"A Comparison of Methods for Imposing Curvature on the Normalized Quadratic Indirect Profit Function."

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Introduction

It is common for a researcher to assume profit maximization criterion exists and impose restrictions on functional forms to ensure these properties hold. Properties of symmetry, homogeneity, monotonicity, and curvature are required in addition to price taking behavior to ensure the existence of an indirect profit function with a one-to-one correspondence with the direct profit function. Symmetry and homogeneity are almost universally imposed however curvature is not always imposed. The appropriate curvature (convex in prices) on a profit function results in a positive semi-definite or definite hessian matrix. Curvature has important economic properties as well as it guarantees positive output supply elasticities and negative input demand elasticities; which conform to the laws of supply and demand under profit maximizing conditions.

Curvature is a more difficult property to impose because of the nonlinearity of the condition and because the curvature conditions are inequality constraints. As with the imposition with symmetry and homogeneity restrictions, the imposition of curvature conditions will not improve the in-sample fit and will likely result in a reduce in-sample fit. However, Featherstone, Moss and Hsu (1997) show that in one case, the out-of-sample forecasting improves with the imposition of curvature and they argue that the imposition of theory based conditions should generally improve out-of-sample fit if that theory is representative of economic behavior.

Objectives

The objective of this paper is to examine empirically the difference from using the Cholesky method and Geweke's method of exact inference to impose curvature in an indirect profit function. The normalized quadratic flexible functional form will be estimated using each of the methods to impose curvature using Kansas Farm Management Association (KFMA) farm level data. The results of this paper will be useful in guiding researchers regarding the advantages and disadvantages of using alternative methods to impose curvature.

Methods

The Cholesky method is used often, because it is simple to understand and can be applied by using nonlinear estimation. This method allows the Hessian to be specified as the product of two identical triangular matrices, so that the leading principle minors are forced to be greater than or equal to zero. However, because of the nonlinear nature of the constraint, the system can be challenging to optimize using some software packages. If there is a number of output supply and input demand functions, the difficulty of estimating the system increase. In addition, with the imposition of curvature and the use of an optimization routine, at least one of the eigenvalues from the Cholesky decomposition method will be zero. While a positive semi-definite matrix is not an issue for certain economic results, it can be an issue for deriving other economic results if the hessian matrix needs to be inverted (Lusk, Featherstone, Marsh, and Abdulkadri; Gao and Featherstone, Lau).

The Geweke method allows for inequality constraints because it is a Bayesian method. This Bayesian method uses an uninformative prior distribution and an indicator function to represent the inequality constraints that are necessary. Sampling is done by bootstrapping and observations that have the correct sign are kept and those that do not are eliminated. The reported coefficients are the means of the final sample with the sample ordered to establish confidence intervals. This will very likely results in the Hessian matrix being positive definite instead of positive semi-definite. Thus, problems with inverting the matrix are avoided.

Data

The farm level data from the KFMA is a balanced panel of 83 farms from 1989 through 2008. The data were aggregated into two output categories: livestock and crops. The data were aggregated into eight inputs: machinery, land, labor, chemical, fertilizer, seed, feed and fuel. The revenue and cost data were divided by a corresponding price index from the USDA to obtain a quantity index. The prices of inputs and outputs were normalized by the machinery price to impose homogeneity.

Results

Elasticities from the Cholesky Method of Imposing Curvature

	Crops	Livestock	Seed	Fertilizer	Chemical	Feed	Fuel	Labor	Land	Machinery
Crops	0.0871	0.0361	-0.0218	-0.0410	-0.0353	-0.1089	0.0030	0.0002	0.0160	0.0645
Livestock	0.0606	0.6768	-0.0176	0.0788	0.0255	-0.4399	-0.0858	-0.0848	0.1416	-0.3553
Seed	0.2685	0.1294	-0.5259	0.0983	-0.7474	0.0047	0.0432	0.2797	0.0908	0.3588
Fertilizer	0.3047	-0.3496	0.0593	-0.6616	0.0425	0.1909	0.3035	0.1984	-0.1081	0.0200
Chemical	0.3924	-0.1691	-0.6741	0.0636	-1.1086	0.0801	0.1104	-0.0774	0.2552	1.1275
Feed	0.6298	1.5188	0.0022	0.1486	0.0417	-2.1374	-0.2025	-0.4403	0.2174	0.2217
Fuel	-0.0306	0.5280	0.0361	0.4210	0.1024	-0.3609	-0.5509	-0.3140	0.4628	-0.2939
Labor	-0.0019	0.4925	0.2209	0.2599	-0.0678	-0.7409	-0.2965	-1.9975	0.9447	1.1867
Land	-0.0426	-0.2250	0.0196	-0.0387	0.0611	0.1000	0.1195	0.2583	-0.2522	0.000038
Machinery	-0.1829	0.6012	0.0825	0.0076	0.2875	0.1086	-0.0808	0.3456	0.000041	-1.1695

Elasticities from the Geweke Method of Imposing Curvature

	Crops	Livestock	Seed	Fertilizer	Chemical	Feed	Fuel	Labor	Land	Machinery
Crops	0.0475	0.2019	0.0290	0.0723	0.0618	0.1755	0.0158	0.0395	-0.0548	-0.5884
Livestock	0.3986	0.6134	0.0619	-0.0473	0.0045	0.5897	0.0853	0.0333	-0.2013	-1.5381
Seed	0.3579	0.3875	-0.7018	0.2629	-1.3636	0.3241	-0.0778	0.9025	0.0351	-0.1267
Fertilizer	0.5231	-0.1733	0.1539	-0.6904	0.0661	0.1133	0.2206	-0.2779	-0.2749	0.3394
Chemical	0.6867	0.0254	-1.2272	0.1016	-1.2753	-0.0835	0.2305	0.4864	0.1375	0.9181
Feed	1.3068	2.2252	0.1953	0.1167	-0.0559	-2.4144	-0.2104	-0.8497	-0.1445	-0.1691
Fuel	0.1613	0.4425	-0.0645	0.3122	0.2122	-0.2893	-0.3963	0.2394	0.6039	-1.2213
Labor	0.4134	0.1766	0.7651	-0.4023	0.4581	-1.1949	0.2449	-1.1007	1.5091	-0.8693
Land	-0.1341	-0.2497	0.0070	-0.0930	0.0303	-0.0475	0.1444	0.3529	-0.1054	0.0952
Machinery	0.2407	0.3188	0.0042	-0.0192	-0.0338	0.0093	0.0488	0.0340	-0.0159	-0.5869

Comparing methods it seems immediately obvious that there are frequent sign disagreements on the cross-elasticity measures. Many of the magnitudes are quite similar. The biggest magnitude differences seem to occur with the recovered estimates of the machinery.

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