Modeling Crop Yield Distributions from Small Samples

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Abstract

Accurately modeling crop yield distributions is important for estimation of crop insurance premiums and farm risk-management decisions. A major challenge in the modeling has been due to small sample size. This study evaluated potentials of L-moments, a recent concept in mathematical statistics, in modeling crop yield distribution. Five candidate distributions were ranked for describing the wheat yields. The selected distribution was robust for small sample and was invariant to de-trending. The result was consistent with that from the maximum likelihood and goodness-of-fit method.

Modeling Crop Yield Distributions from Small Samples

Accurately modeling crop yield distributions is important for estimation of crop insurance premiums and farm risk-management decisions. Accurate modeling of crop yield typically requires longer time series observations. In general, a lack of consistent series of farm-level yields of sufficient length is a major concern in modeling crop yield distribution. Researcher's responses have been to use the available regional aggregate yields or the augmentation of temporal variation with spatial variation.

Various crop modeling approaches are proposed in the agricultural economics literature (Day; Gallagher; Just and Weninger; Goodwin and Ker; Nelson; Ramirez, Misra and Field). The modeling approach varies from non-parametric to parametric. Few researchers favour the semi-parametric and non-parametric approaches for modeling crop yield distribution (Goodwin and ker; Ramiraz, Misra and Field; Chen and Miranda). However, parametric approaches are usually preferred due to small samples. The parametric approach of modeling yields usually involves selection of candidate distributions, parameter estimation and assessment of goodness-of-fit. Several distributions could be selected as candidate distributions based on literature: Beta distribution (Nelson), Gamma distribution (Gallagher), Normal distribution (Just and Weninger), Weibull distribution (Chen and Miranda), Lognormal (Day), and Logistic (Sherrick et al.). Maximum likelihood (ML) is an efficient method for parameter estimation, but when the sample size is small it is difficult to distinguish the distribution precisely. L-moment ratio estimator is a recent development in mathematical statistics (Hosking). L-moment ratio diagrams have been successfully used for distribution selection in hydrology (Kroll and

Vogel; Pandey, van Gelder and Vrijling). L-moment ratios could have a potential use in crop modeling because they do not have sample size related bounds and are more robust to the presence of outliers compared to product moment ratios.

The literature also varies on sources of crop yield: experiment, field, farm and regional aggregate data. Several studies based their modeling on aggregate time series data (Gallagher; Goodwin and Ker). Aggregate farm level data usually does not provide impact of input decisions on crop yield distributions. Just and Weninger (1999) indicated that recent literatures mostly used aggregate time-series data and none used experimental data to evaluate crop yield distribution. Day (1965) used long term experiment data to evaluate the impact of nitrogen fertilizer on wheat yield distribution. However, it is rare to find long term crop yields from the same experiment.

The objective of this study is to rank candidate yield distribution based on L-moment and conventional maximum likelihood based methods. The impact of trend removal will be assessed on the ranking of the distributions. Alternate methods will be tested using wheat yields from a long term experiment at Lethbridge, Alberta. Characteristics of wheat yield distributions will be assessed under fallow and fertilizer decisions.

METHOD

L-moments are summary statistics for probability distributions as ordinary moments. They provide measures of location, scale, and shape of probability distributions. L-moment is a linear combination of order statistics and it is less prone to the effect of sampling variability as compared to conventional moments. Hosking (1990) provides details on

3

underlying theory and application of L-moments as summary statistics for probability distributions.

Let $X_{1:n} \le X_{2:n} \ldots \le X_{n:n}$ be the order statistics of a random sample of size n drawn from a distribution of X. The rth L-moment (λ_r) of the random variable X are defined as:

$$\lambda_r = r^{-1} \sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} E(X_{r-k:r})$$

where $E(X_{r-k:r})$ is the expectation of an order statistic.

The first four L-moments of the random variable X is given as:

$$\lambda_1 = E(X)$$

$$\lambda_2 = 1/2 E(X_{2:2} - X_{1:2})$$

$$\lambda_3 = 1/3 E(X_{3:3} - X_{2:3} - X_{1:3})$$

$$\lambda_4 = 1/4 E(X_{4:4} - X_{3:4} - X_{2:4} - X_{1:4})$$

Unlike L-moments, L-moment ratios are neutral to the scale of measurement. L-moment analogue of the coefficient of variation, skewness and kurtosis are computed as:

$$L - cv = \tau_2 = \lambda_2 / \lambda_1$$

$$L - skewness = \tau_3 = \lambda_3 / \lambda_2$$

$$L - kurtosis = \tau_4 = \lambda_4 / \lambda_2$$

L-moment ratios are bounded. For non-degenerate distributions with finite means, $|\tau_r| < 1$ for $r = 3$ and 4, and for positive random variables, $X>0$, $0 < \tau_2 < 1$ (Stedinger, Vogel and Foufoula-Georgiou).

L-moments from a sample

Let x be a sample of random yields from year 1 to n with mean μ and standard deviation σ. L-moments for the sample are estimated using probability weighted-moments estimator:

$$b_0 = n^{-1} \sum_{i=1}^{n} X_{i:n}$$

$$b_r = n^{-1} \sum_{i=1}^{n} \frac{(i-1)(i-2)...(i-r)}{(n-1)(n-2)...(n-r)} X_{i:n}, r \geq 1$$

For a random sample x, the estimate of first four L-moments are computed as:

$$l_1 = b_0$$

$$l_2 = 2b_1 - b_0$$

$$l_3 = 6b_2 - 6b_1 + b_0$$

$$l_4 = 20b_3 - 30b_2 + 12b_1 - b_0$$

L-moment ratio diagrams

L-moment ratio diagrams are useful for identifying the underlying data distribution (Hosking). L-moment diagrams include plots of τ_2 versus τ_3 for choosing among two-parameter distributions and τ_4 versus τ_3 for choosing among three-parameter distributions (Stedinger, Vogel and Foufoula-Georgiou). If the observations are drawn from a normal distribution, then $l_2 = \sigma / \mu\sqrt{\pi}$ and l_3 has mean 0 and var $[l_3]$ = *(0.1866+0.8/n)/n*. If the sample average L-moment ratio is closer to the expected L-moment ratio for a specific family of distribution then that distribution is implied by the diagram. Polynomial approximations are useful for showing the expected L-moment ratios for different

probability distributions. The polynomial approximation of τ_2 in terms of τ_3 could be

estimated with a reasonable accuracy (Hosking; Vogel and Wilson) by:

$$\tau_2 = \sum_{j=0}^{7} A_j (\tau_3)^j$$

where A_j are given in table 1.

Maximum likelihood estimation

Maximum likelihood (ML) based method involves parameter estimation using ML

and assessing the goodness-of-fit of the simulated distribution with that from sample

distributions. It is an acceptable practice to test for normality before parameter estimation

for non-normal distributions. Identification of distributions is sensitive to trend and

heteroskedasticity in the data. Therefore, data are corrected for any trend and

heteroskedasticity before parameter estimation. Several tests are available for decision on

normality (Just and Weninger). However, decision on normality would be based on

Shapiro-Wilk (S-W), Kolmogorov-Smirnov (K-S), Cramer-von Misses (C-M), and

Anderson and Darling test (A-D).

Goodness-of-fit measure

Anderson and Darling (AD) is an appropriate distribution free goodness-of-fit test

(Sherrick et al.). Yield realizations over time would be judged against the simulated yields

from candidate distributions using the AD statistic. The smaller the AD statistic, closer the

fit of the hypothetical distribution to the historical sample of crop yields.

The AD test statistic is defined as (Law and Kelton, p. 368):

$$AD = n\int_{-\infty}^{\infty}\left[F_n(X) - \hat{F}(X)\right]^2 \psi(X)\hat{f}(X)dy$$

where $\psi(X)$, $\hat{F}(X)$ and $\hat{f}(X)$ are the weight function, the fitted cumulative and the fitted probability density functions, respectively and n is the sample size. The AD statistic can be estimated as:

$$AD = \frac{-\left\{\sum_{i=1}^{n}(2i-1)\left\{\ln\hat{F}(X_i) + \ln(1-\hat{F}(X_{n+1-i}))\right\}\right\}}{n} - n$$

Candidate yield distributions

Any theoretical distribution with desirable characteristics can be selected as a candidate distribution. Candidate distribution can be univariate or multivariate. Univariate distributions can be further classified based on number of parameters. Five two-parameter univariate distributions were selected based on the literature: Gamma, Lognormal, Normal, Weibull, and Pareto. Among them lognormal and normal are symmetric distributions. Gamma, Weibull and Pareto allows the yield distribution to be skewed.

DATA

Wheat yields were obtained from a long term single site experiment since 1906 at Lethbridge Research Centre at Alberta, Canada. However, yields used in this study were from 1972 to 2004, the period of fertilization. Annual yields for 16 treatment decisions were recorded from the entire large plot of about 0.25 acres. Treatment decisions included the frequency of fallow in sequence with wheat and application of nitrogen (N) and/ or phosphorus (P) fertilizer. There were four crop rotations including continuous wheat (W),

wheat followed by fallow (FW) in a two-year rotation and, wheat followed by fallow in a three-year rotation (FWW) and wheat followed by wheat in a three-year rotation (WWF). Each sequence received four fertilizer treatments (zero fertilizer, 45 kg ha^{-1}N, 20 kg ha^{-1}P, 45 kg ha^{-1}N and 20 kg ha^{-1}P).

Trend and normality

      The sample of wheat yields failed to show evidence for higher order polynomial trend and non-normal yield distributions (Table 2). Positive linear trend was found significant for 12 and 14 treatment decisions at 5% and 10% level, respectively. Positive linear trend witnessed the positive impact of technological change on wheat yields. Linear trend was positive but not significant for FW and FW-P decisions. Presence of linear trend in the majority of treatment decisions justified the use of de-trended yields for parameter estimation. De-trending was done by rotating the data around the mean:

$$x_t^d = x_t - (a + b*t) + \mu$$

where x_t^d is the de-trended yield at time t, x_t is actual yield, a is the intercept and b is the slope parameters from the regression, and μ is the average yield.

      There is no evidence of single consistently superior test for normality. Past studies have used multiple tests to overcome this limitation. In this study, yield normality was tested using four recently popular tests: K-S, C-M, A-D and S-W tests. Just and Weninger (1999) cautioned against the use of multiple tests due to the potential for conflicting information. However, this study showed that all tests were consistent in failing to reject normality of actual as well as de-trended wheat yield at Lethbridge (Table 2). This shows that normality was not conditional on de-trending for technological change over time.

Failure to reject normality may not necessarily imply that non-normal models could not fit better. Five candidate distributions were ranked based on their fit to the sample wheat yields.

RESULTS

Wheat yields for 16 treatment decisions over 33 years, before and after trend removal, are summarized in Table 3. Treatment decisions included fallow and fertilizer decisions. Yield distributions are summarized using mean, standard deviation, skewness and kurtosis statistics.

Fallow had an impact on wheat yield distribution. Continuous wheat generated low yields, negative skewness, and negative kurtosis. This indicated the influence of few extreme years with low yields on continuous wheat yield distribution at Lethbridge. Fallow in the rotation increased the mean wheat yield compared to without fallow. Fallow helps conserve soil moisture, which is available for the subsequent wheat crop in this semi-arid region. Frequency of fallow also had an impact on yield. Fallow in a two year rotation compared to in a three year rotation generally increased the mean yield, increased positive skewness and decreased kurtosis. Positive skewness is expected for dry land wheat production in semi-arid regions because of few good years with many not-so-good years in terms of precipitation.

Fertilizer had a consistent positive impact on wheat yield. For any crop and crop-fallow sequence, nitrogen (N) or phosphorus (P) increased the mean yield. Application of N and P together consistently generated the highest mean yields. Application of N, compared to without N, generated a more symmetric wheat yield distribution. When the

wheat yield without N had negative (positive) skewness, N increased (decreased) skewness. Kurtosis generally decreased due to application of N.

Trend removal had an impact on higher moments of the yield distribution (Table 3). As expected, first moment about the origin (mean) was not affected by the trend removal. De-trending the data by rotating around the mean reduced the second moment (variance) and third moment about the origin (skewness), except for FW. The FW treatments also failed to show a significant impact of technological progress (Table 2). The impact on fourth moment about the origin (kurtosis) was not as conclusive as for other lower order moments. However, kurtosis generally increased with the trend removal except for W-N, WWF, WWF-P and WWF-NP treatment decisions.

L-moment diagrams

L-moment diagram compared the sample estimates of L-moment ratios with their population counterparts for the four out of selected five distributions: Gamma, Lognormal, Normal, Weibull, and Pareto. Theoretical relationships among L-cv and L-skewness for two and three-parameter distributions are available in exact form from Hosking (1990) and Stedinger, Vogel and Foufoula-Georgiou (1993). Polynomial approximations used in this study for two-parameter Gamma, Lognormal, Weibull, and Pareto distributions are summarized in Table 1.

L-moment diagrams were generated from historical wheat yields from all 16 treatment decisions using actual (Figure 1) and de-trended (Figure 2) yields. Each series of yields, actual and de-trended, were also grouped into 3 separate subsets of equal lengths (1972-82, 1983-93 and 1994-2004) to analyze the impact of sample size on distribution

ranking. Each diagram shows L-skewness along the horizontal axis and L-cv along the

vertical axis. Since L-moment ratio estimators are approximately unbiased, regardless of

the underlying probability distribution, one expects the best fitted theoretical curve to

separate the sample data points into approximately half on each side of the distribution

(Vogel and Wilson).

Figure 1 compares the observed and theoretical relations between L-cv and L-

skewness from actual wheat yields among four probability distributions and for four yield

samples. A 33 year sample of wheat yield showed that L-cv points for Weibull distribution

separate the sample L-cv points approximately into two halves indicating that Weibull

distribution could be the appropriate wheat yield distribution. Pareto distribution was a

poor fit because it over predicted the observed L-cv. Similarly, Lognormal and Gamma

distributions were poor fit because they consistently under predicted L-cv. The pattern of

result was similar across all samples, except for the period 1972-82.

For the best fitted distribution, sample statistics should approximate the population

parameters. The sample average L-cv and L-skewness for actual wheat yields (1972-2004)

was 0.2438 and 0.0251, respectively (Table 4). For Normal distribution L-cv and L-

skewness are 0.2094 and 0, respectively (Stedinger, Vogel and Foufoula-Georgious).

Comparing sample L-cv with Normal and Weibull (0.2052), Normal distribution seems

more appropriate for wheat distribution. The Normal distribution was consistently better

fitted among selected candidate distributions in all periods.

Similarly, Figure 2 compares the observed and theoretical relations between L-cv

and L-skewness for the de-trended wheat yields. Among the four fitted distributions,

Weibull distribution was a better fit for the sample in all periods, except for 1972-82. In

the exceptional period Gamma and Lognormal had a better fit than Weibull. Pareto distribution was consistently a poor fit due to over prediction of L-cv. When normal distribution was included in the comparison, Normal distribution had consistently better fit in all period (Table 4).

The selected distribution using L-moment ratios and L-moment diagrams were robust across all small samples and de-trending. This consistent pattern of better fit of Normal over other candidate distributions across all sample periods indicated the robustness of L-moment ratios even under small sample condition (Table 4). L-moment diagrams were also consistent in selecting distribution before and after trend removal (Figures 1 and 2). This indicated that selected distribution is invariant to trend removal. However, the L-moment ratios from the de-trended data approximated Normal distribution more closely than the actual data (Table 4). This support the finding of Atwood, Shaik and Watts (2003) that the de-trending of trended data makes the distribution closer to Normal.

Maximum Likelihood and Goodness-of-fit

Parameters for candidate distributions were estimated from de-trended wheat yields generated from 16 treatment decisions over 33 years (Table 5). The Normal and Weibull distributions were found superior to other candidate distributions for representing wheat yield at Lethbridge. The Pareto distribution was consistently the most inferior distribution. Lognormal and Gamma distributions were also generally inferior fitted compared to Normal distribution. Normal or Weibull distribution was selected for 12 out of 16 treatment decisions, which indicated that these management decisions had less roles in

determining the yield distribution compared to environmental factors. However, management decisions did have impact on parameters of the distribution (Table 3).

SUMMARY AND CONCLUSIONS

Several candidate crop yield distributions have been offered in the agricultural economics literature. Accurately identifying the yield distribution is important due to its potential use for farm risk management, policy, and crop insurance premium identification. A major challenge in modeling crop yield distributions has been small sample size. This study seeks to introduce an alternate method of modeling crop yield distribution and apply it to wheat yields.

L-moments are relatively recent introduction in mathematical statistics. The L-moment based method for distribution selection has some advantages over the conventional ML based method. L-moments are linear combination of order statistics and are less biased than ordinary moments. L-moment ratios can be plotted as L-moment diagrams which provide a simple and visual way of identifying a best fitting distribution. Selected distributions based on L-moment are also less likely to be sensitive to trend removal.

Five two-parameter candidate crop yield distributions were ranked for describing the wheat yield distribution: Gamma, Lognormal, Normal, Weibull, and Pareto. Ranking was based on both L-moment and ML based methods. Both methods consistently ranked Normal as the best fitted distribution. L-moment based methods were also tested under small sub samples and de-trending. Best fitted distribution remained robust across all small sub samples. Trend removal had no impact on the ranking of distributions. Normally

13

distributed wheat yield have several economic implications. It simplifies the economic decision making to mean-variance approximation of expected utility. Furthermore, normal yield may imply lower insurance premium compared to some thicker tail distributions.

Several specific conclusions can be drawn for wheat yields at Lethbridge. Continuous wheat was found to generate negative skewness and kurtosis for yields at Lethbridge exposing the farmer to few extreme financial risks. Fallow had positive impact on wheat yield due to increased expected yield and lower risk of being below an expected yield. Fertilizer consistently increased wheat yield. Application of nitrogen fertilizer had an impact on wheat yield distribution making it more symmetric.

This study found that L-moment based method could be useful for distribution identification even in the case of a small sample. The method should also be tested with several crops and locations to gain further insight on its usefulness for crop yield modeling.

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Table 1. Coefficients of polynomial approximations for L-moment diagram

Coefficient	Two-parameter Distributions			
	Lognormal	Gamma	Weibull	Pareto
A_0	.	.	0.17864	0.33299
A_1	1.16008	1.7414	1.02381	0.44559
A_2	-0.05325	.	-0.17878	0.16641
A_3	.	-2.5974	.	.
A_4	-0.10501	2.0991	-0.00894	.
A_5	.	.	.	0.09111
A_6	-0.00103	-0.3595	-0.01443	.
A_7	.	.	.	-0.03625

Source: Vogel and Wilson 1996

Table 2: P-values for linear trend and normality tests for actual wheat yields, 1972-2004.

Treatments	Trend	S-W	K-S	C-V	A-D
W	0.08	0.94	>0.15	>0.25	>0.25
W-N	0.03	0.89	>0.15	>0.25	>0.25
W-P	0.05	0.65	>0.15	>0.25	>0.25
W-NP	0.02	0.67	>0.15	>0.25	>0.25
FW	0.36	0.36	>0.15	>0.25	>0.25
FW-N	0.06	0.09	>0.15	>0.25	>0.25
FW-P	0.25	0.23	>0.15	>0.25	>0.25
FW-NP	0.04	0.21	>0.15	>0.25	>0.25
FWW	0.01	0.54	>0.15	>0.25	>0.25
FWW-N	0.02	0.29	>0.15	>0.25	>0.25
FWW-P	0.03	0.68	>0.15	>0.25	>0.25
FWW-NP	0.05	0.36	>0.15	>0.25	>0.25
WWF	0.02	0.33	>0.15	0.24	>0.25
WWF-N	0.03	0.84	>0.15	>0.25	>0.25
WWF-P	0.01	0.10	0.08	0.08	0.10
WWF-NP	0.05	0.54	>0.15	>0.25	>0.25

Note: Treatments: W= continuous wheat, FW= fallow-wheat, FWW= fallow-wheat-wheat (after fallow), WWF= wheat-wheat-fallow (after wheat), N= 45 kg ha^{-1} N and P= 20 kg ha^{-1}P. Quadratic and higher order trend was not significant for all variables and therefore not reported. S-W, K-S, C-V and A-D stands for Shapiro-Wilks, Kolmogorov-Smirnov, Cramer-von Mises and Anderson and Darling tests, respectively.

Table 3. Summary statistics of wheat yield (t ha^{-1}) distribution, 1972-2004

Treatments		Before trend removal			After trend removal		
	Mean	StDev	Skewness	Kurtosis	StDev	Skewness	Kurtosis
W	1.44	0.58	-0.13	-0.25	0.55	-0.15	-0.09
W-N	1.90	0.89	0.02	-0.60	0.82	-0.07	-0.84
W-P	1.60	0.59	-0.34	-0.40	0.55	-0.37	-0.33
W-NP	2.40	1.15	-0.06	-0.70	0.99	-0.24	-0.56
FW	2.83	0.74	0.46	-0.05	0.73	0.60	0.27
FW-N	2.98	0.93	0.22	-1.06	0.88	0.06	-0.88
FW-P	2.92	0.78	0.36	-0.72	0.77	0.34	-0.16
FW-NP	3.29	0.97	0.30	-0.83	0.91	0.14	-0.75
FWW	2.50	0.75	-0.11	-0.72	0.74	-0.22	-0.38
FWW-N	2.73	0.90	0.51	0.06	0.87	0.41	0.09
FWW-P	2.90	0.88	-0.46	0.85	0.84	-0.89	3.07
FWW-NP	3.22	1.01	0.13	-0.80	0.86	-0.45	-0.09
WWF	1.45	0.72	0.52	-0.19	0.69	0.35	-0.39
WWF-N	1.99	0.88	0.07	-0.57	0.81	-0.09	-0.52
WWF-P	1.34	0.60	0.81	1.11	0.59	0.61	0.59
WWF-NP	2.38	1.08	0.25	0.14	0.95	-0.03	-0.06

Note: Treatments: W= continuous wheat, FW= fallow-wheat, FWW= fallow-wheat-wheat (after fallow), WWF= wheat-wheat-fallow (after wheat), N= 45 kg N ha^{-1} and P= 20 kg P ha^{-1}.

Table 4. Sample L-moment statistics for historical wheat yields at Lethbridge, Alberta

Period	Sample		L-cv for probability distributions			
(Years)	L-cv	L-skewness	Lognormal	Gamma	Weibull	Pareto
---Actual---						
1972-1982	0.199	0.1032	0.118	0.1654	0.2872	0.3838
1983-1993	0.248	0.0061	0.0061	0.0139	0.1823	0.3385
1994-2004	0.1759	-0.0491	-0.0591	-0.0739	0.1181	0.3171
1972-2004	0.2438	0.0251	0.0288	0.0437	0.2052	0.3453
---De-trended---						
1972-1982	0.1599	0.1247	0.1427	0.1993	0.3103	0.3941
1983-1993	0.2839	0.0224	0.0248	0.0353	0.1997	0.3464
1994-2004	0.2094	-0.0083	-0.0116	-0.0082	0.1638	0.3346
1972-2004	0.1983	-0.0003	-0.0006	-0.0006	0.1775	0.3336

Note: For normal distribution: L-skewness =0 but L-cv= 0.2094 for actual historical

data and L-cv = 0.1959 for de-trended data.

Table 5. Ranking of alternate wheat distributions based on goodness-of-fit measure-detrended data

Treatments	Gamma	Lognormal	Normal	Pareto	Weibull
W	3	4	2	5	1
W-N	3	4	1	5	2
WP	3	4	1	5	2
W-NP	3	4	2	5	1
FW	1	2	3	5	4
FW-N	3	4	2	5	1
FW-P	3	4	2	5	1
FW-NP	2	3	1	5	4
FWW	3	4	1	5	2
FWW-N	2	1	4	5	3
FWW-P	3	4	2	5	1
FWW-NP	3	4	2	5	1
WWF	4	1	3	5	2
WWF-N	3	4	1	5	2
WWF-P	2	1	4	5	3
WWF-NP	3	4	1	5	2
Average	2.75	3.25	2	5	2

Treatments: W= continuous wheat, FW= fallow-wheat, FWW= fallow-wheat-wheat (after fallow), WWF= wheat-wheat-fallow (after wheat), N= 45 kg N ha^{-1} and P= 20 kg P ha^{-1}.
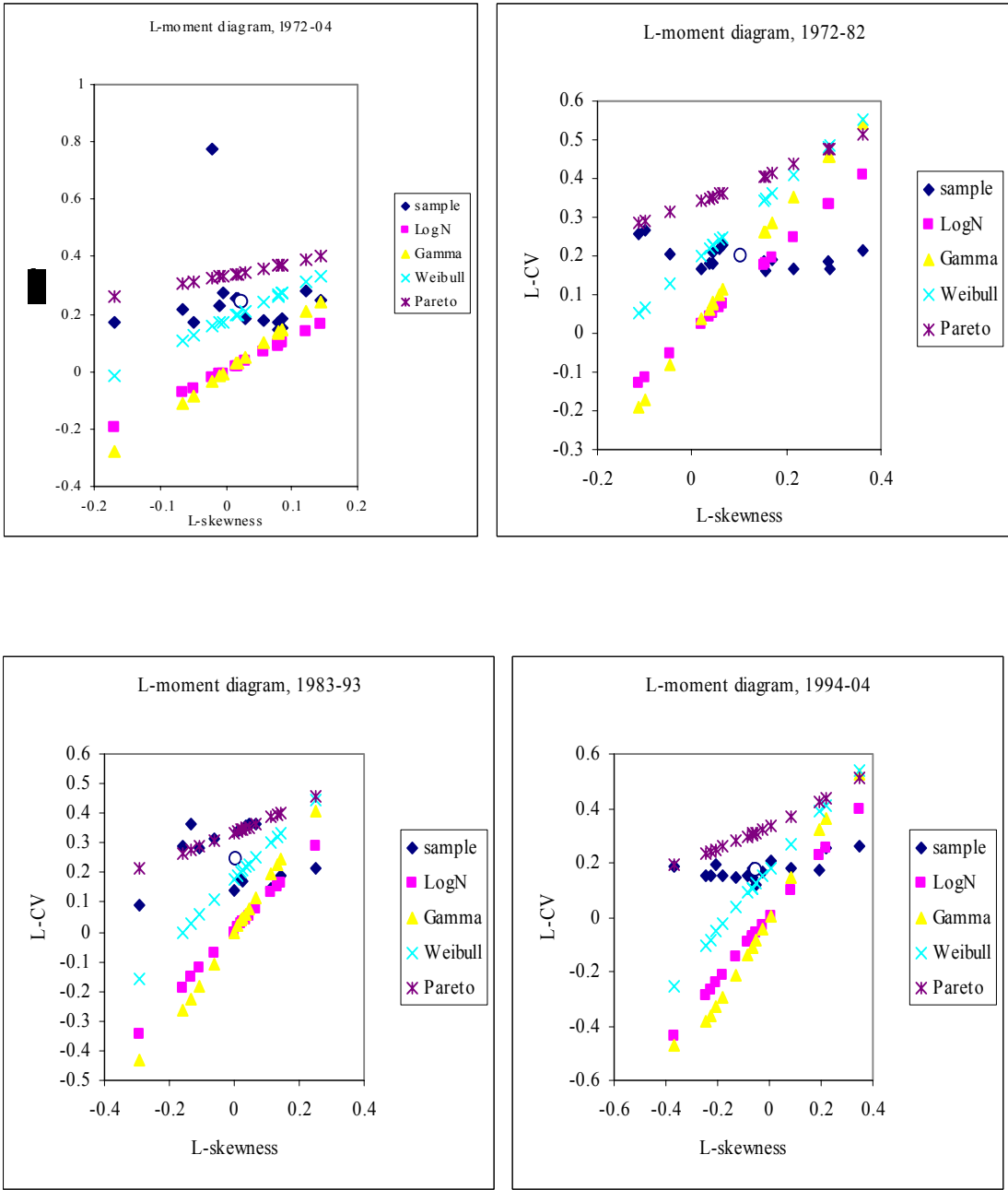
Figure 1. L-moment diagrams for actual historical wheat yields

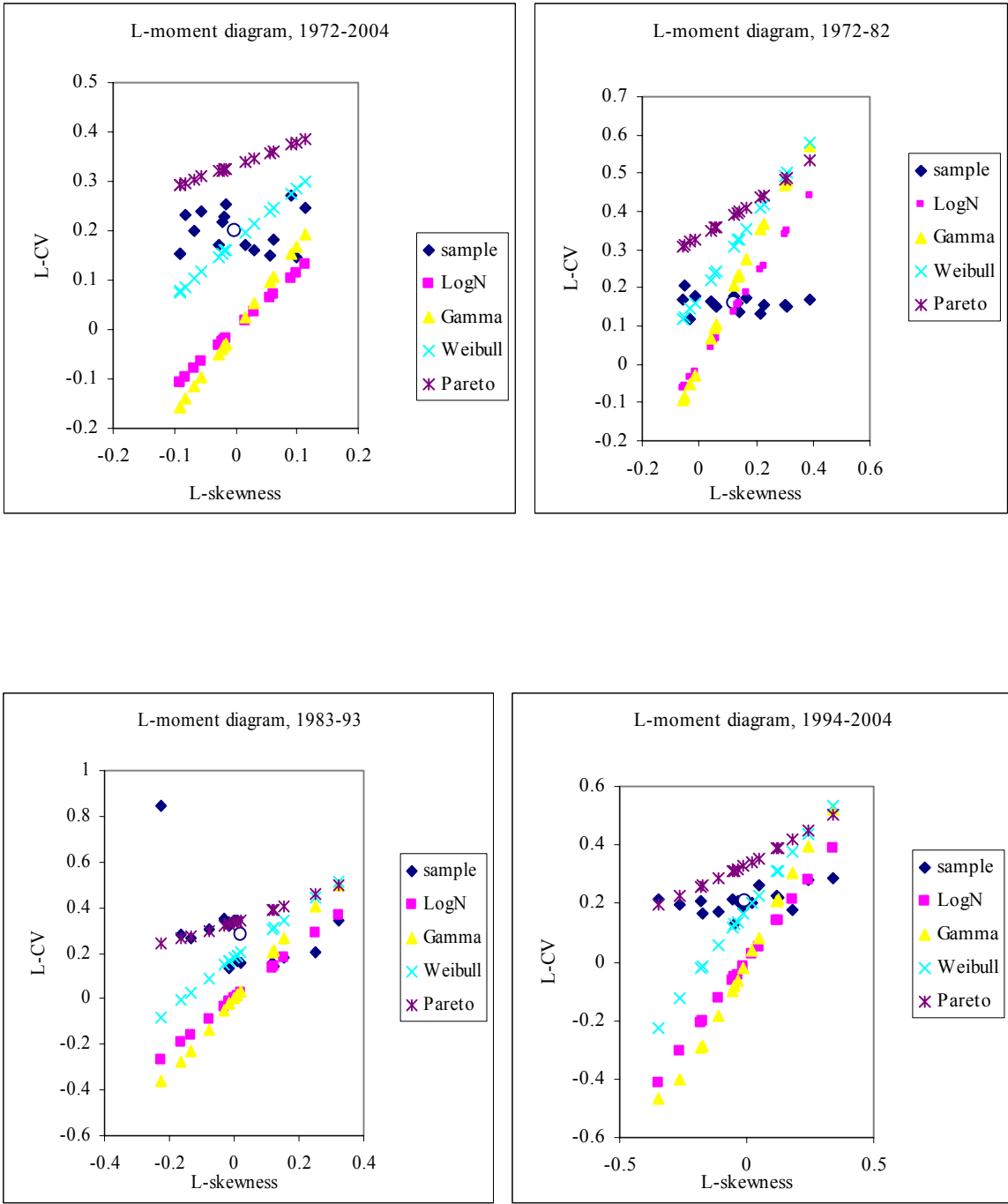Note: o in the diagram represents the sample average L-cv.

Figure 2. L-moment diagrams for de-trended wheat yields at Lethbridge

Note: o in the diagram represents the sample average L-cv.