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# ENVIRONMENT, WELFARE AND GAINS FROM TRADE: A NORTH-SOUTH MODEL IN GENERAL EQUILIBRIUM

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## ABSTRACT

The effects of environment on trade and welfare are analyzed in a modified Heckscher-Ohlin framework using a quasi-homothetic preferences to account for differences in countries' expenditure shares on health. Three types of pollution, local-disembodied, global-disembodied and embodied, result as a by-product of inputs used in production. For each case, the Walrasian, Pareto optimal and the Regulators' problem are analyzed. The optimal tax is shown to improve each country's welfare if the country is small in the world market. Otherwise, changes in the terms of trade may cause one country to be made better off at the expense of the other. Interdependence for the global-disembodied case is explored using a one-shot Nash game. For the embodied pollution, taxing the polluting input only can cause a decline in welfare when the polluting input is intensively used. Instead, a tax on the polluting input in combination with a subsidy to the non-polluting input is optimal. In general, the results suggest compensatory payments may be required to encourage abatement policies. Contrary to other approaches, an abatement policy does not necessarily decrease a country's comparative advantage, i.e., reduce exports of the polluting sector.

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# ENVIRONMENT, WELFARE AND GAINS FROM TRADE:
# A NORTH-SOUTH MODEL IN GENERAL EQUILIBRIUM

Environmental effects on welfare and the gains from North-South trade are modeled by adapting the traditional Heckscher-Ohlin framework to account for environmental externalities in production and their impact on consumption through health. As incomes grow, a greater proportion of income is spent on health including expenditures to mitigate environmental effects. Expenditures on health range from a high of 12% of GNP in the US to an average of about 4% in developing countries (World Bank, 1993, p.4)[2]. Consequently, health has become an important impetus for environmental protection in wealthy countries, a cause of trade disputes as illustrated by the EC ban on beef imports treated with growth hormones, and of particular concern in developing countries (World Bank, 1993). Agricultural pollutants that enter the food chain have received considerable attention in the US (Caswell, 1991). US epidemiological evidence suggests that 2-3 percent of all cancers associated with environmental pollution occurs from exposure to pesticide residues on food stuffs which allegedly presents a greater risk than hazardous waste. Emissions of particulate are alone suspected of causing 20,000 to 30,000 premature deaths each year in the US (Chivian, 1993). It is also well known that high levels of morbidity and shortened life expectancies in developing countries have direct environmental linkages. The World Bank (1992) presents persuasive evidence that unsafe water, inadequate sanitation, and suspended

[2] Based on data from 25 countries, Gertler and van der Gaag (1990) estimate that health care expenditures rise by about 1.32 percent for every one percent increase in a country's GNP.

particulate matter are particularly deleterious to health in these countries[3].

As rich countries tend to be more willing to pursue policies that alleviate negative environmental impacts than poor countries, concern has been expressed about the possible effects of these policies on trade and comparative advantage. The conflicts and potential for conflict between trade and environmental policies, especially the effects of environmental protection on trade patterns and gains from trade[4] have also become a North-South issue. Most of the trade based models tend to predict that more stringent abatement policies negatively affect countries' comparative advantage, thus inducing pollution intensive industries to migrate to the South where environmental standards are more lax. Pethig (1976) and Siebert (1979) were among the first to focus on pollution's effect on productivity in a trade context. After accounting for the externality, comparative advantage is found to lie with the country whose shadow price for pollution is low relative to the other country. By a continuum good model, Copeland and Taylor (1994) find that the higher income country tends to choose stronger environmental protection, and specializes in relatively clean goods. If differences in pollution taxes are the only motive for trade, then a movement from autarky to free trade increases aggregate world pollution. Other contributions focusing on the resource productivity effects are those of McGuire (1982), and Merrifield (1989). The former used a Heckscher-Ohlin framework to obtain more general results than the previous studies, while the latter considered international capital mobility and the likelihood of specialization and the closing of polluting industries among countries in the presence of

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[3] From a 1989 survey of 17,920 households in Brazil, Kassouf (1993) finds water, sewers, electricity and paved streets to be strongly associated with weight to height measures for children under six year of age.

[4] See Patrick Low (1992) for a review of this literature.

externalities. Chichilinisky (1993) studies in an innovative way the effect of property rights on comparative advantage in the presence of a potentially exhaustible resource and obtains a similar result, namely, the region in which property rights for the environmental resource are poorly defined tend to export environmentally intensive goods.

The models upon which these results are based suffer a number of shortcomings. First, they tend to ignore the North-South health-pollution-trade linkages. Pethig (1976) was among the few to consider environmental impacts on utility directly, although he only entered an environmental variable into a utility function of Leontief form without discerning whether its impact was on health, amenities or some other factor. Chichilinisky and Heal (1992) focus on differences in willingness to pay for carbon emission abatement among wealthy and poor countries. Each country's aggregate utility function depends on the quality of the atmosphere (a global public good), and a composite private good. The private good can be transformed into the public good through an abatement technology. The marginal costs of pollution abatement across countries is found to be equal only if countries' marginal valuations of the private good are equal. With diminishing returns to abatement, richer countries should push abatement further as they have a lower marginal valuation of the private good. However, a single composite good and no production precludes insights into production - emission linkages and term of trade effects.

Second, the typical approach to modeling an externality is to treat it proportional to output (Siebert (1979) and Kohn (1991)), or to be an input into the production process (Pethig (1976), McGuire (1982), Merrifield (1988) and Copeland (1994)). However, inputs used in the production process typically yield a pollution by-product, which is not necessarily proportional to output, nor is pollution typically an input per se. Moreover, some forms of pollution affect health through consumption of market goods. The health effects through consumption have direct trade implications if the pollution is *embodied* in the good. Further, pollution as a by-product of inputs used in production will have different effects if pollution remains within the country compared to the case where it is transnational in nature. In both the case of *embodied* and *disembodied* pollution, the direct effect of Pigouvian taxes on welfare can be undone when the terms of trade between North and South are permitted to adjust, suggesting the need for compensatory payments.

The approach developed here addresses these shortcomings. To emphasize the North-South health-pollution-trade linkages, identical but non-homothetic preferences are assumed so that the richer North consumes higher levels of health than the South. Three types of pollution are modeled, local-*disembodied*, global-*disembodied* and *embodied* pollution, in a single analytical framework which treats pollution as a by-product of an input employed in the production process. The Walrasian equilibrium, the Pareto optimal policy, and the Regulators' problem are considered for each type of pollution. For the *embodied* case we find the first best policy instrument is not only a tax on the polluting input, but also a subsidy on the non-polluting input if the polluting input is intensively used. We analyze the effects of pollution abating instruments on trade and welfare for both the small and large country assumptions. We find that an pollution control policy does not necessarily have an adverse effect on the country's comparative advantage. Hence, a country's comparative advantage in trade is still determined by factor proportion theory (the Heckscher-Ohlin theorem). Further, the direct effect of Pigouvian taxes on welfare can be undone when the terms of trade between North and South are permitted to adjust, suggesting the need for compensatory payments.

The basic model and the Walrasian equilibrium are laid out in Section I. The Pareto optimal solution to each of the local, global and embodied cases are analyzed in Section II. Section III focuses on the Regulator's problem and a number of propositions for each of these three cases. In Section IV we develop numerical examples of each case to further clarify the conceptual model and its implications. The numerical examples also serve to illustrate the nature of a number of analytical predictions that are indeterminate, and the Nash game that emerges for the Regulator's problem. The paper thus lays the ground work for the possible next step of constructing a North - South applied general equilibrium model calibrated to world data.

**I. The Basic Model**

There are two open economies, North and South. Each employs labor, L, and capital, K, to produce two tradable goods X and Y. The technologies are constant returns to scale and identical across countries. The inputs are mobile between sectors in each country, but immobile across countries. The North is assumed to be wealthier than the South by being endowed with more capital and equal amounts of labor. The key departures from the Heckscher-Ohlin 2x2x2 model are the assumptions: (1) Pollution is a by-product of input K employed in the production of X; (2) Two countries have identical but non-homothetic preferences over goods X, Y and health. The first assumption is based on the observation that most pollutants are produced by inputs, and the same input used in different industries can release different amounts of pollutants. The health effects of pollution are either through the environmental degradation which we call *disembodied* effects or through the consumption of a good within which contaminants are embodied, which we call *embodied* effects. The second assumption captures the phenomenon that demand for health increases in greater proportion to an increase in income. Our results are also sensitive to the assumption that the production of X is capital intensive.

The production, pollution, health and utility functions are specified as follows.

**1. Production Technologies**

$X^i = F(L_x^i, K_x^i)$

$Y^i = G(L_y^i, K_y^i)$,

where $L_j^i$, $K_j^i$ denote inputs allocated to the production of the j-th commodity, j = X, Y, in the i-th country, i = n (North), s (South). Technology is assumed to be strictly increasing, concave, continuously differentiable and homogeneous of degree one in arguments.

**2. Pollution**

The effect of pollution on the environment can take one of two forms, *embodied* and *disembodied*. *Embodied* pollution affects utility through the consumption of X which, as we show below, maps into utility. Examples are organic and inorganic impurities in food tissues, such as bacteria and bacteriological toxins, pesticides, herbicides and heavy metal deposits. *Disembodied* pollution is not attached or bound to the individual good demanded. *Disembodied* pollution can be local (country specific) or global (world-wide) such as air pollution caused by suspended particulate matter, ozone depletion, toxic gases from manufacturing plants, and diseases caused by airborne bacteria resulting from plant or municipal wastes. We analyze the welfare implications of each of these types of pollution separately.

*Disembodied* pollution, $PO^i$, in the i-th country is assumed to be generated as a by-product from the employment of input $K_x$ in the production X:

$PO^i = f(K_x^i)$,

$f(\cdot)$ is assumed to be identical across countries, differentiable and strictly increasing in $K_x^i$. Local-*disembodied* pollution's effect on environmental degradation is expressed as a departure from some uniform environmental standard $E^*$:

$$E^i = E^* - PO^i,$$

Global-*disembodied* pollution is simply the effects of both countries:

$$E^i = E^* - (PO^n + PO^s). \tag{1.0}$$

*Embodied* pollution is expressed as the concentration, $po^i \equiv PO^i/X^i$, in parts per unit of X produced. Since $F(\cdot)$ is homogeneous of degree one, the concentration of pollutants, $po^i$, is scale neutral, which implies homogeneity of degree zero in $(L_x, K_x)$. Hence, the *pollution concentration function* is expressed as:

$$po^i = g\,(K_x^i/L_x^i), \tag{2.0}$$

where g(.) is identical across countries and strictly increasing in $K_x^i/L_x^i$. Consequently, the level of *embodied* pollution is determined by relative input levels, not their absolute levels. Since pollution $po^i$ is *embodied* in a tradable good, pollution consumed in a country is not necessarily equal to the amount produced. For the X-exporting country, the purity of X consumed is equal to the purity of the X it produces, i.e.,

$$E_i = 1 - po^i, \text{ such that } 0 \le po^i < 1.$$

For the X-importing country, the level of pollution consumed is a weighted average of domestic and foreign production in the country's consumption, i.e.,

$$E_i = (1 - po^s)\gamma + (1 + po^n)(1 - \gamma),$$

where $\gamma = X_i^s/X_i$, the consumed X in the i-th country over produced X in the South.

**3. Utility**

Several considerations affect the specification of utility. The specification should permit identical preferences among agents in the North and South, it should be consistent with the observation that the North consumes higher levels of health relative to other normal goods than the South, and it should avoid problems of aggregation. These considerations are most easily handled by specifying a quasi-homothetic form of utility (e.g. Gorman polar, Gorman (1953), or a Stone-Geary form). Arguments of the identical utility function are goods X, Y and health, H:

$$U_i = U(X_i, Y_i, H_i), \quad i = n, s. \tag{3.0}$$

Health is produced by goods and environmental quality

$$H_i = h(X_i, Y_i, \tilde{E}), \tag{3.1}$$

where $\tilde{E} = \{E^i, E, E_i\}$ depending on the case being analyzed. This function is assumed to be identical across countries, differentiable, strictly increasing in $(X_i, Y_i, \tilde{E})$ and concave in $(X_i, Y_i)$. Hence, environmental degradation affects health and utility negatively.

From this structure, the indirect utility

$$V(P_x, P_y, GNP_i, \tilde{E}) = \max_{(x,y)} \{U(X_i, Y_i, h(X_i, Y_i, \tilde{E})) \mid P_x X_i + P_y Y_i = GNP_i\} \tag{4.0}$$

follow, where gross national product is:

$$GNP_i = P_x X^i + P_y Y^i.$$

Further, we assume health is relatively more responsive to Y than X, i.e., $\partial \ln(H)/\partial \ln(Y) > \partial \ln(H)/\partial \ln(X)$, so that $\partial(Y/X)/\partial GNP > 0$ holds[5]. However, the signs of $\partial X/\partial \tilde{E}$ and $\partial Y/\partial \tilde{E}$ require even more restrictions functional forms.

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[5] An example of a Stone-Geary form for the *disembodied* case is $U = \alpha_1 \ln(X-a_1) + \alpha_2 \ln(Y-a_2) + (1-\alpha_1-\alpha_2)\ln(H)$, where $\ln(H) = b\ln(X-a_1) + (1-b)\ln(Y-a_2) + c\ln(\tilde{E})$. The Marshallian demand functions are: $X = a_1 + (B/P_x)(GNP-a_1P_x-a_2P_y)$, $Y = a_2 + [(1-B)/P_x](GNP-a_1P_x-a_2P_y)$, where $B = \alpha_1 + (1-\alpha_1-\alpha_2)b$. Then, $\partial(H/X)/\partial GNP > 0$ and $\partial(H/Y)/\partial GNP > 0$. Furthermore, if $b < 0.5$, $a_1 > a_2$, and either $\alpha_2 \geq \alpha_1$ or $1-\alpha_1-\alpha_2 > 0.5$, then $\partial(Y/X)/\partial GNP > 0$.

## 4. Competitive Equilibrium with Pollution

The equilibrium levels of commodity supplies and factor prices for this Heckscher-Ohlin 2x2x2 model can be derived following any one of a number traditional methods (e.g., Woodland, 1982). Given that the North is endowed with more K than the South, and the production of X is capital intensive, equilibrium within country's cone of diversification implies the levels of production: $X^n > X^s$ and $Y^n < Y^s$. Given the assumed restrictions on preferences, the consumption levels and ratios

$$X_n > X_s\,, \qquad Y_n/X_n > Y_s/X_s \tag{5.0}$$

are implied. Thus, the North's (South's) trade pattern is to export (import) good X for which it has an excess supply (demand), and import (export) good Y for which it has an excess demand (supply). At equilibrium, world excess demand and supply are zero.

For local-*disembodied* pollution, the levels produced and consumed in the North exceed those in the South, i.e., $PO^n > PO^s$. Hence the environment is more degradated in the North than the South, $E^n < E^s$. For global-*disembodied* pollution, $PO^n > PO^s$ remains, however, from (1.0), both countries face the same level of environmental degradation E. Since factor prices, w and r for L and K, respectively, equilibrate, equation (2.0) implies that pollution concentration per unit of X produced are equal, i.e., $po^n = po^s$.

## II. Optimal Analysis with Three Types of Pollution

Since the externality affects consumer's utility, the competitive equilibrium is not Pareto optimal. By comparing the necessary conditions for Pareto optimality with those for a competitive equilibrium, we are able to identify first best policy instruments and correctly specify the Regulator's problem for each of the three cases, local-*disembodied*, global-*disembodied*, and *embodied.*

**1. Optimal Analysis for the Case of Local-Disembodied Pollution**

A Pareto optimal solution can be derived by maximizing one country's social welfare function subject to its endowments and a constraint which requires that the level of the other country's welfare be at least equal to the level derived in the competitive equilibrium. The problem is:

P: 1 $\quad \text{Max } (\chi)\ U(X_n, Y_n, h(X_n, Y_n, E^* - PO^n))$

$$\chi = \{X, Y, L, K, PO \in \mathbb{R}_+^{14} \mid U_s = U(X_s, Y_s, h(X_s, Y_s, E^* - PO^s)),$$

$$X_n + X_s = F(L_x^n, K_x^n) + F(L_x^s, K_x^s)$$

$$Y_n + Y_s = G(L_y^n, K_y^n) + G(L_y^s, K_y^s)$$

$$PO^i = f(K_x^i)$$

$$L_x^i + L_y^i = L^i$$

$$K_x^i + K_y^i = K^i, \quad i = n, s \ \}.$$

Assuming an interior solution and rearranging the first order conditions, we obtain for each country:

$$\frac{U_{X_i} + U_{H_i} h_{X_i}}{U_{Y_i} + U_{H_i} h_{Y_i}} - \frac{G_{L_{y_i}}}{F_{L_{x_i}}} = 0 \tag{6.0}$$

$$\frac{U_{X_i} + U_{H_i} h_{X_i}}{U_{Y_i} + U_{H_i} h_{Y_i}} - \frac{G_{K_{y_i}}}{F_{K_{x_i}}} = \frac{-\lambda_e^i f_{K_{x_i}}}{F_{K_{x_i}}(U_{Y_i} + U_{H_i} h_{Y_i})}, \tag{6.1}$$

where $\lambda_e$ is the shadow price for pollution and $\lambda_e = -U_H h_E < 0$.

In Walrasian equilibrium, the right hand side of (6.1) is zero. The term $U_H h_E$ takes into account the effect of capital on pollution and environment on utility. It is positive by construction of equation (3.0) and (3.1). Hence, and not surprisingly, the Walrasian equilibrium is not Pareto optimal.

The first order conditions of P:1 suggest that while the technical rate of factor substitution in the production of Y equals the relative shadow prices, $\lambda_l$ and $\lambda_k$, of the resource endowments, this is not the case for the polluting sector X:

$$\frac{G_{L_{y_l}}}{G_{K_{y_l}}} = \frac{\lambda_l^i}{\lambda_k^i}$$

$$\frac{F_{L_{x_l}}}{F_{K_{x_l}}} = \frac{\lambda_l^i}{\lambda_k^i - \lambda_e^i f_{K_{x_l}}}, \tag{6.2}$$

where the rate of substitution departs by the product of the shadow price of pollution and the marginal physical product of pollution, $\lambda_e f_{K_x}$. Result (6.2) implies that a Pareto optimal outcome is characterized by producers of X facing a shadow price of input $K_x$ augmented by the social cost of the effect of pollution on utility, thus increasing the ratio $L_x/K_x$ relative to the Walrasian equilibrium. As pollution only depends on the use of $K_x$, less pollution is generated, and hence a higher environmental quality $E^i$ is obtained. The term $\lambda_e f_{K_x}$ can also be used to formulate an optimal tax on the input $K_x$, as a policy instrument for the Regulator in Section III.

**2. Optimal Analysis for the Case of Global-Disembodied Pollution**

The global-*disembodied* pollution problem is:

P: 2 $\qquad \text{Max}\ (\chi)\ U(X_n,Y_n,h(X_n,Y_n,E^* - PO^n - PO^s))$

$\chi = \{X,Y,L,K,PO \in \mathbb{R}_+^{14} \mid U_s = U(X_s,Y_s,h(X_s,Y_s,E^* - PO^n - PO^s)),$

$$PO^n + PO^s = f(K_x^{\,n}) + f(K_x^{\,s}),\ \ldots\},$$

and other constraints listed in P: 1. The difference between local and global for the optimal problem is that pollution is jointly produced by the two countries, and hence requires a single constraint equation for it. Assuming an interior solution, rearranging the first order conditions, and keeping track of the country index i, we obtain:

$$\frac{U_{X_n} + U_{H_n} h_{X_n}}{U_{Y_n} + U_{H_n} h_{Y_n}} - \frac{G_{K_{y_n}}}{F_{K_{x_n}}} = \frac{-\lambda_e f_{K_{x_n}}}{F_{K_{x_n}}(U_{Y_n} + U_{H_n} h_{Y_n})}\ , \tag{7.0}$$

and similarly for the South. The shadow price of pollution now becomes

$$\lambda_e = -\, U_{H_n} h_E \; - \; \lambda_u^s U_{H_s} h_E < 0. \tag{7.1}$$

As pollution affects the global environmental quality, the shadow price for it is the same for both countries. $\lambda_u^s$ is the shadow price associated with the South's utility constraint in P:2. Further, the difference between (6.0) and (7.0) is that in the global case, the shadow price of pollution has take into account the effects on utility in both countries.

The form of technical rate of factor substitution in sector X is the same. i.e.,

$$\frac{F_{L_{x_n}}}{F_{K_{x_n}}} = \frac{\lambda_l^n}{\lambda_k^n - \lambda_e f_{K_{x_n}}}\ . \tag{7.2}$$

However, as the shadow price of pollution, $\lambda_e$ in (7.2) takes into account the negative impacts on both countries, the ratio $L_x/K_x$ relative to P:1 increases more.

## 3. Optimal Analysis for the Case of Embodied Pollution

The special feature of *embodied* pollution is its negative impact on health associated with the ingestion of X. Thus, we redefine the utility function in the embodied pllution as follows.

$$U_i = U(E_i X_i, Y_i).$$

The effect of pollution on health is included in the argument, $E_i X_i$. For the trade pattern noted in Section I, the maximization problem in the embodied pollution case is:

P: 3 $$\text{Max } (\chi)\ U((1 - po^n)X_n^n, Y_n)$$

$$\chi = \{X,Y,L,K,po \in \mathbb{R}_+^{15} \mid U_s = U((1 - po^s)X_s^s + (1 - po^n)X_s^n, Y_s),$$

$$X_n^n + X_s^n = F(L_x^n, K_x^n)$$

$$X_s^s = F(L_x^s, K_x^s)$$

$$Y_n + Y_s = G(L_y^n, K_y^n) + G(L_y^s, K_y^s)$$

$$po^i = g(K_x^i/L_x^i)\ , \ldots\},$$

and the endowment constraints. The rearranged first order conditions are different from the *disembodied* cases. For the North, they are:

$$\frac{U_{X_n}(1 - po^n)}{U_{Y_n}} - \frac{G_{L_{y_n}}}{F_{L_{x_n}}} = \frac{-\lambda_e^n\, g_{L_{x_n}}}{F_{L_{x_n}} U_{Y_n}}$$

$$\frac{U_{X_n}(1 - po^n)}{U_{Y_n}} - \frac{G_{K_{y_n}}}{F_{K_{x_n}}} = \frac{-\lambda_e^n\, g_{K_{x_n}}}{F_{K_{x_n}} U_{Y_n}} .$$

The shadow price of the *embodied* effect of pollution in the North and South are:

$$\lambda_e^n = -U_{EX_n} X_n^n - \lambda_u^s U_{EX_s} X_s^n < 0$$
$$\lambda_e^s = -U_{EX_s} X_s^s < 0 \quad . \tag{8.0}$$

where $U_{EX_i} = \partial U / \partial(E_i X_i)$, and $\lambda_u^s$ is the shadow price associated with the South's utility constraint.

The second term of $\lambda_e^n$ accounts for the marginal effect of *embodied* pollution on utility in the South associated with exports from the North that are consumed in the South, $X_s^n$. The shadow price of the *embodied* affect of pollution in the South is only associated with contaminants from its own production $X_s^s$. These results, of course, indicate that a competitive equilibrium is not Pareto optimal.

The relationship between factors and shadow prices are given by:

$$\frac{F_{L_{x_n}}}{F_{K_{x_n}}} = \frac{\lambda_l^n - \lambda_e^n g_{L_{x_n}}}{\lambda_k^n - \lambda_e^n g_{K_{x_n}}} \quad . \tag{8.1}$$

Note the result that not only is the denominator increased by the shadow price of *embodied* pollution, but the numerator is decreased by the shadow price of pollution since the marginal physical products of pollution concentration are $g_{L_x} < 0$ and $g_{K_x} > 0$. The policy implication of equation (8.1) is to induce producers in sector X to use more labor and less capital relative to the Walrasian equilibrium. To achieve this objective, taxing on $K_x$ alone is not sufficient for the *embodied* externality. If the cost of labor is reduced by a policy, producers in sector X are motivated to substitute labor for capital. That is the reason for the consideration of a subsidy policy in Section III.

The differences among the first order necessary conditions for the types of externalities considered here suggest that optimal environmental policy cannot be the same for the different types of pollution. For local *disembodied* pollution, the shadow price of environmental quality in each country only depends on its own marginal utility (6.1). As the North has a higher level of GNP and a lower level of environmental quality, $-\lambda_e^n > -\lambda_e^s$. The shadow price for global *disembodied* pollution (7.1) is equal across countries since $\lambda_e$ depends on the summation of two countries' marginal utility of environmental degradation. For *embodied* pollution, the shadow price in the exporting country (the North) depends on the two countries' marginal utility, while in the South the shadow price only depends on its own (8.0). The other mentioned departure is a subsidy for each unit of labor, $L_x$, employed in the production of X.

## III. The Regulator's Problem: Internalizing the Externality

### 1. The Case of Local-Disembodied Pollution

The Regulator's problem of each country is to increase social welfare by inducing producers of X to subsititute labor for capital. The analysis of Section II implies that this result can be accomplished by an optimal tax and its form and level are suggested by (6.2). The form can be an emission tax[6] or a tax on $K_x$. As the shadow price is only dependent on each country's marginal utility, the Regulator can determine the tax policy unilaterally. We first analyze the effects of a tax on $K_x$ for the small country case where world market prices are given. The effects of the input tax in this case are referred to as the direct effects.

Given the assumptions and structure developed in Section I,

[6] It is easily shown that the Regulator could instead choose an emission tax and obtain equivalent results if the pollution production function $f(K_x)$ is linear, i.e., $PO = \alpha K_x$. In this case, the advalorem input tax rate t on capital employed in the production of X is: $t = \alpha T/r$ where $T = -\lambda_e$ is the emission tax.

***Proposition 1:*** *Holding world prices constant, the following conditions hold for the imposition of a positive advalorem tax t on $K_x$ in either country:*

$$\frac{\partial w}{\partial t} > 0, \quad \frac{\partial r}{\partial t} < 0, \quad \frac{\partial r^*}{\partial t} < 0, \tag{9.1}$$

$$\frac{\partial y}{\partial t} > 0, \quad \frac{\partial X}{\partial t} < 0, \tag{9.2}$$

$$\frac{\partial b_{ky}}{\partial t} > 0, \quad \frac{\partial b_{kx}}{\partial t} > 0, \tag{9.3}$$

$$\frac{\partial b_{ly}}{\partial t} < 0, \quad \frac{\partial b_{lx}}{\partial t} < 0, \tag{9.4}$$

$$\frac{\partial K_x}{\partial t} < 0, \tag{9.5}$$

$$\frac{\partial E}{\partial t} > 0, \tag{9.6}$$

$$\frac{\partial Gnp}{\partial t} < 0. \tag{9.7}$$

See Section VI.2 for proof. The advalorem tax is $t = (r^* - r)/r$, and $b_{ij}$ is the input i per unit of output j. Conditions (9) indicate that a tax on $K_x$ affects: wages positively, capital rental rate negatively, cost of capital used in the polluting industry (X) negatively, the production of Y positively, X negatively, the capital used per unit of output in both industries positively, the labor used per unit of output negatively, the total $K_x$ used in the polluting industry negatively, the environment E positively, and GNP which includes the transfer of lump sum tax revenue negatively. These results depend on the factor intensity assumption we made in Section II. Alternatively, if we assume instead that the polluting sector X is labor intensive, the signs in (9) are reversed for (9.1), (9.3) and (9.4).

Since the relaionship between the demand for goods and pollution is not clear, the *direct* effect on consumer's demand (holding world prices constant) of a $K_x$ tax is indeterminate without adding more structure to the model.

***Proposition 2:*** *If the utility function has a Stone-Geary form, then, demand for goods is independent of the disembodied pollution. Further, as after tax GNP falls, demand for both goods falls.*

The demand functions derived from a Stone-Geary utility are as follows:

$$X_i = C + [a_x/(a_x+a_y)](GNP_i - P_xC - P_yD)$$

$$Y_i = D + [a_y/(a_x+a_y)](GNP_i - P_xC - P_yD),$$

where C and D are subsistence quantity of X and Y, respectively, $a_x = \alpha_x + (1 - \alpha_x - \alpha_y)\gamma_x$, $a_y = \alpha_y+(1- \alpha_x- \alpha_y)\gamma_y$. $\alpha_x$, $\alpha_y$ and $1-\alpha_x-\alpha_y$ are coefficients for X, Y and H in utility function, $\gamma_x$ and $\gamma_y$ are coefficients for X and Y in health function, and $\gamma_x + \gamma_y < 0$. Obviously, X and Y are both independtent of E, and a falling in GNP causes demand for X and Y falls.

It is not surprising that the effects of a $K_x$ tax on the price of X cannot be signed either. The change in the price $P_x$ as one country's tax changes can be derived by differentiating the world market equilibrium condition:

$$Q_{xn}(P_x,P_y,E^n,t^n) + Q_{xs}(P_x,P_y,E^s,t^s) = 0\ ,$$

with respect to one country's $t^i$, where $Q_{xi}(.)$ is the i-th country excess demand for X. Treating $P_y$ as numeraire, and allowing the change in $E^i$ to only depend on $t^i$ yields:

$$\frac{\partial P_x}{\partial t^i} = - \frac{\frac{\partial X_i}{\partial Gnp_i}\frac{\partial Gnp_i}{\partial t^i} + \frac{\partial X_i}{\partial E^i}\frac{\partial E^i}{\partial t^i} - \frac{\partial X^i}{\partial t^i}}{\frac{\partial Q_{x_n}}{\partial P_x} + \frac{\partial Q_{x_s}}{\partial P_x}}\ .$$

Recall that $X_i$ and $X^i$ denote demand and supply, respectively. Since the denominator is negative by the stability conditions (Samuelson, 1947), $\partial P_x/\partial t^i$ has the sign of the numerator. The supply effect is negative (by 9.2). Since $\partial X_i/\partial E^i$ cannot be signed in general, $\partial P_x/\partial t^i$ is indeterminate. However, for a Stone-Geary form of utility, Proposition 2 implies that $\partial X_i/\partial E^i = 0$. As the demands for both goods fall and the supply of Y rises, the supply of X should fall more than demand, i.e.,

$$|(\partial X_i/\partial GNP)(\partial GNP_i/\partial t^i)| < |\partial X^i/\partial t^i|$$

Hence $P_x$ rises in this case.

The imposition of a tax on $K_x^i$ affects utility through three channels: change in $E^i$, change in $P_x$ and the change in GNP.

***Proposition 3:*** *For the small country disembodied case, an optimal tax on $K_x^i$ raises the i-th country's utility independently of the other country's choice of a tax on $K_k^{i*}$.*

For given price, the indirect utility is a function of t. Differentiating indirect utility (4.0) with respect to t, yields:

$$\frac{dV_i}{dt} = \frac{\partial V}{\partial GNP_i}\frac{\partial GNP_i}{\partial t} + \frac{\partial V}{\partial E^i}\frac{\partial E^i}{\partial t}$$

**We know that**

$$\frac{\partial E}{\partial t} = -\frac{df(K_x)}{dK_x}\frac{\partial K_x}{\partial t} = -\frac{tr\dfrac{\partial K_x}{\partial t}}{\dfrac{\partial V}{\partial E}}, \text{ at the optimal tax rate, since } t = \left(\frac{\partial V}{\partial E}\frac{df}{dK_x}\right)/r \text{ , and } \frac{\partial K_x}{\partial t} < 0.$$

**In Section VI.2 we prove that:** $\dfrac{\partial GNP}{\partial t} = tr\dfrac{\partial K_x}{\partial t}$. **Hence,**

$$\frac{dV_i}{dt} = \frac{\partial V}{\partial GNP_i}tr\frac{\partial K_x}{\partial t} - tr\frac{\partial K_x}{\partial t} = \left(\frac{\partial V}{\partial GNP_i} - 1\right)tr\frac{\partial K_x}{\partial t}\ .$$

$$\text{If } \frac{\partial V}{\partial GNP} < 1 \text{ , then } \frac{dV}{dt} > 0 \text{ .} \qquad (10)$$

For example, for Stone-Geary preferences, $\partial V/\partial GNP = (a_x + a_y)^{1-ax-ay} a_x^{ax} a_y^{ay} P_x^{-ax} P_y^{-ay} (E/GNP)^{1-ax-ay}$, which is smaller than one as usually $E/GNP < 1$. Hence, (10) is likely.

When prices are permitted to adjust, the welfare of the two countries is affected differentially. The change in utility from abatement and price adjustment is obtained by totally differentiating the indirect utility function:

$dU_i = (\partial U/\partial GNP_i)(X^i - X_i)dP_x + (\partial U/\partial GNP_i)dGNP|_{\text{given P}} + (\partial U/\partial E^i)[dE^i|_{\text{given t}} + dE^i|_{\text{given P}}]$.

Recall that $X^i - X_i$ is positive (negative) for the North (South).

***Proposition 4:*** *Given that an increase in $P_x$ has a "small" effect on $E^i$, i.e., $dE^i|_{\text{given t}}$ is small, the North (an X-exporting country) is made better off by an optimal tax, and the South (an X-importing country) is made worse off if the South's import volume is lager. The South is better off only when the trade volume of X is small and the positive change in the utility from the abatement effects is large.*

The direct effects of the tax is

$(\partial U/\partial GNP_i)dGNP|_{\text{given P}} + (\partial U/\partial E^i)dE^i|_{\text{given P}}$,

which is positive for an optimal tax rate by (10). The sign of $(\partial U/\partial E^i)dE^i|_{\text{given t}}$ is negative if $P_x$ rises. Hence, if this term is small, $dU_n$ is positive for the North. However, the term $(X^i - X_i)dP_x$ is negative for the South (X-importing country). Thus, $dU_s < 0$, if its excess demand for X is large and dominates the positive effect of the policy on its utility. Only when the trade volume is small, and the positive abatement effects on its utility is large, can the South be made better off when $P_x$ rises.

In contrast to the analyses in Section II, if no transfer payments are made among countries, then a country (the South in this case) can be made worse off when the imposition of pollution taxes cause the terms of trade to change in favor of the other country. The South could be made no worse off if part of the environmental tax revenues collected in the North are transferred to the South. For this reason the optimal level of $t^i$ is at least a second best policy. The level and direction of transfers are derived in the numerical analysis.

The implications of these results, and those in the other two cases considered below, suggest that in the absence of international transfers, a country is unlikely to impose environmental taxes if the loss in welfare from changes in the terms of trade dominate welfare gains from environmental enhancement. Countries that experience gains from environmental taxes that improve their terms of trade may be encouraged to over-tax the polluting factors if the incremental losses from over-taxing are smaller than the gains from changes in the terms of trade. These results also contradict the notion that a country's comparative advantage will be compromised by the adoption of a pollution control policy in the sector for which it holds a comparative advantage.

**2. The Case of Global-Disembodied Pollution**

Proposition 1, 2, and 4 of III.1 still hold, while Proposition 2 has to be revised since, for the global-disembodied pollution, intervention in one country affects the other country's welfare directly.

We can use a one shot Nash game to characterize the interdependence of regulator choices on the other country. A strategy for each country's regulator is to abate pollution using the tax suggested by problem P: 2. Hence, using equation (4.0), utility for each country is:

$U_i = V_i\,(GNP_i(t^i),E(t^n,t^s)) = V_i(t^n,t^s)$.

***Proposition 5:*** *For the small country case, if pollution is global, then the Nash equilibrium, if one exists, will not necessarily lead to a Pareto Superior outcome without cooperation between regulators of the two countries.*

Given world price $P_x$, the changes in social welfares of the non-cooperative regulator's behavior can be represented in a Nash table:

| North / South | No pollution abatement | With pollution abatement |
|---|---|---|
| No pollution abatement | $(t^s=0,\ t^n=0) \Rightarrow$ $dU_s=0;\ dU_n=0\}$ | $(t^s = 0,\ t^n > 0) \rightarrow$ $dU_s = \frac{\partial V_s}{\partial E}\frac{\partial E}{\partial t^n}dt^n$ $dU_n = \frac{\partial V_n}{\partial GNP}\frac{\partial GNP_n}{\partial t^n}dt^n + \frac{\partial V_n}{\partial E}\frac{\partial E}{\partial t^n}dt^n$ |
| With pollution abatement | $(t^s > 0,\ t^n = 0) \rightarrow$ $dU_s = \frac{\partial V_s}{\partial GNP}\frac{\partial GNP_s}{\partial t^s}dt^s + \frac{\partial V_s}{\partial E}\frac{\partial E}{\partial t^s}dt^s$ $dU_n = \frac{\partial V_n}{\partial E}\frac{\partial E}{\partial t^s}dt^s$ | $(t^s > 0,\ t^n > 0) \rightarrow$ $dU_s = \frac{\partial V_s}{\partial GNP}\frac{\partial GNP_s}{\partial t^s}dt^s + \frac{\partial V_s}{\partial E}(\frac{\partial E}{\partial t^s}dt^s + \frac{\partial E}{\partial t^n}dt^n)$ $dU_n = \frac{\partial V_n}{\partial GNP}\frac{\partial GNP_n}{\partial t^n}dt^n + \frac{\partial V_n}{\partial E}(\frac{\partial E}{\partial t^s}dt^s + \frac{\partial E}{\partial t^n}dt^n)$ |

If $\quad |(\partial V_i/\partial GNP)(\partial GNP_i/\partial t^i)| > (\partial V_i/\partial E)(\partial E/\partial t^i)$, for the i-th country, (12)

then, $t^i = 0$ is a rational choice for the i-th country's regulator. If (12) holds for both countries, then, $t^n = 0$, $t^s = 0$ are chosen. Thus, the Nash equilibrium is the status quo. Only if (12) does not hold for both countries, would $t^n > 0$, $t^s > 0$ be chosen, and Nash solution is Pareto optimal.

Similar to the local-*disembodied* case, when world market prices are allowed to change, the North (South) experiences an improvement (deterioration) in its terms of trade thus increasing $dU_n$ and decreasing $dU_s$ for any strategy except the status quo. This increases incentives for the North to choose $t^n > 0$ and for the South to choose $t^s = 0$. An example in Section IV shows that it might be a Nash equilibrium in the large country case.

**3. The Case of Embodied Pollution**

There are two distinguishing properties of the *embodied* case. First, equation (8.1) implies that the Regulator needs to tax $K_x$ and to subsidize $L_x$. Thus producers pay w(1-s) for $L_x$ and r(1+t) for $K_x$ where $s = \lambda_e^i g_{L_x}/w$ and $t = -\lambda_e^i g_{K_x}/r$. Second, for the X-exporting country (North), its shadow price of pollution $\lambda_e^n$ depends on both countries' marginal utilities of environmental quality since the pollutants are contained in the tradable good X (8.0). Based on these properties, only Proposition 1 holds for the embodied case. For Proposition 2, demand for goods are not independent of pollution.

***Proposition 6:*** *For a Stone-Geary form utility function, demands for both goods are not independent of pollution if pollution is embodied in X. Further, if X is a necessary good, demand for it rises with a higher level of pollution concentration, while if X is a luxury good, demand for it can fall if the level of concentration is high.*

The demand functions derived from the Stone-Geary utility in the case of embodied pollution are as follows:

$$X_i = C/E_i + a(GNP_i - P_x C/E_i - P_y D)/P_x$$

$$Y_i = D + (1-a)(GNP_i - P_x C/E_i - P_y D)/P_y.$$

If C is positive, then $\partial X/\partial E < 0$, i.e., demand for X falls as X has less contaminants. An example is the need to consume a large quantity of low quality - impure food to obtain a given nutrition level. If C is negative, $\partial X/\partial E > 0$, i.e., demand for X rises as X is cleaner. Proposition 6 depends on the functional form of utility we chose. If utility were of the Cobb-Douglas form, then the demand for goods is independent of E.

***Proposition** 7: If the South (an X-importing country) does not have an abatement policy, then reducing embodied pollution in the North can reduce the excess demand for X in the South if X is a necessary good.*

This is a straightforward result of Proposition 6. As the pollution embodied in the imported X is reduced, then, the demand for X falls in the South. However, as the South does not intervene, its supply does not change. Hence, the decline in its demand for X only causes its import demand to fall. The result of Proposition 7 depends on the sign of C. If X is akin to luxury good (C negative), then the import demand for X rises as pollution falls.

***Proposition 8:** For a small country, an advalorem subsidy s on $L_x$ affects w, r, $X^i$, $Y^i$, the input ratio K/L, and the intensity of pollution, po, in the opposite directions to the effects of a tax on $K_x$, with an exception of the effect on GNP, which is negative for both.*

The proof is similar to the proof of Proposition 1 in Section VI.2. It can also be shown that because the effects of s are opposite to those of t, their joint effects on $L_x$ and $K_x$ are indeterminate.

***Proposition** 9: Given that X is K intensive, only when the concentration of pollution is reduced by taxing $K_x$ and subsidizing $L_x$, can the social welfare for each country be improved; if $t > 0$ and $s = 0$, the concentration of pollution (po) rises.*

The tax effect on the input ratio $K_x/L_x$ can be derived from Proposition 1, (9.3) and (9.4). Since $K_x/L_x = b_{kx}/b_{lx}$, (9.3) and (9.4) imply $\partial(K_x/L_x)/\partial t > 0$. From the concentration function,

$\partial po/\partial(K_x/L_x) > 0$. Hence, $\partial po/\partial t > 0$, which completes the proof of the second part of the Proposition. Holding output prices constant, the change in utility in the embodied case has the form:

$$dV_i = \frac{\partial V_i}{\partial Gnp_i}\left(\frac{\partial Gnp_i}{\partial t^i}dt^i + \frac{\partial Gnp_i}{\partial s^i}ds^i\right) + \frac{\partial V_i}{\partial E_i}\left(\frac{\partial E_i}{\partial t^i}dt^i + \frac{\partial E_i}{\partial s^i}ds^i\right) .$$

By Propositions 1 and 8, $\partial GNP_i/\partial t^i < 0$, and $\partial GNP_i/\partial s^i < 0$. With $s^i = 0$, $dU_i < 0$ as $\partial po^i/\partial t > 0$. From Proposition 8, $\partial E_i/\partial s^i > 0$. Hence, $dU_i > 0$, when the positive effect of $\partial E_i/\partial s^i$ dominates the negative effects of $\partial E_i/\partial t^i$ and $\partial GNP_i/\partial t^i$ plus $\partial GNP_i/\partial s^i$.

***Proposition 10:*** *In the small country case, the South (who imports the polluting good) benefits from the unilateral action of the North.*

The Stone-Geary indirect utility in the embodied case is

$U_i = a^a(1-a)^{1-a}P_x^{-a}P_y^{-(1-a)}E_i^a\,(GNP_i - P_xC/E_i - P_yD)$.

If the North adopts an unilateral abatment policy, the South's utility would change as $po^n$ changes, i.e.,

$\partial U_s/\partial po^n = (\partial E_s/\partial po^n)[aU_s + a^a(1-a)^{1-a}P_x^{1-a}CE_i^{a-1}](1/E_s) < 0$, as $\partial E_s/\partial po^n < 0$.

Hence, a lower level of $po^n$ rises utility in the South. However, an unilateral action in the South cannot benefit the North as the South is an X-importing country.

In the large country case, the world price is re-equilibrated following a country's imposition of an abatement policy. Proposition 1 and 8 imply that if any country (or both) tax $K_x$ only, then the total supply of X falls and Y rises, and $P_x$ might rise. Further, if any country (or both) subsidize $L_x$ only, then the total supply of X rises and Y falls, and $P_x$ might fall.

However, the joint effects of a tax on $K_x$ and a subsidy on $L_x$ are indeterminate. The numerical example in Section IV is used to show the nature of this relationship.

## IV. An Example Economy

Functional forms and parameters of production and pollution technologies, and utility functions assumed are as follows.

### 1. Function forms and parameters assumed

Consistent with the structure presented in Section I, the following is assumed.

a. Production technology and endowments

$$X = L_x^{0.25} K_x^{0.75} \quad , \qquad Y = L_y^{0.75} K_y^{0.25} \ ,$$

$$L^n = L^s = 10 \ , \quad K^n = 18 \ , \ K^s = 12 \ .$$

b. Pollution

Local-disembodied: $PO^i = 0.07(K_{xi})^{0.9}$.

Global-disembodied: $PO = 0.07[(K_x^n)^{0.9} + (K_x^s)^{0.9}]$.

Embodied: $po^i = 0.02(K_x^i/L_x^i)$.

c. Utility

$$U_i = (X_i - 1)^{0.2} (Y_i - 1)^{0.2} (H_i - 0.2)^{0.6}.$$

Health production function

$$H = (X_i - 1)^{0.25} (Y_i - 1)^{0.65} \tilde{E}^{0.1},$$

where, $\tilde{E} = \{4.5\text{-}PO^i, 4.5\text{-}PO^n\text{-}PO^s\}$ for the local and global-*disembodied* cases, respectively. In the *embodied* case, as the health effects of pollution cannot be separated from the consumption of X, the utility functions are defined as follows:

$$U_n = [(1\text{-}po^n)X_n^n - 1]^{0.4} (Y_n - 1)^{0.6}$$

$$U_s = [(1\text{-}po^n)X_s^n + (1\text{-}po^s)X_s^s - 1]^{0.4} (Y_s - 1)^{0.6}.$$

**2. The empirical analysis**

Eight equilibria are calculated for each of the *disembodied* and *embodied* cases:

P.O: Pareto optimal solution, as depicted in Section II,

WE: Walrasian equilibrium with no tax

RSB: both countries tax, world prices fixed

RWB: both countries tax, world prices adjust

RSN: unilateral action: only the North taxes, world prices fixed

RSS: unilateral action: only the South taxes, world prices fixed

RWN: unilateral action: only the North taxes, world prices adjust

RWS: unilateral action: only the South taxes, world prices adjust.

All solutions R·· utilized the Regulator's optimal tax rate on $K_x$ and for *embodied* case, R·· includes a subsidy on $L_x$. The results are presented in the Section VI.1 where the various solutions are reported relative to the Walrasian equilibrium, WE. For brevity, we largely focus on those results that are noted as being indeterminate in the analytical analyses.

**2.1 The local-disembodied case.**

Tables 1 and 2 report the results for the North and South, respectively. The RSB (Column 2) results support the predictions of Proposition 1 - 3. Proposition 4 is shown by RWB, Column 3. Column 1, Table 1 and 2 shows that the Pareto optimal solution results in an increase in the consumption of the Y and H, and a fall in the consumption of X, pollution PO falls and utility rises in the North with the constraint, requiring that the South be made no worse off, binding. The Regulator's advalorem tax t on $K_x$ equals about 1.44 % in the wealthier North and about 1.18 % in the South. Holding prices fixed, the consumption of both X and Y are slightly smaller than for the WE case, pollution falls and welfare rises (column 2, both tables).

When both countries tax and world prices adjust (column 3), the price of $P_x$ rises. But the supply of X still falls and Y rises in both countries relative to WE, and hence, pollution falls. The levels of supplies, pollution and input ratio $K_x/L_x$ in both countries are identical to those obtained in the Pareto optimal solution. The direction of change in consumer demand is also the same as in P.O. However, as $P_x$ increases, the North's comparative advantage in the production of X causes its GNP to rise relative to the South. This causes the demand for X to fall less and for Y to rise more in the North than in the South. Thus, the consumption level of health in the South falls while it rises in the North relative to P.O. In this case, the South's welfare falls because the terms of trade effects dominate the welfare increasing effects of the pollution tax. We study the transfer needed to make the South no worse off in a later Section.

The contrast between unilateral action with world prices fixed (RSN and S, column 4 and 5) and unilateral action with world prices equilibrating (RWN and S, column 6 and 7) provides insights into terms of trade effects. Since pollution is local, when only the North taxes at fixed world prices, the solution is identical to RSB, column 2, where both countries tax at fixed world prices, and likewise for the South. This substantiates the analytical results that, for a small country, unilateral and bilateral actions are equivalent. When prices adjust, the welfare of the South always falls whether or not it imposes a pollution tax. If the South does not tax, the terms of trade effects on the North are smaller than when both tax. When only the South taxes and prices adjust, supply of X in the North rises and pollution rises relative to the WE. Further, the South experiences the smallest negative terms of trade effects which are not dominated by the welfare increasing effects of lower pollution.

**2.2 The global-disembodied case**

Results for this case are reported in Table 3 and 4. The direction of changes found in the Pareto optimal solution (column 1) are similar to the local-*disembodied* case. When both countries tax $K_x$, whether prices are fixed or adjust, the direction and causes of changes in the variables listed in the Tables are similar to the local-*disembodied* case (columns 2 and 3). As pollution is global, the effects of unilateral action differ from the local case (Columns 4 and 5). For the small country (prices fixed), unilateral action to tax $K_x$, yields benefits to the other country from the reduction in pollution that exceed the benefits of the taxing country. Consequently, as mentioned in the analytical model, taking no action permits a country (or region) to free ride on the other's pollution control policy. When world prices adjust, this result is dramatically changed. Changes in the terms of trade make the South worse off when either it acts unilaterally, or when the North acts unilaterally to tax $K_x$. When both tax, the South is made even worse off while the North experiences its largest gain in welfare. Table 10 shows that the Nash solution is for the North to tax and the South not to tax. To obtain a Pareto optimal outcome (where both countries tax), a transfer from North to South is required. However, it can be shown that such a transfer reduces the welfare gain of the North to a level that is below the gain it would experience in the Nash equilibrium.

**2.3 The embodied pollution case**

Results for this case are reported in Tables 5 and 6. The directions of changes found in the Pareto optimal solution are different from the *disembodied* cases. The supply of X rises and Y falls in the North. As the concentration level of pollution embodied in X falls, the demand for a healthier X rises and Y falls in both countries. Utility rises in the South while the constraint requiring the North be made no worse off is binding.

For the small country case (column 2), when both countries take action (i.e., they tax $K_x$ and subsidize $L_x$), the supply of X increases and Y falls in both countries. As the change in GNP is negative, demand for both goods fall, but the decline in demand for X is larger. Hence, utility increases in both countries.

In contrast to the *disembodied* cases, when world prices adjust, the price of $P_x$ falls, column 3. Effectively, the labor subsidy effect on supply of X in the North dominates the negative effects of the $K_x$ tax and the falling in $P_x$, and hence its supply of X rises and Y falls. However in the South, changes in the supplies of both goods are in the opposite direction. As GNP falls less than the decline in $P_x$, demand for X rises and falls for Y in both countries. Utility increases in both countries, and increases more in the South, which implies that environmental effect dominates the terms of trade effect, a rare phenomenon in our simulation.

For the *embodied* case, we also examine the effects where both countries impose the previously determined optimal tax on $K_x$ only:

RSK: both countries tax $K_x$, world prices fixed

RWK: both countries tax $K_x$, world prices adjust

The results are reported in Table 5 and 6, column 4 - 5. We observe that for the small country, if only the pollution input $K_x$ is taxed, the pollution concentration per unit of X rises since the input ratio $K_x/L_x$ rises (column 4). The ratio rises because X is K intensive and tax causes the rental rate r to fall more than the effect of the advalorem tax (see Proposition 1). The welfare in both countries fall, and shown in Proposition 9. This is an important counter-intuitive result.

It suggests that, in the *embodied* case, and for a small country, taxing the pollution input only (no labor subsidy) may cause the tax inclusive price of this input to fall, thus increasing the concentration of pollution per unit of X, and decreasing utility! For the large country, only taxing $\dot{K}_x$ causes $P_x$ to rise (column 5) and pollution concentration to fall, as a rise in $P_x$ causes $(1+t)r/w$ to rise and hence $K_x/L_x$ to fall. The terms of trade effect makes the North better off and the South worse off in this situation.

**2.4 Trade effects**

As mentioned, the total effects of input taxes on trade are analytically indeterminate as well. Simulation results of these effects are reported in Table 7. Exclusive of output price changes, i.e., the small country case, optimal taxes cause the North's exports of X to fall and the South's export of Y to rise in both *disembodied* cases, while exports of X rise and Y fall for the North and South, respectively, in the *embodied* case. After output prices are permitted to re-equilibrate, the North's exports of X rise for the global-*disembodied* and *embodied* cases, but fall for the local-*disembodied* case. The South's exports of Y rise in all three types of pollution. Thus, when world prices re-equilabrate, countrys' comparative advantage is not reduced for the cases of global-*disembodied* and *embodied* types of pollution. Only for the case of local-*disembodied* pollution is that North's comparative advantage in X decreased.

**2.5 Transfer effects**

If the welfare effects caused by the change in the terms of trade dominate the effects caused by environmental policy, so that one country is made worse off, then compensatory payments from the gainer to the looser might be considered. We simulate such scenarios for the *disembodied* cases, where the North transfers tax revenue to the South at a level that leaves the

South no worse off than before the imposition of taxes. These results are reported in Table 8.

Based on the optimal tax rates, the South's total tax revenue is 0.23% and 0.61% of GNP for the local and global disembodied, respectively. Correspondingly, for the North, revenues are: 0.5% and 0.99% of GNP. The amounts of revenue that have to be transferred from the North to the South so that the South is made no worse off are only 10.25% and 10.87% of the North's total tax revenues for the local and global cases, respectively. These amounts are only equivalent to 0.05% and 0.11%, respectively, of the North's GNP. The corresponding change in welfare levels are reported in Table 9. Note that after the transfers, the Pareto optimal levels of welfare reported above are obtained. Hence, these results indicate that the optimal tax rates are second best policies for both countries.

**V. Conclusions**

The effects of environment on trade and social welfare are analyzed in a modified general equilibrium Heckscher-Ohlin framework where health appears as an argument in a quasi-homothetic utility function. This form of the function is used to capture the notion that the North is willing to spend more to alleviate environmental effects on health than the South. Environmental effects on health and welfare depend on three types of pollution which we characterize as local-*disembodied*, global-*disembodied* and *embodied*. Pollution is produced by an input as a by-product of production. The results show that an optimal tax can, in principle, improve each country's welfare if the country is small in the world market. however, for a large country or region, changes in the terms of trade may cause one country to be made better off at the expense of the other. Then, a Pareto improvement can only be reached by an optimal tax with compensation, which suggests that some form of compensatory payment may be required to encounrage the other country to pursue abatement policies. We explore the strategic interdependence that arises in the case of global-*disembodied* pollution. Characterizing the interdependence as a one-shot Nash game, we find that Nash equilbrium is not necessary a Pareto optimal. Under cooperative behavior, both countries can improve their welfare by jointly imposing a pollution control tax with a necessary compensatory transfer.

For the case of *embodied* pollution, the optimal tax for the exporting country not only depends on its own marginal welfare loss of pollution on health, but also on the loss the country's exports cause on consumers in the importing country. Further, if only the polluting input is taxed, then its after tax rental rate falls if this input is intensively used. Hence the effectiveness of this instrument to lower the *embodied* pollutants is limited and even is negative. Instead, a tax on the polluting input in combination with a subisdy to the non-polluting input can reduce pollution and improve country's welfare if the country is small in the world market.

No matter pollution is local or global, disembodied or embodied, an abatement policy adopted by both countries or by one country unilaterally would not necessarily hurt a country's comparative advantage in both small and large country cases, i.e., reduce its exports of polluting good.

## VI. Appendix

### 1. Simulation Results for the Example Economy

Table 1: Local-disembodied, North

| | P.O/WE (1) | RSB/WE (2) | RWB/WE (3) | RSN/WE (4) | RSS/WE (5) | RWN/WE (6) | RWS/WE (7) |
|---|---|---|---|---|---|---|---|
| $X^n$ | 0.997420 | 0.988028 | 0.997420 | 0.988028 | 1.0 | 0.993232 | 1.004169 |
| $Y^n$ | 1.002259 | 1.010485 | 1.002259 | 1.010485 | 1.0 | 1.005933 | 0.996327 |
| $X_n$ | 0.996932 | 0.999975 | 0.997432 | 0.999975 | 1.0 | 0.998562 | 0.998841 |
| $Y_n$ | 1.001871 | 0.999974 | 1.002384 | 0.999974 | 1.0 | 1.001304 | 1.001088 |
| $H_n$ | 1.000546 | 1.000144 | 1.001074 | 1.000144 | 1.0 | 1.000654 | 1.000418 |
| $U_n$ | 1.000044 | 1.000078 | 1.000605 | 1.000078 | 1.0 | 1.000365 | 1.000235 |
| $PO^n$ | 0.996696 | 0.990816 | 0.996696 | 0.990816 | 1.0 | 0.994081 | 1.002576 |
| $K_x/L_x$ | 0.995636 | 1.007197 | 0.995636 | 1.007197 | 1.0 | 1.000779 | 0.994807 |
| $t^n$ | | 0.014442 | 0.014317 | 0.014442 | 0.0 | 0.014358 | 0.0 |
| $r/w$ | | 0.978717 | 0.990204 | 0.978717 | 1.0 | 0.985061 | 1.005221 |
| $r^*/w$ | | 0.992855 | 1.004383 | 0.992855 | 1.0 | 0.999221 | 1.005221 |
| $P_x/P_y$ | | 1.000 | 1.005758 | 1.0 | 1.0 | 1.003184 | 1.002607 |
| $GNP_n$ | | 0.999974 | 1.002682 | 0.999974 | 1.0 | 1.001469 | 1.001223 |

Table 2: Local-disembodied, South

| | P.O/WE (1) | RSB/WE (2) | RWB/WE (3) | RSN/WE (4) | RSS/WE (5) | RWN/WE (6) | RWS/WE (7) |
|---|---|---|---|---|---|---|---|
| $X^s$ | 0.996020 | 0.979728 | 0.996020 | 1.0 | 0.979728 | 1.008835 | 0.987126 |
| $Y^s$ | 1.001464 | 1.007448 | 1.001464 | 1.0 | 1.007448 | 0.996735 | 1.004735 |
| $X_s$ | 0.997003 | 0.999979 | 0.996408 | 1.0 | 0.999979 | 0.998017 | 0.998354 |
| $Y_s$ | 1.001789 | 0.999977 | 1.001176 | 1.0 | 0.999977 | 1.000660 | 1.000517 |
| $H_s$ | 1.000496 | 1.000127 | 0.999848 | 1.0 | 1.000127 | 0.999843 | 0.999996 |
| $U_s$ | 1.000 | 1.000068 | 0.999308 | 1.0 | 1.000068 | 0.999576 | 0.999719 |
| $PO^s$ | 0.995149 | 0.983034 | 0.995149 | 1.0 | 0.983034 | 1.006508 | 0.988548 |
| $K_x/L_x$ | 0.994354 | 1.005888 | 0.994354 | 1.0 | 1.005888 | 0.993663 | 1.000641 |
| $t^s$ | | 0.011809 | 0.011707 | 0.0 | 0.011809 | 0.0 | 0.011775 |
| $r/w$ | | 0.982542 | 0.994040 | 1.0 | 0.982542 | 1.006378 | 0.987740 |
| $r^*/w$ | | 0.994146 | 1.005678 | 1.0 | 0.994146 | 1.006378 | 0.990791 |
| $P_x/P_y$ | | 1.0 | 1.005758 | 1.0 | 1.0 | 1.003184 | 1.002607 |
| $GNP_s$ | | 0.999978 | 1.001542 | 1.0 | 0.999978 | 1.000862 | 1.000683 |

Table 3: Global-disembodied, North

| | P.O/WE (1) | RSB/WE (2) | RWB/WE (3) | RSN/WE (4) | RSS/WE (5) | RWN/WE (6) | RWS/WE (7) |
|---|---|---|---|---|---|---|---|
| $X^n$ | 0.996572 | 0.975514 | 0.997498 | 0.975514 | 1.0 | 0.986287 | 1.011108 |
| $Y^n$ | 1.004322 | 1.021341 | 1.002164 | 1.021341 | 1.0 | 1.011976 | 0.990194 |
| $X_n$ | 0.993052 | 0.999899 | 0.994129 | 0.999897 | 1.0 | 0.997050 | 0.996928 |
| $Y_n$ | 1.004322 | 0.999891 | 1.005478 | 0.999892 | 1.0 | 1.002598 | 1.002919 |
| $H_n$ | 1.001434 | 1.000769 | 1.002610 | 1.000278 | 1.000495 | 1.001166 | 1.001447 |
| $U_n$ | 1.000236 | 1.000428 | 1.001481 | 1.000123 | 1.000308 | 1.000621 | 1.000840 |
| $PO^n$ | 0.994340 | 0.981136 | 0.995038 | 0.981136 | 1.0 | 0.987985 | 1.006841 |
| $K_x/L_x$ | 0.993714 | 1.014626 | 0.987985 | 1.014626 | 1.0 | 1.001510 | 0.986207 |
| $t^n$ | | 0.03175 | 0.03111 | 0.03175 | 0.0 | 0.031380 | 0.0 |
| r/w | | 0.957375 | 0.983759 | 0.957375 | 1.0 | 0.970189 | 1.013986 |
| $r^*/w$ | | 0.985585 | 1.012161 | 0.985585 | 1.0 | 0.998493 | 1.013986 |
| $P_x/P_y$ | | 1.000 | 1.013246 | 1.0 | 1.0 | 1.006456 | 1.006969 |
| $GNP_n$ | | 0.999894 | 1.006164 | 0.999894 | 1.0 | 1.002933 | 1.003280 |

Table 4: Global-disembodied, South

| | P.O/WE (1) | RSB/WE (2) | RWB/WE (3) | RSN/WE (4) | RSS/WE (5) | RWN/WE (6) | RWS/WE (7) |
|---|---|---|---|---|---|---|---|
| $X^s$ | 0.985792 | 0.945174 | 0.983794 | 1.0 | 0.945174 | 1.017867 | 0.965609 |
| $Y^s$ | 1.005206 | 1.020004 | 1.005937 | 1.0 | 1.020004 | 0.993388 | 1.012585 |
| $X_s$ | 0.993072 | 0.999846 | 0.991734 | 1.0 | 0.999841 | 0.996001 | 0.995534 |
| $Y_s$ | 1.003986 | 0.999835 | 1.002638 | 1.09 | 0.999833 | 1.001345 | 1.001294 |
| $H_s$ | 1.001205 | 1.000707 | 0.999775 | 1.000388 | 1.000318 | 0.999917 | 0.999816 |
| $U_s$ | 1.000 | 1.000365 | 0.998464 | 1.000242 | 1.000121 | 0.999291 | 0.999103 |
| $PO^s$ | 0.984655 | 0.953873 | 0.982972 | 1.0 | 0.953873 | 1.013128 | 0.969343 |
| $K_x/L_x$ | 0.988577 | 1.015778 | 0.989084 | 1.0 | 1.015778 | 0.987213 | 1.001602 |
| $t^s$ | | 0.029579 | 0.028977 | 0.0 | 0.029579 | 0.0 | 0.028238 |
| r/w | | 0.954121 | 0.980483 | 1.0 | 0.954121 | 1.012953 | 0.967945 |
| $r^*/w$ | | 0.984467 | 1.011036 | 1.0 | 0.984467 | 1.012953 | 0.998401 |
| $P_x/P_y$ | | 1.000 | 1.011525 | 1.0 | 1.0 | 1.006456 | 1.006969 |
| $GNP_s$ | | 0.999839 | 1.003039 | 1.0 | 0.999836 | 1.001755 | 1.001739 |

Table 5: Embodied, North

| | P.O/WE (1) | RSB/WE (2) | RWB/WE (3) | RSK/WE (4) | RWK/WE (5) |
|---|---|---|---|---|---|
| $X^n$ | 1.034945 | 1.112531 | 1.034945 | 0.937949 | 0.991828 |
| $Y^n$ | 0.961009 | 0.879662 | 0.961009 | 1.060334 | 1.007866 |
| $X_n$ | 1.010880 | 0.993376 | 1.012606 | 0.999700 | 0.986263 |
| $Y_n$ | 0.979127 | 0.997677 | 0.980837 | 0.998910 | 1.014345 |
| $U_n$ | 1.0 | 1.008804 | 1.002009 | 0.996813 | 1.004773 |
| $po^n$ | 0.833744 | 0.760929 | 0.833744 | 1.039937 | 0.973371 |
| $s^n$ | | 0.1232 | | | |
| $t^n$ | | 0.0273 | | | |
| $r/w$ | | 0.994793 | 0.908122 | 0.889158 | 0.953640 |
| $r^*/w^*$ | | 1.475209 | 1.345190 | 0.975253 | 1.041255 |
| $P_x/P_y$ | | 1.0 | 0.960210 | 1.0 | 1.032631 |
| $GNP_n$ | | 0.995924 | 0.977362 | 0.999232 | 1.016017 |

Table 6: Embodied, South

| | P.O/WE (1) | RSB/WE (2) | RWB/WE (3) | RSK/WE (4) | RWK/WE (5) |
|---|---|---|---|---|---|
| $X^s$ | 0.979589 | 1.109601 | 0.979589 | 0.875360 | 0.967025 |
| $Y^s$ | 1.006490 | 0.949718 | 1.006490 | 1.051704 | 1.013840 |
| $X_s$ | 1.022694 | 0.994480 | 1.021159 | 0.999554 | 0.980338 |
| $Y_s$ | 0.993109 | 0.999697 | 0.991099 | 0.998599 | 1.007157 |
| $U_s$ | 1.016235 | 1.011136 | 1.013744 | 0.996407 | 0.996874 |
| $po^s$ | 0.833744 | 0.760929 | 0.771228 | 1.039937 | 0.973371 |
| $s^s$ | | 0.1232 | | | |
| $t^s$ | | 0.0273 | | | |
| $r/w$ | | 0.994793 | 0.908122 | 0.889158 | 0.953698 |
| $r^*/w^*$ | | 1.475209 | 1.343519 | 0.975253 | 1.041255 |
| $P_x/P_y$ | | 1.0 | 0.960210 | 1.0 | 1.032631 |
| $GNP_s$ | | 0.997562 | 0.986773 | 0.998890 | 1.009273 |

Talbe 7: The changes in the excess supplies after tax

| | Local disembodied | | Global disembodied | | Embodied | |
|---|---|---|---|---|---|---|
| | Small country | Large country | Small country | Large country | Small country | Large country |
| Excess supply of X (North) | -0.06148 | -0.00263 | -0.12554 | +0.01146 | +0.6428 | +0.1344 |
| Excess supply of Y (South) | +0.05028 | +0.00311 | +0.13566 | +0.02486 | -0.3190 | +0.0892 |

Table 8: Tax revenue, transfer and its percentages in GNP

| | SOUTH | | NORTH | |
|---|---|---|---|---|
| | Local- | Global- | Local- | Global- |
| Total tax revenue | 0.0255 | 0.0664 | 0.0646 | 0.1294 |
| (Tax revenue/GNP) % | 0.2340 | 0.6067 | 0.4958 | 0.9894 |
| Total transfer | | | 0.0066 | 0.0141 |
| (Transfer/Tax revenue)% | | | 10.2449 | 10.8651 |
| (Transfer/GNP)% | 0.0606 | 0.1285 | 0.0508 | 0.1075 |

Table 9: Comparisons of the utility levels before and after transfer

| | SOUTH | | | NORTH | | |
|---|---|---|---|---|---|---|
| | Local- | Global- | Embodied | Local- | Global- | Embodied |
| No tax | 5.153 | 5.093 | 4.6289 | 6.24399 | 6.205 | 5.6486 |
| Tax without transfer | 5.149 | 5.085 | 4.630 | 6.248 | 6.214 | 5.647 |
| Tax with transfer | 5.153 | 5.093 | 4.6292 | 6.24426 | 6.206 | 5.6486 |
| Pareto optimal | 5.153 | 5.093 | 4.6289 | 6.24426 | 6.206 | 5.6489 |

Table 10: Nash equilibrium, large country case

| NORTH / SOUTH | NO TAX | TAX |
|---|---|---|
| NO TAX | $dU_s = 0$; $dU_n = 0$ | **$dU_s = -0.701$; $dU_n = 0.621$** |
| TAX | $dU_s = -0.897$; $dU_n = 0.840$; | $dU_s = -1.536$; $dU_n = 1.481$; |

where $\{t_s = 0, t_n > 0\}$ is Nash equilbrium.

**2. Proof of Proposition 1 and 5**

**a. Background**

Following the traditional model (e.g., Woodland, 1982), given factor endowments and output prices, the min-unit-cost function for each sector is equal to the output price of this sector, i.e.,

$$c_x(w, r) = P_x$$
$$c_y(w, r) = P_y \quad .$$

The factor market clearing equations are

$$\frac{\partial c_x}{\partial w} X^i + \frac{\partial c_y}{\partial w} Y^i = \bar{L}_i$$
$$\frac{\partial c_x}{\partial r} X^i + \frac{\partial c_y}{\partial r} Y^i = \bar{K}_i \quad .$$

**b. Proof of condition (9.1) for the signs of $\partial w/\partial t$ and $\partial r/\partial t$**

Differentiating min-unit-cost functions with respect to t, holding output prices constant, yields

$$\frac{\partial c_x}{\partial w}\frac{\partial w}{\partial t} + (1+t)\frac{\partial c_x}{\partial r^*}\frac{\partial r}{\partial t} + r\frac{\partial c_x}{\partial r^*} = 0$$
$$\frac{\partial c_y}{\partial w}\frac{\partial w}{\partial t} + \frac{\partial c_y}{\partial r}\frac{\partial r}{\partial t} = 0.$$

where $r^*=(1+t)r$. In the matrix form, we obtain:

$$\begin{vmatrix} b_{lx} & (1+t)b_{kx} \\ b_{ly} & b_{ky} \end{vmatrix} \begin{vmatrix} \frac{\partial w}{\partial t} \\ \frac{\partial r}{\partial t} \end{vmatrix} = \begin{vmatrix} -b_{kx} r \\ 0 \end{vmatrix}$$

$$\frac{\partial w}{\partial t} = -\frac{1}{\Delta_1} b_{ky} b_{kx} r > 0$$
$$\frac{\partial r}{\partial t} = \frac{1}{\Delta_1} b_{ly} b_{kx} r < 0 \qquad \text{(A1)}$$

$$\text{where } \Delta_1 = b_{lx} b_{ky} - (1+t) b_{kx} b_{ly} = b_{ly} b_{lx}\left(\frac{b_{ky}}{b_{ly}} - (1+t)\frac{b_{kx}}{b_{Lx}}\right) < 0.$$

$$b_{lj} = \frac{\partial c_j}{\partial w}, \quad b_{ky} = \frac{\partial c_y}{\partial r}, \quad b_{kx} = \frac{\partial c_x}{\partial r^*}. \qquad \blacksquare$$

**c. Proof of condition (13.1) for the signs of $\partial w/\partial s$ and $\partial r/\partial s$**

$$\frac{\partial w}{\partial s} = \frac{1}{\Delta_1} b_{ky} b_{kx} w < 0$$
$$\frac{\partial r}{\partial s} = -\frac{1}{\Delta_1} b_{ly} b_{kx} w > 0$$

$$\text{where } \Delta_1 = (1-s) b_{lx} b_{ky} - b_{kx} b_{ly} = b_{ly} b_{lx}\left((1-s)\frac{b_{ky}}{b_{ly}} - \frac{b_{kx}}{b_{Lx}}\right) < 0.$$

$$b_{lx} = \frac{\partial c_x}{\partial w^*} \qquad \blacksquare$$

where, $w^*=(1-s)w$.

**d. Proof of condition (9.1) for the sign of $\partial r^*/\partial s$**

$$\frac{\partial r}{\partial t^*} = (1+t)\frac{\partial r}{\partial t} + r. \tag{A2.0}$$

Substituting (A1) into (A2.0) for $\partial r/\partial t$, yields

$$\begin{aligned}\frac{\partial r^*}{\partial t} &= \frac{1}{\Delta_1} b_{ly}\, b_{kx}\, r^* + r \\ &= \frac{1}{\Delta_1}(b_{ly}\, b_{kx}\, r^* + \Delta_1 r) \\ &= \frac{1}{\Delta_1} b_{lx}\, b_{ky}\, \dot{r} < 0 \qquad \blacksquare\end{aligned} \tag{A2.1}$$

Since $\partial w/\partial t$, $\partial r/\partial t$ and $\partial w/\partial s$, $\partial r/\partial s$ have opposite signs, we only prove the signs of $\partial X/\partial t$, $\partial Y/\partial t$, $\partial b_{kx}/\partial t$, $\partial b_{lx}/\partial t$, and $\partial K_x/\partial t$.

**e. Proof of conditions (9.3) and (9.4) for the signs of $\partial b_{kx}/\partial t$, $\partial b_{lx}/\partial t$, $\partial b_{ky}/\partial t$ and $\partial b_{ly}/\partial t$**

$$\frac{\partial b_{kx}}{\partial t} = \frac{\partial b_{kx}}{\partial w}\frac{\partial w}{\partial t} + \frac{\partial b_{kx}}{\partial r^*}\frac{\partial r^*}{\partial t}$$

$$\text{where} \quad \frac{\partial b_{kx}}{\partial w} = \frac{\partial c_x^2}{\partial r^* \partial w}, \quad \frac{\partial b_{kx}}{\partial r^*} = \frac{\partial c_x^2}{\partial r^{*2}}$$

The unit cost function $c_x(w,r^*)$ is quasi concave and homogenous of degree one in $(w,r^*) \in R^2_{++}$. Hence $b_{kx}=\partial c_x/\partial r^*$ and $b_{lx}=\partial c_x/\partial w$ are homogenous of degree zero in $(w,r^*) \in R^2_{++}$. Then, by Euler's theorem we have

$$\frac{\partial b_{kx}}{\partial w} w + \frac{\partial b_{kx}}{\partial r^*} r^* = 0.$$

Since $c_x(w,r^*)$ is quasi concave, $\partial b_{kx}/\partial r^* < 0$. Thus $\partial b_{kx}/\partial w > 0$. Combined the results of (A1) and (A2.1), we get

$$\frac{\partial b_{kx}}{\partial t} = \underset{(+)}{\frac{\partial b_{kx}}{\partial w}} \underset{(+)}{\frac{\partial w}{\partial t}} + \underset{(-)}{\frac{\partial b_{kx}}{\partial r^*}} \underset{(-)}{\frac{\partial r^*}{\partial t}} > 0$$

By the same method, we get

$$\frac{\partial b_{lx}}{\partial t} = \underset{(-)}{\frac{\partial b_{lx}}{\partial w}} \underset{(+)}{\frac{\partial w}{\partial t}} + \underset{(+)}{\frac{\partial b_{lx}}{\partial r^*}} \underset{(-)}{\frac{\partial r^*}{\partial t}} < 0$$

and similar for $\partial b_{ky}/\partial t$ and $\partial b_{ly}/\partial t$

$$\frac{\partial b_{ky}}{\partial t} > 0, \quad \frac{\partial b_{ly}}{\partial t} < 0. \qquad \blacksquare$$

**f. Proof of (9.2) for the signs of $\partial X/\partial t$ and $\partial Y/\partial t$**

Differentiating factor market clearing conditions with respect to t, holding endowments constant, yields

$$b_{lx}\frac{\partial X}{\partial t}+b_{ly}\frac{\partial Y}{\partial t}+\left(\frac{\partial b_{lx}}{\partial w}X+\frac{\partial b_{ly}}{\partial w}Y\right)\frac{\partial w}{\partial t}+\frac{\partial b_{lx}}{\partial r^{*}}\frac{\partial r^{*}}{\partial t}X+\frac{\partial b_{ly}}{\partial r}\frac{\partial r}{\partial t}Y=0$$

$$b_{kx}\frac{\partial X}{\partial t}+b_{ky}\frac{\partial Y}{\partial t}+\left(\frac{\partial b_{kx}}{\partial w}X+\frac{\partial b_{ky}}{\partial w}Y\right)\frac{\partial w}{\partial t}+\frac{\partial b_{kx}}{\partial r^{*}}\frac{\partial r^{*}}{\partial t}X+\frac{\partial b_{ky}}{\partial r}\frac{\partial r}{\partial t}Y=0\quad .$$

In the matrix form, we have

$$\begin{vmatrix} b_{lx} & b_{ly} \\ b_{kx} & b_{ky} \end{vmatrix}\begin{vmatrix} \dfrac{\partial X}{\partial t} \\ \dfrac{\partial Y}{\partial t} \end{vmatrix} = -\begin{vmatrix} \left(\dfrac{\partial b_{lx}}{\partial w}X+\dfrac{\partial b_{ly}}{\partial w}Y\right)\dfrac{\partial w}{\partial t}+\dfrac{\partial b_{lx}}{\partial r^{*}}\dfrac{\partial r^{*}}{\partial t}X+\dfrac{\partial b_{ly}}{\partial r}\dfrac{\partial r}{\partial t}Y \\ \left(\dfrac{\partial b_{kx}}{\partial w}X+\dfrac{\partial b_{ky}}{\partial w}Y\right)\dfrac{\partial w}{\partial t}+\dfrac{\partial b_{kx}}{\partial r^{*}}\dfrac{\partial r^{*}}{\partial t}X+\dfrac{\partial b_{ky}}{\partial r}\dfrac{\partial r}{\partial t}Y \end{vmatrix}$$

Substituting (A1) and (A2) in the above for $\partial w/\partial t$ and $\partial r^{*}/\partial t$:

$$\begin{vmatrix} b_{lx} & b_{ly} \\ b_{kx} & b_{ky} \end{vmatrix}\begin{vmatrix} \dfrac{\partial X}{\partial t} \\ \dfrac{\partial Y}{\partial t} \end{vmatrix} = \frac{r}{\Delta_1}\begin{vmatrix} b_{ky}X\left(b_{kx}\dfrac{\partial b_{lx}}{\partial w}-b_{lx}\dfrac{\partial b_{lx}}{\partial r^{*}}\right)+b_{kx}Y\left(b_{ky}\dfrac{\partial b_{ly}}{\partial w}-b_{ly}\dfrac{\partial b_{ly}}{\partial r}\right) \\ b_{ky}X\left(b_{kx}\dfrac{\partial b_{kx}}{\partial w}-b_{lx}\dfrac{\partial b_{kx}}{\partial r^{*}}\right)+b_{kx}Y\left(b_{ky}\dfrac{\partial b_{ky}}{\partial w}-b_{ly}\dfrac{\partial b_{ky}}{\partial r}\right) \end{vmatrix}$$

Hence,

$$\begin{vmatrix} \dfrac{\partial X}{\partial t} \\ \dfrac{\partial Y}{\partial t} \end{vmatrix} = \frac{r\,dt}{\Delta_1\Delta_2}\begin{vmatrix} b_{ky} & -b_{ly} \\ -b_{kx} & b_{lx} \end{vmatrix}\begin{vmatrix} b_{ky}X\left(b_{kx}\dfrac{\partial b_{lx}}{\partial w}-b_{lx}\dfrac{\partial b_{lx}}{\partial r^{*}}\right)+b_{kx}Y\left(b_{ky}\dfrac{\partial b_{ly}}{\partial w}-b_{ly}\dfrac{\partial b_{ly}}{\partial r}\right) \\ (-)\qquad(-)\qquad\qquad(-)\qquad(-) \\ b_{ky}X\left(b_{kx}\dfrac{\partial b_{kx}}{\partial w}-b_{lx}\dfrac{\partial b_{kx}}{\partial r^{*}}\right)+b_{kx}Y\left(b_{ky}\dfrac{\partial b_{ky}}{\partial w}-b_{ly}\dfrac{\partial b_{ky}}{\partial r}\right) \\ (+)\qquad(+)\qquad\qquad(+)\qquad(+) \end{vmatrix}$$

$$\text{where} \quad \Delta_2 = b_{lx}b_{ky} - b_{kx}b_{ly} = \mathrm{bly}\,b_{lx}\left(\frac{b_{ky}}{b_{ly}} - \frac{b_{kx}}{b_{Lx}}\right) < 0.$$

$$\text{Thus,} \quad \frac{\partial X}{\partial t} < 0 \ , \quad \frac{\partial Y}{\partial t} > 0. \qquad \blacksquare$$

**g. Proof of condition (9.5) for the sign of $\partial K_x/\partial t$**

$$\frac{\partial K}{\partial t_x} = b_{kx}\frac{\partial X}{\partial t} + X\frac{\partial b_{kx}}{\partial t} = b_{kx}\frac{\partial X}{\partial t} + X\frac{\partial b_{kx}}{\partial w}\frac{\partial w}{\partial t} + X\frac{\partial b_{kx}}{\partial r^*}\frac{\partial r^*}{\partial t}$$

$$\text{where} \quad X\frac{\partial b_{kx}}{\partial w}\frac{\partial w}{\partial t} + X\frac{\partial b_{kx}}{\partial r^*}\frac{\partial r^*}{\partial t}$$

$$= \frac{Xr}{\Delta_1\Delta_2}\left[-b_{ky}b_{kx}\frac{\partial b_{kx}}{\partial w}(b_{lx}b_{ky} - b_{kx}b_{ly}) + b_{lx}b_{ky}\frac{\partial b_{kx}}{\partial r^*}(b_{lx}b_{ky} - b_{kx}b_{ly})\right]$$

$$\text{further} \quad b_{kx}\frac{\partial X}{\partial t} \tag{A3.1}$$

$$= \frac{r}{\Delta_1\Delta_2}\Big\{b_{kx}b_{ky}\Big[b_{ky}X\Big(b_{kx}\frac{\partial b_{lx}}{\partial w} - b_{ly}\frac{\partial b_{lx}}{\partial r^*}\Big) + b_{kx}Y\Big(b_{ky}\frac{\partial b_{ly}}{\partial w} - b_{lx}\frac{\partial b_{ly}}{\partial r}\Big)\Big]$$

$$- b_{kx}b_{ly}\Big[b_{ky}X\Big(b_{ky}\frac{\partial b_{kx}}{\partial w} - b_{ly}\frac{\partial b_{lx}}{\partial r^*}\Big) + b_{kx}Y\Big(b_{kx}\frac{\partial b_{ly}}{\partial w} - b_{lx}\frac{\partial b_{ly}}{\partial r}\Big)\Big]\Big\} \tag{A3.2}$$

adding (A3.1) with (A3.2), yields

$$\frac{\partial K_x}{\partial t}$$

$$= \frac{r}{\Delta_1\Delta_2}\Big\{b_{kx}b_{ky}\Big[b_{ky}X\Big(b_{kx}\underset{(-)}{\frac{\partial b_{lx}}{\partial w}} - b_{ly}\underset{(-)}{\frac{\partial b_{lx}}{\partial r^*}}\Big) + b_{kx}Y\Big(b_{ky}\underset{(-)}{\frac{\partial b_{ly}}{\partial w}} - b_{lx}\underset{(-)}{\frac{\partial b_{ly}}{\partial r}}\Big)\Big]$$

$$- b_{ly}b_{kx}^2Y\Big(b_{kx}\underset{(+)}{\frac{\partial b_{ky}}{\partial w}} - b_{ly}\underset{(+)}{\frac{\partial b_{ky}}{\partial r}}\Big) - b_{lx}b_{kx}b_{ky}^2\underset{(-)}{\frac{\partial b_{kx}}{\partial w}}X + b_{lx}^2b_{ky}^2\underset{(-)}{\frac{\partial b_{kx}}{\partial r}}X\Big\} < 0 \qquad \blacksquare$$

As $E = E^{*} - f(K_x)$, $\partial K_x/\partial t < 0$ implies $\partial E/\partial t > 0$.

**h. Proof of condition (9.7) for the sign of $\partial Gnp/\partial t$**

We proceed by showing that the summation of the first four terms of following equation is zero:

$$\frac{\partial Gnp}{\partial t} = \bar{L}\frac{\partial w}{\partial t} + \bar{K}\frac{\partial r}{\partial t} + rK_x + tK_x\frac{\partial r}{\partial t} + tr\frac{\partial K_x}{\partial t}$$

Substituting $\partial w/\partial r$, $\partial r/\partial t$ of (A1) into the above, we obtain for the first two terms

$$\bar{L}\frac{\partial w}{\partial t} + \bar{K}\frac{\partial r}{\partial t} = -\frac{r}{\Delta_1}(b_{ky}b_{kx}\bar{L} - b_{ly}b_{kx}\bar{K})$$

Substituting for $\bar{L},\bar{K}$, we obtain

$$= -\frac{r\,b_{kx}}{\Delta_1}[b_{ky}(b_{lx}X + b_{ly}Y) - b_{ly}(b_{kx}X + b_{ky}Y)]$$

$$= -rK_x\left(\frac{\Delta_2}{\Delta_1}\right)$$

Then, $\bar{L}\dfrac{\partial w}{\partial t} + \bar{K}\dfrac{\partial r}{\partial t} + rK_x + tK_x\dfrac{\partial r}{\partial t}$

$$= rK_x\left(1 - \frac{\Delta_2}{\Delta_1} + t\frac{1}{\Delta_1}b_{ly}b_{kx}\right)$$

$$= \frac{1}{\Delta_1}rK_x[b_{lx}b_{ky} - (1+t)b_{kx}b_{ly} - b_{lx}b_{ky} + b_{kx}b_{ly} + t\,b_{ly}b_{kx}]$$

$$= 0$$

Thus, $\dfrac{\partial Gnp}{\partial t} = \dfrac{\partial Gnp}{\partial t} = \bar{L}\dfrac{\partial w}{\partial t} + \bar{K}\dfrac{\partial r}{\partial t} + rK_x\dfrac{\partial r}{\partial t} + tr\dfrac{\partial K_x}{\partial t}$

$$= tr\frac{\partial K_x}{\partial t} < 0, \quad \text{as} \quad \frac{\partial K_x}{\partial t} < 0. \quad \blacksquare$$

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