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# How Fiscal Decentralization Flattens <br> Progressive Taxes 

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#### Abstract

We study the tension between fiscal decentralization and progressive taxation. We present a multi-community model in which the local income tax rate is determined by an exogenous progressive tax schedule and a tax shifter that can differ across communities. The progressivity of the tax schedule induces a self-sorting process that results in substantial though imperfect income sorting. Rich households are more likely to locate themselves in low tax communities than poor households. The actual tax structure is thus less progressive than the exogenous tax schedule. To investigate the quantitative implications of our model, we calibrate a fully-specified version to the largest metropolitan area in Switzerland. The equilibrium values of the simulation show the same pattern across communities as we observe in this area. The theoretical result is challenged by estimating the actual tax structure faced by the households in this area. We find that the actual tax structure is indeed substantially less progressive than the fixed tax schedule.


Key words: Progressive Taxation, Fiscal Decentralization, Income Segregation

[^0]
## 1 Introduction

In this paper, we study the tension between fiscal decentralization and progressive taxation. We investigate to what extent fiscal decentralization reduces the progressivity of a common tax schedule in a federation in which the level of the tax rates can differ across communities. We show that progressive taxation and fiscal decentralization lead to income sorting, which undermines the progressivity of the tax schedule.

We base our analysis on a multi-community model, in which the income tax rate is determined by an exogenous progressive tax schedule and a local tax shifter that can differ across communities. Local tax revenue is used to finance a local public good. In the basic version of the model, the mobile individual households differ only in their incomes.

In equilibrium, no household wants to move, local housing markets clear and the communities' budgets are balanced. It follows that the local tax shifters must be higher in communities in which housing prices are lower. The progressivity of the tax schedule then induces a self-sorting process that results in perfect income sorting. Poor households locate themselves in high tax communities while rich households locate themselves in low tax communities. Different from most of the previous literature, ${ }^{1}$ sorting is a direct result of the progressive tax schedule and does not require strong assumptions on the preferences for either public goods or housing. The spatial segregation of the population by incomes implies that the actual tax structure must be less progressive than the exogenous tax schedule if progressive at all.

In an extension, we assume that the individual households differ not only in their incomes, but also in their preferences for housing. Since each house-

[^1]hold's location choice depends now on its income and its preferences, the income sorting is imperfect in equilibrium: households with the same income are found in different communities, though rich households are still more likely to locate in low tax communities than poor households.

To investigate the quantitative implications of this model, we calibrate a fully-specified version to the Zurich area, the largest Swiss metropolitan area. Swiss metropolitan areas offer an excellent laboratory for the analysis of fiscal decentralization. In Switzerland, each community can individually set the level of income taxes by a local tax shifter while the cantons (states) fix the (progressive) schedule of income taxes. The equilibrium values of this simulation show the same pattern across communities as we observe in the Zurich area: A high tax shifter and low average incomes in the center, and low tax shifters and high average incomes in the fringe.

We then use data on the spatial distribution of incomes in the Zurich area to estimate the actual tax structure, i.e. average tax rates as a function of income as faced by the households in this area. We find that the actual tax structure is substantially less progressive than the tax schedule implemented by the canton because rich households are more likely to live in low tax communities than poor households. This finding is in line with the predictions of our theoretical model.

This paper is probably most closely related to Feldstein and Wrobel (1998), Epple and Platt (1998) and Schmidheiny (2004). Feldstein and Wrobel show that a shift in a single US state's tax progressivity has no redistributive effects since migration leads to an adjustment of the net wages and the employment structure. Complementary, we show that migration undermines the redistributive effect of progressive taxation in presence of fiscal decentralization even if wages do not adjust. Our theoretical model shares the formal
structure with Epple and Platt. Schmidheiny shares the location choice part of our model and shows empirically that rich households are more likely to move to low tax communities than poor households.

It is a well known normative principle of the literature on fiscal federalism that income redistribution should be centralized. ${ }^{2}$ Our paper relates to this literature as it shows that fiscal decentralization undermines the redistributive effect of progressive taxes.

The paper is organized as follows: Section 2 briefly informs about fiscal decentralization and progressive taxation in Switzerland. Section 3 presents the theoretical model and some results concerning the agents' location choice. It further proves that an (asymmetric) equilibrium exists. Section 4 presents the simulation of a fully-specified version of our model, which is calibrated to the Zurich metropolitan area. Section 5 estimates the actual tax structure faced by the households in this area. Section 6 concludes.

## 2 Fiscal Decentralization and Progressive Taxation in Switzerland

Switzerland is an exemplary federal fiscal system. The Swiss federation comprises 26 states, the so-called cantons. The cantons are divided into roughly 3000 municipalities of varying size and population. All three state levels finance their expenditures essentially by their own taxes and fees. The total tax revenue of all three levels was 93 billion CHF in 2001, of which $46 \%$ is imposed by the federation, $32 \%$ by the cantons and $22 \%$ by the municipalities. ${ }^{3}$ While the federal government is mainly financed by indirect taxes ( $61 \%$ of

[^2]federal tax revenue) such as the VAT, the cantons and municipalities largely rely on direct taxes. Income taxes account for $60 \%$ of cantonal and $84 \%$ of municipal tax revenue.

The cantons organize their tax systems autonomously. For example, they decide upon the level of income and corporate taxes and the degree of tax progression. The individual municipalities in turn can generally set a tax shifter for income and corporate taxes. The municipal tax is then the cantonal tax rate multiplied by the municipal tax shifter. Federal and cantonal systems of fiscal equalization limit the tax differences across cantons and across municipalities within the same canton to some extent, but still leave room for considerable variation.

The above outlined federal system leads to ample differences of income taxes across Swiss municipalities. For example, for a two-child family with a gross income of 80,000 Swiss francs (CHF) combined cantonal and municipal income taxes ranged from $3.6 \%$ to $11.3 \%$ in the year 1997. The federal income tax for this household was $0.7 \%$. With an income of 500,000 CHF a two-child family faced much higher tax rates due to the progressivity of the tax schedules. Combined cantonal and municipal income taxes ranged from $10.9 \%$ to $28.7 \%$ for this household and its federal income tax was $9.4 \%$. Within metropolitan areas the (municipality) tax differences are smaller but still differ by a factor of 1.5 in e.g. the Zurich area.

## 3 The Model

In this section, we introduce and solve the model. After presenting the general setting, we characterize the preferences and derive the resulting allocation of households across distinct communities. We then prove the existence of an asymmetric equilibrium. Finally, we introduce heterogeneity in the
preferences and discuss how this affects our results.

### 3.1 The Setting

Given is a metropolitan area with $J$ communities. This area is populated by a continuum of households, which differ in their income $y \in[\underline{y}, \bar{y}]$. Income follows a distribution function $f(y)>0$.

There are three goods in the economy: private consumption $b$, housing $h$ and a local public good $g$. The housing $h$ is provided by absentee landlords, and the housing market is competitive. Hence, the price for housing $p_{i}$ equates the housing supply $H S_{i}$ with the aggregate housing demand $H D_{i}$. We assume that the housing supply $H S_{i}=H S\left(L_{i}, p_{i}\right.$, ) is a non-decreasing function of the land area $L_{i}$ and the price $p_{i}$.

Each community $i$ spends the amount $n_{i} g$ to provide the local public good $g$, where $n_{i}$ is the measure of households living in community $i$. The communities levy income taxes to finance the public good. In each community $i$, the tax rate consists of two parts, a local tax shifter $t_{i}$ and a progressive tax rate structure $r(y)$. We assume $r(y)$ continuous and increasing in $y, r(y)>0$, the average tax rate $t \cdot r(y) \in[0,1)$ and the marginal tax rate $t\left[r+y r^{\prime}(y)\right] \in[0,1)$. The quantity of the local public good $g$ and the tax rate structure $r(y)$ are both exogenous (to the communities) and identical across communities. In each community $i$, the tax shifter $t_{i}$ is then determined by budget balance.

Each household can move costlessly and chooses the community maximizing its utility as place of residence.

### 3.2 Preferences and Location Choice

The preferences of the households are described by the utility function

$$
\begin{equation*}
U(h, b) \tag{1}
\end{equation*}
$$

where $h$ is the consumption of housing and $b$ the consumption of the private good. We assume the utility function to be strictly increasing, strictly quasiconcave, twice continuously differentiable in $h$ and $b$ and homothetic. ${ }^{4}$

Households face the budget constraint (omitting community indices)

$$
\begin{equation*}
p h+b \leq y_{d}=y[1-t \cdot r(y)] \tag{2}
\end{equation*}
$$

where $p$ is the price of housing; the price of the private good is set to unity. Disposable income $y_{d}$ depends on the local tax shifter $t$ and the tax rate structure $r(y)$.

Maximization of the utility function (1) with respect to $h$ and $b$ subject to constraint (2) yields housing demand $h^{*}=h\left(p, y_{d}\right)=h(t, p, y)$, demand for the private good $b^{*}=y(1-t)-p h(t, p, y)$, and indirect utility

$$
\begin{equation*}
V(t, p, y)=U\left(h^{*}, b^{*}\right) . \tag{3}
\end{equation*}
$$

For later use note that $V$ is continuous in $t, p$ and $y$.
We assume that the elasticity of housing with respect to the disposable income is smaller or equal to unity, i.e.,

$$
\begin{equation*}
\varepsilon_{h, y_{d}}:=\frac{\partial h^{*}}{\partial y_{d}} \frac{y_{d}}{h^{*}} \leq 1 \quad \text { for all } y_{d} \text { and } p \tag{4}
\end{equation*}
$$

Next, we present two properties of the households' indifference curves that will lead to segregation of the population by incomes:

## Property 1

$$
M(t, p, y):=\left.\frac{d t}{d p}\right|_{d V=0}=-\frac{\partial V / \partial p}{\partial V / \partial t}=-\frac{h^{*}}{y \cdot r(y)}<0
$$

[^3]Property 1 follows from the strictly increasing utility function after applying the implicit function theorem and the envelope theorem. It implies that a household can be made indifferent towards an increase in the tax shifter $t$ when it is compensated by decreased housing prices $p$, and vice versa.

## Property 2

$$
\frac{\partial M}{\partial y}=\left[1-\frac{\partial h^{*}}{\partial y_{d}} \frac{y_{d}}{h^{*}} \frac{\partial y_{d}}{\partial y} \frac{y}{y_{d}}\right] \frac{h^{*}}{y^{2} r(y)}+\frac{\partial r(y)}{\partial y} \frac{h^{*}}{y^{2} r^{2}(y)}>0 \text { for all } y, t \text { and } p .
$$

Proof: By assumption, $\left(\partial h^{*} / \partial y_{d}\right)\left(y_{d} / h^{*}\right) \leq 1$. Our assumptions about the bounds of the average and the marginal tax rate guarantee $\left(\partial y_{d} / \partial y\right)\left(y / y_{d}\right)$ $=\left[1-\operatorname{tr}-\operatorname{tyr}^{\prime}(y)\right] /[1-\operatorname{tr}(y)] \in[0,1)$. The assumption that $r(y)$ increases in $y$, implying $\partial r(y) / \partial y>0$, concludes the proof.

Property 2 implies that the decrease in housing prices $p$ which compensates a household for a higher tax shifter $t$ has to be larger for poor households than for rich ones.

Given a set of community characteristics, $\left(p_{i}, t_{i}\right)$ for $i=1 . . J$, a household prefers community $i$ if and only if

$$
\begin{equation*}
V\left(p_{i}, t_{i}, y\right) \geq V\left(p_{j}, t_{j}, y\right) \quad \text { for all } j \neq i \tag{5}
\end{equation*}
$$

From this, the following proposition directly follows:

## Proposition 1 (Order of community characteristics)

If any two populated communities differ in their characteristics $\left(p_{i}, t_{i}\right)$, then the community with the higher housing prices $p_{i}$ must impose a lower tax shifter $t_{i}$.

Proof: Suppose the opposite, i.e., that the housing prices $p_{i}$ and the tax shifter $t_{i}$ are both higher in one community. In this case, no household would
choose to live in this community (for the same reason that leads to property $1)$. This is a contradiction.

In the remaining part of this section, we show how households allocate themselves across distinct communities. Distinct communities differ in both tax shifters and prices. Note that our model allows for groups of communities with identical community characteristics $\left(t_{i}, p_{i}\right)$. Such groups appear as one community in our notation.

## Lemma 1 (Boundary indifference)

There is a 'border' household between any two communities $i$ and $j$ that is indifferent between these two communities. That is, if a household with income $y^{\prime}$ prefers to live in $i$ and another household with income $y^{\prime \prime}>y^{\prime}$ prefers to live in $j$, then there exists a household with income $\hat{y}_{i j}=\hat{y}\left(p_{i}, t_{i}, p_{j}, t_{j}\right)$, $y^{\prime} \leq \hat{y}_{i j} \leq y^{\prime \prime}$, such that $V\left(p_{i}, t_{i}, \hat{y}_{i j}\right)=V\left(p_{j}, t_{j}, \hat{y}_{i j}\right)$.

Proof: Let $V_{i}(y):=V\left(p_{i}, t_{i}, y\right)$ be a household's utility in $i$ and $V_{j}(y):=$ $V\left(p_{j}, t_{j}, y\right)$ in $j$. The household with income $y^{\prime}$ prefers $i$ to $j$, hence $V_{i}\left(y^{\prime}\right)-$ $V_{j}\left(y^{\prime}\right) \geq 0$. The opposite is true for a household with income $y^{\prime \prime}: V_{i}\left(y^{\prime \prime}\right)-$ $V_{j}\left(y^{\prime \prime}\right) \leq 0$. From the continuity of $V$ in $y$ follows the continuity of $V_{i}(y)-$ $V_{j}(y)$ in $y$. The intermediate value theorem proves that there is at least one $\hat{y}$ between $y^{\prime}$ and $y^{\prime \prime}$ such that $V_{i}(\hat{y})-V_{j}(\hat{y})=0$.

## Lemma 2 (Two-community income segregation)

Given two populated communities $i$ and $j$ with distinct characteristics $\left(t_{i}, p_{i}\right) \neq$ $\left(t_{j}, p_{j}\right)$, where $t_{i}<t_{j}$, then any household in $i$ is richer than any household in $j$. That is, if a household with income $\hat{y}$ is indifferent between $i$ and $j$, then any household $y^{\prime}<\hat{y}$ strictly prefers $j$ and any household $y^{\prime \prime}>\hat{y}$ strictly prefers $i$.


Figure 1: Indifference curves in the $(t, p)$ space

Proof: The proof uses figure 1, which shows the indifference curves in the $(t, p)$-space for three different income levels $y^{\prime}<\hat{y}<y^{\prime \prime}$. The indifference curves represent all $(t, p)$ combinations that households consider as good as community $j$ 's $\left(p_{j}, t_{j}\right)$-pair. Each household prefers pairs south-west of its indifference curve. It follows from property 1 that the indifference curves decrease in the $(t, p)$-space and from property 2 that they become flatter as income rises. Imagine now a community $i$, characterized by $t_{i}<t_{j}$ and $p_{i}>p_{j}$, where household $\hat{y}$ is indifferent to $j$. All poorer households, e.g. $y^{\prime}$, prefer $j$ to $i$ and all richer households, e.g. $y^{\prime \prime}$, prefer $i$ to $j$.

## Proposition 2 (Multi-community income segregation)

Given J populated communities with distinct characteristics $\left(t_{i}, p_{i}\right)$, then it holds for any two communities $i$ and $j$ with $t_{i}<t_{j}$ that any household in $i$ is richer than any household in $j$.

Proof: The proposition implies that $[\underline{y}, \bar{y}]$ must be partitioned into $J$ non-empty and non-overlapping intervals. Suppose the opposite, i.e., $y^{\prime}$ as
well as $y^{\prime \prime}$ prefer community $i$, but $y^{\prime \prime \prime}, y^{\prime}<y^{\prime \prime \prime}<y^{\prime \prime}$, strictly prefers another community $j$. Then it follows from lemma 1 that there is a $\hat{y}_{i j}, y^{\prime} \leq \hat{y}_{i j}<y^{\prime \prime \prime}$. Lemma 2 implies that $y^{\prime \prime}>\hat{y_{i j}}$ strictly prefers $j$ to $i$, which is a contradiction.

### 3.3 Equilibrium

In this section, we prove that an asymmetric equilibrium exists. That is, we show that an allocation in which communities exhibit different characteristics ( $p_{i}, t_{i}$ ) can be an equilibrium.

An equilibrium requires that each household is located in the community that maximizes its utility, that each household maximizes its utility within the given community, that the housing market clears in each community, that each community has a balanced public budget and that each community has a positive population.

There always exists a symmetric equilibrium in which all communities have identical characteristics $\left(p_{i}, t_{i}\right)$ and in which the households allocate themselves such that all communities show the same income distribution. ${ }^{5}$ However, we are interested in the case in which at least some communities differ in their characteristics $\left(p_{i}, t_{i}\right)$. We therefore show that an asymmetric equilibrium, i.e., an equilibrium in which $\left(p_{i}, t_{i}\right) \neq\left(p_{j}, t_{j}\right)$ for some $i$ and $j$, exists too. For simplicity, we focus thereby on the case of two distinct communities, 1 and 2.

We assume with no loss of generality that $t_{1}>t_{2}$. Hence, any household in 2 must be richer than any household in 1, as lemma 2 implies. We define

$$
\begin{equation*}
\Delta V(\hat{y})=V_{1}\left(p_{1}(\hat{y}), t_{1}(\hat{y}), \hat{y}\right)-V_{2}\left(p_{2}(\hat{y}), t_{2}(\hat{y}), \hat{y}\right) \tag{6}
\end{equation*}
$$

[^4]where $p_{i}(\hat{y})$ and $t_{i}(\hat{y})$ are the equilibrium housing price and the equilibrium tax shifter, respectively, in $i$ given that households with $y<\hat{y}$ live in 1 and households with $y>\hat{y}$ in 2 . Hence, $V_{i}\left(p_{i}(\hat{y}), t_{i}(\hat{y}), \hat{y}\right)$ is the indirect utility of a household with $\hat{y}$ in $i$ given this allocation of households.

In addition, we assume: ${ }^{6}$
(i) The housing supply $H S\left(L_{i}, p_{i}\right)$ satisfies $H S\left(L_{i}, 0\right)=\underline{L}_{i}>0$ for $i=1,2$.
(ii) The minimum income $\underline{y}>g$.
(iii) If $h_{i} \rightarrow \infty, b_{i}>0, h_{j}<\infty$ and $b_{j}<\infty$, then $U\left(h_{i}, b_{i}\right)>U\left(h_{j}, b_{j}\right)$.

## Proposition 3 (Existence of an asymmetric equilibrium)

There exists an equilibrium in which the communities 1 and 2 exhibit different characteristics, i.e. $t_{1}>t_{2}$ and $p_{1}<p_{2}$.

Proof: We prove proposition 3 by showing (1) that $\Delta V(\hat{y})$ is continuous and (2) that $\Delta V(\hat{y})>0$ as $\hat{y} \rightarrow \underline{y}$ and that $\Delta V(\hat{y})<0$ as $\hat{y} \rightarrow \bar{y}$. It follows then from the intermediate value theorem that there is at least one $\hat{y}$, $\underline{y}<\hat{y}<\bar{y}$, such that $\Delta V(\hat{y})=0$. This implies - from the definition of $\Delta V-$ that the border household $\hat{y}$ is indifferent between the two communities, the prices $p_{1}$ and $p_{2}$ clear the local housing markets and the tax shifters $t_{1}$ and $t_{2}$ balance the community budgets.
(1) The equilibrium housing price $p_{i}$ is determined by $H S\left(L_{i}, p_{i}\right)=H D_{i}$. It follows from lemma 2 that

$$
\begin{equation*}
H D_{i}=\int_{\underline{y}_{i}}^{\bar{y}_{i}} h\left(p_{i}, t_{i} ; y\right) f(y) d x \tag{7}
\end{equation*}
$$

where $\underline{y}_{i}$ and $\bar{y}_{i}$ are the highest and lowest incomes in community $i$. The hereby implicitly defined $p_{i}$ is continuous in $\bar{y}_{i}$ and $\underline{y}_{i}$ given continuity of

[^5]$H S(\cdot), h(\cdot)$ and $f(\cdot)$. The balanced budget requirement and lemma 2 imply that the equilibrium tax shifter in community $i$ is
\[

$$
\begin{equation*}
t_{i}=\frac{n_{i} g}{\int_{\underline{y}_{i}}^{\bar{y}_{i}} r(y) f(y) d x}, \tag{8}
\end{equation*}
$$

\]

where

$$
\begin{equation*}
n_{i}=\int_{\underline{y}_{i}}^{\bar{y}_{i}} f(y) d x \tag{9}
\end{equation*}
$$

Given continuity of $r(\cdot)$ and $f(\cdot), t_{i}$ is continuous in $\bar{y}_{i}$ and $\underline{y}_{i}$. Since the indirect utility $V_{i}$ is continuous in $p_{i}, t_{i}$ and $y$ and since $p_{i}$ and $t_{i}$ are continuous in $y, \Delta V(\hat{y})$ is continuous in $\hat{y}$.
(2) If follows from equations (7) and (9) that $H D_{1} \rightarrow 0$ and $n_{1} \rightarrow 0$ as $\hat{y} \rightarrow \underline{y}$. Since assumption (i) guarantees that $H S\left(L_{1}, 0\right)=\underline{L}_{1}>0$ (and since $\left.\partial H S\left(L_{i}, p_{i}\right) / \partial p_{i} \geq 0\right)$, it holds that $h^{*}\left(p_{1}, t_{1} ; y\right) \rightarrow \infty$ and $p_{1} \rightarrow 0$ as $\hat{y} \rightarrow \underline{y}$. Hence, $b^{*} \rightarrow \underline{y}-g>0$, where the strict inequality follows from assumption (ii). Assumption (iii) then guarantees that $\Delta V(\hat{y})>0$ as $\hat{y} \rightarrow \underline{y}$. Analogously, it can be shown that $\Delta V(\hat{y})<0$ as $\hat{y} \rightarrow \bar{y} .{ }^{7} \square$

### 3.4 Adding Heterogeneous Preferences

So far, we have assumed that households differ only in their incomes $y$. In this section, we extend the model by assuming that households differ in their preferences as well.

The household preferences are now represented by the utility function $U(h, b ; \alpha)$, where the parameter $\alpha$ describes the taste for housing. The higher $\alpha$, the more a household is, ceteris paribus, willing to spend on housing. Hence, the housing demand increase in $\alpha$, i.e.

$$
\begin{equation*}
\frac{\partial h^{*}}{\partial \alpha}=\frac{\partial h(t, p ; y, \alpha)}{\partial \alpha}>0 \text { for all } t, p, y \text { and } \alpha \tag{10}
\end{equation*}
$$

[^6]

Figure 2: Simultaneous income and preference segregation. The areas denoted by $j=1, \ldots, J$ show the attributes of the households that prefer community $j$.

Income and preferences are jointly distributed according to the density function $f(y, \alpha)$.

It follows that income segregation holds, but only within the subpopulation of households with identical preferences. Preference segregation occurs as well: That is, among the subpopulation of households with the same income $y$, households with a high $\alpha$, i.e. a strong taste for housing, tend to allocate themselves to communities with higher tax shifters $t_{i}$ than households with a low $\alpha$.

Simultaneous heterogeneity by incomes and tastes leads to a more realistic pattern of household segregation. Although income groups tend to gather, the segregation is not perfect. Figure 2 shows the resulting allocation of household types to communities. The households on the borders are indifferent between neighboring communities $j$. Community 1 with the lowest housing prices is populated by the poorest households with strong taste for housing, while the richest households with low housing taste are situated in community $J$ with the lowest tax rate and the highest housing price.

However, rich households with strong taste for housing prefer lower-priced communities and poor households with weak taste for housing group with relatively rich households in the lower-tax communities.

## 4 A Specified Version of the Model

To investigate the qualitative and quantitative properties of the model we construct a fully specified example in this section. The specification is kept as simple as possible but still captures all mechanisms of the model. The example is calibrated to the Zurich area, the largest Swiss metropolitan area.

The common tax schedule is taken from Young (1990)

$$
r(y)=r_{0}\left[1-\left(1+r_{2} y^{r_{1}}\right)^{-1 / r_{1}}\right] .
$$

with parameters $r_{0}>0, r_{1}>0$ and $r_{2}>0$. The average local tax rate $\operatorname{tr}(y)$ and the local marginal tax rate $t[y \partial r(y) / \partial y+r(y)]$ is increasing in income $y$. The marginal tax rate is always above the marginal tax rate; both asymptotically approach a maximum $t r_{0}$.

Household preferences are described by a Cobb-Douglas utility function:

$$
U=h^{\alpha} b^{1-\alpha}
$$

where $0<\alpha<1$ stands for the taste parameter of the general model. Utility function and tax schedule satisfy our properties 1 and 2 .

The locus of indifferent households between two communities $i$ and $j$ for any given taste $\alpha$ is

$$
\hat{y}_{i j}=\left\{\left[\left(1-\frac{p_{j}^{\alpha}-p_{i}^{\alpha}}{r_{0}\left(p_{j}^{\alpha} t_{i}-p_{i}^{\alpha} t_{j}\right)}\right)^{-r_{1}}-1\right] \frac{1}{r_{2}}\right\}^{1 / r_{1}}
$$

We adopt the housing supply function

$$
H S_{i}=L_{i}\left(p_{i}\right)^{\theta}
$$



Figure 3: Center and periphery in the metropolitan area of Zurich.
from Epple and Romer (1991). ${ }^{8}$
We calibrate the above outlined model to the metropolitan area of Zurich in Switzerland. The area around the city of Zurich forms the biggest Swiss metropolitan area. The city of Zurich has about 330 thousand inhabitants and is the capital of the canton (state) of Zurich. The canton of Zurich counts 1.2 million inhabitants in 171 individual communities. As described in section 2, each of these communities can choose its own tax shifter.

The analysis is restricted to the city of Zurich and a ring of the most integrated communities around the center. This ring is formed by all communities in the canton of Zurich with more than $1 / 3$ of the working population commuting to the center. ${ }^{9}$ Figure 3 shows a map with the city of Zurich and the thus defined ring of 40 communities. This agglomeration is

[^7]Table 1: Equilibrium values of the specified model.

|  | harmonized | homogeneous preferences |  | heterogeneous preferences |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | center | periphery | center | periphery |
| $L$ : area | 1 | 0.4 | 0.6 | 0.4 | 0.6 |
| $p$ : rent | 11.7 | 6.6 | 13.1 | 9.9 | 12.5 |
| $t$ : tax shifter | 1 | 5.17 | 0.91 | 2.30 | 0.87 |
| $n$ : inhabitants | 1 | 0.12 | 0.88 | 0.23 | 0.77 |
| Ey: mean income | e $\quad 78$ '547 | 30'771 | 85 '010 | 47'755 | 87’703 |
| The calibrated model parameters: $g=5000, E(\ln y)=11.1, S D(\ln y)=0.55$, $y_{\min }=23,000, y_{\max }=500,000, E(\alpha)=0.25, S . A .(\alpha)=0$ (homogeneous preferences), S.A. $(\alpha)=0.11$ (heterogeneous preferences), $\theta=3, r_{0}=0.132, r_{1}=1$ and $r_{2}=0.00001$. |  |  |  |  |  |

modelled as two distinct jurisdictions, which we call center and periphery. The details of the calibration are described in the appendix. The parameters are summarized at the bottom of table 1 .

### 4.1 Simulated Equilibrium

The equilibrium values $p_{i}$ and $t_{i}$ in both communities satisfy equations (7) and (8) and guarantee that no households wants to move. As there is no closed form solution to this nonlinear system of four equations and four unknowns, we solve numerically for the equilibrium values of the model. ${ }^{10}$

Table 1 shows in column 2 and 3 the equilibrium values for the case of homogeneous tastes. There are large differences in both taxes and prices between the two communities. The tax rate in the center community is almost

[^8]

Figure 4: Income and taste segregation in equilibrium. The left figure shows the preferred community for all household types. The right figure shows the resulting income distributions in both communities.
six times higher than in the periphery while housing prices in the center are halve the ones in the periphery. ${ }^{11}$ Households are perfectly segregated: All households in the high-tax center are poorer than all households in the lowtax periphery. The mean income in the periphery is therefore almost three times the mean income in the center. Column 1 in table 1 gives the equilibrium values for the hypothetical case that the two communities merged or harmonized their taxes.

These predictions of segregation and the implied differences of community characteristics are extreme. The consideration of heterogeneous housing tastes leads to a more realistic situation. Table 1 shows in column 4 and 5 the equilibrium values allowing for heterogeneous tastes. The differences between the two communities are still substantial but smaller than with homogeneous tastes: The center exhibits now 2.5 times higher taxes and 20 \% lower housing prices. The left graph in figure 4 shows the segregation

[^9]

Figure 5: Taxes and incomes in the Zurich metropolitan area.
pattern in the income-taste space. The population is now imperfectly sorted by incomes: While it is still true that more rich households are found in the low-tax periphery, rich households with a strong taste for housing prefer the low-price high-tax center and poor households with a low taste for housing prefer the periphery. The right graph in figure 4 shows the resulting income distributions in the two communities. The mean income in the center is about half the one in the periphery.

Figure 5 shows the actual local tax levels and the spatial income distribution in the calibrated area. The left map visualizes the considerable tax differentials in the Zurich area. The right map demonstrates the striking relationship between income taxation and spatial income distribution: the local share of rich households is almost an inverted picture of the local tax levels. ${ }^{12}$ Our simple two-community model captures this empirical pattern

[^10]

Figure 6: Mean average tax rate by income in the case of homogeneous (left) and heterogenous (right) tastes.
well.

### 4.2 The Resulting Tax Schedule

The average tax rate $t_{i} r(y)$ depends not only on the individual household's income but also on its place of residence. As the model shows the place of residence is not random and rich households are more likely to reside in low-tax communities. In this section, we ask what tax schedule is realized after considering the sorting of the population. In other words, we ask what tax rate a household with income $y$ pays on average.

In the case of homogeneous taste this question is trivial. All households with income below the indifferent household ( $\hat{y}=37,000$ ) face the average tax rate in the high-tax community; rich households the one in the low-tax community. The left graph in figure 6 shows the resulting tax schedule. While progressive within the communities, it is actually regressive as the richest households face lower average tax rates than the poorest households.

In general, the expected or mean average tax for a household with in-
come $y$ is

$$
\begin{equation*}
E[\operatorname{tr}(y) \mid y]=\sum_{i}\left[P(i \mid y) \cdot t_{i} r(y)\right] \tag{11}
\end{equation*}
$$

where $t_{i} r(y)$ is the average tax rate for a household with income $y$ in community $i$. The probability that a household with income $y$ lives in community $i$,

$$
\begin{equation*}
P(i \mid y)=\frac{f(y \mid i) P(i)}{f(y)} \tag{12}
\end{equation*}
$$

is calculated from the income density $f(y \mid i)$ in community $i$, the probability $P(i)$ that an arbitrary household resides in community $i$ and the income distribution $f(y)$ of the whole area.

In the case of heterogenous tastes, the marginal income distribution $f(y \mid i)$ in a community $i$ (shown in the right figure 4) is calculated by integrating over tastes in community $i$ :

$$
f(y \mid i)=\int_{\underline{\alpha}_{i}}^{\bar{\alpha}_{i}} f(y, \alpha) d \alpha,
$$

where $\underline{\alpha}_{i}$ and $\bar{\alpha}_{i}$ are the lowest and highest tastes in community $i$.
The right graph in figure 6 shows the mean average tax rate in the case of heterogenous tastes. The realized tax schedule is still progressive, though, much flatter than the tax schedule implemented by the canton.

## 5 Evidence

In this section, we estimate the mean average tax rates that households with a given income face in the Zurich metropolitan area. We then compare our estimates to the results obtained in the previous section.

### 5.1 Method

In principle, the mean average tax rate can be estimated from a random sample of households in the studied area. Knowing each households' income
and community tax rate allows to directly estimate the mean average tax rate with e.g. a kernel regression. The random sampling automatically accounts for the sorting of the population by incomes. Unfortunately, we do not have such microdata with tax information. Furthermore, available survey data suffers from small sample sizes and stratified sampling over communities.

We therefore follow an alternative estimation strategy. The mean average tax rate of a household with income $y$ can be estimated from equation (11):

$$
\hat{E}[\operatorname{tr}(y) \mid y]=\sum_{i}\left[\hat{P}(i \mid y) \cdot t_{i} r(y)\right]
$$

As the canton sets the tax structure $r(y)$ and the individual communities their tax shifters $t_{i}$, the average tax rate $t_{i} r(y)$ for any income $y$ in any community $i$ is known.

The estimated probability that a household with income $y$ lives in community $i$ is given by equation (12):

$$
\hat{P}(i \mid y)=\frac{\hat{f}(y \mid i) \hat{P}(i)}{\hat{f}(y)}=\frac{\hat{f}(y \mid i) n_{i}}{\sum_{j}\left[\hat{f}(y \mid j) n_{j}\right]}
$$

where $n_{i}$ is the known number of households living in community $i$.
It remains, therefore, estimating the income density $\hat{f}(y \mid i)$ of each community $i$ in the area. We estimate $\hat{f}(y \mid i)$ from publicly available local income distribution data. The federal tax administration publishes the number of households with taxable income in seven different income classes. ${ }^{13}$ We assume that incomes are log-normally distributed and estimate mean and variance of this distribution using maximum likelihood. ${ }^{14}$ We estimate a truncated log-normal distribution as the first reported income interval is empty

[^11]for technical reasons. The log likelihood function for any community $i$ is
$$
\log \mathcal{L}_{i}=\sum_{k=1}^{6} s_{k} \cdot \log \left[\frac{\Phi\left(\frac{c_{k+1}-\mu_{i}}{\sigma_{i}}\right)-\Phi\left(\frac{c_{k}-\mu_{i}}{\sigma_{i}}\right)}{1-\Phi\left(\frac{c_{1}-\mu_{i}}{\sigma_{i}}\right)}\right]
$$
where $\mu_{i}$ and $\sigma_{i}^{2}$ are mean and variance of $\log$ income in community $i . s_{k}$ is the number of households in income class $k$ with lower interval limit $c_{k} \in$ $\{\log (15000), \log (20000), \log (30000), \log (40000), \log (50000), \log (75000), \infty\}$. $\Phi($.$) is the cdf of the standard normal distribution. The income density$ in community $i$ is then estimated as
$$
\hat{f}(y \mid i)=\frac{1}{\hat{\sigma}_{i} y \sqrt{2 \pi}} \exp \left[-\frac{1}{2}\left(\frac{\log (y)-\hat{\mu}_{i}}{\hat{\sigma}_{i}}\right)^{2}\right]
$$

### 5.2 Results

Figure 7 shows that the average tax rates that households with income $y$ face in the Zurich metropolitan area. The top line is the average tax rate $t_{i} r(y)$ of households living in the community with the highest tax shifter. The bottom line is the average tax rate of households in the community with the lowest tax shifter. The middle line is the estimated mean average tax rate that households in this area face, $\hat{E}[\operatorname{tr}(y) \mid y]$. This is the expected unconditional, i.e. not conditioned on the place of residence, average tax rate. As one can see, the average poor household faces almost the average tax rate in the highest-tax community (or in the city of Zurich). ${ }^{15}$ This is, of course, because most poor households live in high tax communities. As households become richer, they live more often in the low-tax communities and thus face an average tax rate that is on average substantially smaller than in high-tax communities. The mean average tax rate of households

[^12]

Figure 7: Estimated mean average tax rate by income.
with very high incomes $y$ is even relatively close to the average tax rate of very rich households living in the lowest-tax community.

The results from the estimation (figure 7) are very similar to the predictions of the calibrated model with taste heterogeneity (figure 6). There are though two noteworthy differences: First, the difference between the highest and the lowest tax shifters is in reality smaller than our model predicts. Second, the mean average tax rate of very rich households remains in reality above the average tax rate of very rich households in the lowest-tax community, unlike in our simulation. While polito-economical considerations may account for the first difference, ${ }^{16}$ the second might indicate that the location choice depends also on preference characteristics other than the taste for housing.

## 6 Conclusions

We have focused on the tension between fiscal decentralization and progressive taxation. We have presented a multi-community model in which the

[^13]local income tax rate is determined by an exogenous progressive tax schedule and a tax shifter that can differ across communities. The progressivity of the tax schedule has been shown to induce a self-sorting process that results in substantial though imperfect income sorting. Rich households are found to be more likely to locate themselves in low tax communities than poor households such that the actual tax structure becomes less progressive than the exogenous tax schedule. To investigate the quantitative implications of our model, we have calibrated a fully-specified version to the largest metropolitan area in Switzerland. The equilibrium values of the simulation have shown the same pattern across communities as we observe in this area. We have further estimated the actual tax structure faced by the households in this area. We have found that the actual tax structure is indeed significantly less progressive than the fixed tax schedule. Hence, progressive taxes should be implemented at the state, the national or even the supranational level rather than at the community level given that one wants them to unfold their full redistributive effect.

## Appendix: Calibration

Land Area: The whole area has a physical size of $349 \mathrm{~km}^{2}$, of which $88 \mathrm{~km}^{2}$ $(25 \%)$ form the city of Zurich. $140 \mathrm{~km}^{2}$ are dedicated to development, $53 \mathrm{~km}^{2}$ $(38 \%)$ in the inner city and $87 \mathrm{~km}^{2}$ in the fringe communities. In 1998, the whole area was populated by around $628^{\prime} 000$ inhabitants, of whom 334,000 lived in the city and 294,000 in the fringe communities. ${ }^{17}$ This agglomeration is modelled as two distinct jurisdictions with land area $L_{1}=0.4$ and $L_{2}=0.6$ respectively.

Tax schedule: The parameters $r_{0}=0.132, r_{1}=1$ and $r_{2}=0.00001$ almost perfectly approximate the tax scheme of the canton of Zurich. ${ }^{18}$

Income Distribution: The income distribution is calibrated with data from the Swiss labor force survey. ${ }^{19}$ The 1995 cross-section contains detailed information on 1124 households in the above defined region. These households had average income (after state and federal taxes) of CHF 92,000, median income of CHF 66,700 and a quartile distance of $47,700 .{ }^{20}$ We use a log-normal distribution to approximate this right-skewed distribution. A log-normal distribution with mean $E(\ln y)=11.1$ and standard deviation $S D(\ln y)=0.55$ matches the observed median and quartile distance. For numerical tractability, the model distribution is truncated at a minimum income of $y_{\min }=23,000$ and a maximum income $y_{\max }=500,000 .{ }^{21}$

Taste Distribution: The Swiss labor force survey also contains the monthly

[^14]housing expenditure of renters which allows to calibrate the distribution of tastes for housing. ${ }^{22}$ Note that the taste parameter $\alpha$ in the Cobb-Douglas utility function is the share of housing in a utility maximizing household. We therefore estimate each household taste parameter as $\alpha=(p h) / y_{d}$, where $p h$ is observed households housing expenditure and $y_{d}$ is observed household income minus federal, state and communal taxes. A beta distribution with mean $E(\alpha)=0.25$ and standard deviation $S D(\alpha)=0.11$ describes the distribution of the so calculated taste parameter well. Taste and income are assumed to be uncorrelated.

Housing and Public Good Production: The price elasticity of housing supply is $\theta=3$ as in Epple and Romer (1991) and Goodspeed (1989). The targeted public goods provision is set to 5000 .

[^15]
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[^1]:    ${ }^{1}$ See e.g. Ellickson (1971), Westhoff (1977), Epple and Romer (1991) and the literature surveyed in Ross and Yinger (1999).

[^2]:    ${ }^{2}$ See Musgrave (1959) and Oates (1972). For a recent survey of the literature on fiscal federalism see Oates (1999).
    ${ }^{3}$ All figures in this paragraph apply to 2001. Source: Swiss Federal Tax Administration (2002), Öffentliche Finanzen der Schweiz 2001, Neuchâtel: Swiss Federal Statistical Office.

[^3]:    ${ }^{4}$ Since the local public good $g$ is constant across communities and not of primary interest for our considerations, we assume for simplicity that it does not enter the utility function. Equivalently, we could assume that it enters separably.

[^4]:    ${ }^{5}$ Other equilibria in which all communities have identical characteristics $\left(p_{i}, t_{i}\right)$ might exist as well.

[^5]:    ${ }^{6}$ As it will become evident in section 4, these assumptions are sufficient, but not necessary for the existence of an asymmetric equilibrium.

[^6]:    ${ }^{7}$ The only difference is that $b^{*} \rightarrow \bar{y}-g$, which exceeds $\underline{y}-g$.

[^7]:    ${ }^{8}$ Epple and Romer derive this housing supply function from an explicit production function, where $0 \leq \theta \leq 1$ is the ratio of non-land to land input.
    ${ }^{9}$ The number of commuters to the city of Zurich and the size of the working population in the communities is based on the 1990 Census. This definition of the urban area is chosen to justify the model's assumption that households income is exogenous, i.e. that they choose their place of residence independent of where they work. It results in a set of communities closest to the central business district.

[^8]:    ${ }^{10}$ The aggregation of individual behavior requires double integrals over the community population. These integrals cannot be calculated analytically. We use Gauss-Legendre Quadrature with 40 nodes in each dimension to approximate the various double integrals. We numerically solve for the equilibrium values by minimizing the sum of squared deviations from the equilibrium conditions with the Gauss-Newton method.

[^9]:    ${ }^{11}$ The labels 'center' and 'periphery' are arbitrary. There is always a second equilibrium with lower taxes in the center.

[^10]:    ${ }^{12}$ Data from the following sources: Commuter: Swiss Federal Statistical Office, Census 1990. Tax rates: Statistisches Amt des Kantons Zürich, Steuerfüsse 1997. Income distribution: Swiss Federal Tax Administration. Considered are all communities where more than $1 / 3$ of the working population is commuting to the center community.

[^11]:    ${ }^{13}$ Swiss Federal Tax Administration, Steuerbelastung in der Schweiz, Natürliche Personen nach Gemeinden 1997, Neuchâtel: Swiss Federal Statistical Office.
    ${ }^{14}$ Note that this maximum likelihood estimator corresponds to an ordered probit with known thresholds.

[^12]:    ${ }^{15}$ The tax shifter is 131 in the highest-tax community and 130 in the city of Zurich.

[^13]:    ${ }^{16}$ The threat of a so-called tax harmonization often prevents low-tax communities from further lowering their tax shifters.

[^14]:    ${ }^{17}$ Source: Statistisches Amt des Kantons Zürich, Gemeindedaten per 31.12.1998.
    ${ }^{18}$ Tax scheme according to Steuergesetz vom 8. Juni 1997.
    ${ }^{19}$ Swiss Federal Statistical Office, Schweizerische Arbeitskräfterhebung (SAKE) 1995.
    ${ }^{20}$ State and Federal taxes were deducted from net household income (after social security contribution) assuming a two-child family.
    ${ }^{21}$ The minimum income is subsistence level for a one-person-household as defined by the Schweizerische Konferenz für Sozialhilfe (SKOS) and adjusted for inflation. The maximum income is chosen arbitrarily, but has no influence on the numerical simulation due to the low weight on high incomes.

[^15]:    ${ }^{22}$ Of course, there is a selection bias by only considering renters. This seems nevertheless justified because the proportion of renters is very high in Switzerland ( $65 \%$ in the data set used).

