities arising from one firm's costly investment in new technology, creating beneficial spillovers for other firms, since new knowledge has a public-good nature.There may also be adoption externalities if one firm's use of a technology lowers costs for other firms, through learning effects or network externalities. This double market failure diminishes private incentives for the development of green technologies and strengthens the case for government intervention, preferably through the application of combined policies instead of single instruments.[2]

In this paper, we provide a quantification of externalities in an endogenous growth model with pollution-diminishing R&D and human capital accumulation. In our model, improvements in technological quality mean fewer emissions, i.e. a cleaner technology. Furthermore, we show that considering pollution in an endogenous growth model with physical capital, human capital, and R&D introduces distortions, not only on the allocation of resources throughout sectors in the economy, but also on output and capital growth rates. We quantify these distortions by means of a calibration exercise. The negative effect of R&D on pollution is an additional externality, which drives the decentralized equilibrium further from the social optimum and adds an extra reason to obtain underinvestment in R&D. In this sense, this paper also contributes to a large literature on the optimality of investments in R&D (for a good revision see Alvarez, Paleaz, and Groth [3]).

The following section presents the model, whereas section 3 shows the main relationships between variables from a social planner's point of view. Section 4 presents the decentralized equilibrium. Section 5 presents the allocations of human capital and other macroeconomic variables and shows the distortions in the market solution. In section 6 we calibrate the model and quantify the distortions presented in the previous section, whereas section 7 concludes.

[2] The authors also refer the problem of incomplete information as an additional reason for slow diffusion of better technologies. This market failure may explain, for instance, the widespread under-investment in energy-saving technologies.

2 Model

We build an endogenous growth model of a closed economy with physical and human capital accumulation as well as quality-improving R&D (quality-ladders or vertical R&D), to which we add utility-decreasing pollution. The flow of pollution emissions arises from production in the economy and decreases with the quality index for technologies. The higher the technology index, the cleaner is the technology used in the economy.

2.1 Production Factors and Final Goods

2.1.1 Capital Accumulation

The accumulation of physical capital (K_P) in the economy arises through production that is not consumed, and is subject to depreciation:

$$\dot{K}_P = Y - C - \delta_P K_P \tag{1}$$

where Y denotes production of final goods, C is consumption, and δ_P represents depreciation.

We propose that human capital K_H is produced using human capital allocated to schooling, according to:

$$\dot{K}_H = \xi H_H - \delta_H K_H \tag{2}$$

where H_H are school hours, $\xi > 0$ is a parameter that measures productivity inside schools, and $\delta_H \geq 0$ is the depreciation of human capital.

Individual human capital can be divided into skills in final good production (H_Y), school attendance (H_H), and doing R&D (H_R). Assuming that the different human capital activities aren't done cumulatively, we have:

$$K_H = H_Y + H_H + H_R \tag{3}$$

2.1.2 R&D Technology

Technological capital, or new qualities of the current technology, Q_R, are produced in a R&D sector with human capital employed in R&D labs (H_R) and using the current quality (Q_R). At each point in time, an improvement from the quality level k to $k+1$ occurs with probability $\mu_{k_i} = \frac{H_R^\lambda Q_R^\phi}{q_{k_i}^{\frac{\alpha}{1-\alpha}}}$. If an innovation occurs in a given sector i, quality grows at rate $(\gamma^{(k_i+1)\frac{\alpha}{1-\alpha}} - \gamma^{k_i\frac{\alpha}{1-\alpha}})/\gamma^{k_i\frac{\alpha}{1-\alpha}} = \gamma^{\frac{\alpha}{1-\alpha}} - 1$. At any sector at any time, an innovation occurs with probability $\mu = \frac{H_R^\lambda Q_R^\phi}{Q_R}$. Thus, $\dot{Q}_R$ is given by:

$$\dot{Q}_R = \left(\gamma^{\alpha/(1-\alpha)} - 1\right) H_R^\lambda Q_R^\phi \tag{4}$$

where λ measures duplication ("stepping-on-toes") effects and $0 < \phi < 1$ measures the degree of spillover externalities in R&D across time, as in Jones (1995).

2.1.3 Final Good Production

The final good is a differentiated one, produced with a Cobb-Douglas technology:

$$Y = AD^\eta K_P^\beta,\ \beta < 1,\ \eta > 0 \tag{5}$$

where $A \in [0,1]$ is an index of the technology production potential in a given period of time. Thus A approaches 1 if the economy is approaching it production frontier. D is an index of intermediate goods and is produced using the following Dixit-Stiglitz CES technology:

$$D = \left[\int_0^1 \left(\sum_0^{k_i} q_{k_i} x_{k_i}\right)^\alpha\right]^{\frac{1}{\alpha}} \tag{6}$$

The elasticity of substitution between varieties is measured by $0 < \alpha < 1$. x_i is the intermediate good i and is produced in a differentiated goods sector using human capital: $x_i = H_{Y_{x_i}}$.

This means that (5) can be re-written as:

$$Y = AQ_R^{\eta\frac{1-\alpha}{\alpha}} K_P^{\beta} H_Y^{\eta} \tag{7}$$

where $Q_R = \int_0^1 q_{k_i}^{\frac{\alpha}{1-\alpha}}$ is an aggregate quality index.

2.2 Consumers

The utility function has the following functional form:

$$U(C_t, K_{S_t}) = \frac{\tau}{\tau-1} \int_0^{\infty} e^{-\rho t} \left[C_t^{\frac{\tau-1}{\tau}} - bP_t^{\kappa} \right] dt \tag{8}$$

where ρ is the utility discount rate, $b > 0$ gives the level of disutility that a consumer has from pollution, $\kappa > 1$ describes the intensity of the pollution effect in welfare. This parameter means that the effect of pollution is increasing in its level, as high levels of pollution is proportionally more damaging than low levels of pollution.[3] P is pollution which is equal to:

$$P = \frac{K_P^{\beta} H_Y^{\eta} A^{\chi}}{Q_R^{\epsilon}} = Y \frac{A^{\chi-1}}{Q_R^{\epsilon+\eta\frac{1-\alpha}{\alpha}}} \tag{9}$$

Pollution increases significantly with production, specially since $\chi > 1$, which means that higher values of A mean more production but also more pollution. As the economy approaches the technological frontier, pollution increases. On the other hand, new cleaner technologies decrease pollution. The effect of Q_R^{ϵ} in pollution is crucial as it means that high technological knowledge decreases pollution.

3 Optimal Growth

It is clear that when assets directly provide utility, while simultaneously acting as inputs to the production function, the decentralized equilibrium will in general not

[3] The t subscripts are dropped in the remaining sections for ease of notation.

maximize aggregate welfare. Thus we must solve a social planner's problem. In this section we derive the conditions associated with the maximization of (8) subject to the production function (5) as well as the transition equations for the different types of capital (1), (2), and (4).

The problem gives rise to the following Hamiltonian function:

$$\begin{aligned} \mathcal{H} &= \frac{\tau}{\tau-1}C^{\frac{\tau-1}{\tau}} - b\left[\frac{K_P^{\beta}H_Y^{\eta}A^{\chi}}{Q_R^{\epsilon}}\right]^{\kappa} + \lambda_P\left(AQ_R^{\eta\frac{1-\alpha}{\alpha}}K_P^{\beta}H_Y^{\eta} - C - \delta_P K_P\right) + \\ &\quad + \lambda_H\left(\xi H_H - \delta_H K_H\right) + \lambda_R(\left(\gamma^{\alpha/(1-\alpha)} - 1\right) H_R^{\lambda}Q_R^{\phi}) \end{aligned} \tag{10}$$

where the λ_j are the co-state variables for each stock K_j, with $j = P, H$ and Q_R. Considering choice variables A, C, H_Y, and H_R (and substituting H_H for $K_H - H_Y - H_R$ using (3)), the first order conditions yield:

$$\frac{\partial U}{\partial C} = \lambda_P \tag{11}$$

$$b\kappa\chi\frac{P^{\kappa}}{A} = \frac{\lambda_P Y}{A} \tag{12}$$

$$-b\kappa\eta\frac{P^{\kappa}}{H_Y} - \lambda_H\xi + \frac{\lambda_P\eta Y}{H_Y} = 0 \tag{13}$$

$$\lambda_R = \frac{\lambda_H\xi}{\lambda\left(\gamma^{\alpha/(1-\alpha)} - 1\right)H_R^{\lambda-1}Q_R^{\phi}} \tag{14}$$

as well as:

$$-\dot{\lambda}_P + \rho\lambda_P = \frac{\lambda_P\beta Y}{K_P} - \delta_P\lambda_P - b\kappa\beta\frac{P^{\kappa}}{K_P} \tag{15}$$

$$\frac{\dot{\lambda}_H}{\lambda_H} = \rho + \delta_H - \xi \tag{16}$$

$$\dot{\lambda}_R = \rho\lambda_R - b\kappa\epsilon\frac{P^{\kappa}}{Q_R} - \frac{\lambda_P\eta\left(\frac{1-\alpha}{\alpha}\right)Y}{Q_R} - \lambda_R\left(\gamma^{\alpha/(1-\alpha)} - 1\right)\phi H_R^{\lambda}Q_R^{\phi-1} \tag{17}$$

with $\frac{\partial U}{\partial C} = C^{-\frac{1}{\tau}}$ representing the marginal utility of consumption.

3.1 Optimal Growth Rates

Growth rates will, by definition, be constant, so equation (1) tells us that K_P, Y, and C all grow at the same rate. Furthermore, K_H and all its components will also be growing at that same rate, respecting equations (2) and (3).

Denote the growth rate of technological capital as g_{Q_R} and the growth rate of human capital as g_{K_H}. From equation (4) we can see that these two growth rates have to respect the following relation: $g_{K_H} = \frac{(1-\phi)}{\lambda} g_{Q_R}$.

In the steady-state, we can obtain the human capital growth rate as follows. From (13) we find $g_{\lambda_H} = g_{\lambda_P} + g_Y - g_{K_H}$ and using equation (16) we can then replace the previous two equations into $-\frac{1}{\tau} g_Y = \frac{\dot{\lambda}_P}{\lambda_P}$, which we calculated from (11) and the substitution yields:

$$\left(1 - \frac{1}{\tau}\right) g_Y = g_{K_H} + \rho + \delta_H - \xi \tag{18}$$

To simplify the above expression, we log-differentiate equation (7) and substitute in this result the formula for g_A, which we get from (12), we then get:

$$g_Y \left[(1-\beta) + \left(\frac{\frac{1}{\tau} + \kappa - 1}{\kappa(\chi - 1)} \right) \right] = g_{K_H} \left[\eta \left(1 + \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{\lambda}{1-\phi} \right) \right) + \frac{\kappa \left(-\epsilon - \eta \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{\lambda}{1-\phi} \right) \right)}{\kappa(\chi - 1)} \right] \tag{19}$$

We then substitute this last expression into equation (18) to get the growth rate of human capital:

$$g^*_{K_H} = \frac{\xi - \delta_H - \rho}{1 - \left[\frac{\eta\left(1 + \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\lambda}{1-\phi}\right)\right) + \frac{\kappa\left(-\epsilon - \eta\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\lambda}{1-\phi}\right)\right)}{\kappa(\chi-1)}}{(1-\beta) + \frac{\frac{1}{\tau} + \kappa - 1}{\kappa(\chi-1)}} \right] \left(1 - \frac{1}{\tau}\right)} \tag{20}$$

Using the fact that $g_{Q_R} = \frac{\lambda}{1-\phi} g_{K_H}$ we solve for the growth rate of technological capital:

$$g_{Q_R}^* = \frac{\frac{\lambda}{(1-\phi)}(\xi - \delta_H - \rho)}{1 - \left[\frac{\eta\left(1+\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\lambda}{1-\phi}\right)\right)+\frac{\kappa\left(-\epsilon-\eta\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\lambda}{1-\phi}\right)\right)}{\kappa(\chi-1)}}{(1-\beta)+\frac{\frac{1}{\tau}+\kappa-1}{\kappa(\chi-1)}}\right]\left(1-\frac{1}{\tau}\right)} \tag{21}$$

Substituting equation (20) in equation (19) we get the output growth rate:

$$g_Y^* = \frac{(\xi - \delta_H - \rho)\left[\eta\left(1+\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\lambda}{1-\phi}\right)\right)+\frac{\kappa\left(-\epsilon-\eta\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\lambda}{1-\phi}\right)\right)}{\kappa(\chi-1)}\right]}{\left[1 - \left[\frac{\eta\left(1+\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\lambda}{1-\phi}\right)\right)+\frac{\kappa\left(-\epsilon-\eta\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\lambda}{1-\phi}\right)\right)}{\kappa(\chi-1)}}{(1-\beta)+\frac{\frac{1}{\tau}+\kappa-1}{\kappa(\chi-1)}}\right]\left(1-\frac{1}{\tau}\right)\right]\left[(1-\beta)+\left(\frac{\frac{1}{\tau}+\kappa-1}{\kappa(\chi-1)}\right)\right]} \tag{22}$$

If we use equation(12), after we log-differentiate, we get:

$$g_A^* = g_Y^*\left[\frac{1-\kappa-\frac{1}{\tau}}{\kappa(\chi-1)}\right] + g_{K_H}^*\left[\frac{\kappa\left(-\epsilon-\eta\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\lambda}{1-\phi}\right)\right)}{\kappa(\chi-1)}\right]$$

Then by substitution equations (20) and (22) we reach the following final expression for g_A^*:

$$\begin{aligned} g_A^* \;=\; & \left[\frac{\xi - \delta_H - \rho}{\left[1 - \left[\frac{\eta\left(1+\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\lambda}{1-\phi}\right)\right)+\frac{\kappa\left(-\epsilon-\eta\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\lambda}{1-\phi}\right)\right)}{\kappa(\chi-1)}}{(1-\beta)+\frac{\frac{1}{\tau}+\kappa-1}{\kappa(\chi-1)}}\right]\left(1-\frac{1}{\tau}\right)\right]\left[\kappa(\chi-1)\right]}\right]\times \\ & \times\left[\begin{array}{c}\left(1-\kappa-\frac{1}{\tau}\right)\left(\frac{\eta\left(1+\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\lambda}{1-\phi}\right)\right)+\frac{\kappa\left(-\epsilon-\eta\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\lambda}{1-\phi}\right)\right)}{\kappa(\chi-1)}}{(1-\beta)+\frac{\frac{1}{\tau}+\kappa-1}{\kappa(\chi-1)}}\right)+ \\ +\left(\kappa\left(-\epsilon-\eta\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\lambda}{1-\phi}\right)\right)\right)\end{array}\right] \end{aligned} \tag{23}$$

From log-differentiating equation (9) we get:

$$g_P^* = \beta g_Y^* + \left(\eta - \frac{\epsilon\lambda}{1-\phi}\right) g_{K_H}^* + \chi g_A^* \tag{24}$$

The derivation of the growth rates in the decentralized equilibrium are in Appendix B.

4 Decentralized Equilibrium

In the decentralized equilibrium both consumers and firms make choices that maximize, respectively, their own felicity or profits. Consumers maximize their intertemporal utility function:

$$\frac{\tau}{\tau-1}\int_0^\infty e^{-\rho t}\left[C_t^{\frac{\tau-1}{\tau}} - b\left(\frac{K_{t,P}^{\beta} H_{t,Y}^{\eta} A_t^{\chi}}{Q_{t,R}^{\epsilon}}\right)^{\kappa}\right] dt$$

subject to the budget constraint:

$$\dot{a} = (r-\delta_p)a + W_H H_H - C \tag{25}$$

where a represents the family physical assets, r is the return on physical capital, and W_H is the market wage. The market price for the consumption good is normalized to 1. Since it is making an intertemporal choice, the family also takes into account equation (2) which represents human capital accumulation.[4]

The markets for purchased production factors are assumed to be competitive. From the substitution of the technological index (A) from equation (9) into equation (7) we get:

$$Y = K_P^{\beta\left(\frac{\chi-1}{\chi}\right)} H_Y^{\eta\left(\frac{\chi-1}{\chi}\right)} Q_R^{\left(\eta\frac{1-\alpha}{\alpha}+\frac{\epsilon}{\chi}\right)} P^{\frac{1}{\chi}} \tag{26}$$

[4]See Appendix A for the first order conditions in the decentralized equilibrium.

From this problem we know that returns on production are as follows:

$$W_H = \frac{\eta \left(\frac{\chi - 1}{\chi}\right) Y}{H_Y} \tag{27}$$

$$r = \frac{\beta \left(\frac{\chi - 1}{\chi}\right) Y}{K_P}, \tag{28}$$

where p_D represents the price for the index of intermediate capital goods.

$$T = \frac{1}{\chi} \frac{Y}{P} \tag{29}$$

where T is a constant tax rate which firms have to pay to the Government due to pollution they incur in their production process. This imply that in the decentralized equilibrium $g_Y = g_P$.

Each firm in the intermediate goods sector owns an infinitely-lived patent for selling its variety x_i. Producers of differentiated goods act under monopolistic competition in which they sell their own variety of the intermediate capital good x_i and maximize operating profits, π_i:

$$\pi_i = (p_i - w_H) x_{i,} \tag{30}$$

where p_i denotes the price of intermediate good i and r is the unit cost of x_i. The demand for each intermediate good results from the maximization of profits in the final goods sector. Profit maximization in this sector implies that each firm charges a price of:

$$p_i = p = w_H / \alpha. \tag{31}$$

After insertion of equations (31) and (27) into (30), profits can be rewritten as:

$$\pi = \frac{(1 - \alpha) \eta \left(\frac{\chi - 1}{\chi}\right) q_{k_i}^{\frac{\alpha}{1 - \alpha}} Y}{Q_R} \tag{32}$$

Let ν denote the value of an innovation, defined by:

$$\nu_t = \int_t^{\infty} e^{-[R(\tau)-R(t)]}\pi(\tau)d\tau, \text{ where } R(\tau) = \int_0^t r(\tau)d\tau. \tag{33}$$

Taking into account the cost of an innovation as determined by equation (4), free-entry in R&D implies that,

$$W_H H_R = \nu \frac{H_R^{\lambda} Q_R^{\phi}}{q_{k_i}^{\frac{\alpha}{1-\alpha}}} \text{ if } \dot{Q}_R > 0 \quad (H_R > 0); \tag{34}$$

$$W_H H_R > \nu \frac{H_R^{\lambda} Q_R^{\phi}}{q_{k_i}^{\frac{\alpha}{1-\alpha}}} \text{ if } \dot{Q}_R = 0 \quad (H_R = 0). \tag{35}$$

Finally, the no-arbitrage condition requires that investing in patents has the same return as investing in bonds:

$$\frac{\dot{\nu}}{\nu} = (r - \delta_P) - \pi/\nu. \tag{36}$$

The free-entry condition also means that $\frac{\dot{\nu}}{\nu} = \frac{\dot{w}_H}{w_H} - \phi\frac{\dot{Q}_R}{Q_R} + (1-\lambda)\frac{\dot{H}_R}{H_R}$. This fully describes the economy.

4.1 Growth Rates in the Decentralized Equilibrium

In the steady-state, we can obtain the human capital growth rate of the decentralized equilibrium as follows. By using equation(49) and replacing it in $g_{\lambda'_H} = g_{\lambda_a} + g_W$ which we get by (47), we find $\frac{\dot{\lambda}_a}{\lambda_a} = \rho + \delta_H - \xi - g_W$. From (27)we get $g_W = g_Y - g_{K_H}$. Substituting this last equation in the previous one and introducing both in $-\frac{1}{\tau}g_y = \frac{\dot{\lambda}_a}{\lambda a}$ which we find in (46) we get:

$$\left(\frac{1}{\tau} - 1\right) g_Y + g_{K_H} = \xi - \rho - \delta_H \tag{37}$$

By log-differentiating the production function (26), using the fact that $g_{Q_R} = \frac{\lambda}{1-\phi} g_{K_H}$, and that by equation $g_P = g_Y$ (29), and an assumed constant tax rate in the decentralized equilibrium, we get:

$$g_Y \left[(1-\beta) \left(1 - \frac{1}{\chi} \right) \right] = g_{K_H} \left[\left(\frac{\lambda}{1-\phi} \right) \left(\eta \frac{1-\alpha}{\alpha} + \frac{\epsilon}{\chi} \right) + \eta \left(\frac{\chi - 1}{\chi} \right) \right] \tag{38}$$

Substituting this last expression into (37) we get:

$$g_{K_H}^{DE} = \frac{\xi - \delta_H - \rho}{1 - \left[\frac{\left(\frac{\lambda}{1-\phi}\right)\left(\eta \frac{1-\alpha}{\alpha} + \frac{\epsilon}{\chi}\right) + \eta\left(\frac{\chi-1}{\chi}\right)}{(1-\beta)\left(1-\frac{1}{\varkappa}\right)} \right] \left(1 - \frac{1}{\tau}\right)} \tag{39}$$

By using the fact that $g_{Q_R} = \frac{\lambda}{1-\phi} g_{K_H}$ we solve for the growth rate of technological capital:

$$g_{Q_R}^{DE} = \frac{\left(\frac{\lambda}{1-\phi}\right)(\xi - \delta_H - \rho)}{1 - \left[\frac{\left(\frac{\lambda}{1-\phi}\right)\left(\eta \frac{1-\alpha}{\alpha} + \frac{\epsilon}{\chi}\right) + \eta\left(\frac{\chi-1}{\chi}\right)}{(1-\beta)\left(1-\frac{1}{\chi}\right)} \right] \left(1 - \frac{1}{\tau}\right)} \tag{40}$$

By substituting equation (39) into (38) we find:

$$g_Y^{DE} = \frac{(\xi - \delta_H - \rho) \left[\left(\frac{\lambda}{1-\phi} \right) \left(\eta \frac{1-\alpha}{\alpha} + \frac{\epsilon}{\chi} \right) + \eta \left(\frac{\chi-1}{\chi} \right) \right]}{\left[1 - \left[\frac{\left(\frac{\lambda}{1-\phi}\right)\left(\eta \frac{1-\alpha}{\alpha} + \frac{\epsilon}{\chi}\right) + \eta\left(\frac{\chi-1}{\chi}\right)}{(1-\beta)\left(1-\frac{1}{\chi}\right)} \right] \left(1 - \frac{1}{\tau}\right) \right] \left[(1-\beta) \left(1 - \frac{1}{\chi} \right) \right]} \tag{41}$$

5 Optimality of Human Capital Allocations

The shares of human capital allocated to the different sectors in the social planner framework are:

$$u_H^* \;=\; \frac{H_H}{K_H} = \frac{1}{\xi}\left(g_{K_H}^* + \delta_H\right) \tag{42}$$

$$u_R^* \;=\; \frac{H_R}{K_H} = \frac{\frac{\lambda}{\eta}\left(\frac{\epsilon}{\chi} + \eta\left(\frac{1-\alpha}{\alpha}\right)\right)\left(\frac{\chi}{\chi-1}\right) g_{Q_R}^*}{\xi - \delta_H + g_{Q_R}^*\left(\frac{(1-\phi)(\lambda-1)}{\lambda}\right)} u_Y^* \tag{43}$$

The shares of human capital allocated to the different sectors in the decentralized equilibrium are:

$$u_H^{DE} \;=\; \frac{H_H}{K_H} = \frac{1}{\xi}\left(g_{K_H}^{DE} + \delta_H\right) \tag{44}$$

$$u_R^{DE} \;=\; \frac{H_R}{K_H} = \frac{\frac{(1-\alpha)}{\left(\gamma^{\alpha/(1-\alpha)}-1\right)} g_{Q_R}^{DE}}{\xi - \delta_H + g_{Q_R}^{DE}\left[\frac{(1-\phi)(\lambda-1)}{\lambda} + \phi\right]} u_Y^{DE} \tag{45}$$

The equations that were presented in this section provide a basis for the comparison between the optimal solution with the decentralized equilibrium solution.[5] In fact, this model incorporates several reasons why the decentralized equilibrium solution may be different from the social planner solution:

- The creative destruction effect - or the probability of success of an innovation is internalized by the social planner but not by the agents in the decentralized equilibrium. An increasing rate of creative destruction reduces the time span during which a newly invented technology creates value for the inventor. This is measured by $\left(\gamma^{\alpha/(1-\alpha)} - 1\right)$ and increases the effort in R&D in the market.

- Spillovers in R&D - the R&D activity depends on past knowledge. This is a positive effect that firms do not internalize. It contributes to the sub-optimallity of R&D and is measured by ϕ.

[5] A detailed explanation of the calculation of these shares can be found in Appendix B. In the calibration section we also calculate u_R and u_Y separately, using the fact that $u_R + u_Y + u_H = 1$.

- Duplication effects in R&D - Some R&D efforts would be redundant in comparison with others. This effect will contribute to a higher effort in the decentralized equilibrium than the social planner would do. This is measured by λ.
- Specialization gains from R&D - Having better qualities available increases production and welfare, which is an effect that is not internalized by firms. It is measured by $\eta > \alpha$.
- Externality from Pollution - the lower the effect of the technological index A in pollution and the higher the effect of the technological quality index, the higher the allocation of human capital to R&D in the optimal solution. It is measured by χ (the effect of the technological index A in pollution) and ϵ (the effect of the technological quality index in pollution).

Contrary to what happens in some previous papers, here, due to the introduction of pollution, growth rates in the social planner solutions also deviate from the decentralized equilibrium solution. However, as we could see in the following sections, these deviations are relatively less important than those in shares. As usual in the studies that intend to evaluate distortions between the social planner and decentralized equilibrium solutions, this evaluation is a quantitative issue. Thus we now implement a calibration exercise to evaluate the distortions.

6 Results and Calibration

6.1 Calibration Procedure

In this section, we present and justify the calibrated values for the parameters. For the share of physical capital in the final good production, we use the typical value, $\beta = 0.36$. For the markup in the differentiated goods sector ($1/\alpha$), we have based on Norrbin [22] to use a value of $1/\alpha = 1.33$. We use the output growth rate of Western Europe in the period from 1973 to 2001, reported by Maddison, as a base for our calibration exercise [21]. As in Strulik [32], we use the TFP growth rate to estimate the growth rate of R&D, using the facts that $g_{K_P} = g_Y$ and $g_{K_H} = \frac{1-\phi}{\lambda} g_{Q_R}$. Using the values already defined we reach that $g_{Q_R} = 0.006532$. Following the approach of Strulik, we estimate the parameter of innovation γ such that the lifetime of an innovation is 20 years. This implies that $\gamma = 1.0418$. For the elasticity of intertemporal substitution (τ), we follow Jones at. al. [19] in considering $\tau = 0.8$. We note that this is an important parameter and because of that we have confirmed that a value in this range is also empirically supported (see Guvenen, [13]).

The discount rate is set to 0.01, a value in the range used in the literature. The depreciation rate of physical capital is also set to be in a lower bound of the interval usually seen in the literature (0.01). It is usual to see endogenous growth models with human capital accumulation considering no depreciation of human capital, thus we also set $\delta_H = 0$. Small oscillations in these parameters are not of crucial importance for our results. For the duplication effect (λ) we have followed Strulik [32] and others in considering $\lambda = 0.5$. For the spillover effects, we follow Reis and Sequeira [24] in considering $\phi = 0.4$, which is an appropriate value for models with human capital accumulation, as the authors argue.

The parameter that relates the technological intensiveness A with pollution (χ), the productivity of human capital in the human capital accumulation process (ξ), and the weight given to pollution in the utility function (k) are calibrated according to the following conditions: $g_Y^{DE} = 0.0188$, $u_R^* > 0$, and $g_Y^* > 0$. These conditions give a single value for $\xi = 0.022588$ and a single value for $\chi = 2.1345$. However, given these values, k can assume any value. Thus, for the effect of pollution in the utility, we have adopted the value used in Stokey [31], $\kappa = 1.2$. We also note that ξ is a reasonable value comparing to values for the same parameter used in the economic growth literature. We can also compare the value of χ with evidence for the empirical relationship between output and pollution. This concept predicts an inverted U-shaped relationship between the level of income and pollution, usually known as the Environmental Kuznets Curve (EKC). However, recently, specific country studies have noted that the relationship between income and pollutants is almost linear or, for some pollutants, N-shaped. From Roca et al. [25] we see that the EKC is rejected and the effect of income in pollution is near 1.2. From Akbostanci et al. [1], we learn that this coefficient for

Turkey is about 3.5. From Song et al. [29], we see coefficients from 1.5 to 3, in a work applied to China. Thus, the value of 2.1345 is in the range of plausible values. Finally, for the effect of technological progress on pollution, ϵ, we use 0.5, meaning that it has decreasing returns in influencing pollution. As this parameter governs the externality of technology on pollution, which we want to focus, we will do significant sensitivity analysis on it.

Table 1 summarizes the benchmark values for calibration.

Table 1 - Parameters Values (Benchmark)

Production and Utility					
g_Y	β	δ_P	τ	ρ	α
0.0188	0.36	0.01	0.8	0.01	0.75
Human Capital and R&D					
ξ	δ_H	η	λ	ϕ	γ
0.022538	0	0.64	0.5	0.4	1.0758
Pollution					
χ	ϵ	κ			
2.1345	0.5	1.2			

6.2 Results

In this section, we present results for the calibration exercise. First we present the main economic indicators for the benchmark economy and a comparison with optimal values.

Table 2 - Statistics for the Benchmark Economy

Decentralized Economy					
g_Y	g_{K_H}	g_{Q_R}	u_R	u_H	u_Y
1.88%	0.78%	0.65%	24%	35%	41%
Optimal Solution					
g_Y	g_{K_H}	g_{Q_R}	u_R	u_H	u_Y
0.00%	1.25%	1.04%	15%	56%	29%

This exercise shows that due to pollution, the social planner wants to decelerate the economy, leading to an output growth rate of 0% at the equilibrium. However, due to the effect of technological progress on pollution and of human capital on technological progress, the social planner solution leads to higher growth rates of human capital and R&D. This implies that the social planner allocates more human capital to the human capital production (56%) than the decentralized equilibrium does (35%). Due to a specially high creative destruction effect that results from our calibration exercise, the decentralized equilibrium allocates more human capital to R&D than the social planner.

As we introduce a new positive externality of R&D in the model, which is explained by its positive effect on reducing pollution, we want to know the quantitative effect of this distortion. Thus, we present figures in which we increase this effect (the parameter ϵ) and see what happens to the allocations of human capital to the different sectors of the economy.

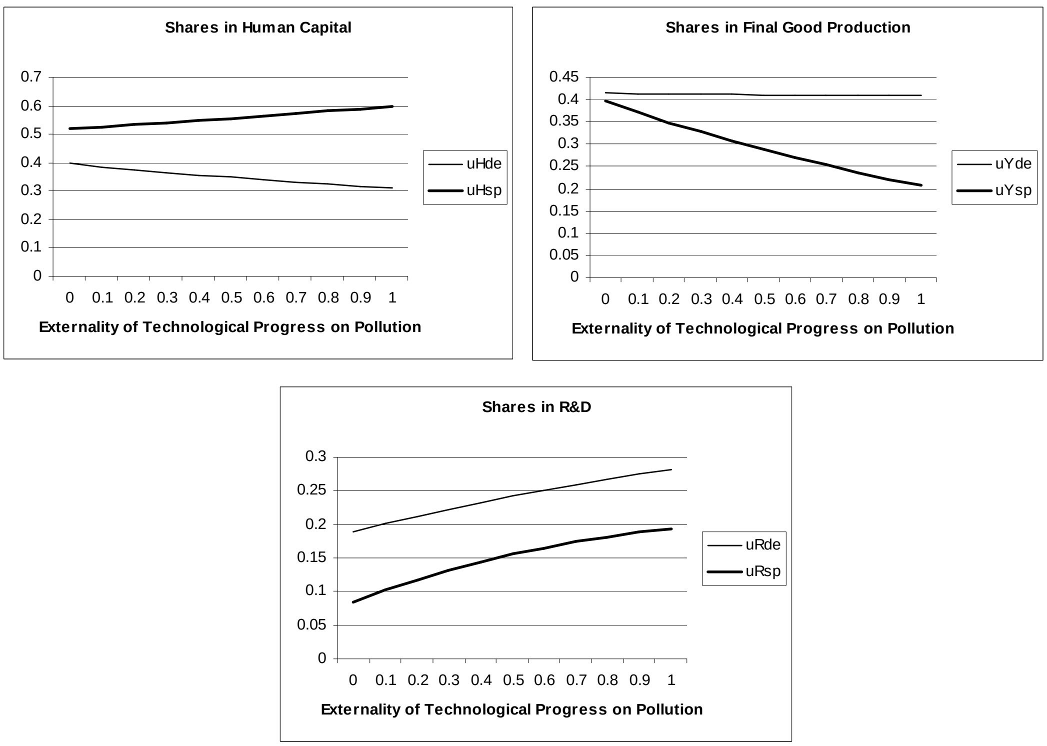

Figure 1: Human Capital Allocations to the Different Sectors in the decentralized equilibrium and in the social planner solution

From these figures, we can show that as the externality of R&D in pollution increases, the optimal allocation of the final good decreases to allow an increase in allocations to the human capital production sector (schools) and to the R&D sector. We note that the difference between the social planner and the decentralized equilibrium allocation to R&D decreases as ϵ increases. This means that the positive effect of R&D in decreasing pollution decreases the overallocation to R&D that results from the strong effect of creative destruction. While the initial difference (for $\epsilon = 0$) is above 10%, the difference for $\epsilon = 1$ is below 9%.

Due to the fact that g_{Q_R} depends on ϵ and that following our calibration strategy γ depends on g_{Q_R}, as ϵ decreases γ also decreases, which increases the creative destruction effect. As this fact results only from the strategy we have followed to calibrate the model, we want to show a sensitivity analysis in which we kept constant the creative destruction effect, as the new externality from technological progress on pollution increases. Thus, we use the value for $\gamma = 1.1$ from Strulik [32] (for $\epsilon = 0.5$, this implies a lifetime of patents of 50 years, which is on the upper bound of the reasonable interval), and maintain this value throughout the exercise. The following figure shows the results.

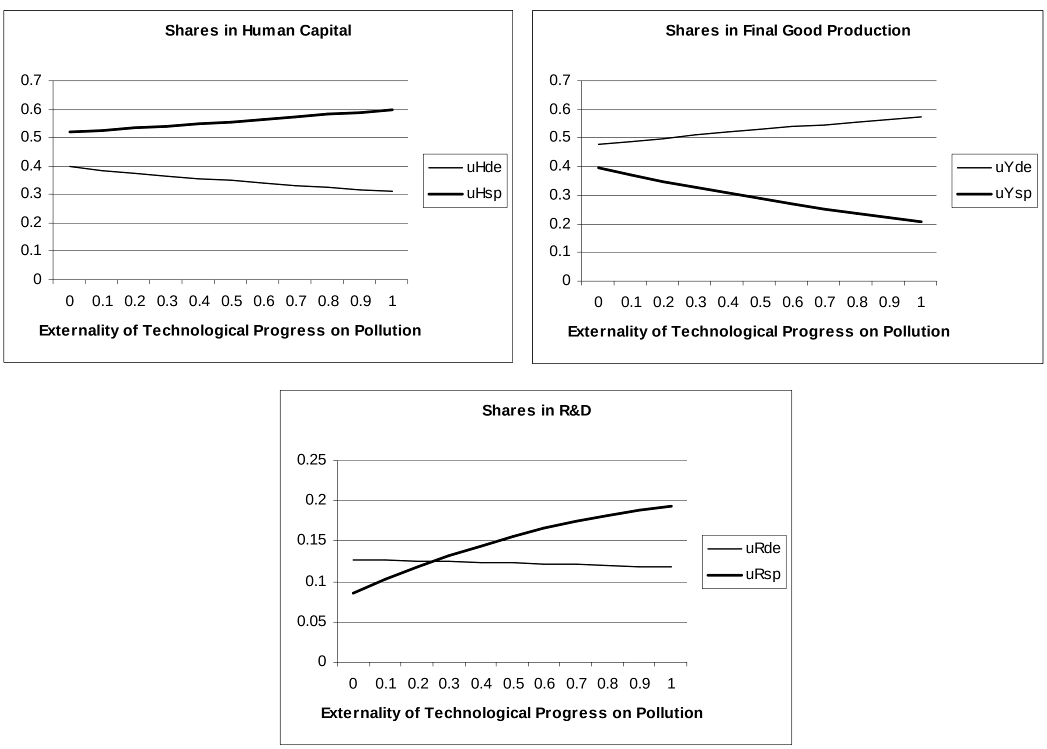

Figure 2: Human Capital Allocations to the Different Sectors in the decentralized equilibrium and in the social planner solution - extension with lower and constant creative destruction

This scenario, which clearly decreases the importance of creative destruction as a distortion of the market economy, shows that the increase in the effect of technological progress in pollution increases the allocation of human capital to R&D activities in the social planner solution and converts a situation of initial overinvestment in R&D to a situation of underinvestment in R&D. This is an important result as it shows the strength of the new externality introduced in this model in controlling if the decentralized economy is investing in R&D under or above the optimal level. The threshold level above which the economy underinvests in R&D is $\epsilon = 0.3$, which is a low threshold value. This means that it is sufficient an improvement of the quality of technologies in 1% to imply a reduction in pollution of 0.3%, to lead to underinvestment in R&D in the decentralized equilibrium. An alternative exercise to show the importance of this externality, would be to depart from the benchmark calibration, to cancel all other distortions (in u_R), and analyze the reasonability of the implied lifetime. Let $\alpha = \eta = 0.64$ (which eliminates the specialization externality), $\lambda = 1$ (which eliminates duplication externality), $\phi = 0$ (which eliminates spillovers externality), and $\gamma = 1.076054$ (which eliminates the creative destruction distortion) - this implies a very reasonable lifetime of 21.308.[6]

This can be understood as an argument in favour of the existence of subsidies to the development of cleaner technologies.

7 Conclusion

We derive a complex model of endogenous growth with physical capital, human capital, and technological progress through quality-ladders, to which we add pollution. In particular, we focus on the effect of technological progress in decreasing pollution, through the development of cleaner technologies. This follows the idea of Balcão Reis [**?**], which models technological progress, both exogenously and endogenously, and in which the possibility of discovering a cleaner technology is taken as exogenous. Contrary to that

[6] In an alternative exercise where the tax rate T is set such as that the growth rates of the decentralized equilibrium would equalize those of the optimal solution, the implied lifetime would be 22.574, also a reasonable value.

author, we model the R&D process as a quality-improving technology in which better qualities are always cleaner qualities, but the impact they have in pollution may differ a lot.[7] Smulders and Gradus [?] also present models in which technological progress reduces pollution. However, none of these authors made a quantitative evaluation of their models. We intend to fill this gap in the literature.

The study of the optimality of investment in R&D has been the focus of a large set of papers. However, none introduces the potential externality that derives from the effect of technological progress on pollution and studied it quantitatively. We also contribute to this literature.

We derive growth rates for output, human capital, technological progress, and pollution both resulting from the decentralized equilibrium and from the social planner choices. We then identify the different distortions in place (spillovers, duplication, specialization returns, creative destruction, and pollution). We have also implemented a calibration exercise. From this exercise we conclude that the economy overinvests in R&D due to a high creative destruction effect. However, we show clear evidence that the new externality from technological progress to pollution considered in this model is sufficiently strong to induce underinvestment in R&D as an outcome of the decentralized equilibrium. The threshold level for the effect of R&D in pollution above which underinvestment occurs is relatively low.

An important policy implication of the main result of this article is a justification to subsidize the research in cleaner technologies.

References

[1] Akbostanci, E., T. Serap, Tunç, G. (2009), "The Relationship Between Income and Environment in Turkey: Is There an Environmental Kuznets Curve?", *Energy Policy*, 37, 861-867.

[2] Aghion, P. and P. Howitt (1998), *Endogenous Growth Theory,* MIT press.

[7] An extension in which better qualities are more polluting is straightforward to do. However, due to our focus on distortions, this would be much less interesting.

[3] Alvarez-Palaez, M. and C. Groth (2005), "Too Little or Too Much R&D?", *European Economic Review,* 49, 437-456.

[4] Benassy, J. P. (1998), "Is There Always Too Little Research in Endogenous Growth With Expanding Product Variety?", *European Economic Review*, 42, 61-69.

[5] Bovenberg, A. and S. Smulders (1995), "Environmental Quality and Pollution-Augmenting Technological Change in a Two-Sector Endogenous Growth Model", *Journal of Public Economics,* 57, 369-391.

[6] Bretschger, L and S. Smulders (2007), "Sustainable Resource Use and Economic Dynamics", *Environmental and Resource Economics*, 36, 1-13.

[7] Cunha-e-Sá, M.A., A. Leitao, and A.B. Reis (2009) "Innovation and Environmental Policy: Clean vs. Dirty Technical Change", unpublished manuscript.

[8] Economides, G. and A. Phillipopoulos (2008), "Growth Enhancing Policy is the Means to Sustain the Environment", *Review of Economic Dynamics*, 11, 207-219.

[9] Funke, M. and H. Strulik (2000), "On Endogenous Growth with Physical Capital, Human Capital and Product Variety", *European Economic Review*, 44, 491-515.

[10] Gillingham, K., R. Newell and W. Pizer (2008) "Modeling endogenous technological change for climate policy analysis", *Energy Economics*, 30, 2734-2753.

[11] González, P. del R. (2009), "The Empirical Analysis of the Determinants of Environmental Technological Change: A Research Agenda", *Ecological Economics*, 861-878.

[12] Grossman, G. and E. Helpman (1991), *Innovation and Growth in the Global Economy*, MIT press.

[13] Guvenen, M. (2006), "Reconciling Conflicting Evidence on the Elasticity of Intertemporal Substitution: A Macroeconomic Perspective", *Journal of Monetary Economics*, 53 (7), 1451-1472.

[14] Ikazaki, D. (2006), "R&D, Human Capital and Environmental Externality in an Endogenous Growth Model", *International Journal of Global Environmental Issues*, 6 (1), 29-46.

[15] Jaffe, A., R. Newell, and R. Stavins (2005), "A tale of two market failures: Technology and Environmental Policy", *Ecological Economics*, 54, 164-174.

[16] Jones, C. (1995), "R&D-Based Models of Economic Growth", *Journal of Political Economy*, Vol. 103, Issue 4, August, 759-784.

[17] Jones, C. and J. Williams (1998), "Measuring the Social Return to R&D", *The Quarterly Journal of Economics*, November, 1119-1135.

[18] Jones, C. and J. Williams (2000), "Too Much of a Good Thing? The Economics of Investment in R&D", *Journal of Economic Growth*, 5, March, 65-85.

[19] Jones, L., R. Manuelli, and H. Siu (2000), "Growth and Business Cycles", *Working Paper No.7633*, National Bureau of Economic Research.

[20] Löeschel, A. (2002), "Technological change in economic models of environmental policy: a survey", *Ecological Economics*, 43, 105-126.

[21] Maddison, A. (2003), *The World Economy: Historical Statistics*, OECD.

[22] Norrbin, S. (1993), "The Relation Between Price and Marginal Cost in U.S. Industry: A Contradiction", *Journal of Political Economy*, Vol. 101, No. 6, 1149-1164.

[23] Pessoa, A. (2005), "Ideas' Driven Growth: the OECD Evidence", *Portuguese Economic Journal*, 4 (1), 46-67.

[24] Reis, A.B. and T. N. Sequeira (2007), "Human Capital and Overinvestment in R&D", *Scandinavian Journal of Economics,* 109 (3), 573-591.

[25] Roca, J., E. Padilla, M. Farré, and V. Galletto (2001), "Economic Growth and Atmospheric Pollution in Spain: Discussing the Environmental Kuznets Curve Hypothesis", *Ecological Economics,* 39, 85-99

[26] Romer, P. (1990), "Endogenous Technological Change", *Journal of Political Economy,* 98, 71-102.

[27] Smulders, S., L. Bretschger, and H. Egli (2005), "Economic Growth and the Diffusion of Clean Technologies: Explaining Environmental Kuznets Curves", *Working Paper 05/42*, August, WIF - Institute of Economic Research.

[28] Smulders, S. (2005) "Endogenous Technological Change, Natural Resources, and Growth", in *Scarcity and Growth Revisited - Natural Resources and the Environment in the New Millennium*, Ed. R.D. Simpson, M.A. Toman, and R. Ayres, Resources for the Future, Washington DC

[29] Song, T., T. Zheng, and L. Tong (2008), "An Empirical Test of the Environmental Kuznets Curve in China: A Panel Cointegration Approach", *China Economic Review,* 19, 381-392.

[30] Stokey, N. (1995), "R&D and Economic Growth", *Review of Economic Studies*, 62, February, 469-489.

[31] Stokey, N. (1998), "Are There Limits to Growth?", *International Economic Review*, 39 (1), February, 1-31.

[32] Strulik, H. (2007), "Too Much of a Good Thing? The Quantitative Economics of R&D Driven Growth Revisited", *Scandinavian Journal of Economics*, 109 (2), 369-386.

[33] World Bank (2006), *Where is the Wealth of Nations? Measuring Capital for the 21st Century*, The World Bank, Washington DC.

A Appendix A - First Order Conditions for the Decentralized Equilibrium

The choice variables for the consumers are C and H_H so the first order conditions for the consumer problem yield:

$$\frac{\partial U}{\partial C} = \lambda_a \quad (46)$$

$$\lambda'_H = \frac{\lambda_a W_H}{\xi} \quad (47)$$

as well as:

$$\frac{\dot{\lambda}_a}{\lambda_a} = \rho + \delta_P - r \quad (48)$$

$$\frac{\dot{\lambda}'_H}{\lambda'_H} = \rho + \delta_H - \xi \quad (49)$$

where λ_a is the co-state variable for the budget constraint and λ'_H is the co-state variable for the stock of human capital.

B Appendix B - Human Capital Shares

B.1 Social Planner

We get the share of human capital allocated to school time from equation (2). The relation between the share of human capital allocated to R&D activities and the share of human capital allocated to work was found as described below:

From (12) we get the expression:

$$b\kappa Y^{(\kappa-1)} K_R^{\kappa\left(-\epsilon-\eta\left(\frac{1-\alpha}{\alpha}\right)\right)} A^{\kappa(\chi-1)} = \frac{\lambda_P}{\chi}$$

which we substitute into (17) to find:

$$\rho\lambda_R - \dot{\lambda}_R = \frac{\epsilon}{\chi}\frac{\lambda_P Y}{K_R} + \frac{\lambda_P Y}{Q_R}\eta\left(\frac{1-\alpha}{\alpha}\right) + \lambda_R\left(\gamma^{\alpha/(1-\alpha)} - 1\right)\phi H_R^{\lambda} Q_R^{\phi-1} \tag{50}$$

Using equations (13) and (14) we find that $\lambda_P Y = \frac{\lambda_R\left(\gamma^{\alpha/(1-\alpha)}-1\right)\lambda H_R^{\lambda-1} K_R^{\phi} H_Y}{\eta}\left(\frac{\chi}{\chi-1}\right)$, and knowing that $g_{Q_R} = \left(\gamma^{\alpha/(1-\alpha)} - 1\right) H_R^{\lambda} K_R^{\phi-1}$, we substitute these two expressions into (50) to find:

$$\frac{\dot{\lambda}_R}{\lambda_R} - \rho + \phi g_{Q_R} = \frac{\lambda}{\eta} g_{Q_R}\left(\eta\left(\frac{\alpha-1}{\alpha}\right) - \frac{\epsilon}{\chi}\right)\left(\frac{u_Y}{u_R}\right) \tag{51}$$

By using equations (14), (16), (17), and also $g_{K_H} = \frac{(1-\phi)}{\lambda} g_{Q_R}$we find $\frac{\dot{\lambda}_R}{\lambda_R} = \rho + \delta_H - \xi + g_{Q_R}\left(\frac{(1-\phi)(1-\lambda)}{\lambda} - \phi\right)$, which we substitute into (51) to get (43).

B.2 Decentralized Equilibrium

As in the previous case, we get the share of human capital allocated to school time from equation (2).

The relation between the share of human capital allocated to R&D activities and the share of human capital allocated to work in the decentralized equilibrium was found as described next. We log-differentiating equation (34) to get $g_{W_H} + g_{K_H} = \frac{\dot{\nu}}{\nu} + \lambda g_{K_H} + \phi g_{Q_R}$ and we also use the referred equation to find $\nu = W_H q_{k_i}^{\frac{\alpha}{1-\alpha}} H_R^{1-\lambda} Q_R^{-\phi}$. We then substitute these two last expressions into (36) to obtain:

$$r - \delta_P - g_{W_H} - g_{K_H} - \lambda g_{K_H} + \phi g_{Q_R} = \frac{\pi}{W_H q_{k_i}^{\frac{\alpha}{1-\alpha}} H_R^{1-\lambda} Q_R^{-\phi}} \tag{52}$$

Since $g_{W_H} = g_{\lambda'_H} - g_{\lambda_a} = r + \delta_H - \xi - \delta_P$ and by substituting this expression and also (32) and (27) - using (4) - into (52) we get equation (45).