Bounded Learning Efficiency and Sources of Firm Level Productivity Growth in Colombian Food Manufacturing Industry

By

Apurba Shee (Penn State University)

and

Spiro E. Stefanou (Penn State University)

Selected Paper prepared for presentation at the Agricultural & Applied Economics Association's 2011 AAEA & NAREA Joint Annual Meeting, Pittsburgh, Pennsylvania, July 24-26, 2011

Abstract

The measurement of productivity fluctuations has been the focus of decades-long interest. In addition to broad structural forces driving productivity changes, there is more recent interest in measuring and identifying the heterogeneous forces driving these changes. A major force is learning-by-doing which is used by economists to describe the phenomenon of productivity growth arising from the accumulation of production experience by a firm.

This paper proposes a bounded learning concept with the learning progress function characterized by the degree of efficiency and the specification of the learning progress as a logistic function capturing both the slow start-up and the limit in learning progress. The inter-firm learning inefficiency is defined as the inability of a firm to reach the optimal plateau relative to the 'best practice' firm from the set of comparable firms. We further differentiate learning efficiency from the technical efficiency. The key contribution of this research is to provide a measure the firm's movement along the learning progress curve and explain the existence of firm-level heterogeneity in learning. The time varying technical efficiency is estimated based on stochastic production frontier methods and firm-specific learning efficiency is disentangled using the residual of the production frontier (productivity).The model is then used to decompose the factor productivity growth into components associated with learning, scale, technical efficiency, technological change and change in allocative efficiency. This productivity growth decomposition provides useful information and policy level insight in firm-level productivity analysis.

The major econometric issue in production function estimation is the possibility that there are some forces influencing production that are only observed by the firm and not by the econometrician. With firm input use being endogenous, inputs might be correlated with unobserved productivity shocks. The measure of technical efficiency by estimating the production frontier directly in presence of endogeneity of input choice can be biased in the sense that the measure of efficiency favors the firms employing higher levels of inputs. The Levinsohn and Petrin (2003) approach is extended to overcome this simultaneity problem in stochastic production frontier estimation to generate consistent estimates of production parameters and technical efficiency.

The model is applied to plant-level panel data on Colombian food manufacturing sector. The dataset is unique longitudinal data on firms in the sense that it has information on both plant-specific physical quantities and prices for both outputs and inputs. In contrast to most of the existing literature which measure productivity by deflating sales by an industry-level price index, these data eliminate a common source of measurement error in production function estimation. Plant-level productivity growth decomposition and the contribution of learning effect are explored by estimating the production frontier and firm-specific learning efficiency.

1. Introduction

Being a source of productivity growth, understanding the influence of learning effect in production by the policy maker and business manager can enhance firm performance. Learning-by-doing is a dynamic process of productivity growth associated with the accumulation of production experience (or cumulative output) by a firm. Production experience yields information or knowledge, which improves decisions and results in productivity enhancement. One representation is a cost-quantity power relationship, $c(t) = c(0)v(t)^{-\alpha}$ where $c(t)$ is current unit cost, $v(t)$ is cumulative past output, and $\alpha > 0$ is the learning coefficient. Another way of representing learning is the productivity-quantity relation where productivity in time period t is an increasing function of cumulative past output or $A(t) = a_0 v(t)^{\alpha}$. On the one hand, the costless by-product of a firm's production activity is called passive learning (Rosen, 1972). When the firm's productivity enhancement is only due to passive learning it is called an *experience curve*. On the other hand, the observed Power relationship of productivity (or unit cost) being an increasing (or decreasing) function of cumulative output is called *progress curve* and it is an umbrella where productivity growth is the result of not only passive learning but also a variety of complex forces like research, training, capital investment and other unmeasured factors. The productivity gain due to learning is used as long run planning and control tool in a variety of manufacturing industries.

The classical learning progress assumes that learning is unbounded. But does the learning continue forever? Differences in management, training, and infrastructure lead to varied learning abilities of the firms (Adler & Clark, 1991; Argote, 1999). But how can we quantify firm's heterogeneous learning abilities? What are the contributions of learning and other sources to the firm level productivity growth?

Considerable empirical research uses the log-linear model to estimate the unbounded learning rates and finds a significant relationship of firm productivity with production experience. Most recent empirical studies such as (Arrow, 1962; Rapping, 1965; Lieberman, 1984; Bahk & Gort, 1993; Lucas Jr, 1993; Luh & Stefanou, 1993; Irwin & Klenow, 1994; Jarmin, 1994; Benkard, 2000; Thompson, 2001; Thornton & Thompson, 2001) find that firms and industries become more productive as they gain more experience of producing goods and services. The estimated results from these researches are varied and average finding is approximately 10 to 20% reduction in average cost of production for every doubling of cumulative output.

Organizational knowledge through experience is embedded in individual workers, technology, and structure of the organization. When passive learning (Rosen, 1972) is the dominant factor in learning process, productivity growth is invariably bounded. Conway & Schultz (1959), Jovanovic & Nyarko (1995), Baloff (1966, 1971) , and Young (1993, p. 445) present evidence that productivity reaches a limit, or a “plateau effect”. On the other hand, the recognition of S-shaped learning curve is not new, having appeared in the literature as early as Carr (1946) and has been useful for planning and control methods for new product introduction. Cochran (1960) also proposes the learning curve as S-shaped, suggesting that an S-shaped pattern appears more appropriate than the classical learning model. The idea is during the early stage a firm attempts various options and explores different alternative production plans and designs which slow down the initial learning rate. After the initial exploration there are fewer changes in the production system leading to a higher learning rate (Cochran & Sherman, 1982). Both the learning bound and its S-shape character are important in the sense that the learning limit captures the diminishing return of learning on a given technology and the S-shape replicates the start-up phase of a firm.

This paper models the learning phenomenon at the micro level to overcome the limitations posed by the classical learning curve literature and to investigate its contribution to the firm level productivity growth. The learning progress function is characterized by the degree of efficiency and the specified as a logistic function capturing both the slow start-up and the limit in learning progress. This paper corrects for endogeneity of input choice problem within the stochastic production frontier estimation to generate consistent estimate of the production parameters and technical efficiency as we estimate time varying technical efficiency using stochastic frontier approach and disentangle learning efficiency from it using the frontier residual. The model is then used to decompose the total factor productivity growth into components associated with learning, scale, technical efficiency, technological change and change in allocative efficiency in Colombian food manufacturing industry.

The rest of the paper is organized as follows. Section 2 presents the idea of bounded learning and learning inefficiency and how to distinguish it from technical inefficiency. Section 3 describes analytic framework for productivity growth decomposition where learning effect is a source of productivity growth. Section 4 describes the methodology for estimation where endogeneity of input choice problem is corrected in stochastic production frontier. Section 5 presents the data and basic estimation results. Section 6 provides concluding comments.

2. Bounded Learning and Learning Inefficiency

The conventional learning model is extended by modeling learning progress as a logistic function which explains both the initial ‘start up’ phase and steady state ‘plateau’. Inter-firm learning efficiency is defined as a relative measure quantifying the learning progress of a particular firm relative to the ‘best practice firm’ from the set of comparable firms in the industry and captures heterogeneous learning abilities. The proposed logistic learning progress is governed by a differential equation given by

$$\frac{dA}{dV} = \alpha A - \frac{\alpha}{\bar{a}} A^2 - \eta \alpha A \tag{1}$$

and the explicit solution is

$$A(V_t) = \frac{(1-\eta)\bar{a}}{1 + \frac{(1-\eta)\bar{a} - a_0}{a_0} e^{-\alpha(1-\eta)V}} \tag{2}$$

where A is productive knowledge arising through experience, α and η are instantaneous learning rate and level of learning inefficiency, respectively.

Understanding the difference between maximum potential frontier and potential frontier given learning is important to distinguish between learning and technical inefficiency. Learning inefficiency parameter ($\eta \in [0,1)$) is firm specific and reflects the inability of a firm to reach the learning progress curve of the ‘best practice firm’ given a set of cumulative past output. The deterministic kernel of the potential production frontier given learning can be represented as $A\left(V_t; \eta = \eta_i\right) f(x,t;\beta)\exp(u)$. The maximum potential frontier is the production frontier of the best learning progress (100% learning efficient) firm and can be represented as $A\left(V_t; \eta = 0\right) f(x,t;\beta)\exp(u)$.

Figure 1 depicts the deterministic production function of both the maximum potential frontier and potential frontier given learning for a single product and one-variable factor of production. Point A depicts a firm that produce y_t using input x_t is technically inefficient because it operates beneath the potential production frontier given learning and the deviation AB is measured as technical inefficiency. The impact of learning inefficiency for the firm is represented by the deviation of the potential frontier given learning from the maximum potential frontier or BC.

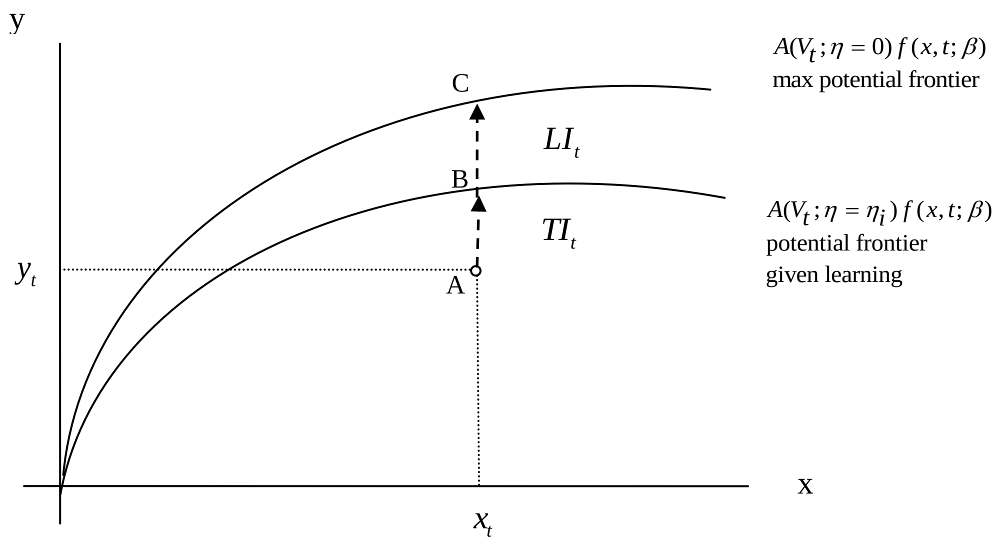

Figure 1: learning and technical inefficiency

Technical inefficiency reflects the inability of a firm to obtain the maximum potential output given learning, from a certain amount of input use. Technical efficiency compares the actual quantity of output achieved to the maximum achievable output for certain inputs given the constant learning inefficiency for the firm. $TE_i = \dfrac{y_{it}}{f(x_{it};\beta)e^{v_{it}}} = e^{-u_{it}} |_{\eta=\eta_i}$. What are the sources of technical efficiency? Increased education and managerial ability to production are widely accepted sources of technical efficiency in a firm. Leibenstein calls the technical efficiency an X-inefficiency (see Leibenstein, 1966; Stigler, 1976; Leibenstein, 1978, 1979) and constructed a theory where difference in motivation was the source of inefficiency. He also points out that differences in knowledge among the firms can lead to firm inefficiency. Mundlak's (1961) covariance analysis to control for managerial bias in production reflects a positive relation between managerial ability and technical efficiency. Stefanou and Saxena (1988) find a significant impact of education and training on allocative efficiency by a non-frontier approach to efficiency. Battese and Coelli (1995) model technical inefficiency effect in a stochastic production frontier approach and finds that age has positive and schooling has negative effect on inefficiency.

If the source of technical inefficiency is the difference in motivation, efficiency can be improved by introducing appropriate incentives in the firm. On the other hand, if the difference in knowledge is the lever of technical inefficiency, its improvement is possible by sustained learning process. That means inefficiency due to learning can lead to technical inefficiency. Hence, in that sense, learning inefficiency might be one of the sources of technical inefficiency. Learning efficiency allow some firms to benefit more than others from equivalent level of experience (cumulative volume of past output). In other words, learning inefficiency reflects the failure of a firm to obtain the maximal state of knowledge achievable from the given amount of experience. Firm-specific learning inefficiency (η) parameter can be estimated from the learning progress function (or productivity experience relationship) where $(1-\eta)$ is the measure of learning efficiency. The learning effect can be realized by the ratio of the actual quantity of output achieved given firm-specific learning to the output achieved by the best learning practice firm given technical efficiency or $\frac{A(V_t;\eta=\eta_i)\, f(x,t;\beta)}{A(V_t;\eta=0)\, f(x,t;\beta)}\Big|_{TE}$. The productivity gain due to learning is not automatic or costless by-product of experience. Sources of the firm-specific learning inefficiency are attributed to the investment in research, training and infrastructure which impacts both the intrinsic learning rate and learning inefficiency.

While technical inefficiency varies with time, the learning inefficiency parameter is constant for a firm. However, as the productivity varies with cumulative past output the effect of learning on production changes over time. Technical efficiency and learning effect over time is illustrated in figure 2, in which a single input is used to produce a single output, and a firm operates from (x_{t_1}, y_{t_1}) to (x_{t_2}, y_{t_2}). The technical inefficiency changes from time t_1 to t_2, and it is measured as the deviation of the production point from the new potential frontier given learning $A(V_{t_2}; \eta = \eta_i) f(x, t_2; \beta)$. The effect due to learning inefficiency is captured by the difference between this potential frontier to the maximum potential frontier $A(V_{t_2}; \eta = 0) f(x, t_2; \beta)$ at time t_2. Notice that the maximum potential frontiers at the two periods will be same if the cumulative volume of the output is such that the learning progress function approaches a plateau.

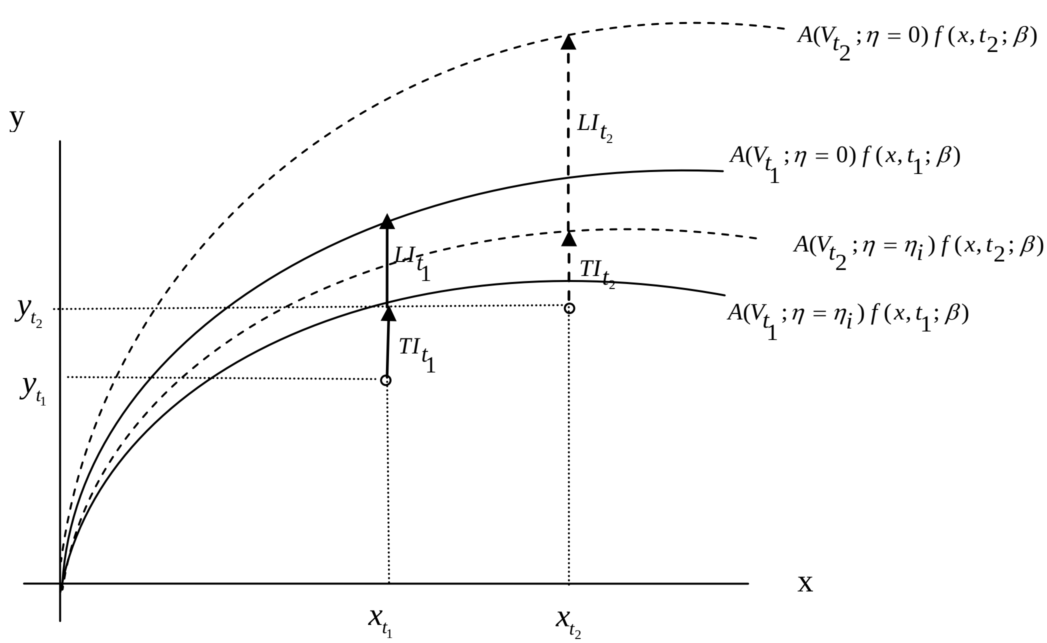

Figure 2: Technical inefficiency and learning effect over time

The two definitions are based on two different reference points; one is the deviation from the production frontier given learning and the other is the deviation from the progress curve of the 'best practice firm'. A firm might face both inefficiencies simultaneously. In next section we disentangle the learning and technical efficiency by two steps: a) estimating the technical efficiency by stochastic frontier approach and b) estimating learning inefficiency from the residual of the production frontier. The change in both the technical efficiency and learning progress in a firm contribute to firm productivity growth. After measuring both the efficiencies it is interesting to measure their influence to the firm level productivity growth. The next section deals with the decomposition of the firm level productivity growth which provides policy perspectives on the firm performance.

3. Analytical framework for productivity growth decomposition

Literature on productivity growth decomposition acknowledges that along with technical change, change in efficiency (both technical and allocative) and scale can contribute to productivity growth (Denny, Fuss, & Waverman, 1981; Nishimizu & Page, 1982; Bauer, 1990; Kumbhakar, 2000; Kim & Han, 2001). But they do not explore the contribution of learning in productivity growth decomposition. This research estimates the contribution of learning to the firm-level productivity growth by using the stochastic production frontier approach.

Single factor productivity reflects the ratio of units of output produced and the units of a particular input used. This measure can be affected by the intensity of other inputs use. For example, with the same technology, two firms might have very different labor productivity levels if one firm uses capital more intensely than another because of different factor prices. To make the productivity measure invariant to the intensity of the factor use, the concept of total factor productivity (TFP) is used. While the TFP variation reflects the shifts in isoquants, the factor price variation reflects the shifts along the isoquants and hence does not affect TFP.

TFP is represented by the often-used formulation of production function where output is the product of a function of inputs and a Hicks-neutral shifter. The deterministic kernel of a stochastic production frontier with Hicks-neutral shifter is written as

$$y_{it} = A(V_{it})\, f(x_{it}, t) e^{-u_{it}} \tag{3}$$

where y_{it} is the scalar output of ith firm at time period t (i = 1,…..,N and t = 1,…..,T), x is input vector, the shifter $A(V_t)$ is the TFP contribution due to learning progress, and $u \geq 0$ reflects the technical inefficiency or the gap between frontier technology (or potential frontier given learning) and a firm's actual production output. Notice that technical efficiency is time varying in equation (3). Logarithmic transformation of (3) yields (omitting subscripts)

$$\ln y = \ln A(V) + \ln f(x,t) - u \tag{4}$$

Totally differentiating with respect to time, and denoting '$\dot{z}$' as the rate of change or its logarithmic time derivative, we obtain

$$\dot{y} = \frac{1}{A(V)} \frac{dA}{dV} \frac{dV}{dt} + \frac{\partial \ln f(x,t)}{\partial t} + \sum_j \frac{\partial \ln f(x)}{\partial x_j} \frac{dx_j}{dt} - \frac{du}{dt} \tag{5}$$

The first term on the right-hand side of equation (5) measures the change in output growth contribution due to learning. The second and third terms measure the change in output caused by technical progress (TP) and by change in input use, respectively. The fourth term captures the change in technical inefficiency. Hence, the overall change in production is not only affected by technical progress, changes in input use, and change in technical inefficiency but also by the change in learning progress. Using $TP = \frac{\partial \ln f(x,t)}{\partial t}$ and the differential equation (1), equation (5) is rewritten as

$$\dot{y} = \frac{1}{A}\left(\alpha A - \frac{\alpha}{\bar{a}} A^2 - \eta \alpha A\right) y + TP + \sum_j \varepsilon_j \dot{x}_j - \frac{du}{dt} \tag{6}$$

where the change in the frontier output due to the change in input use or the output elasticity of input j is $\varepsilon_j = \frac{\partial \ln f(x)}{\partial \ln x_j}$. Total factor productivity growth is defined as output growth less by input growth, where input growth accounts for all factor of production. The familiar definition is

$$T\dot{F}P = \dot{y} - \sum_j s_j \dot{x}_j \tag{7}$$

where input growth is the sum of the growth of all inputs weighted by their respective cost shares (Denny, et al., 1981). Equation (7) can be expanded to

$$T\dot{F}P = \left(\alpha - \frac{\alpha}{\bar{a}} A - \eta \alpha\right) y + TP - \frac{du}{dt} + \sum_j \left(\varepsilon_j - s_j\right) \dot{x}_j \tag{8}$$

The share of marginal product of input j = $\lambda_j = \frac{f_j x_j}{\sum_k f_k x_k} = \frac{\varepsilon_j}{\sum \varepsilon_k}$ because $\varepsilon_j = \frac{df}{dx_j} \frac{x_j}{f} = f_j \frac{x_j}{f}$

Replacing $\varepsilon_j = \lambda_j \sum \varepsilon_j$ equation (8) yields

$$T\dot{F}P = \left(\alpha - \frac{\alpha}{\bar{a}} A\right) y - \eta \alpha y + TP - \frac{du}{dt} + \sum_j \left(\lambda_j \sum \varepsilon_j - s_j\right) \dot{x}_j \tag{9}$$

Rearranging the terms and using the definition of returns to scale ($RTS = \sum_j \varepsilon_j$), equation (9) is written as

$$T\dot{F}P = \alpha y(\tilde{\eta} - A_{it}) + TP - \frac{du}{dt} + \sum_j \lambda_j \dot{x}_j (RTS - 1) + \sum_j \dot{x}_j (\lambda_j - s_j) \qquad (10)$$

where $\tilde{\eta} = (1-\eta)$ is learning efficiency. Hence, productivity growth is influenced by technical progress, learning inefficiency, technical inefficiency, and components related to input use (scale effect and allocative efficiency effect). If $\frac{du}{dt}$ is negative, technical inefficiency falls, meaning technical efficiency increases over time or the production point becomes closer to the frontier. The first and second components of the equation (9) represent the growth and the decay in knowledge absorption, respectively, and thus, reflect the net knowledge growth accounting for the ability to absorb knowledge. The last component of the equation presents the allocative efficiency effect which actually depicts the inefficiency in allocating resources resulting from the deviation of input prices from the value of their marginal product. All in all, productivity change is decomposed into changes in efficiency, both technical and allocative efficiency, change in learning progress, technical change, and change in scale, where the first is measured by how far the firm is from the production frontier given learning, the second by the inability of the firm in allocating resources resulting from the deviation of the input prices from the value of their marginal product, the third by net knowledge growth due to learning, the fourth by the shift in the frontier, the last by the movement of the firm along the curvature of the production frontier.

4. Econometric estimation

Consider a firm in a competitive market has production function

$$y = F\left(X(t), K(t), A(V_t), t\right) \tag{11}$$

where $X(t)$ is vector of variable inputs and $K(t)$ is a vector of quasi-fixed inputs like capital, and $A(V_t)$ is learning progress function reflecting a productivity enhancing factor. How $A(V_t)$ enters the production function depends on the nature of the learning progress function. The question is does it embody in the inputs or embrace in the organization? Bahk and Gort (1993) decompose the firm specific learning-by-doing into labor, capital and organizational learning by modeling learning component as (1) separate arguments in labor and capital augmenting term and (2) productivity shift parameter (also see Rapping, 1965). The production frontier for a sample of N firms for T time periods, can be written as

$$y_{it} = A(V_{it}) f(x_{it}; \beta) e^{v_{it} - u_{it}} \tag{12}$$

y_{it} denotes production of i$^{\text{th}}$ firm at time period t, x_{it} is a vector of input quantities of i$^{\text{th}}$ firm at t time period, β is a vector of unknown parameters to be estimated, $v_{it} \sim N(0, \sigma_v^2)$, and $u_{it} \sim N^{+}(0, \sigma_u^2)$. $A(V)$ is scaling factor that reflects state of organizational knowledge which depends not only on experience (cumulative volume of output) but also on learning efficiency for a firm. Most of the microeconomic studies on production experience assume a Cobb-Douglas form for the production technology (see Rapping, 1965; Irwin & Klenow, 1994; Thompson, 2001). Following (12) a Cobb-Douglas production frontier with time variant technical efficiency can be written as

$$Y_{it} = A_{it} L_{it}^{\beta_l} M_{it}^{\beta_m} E_{it}^{\beta_e} K_{it}^{\beta_k} e^{v_{it} - u_{it}} \tag{13}$$

where Y is output quantity produced, L, M, E, and K are labor, material, energy, and capital inputs, respectively, and β_l, β_m, β_e, and β_k are the their respective coefficients. The productivity shock is denoted by A which is influenced by learning of a firm, technical inefficiency is represented by u, and v is random statistical noise. Writing in log-linear form

$$\ln Y_{it} = \beta_l \ln L_{it} + \beta_m \ln M_{it} + \beta_e \ln E_{it} + \beta_k \ln K_{it} + \ln A_{it} + v_{it} - u_{it} \tag{14}$$

Writing log terms in lowercase letters

$$y_{it} = \beta_l l_{it} + \beta_m m_{it} + \beta_e e_{it} + \beta_k k_{it} + a_{it} + v_{it} - u_{it} \tag{15}$$

Simultaneity problem in stochastic production frontier

It is well documented in the literature (Marschak & Andrews, 1944; Griliches & Mairesse, 1995; Olley & Pakes, 1996; Levinsohn & Petrin, 2003; Ackerberg, Caves, & Frazer, 2006) that quantities of inputs are likely to be correlated with productivity shocks which lead to a biased estimate of production function parameters. Same argument can be applied to the stochastic production frontier that can cause potential identification problem in standard frontier estimation.

The efficiency literature assumes that input choices are independent of the efficiency and productivity term. However, if a firm observes some part of its efficiency and productivity, its input choices may be influenced resulting in a simultaneity problem in stochastic production frontier estimation. Production input decision can be influenced by the common causes affecting efficiency and hence simultaneity problem is arisen. Inputs are likely to be correlated with the components of productivity and efficiency that are observed by the firm but unobserved by the econometrician. The problem is more serious for inputs that adjust quickly like labor and materials. The omission of some explanatory variables makes the likelihood estimation of the stochastic production frontier biased. In estimating the unobserved productivity as residual of the production function and technical efficiency as the deviation from the 'best-practice' production frontier, the frontier estimation encounter omitted and /or simultaneity problem. The anatomy of the error term ($\varepsilon_{it} = a_{it} + v_{it} - u_{it}$) is the following. a_{it} represents shocks to production that are predictable by firms when making input decision and can be thought of as factors like expected rainfall at the firm's location, managerial ability of the firm, expected breakdowns or strikes time. v_{it} represents pure random deviation or measurement error that are not observable by firms when making their input decisions. u_{it} captures the deviation from the 'best-practice' firm. The basic idea is to throw all the predictable components of the productivity and efficiency into a_{it} term and consolidate endogeneity problem into it.

The simultaneity issue is neglected by the efficiency literature; however, ignoring this problem might have profound policy implications on firm performance. Not only might this misspecification lead to a biased inference on the elasticity of inputs and hence the economies of scale, but it also provides a faulty measure of technical efficiency. The measure of technical efficiency by the traditional frontier method in presence of endogeneity of input choice can be biased in the sense that the measure of efficiency favors the firms employing higher level of inputs. The regressors are to be uncorrelated with the error term to obtain consistent parameter estimates. We present the methodology to solve the endogeneity of input bias problem within stochastic production frontier estimation by using the semi-parametric approach proposed by Levinsohn and Petrin.

Semi-parametric estimation approach

To correct for the simultaneity issue in stochastic production frontier estimation the methodology proposed by Levinsohn and Petrin (2003) is extended for obtaining consistent estimates of production parameters and technical efficiency. Olley and Pakes (1996) first introduce the approach of using investment as proxy for unobserved productivity shock to overcome the simultaneity problem. Levinsohn and Petrin (2003) suggest that investment being non continuous, may not respond fully to the productivity shocks and show that intermediate inputs can be used to control for the simultaneity problem. The estimation stages are presented below.

<u>Stage 1:</u>

Selecting energy as a proxy for the unobserved productivity shocks, equation (15) is estimated using this approach. a_{it} is consolidated as the observed part of productivity and efficiency. This is called predictable 'productivity shock'. We assume $v_{it} \sim N(0,\sigma_v^2)$ and $u_{it} \sim N^+(0,\sigma_u^2)$. Following Battese and Colli (1992) we assume time varying technical efficiency is defined by $u_{it} = u_i \exp(-\zeta[t-T])$. The main difference between this productivity shock and the composed error is that the former is a state variable, and hence influence firm's decision while the later has no impact on firm's decision. Putting a constant term in equation (15) such that $\ln A_{it} = \beta_0 + a_{it}$

$$y_{it} = \beta_0 + \beta_l l_{it} + \beta_m m_{it} + \beta_e e_{it} + \beta_k k_{it} + a_{it} + v_{it} - u_{it} \tag{16}$$

We use intermediate input electricity as proxy for the unobserved productivity shock. The input demand function for electricity can be written as (assuming perfect competition firms facing identical prices)

$$e_t = e_t(a_t, k_t)$$

Imposing monotonicity condition this demand function can be inverted

$$a_t = a_t(e_t, k_t)$$

Equation (16) can be written as

$$y_{it} = \beta_l l_{it} + \beta_m m_{it} + \phi_t(e_{it}, k_{it}) + v_{it} - u_{it} \tag{17}$$

where

$$\phi_t(e_{it}, k_{it}) = \beta_0 + \beta_k k_{it} + \beta_e e_{it} + a_t(e_t, k_t) \tag{18}$$

Using the method proposed by Robinson (1988) we take the expectation of equation (17) conditional on k_{it}, e_{it}.

$$E\left(y_{it} \mid k_{it}, e_{it}\right) = \beta_l E\left(l_{it} \mid k_{it}, e_{it}\right) + \beta_m E\left(m_{it} \mid k_{it}, e_{it}\right) + \phi_t(e_{it}, k_{it}) \tag{19}$$

We use the fact that $E(v_{it} - u_{it} \mid k_{it}, e_{it}) = 0$ and $E\left(\phi_t(e_{it}, k_{it}) \mid k_{it}, e_{it}\right)$ is itself

Subtracting (19) from (17)

$$y_{it} - E\left(y_{it} \mid k_{it}, e_{it}\right) = \beta_l\left[l_{it} - E\left(l_{it} \mid k_{it}, e_{it}\right)\right] + \beta_m\left[m_{it} - E\left(m_{it} \mid k_{it}, e_{it}\right)\right] + v_{it} - u_{it} \tag{20}$$

This difference makes $\phi_t(e_{it}, k_{it})$ out of the regression equation. Using maximum likelihood estimation with no intercept we can obtain consistent estimates of the coefficients of freely variable inputs except the proxy. Time varying technical efficiency is also estimated in this stage. The dependent and independent variables in this regression are based on the local least square estimates. Using bootstrap approach we can estimate the standard errors. An alternative approach to this is to use polynomial approximation for $\phi_t(e_{it}, k_{it})$.

Stage 2:

In the stage 2 coefficients of the proxy input and capital are identified. Coefficients of capital and electricity enter twice in equation (18) and hence are not identified without further restrictions. For the identification we assume that capital is a state variable and does not instantaneously adjust to the unexpected part of productivity shock while it might adjust to the predicted part. To formalize the notion we assume that productivity is governed by first order Markov process, or

$$a_t = E\left(a_t \mid a_{t-1}\right) + \xi_t \tag{21}$$

Further we assume that the non-forecastable part of productivity is uncorrelated with capital. Two moment conditions can be formed from the above two assumptions

$$E\left(\xi_t + v_t \mid k_t\right) = E(\xi_t \mid k_t) + E(v_t \mid k_t) = 0 \tag{22}$$

$$E\left(\xi_t + v_t \mid e_{t-1}\right) = E(\xi_t \mid e_{t-1}) + E(v_t \mid e_{t-1}) = 0 \tag{23}$$

The first moment condition states our assumption that capital does not respond to the innovation in productivity. Capital stock in period t is determined by the investment decisions of the previous periods, it does not respond to this period's productivity innovation ξ_t. The second moment says that last period's electricity choice is uncorrelated with innovation in productivity. We employ Generalized Method of Moments to estimate the parameters of capital and energy. The GMM estimation steps are the following. First we choose a starting value β_e^* and β_k^* for the estimation algorithm. For this value we write (15) as

$$y_{it} - \hat{\beta}_l l_{it} - \hat{\beta}_m m_{it} - \beta_e^* e_{it} - \beta_k^* k_{it} + \hat{u}_{it} = a_{it} + v_{it} \tag{24}$$

Substitution (21) into (24) yields

$$y_{it} - \hat{\beta}_l l_{it} - \hat{\beta}_m m_{it} - \beta_e^* e_{it} - \beta_k^* k_{it} + \hat{u}_{it} - E(a_{it} \mid a_{i,t-1}) = \xi_{it} + v_{it} \tag{25}$$

If we knew $E(a_{it} \mid a_{i,t-1})$ we could compute $\xi_{it} \hat{+} v_{it}$ but we do not know it and hence we estimate $E(a_{it} \mid a_{i,t-1}) = E\left(a_{it} + v_{it} \mid a_{i,t-1}\right)$.

From (24) we get

$$a_{it} \hat{+} v_{it} = y_{it} - \hat{\beta}_l l_{it} - \hat{\beta}_m m_{it} - \beta_e^* e_{it} - \beta_k^* k_{it} + \hat{u}_{it} \tag{26}$$

From stage 1 equation (19) and (18) we get

$$\hat{a}_{i,t-1} = \hat{\phi}_{i,t-1} - \beta_e^* e_{i,t-1} - \beta_k^* k_{i,t-1} \tag{27}$$

By performing local least squares regression on $a_{it} \hat{+} v_{it}$ by $\hat{a}_{i,t-1}$ we get $E(a_{it} \mid a_{i,t-1})$. We can now compute an estimate of the residual $\xi_{it} + v_{it}$ using (25). We then use the GMM criterion to estimate the unknown parameters.

$$\min_{\beta} \left[\left(\sum_i \sum_t \left(\xi_{it} \hat{+} v_{it} \right) k_{it} \right)^2 + \left(\sum_i \sum_t \left(\xi_{it} \hat{+} v_{it} \right) e_{i,t-1} \right)^2 \right] \tag{28}$$

Ultimately the productivity term can be recovered from the residual using the estimated coefficients

$$\hat{A}_{it} = \exp\left(y_{it} - \hat{\beta}_0 - \hat{\beta}_l l_{it} - \hat{\beta}_m m_{it} - \hat{\beta}_e e_{it} - \hat{\beta}_k k_{it} \right) \tag{29}$$

This can be thought of as unexplained residual. To understand the productivity growth better our effort is to minimize the unexplained residual.

Stage 3:

From this residual we estimate the firm specific learning parameters including the learning efficiency. In stage 3 we estimate the firm specific parameters instantaneous learning rate α and learning inefficiency η by using the estimation equation given below.

$$\frac{d\hat{A}}{dV} = \alpha\hat{A} - \frac{\alpha}{\bar{a}}\hat{A}^2 - \eta\alpha\hat{A} \tag{30}$$

The discrete analogue of the derivative term $\frac{d\hat{A}}{dV} = \frac{\hat{A}_{it} - \hat{A}_{it-1}}{\hat{y}_{it}}$

The estimation equation becomes

$$\frac{d\hat{A}_{it}}{\hat{y}_{it}} = \sum(\alpha_i - \eta_i\alpha_i)D_i\hat{A}_{it-1} - \sum\alpha_i D_i\hat{A}_{it-1}^2 + \varepsilon_{it} \tag{31}$$

Alternatively, non linear least square technique can be used to estimate the parameters from the equation given below.

$$\hat{A}_{it} = \frac{(1-\eta)\bar{a}}{1 + \frac{(1-\eta)\bar{a} - a_0}{a_0}\exp\left(-\alpha(1-\eta)V_{it-1}\right)} \tag{32}$$

We estimate η and α by using nonlinear optimization and by assuming a given initial level of endowment of knowledge a_0.

Stage 4:

The decomposition of productivity growth following (10) is presented below. In the regression (13) we put $A_{it}e^{\delta t}$ instead of A_{it} to account for exogenous technical change.

1) The learning component $LC = \alpha y(\tilde{\eta} - A_{it})$

2) Rate of technical progress $TP = \dfrac{\partial \ln f(x_{it}, t)}{\partial t} = \delta$

3) Technical efficiency change can be obtained by $TEC = -\dfrac{\partial u_{it}}{\partial t}$

where $u_{it} = TE_{it} = E\left(\exp(u_{it}) \mid v_{it} - u_{it}\right)$

4) To find the change of scale component, output elasticity with respect to j-th input is defined by $\varepsilon_j = \dfrac{\partial \ln f(x,t)}{\partial \ln x_j} = \beta_j$. Share of marginal product of input j is $\lambda_j = \dfrac{\varepsilon_j}{RTS}$, where $RTS = \sum_j \varepsilon_j$.

The scale component $SC = (RTS - 1)\sum_j \lambda_j \dot{x}_j$

5) Allocative efficiency change can be found by $AE = \sum_j \dot{x}_j(\lambda_j - s_j)$

where S_j can be directly calculated from the data if price information is available.

5. Data and Empirical Results

The dataset used for this application is the Colombian Annual Manufacturers Survey (AMS) and covers the period 1982 to 1998. The data is provided by *Departamento Administrativo Nacional de Estadistica* (DANE) and originally created in a study of the effect of structural reforms on productivity and profitability enhancing reallocation in Colombian manufacturing industry (Eslava, Haltiwanger, Kugler, & Kugler, 2004). The same data is also used by Eslava, Haltiwanger, Kugler, and Kugler (2010) to investigate the plant-level adjustment dynamics of capital and labor and their joint interactions in the context of deregulated Colombian manufacturers.

The dataset is an unbalanced panel of Colombian manufacturing plants with more than 10 employees or sales over US$35,000 in 1998[1]. The dataset contains annual plant-level information on the value of output and prices charged for each product; cost and prices paid for each material used; energy consumption in kilowatt per hour and energy prices; number of workers and payroll; and book values of capital stock (buildings, structures, machinery, and equipment)[2]. The AMS dataset is a unique longitudinal data on plants in the sense that it has information on both plant-specific physical quantities and prices for both outputs and inputs. In contrast to most of the existing literature which measure productivity by deflating sales by an industry-level price index, these data eliminate a common source of measurement error in production function estimation.

We estimate the production parameters and the technical efficiency by using a capital-labor-energy-materials (KLEM) production frontier. The plant-level price indices of output and materials are constructed using Tornqvist indices where 1982 prices are considered base price 100. While the quantities of materials and output are constructed by dividing the cost of materials and value of output by the corresponding price indices, the quantities of energy consumption are directly reported in the data. The capital stock variable is constructed by the perpetual inventory method using the book values and capital expenditure together with gross capital deflators and depreciation rate of capital. Labor is measured as total hours of employment which is an improvement over the number of employees as a labor variable. Since the data does not have worker hours, a sector-level measure of average hours per labor is constructed as the

[1] For detailed description of the data see (Eslava, et al., 2004)
[2] We treat plants as firms although there are multi-plant firms in the sample because of data restriction. We do not aim to capture the scale or scope economies generally experienced by multi-plant firms.

ratio of earning per worker and the sectoral wage which is obtained from Monthly Manufacturing Survey of various years.

The application focuses on Colombian food industries. Summary statistics for the key variables are presented in table 1 where the means and standard deviations of the logarithm of plant-level physical quantity and price of output and input variables are presented. The units for energy consumption and labor use are kilowatt hours and hours of employment, respectively. Output, capital, and materials are expressed by thousands of pesos based on the constant price index for 1982 being 100. The prices for output, materials, and energy are expressed as prices relative to the yearly producer price index to make the prices inflation free (logarithmic difference between price index and PPI).

Table 1: No. of observations per ISIC 4 digit level and summary statistics of key variables

ISIC Code	Food Industry	no. of plant	no. of obs
3111	Butchering and meat canning	166	1474
3112	Dairy products	170	1619
3113	Vegetable and fruit canning	70	517
3114	Fish, crustaceans, and other seafood	30	198
3115	Oils, and vegetable and animal fats	89	792
3116	Grain mill products	449	4278
3117	Bakery products	630	5177
3118	Sugar refining and sugar products	63	565
3119	Cocoa, chocolate and confectionary	84	847
Sum		1751	15467

Variables	Mean	Std. Dev.
Output	11.002	1.873
Capital	8.644	2.115
Labor	10.774	1.192
Energy	12.034	1.781
Materials	10.558	1.952
Output prices	0.005	0.425
Energy prices	0.354	0.505
Material prices	-0.021	0.304

Notes: This table reports mean and standard deviations (in the brackets) of the log of quantity variables and log of prices deviated from yearly producer price indices to discount inflation. The units of the labor and energy variables are hours of employment and kilowatt hours respectively. Rests of the variables are expressed by thousands of pesos based on constant price index for 1982 being 100.

Estimation results with this dataset are in progress and preliminary results will be presented at the AAEA session.

6. Concluding Comments

Decomposition of the productivity change can provide useful information and policy level insight of firm-level productivity analysis by quantifying the sources of TFP growth. For example, if low productivity growth results due to poor learning efficiency then the policy recommendation is to invest in training and infrastructure so that the firm can advance to the learning progress function of the ‘best practice firm’. On the other hand, if there is low productivity because of poor technical efficiency, then the recommendation for the firm is to improve managerial practices. The productivity growth decomposition directs the firm managers to make decisions for improving firm performance.

7. References

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