Application of Weather Derivatives in Multi-Period Risk Management

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1. Introduction

Crop production involves a complex decision process. During each crop season, farmers must decide what, how, and how much they should produce. Usually the "what and how" is inherited from their ancestors, who transmitted their knowledge about cropping. However, the "how much" depends on what position the farmer has in the market. Assuming that farmers are price takers, farmers will tend to produce as much as they can, only restricted by how much land they have available and how much they need to consume (of course, cost are also involved in the decision process).

The consumption and production decisions are always linked. Once the production is sold, farmers decide how much they should consume, and therefore how much production will be saved to be consumed in the future. The consumption allocation that farmers make is made more complex because crop production is highly dependable on random weather conditions. Thus, when extreme weather conditions are present, production shortfall may become frequent, which in turn affects the way farmers allocate consumption between today and tomorrow.

Developing countries tend to be more vulnerable to natural disasters because they are more dependent on agriculture (Barnett,Barrett and Skees, 2008). Indeed, losses due to extreme weather as a proportion of GDP in these countries have been historically higher than in developed countries (Linnerooth-Bayer,Mechler and Pflug, 2005). In order to deal with the effects of the disasters, governments often have to redirect resources that would be otherwise used in activities such as education, health, etc (Barnett,Barrett and Skees, 2008).

The lack of insurance markets could be one of the causes why producers in the developing countries find themselves in a poverty trap (USAID, 2006). The extreme weather events strangle rural household economy which owns few assets. Barnett,Barrett and Skees, 2008) say that "…shocks that push people below the threshold can set them onto a downward spiral into destitution…" — a situation that could be irreversible. Due to high risk exposure, rural household become more risk averse than otherwise reasonable. Thus, they may adopt low risk investment strategies associated with low return, which is not enough to allow rural households to escape of the poverty trap.

An approach that emerged in recent years uses index insurance products as a way to alleviate the effect of natural disasters in developing countries (Barnett and Mahul, 2007). In 2001 the World Bank designed a rainfall insurance program in Morocco, where a basic cumulative rainfall contract could reduce basis risk for cereal, and at the same time provide income protection to farmers (Skees, et al., 2001). Since then, World Bank has also sponsored several projects in Nicaragua, India, Ukraine, Ethiopia, Malawi, Peru, and Mongolia that attempted to facilitate risk transfer by means of index insurance (World-Bank, 2005).

This work represents a first attempt to analyze the effect of weather derivative availability on the risk management strategies in a multi-period setting. Rice production in Ecuador will be used as a case study.

Weather derivatives can help farmers when production shortfalls are the result of climate conditions. When crop activities take places twice a year, farmer may be better off if they purchase a weather derivatives contract in each season. Although the contract for each season will be written on the realization of different weather index, it allows to farmers be cover during each crop season. This insurance decision will affect how much farmers will consume and save in each period and may help improve their long-term well-being.

2. Some aspects about Rice Production in Ecuador

R ice is one of the largest cereal crops in Ecuador. It is cultivated on the coast and employs 11% of labor force in agriculture. The provinces of Guayas and Los Rios produce 47% and 40%, respectively of the Ecuador total production of rice. Together these two provinces account for 83% of hectares planted with rice.

According to the 2000 Census, 45% of the production units (UPA[1]) that are dedicated to rice production have at most 5 hectares, and 75% of the UPA's are small producers with less than 20 hectares.

Ecuador exports rice mainly to Colombia, Peru and Venezuela. The volume of the rice trade does not exhibit a sustained trend over time. Rather, it depends on domestic supply, domestic producer price paid relative to exports, the supply situation in neighboring countries, and formal and informal current regulations at the northern and southern borders regarding the trade of rice.

The existing price support policy contributes to distortions in Ecuadorian rice market. In 2009, the government of Ecuador signed the ministerial agreement No.0071 with the rice producers, which

[1] UPA stands for Unidad Productiva Agropecuaria (Agricultural Production Unit).

established a price support for rice at USD 28 per a 200-pound bag. The policy is used to guarantee a minimum price in case of overproduction, with the government buying excess rice in order to keep the price at the established level. A government agency — Unidad Nacional de Almacenamiento[2] (UNA) — was created to provide a nationwide network of grain storage that would meet the domestic requirements and also serve as a resource in times of surplus production in order to supply international markets.

However, small farmers do not always benefit from this policy. When the government purchases rice due to overproduction, the producers must bring their production to UNA in order to receive the support price. Small farmers usually do not have access to the infrastructure to bring their rice to the storage units. Thus, they lose the opportunity to receive the price at the support level. On the other hand, the farmers borrow money to cover their planting cost. Intermediaries who lend them money then buy small farmers' production at a lower price than the price support.

There are other threats to small farmers' well-being. The extreme weather often affects rice yields in Ecuador. During the winter season (February and March) excessive rains could affect the growth of the plant. When El Niño occurs rains become more intensive. On the other hand, low temperatures are primary concerns during the summer season (August and September), especially with La Niña.

Agronomists have observed evidence of relationship between low rice yield and El Niño events. In the years preceding the 1998 El Niño, cereals represented 13.1% of the total agricultural GDP, while after the El Niño, they only contributed 9.56%. This is just one example of how extreme weather event could push rural and smallholder farm households into a cycle of poverty (Skees, 2008), especially when they have poor access to infrastructure.

Since Ecuador lacks both the traditional crop insurance and general insurance markets, weather derivatives, if appropriately designed, could be used as risk management tools for rice production.

3. Model

For the purposes of the model, a representative farmer approach is used. The model introduces the possibility for the representative farmer to self-insure risks by accumulating precautionary savings. The farmer is assumed to maximize the discounted sum of expected utility during the years $t = 1 \dots T$. Crop

[2] National Storage Unit

planting activities take place twice a year during the winter and summer seasons. For definiteness sake, we assume that the year starts in September and includes a fall/winter season ($i = 1$) and a spring/summer season ($i = 2$).

At the beginning of the year t (prior to the start of winter season) farmer is assumed to own a wealth $w_{2,t-1}$ (final wealth at the end of the previous year's summer season) from which he consumes $c_{1,t}$ and saves the remainder at the interest rate r.[3]

The farmers receive random revenue $\left(y_{i,t}\right)$ from crop activities at the end of each season, $i = 1,2$. Yields are assumed to be affected by both current and previous season weather conditions. More precisely, climatology conditions prevailing during the summer of year t will affect crop production both in this and the following — winter of $t + 1$—season.

In the baseline scenario (no insurance), the farmer's wealth at the end of the winter in period t is

(1) $$w_{1,t} = (1 + r)\left(w_{2,t-1} - c_{1,t}\right) + y_{1,t}$$

where $w_{1,t}$ is the farmer's wealth at the end of winter of year t, $w_{2,t-1}$ is the farmer's wealth at the end of summer $t - 1$, $c_{1,t}$ is the farmer's consumption during the winter t, and $y_{i,t}$ is the yield received at the end of winder t. The farmer's wealth at the end of summer t can be expressed similarly as

(2) $$w_{2,t} = (1 + r)\left(w_{1,t} - c_{2,t}\right) + y_{2,t}$$

Here again $w_{2,t}$ is the farmer's wealth at the end of summer-t, and $c_{2,t}$ is the farmer's consumption during the summer-t. We assume that the farmer is risk averse and derives utility from consumption, with his preferences described by a utility function $u(c)$.[4] Note that by substituting (1) into (2), we can express the evolution of end-period wealth on an annual basis, viz.

(3) $$w_{2,t} = (1 + r)\left((1 + r)\left(w_{2,t-1} - c_{1,t}\right) + y_{1,t} - c_{2,t}\right) + y_{2,t}$$

The objective of the farmers is then to find the optimal consumption/savings allocation strategy in both seasons and over time so as to maximize the sum total of discounted expected utility over the planning

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[3] In this model, the interest rate is compounded semiannually and is assumed to be constant over time.

[4] It is assumed that u is a concave function.

horizon (i.e. the sum of all utilities the farmer is expected to derive from consumption due to farming over a given number of periods expressed in present-day monetary units). The optimal allocation strategy then has to satisfy the Bellman equation

(4) $$V(w_{2,t-1}) = \max_{c_{1,t},c_{2,t}} \{u(c_{1,t}) + u(c_{2,t}) + \delta EV(w_{2,t})\} \qquad \text{for all} \quad t = 1, \dots, T.$$

subject to (3), where

(5) $$V(w_{2,t-1}) = \max \sum_{\tau=t}^{T} E[u(c_{1,\tau}) + u(c_{2,\tau})]$$

The Bellman equation in (4) can be rewritten as the Euler equilibrium condition on shadow price of wealth $\lambda(w_{2,t-1})$ so that

(6) $$u_1^{'}(c_{1t}) - (1+r)^2 \delta E\lambda \Big((1+r)\big((1+r)(w_{2,t-1} - c_{1,t}) + y_{1,t} - c_{2,t}\big) + y_{2,t}\Big) = 0$$

(7) $$u_2^{'}(c_{2t}) - (1+r) \delta E\lambda \Big((1+r)\big((1+r)(w_{2,t-1} - c_{1,t}) + y_{1,t} - c_{2,t}\big) + y_{2,t}\Big) = 0$$

(8) $$\lambda(w_{2t-1}) = (1+r)^2 \delta E\lambda\big((1+r)^2(w_{2,t-1} - c_{1,t}) + (1+r)(y_{1,t} - c_{2,t}) + y_{2,t}\big)$$

It follows that along the optimal path

(9) $$u_1^{'}(c_{1t}) = u_2^{'}(c_{2t}) + \delta E\lambda \Big((1+r)\big((1+r)(w_{2,t-1} - c_{1,t}) + y_{1,t} - c_{2,t}\big) + y_{2,t}\Big)$$

Thus, the optimal path strategy for consumption in both season should be selected by the farmer so that, on the margin, the benefit received by the farmer from giving up one unit of consumption in the winter is equal to the marginal benefit received from consuming the unit in the summer plus the expected benefits of having that unit of consumption available for either season in the following year.

The equation (9) together with constraint (3) allows us to calculate the optimal consumption decision for the farmer in both seasons. Once we know these consumption strategies, we can project the amount of wealth available to the farmer over the planning horizon, and calculate the state-contingent consumption plans $\{c_{i,t}\}_{t=1}^{T}$.

Depending on the functional forms chosen to represent the farmers' utility function, the Bellman and/or Euler equations generally do not possess closed-form solutions. Thus, the optimal allocation strategies have to be found numerically. In this work, we assume a CRRA[5] power utility function

(10) $$u(w_i;\gamma) = \frac{w_i^{1-\gamma}}{1-\gamma} \;, \; \forall\, i = 1,2$$

Incorporation of Weather Insurance

The general idea of a weather-based insurance (or weather derivatives) is to compensate producers for a particular weather event (e.g. excess rain or low temperatures) measured over a certain time period. If the weather variable is sufficiently correlated with the farmer's yields, the payoff of the weather derivate would then offset the producer's loss. For the purposes of the present project, the weather derivatives constructed in Essay 1 will be used.

The farmers can purchase an insurance contract to cover the risk of low yield. The insurance policy is characterized by a premium rate *P* and indemnity function $I(\cdot)$, which is paid conditional on a weather variable or an index ε_t.[6] Under this new scheme the farmer's wealth at the end of winter season is

(11) $$w_{1,t} = (1+r)\big(w_{2,t-1} - c_{1,t}\big) + y_{1,t} + I_1\big(\varepsilon_{1,t}\big) - P_1(I_1)$$

where $I_1(\varepsilon_{1,t})$ is the indemnity function which depends on the index measured on weather variables that affect winter crop production, $P_1(I_1)$ is the associated insurance premium, and all other variables are as in (1). On the other hand, the farmers' wealth in the summer is

(12) $$w_{2,t} = (1+r)\big(w_{1,t} - c_{2,t}\big) + y_{2,t} + I_2\big(\varepsilon_{2,t}\big) - P_2(I_2)$$

where $I_2(\varepsilon_{2,t})$ is the indemnity function which depends on the index measured on weather variables that affect summer crop production, $P_2(I_2)$ is the associated insurance premium, and all other variables are as in (2). Under this assumption, the farmer's decision problem is to select a state-contingent consumption plan $\{c_t\}_{t=1}^{T}$, and insurance plan $\{I_{1,t}, I_{2,t}\}_{t=1}^{T}$ by solving equations (9), (11), and (12).

[5] CRRA stands for Constant Relative Risk Aversion.
[6] For simplicity sake, we assume that y, $I(\varepsilon)$, and P are all denominated in the same (monetary) units.

4. Data

The province of Guayas and Los Rios produces around 87% of the total rice production. In this study we focus on the information reported for two counties: Daule and Babahoyo, which are located in Guayas and Los Rios respectively. The county-level rice data per season are obtained from MAGAP in the period 1990-2008. MAGAP reports the total rice production for each county.

The county-level weather data used for analysis are observations of average monthly total rainfall and temperature for the two winter months (February and March) and two summer months (August and September). Data for each weather station are collected from databases of the Instituto Nacional de Meteorología e Hidrología[7] (INAMHI) which gather this information from the weather stations located in both provinces.

Data Exploration

According to MAGAP, since 2000 these two counties have represented more than 35% of rice production in Ecuador. Descriptive statistics of the collected data are presented in Table 1. The table provides basic information about these counties. Figures 1-2 present historical rice yields in each county. Two unit root tests — Augmented Dickey-Fuller and Phillip-Perron — were performed on the data (Table 2). The test results suggest that there is no evidence of unit root in all counties in both seasons.

Detrending Data

The historical yield graphs in Figure 1-2 also suggest that the rice production is affected by a trend. To account for this effect, yields and weather data are detrended using a log-linear trend model:

$$log(Y_t^{tr}) = \beta_0 + \beta_1(t - 1990)$$

The detrended yields were calculated as:

$$Y_t^{det} = Y_t \frac{Y_t^{tr}}{Y_{1990}^{tr}}$$

Yield-Weather Relationship

[7] National Institute of Meteorology and Hydrology.

The weather models used to define the weather index are constructed based on the relationship between rice yield and weather variables. As mentioned above, winter production is affected mainly by rainfall, while low temperatures are the primary concern during the summer. Tables 3 and 4 list weather models estimated as a part of the preliminary analysis. Of the models analyzed, the best one has a goodness of fit of only R^2 = 0.33 (for Babahoyo in winter season).

5. Implementation of the Methodology

Once the weather-yield models are estimated, the weather derivatives contracts for the counties selected are designed and pricing. After, a numerical solution program for the Bellman equation describing the consumption/savings strategies will be constructed. A Monte Carlo simulation approach will be used to construct the evolution of the optimal path of consumption and savings with and without the insurance. Finally, the optimal consumption paths will be compared and the effect of basis risk on changes in optimal consumption allocation patterns will be evaluated.

5.1 Design and Pricing of Weather Derivatives

Following Vedenov and Barnett (2004), a weather derivative is modeled as an “elementary contract” with the payoff according to the schedule:

$$(13)\qquad I(\varepsilon|x,\varepsilon^*,\mu) = x \times \begin{cases} 0 & if\ \varepsilon > \varepsilon^* \\ \frac{\varepsilon^*-\varepsilon}{\varepsilon^*-\mu\varepsilon^*} & if\ \mu\varepsilon^* < \varepsilon \leq \varepsilon^* \\ 1 & if\ \varepsilon \leq \mu\varepsilon^* \end{cases}$$

where ε is a realization of the index. The contract starts to pay when the index ε falls below the specified “strike” ε^*. Once the index falls below the limit $\mu\varepsilon^*$, the insured receives the maximum indemnity x. When the index falls between the strike and the limit, the contract pays a proportion of the maximum indemnity. The parameter μ varies between 0 and 1, with the limiting case of 0 corresponding to the conventional proportional payoff with deductible, and 1 corresponding to a “lump-sum” payment once the contract is triggered regardless of the severity of the shortfall. The contract is completely designed once the values of strike, limit and maximum indemnity are specified.

In order to price the designed contract for a given set of parameter values, the probability distribution $h_\varepsilon(\varepsilon)$ of the index should also be specified. Using historical weather observations from each selected location, the weather-yield models are used to calculate the "historical realizations" of the index for each location. In this study it is assumed that the index can be modeled with a normal probability function.

The actuarially-fair premium is set equal to the expected payoff of the contract, i.e.

$$(14) \qquad P(x, \varepsilon^{*}, \mu) = \int I(\varepsilon | x, \varepsilon^{*}, \mu)\, h_{\varepsilon}(\varepsilon) d\varepsilon$$

The parameters in equation (1) are selected for each location/index analyzed so as to provide the maximum risk reduction for the buyers who are exposed to the risk area-wide yield loss. In particular, the parameters are selected so as to maximize the expected utility

$$(15) \qquad max_{\ \mu, x, \varepsilon*}\ Eu\big(y + I(\varepsilon | x, \varepsilon^{*}, \mu) - P(x, \varepsilon^{*}, \mu)\big)$$

The strikes, limits, and maximum liabilities for optimal contracts are reported in table 5. A CRRA[8] power function was used to get the parameters on the contracts. The weather models estimated (table 3-4) suggest that each season/county combination has its own index and the models fail to capture the relationship between weather and yields. The result shown in table 5 suggests that weather derivatives vary across county and season (e.g. Daule during the winter vs Daule during the summer). Thus, location, season, edaphology, and agricultural practices imply the need for different weather derivatives.

The set of parameters that maximizes equation (15) are used to implement the dynamic algorithm for this paper. Thus, it is assumed that once the parameters for a standard contract are set up, they never change.

5.2 Numerical Solution

The numerical solution is implemented by the collocation method. The collocation method employs a conceptually straightforward strategy to solve functional equations (equation 4). Specifically, the bellman equation is approximated using a linear combination of n known basis functions whose n coefficients are fixed by requiring the approximant to satisfy the bellman equation. Chebychev

[8] CRRA stands for Constant Relative Risk Aversion.

polynomial is the collocation basis-node schemes chosen to approximate the bellman equation. The Newton finding-root method is used to evaluate the collocation function and its derivatives.

At this time the code has been implemented, but some adjustment have been made. Thus, final results will be presented during the conferences.

Post optimality Analysis

The optimal policy and shadow price functions (equations 8-9) reveal the nature of the optimized dynamic process, but it is not a complete picture of the model's implications. As Miranda and Fackler, 2002) say: *A more revealing analysis of the dynamics generated by a stochastic model is to draw not a single representative path, but rather the expected path of the process*. The expected path for consumption in both seasons is computed based on a large number of independent representative paths and averaging the results at each point in time. Thus, the expected path converges to a steady-state mean value. The Monte Carlo Simulation approach is used to compute the steady-state means and variances

At this time the code has been implemented, but some adjustment have been made. Thus, final results will be presented during the conferences.

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Figure 1: Dynamics of Rice Production in Babahoyo, 1990-2008

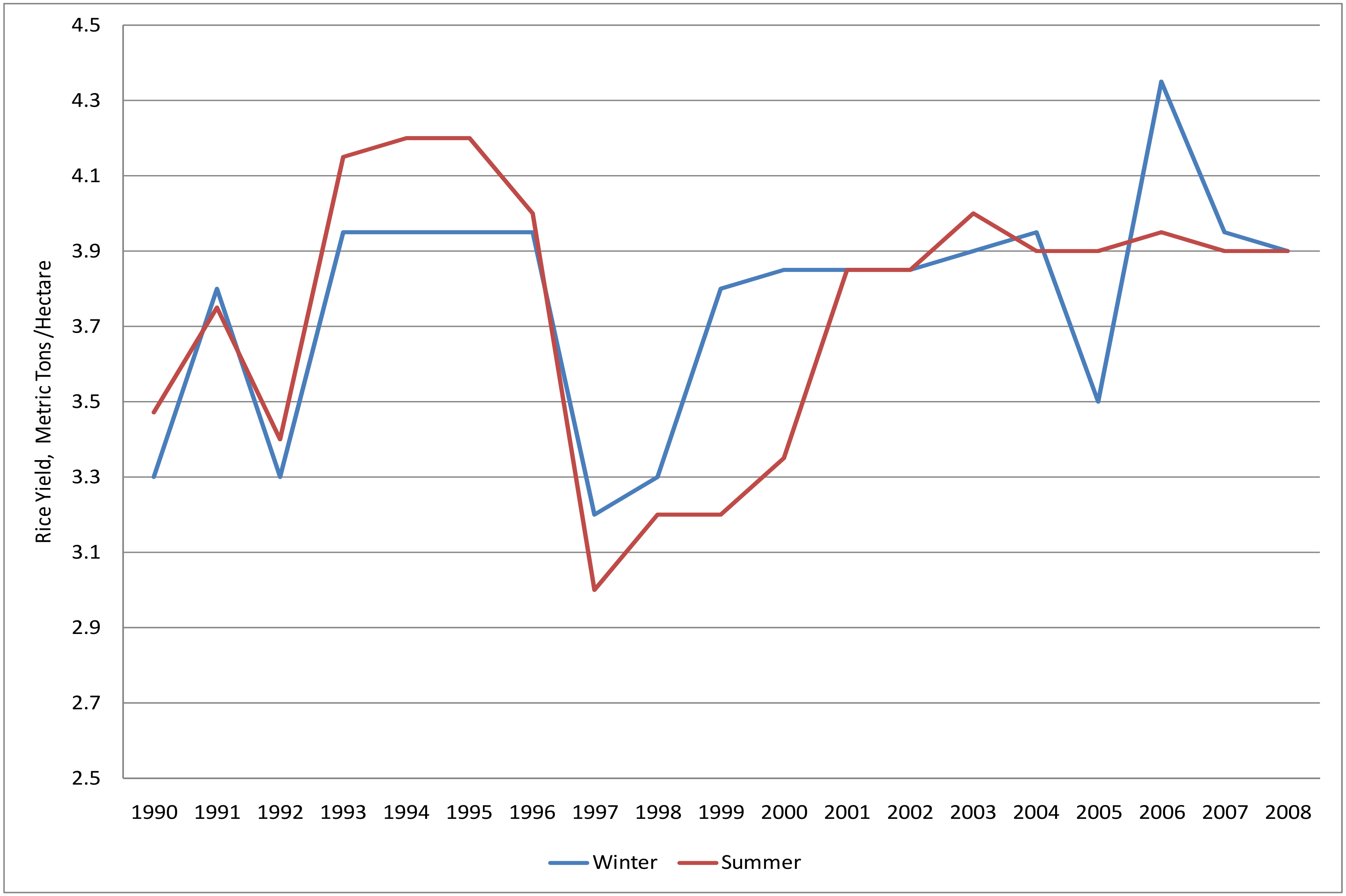

Figure 2: Dynamics of Rice Production in Daule, 1990-2008

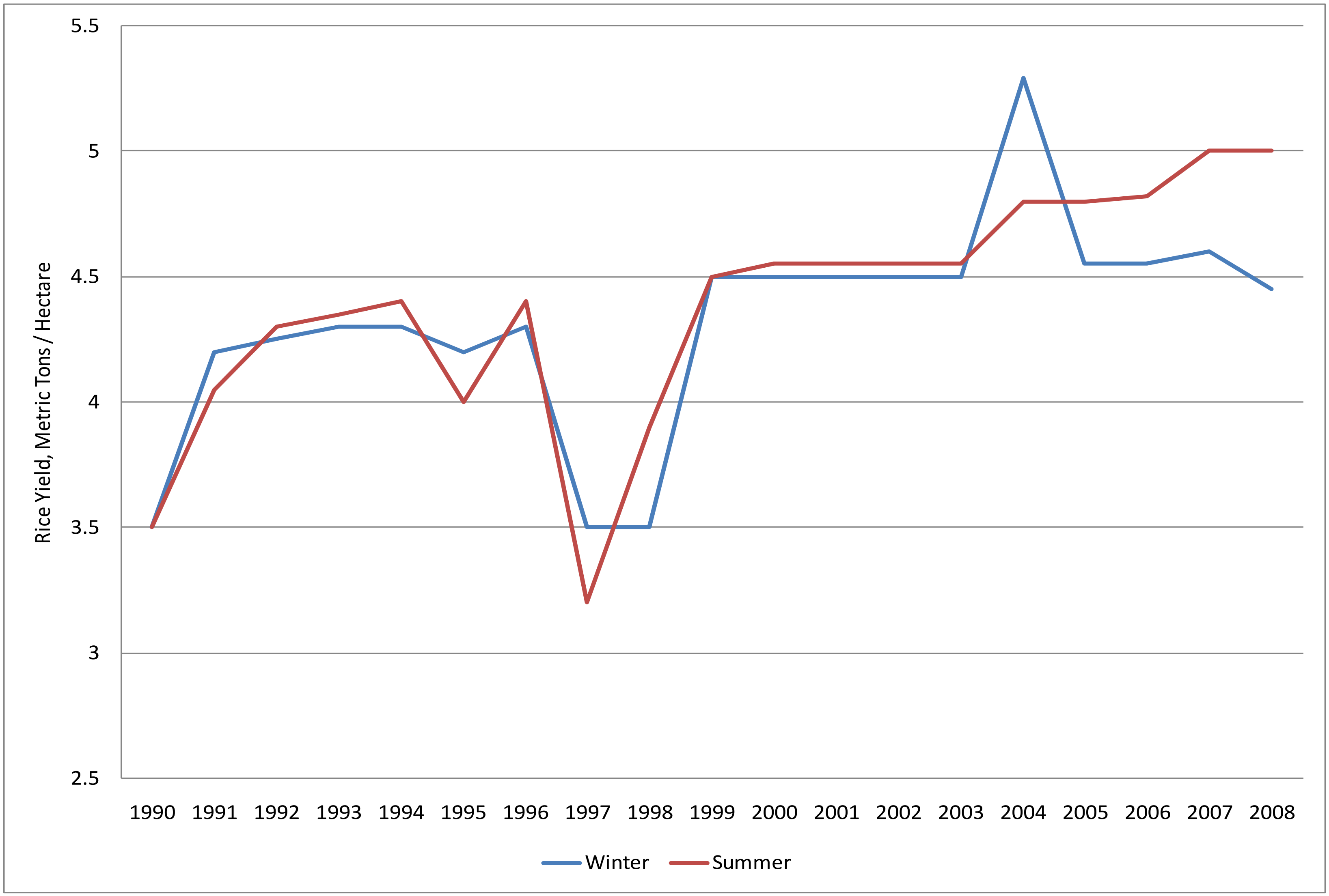

Table 1: Descriptive Statistics of Rice Yields for Main Producing Counties

	Winter		Summer	
	Babahoyo	**Daule**	**Babahoyo**	**Daule**
Sample Statistics				
Mean	3.768	4.315	3.746	4.380
Median	3.850	4.450	3.900	4.500
Maximum	4.350	5.290	4.200	5.000
Minimum	3.200	3.500	3.000	3.200
Std. Dev.	0.302	0.432	0.363	0.477
Skewness	-0.562	-0.440	-0.661	-0.941
Kurtosis	2.634	3.792	2.222	3.411
Jarque-Bera	1.107	1.108	1.862	2.939
Probability	0.575	0.575	0.394	0.230
Observations	19	19	19	19
Correlation Matrix				
Babahoyo	1.000		1.000	0.512
Daule	0.689	1.000	0.512	1.000

Table 2: Unit Root Tests of Yield Data Series

	Winter		Summer	
	Babahoyo	Daule	Babahoyo	Daule
ADF				
c	-3.98	-3.045	-2.26	-2.6
	(-3.04)	(-3.04)	(-3.04)	(-3.04)
ct	-4.19	-3.15	-2.2	-3.41
	(-3.69)	(-3.69)	(-3.69)	(-3.69)
PP				
c	-3.98	-3.06	-2.3	-2.57
	(-3.04)	(-3.04)	(-3.04)	(-3.04)
ct	-4.2	-3.21	-2.25	-3.42
	(-3.69)	(-3.69)	(-3.69)	(-3.69)
Lags	3	3	3	3

Note: The variables are expressed in logarithms. Lags is the number that minimize the Schwartz criterio. ADF= Augmmented Dickey-Fuller Test; PP= Phillips-Perron Test.

Table 3: Weather Models for Winter Season Based on 1990-2008 Data

	Winter	
	Babahoyo	Daule
Rainfall February	0.00704	0.0110
	(0.0313)	(0.0426)
Rainfall March	0.225	0.0390
	(0.154)	(0.104)
Rainfall February square	-0.00183	-0.00209
	(0.00267)	(0.00446)
Rainfall March square	-0.0155	-0.00400
	(0.0121)	(0.00793)
Constant	0.570	1.388***
	(0.471)	(0.284)
Observations	19	19
R-squared	0.334	0.199

Standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 4: Weather Models for Summer Season Based on 1990-2008 Data

	Summer	
	Babahoyo	Daule
Temperature August	4.816	-6.222
	(9.313)	(16.19)
Temperature September	-11.71	8.533
	(13.55)	(20.47)
Temperature August square	-0.0302	0.0417
	(0.0596)	(0.109)
Temperature September square	0.0742	-0.0572
	(0.0865)	(0.137)
Constant	271.8	-85.13
	(322.9)	(317.6)
Observations	19	19
R-squared	0.186	0.029

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 5: Parameters of Optimal Weather Instruments

Season/County	Strike (qq/hec)	Limit: Absolute Value (qq.hec)	Limit: % of Strike	Maximum Liability (qq/hec)	Premium (qq/hec)	Premium Rate
			γ=1			
Winter/Daule	3.8	2.014	0.53	1	1.43	143.0%
Winter/Babahoyo	3.5	2.66	0.76	1	1.54	146.3%
Summer/Daule	3.9	3.12	0.8	1	3.34	317.3%
Summer/Babahoyo	3	1.95	0.65	0.75	0.26	22.1%
			γ=2			
Winter/Daule	3.5	1.925	0.55	1	0.87	87.0%
Winter/Babahoyo	3.1	2.17	0.7	1.5	0.38	25.3%
Summer/Daule	4.2	3.318	0.79	1.25	4.14	331.2%
Summer/Babahoyo	2.5	1.625	0.65	2	0.01	0.5%
			γ=3			
Winter/Daule	3.6	1.8	0.5	1.25	0.96	76.8%
Winter/Babahoyo	3	1.95	0.65	1.75	0.22	12.6%
Summer/Daule	4	2.8	0.7	1.25	2.82	225.6%
Summer/Babahoyo	3.2	1.888	0.59	0.75	0.44	58.7%