

An Optimal Rule for Switching over to Renewable fuels with Lower Price Volatility: A Case of Jump Diffusion Process

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Abstract

This study investigates the optimal switching boundary to a renewable fuel when oil prices exhibit continuous random fluctuations along with occasional discontinuous jumps. In this paper, oil prices are modeled to follow jump diffusion processes. A completeness result is derived. Given that the market is complete the value of a contingent claim is risk neutral expectation of the discounted pay off process. Using the contingent claim analysis of investment under uncertainty, the Hamilton-Jacobi-Bellman (HJB) equation is derived for finding value function and optimal switching boundary. We get a mixed differential-difference equation which would be solved using numerical methods.

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1. Introduction

In the celebrated paper of option pricing, Black and Scholes (1973) and Merton (1973) provide an ideal benchmark model to analyse the asset price movements. In their paper, the underlying assets price is modeled to follow a geometric Brownian motion with constant drift and volatility. Additionally, the assumptions of frictionless market and no arbitrage opportunity ensure completeness of the market. Market is complete in the sense that the portfolio return can be made riskless. Consistent with Black and Scholes (1973) and Merton (1973), oil price

movements are modeled to follow a geometric Brownian motion to capture high degree of random fluctuations. However, for empirical analysis, constant volatility or homoskedasticity is a restrictive assumption. Hull and White (1987), Scott (1987), Wiggins (1987), Stein and Stein (1991) and Heston (1993) allows for a time-varying volatility. The particular specification of time-varying volatility varies across literature. One famous specification is the ARCH-type model. Kallsen and Taqqu (1998) modeled time varying volatility as a GARCH-type model to allow smooth persistent changes in volatility. In their paper, the conditional variance is modeled as a function of past variances and past innovations. Even a Geometric Brownian Motion with time varying volatility is restrictive as it fails to capture occasional significant discontinuities or structural breaks explicitly present in time series data. In modeling oil prices, these structural breaks have been experienced in the world market for oil over the history of oil prices due to the occurrence of several sudden major events, starting from the Yom-Kippur war in 1973, to the Iraq-Iran war in 1980 to some components of the post embargo US-energy policy. On a less significant basis, Merton (1976) pointed out that these discontinuities can be a consequence of arrival of some new information. These large sudden changes can be modeled by a stochastic jump process to capture occasional discontinuities that are not captured by the continuous path of Brownian motion. In such a model, the total change in price is a composition of two components- the normal vibrations in price, which is modeled by a diffusion process with continuous paths and unusual significant discontinuous changes in price modeled by a jump process. The mixture of diffusion and jump process is called the jump-diffusion process. Option pricing in a jump-diffusion model was introduced by Merton (1976), where he extended the basic Black and Scholes (1973) model to allow for jumps in asset price. In an incomplete market model with one stock and

one riskless asset, Merton derived a formula for the value of a call option on a non-dividend paying stock whose price follows a jump-diffusion process.

In this paper, we extend the Merton (1976) results for two asset prices: natural petroleum fuel or gasoline and a renewable substitute, namely ethanol blended gasoline. Prices of both these fuels exhibit high degree of volatility combined with significant discontinuous jumps over a long period of time and therefore modeled as jump-diffusion process. Gasoline prices, though relatively cheaper, are highly volatile when compared to its ethanol blended substitute. The purpose of this paper is to model the two prices as jump-diffusion processes to derive an optimal time to switch from gasoline to ethanol blended gasoline. In a stochastic environment, the decision to switch over would be influenced not only by the current price but also by expected future prices, which is crucially contingent on the drift and volatility of the underlying process. Therefore, the decision rule will take the form of an optimal exercise boundary which will be of threshold type [Refer to Dixit and Pindyck (1994) for details].

Previous studies include Tareen et al.(2000) and Vedenov et al. (2006). Tareen et al. (2000) use the contingent claim analysis of investment under uncertainty. Authors develop a decision rule to switch from petroleum to biodiesel, by modeling prices of biodiesel and petroleum to follow geometric Brownian motion with constant drift and volatility. However, their decision threshold is high and therefore impracticable. This is because Biodiesel is more of a niche fuel which requires an engine modification. Vedenov et al. (2006) used the same model to derive an optimal switching rule from gasoline to ethanol blended gasoline by minimizing the future cost of fuel over a certain time horizon.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 presents the derivation of a completeness result and the HJB equation to determine the value function and optimal switching boundary. Section 4 and 5 summarize the conclusions and limitations of this study respectively. Appendix to this paper presents the proof of the Lemma presented in Section 3.

2. Model

We consider a filtered probability space $(\Omega, \mathcal{F}, \mathbf{P})$ and let $\mathcal{F}(t), t \geq 0$ be the associated filtration. Let $W(t)$ be a Brownian motion relative to this filtration $\mathcal{F}(t)$ such that $W(t)$ is $\mathcal{F}(t)$ measurable for every t and for every $u > t$, the Brownian increment $W(u) - W(t)$ is independent of $\mathcal{F}(t)$. Let us define two independent Poisson processes $N_1(t)$ and $N_2(t)$ with intensities λ_1 and λ_2 respectively, adapted to the same filtration $\mathcal{F}(t)$. From the basic theory of stochastic processes, we know that by construction the Brownian motion, $W(t)$ and the Poisson processes $N_1(t)$ and $N_2(t)$ are independent to each other (Corollary 11.5.3, pg. 487, Shreve, 2008). We define two sequences of independent and identically distributed random variables $(Y_1, Y_2 \dots)$ and $(Z_1, Z_2 \dots)$ with mean $\beta_1 = EY_i$ and $\beta_2 = EZ_i$. The sequence of random variables are assumed to be independent to each other and also independent to the Brownian motion $W(t)$ and the Poisson processes $N_1(t)$ and $N_2(t)$. We construct the following process:

$$Q_1(t) = \sum_{i=1}^{N_1(t)} Y_i, t \geq 0$$

Then, $Q_1(t)$ is a compound Poisson process where jump arrives at the rate $\lambda_1 dt$ and Y_i denotes the size of the i^{th} jump. An immediate implication is the compen-

sated compound Poisson process

$$Q_1(t) - \beta_1 \lambda_1 t$$

is a \mathbf{P} - martingale. Similarly, another compound Poisson process is constructed with the Poisson process $N_2(t)$ and the sequence of random variables $(Z_1, Z_2 ..)$ as follows

$$Q_2(t) = \sum_{i=1}^{N_2(t)} Z_i, t \geq 0$$

By construction, $(Q_2(t) - \beta_2 \lambda_2 t)$ is a \mathbf{P} - martingale and $Q_1(t)$ and $Q_2(t)$ are independent of each other and also the Brownian motion $W(t)$.

With this mathematical setting, we model oil prices as a jump diffusion process, where the total change in price is a mixture of a continuous change, modeled by diffusion process and, discontinuous jumps modeled as compound Poisson process. Specifically, gasoline prices are modeled to follow the following stochastic differential equation:

$$dP(t) \begin{cases} = P(t)[\mu_1 dt + \sigma_1 dW(t)] + P(t-)[dQ_1(t) - \lambda_1 \beta_1 dt] \\ = P(t)[(\mu_1 - \lambda_1 \beta_1) dt + \sigma_1 dW(t)] + P(t-)dQ_1(t) \end{cases} \quad (1)$$

The continuous path of the process is modeled as a diffusion process, with constant drift and volatility and denoted by

$$dP(t)^c = P(t)[(\mu_1 - \lambda_1 \beta_1) dt + \sigma_1 dW(t)] \quad (2)$$

The discontinuous part is modeled by a compound Poisson process with random jump size and is assumed to be the result of arrival of some important information

(Merton 1976). If such an event occurs then the process exhibits a proportional jump of random size Y_i . Within the time interval "dt" the mean rate of arrival of jumps is $\lambda_1 dt$ and the probability of the event occurring more than once is zero. In other words, the stochastic price of the gasoline $P(t)$ jumps at random times t_1, t_2, \dots, t_{N_1} and the proportional change in its value at a jump time is given by Y_1, Y_2, \dots, Y_{N_1} . Between jump times, the gasoline price follows the standard diffusion process.

Applying Generalized Ito's Lemma, it can be shown that, the solution to the stochastic differential equation (SDE) is given by

$$dP(t) \begin{cases} = P(0)e^{\sigma_1 W(t) + (\mu_1 - \beta_1 \lambda_1 - \frac{\sigma_1^2}{2})t} \prod_{i=1}^{N_1(t)} (y_i + 1) \\ = P(0)e^{\sigma_1 W(t) + (\mu_1 - \beta_1 \lambda_1 - \frac{\sigma_1^2}{2})t + \sum_{i=1}^{N_1(t)} \log(y_i + 1)} \end{cases} \quad (3)$$

The price process of ethanol blended gasoline is modeled to follow a different jump-diffusion process as follows -

$$dB(t) = B(t)[(\mu_2 - \lambda_2 \beta_2)dt + \sigma_2 dW(t)] + B(t-)dQ_2(t) \quad (4)$$

Though underlying Brownian motion governing both the price processes is taken to be the same, the deterministic mean rate of return and volatility is different across the oil prices. This feature of the model captures the idea that gasoline prices though relatively cheaper are more volatile compared to its renewable substitute. However, modeling same Brownian motion governing both the prices is another way of saying that though mean and volatility are different, the underlying source of continuous uncertainty is the same. The discontinuous jumps in both price processes are independent Poisson with independent and identically

distributed random jump sizes. Therefore the individual oil prices are driven by two independent sources of randomness, whereas the continuous randomness across the prices are perfectly correlated. This assumption is made to obtain the completeness of the market model which will be explained subsequently.

The resulting sample path of ethanol blended gasoline, which is continuous most of the time, with finite jumps of random size at discrete points in time is :

$$B(t) \begin{cases} = B(0)e^{\sigma_2 W(t) + (\mu_2 - \beta_2 \lambda_2 - \frac{\sigma_2^2}{2})t} \prod_{j=1}^{N_2(t)} (z_j + 1) \\ = B(0)e^{\sigma_2 W(t) + (\mu_2 - \beta_2 \lambda_2 - \frac{\sigma_2^2}{2})t + \sum_{j=1}^{N_2(t)} \log(z_j + 1)} \end{cases} \quad (5)$$

We take the usual money market account $D(t)$ as numeraire that satisfies the following differential equation

$$dD(t) = r(t)D(t)dt$$

where $r(t)$ is the instantaneous risk neutral rate of interest.

3. Results

3.1 Completeness of the Market

Having described the price processes, before going into the option value of the investment opportunity, we would analyze the issue of completeness of the market. According to the first fundamental theorem of asset pricing, a market model is free of arbitrage and therefore complete if there exists a unique risk neutral probability measure. When the asset prices are modeled as geometric Brownian

motion, then in general, market model is complete. On the contrary, in the models where individual asset prices are driven by two independent source of randomness (for example, a jump-diffusion model where price process is governed by both a Brownian motion and an independent Poisson process), then there exists more than one risk neutral measures and thus, corresponding market models are incomplete. To this end, in our framework, we assumed that both the price processes are dependent on a common Brownian motion, while having independent process-specific jump processes. Within this setting, we make the following assumption to make the market model complete and this result is documented in the following lemma.

Assumption: An absolutely continuous change in measure from the Original to the Risk neutral would change the intensities of both the independent Poisson processes by same proportion, denoted by ψ . In other words, under the risk neutral measure \tilde{P} , Poisson processes $N_1(t)$ and $N_2(t)$ will have intensities $\psi\lambda_1$ and $\psi\lambda_2$ respectively.

Now, having made this assumption, we invoke Girsanov's theorem regarding change of measures to obtain the following result which shows that the market model is complete.

Lemma

(1) There exists a unique risk neutral measure \tilde{P} , equivalent to P , such that, the Radon-Nikodym derivative is given by,

$$Z(t) = e^{(\lambda_1(1-\psi)+\lambda_2(1-\psi)-\frac{\theta^2}{2})t-\theta W(t)+(N_1(t)+N_2(t))\log\psi} \quad (6)$$

$0 \leq t \leq T$, where θ and ψ are uniquely determined from the following system of equations

$$\mu_1 - \theta\sigma_1 - \lambda_1\beta_1\psi = r(t)$$

and

$$\mu_2 - \theta\sigma_2 - \lambda_2\beta_2\psi = r(t)$$

And thus market model is complete.

(2) Under the risk neutral measure \tilde{P}

$$\tilde{W}(t) = W(t) + \theta t$$

is a Brownian motion and $Q_1(t)$ and $Q_2(t)$ are Poisson processes with intensities $\psi\lambda_1\beta_1$ and $\psi\lambda_2\beta_2$ respectively. Consequently,

$$\tilde{Q}_1(t) = Q_1(t) - \psi\lambda_1\beta_1 t$$

and

$$\tilde{Q}_2(t) = Q_2(t) - \psi\lambda_2\beta_2 t$$

are \tilde{P} -martingales. Moreover, $\tilde{W}(t)$, $\tilde{Q}_1(t)$ and $\tilde{Q}_2(t)$ are independent to each other.

Proof: See Appendix.

3.2 Optimal Decision Threshold

Following Tareen et al. (2000), we assume that the objective of an agent is to minimize cost while maintaining a reservation level of quality. Given the price

processes modeled as jump-diffusion process, central problem faced by the agent is what is the optimal time to switch over to relatively more expensive ethanol blended gasoline with less volatility. In the stochastic framework described above, this is an optimal stopping problem, where a threshold type optimal exercise boundary is determined.

Assuming both fuels are perfect substitutes and the longevity of the machine and its salvage value is not affected by the fuel used, the expected present value of the project is:

$$V = E \int_0^T e^{-r(t)t} [P(t) - B(t)] dt \quad (7)$$

where, T is the finite lifetime of the machine, and $r(t)$ is the instantaneous risk neutral rate of return to the capital. Following Dixit and Pindyck (1994), the option value of adopting ethanol blended gasoline, at a random time ζ is given by

$$F(t, P(t), B(t)) = \tilde{E} \int_{\zeta}^{T+\zeta} e^{-r(t)t} [P(t) - B(t)] dt \quad (8)$$

where $F(\cdot)$ is assumed to be a twice continuously differentiable function of the oil price process and time. \tilde{E} is the expectation operator with respect to the risk neutral probability \tilde{P} . In a finite lifetime framework, the option value of optimally switching over to ethanol blended gasoline will be contingent on the remaining lifetime of the machine and thus, becomes a function of time.

Following the option pricing of investment under uncertainty, by Dixit and Pindyck (1994), The Bellman equation for the determination of optimal threshold satisfies the following:

$$r(t)Fdt = E[d[F(t, P(t), B(t))]]$$

Using the multidimensional Ito-Doebelin formula for processes with jumps (Theorem 11.5.4, p.489, Shreve), we have

$$\begin{aligned} dF(t, P, B) &= F_t dt + F_p P [(\mu_1 - \beta_1 \lambda_1) dt + \sigma_1 dW] + F_B B [(\mu_2 - \beta_2 \lambda_2) dt + \sigma_2 dW] \\ &\quad + 1/2 F_{pp} P^2 \sigma_1^2 dt + F_{PB} P B \sigma_1 \sigma_2 dt + 1/2 F_{BB} B^2 \sigma_2^2 dt \\ &\quad + d\left[\sum_{0 < s \leq t} [F(S, P(S), B(S)) - F(S, P(S-), B(S-))] \right] \end{aligned}$$

Now following Dixit and Pindyck (1994),

$$\begin{aligned} E d \sum_{0 < s \leq t} [F(S, P(S), B(S)) - F(S, P(S-), B(S-))] &= E_y [\lambda_1 F(t, P(1 + y_i), B) - F(t, P, B)] \\ &\quad + E_z [\lambda_2 F(t, P, B(1 + z_i)) - F(t, P, B)] \end{aligned}$$

where E_y is the expectation with respect to jump size Y_i

where E_z is the expectation with respect to jump size Z_i

3.3 Derivation of Hamilton-Jacobi-Bellman (HJB) equation

Therefore, in case of multidimensional jump diffusion processes, we have,

$$\begin{aligned} E[dF(t, P, B)] &= [F_t + (\mu_1 - \lambda_1 \beta_1) P F_p + (\mu_2 - \lambda_2 \beta_2) B F_B + \frac{\sigma_1^2}{2} P^2 F_{pp} \\ &\quad + \frac{\sigma_2^2}{2} B^2 F_{BB} + \sigma_1 \sigma_2 P B F_{PB}] dt \\ &\quad + E_y [\lambda_1 F(t, P(1 + y_i), B) - F(t, P, B)] dt + E_z [\lambda_2 F(t, P, B(1 + z_i)) - F(t, P, B)] dt \end{aligned}$$

$$\begin{aligned}
\frac{E[dF(t, P, B)]}{dt} &= [F_t + (\mu_1 - \lambda_1\beta_1)PF_P + (\mu_2 - \lambda_2\beta_2)BF_B + \frac{\sigma_1^2}{2}P^2F_{PP} \\
&\quad + \frac{\sigma_2^2}{2}B^2F_{BB} + \sigma_1\sigma_2PBF_{PB}] \\
&\quad + \lambda_1E_y[F(t, P(1 + y_i), B) - F(t, P, B)] + \lambda_2E_z[F(t, P, B(1 + z_i)) - F(t, P, B)]
\end{aligned}$$

Since the option to adopt ethanol blended gasoline has no return till the investment is undertaken, other than the expected capital appreciation, therefore, the Bellman equation to derive optimal threshold satisfies

$$r(t)Fdt = E[dF(t, P, B)]$$

$$\begin{aligned}
&\rightarrow -r(t)F + F_t + (\mu_1 - \lambda_1\beta_1)PF_P + (\mu_2 - \lambda_2\beta_2)BF_B + \frac{\sigma_1^2}{2}P^2F_{PP} \\
&\quad + \frac{\sigma_2^2}{2}B^2F_{BB} + \sigma_1\sigma_2PBF_{PB} \\
&\quad + \lambda_1E_y[F(t, P(1 + y_i), B) - F(t, P, B)] + \lambda_2E_z[F(t, P, B(1 + z_i)) - F(t, P, B)] \\
&= 0
\end{aligned}$$

The initial value function satisfies the above mixed partial differential-difference equation with the free boundary and smooth fit condition. Although linear these equations are difficult to solve and therefore one should use numerical methods to solve such equations to obtain optimal switching boundary.

4. Conclusions and Implications

In this paper, gasoline and ethanol blended gasoline prices are modeled as jump-diffusion processes and result regarding the completeness of the market is derived. Also, to obtain the optimal switching boundary to renewable fuel, the HJB equation is derived. For our analysis, the HJB equation becomes a linear mixed

differential difference equation. This would be solved using numerical methods.

5. Limitations and Future Research

For our analysis and mathematical convenience, we have made a few assumptions which are restrictive. For our analysis, we assumed that the jump intensity, parameters of jump size are independent of time. Future research can derive optimal threshold relaxing these assumptions to see how results change when the jump intensity and size parameters are made a function of time.

Appendix

Proof of Lemma Let θ be a constant and $\tilde{\lambda}_1 = \psi\lambda_1$ and $\tilde{\lambda}_2 = \psi\lambda_2$. Then, we define,

$$Z_0(t) = e^{-\theta W(t) - \frac{\theta^2}{2}t}$$

and

$$Z_m(t) = e^{(\lambda_m - \tilde{\lambda}_m)t} \left[\frac{\tilde{\lambda}_m}{\lambda_m} \right]^{N_m(t)}; m = 1, 2$$

or

$$Z_m(t) = e^{\lambda_m(1-\psi)t} \psi^{N_m(t)}; m = 1, 2$$

Then we define,

$$\begin{aligned}
Z(t) &= Z_0(t) \prod_{m=1}^2 Z_m(t) \\
&= e^{\lambda_1(1-\psi)t + \lambda_2(1-\psi)t - \frac{\theta^2}{2}t - \theta W(t)} \psi^{N_1(t) + N_2(t)} \\
&= e^{\lambda_1(1-\psi)t + \lambda_2(1-\psi)t - \frac{\theta^2}{2}t - \theta W(t) + (N_1(t) + N_2(t)) \log \psi}
\end{aligned}$$

By construction, since $W(t)$, $N_1(t)$ and $N_2(t)$ are independent processes, the process $Z(t)$ is a martingale and $E(Z(t)) = 1$ for all t (Lemma 11.6.8, p.502, Shreve). We invoke Girsanov's theorem of change of measure to define a probability measure \tilde{P} s.t.

$$\tilde{P}(A) = \int_A Z(T) dP, \forall A \in \mathcal{F}$$

Then under the risk-neutral measure \tilde{P}

- $\tilde{W}(t) = W(t) + \theta t$ is a Brownian motion
- $N_m(t)$ is a Poisson process with intensity $\tilde{\lambda}_m = \psi \lambda_m$
- $\tilde{W}(t)$ and $N_1(t), N_2(t)$ are independent to each other.

Therefore, under the risk-neutral measure \tilde{P} , $\tilde{W}(t) = W(t) + \theta t$ is a Brownian motion. Moreover, $N_1(t) \sim \text{Poisson}(\psi \lambda_1)$, which implies, $Q_1(t) \sim \text{Poisson}(\psi, \lambda_1 \beta_1)$ and therefore,

$$[Q_1(t) - \lambda_1 \beta_1 \psi]$$

is a \tilde{P} -martingale

Also, since $N_2(t) \sim \text{Poisson}(\psi \lambda_2)$ therefore

$$Q_2(t) \sim \text{Poisson}(\psi \lambda_2 \beta_2).$$

Therefore, the oil price dynamics under \tilde{P} is given by

$$dP(t) = P(t)[\mu_1 dt + \sigma_1(d\tilde{W}(t) - \theta dt) + dQ_1(t) - \lambda_1\beta_1\psi dt]$$

$$dB(t) = B(t)[(\mu_2 - \sigma_2\theta - \lambda_2\beta_2\psi)dt + \sigma_2 d\tilde{W}(t) + dQ_2(t)]$$

similarly,

$$dB(t) = B(t)[(\mu_2 - \sigma_2\theta - \lambda_2\beta_2\psi)dt + \sigma_2 d\tilde{W}(t) + dQ_2(t)]$$

Since \tilde{P} is a martingale measure, then discounted oil prices would be a martingale for both the prices if θ and ψ are chosen such that

$$\mu_1 - \sigma_1\theta - \lambda_1\beta_1\psi = r(t)$$

and

$$\mu_2 - \sigma_2\theta - \lambda_2\beta_2\psi = r(t)$$

These two equations uniquely determine θ and ψ , confirming the existence and uniqueness of risk neutral measure. Q.E.D

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