

COMMODITY MARKETS: RATIONAL EXPECTATIONS IN MARKETS WITH IRRATIONAL INVESTORS

Teddy Wong , Aaron Smith

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Abstract

The "financialization" of commodity markets have become a concern for policy makers and market participants. What was once a market for the hedging of holding physical commodities has expanded to become a market for the diversification of financial assets. When financial assets diversification goals are decoupled from the fundamental factors that affect producers and consumers of physical goods futures markets may not be as efficient in aggregating information concerning the economics of the underlying commodity. Theoretical understanding of whether commodity futures market function well under exogenous shifts in demand for futures contracts depend on our assumptions of how market participants behave, including their level of risk aversion. This paper builds a competitive storage model with an explicit futures market that incorporates irrational shocks to demand for futures contracts. This model is flexible enough to investigate the impact of the "financialization" of commodity futures markets and the resulting impacts.

keywords: commodity, futures, financialization, competitive storage, rational expectations.

Commodity Markets: Rational Expectations in Markets With Irrational Investors

Teddy Wong , Aaron Smith *

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*University of California, Davis. Email: wong@primal.ucdavis.edu.

1 Rational Expectations Inventory Model with Futures Contracts

In the summer and fall of 2008, high commodities prices drew the scrutiny of Congress. The Committee on Homeland Security held hearings on what role institutional investors may have played in causing commodity futures prices to rise sharply between 2005 to 2008. In particular, large increases in the volume and position of long-only commodity index funds were thought to play a role in the rise of futures prices. (cite masters, leiberman)

Furthermore, the increase in commodity futures prices were seen as a threat to the cost of food in developing countries with countries considering raising export barriers in order to keep enough foodstuffs in countries to avoid shortages.

A number of academic economists argued that speculative bubbles in futures markets did not exist according to the available data at the time and that increases in futures prices are a result of an increase in demand for physical commodities. (Sanders and Irwin, 2010; Krugman, 2008) In some sense the argument relied on the separation of the futures market with the physical market, and rest on the fact that the futures price should not have an independent impact on spot prices absent an accompanying increase in storage and demand or a decrease in supply. Therefore, it is argued, all changes in the futures price should come from changes in expectations about the physical world.

However, it is possible that the rise in demand for futures contract itself be used as basis for forming expectations about the future; expectations that affect choices of production, storage and consumption. If the exogenous shock that impacts futures markets affect expectations of production, consumption and storage, then it is highly likely that exogeneous shocks to financial markets alone can engendered a change in behavior among market participants and will thus result in real impacts.

This paper contributes to the debate on the impact of "financialization" of commodity futures market by building a competitive storage model linked to an explicit futures market in order to investigate the theoretical impacts of exogenous shocks to the demand for futures contracts on our variables of interest: spot price, the quantities of futures contracts held, futures prices, and inventories. We also examine the dynamics of the price series and compare the characteristics of our variables of interest under different regimes.

2 Model

Our model follows the competitive rational expectations commodity storage literature of Williams and Wright (1991); Deaton and Laroque (1992), where a representative firm uses storage to arbitrage between the current price and the expected future price. More recent work by Routledge et al. (2000) uses a similar model with a mean-reverting Markov process. We extend that model by assuming risk aversion on the part of the representative storage

firm. Moreover, we explicitly model the holdings of futures contracts by the firm and a financial sector that has exogenous demand for futures contracts. We do this to investigate the effects of exogenous demand shocks for futures contracts on the spot price, inventory holdings and quantities of futures contracts held by the representative firm.

The model is a discrete-time, infinite horizon model of a risk averse profit-maximizing firm that holds inventories and futures contracts in a homogeneous commodity. In each period t the utility gained from profit Π is represented by the constant relative risk aversion (CRRA) utility function

$$U(\Pi_t) = \frac{\Pi_t^{1-\alpha}}{1-\alpha}, \quad (1)$$

where the α is the risk aversion parameter ($\alpha = 0$ being the risk neutral case and $\alpha = 1$ being the equivalent of log utility, $\ln \Pi_t$). Also note that $1/\alpha$ is the inter-temporal elasticity of substitution in this formulation, the willingness to substitute aggregate profit over time. In each period, the profit of each firm is given by the equation

$$\Pi_t = P_t(I_{t-1}(1-\delta) - I_t) + Q_{t-1}(F_{t-1} - P_t). \quad (2)$$

At time t , the firm receives current price P_t for all inventory I_{t-1} carried into the period and pays the same P_t for all inventory I_t carried out. Inventory must satisfy the non-negativity constraint of storage and therefore $I_t \geq 0$, for all time periods. A spoilage or volumetric storage cost of $\delta \in [0, 1]$ is assessed on inventory carried into the period. For every I_{t-1} units of inventory stored last period, only $(1-\delta)I_{t-1}$ units are available this period. On the financial side of the ledger, the firm comes into the period holding Q_{t-1} contracts short. The firm receives the futures price F_{t-1} , and pays P_t at expiration, assuming the futures and spot price converge at the expiration of the futures contract. Futures contracts are not subject to any non-negativity constraints. A negative Q_t would indicate long positions in this model.

The representative firm faces a constant discount rate β and decides the level of inventory I_t and number of futures contracts Q_t to hold in order to maximize the sum of its expected utility over all future time periods,

$$\max_{I_t, Q_t} E \left[\sum_{t=1}^{\infty} \beta^t U(\Pi_t) \right]. \quad (3)$$

From these equations, the first order necessary conditions for profit maximization are:

$$P_t = \beta(1-\delta)E \left[\left(\frac{\Pi_t}{\Pi_{t+1}} \right)^\alpha P_{t+1} \right] \quad (4)$$

and

$$F_t = \frac{E [\Pi_{t+1}^{-\alpha} P_{t+1}]}{E [\Pi_{t+1}^{-\alpha}]}.$$
 (5)

Equations 4 and 5 relate the demand for inventories to the current price of the commodity and the demand for futures contracts to the current futures price. The intuition of Eq. 4 is clear if we set $\alpha = 0$, the risk neutral case. The equation would simplify into the standard arbitrage condition $P_t = \beta(1 - \delta)E[P_{t+1}]$, where the firm is equating the benefits of selling the inventories today with the expected benefits of selling the inventory next period taking into account the discount rate and the cost of storage.¹ With the addition of risk aversion, the firm is now also concerned with the inter-temporal substitutability of profits, and risk aversion offers a channel for the purchasing of futures contracts to affect the decision to store through its impacts on the profit function. Similarly, Eq.5 without risk aversion, is the standard arbitrage condition that equates the current futures price with the expected price next period. Equations 4 and 5 describe the equilibrium relationships of prices and futures prices between periods but does not generate the equilibrium prices themselves. In order to generate the equilibrium prices we need to specify the demand schedule in both markets.

In each period, the price of the commodity in the goods (as opposed to the financial market) will be determined by the equality between the two types of supply, current production S_t and incoming inventory I_{t-1} , with the two types of demand, current use D_t plus the outgoing inventory I_t . If demand and supply are functions of price, the equation would be

$$S_t(P_t) + I_{t-1} = D_t(P_t) + I_t.$$
 (6)

Rearranging the terms, would yield $D_t(P_t) - S_t(P_t) = I_{t-1} - I_t$, net demand as a function of the change in inventories. If we take the inverse of the left hand side and rewrite the right hand side as a function of ΔI , then the resulting function $P_t = f(\Delta I_t)$ shows that the price of the commodity in each period depends only on the difference between incoming inventories and outgoing inventories. We assume that the price arising from this inverse net demand function is subject to an exogenous shock η_t . We model η_t as a two-state Markov process.

$$P_t = f(\Delta I_t, \eta_t).$$
 (7)

¹To be clear, these are not risk-free arbitrage conditions. Here and throughout the paper, arbitrage occurs when a firm is able to buy or sell with a positive *expected* gain in utility.

In the financial market, the net demand for futures contracts is assumed to be a function of the number of contracts Q_t transacted in the market, with an exogenous demand shifter parameter ν_t .

$$F_t = f(Q_t, \nu_t) \tag{8}$$

The variable ν can be thought of as the exogenous shock that might cause the futures demand schedule to rise or fall, a proxy for demand changes in the futures market .

Equations 4, 5, 7 and 8 make up the system of equations that solves the four unknowns: P_t , I_t , F_t , and Q_t . However, we cannot solve for the equilibrium values using analytical methods and so we rely on a numerical function iteration algorithm to approximate the solution.

3 Numerical Simulation

We develop a numerical simulation of the model described in the previous section in Matlab to calculate the equilibrium spot prices, futures prices, holdings of inventory and futures contracts. The model finds the infinite horizon equilibria but only models three periods to do so. This is possible by appealing to rational expectations and noting that each period's equilibrium prices and decision variables only depend on the five state variables ν_{t-1} , ν_t , η_t , Q_{t-1} , and I_{t-1} . These five variables are the shock to the futures market in the previous period, the current shock to the futures market and physical market, the incoming level of inventories and the incoming holdings of futures contracts. For clarity of exposition, the three periods will be subscripted by $t \in \{0, 1, 2\}$; where $t = 0$ is the previous time period, $t = 1$ is the current time period, and $t = 2$ is the future time period.

We use simple functional forms for net demand of physical goods and futures contracts, the simulation analogs of equations 7 and 8. For the physical market, we assume a linear net demand that is a function of inventory changes,

$$P_1 = b(\eta_1 + I_1 - I_0). \tag{9}$$

We capture the effect of exogenous shocks either to demand or supply with η_t , an additive shock that can take on values $\eta_t \in \{\eta_t^H, \eta_t^L\}$, and b is a parameter that adjusts the slope of the net demand function.

For the futures market, the equilibrium futures price F_1 will be determined by equation 5 and the net demand function in the financial markets. For the simulation, we use a logistic net demand function of the following form:

$$F_1 = \frac{\eta^h}{1 + e^{-(Q_1/c + \nu_1)}}. \quad (10)$$

We use a logistic function so that futures prices are nonnegative, and never above the highest price seen in the simulation.² Just as with the shock to the physical market, the random shock in the futures market, ν , is binary and takes values $\nu_t \in \{\nu_t^L, \nu_t^H\}$, while the parameter c adjusts the slope of the net demand for futures contracts.

Coming into the present period, $t = 1$, our representative firm holds a certain quantity of inventory, I_0 , and futures contracts, Q_0 from the previous period, and is aware of the previous shock in the futures market, ν_0 . The firm needs to know the previous futures market shock because it affected the price at which the contracts were purchased and the profitability of contracts Q_0 . The profitability in the current period, Π_1 affects the degree of risk aversion of the firm through the ratio $\frac{\Pi_1}{\Pi_2}$ in equation 4. The representative firm also knows the current shock η_1 and ν_1 . With this information it chooses I_1 and Q_1 to maximize the expected utility of the sum of current and future profits given by equation 3.

The representative firm In each period finds itself facing a world described by its own holdings of I_0 and Q_0 and three shocks, ν_0 , η_1 , and ν_1 . Each of these shocks are binary and can be high or low giving us eight combinations. These eight combinations (or scenarios) are:

| Scenario | Shocks |
|----------|----------------------------|
| 1 | $\nu_0^H \eta_1^H \nu_1^H$ |
| 2 | $\nu_0^H \eta_1^H \nu_1^L$ |
| 3 | $\nu_0^H \eta_1^L \nu_1^H$ |
| 4 | $\nu_0^H \eta_1^L \nu_1^L$ |
| 5 | $\nu_0^L \eta_1^H \nu_1^H$ |
| 6 | $\nu_0^L \eta_1^H \nu_1^L$ |
| 7 | $\nu_0^L \eta_1^L \nu_1^H$ |
| 8 | $\nu_0^L \eta_1^L \nu_1^L$ |

The transition matrix below gives us the probability of moving from the current regime (column)³

²This is useful in setting up the simulation since we use a grid space spanned by I_0 and Q_0 , and this functional form ensures that we enter the current period with reasonable futures prices.

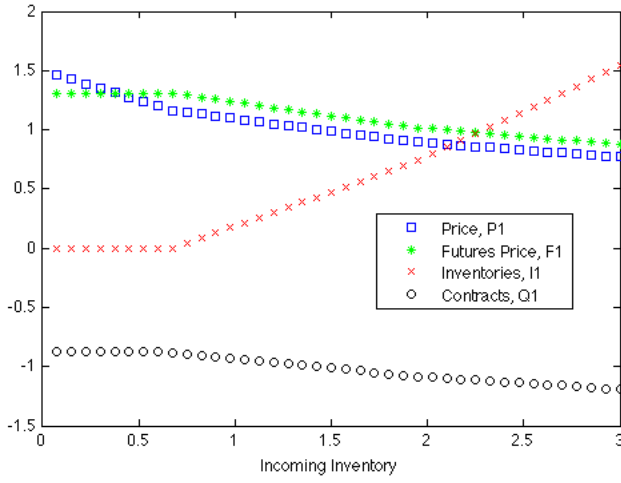
³HHH will be used interchangeably with $\nu_0^H \eta_1^H \nu_1^H$ when the meaning is clear. As shown, the probability of transitioning from HHH to HHL is .1 or 10

| | <i>HHH</i> | <i>HHL</i> | <i>HLH</i> | <i>HLL</i> | <i>LHH</i> | <i>LHL</i> | <i>LLH</i> | <i>LLL</i> |
|----------------------------|------------|------------|------------|------------|------------|------------|------------|------------|
| $\nu_0^H \eta_1^H \nu_1^H$ | .6 | 0 | .2 | 0 | .6 | 0 | .2 | 0 |
| $\nu_0^H \eta_1^H \nu_1^L$ | .1 | 0 | .1 | 0 | .1 | 0 | .1 | 0 |
| $\nu_0^H \eta_1^L \nu_1^H$ | .1 | 0 | .1 | 0 | .1 | 0 | .1 | 0 |
| $\nu_0^H \eta_1^L \nu_1^L$ | .2 | 0 | .6 | 0 | .2 | 0 | .6 | 0 |
| $\nu_0^L \eta_1^H \nu_1^H$ | 0 | .6 | 0 | .2 | 0 | .6 | 0 | .2 |
| $\nu_0^L \eta_1^H \nu_1^L$ | 0 | .1 | 0 | .1 | 0 | .1 | 0 | .1 |
| $\nu_0^L \eta_1^L \nu_1^H$ | 0 | .1 | 0 | .1 | 0 | .1 | 0 | .1 |
| $\nu_0^L \eta_1^L \nu_1^L$ | 0 | .2 | 0 | .6 | 0 | .2 | 0 | .6 |

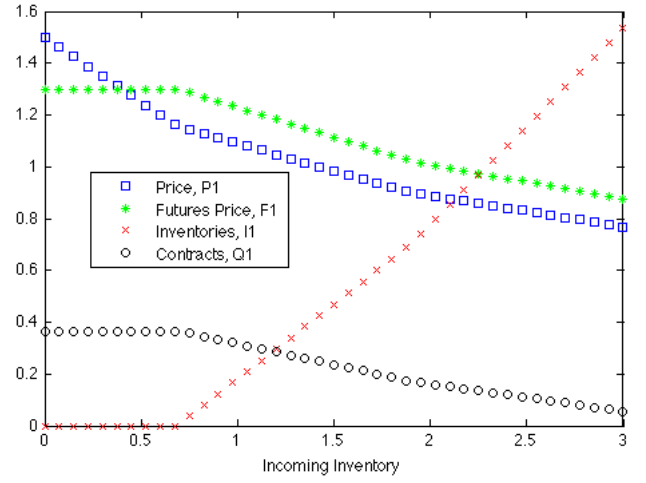
The probabilities in the transition matrix are chosen to simulate the higher likelihood of physical and futures market to be in similar states, i.e., when the physical market demand is high, the futures market demand for contracts will also be high.

Two-dimensional graphs of equilibrium P_1 , F_1 , I_1 , and Q_1 as a function of incoming inventory, I_0 , when $Q_0 = -0.5$ is graphed for the eight possible scenarios. (Fig.1,2)⁴

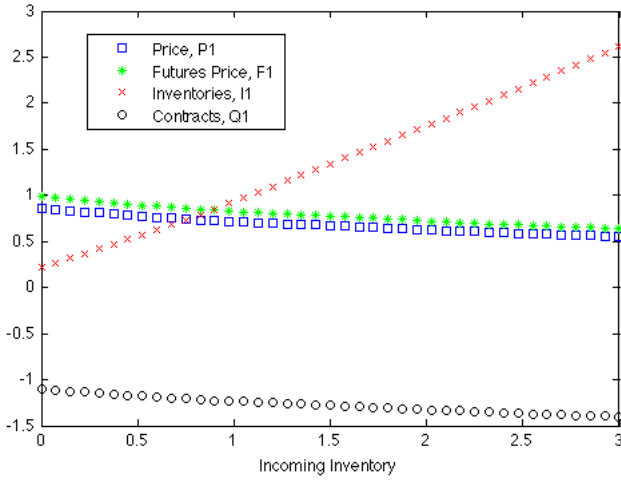
⁴The relevant parameters for these graphs are: $\alpha = 0.15$, $b = .5$, $c = .5$, $\delta = 0.1$, $r = 0.05$, $\nu^H = 1.5$, $\nu^L = -1$, $\eta^H = 3$, $\eta^L = 1.5$, and a *constant* = 4 is added to the profit function in each period to speed convergence



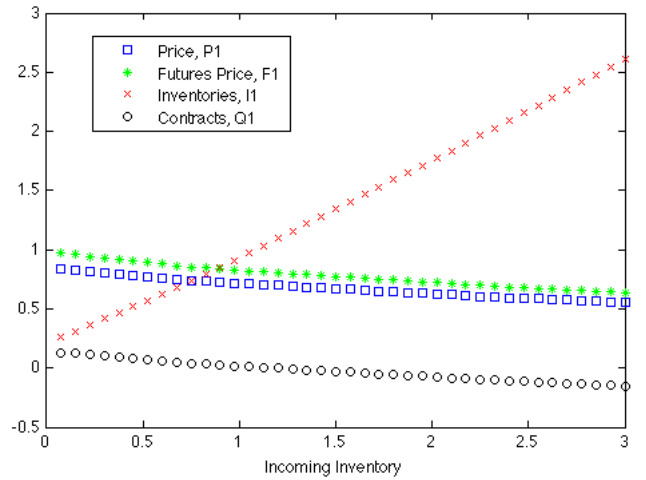
(a) scenario 1, $\nu_0 = \nu_0^H$, $\eta_1 = \eta_1^H$, $\nu_1 = \nu_1^H$



(b) scenario 2, $\nu_0 = \nu_0^H$, $\eta_1 = \eta_1^H$, $\nu_1 = \nu_1^L$

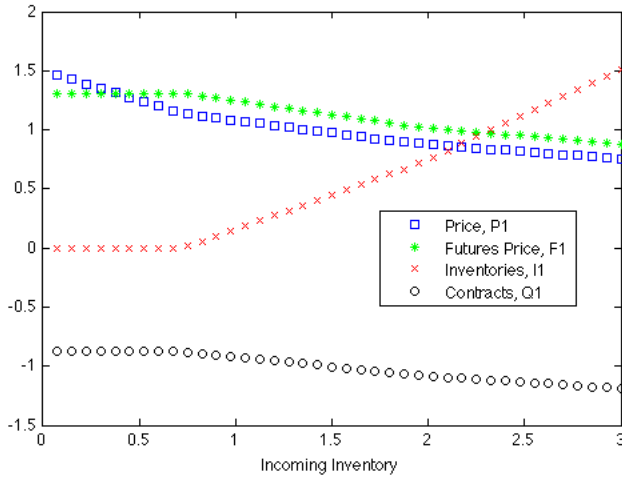


(c) scenario 3, $\nu_0 = \nu_0^H$, $\eta_1 = \eta_1^L$, $\nu_1 = \nu_1^H$

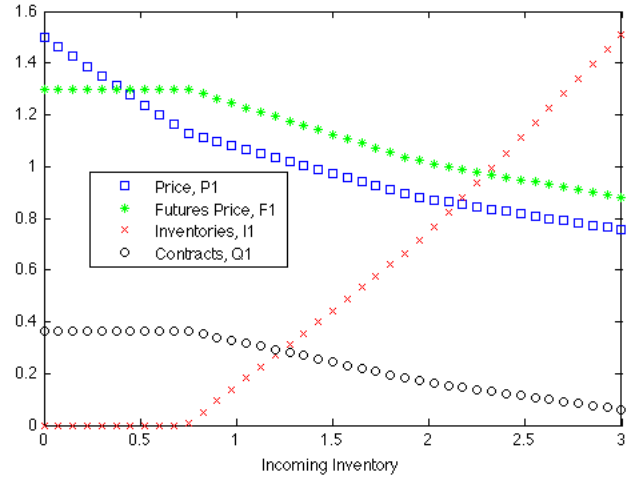


(d) scenario 4, $\nu_0 = \nu_0^H$, $\eta_1 = \eta_1^L$, $\nu_1 = \nu_1^L$

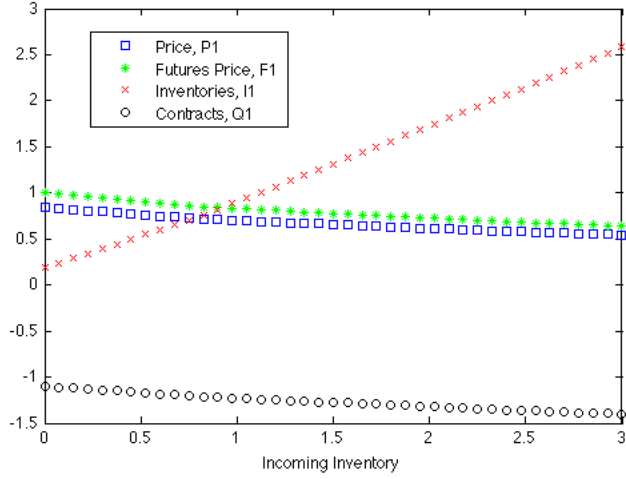
Figure 1: Scenarios 1 to 4



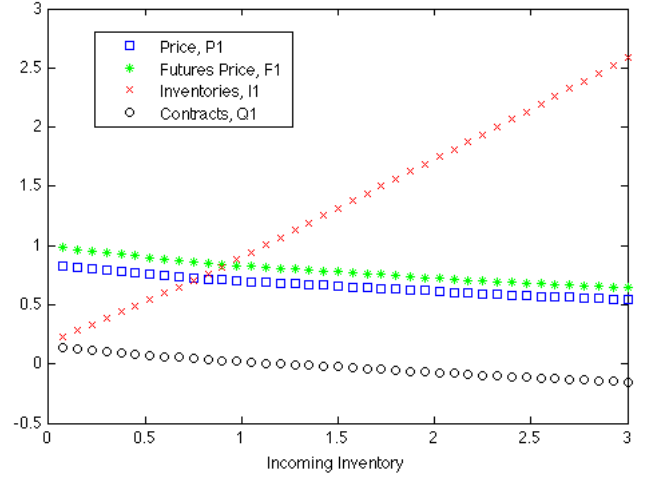
(a) scenario 5, $\nu_0 = \nu_0^L$, $\eta_1 = \eta_1^H$, $\nu_1 = \nu_1^H$



(b) scenario 6, $\nu_0 = \nu_0^L$, $\eta_1 = \eta_1^H$, $\nu_1 = \nu_1^L$



(c) scenario 7, $\nu_0 = \nu_0^L$, $\eta_1 = \eta_1^L$, $\nu_1 = \nu_1^H$



(d) scenario 8, $\nu_0 = \nu_0^L$, $\eta_1 = \eta_1^L$, $\nu_1 = \nu_1^L$

Figure 2: Scenarios 5 to 8

4 dynamics

forthcoming...

5 conclusion

forthcoming...

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